Skilled Workers, Immigration Options and Optimal Investment in Human Capital

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Motivation

- Investment in human capital as a highly irreversible decision
- Uncertainties about the future pay-off of investment
- Human capital investment and real options: education as a multi stage growth option
- Skilled labor immigration as a global problem affecting both developed and developing countries
Forced vs selective immigration

Immigration possible only with human/financial capital (the US, Canada, Australia, Germany’s IT programme) or without this requirement (most of Europe)

Benefits/costs to both sender and host countries
Our Work

- Impact of immigration option on investing in human capital
- Innovation: two types of human capital, *local* and *global*
- Tradeoff between universal and local human capital
  - Full transferability of universal human capital
  - Only a portion $\alpha \in [0, 1]$ of local human capital can be put to productive use in the destination country
- Expected results: Immigration option affects the rate of investment in global human capital positively and the rate of investment in local one adversely
- Total effect?
- A partial not general equilibrium model, wage differential is exogenous
Real Options and Investment in Human Capital: Mainly discrete time

- Human capital and exit option: Katz and Rapoport (2005)
- Higher return on human capital due to the existence of option to wait: Jacobs (2007)
- Education and option to shutdown: Hogan and Walker (2007)


- **Immigration and Real Options**
  - Immigration quotas and option value: Moretto and Vergalli (2008)
  - Uncertainty and option to wait before immigration: Locher (2002)

- **Immigrants Human Capital**
  - Complementarity of language: Chiswick and Miller (2003), Berman et al. (2002)
  - No positive economic return from homeland education: Hartog and Zorlu (2007)
Variations in Modelling

- Finite vs. infinite time horizon
  - Likely to affect the optimal investment policy
  - May not be optimal to accumulate global human capital if "close" to the termination time
  - May not be optimal to migrate if close to the end of career
  - Acknowledge and start with the infinite horizon case

- Probability of immigration, e.g. Quotas
  - Aim is to capture the immigration policy of destination: friendly or hostile
  - Either add an exogenous probability, $p$ of being able to immigrate or assume that there is some underlying process by which the destination becomes friendlier (e.g. a Poisson process)
  - Acknowledge and ignore for the moment

- Continue or stop accumulating global human capital after immigration
The Agent and Her Decision Problem

- A risk-neutral skilled person with an option to work abroad
  - Interim or original country
- Two types of skills accumulation
  - Stock of *universal* human capital, \( g(t) \)
    \[
    dg(t) = u(t)dt
    \] (1)
  - Stock of *local* human capital, \( k(t) \)
    \[
    dk(t) = q(t)dt
    \] (2)
- Investment in human capital is costly:
  \[
  c(u, q) = \frac{c_1}{2} u^2 + \frac{c_2}{2} q^2
  \] (3)
Normalize the wage in the host country to 1

Exercise of option leads to a wage gain (destination/host):

\[ dw(t) = \mu w(t)dt + \sigma w(t)dz(t) \]  

There is a lump-sum (opportunity) cost of moving of \( I \) due, for instance, to losing one’s social network, sentiments and memories, permanent residence, any current pension plans etc.
In the host country, before immigration, the agent’s payoff is:

\[ \Pi^h(g, k, w) = \left[ g(t) + k(t) - c(u, q) \right] \]  \hspace{2cm} (5)

After immigration, the payoff is given by:

\[ \Pi^d(g, k, w) = w \left[ g(t) + \alpha \bar{k} \right] - c(u) \]  \hspace{2cm} (6)

Note: after immigration, only investment in global human capital continues, that is:

\[ \bar{k} = k_T \]
Statement of the Problem

- Before immigration:

\[ \max_{u,q,\tau} Z(g, k, w) = E_0 \left\{ \int_0^\tau \Pi^h e^{-rt} dt + e^{-r\tau} [V(g, w) - l] \right\} \]

s.t. (1), (2), (4)  \hspace{1cm} (7)

- After immigration:

\[ \max_u V(g, w) = E \left\{ \int_0^\infty \Pi^d e^{-rt} dt \right\} \]

s.t. (1), (4)  \hspace{1cm} (8)
Suppose the option to immigrate has been taken. The Bellman equation is:

\[ V = \left[ w(g + \alpha k) - \frac{c_3}{2} u^2 \right] dt + (1 - r dt) E[V(g', w + dw)] \]  

(9)

Using Itô and optimizing over \( u \) yields:

\[ u^* = \frac{V_g}{c_3} \]  

(10)

Analogous arguments establish that before the immigration decision, the agent accumulates according to:

\[
\begin{align*}
& u^* = \frac{W_g}{c_1} \\
& q^* = \frac{W_k}{c_2}
\end{align*}
\]  

(11)
Plugging the optimal policies into the Bellman equations we get

- After immigration

\[
\frac{1}{2} \sigma^2 w^2 V_{ww} + \mu w V_w + \frac{1}{2c_3} V_g^2 - rV + w(g + \alpha \bar{k}) = 0 \tag{12}
\]

- Before immigration

\[
\frac{1}{2} \sigma^2 w^2 Z_{ww} + \mu w Z_w + \frac{1}{2c_1} Z_g^2 + \frac{1}{2c_2} Z_k^2 - rZ + (g + k) = 0 \tag{13}
\]
An Attempt to Make the Model More Tractable

- Rewrite the motion of deterministic states

\[
\begin{align*}
\frac{dg(t)}{dt} &= ug(t)dt \\
\frac{dk(t)}{dt} &= qk(t)dt
\end{align*}
\]  

(14)

- Also change the payoff functions. Assume, respectively, before and after immigration:

\[
\begin{align*}
\Pi^h &= p(g, k) - 0.5c_1u^2 - 0.5c_2q^2 \\
\Pi^d &= wg - 0.5u^2
\end{align*}
\]  

(15)

with

\[
\frac{dy(t)}{dt} = (\mu + u)y(t)dt + \sigma y(t)dz(t)
\]  

(16)
Recast of the Problem
Move backwards: suppose the option has been exercised

The problem is:

\[
\max_u \mathbb{E} \left\{ \int_0^\infty (y - 0.5u^2)e^{-rt} \, dt \right\} \quad \text{s.t. (16)}
\]  

Optimization yields:

\[
 u^* = yV_y
\]  

The HJB now satisfies:

\[
0.5\sigma^2 y^2 V_{yy} + \mu yV_y + \frac{y^2 V_y^2}{2} - rV + y = 0
\]  

⇒ Second-order nonlinear ODE!
Before Immigration

- Separate value function into "assets-in-place" and the option:

\[ Z(g, k, w) = f(g, k) + h(y) \]  \hspace{1cm} (20)

- Assets-in-place have the following structure:

\[ f(g, k) = p(g, k) - 0.5c_1 u^2 - 0.5c_2 q^2 \]  \hspace{1cm} (21)

- Optimization yields:

\[
\begin{align*}
  u^* &= gf_g \\
  q^* &= kf_k
\end{align*}
\]  \hspace{1cm} (22)
The solution depends on the form of \( p(g,k) \). Some alternatives:

1. **Multiplicative**
   \[
   p(g, k) = gk
   \]  
   \[(23)\]

2. **Cobb-Douglas**
   \[
   p(g, k) = g^\theta k^\gamma, \theta, \gamma < 1, \theta + \gamma \leq 1
   \]  
   \[(24)\]

3. **Additive**
   \[
   p(g, k) = g + k
   \]  
   \[(25)\]

Then, conjecture \( f(g, k) = f_1(g) + f_2(k) \)

\[
\begin{cases}
0.5g^2\left(\frac{df_1}{dg}\right)^2 - rf_1 + g = 0 \\
0.5k^2\left(\frac{df_2}{dk}\right)^2 - rf_2 + k = 0
\end{cases}
\]  
\[(26)\]
What about the Option Component?

Analogous to an investment option à la Dixit&Pindyck

\[
\begin{align*}
0.5\sigma^2 y^2 h_y y + (\mu + u^*) y h_y - rh &= 0 \\
\text{s.t.} \\
h(0) &= 0 \\
h(y^*) &= V(y^*) \\
h_y(y^*) &= V_y(y^*)
\end{align*}
\]

BUT: \( u^* = gf_g \Rightarrow \) Not so trivial to solve!
Policy Implications

- An individual immigrant is modeled. What drives her decision?
- Goal: How could countries attract more skilled labor? What are the tools to accomplish that?
  - Providing tax relief?
  - Subsidies (e.g. reducing cost) for integration to the country: ease local human capital investment
  - Pension plans
  - Ease of immigration/bureaucracy
  - Force immigrants to gain local human capital prior to immigration
  - Make labor market requirements (specially in highly skilled sectors) more international
- Effect of transferability of local human capital: France vs Denmark
Conclusion

- Option based model for skilled workers’ human capital investment decision

Comments on:
- Is the problem interesting enough?
- Any idea for analytical solutions?
- Further insights and policy analysis from the model?