Valuation of a Municipal Wastewater Plant Expansion: An Application to a High Growth Resort Area in Canada

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Abstract

The municipal water and wastewater sector is considered to be the most capital intensive industrial sector. Naturally, any methodology that has the potential to improve capital allocation decision making, has the potential to make a positive financial contribution to this sector. Most managers are aware of the power of calculating the Net Present Value (NPV) of an investment decision using Discounted Cash Flows (DCF). The problem with DCF based NPV analysis is that the inherent value of future project options is not modeled. In this study, we consider a small resort-based municipality faced the question of how big to make their new wastewater treatment facility to meet the expanding demand of 10% growth in the number of new residential connections to the wastewater treatment infrastructure. Since a significant number of new dwellings are second “weekend” homes, the planners felt strongly that growth rates were tied to the strength of the market index. Here we set the model framework for considering optimal plant size based on correlation assumptions of municipal growth to the market index. The model takes on the form of an Asian option. The results show that the greater the (assumed) correlation, the smaller the required plant size. Penalty costs associated with not building a large enough plant are hedged in the market. This paper sets that basis for future analysis of staged plant expansion analysis.

Introduction

The municipal water and wastewater industrial sector is considered to be one of the most capital intensive industrial sectors and unfortunately the American Society of Civil Engineers (2005) rated the condition of the drinking water and wastewater infrastructure systems as poor, citing specifically a lack of investment in capital assets over a prolonged period of time. Clearly, methods based on sound financial principles that enhance capital asset allocation strategies can add significant value to municipal decision makers. It is recognized that projects face future uncertainties. The ability of project managers to react to these uncertainties at a future time adds intrinsic value to the project, and this value is not captured by standard discounted cash flow (DCF) / net present value (NPV) methods. To adequately account for the uncertainty and its impact on the project value, financial engineering methods applied in the financial markets can be utilized in “real” capital investment projects. Trigeorgis (1996) provides a thorough introduction and review of real option theory and how it can be utilized to enhance an entity’s strategy in resource allocation.

While capital asset and project valuation using real options has seen a significant research focus over the last 15 years (see, for example Jacoby and Laughton (1992), Ingersoll and Ross (1992), Emhiellen and Alaouze (2003), and van Putten and MacMillan (2004)), real options theory has seen limited application in the municipal infrastructure sector. Of note, Schubert and Barenbaum (2007) discuss how public managers can employ real options technique to better value their capital budgeting opportunities and improve the efficacy of capital budgeting decisions.

Other studies, Ho and Liu (2002), Garvin and Cheah (2004), consider the application of real options to value public infrastructure projects under private management arrangements. Arboleda and Abraham (2006) propose a method using real option analysis to evaluate capital investments in public infrastructure projects managed by private operators. The proposed methodology develops a valuation based on deterioration curves of infrastructure and the associated value of flexibility to invest at optimal states within the model.

In this study, real option valuation is used to determine the optimal size for a wastewater plant expansion required for a small municipality undergoing significant residential growth. Specifically, the community is located in a resort area and has experienced increases in the growth rates of approximately 10% over the last 15 years. Since a significant number of new dwellings are second “weekend” homes, the planners felt strongly that growth rates were tied to the strength of the market index[1]. Assumptions based on the strength of the correlation will be examined.

Cost Description

The municipality charges a fixed fee (currently set at $5000) per new connection. As well, customers are charged a fixed and variable rate per usage (an average value per customer is assumed in this analysis). The premise of the pricing structure for the municipality is that only new customers should pay for the capital cost burden of a plant expansion. Thus, a pre-specified margin % of the revenues from connections and operations from new customers are dedicated to plant expansion.

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[1] Analysis of the data showed some correlation to the general stock index, but due to the limited amount of total connections, confidence intervals on the correlations were very broad. It is likely that a lag exists between the market index and the rate of connectivity and analysis of the data showed a lag of approximately 9 months. For the analysis presented here the lag was ignored.

Gillot et al. (1999) provide a detailed methodology to optimize a wastewater plant based on cost. For simplicity, the capital cost to build / expand a plant in this study is assumed to have a fixed and variable component only, as follows

$$C_{plant}(K) = \alpha_{plant} + \phi_{plant} K \tag{1}$$

where C_{plant} is the present value of the capital cost to build the plant (including any salvage value component), K is the design size of the new plant based on number of (new) customers, α_{plant} is the fixed cost and Φ_{plant} is the variable cost.

Model Development

The market index S_t is assumed to follow a Geometric Brownian Motion (GBM):

$$dS_t = \mu_S S_t dt + \sigma_S S_t dW_t \tag{2}$$

Here, μ_S and σ_S are constants representing the rate of growth and volatility of the index, respectively, and W_t is a standard Brownian motion (or Wiener process) representing the fluctuations.

Similarly, GBM is assumed as an appropriate model for the wastewater connection rate X_t – i.e. the number of connections per unit of time,

$$dX_t = \mu_X X_t dt + \sigma_X X_t dW_t^X \tag{3}$$

μ_X and σ_X are assumed to be constant, and W_t^X is a second, correlated, Weiner process. The correlation between the growth of the connection rate and the market index is captured by the correlation coefficient ρ. It is often convenient to decompose the correlationed Brownian motions into two independent motions as follows

$$dW_t^X = \rho dW_t + \sqrt{1-\rho^2}\, dW_t^{\perp} \tag{4}$$

where $W^{\perp}_t$ is a Weiner process independent of W_t. Under the risk-neutral measure, the risk-adjusted process

$$d\tilde{W}_t \equiv \frac{\mu_S - r}{\sigma_S} dt + dW_t \tag{5}$$

is a standard Browian motion. Here r is the risk-free rate. Assuming that the market price of risk associated with fluctuations uncorrelated to the market is zero (since they are not hedgible), we do not risk-adjust the perpendicular component $W^{\perp}_t$. In this case, the connection rate becomes

$$dX_t = \left(\mu_X - \frac{\rho\sigma_X}{\sigma_S}(\mu_S - r)\right) X_t dt + \sigma_X X_t \left(\rho d\tilde{W}_t + \sqrt{1-\rho^2}\, d\tilde{W}_t^{\perp}\right) \tag{6}$$

or

$$dX_t = \bar{r} X_t dt + \sigma_X X_t \left(\rho d\tilde{W}_t + \sqrt{1-\rho^2}\, d\tilde{W}_t^{\perp}\right). \tag{7}$$

Here $\bar{r} \equiv \mu_X - \frac{\rho\sigma_X}{\sigma_S}(\mu_S - r)$.

Defining N_t as the total number of connections to the plant, then

$$N_t = N_0 + \int_0^t X_u du\,. \tag{8}$$

The rate of income generation from connections is given by

$$I_t = \left(P_0 m_{P_0} X_t + Op_0 m_{Op_0} N_t\right) e^{r_{cpi} t} \tag{9}$$

where P_0 is the price to connect at time t = 0, Op_0 is the operating price charged per customer per unit time at time t = 0, m_{P0} and m_{Op0} are the margin % associated with income dedicated to the new plant capital requirements from the revenue generated from connections and operation, respectively, and r_{cpi} is the rate at which prices increase (i.e. the rate of inflation as measured e.g. by the consumer price index).

The present value of the generated income from time t_1 to time T is

$$I_{t_1,T}^{PV} = \int_{t_1}^{T} e^{-ru} I_u du = \int_{t_1}^{T} e^{-(r-r_{cpi})u} \left(P_0 m_{P_0} X_u + Op_0 m_{Op_0}\left(N_0 + \int_0^u X_s ds\right)\right) du\,. \tag{10}$$

Note that N_0 denotes the number of existing customers whose m_{Op0} fraction of operating revenues is dedicated to the new plant expansion. As discussed above, if the number of connections exceeds plant capacity, the municipality will face extra costs associated with wastewater removal. The cost per each extra connection over capacity is $PC_0 e^{r_{cpi} t}$ and thus the penalty cost (rate) is

$$PC_t = \max\left(0, (N_t - K) \cdot PC_0 e^{r_{cpi} t}\right) \tag{11}$$

where K is the number of connections at capacity. Assuming the plant expansion will be completed by t_2, the present value of the penalty cost associated with the plant expansion is[2]

$$PC_{t_2,T}^{PV} = \int_{t_2}^{T} PC_0 e^{-(r-r_{cpi})u} \cdot \left(N_u - K\right)_+ du\,. \tag{12}$$

Using equations (10) and (12), the net present value of project is

$$PV = I_{t_1,T}^{PV} - PC_{t_2,T}^{PV} - C_{plant} \tag{13}$$

and the expected value of the NPV under the risk-neutral measure becomes

$$\tilde{E}[PV] = \tilde{E}\left[I_{t_1,T}^{PV}\right] - \tilde{E}\left[PC_{t_2,T}^{PV}\right] - C_{plant}\,. \tag{14}$$

It should be emphasized that the maximum capacity K is the variable of optimization and since the income I is independent of K, the optimization exercise amounts to minimizing $\tilde{E}\left[PC_{t_2,T}^{PV}\right] + C_{plant}$.

The first term on the right hand side of equation (14) is calculated in a straight forward manner, i.e.

[2] It is assumed that the size of the plant expansion does not impact the construction time. Clearly, penalty costs incurred before the plant expansion have no bearing on the optimization.

$$\tilde{E}\left[I_{t_1,T}^{PV}\right] = \int_{t_1}^{T} e^{-(r-r_{cpi})u}\left(P_0 m_{P_0}\tilde{E}\left[X_u\right] + Op_0 m_{Op_0}\left(N_0 + \int_0^u \tilde{E}\left[X_s\right]ds\right)\right)du$$

$$= X_0\left(P_0 m_{P_0} + \frac{1}{\bar{r}}Op_0 m_{Op_0}\right)\frac{\left(e^{-\left(r-r_{cpi}-\bar{r}\right)t_1} - e^{-\left(r-r_{cpi}-\bar{r}\right)T}\right)}{r - r_{cpi} - \bar{r}} \tag{15}$$

$$+ Op_0 m_{Op_0}\left(N_0 - \frac{1}{\bar{r}}X_0\right)\frac{\left(e^{-\left(r-r_{cpi}\right)t_1} - e^{-\left(r-r_{cpi}\right)T}\right)}{r - r_{cpi}}$$

where X_0 is the initial connection rate. The second right hand side term of equation (14) can be written as

$$\tilde{E}\left[PC_{t_2,T}^{PV}\right] = \int_{t_2}^{T} PC_0 e^{-(r-r_{cpi})u}\cdot\tilde{E}\left[\left(N_u - K\right)_+\right]du$$

$$= \int_{t_2}^{T} PC_0 e^{-(r-r_{cpi})u}\cdot\tilde{E}\left[\left(N_0 + \int_0^u X_s ds - K\right)_+\right]du. \tag{16}$$

The term $\tilde{E}\left[\left(N_0 + \int_0^u X_s ds - K\right)_+\right]$ takes on the form of an Asian option's payoff. Defining

$$v(t, X_t, N_t) \equiv \tilde{E}\left[\left(N_t + \int_t^T X_s ds - K\right)_+ \middle| \mathscr{F}_t\right], \tag{17}$$

the solution to v(t,x,y), where X_t and N_t are replaced by the dummy variables x and y, is given by the following partial differential equation (PDE)

$$\frac{\partial v}{\partial t} + \bar{r}x\frac{\partial v}{\partial x} + x\frac{\partial v}{\partial y} + \tfrac{1}{2}\sigma_X^2 x^2\frac{\partial^2 v}{\partial x^2} = 0\,. \tag{18}$$

The boundary conditions are determined as follows. For x = 0, v will only depend on the total connections, thus

$$v(t,0,y) = \left(y - K\right)_+,\ 0 \le t \le T\,. \tag{19}$$

and similarly at the terminal time, t = T, v is determined by

$$v(T,x,y) = \left(y - K\right)_+. \tag{20}$$

As per Shreve (2004), there is no mathematical reason to restrict y to positive values and by allowing the computational domain to include values of y < 0, the boundary condition for $y \to -\infty$ gives

$$\lim_{y\to-\infty} v(t,x,y) = 0,\ 0 \le t \le T\,. \tag{21}$$

Note that $y_{\tau_2} = y_{\tau_1} + \int_{\tau_1}^{\tau_2} x_s ds$, and since x >0 then $y_{t_2} \ge y_{t_1}$ for $\tau_2 > \tau_1$. Define $y_{\tau_1} \equiv y_{max}$ with $y_{max} > K$, then y_{τ_2} will always be in the money and

$$
\begin{aligned}
v(t,x,y_{max}) &= \tilde{E}\left[y_u - K \middle| X_t = x, y_t = y_{max}\right] \\
&= \tilde{E}\left[y_u \middle| X_t = x, y_t = y_{max}\right] - K \\
&= \tilde{E}\left[y_{max} + \int_t^u X_s ds \middle| X_t = x, y_t = y_{max}\right] - K \\
&= y_{max} - K + \int_t^u \tilde{E}\left[X_s \middle| X_t = x\right] ds.
\end{aligned} \tag{22}
$$

From equation (6),

$$
\tilde{E}\left[X_s \middle| X_t = x\right] = x e^{\bar{r}(u-t)} \tag{23}
$$

and so equation (22) gives

$$
v(t,x,y_{max}) = y_{max} - K + \frac{x}{\bar{r}}\left(e^{\bar{r}(u-t)} - 1\right). \tag{24}
$$

Finally, for large x values, set x = x_{max}, a constant. Thus

$$
\begin{aligned}
v(t,x_{max},y) &= \tilde{E}\left[\left(y + \int_t^u X_s ds - K\right)_+ \middle| X_t = x_{max}, y_t = y\right] \\
&\sim \left(y + (u-t)x_{max} - K\right)_+ .
\end{aligned} \tag{25}
$$

The finite difference method was used to solve equation (18). Here, an approximate solution to the Asian option is introduced.

Hedging Strategy

Clearly for values of ρ different from zero, the optimal new plant size will be reduced and the costs “extra” penalty costs associated with building a smaller plant can be hedged. In this section, the hedging strategy is developed.

Defining the expected present value of the penalty cost at time t, including the (known) incurred penalty to t, as

$$
\begin{aligned}
G_t &\equiv \tilde{E}\left[PC^{PV}_{t,T} \middle| \mathscr{F}_t\right] + \int_0^t PC_0 e^{-(r-r_{cpi})u} \cdot \left(N_u - K\right)_+ du \\
&= \int_t^T PC_0 e^{-(r-r_{cpi})u} \cdot v\left(t, X_t, N_t\right) du + \int_0^t PC_0 e^{-(r-r_{cpi})u} \cdot \left(N_u - K\right)_+ du
\end{aligned} \tag{26}
$$

then

$$
\begin{aligned}
dG_t = PC_0 \Bigg[\Bigg(& e^{-(r-r_{cpi})t}\left(\left(N_t - K\right)_+ - v\right) + \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial t} du \\
& + \bar{r} X_t \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial x} du + X_t \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial y} du + \frac{\sigma_X^2}{2} X_t^2 \int_t^T e^{-(r-r_{cpi})u} \frac{\partial^2 v}{\partial x^2} du \Bigg) dt \\
& + \rho \sigma_X X_t \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial x} du \, d\tilde{W} + \sqrt{1-\rho^2}\, \sigma_X X_t \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial x} du \, dW^{\perp} \Bigg].
\end{aligned} \tag{27}
$$

The hedge portfolio will consist of the market index and a money market account, such that the value of the portfolio is

$$\Pi_t = a_t S_t + M_t \tag{28}$$

and (in order for the hedge to be self-financing)

$$d\Pi_t = a_t r S_t dt + r M_t dt + a_t \sigma_S S_t d\tilde{W}\ . \tag{29}$$

Since we aim to hedge away all tradable risks, the appropriate hedge requires equating the $d\tilde{W}$ coefficients resulting in

$$a_t = PC_0 \frac{\rho\sigma_X X_t}{\sigma_S S_t} \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial x} du\ . \tag{30}$$

Clearly, the dollar volatility of this strategy will be $PC_0\sqrt{1-\rho^2}\sigma_X X_t \int_t^T e^{-(r-r_{cpi})u} \frac{\partial v}{\partial x} du$ - the last term in equation (27) which cannot be hedged away.

Results

As mentioned above, the municipality in this study is located in a resort area and has seen its growth rate increasing at approximately 10% (i.e. $\mu_X = 0.1$) over a 15 year time span. The volatility in the growth rate was approximately 0.16 during this period. For the last year, the connection rate, N_0, was 81. Current plant capacity is estimated to have the ability to accommodate another approximately 200 new connections. The penalty cost rate was estimated to be $5000 per connection not served per year[3]. Construction for the wastewater capacity expansion project is estimated to be 3 years. For simplicity of analysis, the plant is assumed to have a 20 year lifespan[4]. Feasibility studies conducted by consulting engineers provided estimates of the capital cost for plant expansion (and salvage) and for the type and size of plant considered here, α_{plant} was estimated to be $3,500,000 and ϕ_{plant} to be $860 per connection served (c.f. equation (1)).

In this study, the risk-free rate, r, was take as 5% and the inflation rate, r_{cpi}, as 3%. Market index parameters were determined to be $\mu_S = 0.08$ and $\sigma_S = 0.1$. As previously noted, due to the sparseness of the data, the correlation between the growth rates of the market index and connection rates, ρ, was not determinable with statistical significance, but values in the 0.1 to 0.5 range were observed using a number of standard methods.

Figure 1 plots the present value of capital cost versus new plant size and the risk-neutral expected present value of the penalty costs versus new plant size for varying ρ. The optimal plant size, i.e. $\min\left(C_{plant} + PC^{PV}_{t_2,T}\right)$, is plotted in Figure 2. As expected, the

[3] It is assumed that if not enough capacity is installed, extra sewage entering the treatment facility will need to be hauled to other locations, thus bearing significant yearly costs associated with under design.
[4] The analysis presented here assumes a 23 year time horizon (3 years for construction plus 20 years for the life of the plant). Penalty costs after 23 years are ignored – it is assumed that future plant construction / expansion will negate future penalty costs.

higher the correlation of the connection rate growth to market growth, the smaller the optimal plant size.

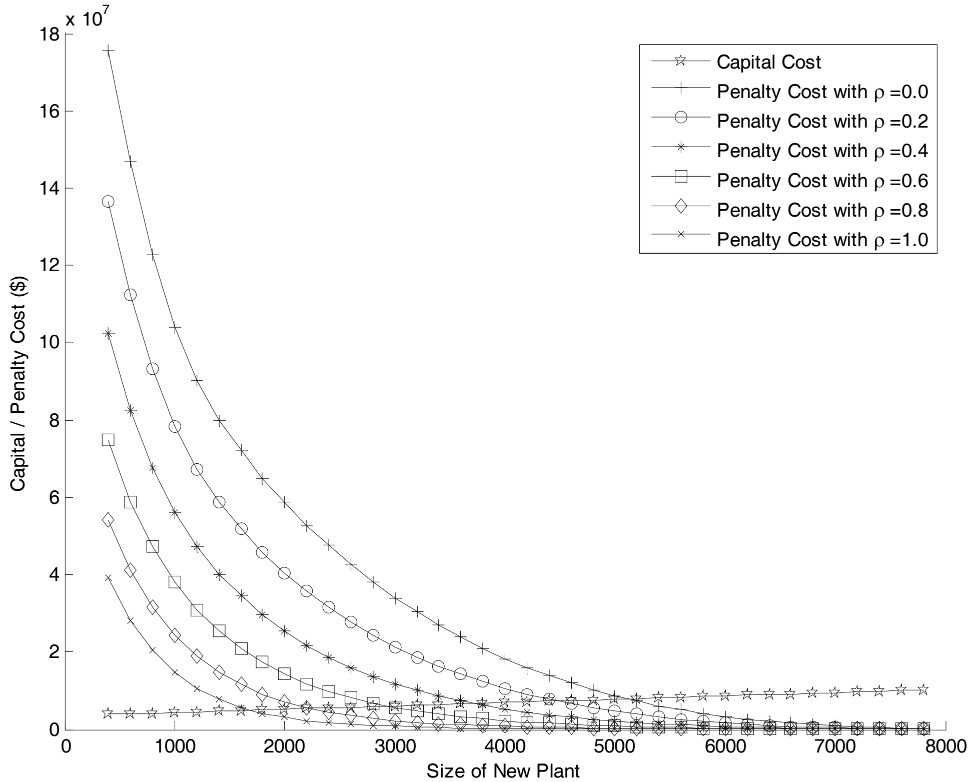

Figure 1. Present value of capital and risk-neutral expected penalty costs versus size of the new plant.

To test the effectiveness of the delta-hedging strategy, simulations were run with ρ = 1.0 and ρ = 0.5. For ρ = 1.0, the optimal plant size was determined be approximately 3200 and for ρ = 0.5, 5500. For the results presented here, 1000 sample paths were simulated, and the delta hedge was re-adjusted once per week. A sample of 10 paths of X, N and S are presented in Figure 3 for ρ = 1.0 and K = 3200 and ρ = 0.5 and K = 5500.The corresponding delta hedge, value of the money market and the penalty cost paths are given in Figure 4. It should be noted that the “money market”, when negative, represents the cost associated with paying for the hedge and the penalty. The histograms of the accumulated negative of the penalty cost (this amount is equal to - $e^{rT}PC^{PV}_{t_2,T}$) and the final money market value are given in Figure 5. Clearly, the hedging strategy is not perfect, but increasing the trading frequency and applying a gamma hedge would improve it.

The total present value of the “savings” for applying the hedging strategy is given by

$$\text{Savings} = C_{plant}(7400) - C_{plant}(K) + M_T e^{-rT} + PC^{PV}_{t_2,T}(7400) - PC^{PV}_{t_2,T}(K) \tag{31}$$

and the associated histogram is given in Figure 6. For both ρ = 1.0 and ρ = 0.5, the municipality has effectively hedged its penalty cost risk in the market, such that if the savings associated with building a smaller plant are considered, the municipality comes out significantly ahead (approximately $3.5 million for ρ = 1.0 and $1.7 million for ρ = 0.5).

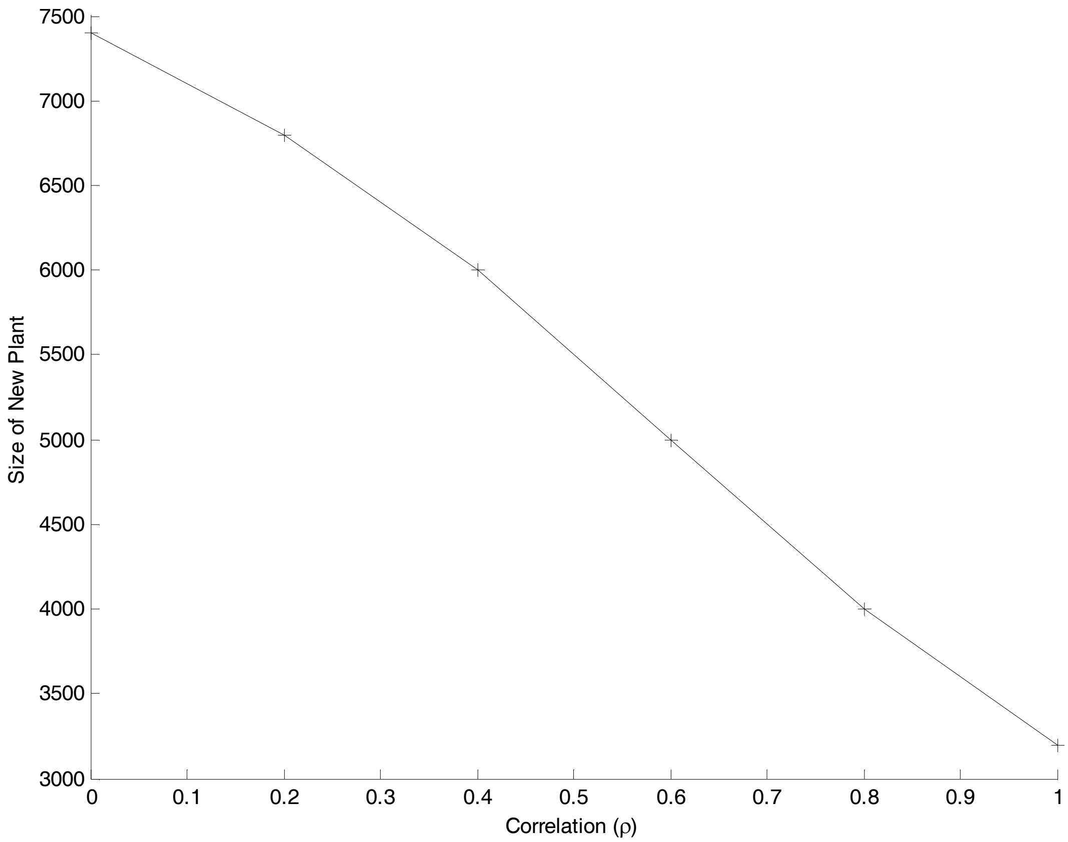

Figure 2. Optimal new plant size versus correlation, ρ.

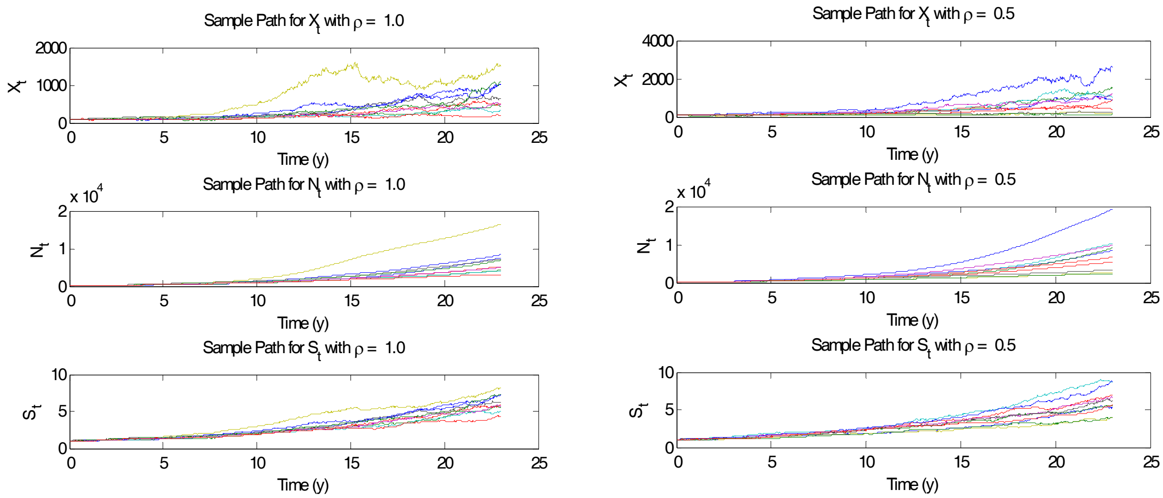

Figure 3. Sample Paths for X_t, N_t and S_t.

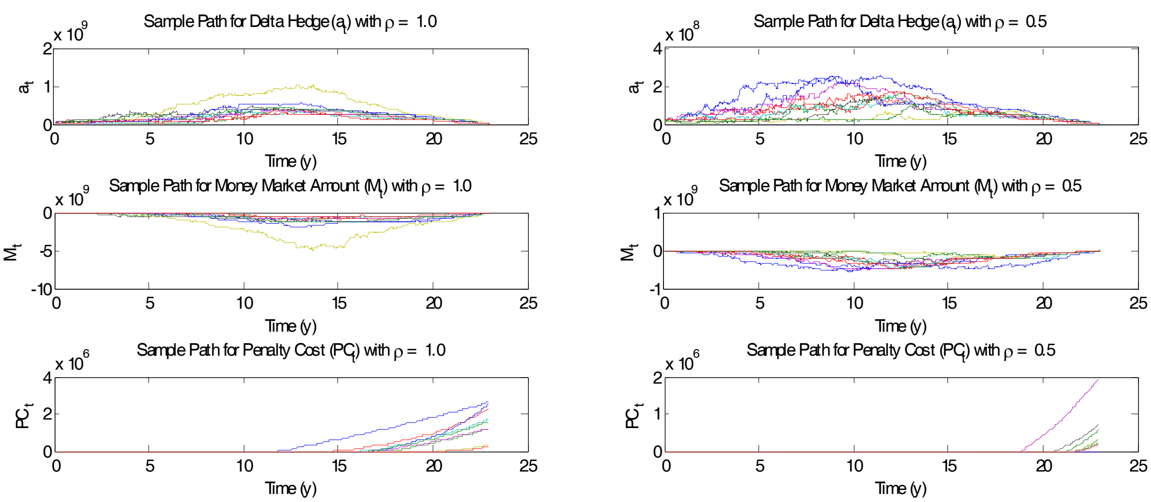

Figure 4. Sample Paths for the Delta Hedge, a_t, Money Market Value, M_t, and Penalty Cost, PC_t.

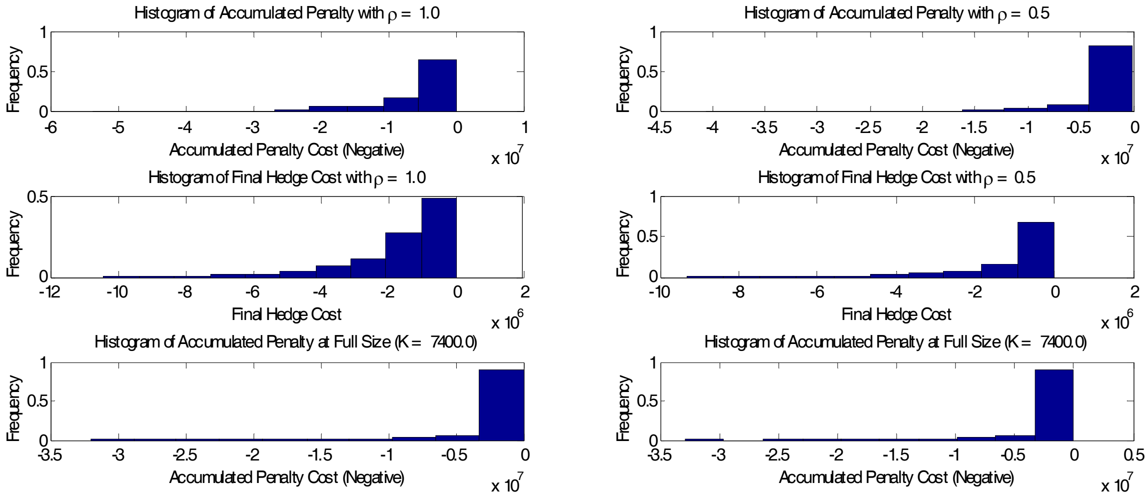

Figure 5. Histogram of (the Negative of) the Accumulated Penalty Cost, Final Money Market Value and (the Negative of) the Accumulated Penalty Cost for Full Plant Size.

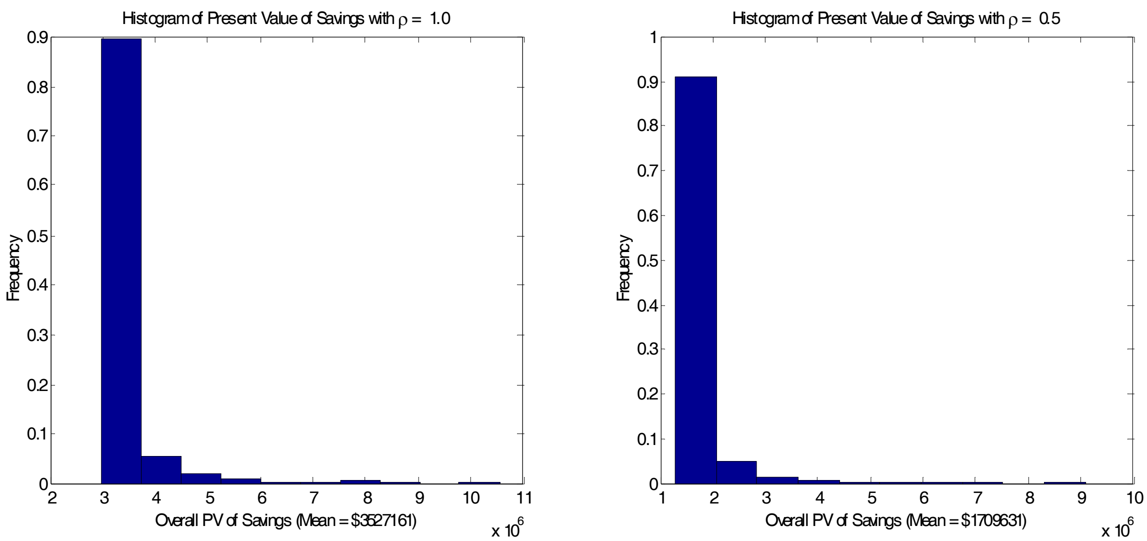

Figure 6. Histogram of the Present Value of Total Savings.

Discussion and Conclusions

It is not likely that many municipalities will now build smaller wastewater treatment plants and try to hedge away their potential penalty costs in the stock market. However, the methodology developed here provides an initial framework for helping municipal planners improve their capital expenditures and an alternative to the current practice of extrapolating the growth and building for extreme events forecasted far into the future. Shorter term hedging strategies may be applicable with local real estate market indices to ward off massive capital requirements in the shorter term. Real hedges can be introduced through development / real estate taxes, user fees, or even strategic agreements with neighbouring municipalities with respect to load and fees sharing.

More importantly, however, the model developed here can now be easily expanded to consider staged investment of the facilities.

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