Investments in energy efficiency under climate policy uncertainty

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1 Introduction

Efficiency gains have put a limit to fuel consumption growth in the past. Thus, according to Geller et al. [4], final energy use in major OECD countries would have been 49% higher in 1998 if intensities (i.e., energy use per unit of activity) of the different sub-sectors and end-uses had remained at their 1973 levels. This trend did not start in 1973, and it is not merely a response adopted by industrialized countries in the wake of the 1970s oil crises. Energy efficiency improvements since 1990 have met 52% of new energy service demands in the world, while new energy supplies have contributed 48% (UNF[14]).

In addition to energy savings, these improvements have another basic impact, namely the avoiding of greenhouse gas (GHG) emissions that go hand in hand with fossil fuel combustion. To the extent that there is a price for these emissions, avoiding them has economic value for firms that operate in an emissions-constrained environment. This holds also where no such price exists currently but there is a chance that climate restrictions will be imposed in the future. When investment in long-lived energy assets is assessed, this second impact clearly cannot be overlooked.

There is a broad consensus (IPCC [9], IEA [6]) that energy efficiency can play a significant role in curbing GHG emissions while paying for itself. However, investments that at first glance seem worthwhile usually are not undertaken. This situation traces in part to the challenge of attracting sufficient interest from the investment community. Mills et al. [11] point out that energy-efficiency experts (as scientists and engineers) and investment decision-makers simply do not speak the same language. Fortunately, though, energy-efficiency investments lend themselves to financial analysis. The aim of this paper is to further contribute to bridging this gap.

Typically, financing of these investments will be based on the creditworthiness of the energy user. Therefore, we analyze investments in efficiency (and savings in energy) from the viewpoint of a firm or individual that behaves rationally, i.e., in her best economic interest. The investment or project is valued like a (real) option that is only exercised at the optimal time and is irreversible (the firm cannot disinvest should market conditions turn). The return on this investment is highly uncertain; this fact, in turn, would lead naturally to a high discount rate for the cash flows from the project. Uncertainty emanates from energy prices and emission allowance prices, but regulatory uncertainty may come on top of them. Indeed, any action that contributes to lower uncertainty enhances the opportunity to invest in efficiency gains and energy savings. We aim to determine the optimal time to invest or, in other words, to learn the conditions under which the investment should be undertaken.

Note, though, that the current economic downturn does not promote investments in energy efficiency. At one level, a weaker industrial activity induces lower carbon emissions. This is already evident in energy markets and carbon markets; opportunistic selling by cash-strapped industrial firms with spare emission allowances has also played a role. On the other hand, a more cautious credit market bodes ill for these investments if they are perceived as entailing greater doses of risk than conventional ones. At least for now, funding costs could include a higher credit-risk premium. In this chapter we deal with the cost that these investments should involve, given their energy savings and emissions reductions, for their immediate undertaking to be optimal. This cost must be interpreted as a total amount, i.e., including whatever public subsidies or other support measures are established.

Our theoretical model comprises two stochastic processes for fuel (say, natural gas) price, and emission (say, carbon) allowance price, respectively. With regard to the carbon price, we consider a standard geometric Brownian motion (GBM) in two different scenarios: within a given commitment period (e.g., 2008-2012), and between two succeeding periods (i.e., the current one and the immediate post-Kyoto period), presumably separated by a change in climate regulation with an ensuing jump in carbon prices. In both cases, we are interested on the value of the emissions avoided yearly over a certain time horizon (thanks to an investment in energy efficiency). With this goal in mind we derive the value of a contract that fits well for our purposes, namely an annuity; if the underlying variable follows a GBM this is relatively easy. As for the gas price, we assume a mean-reverting process in which the long-term equilibrium level grows deterministically over time. This process is not as general as others available in the literature (e.g., Schwartz & Smith [12]). However, it has the advantage that an analytic solution for the value of an annuity can be derived. Thus, the value of future savings in natural gas consumption (due to an investment in energy efficiency) over a given time horizon can be computed directly. The key underlying parameters in these processes are estimated from actual market prices. We can then assess energy-efficiency investments in a fairly realistic setting.

The paper is organized as follows. Section 2 presents the analytical framework. In particular, the stochastic processes for the price of carbon dioxide and natural gas are specified. Section 3 briefly introduces some features of the markets involved. This sets the ground for Section 4, where the underlying parameters of the carbon price process are estimated, followed by those of the natural gas price (a detailed description can be found in the Appendix). Section 5 first addresses investments that lead to lower carbon emissions. Initially the deterministic case is considered. We show that, even in this deterministic world, it may be optimal to postpone investment if profit margins are going to increase in the future. Then we turn to infinite-lived investments either with constant costs or increasing costs. Later on, we focus on finite-lived investments without and with jumps in carbon prices. At a second stage, this section addresses investments that bring about savings in natural gas both when its equilibrium price is a constant and grows steadily. Section 6 deals with a case study, namely a potential investment in either one of two different gas-fired power stations that differ in their efficiency levels. In particular, we derive the total return to an increase of a percentage point in the thermal efficiency as a function of the plant's production factor (i.e., the time that the plant operates on average over the year). Section 7 concludes.

2 Stochastic price models

We assume that the emission allowance price follows a non-stationary process (much like a common stock). Natural gas price, instead, is assumed to be governed by a stationary process; in particular, there is an anchor value which may be somehow related to the marginal cost of extraction or the price of a substitute fuel.

2.1 The emission allowance price

We distinguish two different settings. In the first one the total cap on emission allowances remains fixed; in other words, the relevant variables fall within a given emissions trading period (e.g., the current commitment period 2008-2012 of the Kyoto Protocol). In the second, the total cap is tightened and a jump in the allowance price ensues. This may well be the relevant setting for investments that have a useful life spanning many years or decades.

2.1.1 Within a given commitment period

We adopt a GBM process. The allowance price C (say, in €/tCO_2) is thus assumed to evolve along the following path in a risk-neutral world over a given commitment period:[1]

$$dC_t = \alpha^* C_t dt + \sigma_C C_t dW_t^C, \tag{1}$$

where $\alpha^* \equiv \alpha - \lambda_C$ is the expected growth rate of carbon price, which equals the instantaneous growth rate in the physical world (α) less the risk premium associated to uncertainty on carbon prices (λ_C).[2] Besides, σ_C is the instantaneous volatility of carbon price changes, and W_t^C is a standard Wiener process.

Let $X_t \equiv \ln(C_t)$; by Ito's Lemma:

$$dX_t = (\alpha^* - \frac{1}{2}\sigma_C^2)dt + \sigma_C dW_t^C. \tag{2}$$

The solution to this differential equation is:

$$X_t = X_0 + (\alpha^* - \frac{1}{2}\sigma_C^2)t + \sigma_C \int_0^t dW_s^C. \tag{3}$$

It may be shown that changes in X_t are normally distributed with finite mean and standard deviation:

$$X_t - X_0 \equiv \ln(C_t) - \ln(C_0) \sim N((\alpha^* - \frac{1}{2}\sigma_C^2)t, \sigma_C\sqrt{t}). \tag{4}$$

The time-0 mathematical expectation and the variance are:

$$E_0(X_t) = X_0 + (\alpha^* - \frac{1}{2}\sigma_C^2)t, \tag{5}$$

$$Var(X_t) = \sigma_C^2 t. \tag{6}$$

Now the properties of the log-normal distribution imply that:

$$\begin{aligned} E(C_t) &= e^{[E(X_t)+\frac{Var(X_t)}{2}]} = e^{[X_0+\alpha^* t]} = C_0 e^{\alpha^* t}, &(7)\\ Var(C_t) &= C_0^2 e^{2\alpha^* t}(e^{\sigma_C^2 t} - 1). &(8) \end{aligned}$$

On the other hand, the time-0 futures price for delivery at time t equals the expected spot price at that time, $E(C_t)$, in a risk-neutral world:

$$F(t,0) = C_0 e^{\alpha^* t} \equiv F(C_0, t, 0). \tag{9}$$

[1] Thus, all investors are assumed to be risk-neutral, i.e. the only inputs to their decisions are average values or expected returns; any consideration about risk is dismissed. Risk-neutral valuation, and the contexts suitable for it, are explained in Dixit and Pindyck [3], Trigeorgis [13], among others.

[2] In the actual or physical world, investors' preferences are sensitive to risk, so risk matters. This is why -in this world- the emission allowance is expected to command a higher return, $\alpha = \alpha^* + \lambda_C$, to reward investors for the risk they bear.

For a futures contract with maturity at time T we have: $F(T,0) = C_0 e^{\alpha^* T}$. From this formula and applying Ito's Lemma we can derive the differential equation that the futures price satisfies:

$$\frac{dF}{dC_0} = e^{\alpha^* T}; \quad \frac{d^2F}{dC_0^2} = 0; \quad \frac{dF}{dT} = \alpha^* C_0 e^{\alpha^* T} = \alpha^* F.$$

Hence:

$$dF_t = \sigma_C F_t dW_t^C. \tag{10}$$

This means that the time-t expected value of the futures contract maturing at T is exactly the same as it had at time 0. And so it must be, since the contract involves no payment at the outset.

Let $Y_t \equiv \ln(F_t)$. Hence, by Ito's Lemma:

$$dY_t = -\frac{1}{2}\sigma_C^2 dt + \sigma_C dW_t^C. \tag{11}$$

Therefore, we have two alternative procedures to simulate $F(T,t)$ by Monte Carlo:

a) one is to simulate the allowance price C in the risk-neutral world. Once we have a simulated price at t, the futures price for delivery at T would be: $F(T,t) = C_0 e^{\alpha^*(T-t)}$.

b) the other one is to directly simulate the futures price at time t for delivery at T by means of equation (11).

The solution to equation (10) is:

$$F(T,t) = F(T,0)e^{[-\frac{1}{2}\sigma_C^2 t + bW_t]}. \tag{12}$$

This implies that it is not necessary to set a step-by-step path for some valuations. By equation (11), the change in Y from 0 to t is normally distributed with mean $-\frac{1}{2}\sigma_C^2 t$ and variance $\sigma_C^2 t$:

$$Y_t - Y_0 \equiv \ln(F(T,t)) - \ln(F(T,0)) \sim N(-\frac{1}{2}\sigma_C^2 t, \sigma_C\sqrt{t}) \tag{13}$$

In the valuation of a European call option (on a futures contract) with expiration at t, the former result in equation (13) can be directly applied.

Last, the expected value of an annuity from time τ_1 to τ_2 is:

$$V_{\tau_1,\tau_2} = \int_{\tau_1}^{\tau_2} e^{-rt}E(C_t)dt = C_0 \int_{\tau_1}^{\tau_2} e^{-rt}e^{\alpha^* t}dt = \frac{e^{(\alpha^*-r)\tau_2} - e^{(\alpha^*-r)\tau_1}}{\alpha^* - r}C_0. \tag{14}$$

This result will prove useful when we try to compute the value of 1 tCO_2 avoided yearly between two given dates.

2.1.2 Between two succeeding periods

When one emissions trading period expires, if there is a drop in the number of allowances for the next one, then presumably the carbon price will jump. The size of the jump in a risk-neutral world can be inferred from the quotes on futures markets. For two consecutive periods, we could assume the following two equations:

$$dC_t^1 = \alpha_1^* C_t^1 dt + \sigma_{C1} C_t^1 dW_t^{C1}, \quad \text{for } 0 \leqslant t < \tau, \tag{15}$$

$$dC_t^2 = \alpha_2^* C_t^2 dt + \sigma_{C2} C_t^2 dW_t^{C2}, \quad \text{for } \tau \leqslant t, \tag{16}$$

where τ denotes the time when one commitment period expires and the next one starts (there is no uncertainty regarding this time). In this setting $C_\tau^2 = J^* C_\tau^1$, where $J^* = J - \lambda_J$ stands for the percentage jump; J would be the jump in the physical world whereas λ_J would be the risk premium associated to jump size uncertainty. Note that the two commitment periods do not overlap in time, so that the differentials to the Wiener processes dW_t^{C1} and dW_t^{C2} are unrelated.

By Ito's Lemma,

$$dX_t^1 = (\alpha_1^* - \frac{1}{2}\sigma_{C1}^2)dt + \sigma_{C1} dW_t^{C1}, \tag{17}$$

$$dX_t^2 = (\alpha_2^* - \frac{1}{2}\sigma_{C2}^2)dt + \sigma_{C2} dW_t^{C2}. \tag{18}$$

The solution to the differential equation (17) at time τ when the first period expires is:

$$\ln(C_\tau^1) \equiv X_\tau^1 = X_0^1 + (\alpha_1^* - \frac{1}{2}\sigma_{C1}^2)\tau + \sigma_{C1}\int_0^\tau dW_s^{C1}. \tag{19}$$

Therefore,

$$E_0(X_\tau^1) = X_0^1 + (\alpha_1^* - \frac{1}{2}\sigma_{C1}^2)\tau, \tag{20}$$

$$Var(X_\tau^1) = \sigma_{C1}^2 \tau. \tag{21}$$

If now we let $L_t \equiv \ln(C_t^2) = \ln(J^* C_t^1)$, then we have for time τ when change happens:

$$L_\tau \equiv \ln(J^* C_\tau^1) = \ln J^* + X_0^1 + (\alpha_1^* - \frac{1}{2}\sigma_{C1}^2)\tau + \sigma_{C1}\int_0^\tau dW_s^{C1} \equiv X_\tau^2. \tag{22}$$

The solution to the differential equation (18) is:

$$X_t^2 = X_\tau^2 + (\alpha_2^* - \frac{1}{2}\sigma_{C2}^2)(t-\tau) + \sigma_{C2}\int_\tau^t dW_s^{C2}. \tag{23}$$

Substituting the right-hand side of (22) into (23) we get:

$$X_t^2 = \ln J^* + X_0^1 + (\alpha_1^* - \frac{1}{2}\sigma_{C1}^2)\tau + \sigma_{C1}\int_0^\tau dW_s^{C1} + (\alpha_2^* - \frac{1}{2}\sigma_{C2}^2)(t-\tau) + \sigma_{C2}\int_\tau^t dW_s^{C2}. \tag{24}$$

The time-0 mathematical expectation is:[3]

$$E_0(X_t^2) = \ln J^* + X_0^1 + (\alpha_1^* - \frac{1}{2}\sigma_{C1}^2)\tau + (\alpha_2^* - \frac{1}{2}\sigma_{C2}^2)(t-\tau). \tag{25}$$

The variance is:

$$Var(X_t^2) = \sigma_{C1}^2 \int_0^\tau ds + \sigma_{C2}^2 \int_\tau^t ds = \sigma_{C1}^2\tau + \sigma_{C2}^2(t-\tau); \tag{26}$$

thus, the standard deviation equals:

$$\sqrt{\sigma_{C1}^2\tau + \sigma_{C2}^2(t-\tau)}. \tag{27}$$

The futures price at time 0 for delivery at $t \geq \tau$ is therefore:

$$F(t,0) = e^{[E(X_t) + \frac{Var(X_t)}{2}]} = e^{[\ln(J^*) + X_0^1 + \alpha_1^*\tau + \alpha_2^*(t-\tau)]} = J^* C_0^1 e^{[\alpha_1^*\tau + \alpha_2^*(t-\tau)]}. \tag{28}$$

Or, equivalently,

$$\ln(F(t,0)) = \ln(J^*) + \ln(C_0^1) + \alpha_1^*\tau + \alpha_2^*(t-\tau). \tag{29}$$

The expected value of an annuity enjoyed from τ_1 to τ_2 when we go from one commitment period to the next one ($\tau_1 \leq \tau < \tau_2$) is:

$$V_{\tau_1,\tau_2} = C_0^1 \int_{\tau_1}^{\tau} e^{(\alpha_1^* - r)t}\, dt + \int_\tau^{\tau_2} e^{-rt} J^* C_0^1 e^{[\alpha_1^*\tau + \alpha_2^*(t-\tau)]} dt = \tag{30}$$

$$= C_0^1 \frac{e^{(\alpha_1^*-r)\tau} - e^{(\alpha_1^*-r)\tau_1}}{\alpha_1^* - r} + J^* C_0^1 e^{(\alpha_1^* - \alpha_2^*)\tau} \int_\tau^{\tau_2} e^{(\alpha_2^*-r)t} dt = \tag{31}$$

$$= C_0^1 \frac{e^{(\alpha_1^*-r)\tau} - e^{(\alpha_1^*-r)\tau_1}}{\alpha_1^* - r} + J^* C_0^1 e^{(\alpha_1^* - \alpha_2^*)\tau} \frac{e^{(\alpha_2^*-r)\tau_2} - e^{(\alpha_2^*-r)\tau}}{\alpha_2^* - r} \tag{32}$$

Thus, the higher the initial allowance price on the spot market, C_0^1, the size of the jump, J^*, and the drift rates α_1^* and α_2^*, the higher will be the value of avoiding the yearly emission of one tonne of carbon dioxide between τ_1 and τ_2.[4]

[3] Note that, in the particular case in which no jump takes place, i.e., $J^* = 1$, and $\alpha_1^* = \alpha_2^* = \alpha^*$ and $\sigma_{C1} = \sigma_{C2} = \sigma_C$, it follows that: $E_0(X_t^2) = X_0^1 + (\alpha^* - \frac{1}{2}\sigma_C^2)t$, which closely matches equation (5).

[4] Note that, in the particular case in which $J^* = 1$, $\alpha_1^* = \alpha_2^* = \alpha^*$, and $\sigma_{C1} = \sigma_{C2} = \sigma_C$, it follows that: $V_{\tau_1,\tau_2} = C_0^1 \frac{e^{(\alpha^*-r)\tau_2} - e^{(\alpha^*-r)\tau_1}}{\alpha^*-r}$, which closely resembles equation (14).

2.2 The price of natural gas

We assume a mean-reverting process for gas price G (say, in €/MWh) in which the long-run equilibrium level ($G_m(t)$) grows deterministically over time at a rate θ from its initial value $G_m(0)$. One advantage of this process is that, unlike others available in the literature (for instance, Schwartz and Smith [12]), it is possible to derive an analytic solution for the value of an annuity between two certain dates. Specifically, in a risk-neutral world:

$$\begin{aligned} dG_t &= k_G(G^*_m(t) - G_t)dt + \sigma_G G_t dW^G_t = & (33)\\ &= k_G(G_m(t) - \frac{\lambda_G}{k_G} - G_t)dt + \sigma_G G_t dW^G_t, & (34) \end{aligned}$$

where $G^*_m(t) \equiv G_m(t) - \lambda_G/k_G = G_m(0)e^{\theta t} - \lambda_G/k_G$. In this expression k_G denotes the speed of reversion towards the equilibrium level, and λ_G stands for the risk premium related to gas price uncertainty. σ_G is the instantaneous volatility of gas price changes, and W^G_t is a standard Wiener process.

The expected value satisfies:

$$\frac{E(dG_t)}{dt} = k_G G_m(0)e^{\theta t} - \lambda_G - k_G E(G_t), \tag{35}$$

so we know that:

$$\frac{E(dG_t)}{dt}e^{k_G t} + k_G E(G_t)e^{k_G t} = k_G G_m(0)e^{(\theta + k_G)t} - \lambda_G e^{k_G t}. \tag{36}$$

After integration:

$$E(G_t)e^{k_G t} = \frac{k_G G_m(0)e^{(\theta+k_G)t}}{\theta + k_G} - \frac{\lambda_G e^{k_G t}}{k_G} + K. \tag{37}$$

At time $t = 0$:

$$K = G_0 - \frac{k_G G_m(0)}{\theta + k_G} + \frac{\lambda_G}{k_G} \tag{38}$$

Therefore,

$$E(G_t) = G_0 e^{-k_G t} + \frac{k_G G_m(0)(e^{\theta t} - e^{-k_G t})}{\theta + k_G} + \frac{\lambda_G}{k_G}(e^{-k_G t} - 1). \tag{39}$$

As a consequence, the present value of an annuity from $\tau_1$to τ_2 would be:[5]

[5] It would not be the value of a futures contract for delivery between τ_1 and τ_2. Rather, it would be the present value of such a contract.

$$
\begin{aligned}
V_{\tau_1,\tau_2} &= \int_{\tau_1}^{\tau_2} e^{-rt} E(G_t)dt = && (40)\\
& \frac{e^{-(k_G+r)\tau_1} - e^{-(k_G+r)\tau_2}}{k_G + r}[G_0 + \frac{\lambda_G}{k_G} - \frac{k_G G_m(0)}{\theta + k_G}] + && (41)\\
& + \frac{k_G G_m(0)}{\theta + k_G} \frac{e^{-(r-\theta)\tau_1} - e^{-(r-\theta)\tau_2}}{r - \theta} - \frac{\lambda_G}{k_G} \frac{e^{-r\tau_1} - e^{-r\tau_2}}{r}. && (42)
\end{aligned}
$$

Now, let $Y_t \equiv \ln(G_t)$; then:

$$
dY_t = k_G(\frac{G_m^*(t)}{G_t} - 1 - \frac{1}{2}\sigma_G^2)dt + \sigma_G dW_t^G. \tag{43}
$$

Consider the specific case when $\theta = 0$ (so $G_m(t) = G_m\ \forall t$) and $\tau_2 = \tau_1 + 1$; we would have:

$$
E(G_t) = G_0 e^{-k_G t} + G_m(1 - e^{-k_G t}) + \frac{\lambda_G}{k_G}(e^{-k_G t} - 1).
$$

Hence,

$$
E(G_\infty) = G_m - \frac{\lambda_G}{k_G} \equiv G_m^*, \text{ with } k_G > 0.
$$

Therefore, the value a futures contract for delivery in one year

$$
F_{\tau_1,\tau_1+1} = \int_{\tau_1}^{\tau_1+1} E(G_t)dt = G_m^* + \frac{G_m^* - G_0}{k_G}[e^{-k_G(\tau_1+1)} - e^{-k_G\tau_1}]. \tag{44}
$$

This would be the price of a contract for delivery of one unit of the underlying commodity (natural gas) in one year. For instance, if the unit is meant to be delivered every hour over a year, multiplying this amount by the number of hours in a year ($365 \times 24 = 8{,}760$) we would get the value of receiving hourly the amount of gas equivalent to 1 MWh. Besides, if the reversion speed k_G is very high, then:

$$
F_{\tau_1,\tau_1+1} \approx G_m^* \equiv G_m - \frac{\lambda_G}{k_G}, \tag{45}
$$

since in this case $E(G_t) \approx G_m - \frac{\lambda_G}{k_G}$. And the model becomes quasi-deterministic.[6] Last, when $G_m^* = G_0$ we get an exact (as opposed to approximate) value:

$$
F_{\tau_1,\tau_1+1} = G_m^* \equiv (G_m - \frac{\lambda_G}{k_G}).
$$

[6]This is so becuase, in our model, there is no uncertainty on the long-term equilibrium level G_m. In a model with such a source of risk, the solution would not be that simple, but this is beyond the scope of our paper.

Table 1 shows the value of the term in brackets $[e^{-k_G(\tau_1+1)} - e^{-k_G\tau_1}]$ in equation (44) as a function of k_G and τ_1. For reversion speeds greater than 10, in practice the value of the coefficient is negligible.

Table 1. Value of the coefficient in equation (44).

k_G	$\tau_1 = 0.5$	$\tau_1 = 1$	$\tau_1 = 2$	$\tau_1 = 3$	$\tau_1 = 4$	$\tau_1 = 5$	$\tau_1 = 10$	$\tau_1 = 20$
1	0.3834	0.2325	0.0855	0.0315	0.0116	0.0043	0.0000	0.0000
2.5	0.1052	0.0301	0.0025	0.0002	0.0000	0.0000	0.0000	0.0000
5	0.0163	0.0013	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
10	0.0007	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
20	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

3 Basics of the two markets involved

3.1 The EU carbon market

The EU Emissions Trading Scheme (ETS) is the world's largest market in emissions. It is a system whereby CO_2 emission permits are traded. It covers over 10,000 installations which are collectively responsible for some 45% of CO2 emissions.[7] Thus, EU firms now face a carbon-constrained reality in form of legally binding emission targets.[8] Operations started officially on January 1st 2005. The first phase went from 2005 to 2007 and was considered as a trial or "warm-up" period aimed at getting the scheme "up and running". The second allocation phase, 2008-2012, coincides with the Kyoto commitment period.

EU member states face a cap on annual emissions, namely the ETS total, i.e. the quantity of allowances that are allocated to each country.[9] Right now EU member states' National Allocation Plans for the period 2008-2012 are being approved by the European Commission. Significant uncertainty remains concerning the emissions reductions targeted in them. Still greater uncertainty surrounds the post-Kyoto scenario. The negotiations on the successor to KP are scheduled to conclude in December 2009 (Copenhagen). In addition, a number of institutional arrangements are being proposed elsewhere by different players to cut their emissions. If and how these schemes will be ultimately linked to the EU ETS is the subject of much debate.

These markets periodically face a restriction in supply, with prices jumping accordingly at those times. However, the time of the jump is usually known in

[7] Currently the ETS covers four broad sectors: iron and steel, certain mineral industries, energy production, and pulp and paper. Transport and households, among other sectors, are not covered by the ETS.

[8] Firms whose emissions exceed the allowances they hold at the end of the accounting period must pay a fine (during the pilot period, €40 for each extra metric ton of CO2 emitted, and €100 during the commitment period). Those fined must also make up the deficit by buying the relevant volume of allowances (Convery and Redmond [2]).

[9] See Convery and Redmond [2] for a thorough description of the ETS including its institutional and legal framework.

advance, and coincides with the end of a trading period in allowances.[10] If the latest EC proposal goes ahead, this framework is going to change. Thus, over the third period, there will be a yearly jump as the total cap on the number of allowances is reduced on a yearly basis.[11] If we focus on the current commitment period of the Kyoto Protocol (2008-2012), presumably the price would show an increasing profile if demand grows while supply remains the same.

In anticipation of the establishment of the ETS, beginning in 2004, a futures market in allowances developed. Contracts on emission allowances are traded on different platforms (in addition to over-the-counter markets). Due to its volume of operations and liquidity, the European Climate Exchange (ECX, London UK) stands apart. The ICE ECX CFI futures contract is a deliverable contract where each Clearing Member with a position open at cessation of trading for a contract month is obliged to make or take delivery of emission allowances to or from National Registries. Contract size is 1,000 metric ton of carbon dioxide equivalent gas. Prices are quoted in euros/metric ton. As this paper was developed, there were futures contracts for both the commitment period (2008-2012) along with contracts maturing in December 2013 and December 2014. Therefore, when proposing a model for the behavior of permit prices, one must allow for two possibilities: either only one period is involved because our relevant period is devoid of allowance cuts (e.g., operations over 2008-2012), or more than one period is involved (as in the May-2008 price of a futures contract maturing in May-2013).

3.2 The EEX natural gas market

The European Energy Exchange AG (EEX) was established in 2002 as a result of the merger of the two German power exchanges in Frankfurt and Leipzig. It has established a leading trading market in European energy trading. Despite being one and only one exchange, it operates several sub-markets: (a) EEX spot market (Day ahead and intraday): power; (b) EEX spot market (Day ahead): natural gas, emission allowances; (c) EEX derivatives market (futures and options): power, natural gas, emission allowances, coal. Regarding natural gas, in particular, EEX holds an interest in store-x GmbH (Storage Capacity Exchange), an internet platform for secondary trading in storage capacities for natural gas, and in trac-x GmbH (Transport Capacity Exchange GmbH), an internet platform for natural gas transport capacities.

[10] There may be some uncertainty with regard to the size of the jump. Yet, once the National Allocation Plans have been approved by the European Commission, little (if any) uncertainty remains about when the jump will happen.

[11] In a Poisson distribution jumps occur randomly. Thus, it does not seem to be adequate for our purposes.

4 Estimation of the price models

4.1 The GBM process for the allowance price

Figure 1 shows the time profile of the allowance price. We have plotted the futures prices for maturities Dec-2008, Dec-2012, and Dec-2014. Thus there are already contracts that fall within two succeeding periods. We have also plotted a spot price series. The first part of it has been computed by means of cubic splines; specifically, each day we draw a curve that passes through all the points in the price/time space for which futures prices are available. This task ceased to be necessary when a spot price became available (from BlueNext, in the beginning of the commitment period).

[INSERT FIGURE 1 ABOUT HERE]

4.1.1 Within a given commitment period

If the initial spot price C_0 is known, it is necessary to estimate only α^* and σ_C (see equation (5)). In general, it will not be possible to get a consistent estimate of the drift rate in the actual world, α, since the estimator variance is going to be very large. However, the parameter $\alpha^* \equiv \alpha - \lambda_C$ can be estimated from (the natural log of) futures prices. Our sample consists of ICE ECX CFI futures prices from May-1-2006 to December-23-2008. In our computations below we use the estimate $\widehat{\alpha}^* = 0.039229$.

[INSERT FIGURE 2 ABOUT HERE]

Allowance price volatility can be estimated in several ways from different sources, among them the historical volatility from spot prices and futures prices; see Figure 3. We choose the volatility of the spot price over the second semester of 2008, which is $\sigma_C = 0.4393$. With these values of drift rate and volatility we can evaluate investments that avoid carbon emissions within a given commitment period only.

[INSERT FIGURE 3 ABOUT HERE]

4.1.2 Between two commitment periods

If the initial spot price C_0^1 is known, it suffices to estimate α_1^*, α_2^*, J^*, and σ_C ($= \sigma_{C1} = \sigma_{C2}$ by assumption), as can be seen in equation (25). We have three parameters to estimate (plus volatility). We have run three regressions with the whole sample (3,790 observations), the year 2008 (1,635 observations), and the second half of 2008 (namely, 875 observations). Again, the sample consists of ICE ECX CFI futures prices. Since the estimates of $\widehat{\alpha}_1^*$ and $\widehat{\alpha}_2^*$ derived from the last sub-samples are very similar, we run another regression assuming they are the same. The results are: $\widehat{\alpha}_1^* = \widehat{\alpha}_2^* = 0.039098$, and $\widehat{lnJ^*} = 0.035701$. We are going to use these estimates for the valuation of investments that enhance energy efficiency over two carbon trading periods. As for volatility, σ_C ($= \sigma_{C1} = \sigma_{C2}$) is assumed to be the same as within a given commitment period; thus $\sigma_C = 0.4393$. It is not obvious to us why or how it should differ from it.

4.2 The process for natural gas price

Figure 4 shows both the spot price and the Dec-12 futures price (source: EEX, Leipzig). As shown in equation (39), we need estimates of G_m, k_G, λ_G, σ_G; for discounting purposes we will also need a value of r. We derive the parameters sequentially. In particular, G_m, k_G, and σ_G are estimated with data from the physical world, whereas λ_G is derived from futures market data. We assume throughout that $\theta = 0$; in the sections below we consider investments when gas prices are expected to increase at a certain rate $\theta \neq 0$.

[INSERT FIGURE 4 ABOUT HERE]

Our sample consists of EEX natural gas contracts from October-1-2007 to December-30-2008. From the whole sample with all the contracts we get an average of $\widehat{\lambda}_G = -96.5439$. This suggests that the futures price has been well above the long-run equilibrium level G_m over the period considered. See Figure 5.

[INSERT FIGURE 5 ABOUT HERE]

In the sections below we use the estimates: $G_m = 25.0146$, $k_G = 20.0103$, $\lambda_G = 13.97$,$\sigma_G = 0.6742$, and $r = 0.045$. The value of λ_G corresponds to 12/30/2008, the last day of the series; also, the price of natural gas is 24.40 €/MWh ($= G_0$ in our computations below). Last, regarding the risk-free rate, r, we adopt an *ad hoc* value assumed to apply for the long term.

5 Valuation of projects to improve efficiency

Many investment projects are not projects to be assessed on a now-or-never basis. Therefore, benefits and costs arising from immediate investment are not the only ones to be computed. In general, projects to enhance efficiency can be considered as American-style options. In principle, it would be possible to decide when to undertake them depending on the economic pros and cons. Once the decision is made, we will assume that certain investment costs are incurred; if these are disbursed over time, their present value must be computed. Conversely, (market-valued) benefits will start only upon completion of the project. From then on, they will be enjoyed over the whole useful life of the facility where efficiency is meant to be enhanced. We must also consider the effect of uncertainty on carbon and gas prices as shown below.

5.1 Investment in carbon-avoiding projects

5.1.1 Within a given commitment period: The deterministic case

Constant investment cost We assume that the process for the allowance price is known and conforms exactly to the futures market curve. In this case, the optimal time to invest T^*must be determined. We have to asses whether it is better to invest immediately or rather to wait.

Let I denote the disbursement that is necessary to avoid 1 tCO_2 yearly. We assume that, one year upon that outlay, the savings (or revenues) from spare allowances will accrue over the next 30 years. The present value of the investment if it is undertaken at a future time T is:

$$[V(C_T) - I]e^{-rT} = aC_0 e^{(\alpha^* - r)T} - Ie^{-rT}, \tag{46}$$

where, from equation (14), we have $a \equiv \left[e^{31(\alpha^*-r)} - e^{(\alpha^*-r)}\right]/(\alpha^* - r) = 27.3881$. Differentiating with respect to T, the optimal time to invest happens to be:

$$aC_0(r - \alpha^*)e^{\alpha^* T^*} = rI \Rightarrow T^* = \frac{\ln(rI) - \ln[aC_0(r - \alpha^*)]}{\alpha^*}. \tag{47}$$

In particular, for immediate investment to be optimal ($T^* = 0$), it must be $rI \leq aC_0(r - \alpha^*)$; hence, $I \leq aC_0(r - \alpha^*)/r$. If the current (time-0) spot price is $C_0 = 15.23$ €/tCO_2, with a growth rate $\alpha^* = 0.0392$ and $r = 0.045$, we can compute $I = 53.4935$ €/tCO_2. This is significantly lower than the threshold derived when it is possible to invest only at time 0: $I = aC_0 = 417.12$ €/tCO_2.

In other words, when there is an option to delay investment in a carbon-avoiding project, immediate investment (overlooking the option to wait) only makes financial sense if it significantly cheaper than in the future. Thus, the possibility to choose the optimal time to invest is very relevant even in a deterministic setting. This is mainly due to the lower present value of the investment, I.

Increasing investment cost Assume that the initial outlay is determined by $I_T = I_0 e^{\beta T}$, where β is the rate of growth. In this case, the present value of the investment will be:

$$[V(C_T) - I_T]e^{-rT} = aC_0 e^{(\alpha^* - r)T} - I_0 e^{(\beta - r)T}. \tag{48}$$

Differentiating with respect to T, the optimal time to invest can be derived

$$e^{(\alpha^* - \beta)T^*} = \frac{I_0(r - \beta)}{aC_0(r - \alpha^*)} \Rightarrow T^* = \frac{\ln(I_0(r - \beta)) - \ln[aC_0(r - \alpha^*)]}{\alpha^* - \beta}. \tag{49}$$

If $\beta = r$, i.e., the investment cost grows at the risk-less interest rate, we have:

$$[V(C_T) - I_T]e^{-rT} = aC_0 e^{(\alpha^* - r)T} - I_0. \tag{50}$$

If $\alpha^* - r < 0$, then the optimal time to invest (assuming NPV>0) would be $T = 0$. Therefore, at time 0 the investment cost should be equal to or less than $I = aC_0 = 417.12$ €/tCO_2. And if $\alpha^* - r > 0$, then the optimal decision would be to postpone investment as further into the future as possible.

For a growth rate $0 \leq \beta \leq r$, we get values of I ranging between 53.4935 €/tCO_2 and 417.12 €/tCO_2. This attests to the importance of the expected cost path for investing in efficiency technologies, even in a deterministic case.

5.1.2 Within a given commitment period: Infinite-lived American options

It is hardly realistic to think that we have an option to undertake an investment at any time whatever it stretches into the future. However, if the time to maturity or possible exercise is long enough, the perpetual option (which has an analytic solution) may be both a good approximation and a benchmark against which we can assess the goodness or reliability of finite-lived option values.

Constant investment cost Let H denote the value of a perpetual option to invest in a project the value of which V depends in turn on the emission allowance price C governed by a GBM. Then, $H(C)$ satisfies the following differential equation (Dixit and Pindyck [3]):

$$\frac{1}{2}\sigma_C^2 C^2 H_{CC} + \alpha^* C H_C = rH,$$

Let the solution be $H(C) = A_1 C^{\gamma_1} + A_2 C^{\gamma_2}$, where A_1 and A_2 are constants to be determined, with $\gamma_1 > 0$ and $\gamma_2 < 0$. Since $H(0) = 0$ it must be $A_2 = 0$; therefore: $H(C) = A_1 C^{\gamma_1}$. The first and second partial derivatives are: $H_C = \gamma_1 A_1 C^{\gamma_1 - 1}$; $H_{CC} = \gamma_1(\gamma_1 - 1) A_1 C^{\gamma_1 - 2}$. So the differential equation can be rewritten as:

$$\frac{1}{2}\sigma_C^2 \gamma_1(\gamma_1 - 1) + \alpha^* \gamma_1 - r = 0. \tag{51}$$

Hence, γ_1 is a function of α^*, σ_C, and r. With $\alpha^* = 0.039229$, $\sigma_C = 0.4393$ and $r = 0.045$ we get $\gamma_1 = 1.0413074$.

The above solution is valid on the range of carbon prices for which it is optimal to hold the option (as opposed to exercise it and invest in the project). This range extends from zero to an investment threshold C^*. Of course, C^* is itself an unknown, to be determined as a part of the solution along with A_1. We consider the behavior of $H(C)$ at C^*, i.e., the so-called value-matching and smooth-pasting conditions. The value-matching condition states that the value of the option must equal the net value obtained by exercising it:

$$\text{Value-Matching}: A_1(C^*)^{\gamma_1} = V(C^*) - I.$$

Again, we can interpret I as the investment required to avoid 1 tCO_2 per year. We assume that, starting one year upon this outlay, revenues are received over 30 years. Therefore:

$$V(C^*) = C^* \frac{e^{31(\alpha^* - r)} - e^{(\alpha^* - r)}}{\alpha^* - r} = aC^* = 27.3881C^*,$$

where $a = \left[e^{31(\alpha^* - r)} - e^{(\alpha^* - r)}\right] / (\alpha^* - r)$. The value-matching condition thus reduces to:

$$A_1(C^*)^{\gamma_1} = aC^* - I.$$

The smooth-pasting condition states that the graphs of $H(C)$ and $V(C) - I$ should meet tangentially at C^*:

$$\text{Smooth-Pasting} : \gamma_1 A_1 (C^*)^{\gamma_1 - 1} = a.$$

Solving for A_1 in both conditions and equalizing we get:

$$aC^*(1 - \frac{1}{\gamma_1}) = I. \tag{52}$$

If we were dealing with a certain project with a known investment cost, I, from this expression we could derive the trigger carbon price, C^*, above which investment would be optimal. And the other way round; given a certain value C^*, investment will take place if its cost is less than or equal to I^*:

$$I^* = aC^*(1 - \frac{1}{\gamma_1}) = 1.0864653C^*.$$

It is possible to analyze how volatility σ_C affects this result. See Table 2. In the case of null volatility ($\sigma_C = 0$), we are back in the deterministic setting, case with $I^*/aC^* = 53.4935/417.12 = 0.1282$. From equation (51), as σ_C increases γ_1 decreases, and therefore $(1 - 1/\gamma_1)$ decreases. The greater is the uncertainty on future carbon prices, the smaller is the wedge between I and V^*, that is, the lower is the investment threshold that the firm will demand before it is willing to undertake the investment.

Table 2. Sensitivity of investment threshold to carbon price volatility.

σ_C	0.00	0.10	0.20	0.30	0.40	0.4393	0.50
I^*/aC^*	0.1282	0.1140	0.0863	0.0621	0.0448	0.0397	0.0331

Increasing investment cost In this case we have $dI = \beta I dt$ or, equivalently, $I_t = I_0 e^{\beta t}$. The value of the perpetual option $H(C, I)$ must satisfy the differential equation:

$$\frac{1}{2}\sigma_C^2 C^2 H_{CC} + \alpha^* C H_C + \beta I H_I = rH, \tag{53}$$

with boundary conditions:

$$\begin{aligned} H(C^*, I) &= V(C^*) - I = aC^* - I, & (54) \\ H_C(C^*, I) &= a, & (55) \\ H_I(C^*, I) &= -1. & (56) \end{aligned}$$

Following Dixit & Pindyck [3], if the current values of both C and I are doubled, that will merely double the value of the project and also the cost of investing. The optimal decision should therefore depend only on the ratio $x \equiv C/I$. Correspondingly, the value of the option $H(C, I)$ should be homogeneous of degree 1 in (C, I) enabling us to write:

$$H(C,I) = Ih(\frac{C}{I}) = Ih(x), \tag{57}$$

where h is now the function to be determined. The derivatives are:

$$H_C = h'(x), \tag{58}$$

$$H_{CC} = \frac{1}{I}h''(x), \tag{59}$$

$$H_I = h(x) - xh'(x). \tag{60}$$

Substituting the partial derivatives in the differential equation (53) we get:

$$\frac{1}{2}\sigma_C^2 x^2 h''(x) + (\alpha^* - \beta)xh'(x) + (\beta - r)h(x) = 0. \tag{61}$$

This is an ordinary differential equation. Let the solution be $h(x) = A_1x^{\gamma_1} + A_2x^{\gamma_2}$, with $\gamma_1 > 0$ and $\gamma_2 < 0$. If $x \to \infty$ then $A_2 = 0$. Therefore $h(x) = A_1x^{\gamma_1}$. In this case the value-matching and smooth-pasting conditions would be:

$$\begin{aligned} H(C^*, I) &= V(C^*) - I = aC^* - I \Longrightarrow h(x^*) = ax^* - 1, & (62)\\ h'(x^*) &= a. & (63) \end{aligned}$$

Hence, using $h(x) = A_1x^{\gamma_1}$, these conditions imply:

$$\begin{aligned} A_1 &= (ax^* - 1)(x^*)^{-\gamma_1}, & (64)\\ A_1 &= \frac{a}{\gamma_1}(x^*)^{-\gamma_1+1}. & (65) \end{aligned}$$

Equating both equations:

$$x^* \equiv \frac{C^*}{I^*} = \frac{\gamma_1}{a(\gamma_1 - 1)}. \tag{66}$$

On the other hand, substituting $h(x)$ in equation (61) we get:

$$\frac{1}{2}\sigma_C^2\gamma_1^2 + (\alpha^* - \beta - \frac{1}{2}\sigma_C^2)\gamma_1 + (\beta - r) = 0, \tag{67}$$

which allows to compute γ_1 as a function of α^*, β, σ_C, and r.

If $\beta = r$ then we have:

$$\gamma_1 = \frac{+2\beta + \sigma_C^2 - 2\alpha^*}{\sigma_C^2}.$$

With the same data as in the former case we compute $\gamma_1 = 1.05980792$. Therefore $x^* \equiv C^*/I^* = \gamma_1/a(\gamma_1 - 1) = 0.6470$ or, equivalently, $I^* = 1.5456C^*$.

Table 3 shows how carbon price volatility σ_C influences this result. For a volatility that approaches zero, we get the result of the deterministic case: with $\beta = r = 0.045$, and $\alpha^* = 0.039229$, we have $(\alpha^* - r) < 0$ and immediate investment would be optimal, so $I = aC_0$, or $I^*/aC^* = 1$. A high volatility translates into the requirement of a lower investment cost in comparison to profits (as measured by aC^*).

Table 3. Sensitivity of investment threshold to carbon price volatility.								
σ_C	0.00	0.01	0.10	0.20	0.30	0.40	0.4393	0.50
I^*/aC^*	1.000	0.9914	0.5358	0.2239	0.1137	0.0673	0.0564	0.0441

Comparing these figures with those in Table 2 we can see that the wedge between I and V^* widens. Given that the investment cost is now assumed to increase over time, waiting is less attractive than before, and the firm is ready to invest at higher investment thresholds.

5.1.3 Within a given commitment period: Finite-lived American options

Without jumps in carbon price Now we want to determine the value of an option to invest in avoiding carbon and the optimal investment rule when the option is available between time 0 and T. Again, the costs of the initial disbursement increase over time: $I_t = I_0 e^{\beta t}$, with $0 \leq t \leq T$.

Consider the value of the opportunity to invest when the time to do it can be chosen optimally. At the final moment $t = T$, there is no other option but to invest right then or not to invest. The decision to undertake the investment is made if the present value of revenues exceeds that of costs:

$$NPV = PV(Revenues) - PV(Expenditures) > 0.$$

A binomial lattice is arranged with the following values in the final nodes:

$$W = max(NPV, 0).$$

In previous moments, that is, when $0 \leq t < T$, depending on the current carbon price, we compute the present value of investing (NPV) and compare it with the value of waiting to invest (i.e., keeping the option alive). We choose the maximum between them:

$$W = Max(NPV, e^{-r\Delta t}(p_u W^+ + p_d W^-)),$$

where p_u stands for the probability that carbon price will move upwards (+) in the next step, and $p_d \equiv (1 - p_u)$ is the probability that it will move downwards (−).

The lattice is solved backwards, which provides the time-0 value of the option to invest. If we compare this value with that of an investment made at the outset (on a now-or-never basis), the difference will be the value of the option to wait. Obviously, this option's value will always be nonnegative.

The lattice can be arranged so that the risk-neutral probabilities amount to one:

$$p_u + p_d = 1, \tag{68}$$

and the second non-centered moment satisfies:

$$E(\Delta X^2) = \Delta X^2 = (\alpha^* - \frac{\sigma_C^2}{2})^2 \Delta t^2 + \sigma_C^2 \Delta t; \tag{69}$$

if we ignore the effect of Δt^2 then:

$$\Delta X = \sigma_C \sqrt{\Delta t}. \tag{70}$$

Similarly, for the average values:

$$E(\Delta X) = (\alpha^* - \frac{\sigma_C^2}{2})\Delta t = p_u \sigma_C \sqrt{\Delta t} - p_d \sigma_C \sqrt{\Delta t} = \sigma_C \sqrt{\Delta t}(2p_u - 1); \tag{71}$$

hence we get:

$$p_u = 0.5 + \frac{(\alpha^* - \frac{\sigma_C^2}{2})\sqrt{\Delta t}}{2\sigma_C}. \tag{72}$$

By changing the initial level of the emission allowance price it is possible to determine the carbon price at which the option value switches from positive to zero. This will be the optimal exercise price at $t = 0$, i.e., the first time that C drops to this level the investment must be undertaken immediately. Similarly, arranging a binomial lattice for the investment opportunity with maturity $t < T$ and changing the carbon price, the optimal exercise price for intermediate moments is determined.

At the initial date with $(C^* =)$ $C_0 = 15.23$ €/tCO_2, investment is realized only if $NPV > 0$. The optimal point to invest will be found by computing the investment threshold I^* for which $NPV = 0$. For an investment opportunity with a time horizon of 20 years, and 120 steps per year, we get the values in Table 4.[12] Again, with $\sigma_C = 0$ we are in the deterministic case: $I^* = aC^* = 417.12$ €/tCO_2 avoided. One carbon tonne worth 15.23€ avoided yearly over 30 years implies gross (undiscounted) savings of 456.9€, so investing when the cost is 417.12 € per tonne avoided looks sensible. The third column shows that this amount falls steadily as volatility increases. Regarding the second column, consider for example the case when $\sigma_C = 0.10$. With a constant investment cost ($\beta = 0$) we get $I^* = 47.9353$. Dividing it by $aC^* = 417.12$ we get a ratio of 0.1149, which is rather close to the amount of $I^*/aC^* = 0.1140$ that appears in Table 2 for an infinite-lived option to invest.

[12] The time period during which the oportunity to invest is available is assumed to be 20 years. This is not to be confused with the useful life of the project, i.e., the number of years that it will save costs or bring revenues, which is 30 years (starting one year after completion).

Table 4. Trigger investment cost I^* (€/tCO_2) with 20 years to invest.		
σ_C	$\beta = 0$	$\beta = r = 0.045$
0	53.4935	417.1213
0.01	53.5188	414.1991
0.05	52.0322	356.0297
0.10	47.9353	268.1841
0.15	43.0379	196.2400
0.20	38.2911	142.8354
0.25	33.8406	104.4757
0.30	29.6311	77.2141
0.35	25.7001	57.8435
0.40	22.1420	43.9916
0.4393	19.6494	35.8828
0.45	19.0178	34.0063
0.50	16.3390	26.7303

5.1.4 Between two trading periods: Jumps in carbon price

In this case we use the parameter estimates $\widehat{\alpha}_1^* = \widehat{\alpha}_2^* = 0.039098$ and $\widehat{lnJ^*} = 0.035701$, along with $\sigma_C = 0.4393$. We assume that the investment takes one year to build so profits do no start flowing until this period is over.

If investment takes place in the second period, or in the first one with less than one year to its end, the value of revenues at the time the decision is made is (see equation (14)):

$$V_{\tau_1,\tau_2} = C_0^2 \frac{e^{(\alpha^*-r)\tau_2} - e^{(\alpha^*-r)\tau_1}}{\alpha^* - r}. \tag{73}$$

However, if the decision is made in the first period a year or more before its end, the value of revenues would be (see equation (30)):

$$V_{\tau_1,\tau_2} = C_0^1 \frac{e^{(\alpha_1^*-r)\tau} - e^{(\alpha_1^*-r)\tau_1}}{\alpha_1^* - r} + J^* C_0^1 e^{(\alpha_1^*-\alpha_2^*)\tau} \frac{e^{(\alpha_2^*-r)\tau_2} - e^{(\alpha_2^*-r)\tau}}{\alpha_2^* - r}.$$

At the time when the jump in carbon price happens, the value of the underlying asset increases, but the lattice for the option's value recombines as before. With the above parameter values we get the figures in Table 5. The bold figure $I^* = 19.47$ corresponds to the case with $\sigma_C = 0.4393$, $\beta = 0$, and a mild jump $J^* = 1.0363$. If, instead, we assume no jump ($J^* = 1$), we get $I^* = 19.65$, the same amount (for $\sigma_C = 0.4393$ and $\beta = 0$) that appears in Table 4. In a sense, the jump implies that the greatest savings from emissions avoided will accrue later in time. Investing at the same time as before would mean that a portion of the useful life of the investment is spent during years in which the allowance price was relatively low. Therefore, investing will be justified only if it is comparatively cheaper than before (without jumps).

Table 5. Trigger cost I^* (€/tCO_2) with finite time.					
σ_C	I^*	β	I^*	J^*	I^*
0.05	48.76	0.000	**19.47**	1.00	19.65
0.10	45.95	0.005	20.57	1.01	19.66
0.15	41.81	0.010	21.82	1.02	19.60
0.20	37.44	0.015	23.22	1.03	19.53
0.25	33.22	0.020	24.79	1.0363	**19.47**
0.30	29.18	0.025	26.55	1.04	19.42
0.35	25.38	0.030	28.53	1.05	19.29
0.40	21.91	0.035	30.75	1.06	19.13
0.4393	**19.47**	0.040	33.21	1.07	18.94
0.45	18.85	0.045	35.95	1.08	18.71
0.50	16.21	0.050	38.98	1.09	18.45

The left side shows that greater uncertainty on carbon prices presses the firm to require lower investment costs in order to invest. The figures here are lower than in Table 4 because now we are assuming that there will be a jump $J^* = 1.0363$; so investment will have to be cheaper than before. The central part, instead, suggests that increasing investment costs translate into less stringent requirements to invest. And the right hand suggests that the cost required falls as the size of the (upward) jump gets larger.

5.2 Investment in natural gas-saving projects

5.2.1 Constant equilibrium price level

From equation (40), when $\theta = 0$, the value of an annuity from $\tau_1 = 1$ to $\tau_2 = 31$ is:

$$\begin{aligned} V_{\tau_1,\tau_2} &= \int_{\tau_1}^{\tau_2} e^{-rt} E(G_t)dt = G_0 \frac{e^{-(k_G+r)} - e^{-31(k_G+r)}}{k_G + r} + \\ &+ (G_m - \frac{\lambda_G}{k_G})[\frac{e^{-r} - e^{-31r}}{r} - \frac{e^{-(k_G+r)} - e^{-31(k_G+r)_2}}{k_G + r}]. \end{aligned}$$

Adopting the values mentioned above, this annuity is worth 382.6677 €.[13] This amount is almost independent from the initial gas price G_0, since the reversion speed $k_G = 20.0102$ is so high that it renders the coefficient of G_0 very small for a thirty-year investment next year. Also, the value of k_G allows to compute the half-life of the price process, $t_{1/2}$, i.e., the time that is necessary for the expected value $E(G_{t_{1/2}})$ to reach the average value between G_0 and G_m:

$$E(G_{t_{1/2}}) = G_o e^{-k_G t_{1/2}} + G_m(1 - e^{-k_G t_{1/2}}) = G_0 - \frac{G_0 - G_m}{2}.$$

[13] Note that the initial gas price times the useful life of the investment gives: $24.40 \times 30 = 732$ euros. Since the investment wold be working from year 1 to year 31, a gross discounting over 15.5 years at a rate $r = 4.5\%$ gives an amount of some 370 euros, which is close to the computed annuity. Of course, the other parameters also play a role.

Operating we have:

$$t_{1/2} = \frac{\ln(2)}{k_G}.$$

For a value $k_G = 20.0102$ the half-life is $t_{1/2} = 0.0346$ years, so that it takes just 12.6 days to bridge 50% of the gap $(G_0 - G_m)$.

This implies that the strongest effect of any gap between natural gas current price and the equilibrium price shows up during the first year. As it happens, this period is not taken into account because this is the time lapse when the investment would be under construction, so that it would not be working until the first year is over. In other words, the gas price at the initial moment G_0 has almost no effect on the value of an annuity that starts in one year's time and lasts for thirty years.[14]

In this model, a yearly saving of natural gas equivalent to 1 MWh would be worth 382.66 € now or later. Since investing after Δt would have a present value $382.66e^{-r\Delta t}$, as opposed to 382.66 € if we invest today, immediate investment would be optimal whenever the cost to a MWh saved yearly involves an amount lower than 382.66 €.[15]

5.2.2 Equilibrium price level with drift

The situation is different if natural gas prices are assumed to show a positive drift. As can be seen in Figure 6, gas prices, although volatile, have shown an increasing profile over the last seven years. This trend could continue in the future. Yet the exact amount is hard to predict and is not the aim of this paper.

[INSERT FIGURE 6 ABOUT HERE]

With a non-zero drift in gas prices, and given the negligible value of the coefficient of G_0, at the initial moment we have (see equation (40)):

$$V_{1,31} = \frac{k_G G_m(0)}{\theta + k_G}\frac{e^{-(r-\theta)} - e^{-31(r-\theta)}}{r-\theta} - \frac{\lambda_G}{k_G}\frac{e^{-r} - e^{-31r}}{r}.$$

Table 6 shows the present value of an investment that saves the natural gas equivalent to 1 MWh yearly as a function of the drift rate in gas prices θ, all else remaining constant. The value of the annuity (gas saved) increases as the gas price has a higher drift rate.

Table 6. Investment value I (€/MWh) and growth rate in gas prices.

θ	-0.025	0.000	0.025	0.050	0.075	0.100
$V_{1,31}$	281.76	382.66	541.46	800.68	1,238.44	2,000.52

[14] This holds true in our particular model. In a more complex setting, with uncertainty in the equilibrium price G_m , things could be different depending on the specific modeling of uncertainty. This issue falls beyond the scope of our paper.

[15] Note that this value 382.66 € has been derived taking into account the risk premium λ_G. Therefore, it can be discounted at the risk-free rate r.

If, instead, investment is undertaken at time T, the net present value (accounting for the investment cost I that is required to save one MWh of natural gas yearly, and given $G_m(T) = G_m(0)e^{\theta T}$) would be:

$$[\frac{k_G G_m(0)e^{\theta T}}{\theta + k_G}\frac{e^{-(r-\theta)} - e^{-31(r-\theta)}}{r-\theta} - \frac{\lambda_G}{k_G}\frac{e^{-r} - e^{-31r}}{r} - I]e^{-rT}.$$

This can be rewritten as:

$$[ae^{\theta T} + b - I]e^{-rT},$$

where:

$$\begin{aligned} a &= \frac{k_G G_m(0)}{\theta + k_G}\frac{e^{-(r-\theta)} - e^{-31(r-\theta)}}{r-\theta} \\ b &= -\frac{\lambda_G}{k_G}\frac{e^{-r} - e^{-31r}}{r}. \end{aligned}$$

Differentiating this expression with respect to T we derive the optimal time to invest:

$$a(\theta - r)e^{(\theta - r)T^*} + r(I - b)e^{-rT^*} = 0.$$

Therefore, for investing in $T^* = 0$ to be optimal, the investment cost must be lower tan the value I implicit in the next equation:

$$a(\theta - r) + r(I - b) = 0.$$

Hence we have:

$$I \leqq \frac{r(a+b) - a\theta}{r}.$$

And $I \leqq V_{1,31}$ must also apply, i.e., the NPV must be positive.

Obviously, if $\theta = 0$ then it must be $I \leqq a + b$ ($= V_{1,31}$ in Table 6). For the values of θ considered in Table 6, the corresponding values appear in Table 7.

Table 7. Investment value I (€/MWh) under different scenarios.

θ	a	b	$V_{1,31}$	$\frac{r(a+b)-a\theta}{r}$
-0.025	292.7565	-10.9868	281.7697	444.4122
0.000	393.6545	-10.9868	382.6677	382.6677
0.025	552.4504	-10.9868	541.4636	234.5467
0.050	811.6740	-10.9868	800.6872	-101.1728
0.075	1,249.4278	-10.9868	1,238.4410	-843.9387
0.100	2,011.51	-10.9868	2,000.5232	-2,469.5009

For a growth rate $\theta = r(a+b)/a$, we compute $\theta = 0.0443$. With this figure, a non-positive investment cost $I \leqq 0$ would be necessary. As shown in the last column, a higher expected growth in the price of natural gas leads to postponing investments, unless their costs are lower.

Increasing investment cost If the investment cost I grows over time at a rate φ, then the net present value of the investment at time T would be:

$$[ae^{\theta T} + b - Ie^{\varphi T}]e^{-rT}.$$

Differentiating and equating to zero, the optimal time to invest T^* is:

$$a(\theta - r)e^{(\theta-r)T^*} - rbe^{-rT^*} - (\varphi - r)Ie^{(\varphi-r)T^*} = 0.$$

Investing at $T^* = 0$ will be optimal depending on the investment cost I:

$$a(\theta - r) - rb - (\varphi - r)I = 0.$$

It would we optimal provided that:

$$I \leqq \frac{r(a+b) - a\theta}{r - \varphi}.$$

Also in this case, investment will be undertaken at the initial moment if the inequality $I \leqq V_{1,31}$ holds.

Take as a reference the case $\theta = 0.025$. Then we get the values in Table 8. An increase in the expected growth rate of investment cost brings forward the decision to undertake the investment. This could be the case, for instance, if public subsidies are available currently but they are not expected to last forever, so a rising cost at a rate φ for the future is envisioned. In any case, the cost I must be interpreted as including any subsidies or fiscal measures, and computed as a present value.

Table 8. Investment value I (€/MWh).

φ	$V_{1,31}$	$\frac{r(a+b)-a\theta}{r-\varphi}$
0.000	541.4636	234.5467
0.005	541.4636	263.8651
0.010	541.4636	301.5601
0.015	541.4636	351.8201
0.020	541.4636	422.1841
0.025	541.4636	527.7301

6 Case study: two power technologies with different efficiency rates

It is well known that successive energy conversions and losses require about 10 units of fuel to be fed into a conventional thermal power station in order to deliver one unit of flow in a pipe (Lovins [10]). The possibilities offered by higher efficiency levels are therefore tremendous. Let us consider the choice today between two competing technologies for generating electricity from a given fuel (natural gas) in a power plant with a given total capacity (in MW). The first one allows the power station to operate at a certain efficiency rate (say, 55%). The second technology, though, allows to raise thermal efficiency by one percentage point (to 56%) in exchange for a certain amount of money. This technology thus brings some savings in gas consumption and carbon allowances relative to the first one. We further assume that the choice between them is available only today. An increase in the demand for electricity is taken for sure and, if the firm refuses to build the power plant, a rival utility will seize the chance. We adopt a typical construction period of 30 months and a useful life of 25 years.

The variable costs to producing 1 MWh of electricity from natural gas in a carbon-constrained environment (apart from other inputs), in €/MWh, amount to:

$$\frac{P_G}{E_G} + P_{CO_2} I_G, \tag{74}$$

where P_G is the price of natural gas (€/MWh),[16] E_G is the net thermal efficiency of the gas-fired plant, and P_{CO_2} is the price of a EU emission allowance (€/tCO_2). Last, I_G stands for the emission intensity of the plant (tCO_2/MWh); this in turn depends on the net thermal efficiency of each gas-fired plant.

According to IPCC [8], a plant burning natural gas has an emissions factor of 56.1 $kgCO_2/GJ$.[17] Since under 100% efficiency conditions 3.6 GJ would be consumed per megawatt-hour, we get

$$I_G = \frac{0.20196}{E_G} \frac{tCO_2}{MWh}. \tag{75}$$

Thus the complete formula for variable costs is

$$\frac{1}{E_G}(P_G + 0.20196 \times P_{CO_2}). \tag{76}$$

With $E_{G_1} = 0.55$ and $E_{G_2} = 0.56$, the costs are:

$$(P_G + 0.20196 \times P_{CO_2})(\frac{1}{0.55} - \frac{1}{0.56}) = 0.032468 \times P_G + 0.006557 \times P_{CO_2}. \tag{77}$$

The power station is assumed to be designed as a base load plant. It is expected to operate 80% of the time. Therefore, it will produce 7,008 MWh yearly for each MW of capacity installed.

First, we address the savings from the carbon emissions avoided. Carbon price evolves over time according to the values estimated above: $\widehat{\alpha}^* = 0.039098$, $\widehat{lnJ^*} = 0.035701$. Assuming that four years are left to the end of the Kyoto commitment period (2012), with $C_0^1 = 15.23$ €/tCO_2 and $r = 0.045$, by equation (30) the present value of savings (for each MWh/year) since completion until closure is:

[16] 1 MWh = 3.412 mmBTU, and 1 mmBTU = 0.293083 MWh under 100% efficiency.

[17] This corresponds to 15.311 kgC/GJ, since one ton of carbon is carried on 3.67 tons of CO_2.

$$V_{2.5,27.5} = C_0^1 \frac{e^{4(\alpha^*-r)} - e^{2.5(\alpha^*-r)}}{\alpha_1^* - r} + J^* C_0^1 \frac{e^{27.5(\alpha^*-r)} - e^{4(\alpha^*-r)}}{\alpha_2^* - r} = 360.67 \text{ €}.$$

Following equation (77), this amount must be multiplied 0.006557. Also, the station operates 7,008 hours/year. Thus, the total savings from emissions avoided are 16,573 € (for each MW installed).[18]

Now let us turn to the savings from a lower consumption of natural gas to generate the same amount of electricity. With the parameter values :$G_m = 25.0146$, $k_G = 20.0103$, $\lambda_G = 13.97$, and $\theta = 0.025$ the value of an annuity (for each MWh/year) is (see equation (40)):

$$V_{2.5,27.5} \approx \frac{k_G G_m(0)}{\theta + k_G} \frac{e^{-2.5(r-\theta)} - e^{-27.5(r-\theta)}}{r-\theta} - \frac{\lambda_G}{k_G} \frac{e^{-2.5r} - e^{-27.5r}}{r} = 458.15 \text{ €}.$$

As shown in equation (77), this amount, multiplied by 0.032468 and 7,008 hours, implies a present value of savings equal to 104,251 €.[19]

In sum, for each MW installed and a capacity factor of 80%, total savings amount to 120,825 €. If the net present value of the project is positive, the more efficient technology would be chosen provided its incremental cost (in €/MW) is lower than 120,825 €. Obviously, if the plant does not reach a usage of 80%, there is less time to recover this extra cost. Table 9 shows the savings enjoyed as a function of the production factor. As could be expected, more efficient technologies are harder to be adopted when they do not work as much time as possible. In this respect, the current downturn does not seem to be the most favorable scenario for cleaner technologies.

Table 9. Total savings (€/MW) from 1% additional efficiency.

Production factor (%)	80	70	60	50	40
Incremental Value	120,825	105,722	90,619	75,516	60,413

7 Conclusions

Efficiency improvements have brought about important savings to firms and households alike, particularly in periods of high fuel prices in energy markets. Lower bills, in turn, have been reflected in electricity prices and other output prices. In addition, energy efficiency measures could account for more than 65% of energy-related emissions savings up to 2030 (under the IEA's [5] Alternative Policy Scenario). This is far more than the 22% saving expected from switching to nuclear and renewable energy. If there is a price to be paid for these emissions (or it is expected to be in the future), curbing them has economic value for firms

[18] If the plant size is 500 MW, this amount must be multiplied by 500. The present value of savings is 8,286,500 €, which is a significant figure.

[19] For a capacity of 500 MW installed, savings would amount to 52,125,703 €.

However, the goal of harnessing the enormous potential for energy savings has proven elusive (IPCC [7]).The aim of this paper is to assess energy-efficiency investments in terms of well-established financial principles. We hope that this will further contribute to attract the interest of the investment community.

We envisage an investment in energy efficiency as a project that simultaneously saves fuel consumption and avoids carbon emissions. Investing in this project is risky, not least because the prices of natural gas and emission allowances vary stochastically over time. Besides, this investment is at least partially irreversible, i.e., should market conditions turn, the firm cannot "uninvest" and recover the full expenditure. Further, usually the firm has the opportunity (not the obligation) to invest in this project over some pre-specified period of time. In sum, the firm has to decide whether and when to invest.

We address these capital budgeting decisions following the so-called Real Options approach. Options on financial assets have been trading for a long while. This approach assesses opportunities to acquire real assets by stressing the analogy with financial options. Accordingly, our starting point is a stochastic process for the natural gas price and similarly for the emission allowance price. We estimate the underlying parameters from actual market data, in particular, from spot and futures markets for both commodities (carbon dioxide and natural gas). One difficulty emerges from the fact that the EU carbon market is not mature yet. Indeed, climate policy affects both carbon prices and price volatility. Therefore, changes in policy may give rise to sudden jumps in relevant variables. We explicitly model this feature regarding allowance prices. Upon estimation of the underlying parameters from actual market data, we are able to value cash flows emanating from fuel saved or emissions avoided over time. As a final step, we assess the option to invest in a project and the optimal time to invest. We proceed, somewhat pedagogically, from investments that only help avoid carbon emissions to those that only save fuel consumption, to a case study involving both effects. This is ultimately intended as a rigorous framework to make practical decisions.

A first lesson to be learned regards the value of waiting. At first glance, one might expect that waiting makes sense only when doing so allows learning something new about the future, i.e., in an uncertain context. We show that, even in a deterministic setting, the option to wait has value. Note that we deal with investments that are at least partially irreversible; in other words, upon investment a portion of the costs is "sunk" or hardly recoverable at all should one change her/his mind. Consequently, the requirement of a NPV merely greater than zero is not enough for investing to be optimal. Specifically, the gross value of the investment must surpass its cost by an amount equal to the value of the option to wait (which is lost as soon as the investment expenditure takes place). In our context, since investments in energy efficiency are not mandatory, firms may find it quite reasonable (from a financial point of view) to delay them. If sustainability is to be sought consistently, regulators should be aware of it.

Obviously, the former fact is even more pronounced under uncertainty (e.g., on the future value of the investment). We show that, within a given emissions trading period, rational investments that avoid carbon emissions require an investment value several times as large as the investment cost. This is so both when the option to invest is perpetual and finite-lived. The high volatility in carbon prices plays a major role in this result. In this sense, regulatory uncertainty exacerbates the problem. If there is a change in climate policy, prices may jump (presumably upwards) in the future. The natural response would be to delay investments so as to match facilities' useful lives to the highest carbon prices. In other words, early investments require even higher value-to-cost ratios, or even lower costs.

As for projects that afford fuel savings, we provide reasonable estimates of the value of those savings for a generic facility with thirty years of life. That value increases with the expected growth rate of natural gas price. A higher drift rate, in turn, leads to postponing investments, unless their costs are lower. This trend may be reversed, i.e., investments can be brought forward in time, if their cost is expected to increase in the future, or public subsidies are available but only for a limited period of time.

Last, our case study concerns two different power technologies based on firing natural gas with distinct efficiency rates and covered by a cap-and-trade emissions system. We find that one percentage point of additional efficiency can bring significant savings to the electric utility. The amount of these savings, though, is affected by the availability of the plants, i.e., the proportion of time that they operate over the year. Generation technologies in general have an important upside potential in efficiency, and sustainability goals can be served by efficiency gains at lower (private and social) cost than other alternative instruments.

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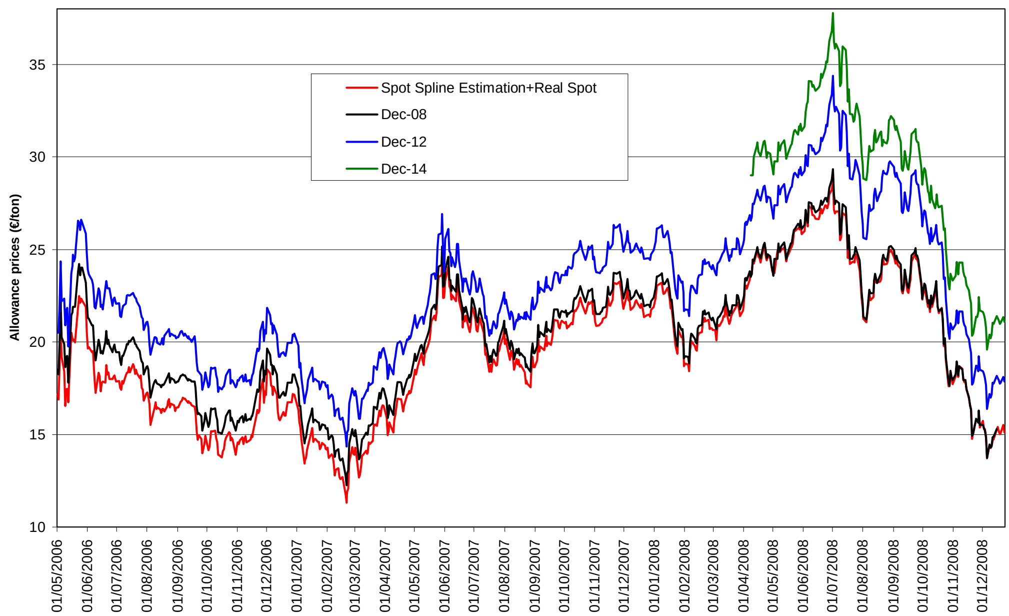

Figure 1: Futures and spot allowance prices.

8

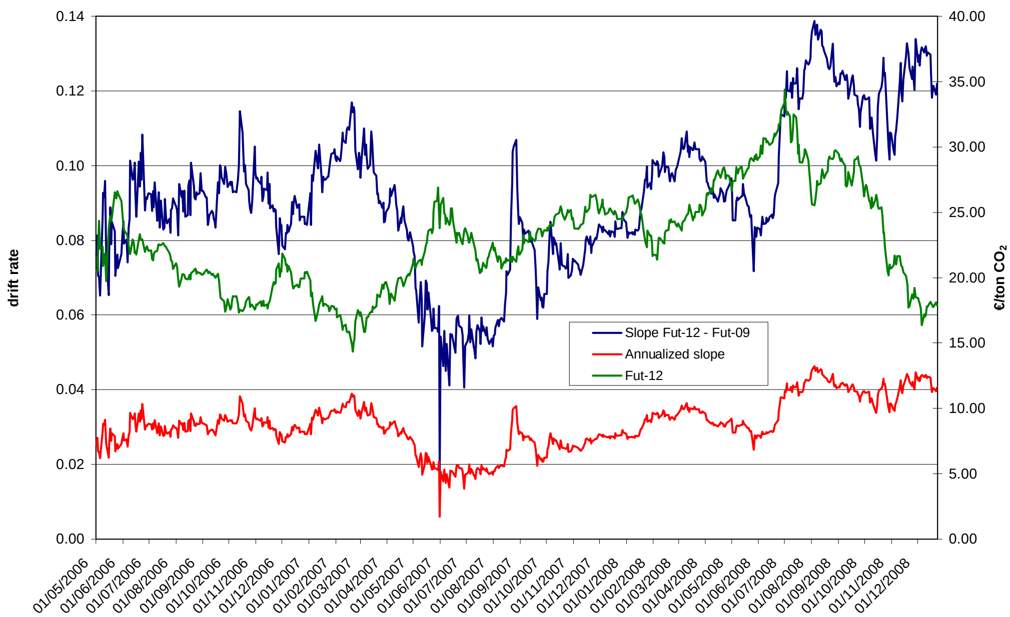

Figure 2: Dec-2012 futures allowance price (right vertical axis) and slope α^* of the (log) futures curve (left vertical axis).

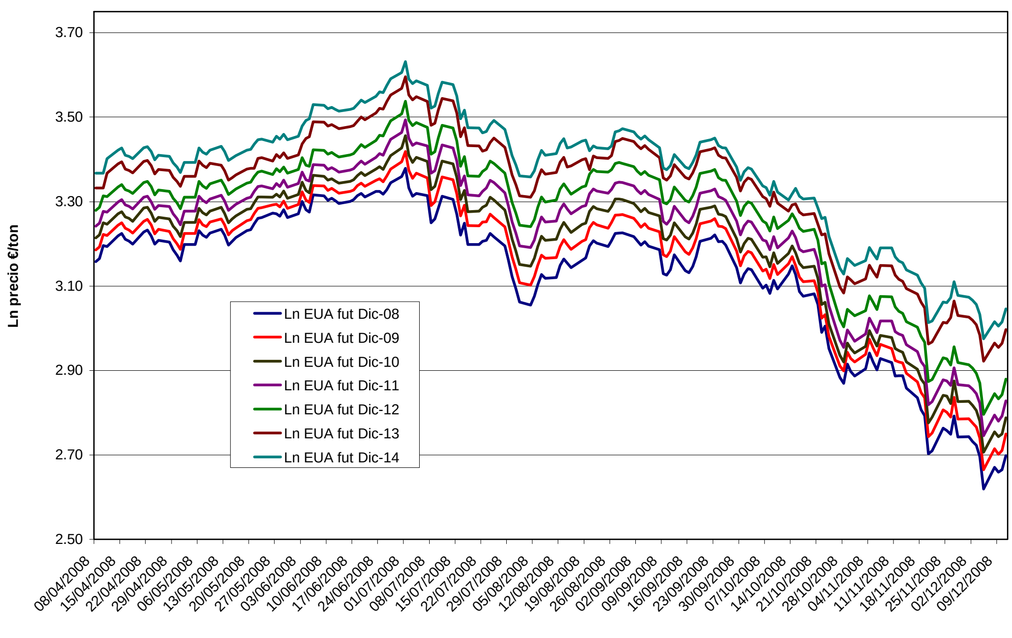

Figure 3: Futures allowance (log) prices.

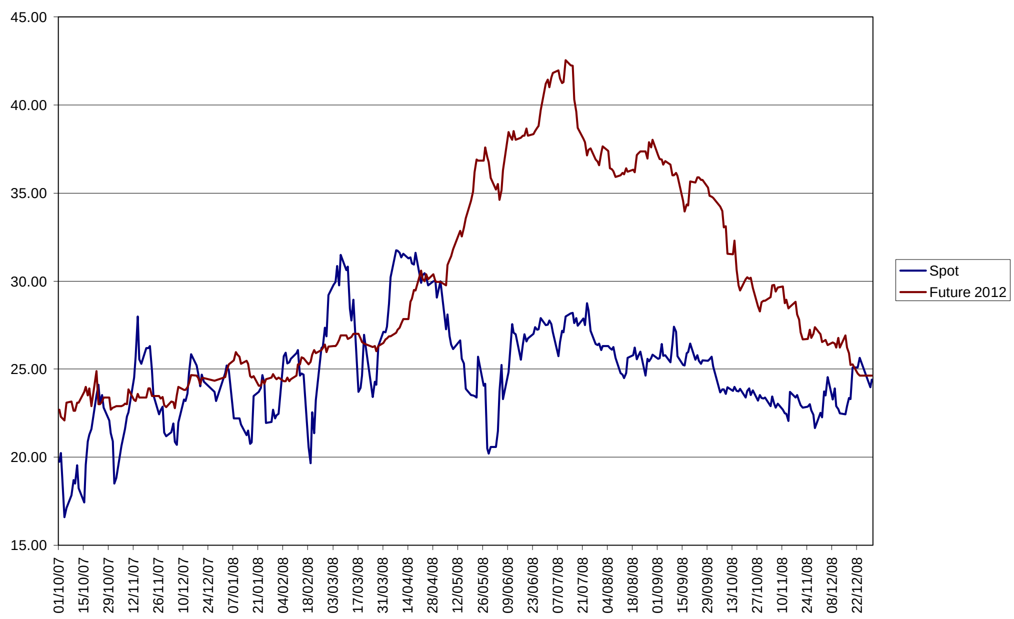

Figure 4: Futures and spot natural gas price.

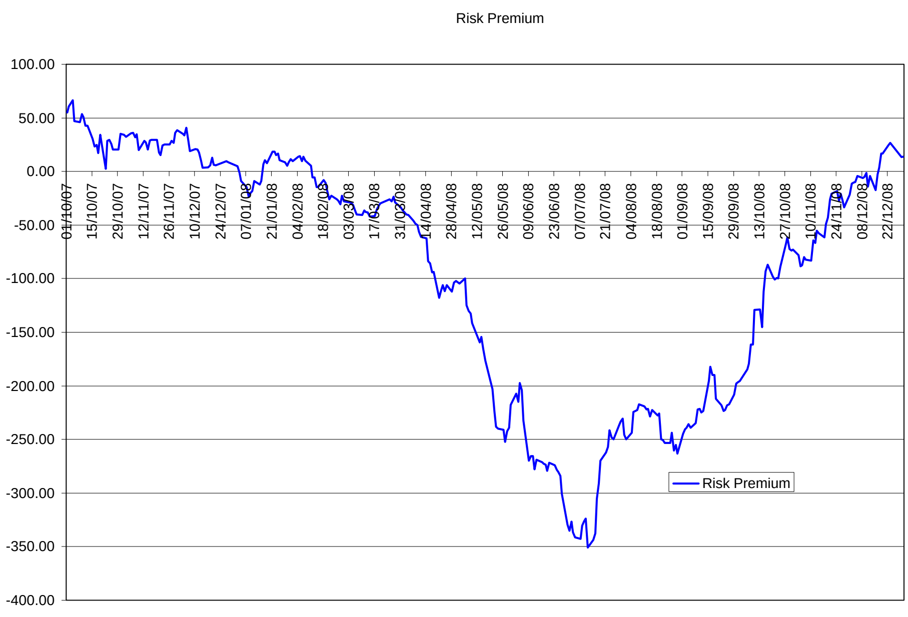

Figure 5: Risk premium in the futures market for natural gas.

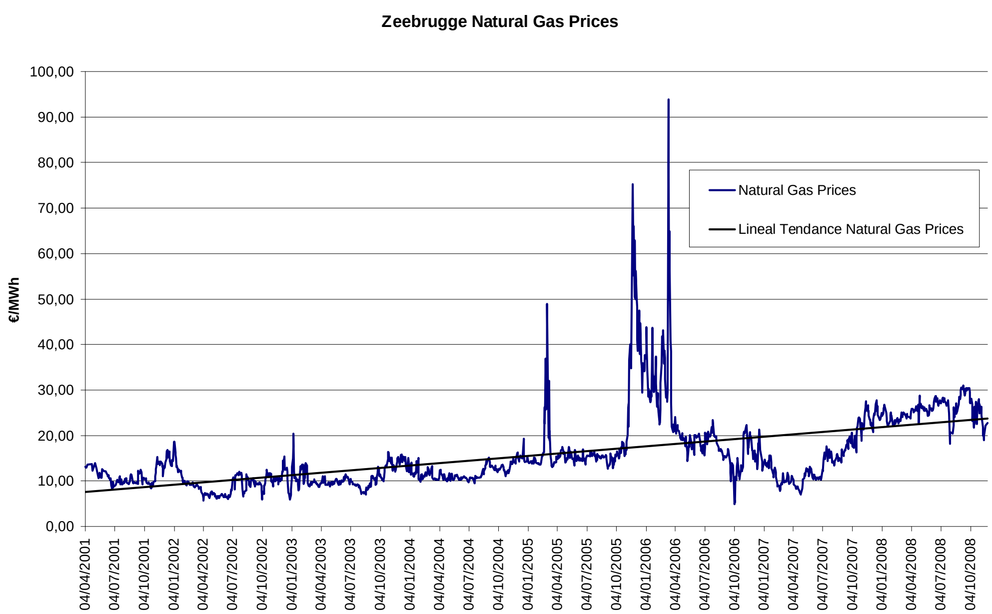

Figure 6: Natural gas price in the Zeebrugge (Belgium) market.

9 Appendix: Estimation of the price models

9.1 The GBM process for the allowance price

9.1.1 Within a given commitment period

If the initial spot price C_0 is known, it is necessary to estimate only α^* and σ_C (see equation (5)). In general, it will not be possible to get a consistent estimate of the drift rate in the actual world, α, since the estimator variance is going to be very large. However, the parameter $\alpha^* \equiv \alpha - \lambda_C$ can be estimated from (the natural log of) futures prices.

The futures market price $F(t, 0)$ gives us the value of an emission allowance at time t. Modeling this price requires the spot price C_0 and the slope of the curve of (log) futures prices.[20] We also need the allowance volatility to model futures prices and to value real options.

Drift. The EU ETS allows us to gauge the difference $\alpha^* \equiv \alpha - \lambda_C$. From equation (9) we compute:

$$\ln F(C_0,t_2) - \ln F(C_0,t_1) = (\alpha - \lambda_C)(t_2 - t_1), \qquad t_2 > t_1. \tag{78}$$

Thus, $\alpha - \lambda_C$ equals the difference between the (log) futures prices of contracts with December maturity when $t_2 - t_1 = 1$.[21] The differences between the (log) futures prices are rather stable, for instance between the EUA Fut Dec-12 and EUA Fut Dec-09;[22] see Figure 2. It can also be seen that the slope α^* has been around 3% over 2007 and the first half of 2008. However, since then it seems to fluctuate around 4%.

A cursory look at the market (see Figure 3) suggests that, if we use data from late last year, our estimate must be very close to some 4%, and that this slope has remained almost unchanged despite the abrupt falls in prices during the last months of 2008.

Let T be the maturity date of the futures contract. In general we could state:

$$\ln\left(F(C_t, T, t)\right) = \ln(C_t) + \alpha^*(T - t) = X_t + \alpha^*(T - t).$$

Therefore we propose to estimate:

$$\ln\left(F(T, t)\right) - X_t = (\alpha - \lambda_C)(T - t) + \varepsilon_t. \tag{79}$$

Our sample consists of ICE ECX CFI futures prices from May-1-2006 to December-23-2008. Table A1 shows our estimate of the slope α^* by Ordinary Least Squares (OLS) in three cases: (i) using all 3,418 observations in our sample, [23] (ii), using

[20] Remember that both the spot price and the futures contract's maturity must fall within a period over which there is no reduction in the number of allowances (e.g., 2008-2012).

[21] Obviously, since we have several futures prices, it is possible and convenient to aim at a more precise estimation.

[22] This value would correspond to three times the slope.

[23] Each day we have 5 futures prices (for maturities Dec-08 to Dec-12), except for the last days of Dec-08, when only four are available.

only the 1,263 observations from the year 2008, and (iii) using only those 623 observations from the second semester of 2008.

Table A1. OLS estimation of the drift in carbon prices within a given period.				
	$\widehat{\alpha}^*$	std. dev.	t-statistic	p-value
Whole sample	0.029843	0.000116	258.2	0.0000
2008 data	0.031852	0.000209	152.3	0.0000
III-IV 2008 data	0.039229	0.000168	233.0	0.0000

In our computations below we use the estimate $\widehat{\alpha}^* = 0.039229$. As already suggested, this is certainly close to 4%.

Volatility. Allowance price volatility can be estimated in several ways from different sources, among them the historical volatility from spot prices and futures prices; see Figure 3. The first step is to compute the natural logarithm of spot prices, then the standard deviation of price changes, and finally multiply by $\sqrt{250}$, since: $\ln(C_t) - \ln(C_0) \sim N((\alpha_C^* - \frac{1}{2}\sigma_C^2)t, \sigma_C\sqrt{t})$, where $t = 1/250$. And similarly with futures prices, since: $\ln(F(T,t)) - \ln(F(T,0)) \sim N(-\frac{1}{2}\sigma_C^2 t, \sigma_C\sqrt{t})$.

We choose the volatility of the spot price over the second semester of 2008, which is $\sigma_C = 0.4393$. With these values of drift rate and volatility we can evaluate investments that avoid carbon emissions within a given commitment period only.

9.1.2 Between two commitment periods

If the initial spot price C_0^1 is known, it suffices to estimate α_1^*, α_2^*, J^*, and σ_C ($= \sigma_{C1} = \sigma_{C2}$ by assumption), as can be seen in equation (25). Similarly to equation (79), we start from:

$$\ln(F(T,t)) - X_t = \alpha_1^* T_1 + d \ln J^* + \alpha_2^* T_2 + \varepsilon_t,$$

where d is a dummy variable which takes on a value of 1 for contracts maturing in the second period, and 0 for those maturing in the first period. T_1 is the term until maturity that falls within the first period. If the contract expires in the first period $T_1 = T - t$; if it expires in the second, then it is the difference between the end of this period τ and time t. Last, T_2 is the term corresponding to the second period. If the contract expires in the first period $T_2 = 0$; if it expires in the second, then it is the difference between the contract's maturity and the beginning of the second period, $T - \tau$.

We have three parameters to estimate (plus volatility). We have run three regressions with the whole sample (3,790 observations), the year 2008 (1,635 observations), and the second half of 2008 (namely, 875 observations); see Table A2. Again, the sample consists of ICE ECX CFI futures prices.

Table A2. OLS estimation of parameters in carbon prices over two periods.				
Whole sample	Value	stad. dev.	t-statistic	p-value
$\widehat{\alpha}_1^*$	0.029783	0.000129	230.0	0.0000
$\widehat{\alpha}_2^*$	0.038272	0.002892	13.2	0.0000
$\widehat{lnJ^*}$	0.051484	0.004608	11.2	0.0000
2008 data	Value	stad. dev.	t-statistic	p-value
$\widehat{\alpha}_1^*$	0.031581	0.000288	109.5	0.0000
$\widehat{\alpha}_2^*$	0.038277	0.003053	12.5	0.0000
$\widehat{lnJ^*}$	0.043676	0.004987	8.8	0.0000
III-IV 2008 data	Value	stad. dev.	t-statistic	p-value
$\widehat{\alpha}_1^*$	0.039114	0.000312	125.4	0.0000
$\widehat{\alpha}_2^*$	0.038009	0.002607	14.6	0.0000
$\widehat{lnJ^*}$	0.037270	0.004328	8.6	0.0000

Since the estimates of $\widehat{\alpha}_1^*$ and $\widehat{\alpha}_2^*$ derived from the last sub-samples are very similar, we run another regression assuming they are the same:

$$\ln(F(T,t)) - X_t = \alpha^*(T-t) + d \ln J^* + \varepsilon_t.$$

The results are as shown in Table A3. We are going to use these estimates for the valuation of investments that enhance energy efficiency over two carbon trading periods. As for volatility, σ_C $(= \sigma_{C1} = \sigma_{C2})$ is assumed to be the same as within a given commitment period; thus $\sigma_C = 0.4393$. It is not obvious to us why or how it should differ from it.

Table A3. OLS estimation of parameters in carbon prices assuming equal drifts.				
III-IV 2008 data	Value	std. dev.	t-statistic	p-value
$\widehat{\alpha}_1^* = \widehat{\alpha}_2^*$	0.039098	0.000310	126.3	0.0000
$\widehat{lnJ^*}$	0.035701	0.002199	16.2	0.0000

9.2 The process for natural gas price

Figure 4 shows both the spot price and the Dec-12 futures price (source: EEX, Leipzig). As shown in equation (39), we need estimates of G_m, k_G, λ_G, σ_G; for discounting purposes we will also need a value of r. We derive the parameters sequentially. In particular, G_m, k_G, and σ_G are estimated with data from the physical world, whereas λ_G is derived from futures market data.

Physical world. The stochastic process in the actual world, assuming $\theta = 0$ and $\lambda_G = 0$, can be expressed as:

$$G_{t+\Delta t} \simeq G_t e^{-k_G \Delta t} + G_m(1 - e^{-k_G \Delta t}) + \sigma_G \sqrt{\Delta t} G_t \epsilon_t \Leftrightarrow \tag{80}$$

$$\Leftrightarrow \frac{G_{t+\Delta t}}{G_t} \simeq e^{-k_G \Delta t} + \frac{G_m}{G_t}(1 - e^{-k_G \Delta t}) + \sigma_G \sqrt{\Delta t} \epsilon_t. \tag{81}$$

Thus, the econometric model to estimate is:

$Z_t = \beta_1 + \beta_2 X_{2t} + u_t$, where $\beta_1 = e^{-k_G \Delta t}$, $\beta_2 = G_m(1 - e^{-k_G \Delta t})$, $X_{2t} \equiv 1/G_t$. (82)

Our sample consists of EEX natural gas contracts from October-1-2007 to December-30-2008. The estimates appear in Table A4. Since we use daily data we have $\Delta t = 1/250$.

Table A4. OLS estimation of underlying parameters in gas price.				
Coefficient	Estimate	Std. dev.	t-statistic	p-value
$\widehat{\beta}_1$	0.9231	0.0202	45.7562	0.0000
$\widehat{\beta}_2$	1.9242	0.4905	3.9226	0.0001

Therefore: $k_G = -ln(\widehat{\beta}_1)/\Delta t = 20.01$ and $G_m = \widehat{\beta}_2/(1 - \widehat{\beta}_1) = 25.014$.[24] The estimate for volatility, as computed from the regression residuals, is $\sigma_G = 0.6742$.

Risk-neutral world. In the futures market, for contracts with delivery over a year, from equation (44) we have (with $\theta = 0$):

$$F_{\tau_1,\tau_1+1} = G_m^* + \frac{G_m^* - G_0}{k_G}[e^{-k_G(\tau_1+1)} - e^{-k_G\tau_1}]. \tag{83}$$

As shown above, with $k_G = 20.01$ we have $[e^{-k_G(\tau_1+1)} - e^{-k_G\tau_1}] \simeq 0$. Therefore,

$$F_{\tau_1,\tau_1+1} \simeq G_m^* \equiv (G_m - \frac{\lambda_G}{k_G}),$$

which can be rewritten as:

$$\lambda_G \simeq k_G(G_m - F_{\tau_1,\tau_1+1}). \tag{84}$$

The availability of several futures prices on every day allows, in principle, to compute a daily average for each contract. From the whole sample with all the contracts we get an average of $\widehat{\lambda}_G = -96.5439$. This suggests that the futures price has been well above the long-run equilibrium level G_m over the period considered.[25] See Figure 5.

This short-term behavior can be grasped from the evolution of spot prices, whose average would be very close to G_m (as estimated from spot prices), and the evolution of futures prices, for instance the contract deliverable in 2014.

In the sections below we use the estimates in Table A5. The value of λ_G corresponds to 12/30/2008, the last day of the series; also, the price of natural gas is 24.40 €/MWh (= G_0 in our computations below). Last, regarding the risk-free rate, r, we adopt an *ad hoc* value assumed to apply for the long term.

[24] The estimated value for G_m is very close to the sample average 24.83. This is to be expected when, as in our case, the reversion speed is strong. In the physical world the long-term value G_m must be very close to the sample average, and the other way round.

[25] On average $\widehat{\lambda}_G/\widehat{k}_G = -4.8247$. Since this appears subtracting in equation (83) for the futures price, it amounts to $+4.8247$.

Table A5. Parameter values of the natural gas price.				
G_m	k_G	λ_G	σ_G	r
25.0146	20.0103	13.97	0.6742	0.045