CAPITAL STRUCTURE, LIQUIDITY AND TRANSFERABLE HUMAN CAPITAL IN COMPETITIVE EQUILIBRIUM

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Abstract

This paper analyzes how human capital and economic uncertainty affect capital structure and managerial compensation. We model a competitive industry where wealth constrained managers provide human capital that can be transferred across firms, and where equityholders give managers access to the physical assets of the firm. Equityholders and managers bargain for the firm's stochastic free cash flows. We show that the level of net debt acts as a tool to attract and retain human capital. Negative net debt occurs in volatile and human capital intensive industries. Cash holdings (or unused lines of credit) in booms serve as a costly hedge against liquidity shocks in recession. The cost of holding cash is internalized by managers, unlike the cost associated with raising cash in recession through a dilutive equity issue. We obtain simple expressions for the equilibrium payout rate and the managerial compensation rate and we show how, in recessions, they are influenced by each party's outside option.

Keywords: capital structure, liquidity policy, agency, human capital (JEL: G31, G32)

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1 Introduction

How does the presence (or relative absence) of human capital in a firm affect corporate capital structure? This is the central question of this paper. The question is particularly relevant given the changing role of human capital over the past decades. First, the relative importance of human capital in the economy has increased over time. The rise of the high-tech, bio-tech, health, media, services and knowledge based industries has shifted the emphasis towards human capital, away from physical capital. Furthermore, increased efficiency (e.g. in the supply chain) has reduced the relative importance of physical capital (e.g. inventory) even for more traditional industries such as manufacturing. Second, the nature of human capital itself has changed over time. The proportion of the work force that has a university degree has increased over the past decades, and so has the amount of money that individuals invest in their human capital in terms of education and training. These investments only make sense if the owners of human capital can expect to get a higher compensation ex post. Third, human capital has become much more transferable and mobile. Human capital is much less tied to a particular firm and has, in a globalized world, also become more mobile in a geographical sense.

Many firms now consider human capital as their most important "asset". Yet, human capital is not recorded as an asset on the firm's balance sheet.[1] Furthermore, providers of human capital are through their personal investment in human capital (often paid for by personal loans) indirectly financing firms. "Knowledge workers" are therefore rightly considered to be the new capitalists (Economist (2001)). However, while they clearly have a stake in the firm, they do not feature in the firm's liabilities, unlike bond- and equityholders.[2] Furthermore, unlike physical capital, human capital cannot serve as collateral for debt financing. The mobility of human capital across firms and industries makes it difficult even to assign human capital to one specific firm. Human capital has in many way blurred the boundaries of the firm. Zingales (2000) argues that "the nature of the firm is changing", that "human capital is emerging as the most crucial asset", and that "existing corporate finance theories seem to be quite ineffective in helping us cope with the new type of firm that is emerging". The case of the British advertising agency Saatchi and Saatchi, described in Rajan and Zingales (2000) provides a stark illustration of the issues raised.[3]

[1] One exception are certain types of sport clubs that buy and sell players.

[2] While the providers of human capital may be given shares in the firm, they are clearly different from the outside equity investor.

[3] After their generous compensation package was voted down by shareholders, the chairman and several senior executives of Saatchi and Saatchi left the firm and started a rival firm that in a short time captured a

Given the special nature of human capital, in what way would one expect the capital structure and asset structure of human capital intensive firms to differ from firms that are more heavily based on physical capital? How does an industry's human capital intensity affect equilibrium profit rates, dividend payout and managerial compensation? This paper presents a theory that addresses these questions. We consider a tax-free, competitive industry where demand switches back and forth between a high ("boom") and low ("recession") state according to a Poisson arrival process. The start of each recession coincides with an adverse liquidity shock that affects all firms in the industry. Firms need both physical capital (e.g. working capital, plant and equipment) and human capital. Upon entry, firms invest in physical capital, part of which can be recovered if the firm is subsequently closed down. Managers need to make a one-off sunk investment in industry specific human capital prior to entering the industry for the first time. The investment in human capital is completely sunk, but is transferable across firms within the industry. Managers have the option to leave the industry and, subsequently, to return. Outside the industry, managers earn a reservation wage that varies according to the state of the economy. Our model contains two types of economic uncertainty: industry demand shocks and liquidity shocks. Both shocks are perfectly correlated and systematic in the sense that they affect all firms in the industry at the same time.

We first consider the case of an industry in which each firm is run by an owner-manager who makes the required investment in both physical capital and human capital. Firms enter in booms, and some firms leave in recession. The equilibrium profit rate in booms and recessions equals the sum of the efficient compensation rate for physical capital and human capital, for that particular state of nature. We obtain simple, intuitive expressions that show how the equilibrium profits depend on the opportunity cost of physical and human capital, sunk costs and economic uncertainty. Fewer firms leave if demand volatility and sunk costs are high, with no firms leaving the industry in extreme cases.

If physical capital and human capital are owned by the same person(s), then there is no role for corporate debt. There may, however, be a role for holding cash (or a line of credit) as a hedge against a future liquidity shock. It is optimal for firms to hold cash if the expected cost of holding cash during booms in terms of interest lost is lower than the expected cost of raising funds at short notice in recession. Interestingly, costly cash holdings create slack for firms in recession: firms that meet the liquidity shock through cash reserves ex post strictly prefer staying to leaving the industry in recession. The presence of slack has important implications when debt is introduced (see below).

substantial part of the business of the original firm.

Next, we consider the case where ownership of equity and human capital are separated. The capital investment is financed by equityholders and bondholders, whereas managers make the prior investment in human capital. Both parties have an outside option. Managers can always earn a reservation wage outside the industry, whereas equityholders can liquidate the firm. While the firm is operational, free cash flows (i.e. profits after interest repayment) are shared between equityholders and managers according to a strategic bargaining model. In particular we adopt a continuous-time version of Rubinstein (1982) that incorporates each parties' outside options and allows for stochastic free cash flows.

With separation of equity and human capital, debt is no longer irrelevant but now acts as a "balancing" variable. Higher debt levels benefit equityholders because for every dollar of debt raised, equityholders need to contribute one dollar less of their own money, while at the same time the disciplining effect of increased interest repayments is shared with managers. A higher debt level obviously harms managers as it reduces the free cash flows to be shared. In a competitive industry, the optimal debt level ensures that the demand for human capital by firms equals the supply of human capital. In equilibrium, the optimal debt level decreases (increases) with the cost of investment in human (physical) capital and can become negative for human capital intensive industries. Negative debt creates the mirror effect of standard debt: for every extra dollar of negative debt, equityholders put up a full dollar worth of high yield liquid assets, but they only capture a fraction of the interest that these assets subsequently generate.

Competition ensures that the first-best industry output level is achieved for as long as the optimal debt level remains below the firm's liquidation value (i.e. the debt is fully secured). In this case both equityholders and managers get the efficient compensation rate in booms and recessions. Inefficiencies arise when the firm requires a lot of physical capital and relatively little human capital. In that case firms would like to put in place a high debt level that is not fully secured by the firm's assets in liquidation in order to reduce free cash flows. However, risky debt brings with it two sources of inefficiency. First, there is the standard Myers (1977) debt overhang problem, which causes equityholders to leave the firm too soon, and therefore creates underinvestment. Second, there can also be deadweight bankruptcy costs that are borne by bondholders in liquidation. These two costs deteriorate the terms at which equityholders can raise debt financing for the firm. As a result, firms may decide instead to cap the debt level to the value of the firm in liquidation, so as to keep the debt safe. The cost of "under-leverage" is that managers can extract more than their fair share of rents, which leads to underinvestment in booms. If the investment in human capital is sufficiently large then the cost of under-leverage is smaller than the costs associated with risky debt. If, however, managers contribute very little human capital then the cost of under-leverage is larger than the cost associated with risky debt, and firms therefore adopt a debt level that exceeds the liquidation value of the firm. Even though all firms are ex ante identical, under-collateralized debt introduces heterogeneity in firms' capital structure. Some firms (i.e. the second movers into the industry) adopt a higher debt principal, have a higher market leverage and default in recession. Other firms (i.e. the first movers) adopt a lower debt level, survive in recession and therefore avoid bankruptcy costs. But, lower leverage allows managers to capture excess rents in booms. These excess rents are partially clawed back in recession when managers of under-levered firms accept a salary rate that is *below* their outside reservation wage. These managers are willing to stick with the firm in recession, because they know that they will enjoy again excess rents once the economy reverts to a boom.

With undercollateralized debt, firms that enter the industry first exploit their first movers' advantage by adopting a lower debt level that allows them to survive in recession. The equity market capitalization of these firms is higher, their managers enjoy excess rents and equity-holders break even. Second movers adopt a higher leverage and default in recession. Their equityholders and managers break even. The expected bankruptcy costs lead to underinvestment in booms. The managerial excess rents in booms enjoyed by first movers increase with the level of bankruptcy costs and the resulting degree of underinvestment in booms. In recession we can observe underinvestment as well as overinvestment when debt is risky. Possible underinvestment results from the Myers (1977) debt overhang effect. Overinvestment results from the fact that the salary rate of surviving managers can be cut in recession below the reservation wage. This cut provides space for more firms to survive. The salary cut is proportional to the amount of excess rents that managers enjoy in booms, and is therefore increasing in the level of bankruptcy costs. Higher bankruptcy costs therefore create an overinvestment effect in recession. If investment in human capital is very low (compared to investment in physical capital) then debt levels are very high, and as a result the debt overhang effect dominates. For somewhat lower (but still risky) debt levels the bankruptcy effect dominates, and we have overinvestment in recession. For very high bankruptcy cost levels we can end up in a situation where few firms enter the industry and few (or even no) firms leave the industry in recession. Bankruptcy costs therefore increase inertia and have a similar effect as sunk costs.

Cash holdings in booms create slack for equityholders in recessions. This slack provides space for debt levels to be raised. We show that in equilibrium cash holdings are at least partly financed by debt. When managers' participation constraint is binding, then an extra dollar of cash increases debt by less than a dollar. In cases where managers enjoy excess rents, cash holdings lead to an increase in *net* debt: the debt level is raised not only by the amount of cash, but also by an additional amount that is proportional to the expected cost of holding cash. This increase in net debt reduces free cash flow and forces equityholders and managers to internalize the cost of hedging against the liquidity shock. Cash holdings can therefore discipline managers. Competitive pressure forces firms to hold cash, if it is optimal to do so. If, however, the cost of holding cash is too high then firms raise the funds in recession through a dilutive equity issue. Equity dilution imposes the cost of raising funds in recession on existing shareholders and – unlike cash holdings – does not have a disciplining effect on managers. All else equal, separation of human capital and equity capital creates a bias in favor of firms holding cash. Compared to equity issues, cash holdings can have a smoothing effect because they tend to decrease (increase) managers' and equityholders' payout in booms (recessions).

Our theory makes a formal distinction between high yield liquid assets and cash. The former are held on a long term basis to provide a stable income stream that can be shared between equityholders and managers, whereas the latter is held on a short term basis to hedge against an adverse liquidity shock. As the return shortfall on cash goes zero, cash becomes indistinguishable from negative debt, and an increase (decrease) in cash holdings is equivalent to a decrease (increase) in debt. We show that a (costly) cash holding is equivalent to a non-revokable line of credit that charges a fee rate proportional to the size of the credit line.

In the existing literature only a few papers analyze the role of human capital for capital structure. Hart and Moore (1994) consider an entrepreneur who needs to raise finance from an investor, but cannot commit not to withdraw his human capital from the project. They show that the threat of repudiation means that some profitable projects will not be financed. This type of underinvestment does not occur in our model because the availability of profitable projects would induce more individuals to invest in human capital and offer their services. The transferable nature of human capital in our model is a double-edged sword for managers: while it allows managers to withdraw and transfer their human capital elsewhere, it also means that other people can be called in to fill their seat. Competition between managers therefore restores efficiency in our model, but underinvestment in booms can result from other frictions such as anticipated bankruptcy costs and wealth constraints. In order to avoid bankruptcy costs, firms may underlever, allowing managers to capture excess rents, with underinvestment as a result. This type of inefficiency cannot be competed away because managers cannot commit not to capture less free cash flows than is efficient. Wealth constraints also makes it impossible for managers to co-invest by making an upfront payment as a compensation for all the excess rents they will capture in future. Jaggia and Thakor (1994) study the link between capital structure and investment in firm-specific human capital (or relation-specific capital, more generally). High leverage increases the likelihood of firms going bankrupt and employees losing their job. High leverage may therefore undermine employees' incentives and propensity to invest in firm-specific human capital, resulting in a loss of efficiency that can be traded off against a debt tax shield. Human capital is not firm-specific in our model. Qian (2003) studies the incentives of non-management personnel and shows that the feasible set of renegotiation-proof contracts is reduced as debt rises. Since debt leads to inferior risk-sharing, the optimal debt level decreases with the human capital intensity of the firm. Our paper does not follow a contractual approach. Instead our equilibrium sharing rule between equityholders and managers is supported by a self-enforcing "implicit contract". Furthermore, competition between firms ensures that the equilibrium does not depend on whether managers or equityholders set debt policy. Finally, Berk, Stanton, and Zechner (2007) identify the negative effect of bankruptcy on the welfare of workers as the key component of indirect bankruptcy costs. To compensate risk-averse workers for a higher risk of bankruptcy, highly levered firms offer higher compensation.

A related strand of papers analyzes how capital structure affects managerial compensation and employees' wages. The idea that capital structure can affect implicit contracts was put forward by Titman (1984) who showed that appropriate selection of capital structure assures that incentives between the firm and its stakeholders (such as workers, customers and suppliers) are aligned so that the firm implements the ex-ante value maximizing liquidation policy. Jensen (1986) argues that debt disciplines managers and reduces the agency costs of free cash flows. In Perotti and Spier (1993) shareholders issue junior claims (debt) to obtain concessions from more senior claimants (workers). In this case, new debt allows the shareholders to capture the entire surplus from a new investment project. As workers (weakly) benefit from shareholders undertaking the investment, they are willing to accept reduction in wages despite their claim being senior to debt. In our model, manager's claim is junior to debt so by increasing the latter the size of the object of bargaining (the "pie") is reduced. The effect of debt on the distribution of surplus in bargaining is central in the analysis by Dasgupta and Sengupta (1993). On the one hand, debt provides a bargaining advantage to equityholders when negotiating the level of wages with workers. On the other, it distorts incentives to exert optimal level of effort when the probability of bankruptcy is strictly positive. The optimal level of debt reflects the trade-off between the two effects. Hennessy and Livdan (2007) examine optimal leverage for a downstream firm relying on implicit contracts with a supplier. The optimal leverage level reflects the trade-off between the advantage of debt when bargaining with the agent and the efficiency cost of debt following from a restricted set of payments that the principal can make to the agent. In Lambrecht and Myers (2008) managers maximize the present value of their future rents subject to a threat of collective action by outside shareholders. Absent bankruptcy costs and managerial wealth constraints managers follow an optimal debt policy that generates efficient investment and disinvestment. Unlike Lambrecht and Myers (2008), in our model outside investors cannot discipline managers by taking over the firm because investors rely on managers' human capital to run the firm.[4]

The structure of the paper is as follows. In section 2 we derive the first-best investment policy in a competitive industry equilibrium where firms are run by owner-managers. Firms are subject to demand shocks. Section 3 develops a strategic bargaining model that determines how a firm's free cash flows are shared between equityholders and managers when there is a separation between equity capital and human capital. Section 4 analyzes the investment policy in a competitive industry equilibrium with separation of equity capital and human capital. We derive closed form solutions for the optimal debt policy, payout policy and managerial compensation, and we discuss efficiency implications. Section 5 introduces cash holdings and adverse liquidity shocks, and analyzes how they alter the results obtained in previous sections. Sections 6 and 7 present the paper's empirical implications and conclusions, respectively.

2 Competitive industry equilibrium with owner-managers

Consider a competitive industry populated with atomistic firms. Let Q denote the total industry output. At any moment in time the industry can be in one of the following two states: boom or recession. When the industry is in the boom (recession), recession (boom) arrives according to a Poisson process with parameter $\overline{\lambda}$ ($\underline{\lambda}$). Each firm's profits in booms and recessions are given by $\overline{\pi}(\overline{Q})$ and $\underline{\pi}(\underline{Q})$, respectively, where $\overline{Q}$ ($\underline{Q}$) denotes the industry output in booms (recessions). For a given industry output level Q, firms enjoy higher profits in booms than in recessions, i.e. $\overline{\pi}(Q) > \underline{\pi}(Q)$. Furthermore, firm profits are decreasing in the total industry output ($\overline{\pi}'(Q) < 0$ and $\underline{\pi}'(Q) < 0$). For an analytical convenience, we assume that $\overline{\pi}(0) = \underline{\pi}(0) = \infty$ and that $\forall Q : \overline{\pi}'(Q) < \underline{\pi}'(Q)$.

Firms need both physical capital *and* human capital to be operational. Investment in the required physical capital costs a fixed, exogenous amount I. The stock of capital can at any time be liquidated for a constant amount L, should the firm wish to leave the industry. In

[4] Our paper also overlaps with the literature that models corporate liquidity policy. Recent papers include Hennessy and Whited (2005), Anderson and Carverhill (2007) and Acharya, Almeida, and Campello (2007), among others. The objective of our paper is, however, not to explain why firms hold cash, or how much cash firms should have. Instead, we take the need for cash as given and examine how the presence of cash in the firm affects debt policy, managerial compensation and economic efficiency.

order to rule out the existence of a money machine we assume that $I \geq L$. Individuals who want to manage a firm have to invest a fixed, exogenous amount H in human capital *before* they enter the industry. Importantly, this investment, while sunk, only needs to be made once. In other words, should a manager leave the firm and join another firm, then there is no need for her to incur the investment in human capital again. Human capital is therefore perfectly transferable in our model. However, unlike physical capital, human capital cannot be sold or liquidated. The cost H can in our model be thought of as investment in time, education, training, knowledge, networking and experience necessary for running the firm.[5]

A key difference of our model with the existing literature is that human capital cannot be acquired in a gradual way through on-the-job training (see Becker (1962) and the substantial literature in labor economics that resulted from this seminal paper).[6] Instead, all required investment in human capital happens upfront. If human capital were firm specific this would leave managers exposed to a severe hold-up problem, and undermine their incentive to invest in human capital. Transferability of human capital is therefore a key ingredient of our model. We assume for the moment that human capital and physical capital are owned by the same person or group and relax this assumption from section 3 onwards.

Human capital is homogenous in that all managers have the same profit generating potential. Each firm requires exactly the same amount H of human capital to be operational. Physical capital and human capital cannot be substituted for one another. If a manager fails to be hired by a firm, or if she loses her job then she can fall back on her opportunity wage. The manager's opportunity wage rate during booms and recessions is $\overline{w}$ and $\underline{w}$, respectively. We assume that this is the same opportunity wage that was available to the manager prior to her incurring the cost H.[7] The basic model framework is summarized in figure 1.

[5]In this paper human capital refers in the first instance to the manager – or a team of managers – who runs the firm. The model is, however, applicable to a broader interpretation of human capital that refers to any collection of individuals within the firm that have transferable skills and that can bargain as one cohesive entity (e.g. through a union) with the equityholders.

[6]In these models investment in human capital is incurred gradually, which makes it possible for the cost to be charged against the employee's wage while the training period lasts.

[7]To avoid a perverse situation where managers want to enter in recession and leave in booms, we make the reasonable assumption that managers' reservation wage is higher in booms than in recessions (i.e. $\overline{w} \geq \underline{w}$). One could also easily allow the manager's opportunity wage to be different before and after the investment in H, but that would not generate any new insights.

2.1 Some building blocks for valuing claims

Before we solve for a global optimizer's investment policy in industry equilibrium, we introduce the following proposition that provides the main building blocks for our future valuation problems (proofs are given in the appendix).

Proposition 1 *(a) The value of a claim that pays 1 dollar the first time when the economy switches from a boom (recession) to a recession (boom) equals* $\overline{\delta} \equiv \frac{\overline{\lambda}}{r+\overline{\lambda}}$ $(\underline{\delta} \equiv \frac{\underline{\lambda}}{r+\underline{\lambda}})$
(b) The value of a claim that pays a cash-flow rate $\overline{\pi}$ $(\underline{\pi})$ *for as long as the current boom (recession) lasts, and nothing thereafter, equals* $\frac{\overline{\pi}}{r+\overline{\lambda}}$ $(\frac{\underline{\pi}}{r+\underline{\lambda}})$.
(c) The value of a perpetual claim that pays a cash flow $\overline{\pi}$ *during booms and* $\underline{\pi}$ *during recessions equals:*

$$\begin{aligned} \overline{V}_s(\overline{Q},\underline{Q}) &\equiv \tfrac{\overline{\pi}(\overline{Q})}{r}(1-\overline{p}) + \tfrac{\underline{\pi}(\underline{Q})}{r}\overline{p} \quad \textit{when currently in a boom} \\ \underline{V}_s(\overline{Q},\underline{Q}) &\equiv \tfrac{\underline{\pi}(\underline{Q})}{r}(1-\underline{p}) + \tfrac{\overline{\pi}(\overline{Q})}{r}\underline{p} \quad \textit{when currently in a recession} \end{aligned} \tag{1}$$

where $\overline{p} \equiv \frac{\overline{\lambda}}{r+\overline{\lambda}+\underline{\lambda}}$ *and* $\underline{p} \equiv \frac{\underline{\lambda}}{r+\overline{\lambda}+\underline{\lambda}}$

The valuation formulas for $\overline{V}_s$ and $\underline{V}_s$ have simple, intuitive interpretations. For example, $\overline{V}_s$ is a weighted average of the perpetuities $\frac{\overline{\pi}}{r}$ and $\frac{\underline{\pi}}{r}$, where the former (latter) perpetuity denotes the present value of receiving the profit flow $\overline{\pi}$ $(\underline{\pi})$ forever.[8] The weights are given by $(1-\overline{p})$ and $\overline{p}$, where $0 \le \overline{p} = \frac{\overline{\lambda}}{r+\overline{\lambda}+\underline{\lambda}} \le 1$. If the likelihood of a switch to a recession is zero ($\overline{\lambda} = 0$ and hence $\overline{p} = 0$) then $\overline{V}_s$ simply equals $\frac{\overline{\pi}}{r}$. As the hazard of switching from a boom to a recession becomes extremely large compared to the hazard of switching from a recession to a boom, the firm value $\overline{V}_s$ converges to $\frac{\underline{\pi}}{r}$.

2.2 First-best investment policy

We now study entry and exit decisions in a competitive industry where each firm is run by an owner-manager who provides both the required physical capital (I) and human capital (H). The opportunity cost to each investor of not investing in human capital is the opportunity wage rate that could be earned outside the industry. Should a potential owner-manager decide never to invest then statement (3) in proposition 1 implies that the value of her claim in booms, $\overline{W}$, and in recessions, $\underline{W}$, is given respectively by:

$$\overline{W} \;=\; \frac{\overline{w}}{r}(1-\overline{p}) + \frac{\underline{w}}{r}\overline{p} \;\text{ and }\; \underline{W} \;=\; \frac{\underline{w}}{r}(1-\underline{p}) + \frac{\overline{w}}{r}\underline{p}$$

[8] We drop the argument of $\overline{\pi}$ $(\underline{\pi})$ if doing so does not introduce an ambiguity.

Consider next an owner-manager who operates in the industry during booms, but leaves the industry during recessions. In that case, she incurs a one-off investment cost H in human capital. She pays I at the start of each boom and receives cash flows at a rate $\overline{\pi}$ during each boom. She also receives the liquidation value L at the start of each recession and her opportunity wage rate $\underline{w}$ during recessions. The present value of all cash flows generated by this investor (in perpetuity) equals:

$$PV \;=\; \Sigma_{j=0}^{\infty} \left[\frac{\overline{\pi}(\overline{Q})}{r\,+\,\overline{\lambda}} \,-\, I \,+\, \overline{\delta}\left(L \,+\, \frac{\underline{w}}{r+\underline{\lambda}}\right)\right] \left(\overline{\delta}\,\underline{\delta}\right)^{j} \;-\; H \tag{2}$$

$$=\; \frac{\frac{\overline{\pi}(\overline{Q})}{r+\overline{\lambda}} \,-\, I \,+\, \overline{\delta}\left(L \,+\, \frac{\underline{w}}{r+\underline{\lambda}}\right)}{1 \,-\, \overline{\delta}\underline{\delta}} \;-\; H \tag{3}$$

where $\overline{Q}$ denotes the industry output during booms and where $\overline{\delta} \,=\, \frac{\overline{\lambda}}{r+\overline{\lambda}}$ and $\underline{\delta} \,=\, \frac{\underline{\lambda}}{r+\underline{\lambda}}$ are discount factors previously defined in proposition 1. Entry is preferable to no entry if $PV \geq \overline{W}$. In a market with competitive entry, firms break even in equilibrium (i.e. $PV \,-\, \overline{W} \,=\, 0$). Consequently, the equilibrium profits in booms must satisfy the following condition:

$$\overline{\pi}(\overline{Q}) \;=\; r\,I \,+\, \overline{\lambda}\,(I \,-\, L) \,+\, \overline{w} \,+\, \frac{r\,H}{1-\overline{p}} \tag{4}$$

We know that in competitive equilibrium the value $\overline{V}_l$ of a firm that leaves in recession is given by $\overline{V}_l \;=\; \frac{\overline{\pi}}{r+\overline{\lambda}} \,+\, \underline{\delta V}_l \;=\; I + \overline{W} + H$.[9] Solving for $\underline{V}_l$ gives:

$$\underline{V}_l \;=\; L \,+\, \underline{W} \,+\, \underline{\delta}\,H \tag{5}$$

The equilibrium profits in recession are determined by the industry output during recessions ($\underline{Q}$). If some exit is optimal when the industry switches from a boom to a recession ($\overline{Q} \,>\, \underline{Q}$), then firms keep leaving the market till in equilibrium their owners are indifferent between staying in the market and leaving. On the other hand, it could be that no firms leave the market ($\overline{Q} \,=\, \underline{Q}$). This happens if at the existing output level $\overline{Q}$ all firms are strictly better off staying than leaving.

Consider first the case where some firms leave the market ($\overline{Q} \,>\, \underline{Q}$). Assuming we are in a recession, then the present value, $\overline{V}_s$, of all profits generated by staying forever in a competitive market is given by:

$$\underline{V}_s(\overline{Q},\underline{Q}) \;=\; \frac{\underline{\pi}(\underline{Q})}{r}\,(1-\underline{p}) \,+\, \frac{\overline{\pi}(\overline{Q})}{r}\,\underline{p} \;=\; L \,+\, \underline{W} \,+\, \underline{\delta}\,H \;=\; \underline{V}_l \tag{6}$$

Solving for the equilibrium profits $\underline{\pi}(\underline{Q})$ gives:

$$\underline{\pi}(\underline{Q}) \;=\; r\,L \,-\, \underline{\lambda}\,(I \,-\, L) \,+\, \underline{w} \tag{7}$$

[9]The subscripts 'l' and 's' refer respectively to firms that leave and survive in recession.

For given profit functions $\overline{\pi}(Q)$ and $\underline{\pi}(Q)$), the above equilibrium conditions yield $\overline{Q}$ and $\underline{Q}$. One can verify that $\overline{V}_s = I + \overline{W} + H$.

Proceeding next to the case where no firm leaves during recessions ($\overline{Q} = \underline{Q} = \tilde{Q}$), competitive entry implies that the value $\overline{V}_s$ obtained from entry equals the sum of all investment costs (I, H) and opportunity costs ($\overline{W}$):

$$\overline{V}_s(\tilde{Q}, \tilde{Q}) = \frac{\overline{\pi}(\tilde{Q})}{r}(1 - \overline{p}) + \frac{\underline{\pi}(\tilde{Q})}{r}\overline{p} = I + H + \overline{W} \tag{8}$$

Furthermore, during recessions the present value of staying in the industry exceeds the present value of leaving:

$$\underline{V}_s(\tilde{Q}, \tilde{Q}) = \frac{\underline{\pi}(\tilde{Q})}{r}(1 - \underline{p}) + \frac{\overline{\pi}(\tilde{Q})}{r}\underline{p} \geq L + \underline{W} + \underline{\delta} H \tag{9}$$

Consequently,

$$\overline{V}_s - \underline{V}_s = \frac{\left(\overline{\pi}(\tilde{Q}) - \underline{\pi}(\tilde{Q})\right)}{r}\left(1 - \overline{p} - \underline{p}\right) \leq I - L + \left(\overline{W} + H - \underline{W} - \underline{\delta} H\right) \tag{10}$$

After rearranging and simplifying, we obtain:

$$\overline{\pi}(\tilde{Q}) - \underline{\pi}(\tilde{Q}) \leq (I - L)\left(r + \overline{\lambda} + \underline{\lambda}\right) + \overline{w} + \frac{r\,H}{1 - \overline{p}} - \underline{w} \equiv \Delta(I, L, H) \tag{11}$$

In the remainder of the paper we focus on the case in which some firms leave in recession.[10] We therefore adopt the following assumption throughout the paper:

Assumption 1 *Demand shocks are sufficiently high such that some firms leave in recession, i.e.:*

$$\overline{\pi}(\tilde{Q}) - \underline{\pi}(\tilde{Q}) > \Delta(I, L, H) \tag{12}$$

where $\tilde{Q}$ is the solution to $\overline{V}_s(\tilde{Q}, \tilde{Q}) = I + H + \overline{W}$.

How big demand shocks have to be depends on the other model parameters, such as the sunk cost of investment in human (H) and physical ($I - L$) capital. Higher sunk costs discourage exit and therefore need to be accompanied by relatively higher demand shocks for exit to occur. One can show, for example, that there exists a critical threshold $\tilde{H}$ such that exit occurs for $H < \tilde{H}$ and no exit occurs for $H \geq \tilde{H}$ (holding all else constant).

The results can be summarized in the following proposition:

[10] The interested reader can find the treatment of the no-exit case in the supplementary appendix 2.

Proposition 2 *The first-best industry output in booms ($\overline{Q}$) and recessions ($\underline{Q}$) are the solution to the following equations:*

$$\begin{aligned} \overline{\pi}(\overline{Q}) &= \Big[r\,I + \overline{\lambda}\,(I\, -\, L)\Big] + \left[\overline{w} + \frac{r\,H}{1-\overline{p}}\right] \equiv \overline{\pi}^o(L) && (13)\\ &= \text{charge for physical capital} \;+\; \text{charge for human capital} \\ \underline{\pi}(\underline{Q}) &= [r\,L\, -\, \underline{\lambda}\,(I\, -\, L)] + \underline{w} \equiv \underline{\pi}^o(L) && (14)\\ &= \text{charge for physical capital} \;+\; \text{charge for human capital}\end{aligned}$$

The value of survivors and leavers in respectively booms and recessions are:

$$\overline{V}_s(\overline{Q},\underline{Q}) \;=\; V_l(\overline{Q},\underline{Q}) \;=\; I + H + \overline{W} \quad \text{and} \quad \underline{V}_s(\overline{Q},\underline{Q}) \;=\; L + \underline{W} \,+\, \underline{\delta}\,H \qquad (15)$$

The proposition gives simple, intuitive expressions for the equilibrium profit rates in booms and recessions. The equilibrium profits can be decomposed in a charge for physical capital and a charge for human capital. During booms the charge for physical capital equals the opportunity cost of the capital invested ($r\,I$) plus a risk premium for the hazard of recession ($\overline{\lambda}\,(I\, -\, L)$). The risk premium equals the difference between the capital investment cost and the value of the physical assets in liquidation ($I - L$) multiplied by the hazard rate $\overline{\lambda}$ of a recession occurring. Conversely, during recessions the charge for physical capital equals the opportunity cost of liquidating the firm ($r\,L$) minus a discount for the hazard of economic recovery.

During booms the charge for human capital consists of the opportunity wage $\overline{w}$ plus a charge equal to $\frac{rH}{1-\overline{p}} \;=\; r\,H\,\left(1\,+\,\frac{\overline{\lambda}}{r+\underline{\lambda}}\right)$ for the investment in human capital. In the limiting case where the industry stays in a boom forever ($\overline{\lambda} \;=\; 0$), the required rate of return on H is just the risk-free rate r. If the hazard rate of switching from a boom to a recession is strictly positive ($\overline{\lambda} \;>\; 0$) then the required rate of return increases by $\frac{\overline{\lambda}\,r}{r+\underline{\lambda}}$, which reflects the discounted value of forgoing $r\,H$ during recession ($\frac{r\,H}{r+\underline{\lambda}}$) times the hazard rate of a recession arriving ($\overline{\lambda}$) when the economy is in a boom. Since managers that lose their job in recession merely earn the opportunity wage $\underline{w}$, the longer (shorter) recessions (booms) are expected to last, the larger profits have to be during booms to recover the investment in human capital. Consider the limiting case where a recession is expected to last forever ($\underline{\lambda} = 0$), once arrived. In that case human capital becomes useless for those managers that leave the industry, and as a result the required rate of return on human capital becomes $H\left(r + \overline{\lambda}\right)$: the interest rate r is simply augmented by $\overline{\lambda}$, where $\overline{\lambda}$ can now be interpreted as a risk of 'ruin'.

3 A strategic bargaining model

In this section we develop a strategic bargaining model that determines how firm value (and therefore cash flows) is shared between managers and equityholders when there is separation between ownership of human capital and equity capital. We adopt the following assumption.

Assumption 2 *The firm needs both physical capital (owned by shareholders) and human capital (owned by wealth constrained managers) to be operational. No cash flows are generated while either party abstains. Each party can temporarily abstain whenever it wishes. Each party can permanently withdraw from the firm by exercising its outside option.*

Assumption 2 implies that physical capital and human capital cannot be supplied by the same person (or group of people). Equityholders are capable of providing the physical capital, but they are unable to run the firm profitably. Conversely, managers can provide the human capital, but are unable to put up the required physical capital because of wealth constraints or because of their inability to obtain access to the specific assets that are required (e.g. land and licenses).[11]

Either party can suspend operations. Managers can suspend operations by refusing to work, or by putting up passive resistance. Equityholders could suspend managers, deny them access to the physical capital in place, or simply stop paying managers. As a result, either party can at any time impose a stalemate. This stalemate is costly to *both* parties while it lasts because profits are being forgone. It is this mutual cost of a stalemate that forces both parties to negotiate an agreement.

Each party can permanently withdraw by exercising its outside option. The value of this outside option in booms (recessions) is denoted by $\overline{o}_e$ and $\overline{o}_m$ ($\underline{o}_e$ and $\underline{o}_m$) for equityholders and managers, respectively. Equityholders can liquidate the physical assets. Managers can leave the firm to receive their outside reservation wage. While managers cannot liquidate their human capital, they can re-enter the industry at some future point without having to incur the investment in human capital again. Managers' outside option to leave the industry in recession therefore embeds an option to return in booms.

Even though huge penalties may *stop* the manager from leaving the firm, penalties cannot *make* managers do a good job. Managers that are kept in the firm against their will can easily put up passive resistance (by doing a lousy job, taking sick leave, etc.) and as such damage

[11] Our assumption does not exclude the possibility of managers being awarded shares in the company. In that case managers would simply maximize a weighted average of their managerial claim and equity stake.

the firm. Faced with this prospect firms are often much better off letting the managers go. As a consequence, agreements between firms and managers essentially have to be sustained on a mutually voluntary basis. In this paper we focus therefore not on contracts, but on self-enforcing agreements and remuneration that, at each moment in time, are the outcome of bargaining between the equityholders and the managers (also known as 'implicit contracts'). Implicit contracts are quite common. Gillian, Hartzell, and Parrino (2009) find that less than half of the S&P 500 CEOs are employed under explicit agreements (agreements that specify the terms of the employment relationship) rather than implicit agreements.

We now develop a bargaining model through which equityholders and managers share the total pie $\overline{v}$ ($\underline{v}$) in booms (recessions). In order to ensure that both parties have an incentive to bargain with each other, the following constraint has to be satisfied in booms (recessions): $\overline{o}_e + \overline{o}_m \leq \overline{v}$ ($\underline{o}_e + \underline{o}_m \leq \underline{v}$). Since equityholders and managers leave no money on the table (there are no bargaining costs or other frictions) it has to be the case that $\overline{v} \equiv \overline{E} + \overline{M}$ and that $\underline{v} \equiv \underline{E} + \underline{M}$, where $\overline{E}$ ($\overline{M}$) and $\underline{E}$ ($\underline{M}$) denote equitholders' (managers') claim in booms and recessions, respectively. We show later that the sharing rule obtained for the firm value $\overline{v}$ and $\underline{v}$ maps into a sharing rule for the firm's cash flows $\overline{\pi}$ and $\underline{\pi}$. Note that v is in general not equal to the present value of gross profits (V), because there may be debt, in which case bondholders have a prior claim on the firm's cash flows. The game and its bargaining protocol are summarized in the following assumption.

Assumption 3 *During booms (recessions) equityholders and managers bargain how to share a pie with value $\overline{v}$ ($\underline{v}$). Both parties make offers in turn for as long as no outcome has been achieved. If equityholders (managers) make an offer then managers (equityholders) who receive the offer can take one of the following 3 decisions: (a) accept the offer, (b) reject the offer and make a counteroffer after a time lag τ has elapsed, or (c) reject the offer and exercise immediately the outside option. If a boom (recession) prevails at the end of a bargaining interval τ then each party receives at the end of that interval its claim value associated with a boom (recession), namely $\overline{E}$ and $\overline{M}$ ($\underline{E}$ and $\underline{M}$). The time lag τ between offers and counteroffers is arbitrarily small.*

In this paper we focus on the limiting case where bargaining can take place continuously (i.e. $\tau \to 0$). In that case the sharing rule does not depend on who makes the first offer (see Rubinstein (1982) and Shaked and Sutton (1984)). This makes the solution much more tractable. Furthermore, continuous bargaining is consistent with our assumption that the profits to be shared come as a stochastic flow variable in continuous time.

The following proposition states how during a boom the value $\overline{v}$ is shared between equityholders and managers (the proposition for the sharing of $\underline{v}$ in a recession is analogous).

Proposition 3 *Assume that managers share $\overline{v}$ according to the bargaining game described in assumption 3. As the time lag between offers and counteroffers becomes infinitesimally small ($\tau \to 0$), the equilibrium payoffs to equityholders ($\overline{E}$) and managers ($\overline{M}$) converge to:*

Case 1) *If $\overline{o}_e \leq \overline{o}_e{}^*$ and $\overline{o}_m \leq \overline{o}_m{}^*$ then $\overline{E} = \overline{o}_e{}^*$ and $\overline{M} = \overline{o}_m{}^*$.*

Case 2) *If $\overline{o}_e \geq \overline{o}_e{}^*$ and $\overline{o}_e + \overline{o}_m \leq \overline{v}$ then $\overline{E} = \overline{o}_e$ and $\overline{M} = \overline{v} - \overline{o}_e$*

Case 3) *If $\overline{o}_e + \overline{o}_m \leq \overline{v}$ and $\overline{o}_m \geq \overline{o}_m{}^*$ then $\overline{E} = \overline{v} - \overline{o}_m$ and $\overline{M} = \overline{o}_m$*

Case 4) *If $\overline{o}_e + \overline{o}_m = \overline{v}$ then $\overline{E} = \overline{o}_e$ and $\overline{M} = \overline{o}_m$*

$$\textit{where} \quad \overline{o}_e{}^* = \frac{\overline{v}}{2} + \overline{\delta}\left(\frac{\underline{E} - \underline{M}}{2}\right) \quad \textit{and} \quad \overline{o}_m{}^* = \frac{\overline{v}}{2} + \overline{\delta}\left(\frac{\underline{M} - \underline{E}}{2}\right)$$

The equilibrium payoffs in recession ($\underline{E}$ and $\underline{M}$) are defined analogously.

The proposition shows that the sharing rule depends on the value of the negotiating parties' outside options. We can distinguish 4 different cases depending on which parties' outside option 'binds': (1) neither party's outside option binds, (2) equityholders' outside option binds, (3) managers' outside option binds, or (4) both parties' outside options bind. Figure 2 illustrates the geometry of the regions that correspond to the above 4 cases.

Consider first case 1 for which both parties' outside options are so small (i.e. $\overline{o}_e \leq \overline{o}_e{}^*$, $\overline{o}_m \leq \overline{o}_m{}^*$) that neither party can credibly threaten to exercise its outside option. The equilibrium payoffs ($\overline{E}$ and $\overline{M}$) are therefore independent of $\overline{o}_e$ and $\overline{o}_m$. The equilibrium payoffs depend on the value of the pie to be shared ($\overline{v}$), the stochastic discount factor ($\overline{\delta}$) and the value of each party's claim in recession ($\underline{E}$, $\underline{M}$). The stochastic discount factor $\overline{\delta}$ not only depends on the discount rate (r) but also on the hazard ($\overline{\lambda}$) of a recession arriving in the next instant. In equilibrium each party gets half of the cake ($\frac{\overline{v}}{2}$) and half the present value of the difference between each party's claim value in recession (eg. $\overline{\delta}\left(\frac{\underline{E} - \underline{M}}{2}\right)$ for equityholders). The sharing rule is therefore an example of the "split-the-difference" rule in bargaining theory (see Shaked and Sutton (1984) and Binmore, Rubinstein, and Wolinsky (1986)).

The other polar case (case 4) covers the situation where the value of each party's outside options is so "large" that they add up exactly to the value of the pie to be shared. In that case each party gets exactly the value of its outside option. This case is represented in the figure by the diagonal straight line for which $\overline{o}_e + \overline{o}_m = \overline{v}$. Note that to the right of this line, each party exercises its outside option because an agreement between both parties is no longer feasible.

Finally, there are the intermediate cases (cases 2 and 3) where exercising the outside option is credible for only one of the two parties. The party whose outside option binds gets exactly its outside option value. The counterparty gets the residual share.

Proposition 3 defines the equity value $\overline{E}$ and managerial claim value $\overline{M}$ as a function of $\overline{v}$, $\underline{E}$ and $\underline{M}$ which remain to be determined. To proceed we need to define explicitly the pie ($\overline{v} \equiv \overline{E} + \overline{M}$) to be shared between equityholders and managers. This pie depends on whether there are other claimants on the firm's cash flows, such as bondholders. We therefore introduce the following assumption that defines the concept of 'net debt' and specifies the priority structure at closure among the firm's stakeholders:

Assumption 4 *The firm's net debt, D, is defined as the difference between the firm's debt liabilities and its liquid assets. The firm pays (receives) a coupon flow $r\,D$ for positive (negative) D until the firm is closed. Managers have no claim at closure. If net debt is negative ($D < 0$) then equityholders receive upon closure L plus the liquid assets $-D$ (i.e. $L - D$). If net debt is positive then bondholders have a first claim (up to D) on the assets L in liquidation, with equityholders having the entire residual claim $(L - D)^+$. If the firm defaults on its debt obligations ($D > L$), then bankruptcy costs amount to ϕL.*

If D is positive then the firm's net debt position is equivalent to a standard perpetual debt contract with coupon rD that is terminated when the firm defaults. We assume that the debt is secured by the firm's physical assets, which means that at closure bondholders receive $\min[D\,,\, L\,(1 - \phi\xi)]$, whereas equityholders receive $(L - D)^+$ (with $\xi = 1$ if $D > L$, and $\xi = 0$ otherwise). This payoff follows from the fact that upon default bondholders liquidate the firm (like equityholders, bondholders are unable to run the firm as a going concern). Equityholders default on the debt contract if $D > L$. Bankruptcy costs associated with default are a fraction ϕ of the liquidation value L and reduce bondholders' payoff. Importantly, human capital cannot serve as collateral for the firm's debt (unlike physical capital).

If D is negative then the firm has a net surplus of liquid assets that generate a perpetual flow of interest $r\,D$. When the firm is liquidated, equityholders receive $L - D$, which equals the liquidation value (L) plus the value of the net surplus of liquid assets ($-D$). For the moment we assume that the firm can invest on a long term basis in high yield marketable securities that generate the risk-free rate of return r. In section 5 we also consider the case where the firm needs precautionary cash holdings (that generate a return ρ less than r) in order to meet adverse liquidity shocks.

Having defined the priority structure among the firm's stakeholders, we can now translate the previously derived sharing rule (see proposition 3) into a sharing rule for the firm's profit flow π. The presence of sunk entry costs for both parties (H for managers and $I - L$ for equityholders) implies that their outside option can never bind during booms.[12] From proposition 3, we know that equityholders' claim in booms is given by:

$$\overline{E} \;=\; \frac{\left(\overline{E} \,+\, \overline{M}\right)}{2} \;+\; \frac{\overline{\delta}}{2}\,(\underline{E} \;-\; \underline{M}) \tag{16}$$

Simplifying gives: $\overline{E} \;-\; \overline{M} \;=\; \overline{\delta}\,(\underline{E} \;-\; \underline{M})$. The equity value can also be written as:

$$\overline{E} \;=\; \frac{\overline{\pi}(\overline{Q}) - rD \,-\, \overline{s}}{r \,+\, \overline{\lambda}} \;+\; \overline{\delta}\underline{E} \tag{17}$$

Note that interest repayments on debt (for $D \,>\, 0$) are a prior claim on profits, whereas the return generated by liquid assets (for $D < 0$) *increase* the profits to be shared between managers and equityholders. Using the fact that $\overline{M} \;=\; \frac{\overline{s}}{r+\overline{\lambda}} \;+\; \underline{\delta M}$, gives:

Proposition 4 *During booms the compensation rate for human capital ($\overline{s}$) and the payout rate to equityholders ($\overline{d}$) both equal one half of the free cash flows:* $\overline{s} \;=\; \overline{d} \;=\; \frac{\overline{\pi}-r\,D}{2}$

The proposition implies that during booms, equityholders and managers each get half of the profits after interest repayments if net debt is positive ($D \;\geq\; 0$), or half of the combined value of operating profits and the interest generated by liquid assets if net debt is negative ($D < 0$). This property is valid irrespective of whether any outside options bind in recession, or whether the debt is risky. This does, however, not mean that the payout to equityholders and managers in booms is independent of what happens in recession or of how much each party invests. We show in next section that the equilibrium profits $\overline{\pi}$ and the optimal debt level D are very much influenced by the other model parameters.

Consider next the sharing rule in recession. Competition between firms generates industry exit up to the point where the remaining firms are indifferent between staying or leaving, i.e. $\underline{v} \;=\; \underline{o}_e \;+\; \underline{o}_m$. This corresponds to case 4 in proposition 3, and therefore equityholders' and managers' claim in recession equal their outside option.

Finally, we should stress that the sum of the equityholders', managers' and bondholders' claims do not necessarily add up to the total value $\overline{V}$ of the firm. If a firm stays in the market

[12] For equityholders and managers to invest, market conditions and the anticipated cash flows need to be sufficiently favorable to justify the sunk costs $I - L$ and H. If investment is preferable to exercising the outside option prior to incurring the sunk cost, then staying in the market will certainly also be preferable to leaving after the sunk cost has been incurred, for as long as market conditions do not change.

forever, then $\overline{V}_s = \overline{E}_s + \overline{M}_s + D_s$, where $\overline{V}_s = \frac{\overline{\pi}}{r}(1-\overline{p}) + \frac{\pi}{r}\,\overline{p}$. If, however, the firm leaves the industry then this equality no longer holds. If a firm leaves the industry in recession then the value of the firm $\overline{V}_l$ and its debt $\overline{B}_l$ are given by:

$$\overline{V}_l = \frac{\overline{\pi}}{r+\overline{\lambda}} + \overline{\delta}\,L\;(1-\phi\,\xi) \tag{18}$$

$$\overline{B}_l = \frac{r\,D_l}{r+\overline{\lambda}} + \overline{\delta}\,[D_l\,(1-\xi) + L\,(1-\phi)\xi] \tag{19}$$

where ξ equals 1 if $D_l > L$ and zero otherwise. We also know that:

$$\overline{E}_l + \overline{M}_l = \frac{\overline{\pi} - r\,D_l - \overline{s}_l}{r+\overline{\lambda}} + \overline{\delta}\underline{E}_l + \frac{\overline{s}_l}{r+\overline{\lambda}} + \overline{\delta}\underline{M}_l \tag{20}$$

$$= \frac{\overline{\pi} - rD_l}{r+\overline{\lambda}} + \overline{\delta}(L-D_l)(1-\xi) + \overline{\delta}\underline{M}_l = \overline{V}_l - \overline{B}_l + \overline{\delta}\underline{M}_l \tag{21}$$

It follows that: $\overline{E}_l + \overline{M}_l + \overline{B}_l - \overline{V}_l = \overline{\delta}\underline{M}_l$. Therefore, if a firm expects to leave the industry in recession then the sum of its stakeholders' claims *exceeds* the value of the firm by an amount equal to the present value of the managers' claim in recession. This important result follows from the fact that the managers' lifespan stretches beyond the life of the firm. While the firm ceases to exist once the recession arrives, managers carry on their activities elsewhere. Transferability of human capital can therefore have important implications for the boundaries of the firm, and potentially also for managers' incentives during booms. We leave this latter issue as an interesting topic for future research.

4 Industry equilibrium with separation of equity and human capital

In the previous section we determined the equityholders' and managers' claim for given firm values $\overline{V}$ and $\underline{V}$, and a given debt principal D. In this section we determine the equilibrium investment output in booms and recessions (and therefore the equilibrium profits $\overline{\pi}(\overline{Q})$ and $\underline{\pi}(\underline{Q})$), and the debt level that firms adopt.

Managers and equityholders of firms that are closed in recession exercise their outside option. Before we can proceed with our analysis we first need to derive the value of $\underline{o}_e$ and $\underline{o}_m$. It follows immediately from equityholders' limited liability that their payoff from leaving the firm in recession is given by $\underline{o}_e = (L-D)^+$. The value of managers' outside option in recession equals the maximum value of two possible strategies. A first strategy is that managers leave the industry in recessions but return in booms. A second strategy is for managers to stay out of the industry in both booms and recessions. This latter strategy results in a lower bound for the value of the managers' outside option given by $\underline{o}_m = \underline{W} = \frac{\underline{w}}{r}(1-\underline{p}) + \frac{\overline{w}}{r}\underline{p}$. Under the former strategy, the value of the outside option equals: $\underline{o}_m = \frac{\underline{w}}{r}(1-\underline{p}) + \frac{\overline{s}_l}{r}\underline{p}$, where $\overline{s}_l$ is managers' salary rate in booms, conditional on managers leaving in recessions.

We know from proposition 4 that $\overline{s}_l = \frac{\overline{\pi} - r D_l}{2}$. In a competitive industry with atomistic firms, each firm takes the industry output and therefore $\overline{\pi}$ as given. The debt principal D_l is determined by the boundary condition $\overline{E}_l = I - \overline{B}_l$, which reflects the fact that the market for outside equity is competitive. One can show that:

$$\overline{E}_l + \overline{B}_l = \frac{\overline{\pi} + r D_l + 2\overline{\lambda}L(1-\phi\xi)}{2(r+\overline{\lambda})} \quad \textit{where } \xi = 1 \textit{ if } D_l > L \textit{ and } \xi = 0 \textit{ otherwise}$$

Equityholders' payoff $\overline{E}_l - \left(I - \overline{B}_l\right) = \overline{E}_l + \overline{B}_l - I$ is monotonically increasing in D_l, except at $D_l = L$ where there is a discrete downward jump because of the deadweight bankruptcy costs ϕL. Consequently, in a competitive equity market

$$\begin{aligned} D_l \le (>)\, L &\iff \overline{E}_l(D_l = L) + \overline{B}_l(D_l = L) \ge (<)\, I \\ &\iff \overline{\pi} \ge (<)\, r\,I + \left(r + 2\overline{\lambda}\right)(I - L) \iff \xi = 0 \;(\xi = 1) \end{aligned}$$

Solving (i) $\overline{s}_l = \frac{\overline{\pi} - rD_l}{2}$ and (ii) $\overline{E}_l = I - \overline{B}_l$ for $\overline{s}_l$ and D_l gives:

$$\overline{s}_l = \overline{\pi} - r\,I - \overline{\lambda}\,[I - (1-\phi\xi)\,L] \quad \text{and} \quad r\,D_l = 2\left[rI + \overline{\lambda}\,(I - (1-\phi\,\xi)L)\right] - \overline{\pi}$$

Substituting the expression for $\overline{s}_l$ into $\underline{o}_m$ gives the expression for $\underline{o}_m$. By comparing the value for $\underline{o}_m$ under both strategies (as the outside option value equals the maximum value of the two strategies) one can show (see appendix) that in equilibrium the value of $\underline{o}_m$ always corresponds to that of the first strategy. This leads to the following proposition:

Proposition 5 *In recessions, the value of equityholders' outside option ($\underline{o}_e$) and of managers' outside option ($\underline{o}_m$) are given respectively by:*

$$\begin{aligned} \underline{o}_e &= (L - D)^+ \\ \underline{o}_m &= \frac{\underline{w}}{r}(1-\underline{p}) + \frac{\left[\overline{\pi} - r\,I - \overline{\lambda}(I - (1-\phi\xi)L)\right]}{r}\,\underline{p} \end{aligned}$$

$$\begin{aligned} \textit{where} \quad \xi &= 0 \iff \overline{\pi} \ge r\,I + \left(r + 2\overline{\lambda}\right)(I - L) \\ &= 1 \iff \overline{\pi} < r\,I + \left(r + 2\overline{\lambda}\right)(I - L) \end{aligned}$$

Note that the value of managers' outside option in recession depends on the profit rate in booms because managers' option to leave the industry in recession includes an option to return in booms.

Armed with our expressions for $\underline{o}_e$ and $\underline{o}_m$ we can now solve for the industry equilibrium. We first derive the claim value of those firms that, in equilibrium, leave the market. To do so we need to pin down the following 3 unknowns: $\overline{\pi}$, $\overline{s}_l$, and D_l (remember that managers leave the industry during recession and therefore $\underline{s}_l = \underline{w}$). As before $\overline{s}_l$ is determined by the bargaining solution (i) $\overline{s}_l = \frac{\overline{\pi} - rD_l}{2}$. Equity capital is supplied competitively, causing equityholders to break even upon investment, i.e. (ii) $\overline{E}_l = I - \overline{B}_l$. This gives an equilibrium condition for $\overline{\pi}$. Finally we need to determine the debt principal D_l of firms that leave the industry. If equityholders set debt policy then they choose the firm's debt principal D_l (or coupon level rD_l) so as to maximize their payoff at investment, subject to the managers' participation constraint. Or, equivalently, D_l is the solution to the following constrained optimization problem:

$$\max_{D_l}\left[\overline{E}_l - \left(I - \overline{B}_l\right)\right] \text{ subject to } \overline{M}_l \geq \overline{W} + H \tag{22}$$

Similarly, if managers set debt policy then they solve:

$$\max_{D_l}\left[\overline{M}_l - \overline{W} - H\right] \text{ subject to } \overline{E}_l - \left(I - \overline{B}_l\right) \geq 0 \tag{23}$$

In a competitive industry one obtains the same debt level in equilibrium irrespective of who gets to set debt policy. The argument goes as follows. A higher debt level unambiguously lowers managers' claim because debt reduces free cash flows and therefore managers' salary rate $\overline{s}_l = \frac{\overline{\pi} - rD_l}{2}$ (note that an atomistic firm cannot influence $\overline{\pi}$ through its debt policy). Managers therefore prefer the lowest debt level that is acceptable to equityholders. On the other hand a higher debt level increases equityholders' payoff from investment $\overline{E} - \left(I - \overline{B}\right)$. For every dollar of debt raised, equityholders have to contribute one dollar less to the investment, but they share the pain of the subsequent interest repayment with managers (remember that $\overline{d}_l = \overline{s}_l$). Since equity contracts can be traded, competition in the equity market ensures that equityholders always break even. In addition, competition forces firms to adopt the debt level that accommodates the largest number of firms in the industry. It follows that the resulting debt policy is constrained optimal from managers' and equityholders' viewpoint because it is not possible to reduce (increase) the debt level without compromising equityholders' (managers') participation.

We show in the appendix that equityholders' payoff $\overline{E}_l - (I - \overline{B}_l)$ is monotonically increasing in D_l, except at $D_l = L$, where there can be a discrete downward jump if bankruptcy costs are strictly positive. At $D_l = L$, a marginal increase in the debt level leads to a discrete fall in the debt value $\overline{B}_l$ because of the deadweight cost of bankruptcy. Depending on the value of H, this leads to 3 possible regimes for the optimal debt level:

(1) For sufficiently high levels of H (i.e. $H^* < H$, where H^* is defined below) debt is constrained to a level $D < L$ because managers' participation constraint already binds (i.e. $\overline{M}_l = \overline{W} + H$) at a debt level below L.
(2) For sufficiently low levels of H (i.e. $H < H^{**}$, where H^{**} is defined below) the debt level D for which managers' participation constraint binds exceeds L.
(3) For an intermediate region (i.e. $H^{**} \leq H \leq H^*$) a risky debt level ($D > L$) could be adopted while still ensuring managers' participation. However, the gain from constraining managers is wiped out by the deadweight costs of bankruptcy, which reduce the proceeds from the debt issue in a discrete fashion. The debt level is therefore restricted to $D_l = L$ because this debt level enables the highest number of firms to enter the industry. By constraining debt to the firm's liquidation value, managers' participation constraint is no longer binding (i.e. $\overline{M}_l > \overline{W} + H$). Only when H is below H^{**} does it pay off to issue risky debt and to raise the principal by a discrete amount over and beyond L. Why can managers enjoy excess rents ($\overline{M}_l > \overline{W} + H$) whereas equityholders cannot? Managers cannot commit to take less than $\overline{s}_l$ of the free cash flows. Given that managers' claim cannot be traded in financial markets (unlike equity) it is not possible for investors to compete away excess value. Managers can neither compete away among themselves the excess value because this would require that they co-invest an amount equal to the present value of their excess rents when joining the firm. Wealth constraints prevent managers from doing this.

We now sketch the derivation of the claim values of those firms that do not leave in recession. To identify the claim values we need to solve for 4 unknowns: $\underline{\pi}$, $\overline{s}_s$, $\underline{s}_s$ and D_s. Competitive exit ensures that firms leave up to the point where both parties' outside options are binding in recession (i.e. managers and equityholders are indifferent between staying or leaving in recession): (i) $\underline{E}_s = \underline{o}_e$, and (ii) $\underline{M}_s = \underline{o}_m$. The bargaining solution for booms implies that (iii) $\frac{\overline{\pi} - rD_s}{2}$. Conditions (i), (ii) and (iii) determine $\underline{\pi}$, $\underline{s}_s$ and $\overline{s}_s$, for a given level of debt D_s. We show in the appendix that equityholders' payoff from investment is monotonically increasing in D_s, while managers' claim value is monotonically decreasing (i.e. $\frac{\partial \overline{M}_s}{\partial D_s} < 0$). In a competitive industry and equity market, firms that survive adopt the debt level that ensures equityholders' and managers' participation in booms as well as in recessions. This participation constraint allows us to solve for the optimal debt level D_s for each of the previously described 3 regions for H. As previously explained the debt level is determined by competitive forces and does *not* depend on whether it is set by managers or equityholders. In what follows propositions 6, 7 and 8 state and discuss the industry equilibrium for different levels H of investment in human capital. The solution naturally splits up in 3 cases: highly human capital intensive firms ($H^* \leq H$), intermediate human capital intensity ($H^{**} \leq H < H^{*}$), and low human capital intensity ($H < H^{**}$).

Proposition 6 *If firms are highly human capital intensive (i.e. if the investment H in human capital satisfies $H^{*} \leq H$) then we observe 'regime 1' in which all firms adopt the same risk-free debt level and some firms leave in recession. The debt, firm profits and managerial compensation are given by:*

$$\begin{aligned} D_1(L) &= D_{1s}(L) = D_{1l}(L) = I + \frac{\overline{\lambda}}{r}(I-L) - \frac{\overline{w}}{r} - \frac{H}{1-\overline{p}} = L - \frac{(H-H^{*})}{1-\overline{p}} \\ \overline{\pi}_1(L) &= r\,I + \overline{\lambda}(I-L) + \overline{w} + \frac{r\,H}{1-\overline{p}} = \overline{\pi}^o(L) \\ \underline{\pi}_1(L) &= r\,L - \underline{\lambda}(I-L) + \underline{w} = \underline{\pi}^o(L) \\ \overline{s}_{1s} &= \overline{s}_{1l} = \overline{w} + \frac{r\,H}{1-\overline{p}} \qquad \text{and} \qquad \underline{s}_{1s} = \underline{s}_{1l} = \underline{w} \end{aligned}$$

The equityholders' and managers' claim values satisfy the following conditions:

$$\begin{aligned} \overline{E}_{1s} &= \overline{E}_{1l} = I - D_1(L) \quad \textit{and} \quad \overline{M}_{1s} = \overline{M}_{1l} = \overline{W} + H \\ \underline{E}_{1s} &= \underline{E}_{1l} = L - D_1(L) \quad \textit{and} \quad \underline{M}_{1s} = \underline{M}_{1l} = \underline{o}_m = \underline{W} + \underline{\delta}H \end{aligned}$$

where H^{*} is the solution to:

$$\overline{w} + \frac{r\,H^{*}}{1-\overline{p}} = (I-L)\left(r + \overline{\lambda}\right) \tag{24}$$

The above proposition derives the optimal investment and debt policy when the investment in human capital H is relatively large (i.e. $H^{*} < H$). We find that all firms adopt the same safe debt level (i.e. $D_s = D_l = D^o < L$). The optimal debt level has a very simple interpretation. It is the debt level that sets equityholders' (leverage adjusted) payout rate in booms equal to managers' salary rate, i.e.: $\overline{d}^o \equiv r\,(I - D^o) + \overline{\lambda}\,(I-L) = \overline{w} + \frac{r\,H}{1-\overline{p}} \equiv \overline{s}^o(H)$. Managers (equityholders) receive the efficient salary (payout) rate at all times.

The equilibrium profit levels in booms and recession are equal to the first-best profit rates derived in section 2. Competitive entry ensures that all firms break even in equilibrium, while debt is a balancing variable that makes entry a zero NPV investment for both equityholders and managers. Both parties are indifferent between staying or leaving in recessions, i.e. $\underline{E}_s = \underline{E}_l = L - D$ and $\underline{M}_s = \underline{o}_m(\overline{\pi}) = \underline{M}_l = \underline{W} + \underline{\delta}H$.[13]

The equilibrium is identical to the first-best solution in section 2 where there was no separation between equityholders and managers. Full efficiency is achieved thanks to competition

[13]The proposition allows us to verify our expression for $\underline{o}_m(\overline{\pi})$ derived in proposition 5. We derived $\underline{M}_l$ and $\overline{\pi}$ without explicitly using the expression for $\underline{o}_m(\overline{\pi})$. Substituting $\overline{\pi}$ into $\underline{o}_m(\overline{\pi})$, we find that $\underline{M}_l = \underline{o}_m(\overline{\pi})$, as to be expected.

and an optimal debt policy that ensures that equityholders and managers each get a fair return on their investment. If a firm requires more investment in human capital then, other things equal, the level of debt in equilibrium is lower. For sufficiently high levels of investment in human capital, net debt can even get *negative*. In particular, negative debt occurs if $r\,I + \overline{\lambda}(I - L) < \overline{w} + \frac{rH}{1-\overline{p}}$, or equivalently if the sunk investment in human capital (H) and the opportunity cost of human capital ($\overline{w}$) are sufficiently large compared to the sunk cost ($I - L$) and the opportunity cost ($r\,I$) of physical capital.[14]. But what does it mean for firms to have negative debt in this context? Negative debt implies that equityholders not only pay the investment cost I but, in addition contribute a net amount ($-D$) of cash and liquid assets that generate a constant positive cash flow $-r\,D$, while the firm is operational. This increases the cash flow to be shared between equityholders and managers from π to $\pi - rD$. Equityholders are prepared to put up this money in order to attract the required amount of human capital. The equilibrium (negative) debt level ensures that the demand for human capital (by equityholders) exactly equals the supply of human capital. The optimal debt level is increasing (decreasing) in the investment in physical (human) capital, decreasing in the outside reservation wage and decreasing in the firm's liquidation value. This last result may come as a surprise as it implies that leverage is negatively related to tangibility. Since debt is overcollaterlized ($D < L$) default is not an issue, and higher tangibility means that equityholders get more of their capital investment back upon closure and, as a result, are willing to accept a lower debt level allowing more value to shift towards managers. We show below that this negative relation between tangibility and leverage can be reversed when firms constrain their debt level because of bankruptcy cost considerations.

Is the equilibrium sustainable ex-post, or is there a potential hold-up problem? In recession competitive exit ensures that both equityholders' and managers' outside option bind. As a result there is no scope for either party to hold up the other without jeopardizing the firm's existence. In booms, neither party's outside option binds. The sharing of firm value (net of debt) happens according to a standard bargaining game. But what if equityholders were to reduce the cake that parties bargain for by issuing ex-post more debt and pocketing the proceeds? In that case managers could walk away and approach new investors, whereas the existing firm would be unable to attract new managers. Conversely, if managers ex post wanted to reduce the debt level then they would have to pay off the debt themselves (existing shareholders would veto an equity issue). Even if they could (managers are wealth constrained) this would not be in their interest as it would allow equityholders to freeride. It follows that

[14] Even if the previous inequality is not satisfied and negative debt does not occur in the exit region (i.e. for $H < \tilde{H}$), one can show that negative debt always occurs in the no-exit region (i.e. for $H \geq \tilde{H}$) for a sufficiently high level H of investment in human capital

the debt level (and therefore the cake to be shared) cannot be altered ex post by either party. But what if managers (or equityholders) tried to get a larger share of the cake by threatening to leave? This would neither work. First, neither party's outside option binds in recession and abandonment is therefore not a credible threat. Second, physical capital and human capital are complementary. Therefore, the firm cannot function without the approval of *both* parties. Third, even if managers were to leave (because of personal reasons, for example) then it should still be possible for the firm to attract new managers. The transferability of human capital implies that no manager is irreplaceable.

The following proposition describes the solution for regime 2, which prevails for intermediate levels of human capital intensity (i.e. $H \in [H^{**}, H^{*}[$).

Proposition 7 *Assume that bankruptcy costs are sufficiently low (i.e. $\phi < \phi^*$, where ϕ^* is defined in proposition 8). If the investment H in human capital satisfies $H^{**} \leq H < H^*$ then we observe 'regime 2' in which all firms adopt the same risk-free debt level L and some firms leave in recession. The debt, firm profits and managerial compensation are given by:*

$$
\begin{array}{lllll}
D_2(L) & = & D_{2s}(L) \,=\, D_{2l}(L) \,=\, L & & \\
\overline{\pi}_2(L) & = & r\,I + \left(r + 2\overline{\lambda}\right)(I - L) > \overline{\pi}^o(L) & \text{and} & \underline{\pi}_2(L) \,=\, r\,L - \underline{\lambda}\,(I - L) + \underline{w} \,=\, \underline{\pi}^o(L) \\
\overline{s}_{2s}(L) & = & \overline{s}_{2l}(L) \,=\, \left(r + \overline{\lambda}\right)(I - L) & \text{and} & \underline{s}_{2s} \,=\, \underline{s}_{2l} \,=\, \underline{w}
\end{array}
$$

The equityholders' and managers' claim values satisfy the following conditions:

$$
\begin{array}{lll}
\overline{E}_{2s} \,=\, \overline{E}_{2l} \,=\, I - L & \textit{and} & \overline{M}_{2s} \,=\, \overline{M}_{2l} \,>\, \overline{W} + H \\
\underline{E}_{2s} \,=\, \underline{E}_{2l} \,=\, 0 & \textit{and} & \underline{M}_{2s} \,=\, \underline{M}_{2l} \,=\, \underline{o}_m \,>\, \underline{W} + \underline{\delta} H
\end{array}
$$

where H^{**} is the solution to:

$$
\overline{w} + \frac{r\,H^{**}}{1 - \overline{p}} \;=\; (I - L)\left(r + \overline{\lambda}\right) - \overline{\lambda}\phi L \qquad (25)
$$

Regime 2 arises for intermediate levels of human capital (i.e. if $H^{**} \leq H < H^*$). Since $\frac{r(H^* - H^{**})}{1-\overline{p}} = \overline{\lambda}\phi L$, regime 2 only occurs if there are strictly positive bankruptcy costs. The optimal debt policy for all firms is to adopt a debt level $D_s = D_l = L$. By constraining the debt level to L, managers' investment in human capital has a strictly positive NPV (i.e. $\overline{M}_s = \overline{M}_l > \overline{W} + H$). Bankruptcy costs make it, however, prohibitively costly to raise debt levels. Equityholders break even in booms and recessions. Since $D = L$, equity has a zero (or arbitrarily small) value in recessions. Managers' salary rate exceeds the efficient compensation rate during booms, and equals the outside wage rate during recessions. Equityholders payout rate equals the efficient rate at all times. Since managers can look forward to earning excess rents in booms, their claim values in recession exceeds $\underline{W} + \underline{\delta}H$. The total profit rate in booms exceeds the first-best profit rate ($\overline{\pi} > \overline{\pi}^o$), which implies that there is underinvestment in booms. The industry output level is, however, efficient in recessions.

Proposition 8 presents the solution for low values of H (i.e. $H < H^{**}$).

Proposition 8 *Assume that bankruptcy costs are sufficiently low (i.e. $\phi \leq \phi^*$). If $H < H^{**}$ then we observe 'regime 3' in which some firms adopt a high debt level and some firms adopt a lower debt level. The former firms leave the industry in recession. The debt value exceeds L for all firms. The debt, profits and managerial compensation are given by:*

$$\begin{aligned}
D_{3l}(L) &= I + \frac{\overline{\lambda}}{r}(I-L) + \frac{\overline{\lambda}}{r}\phi L - \frac{\overline{w}}{r} - \frac{H}{1-\overline{p}} = L + \frac{(H^{**}-H)}{1-\overline{p}} + 2\frac{\overline{\lambda}}{r}\phi L \\
D_{3s}(L) &= \frac{1}{r+2\overline{\lambda}}\left[(r+\overline{\lambda})\,I + \overline{\lambda}L(1-\phi) - \overline{w} - \frac{r\,H}{1-\overline{p}}\right] = L + \frac{r(H^{**}-H)}{(r+2\overline{\lambda})(1-\overline{p})} \\
\overline{\pi}_3(L) &= r\,I + \overline{\lambda}\,(I - L) + \overline{\lambda}\phi L + \overline{w} + \frac{r\,H}{1-\overline{p}} > \overline{\pi}^o(L) \\
\underline{\pi}_3(L) &= r\,D_{3s} - \underline{\lambda}\,(I - D_{3s}) + \underline{s}_{3s} = \underline{\pi}^o(L) + \frac{r^2(H^{**}-H)(r+\overline{\lambda}+\underline{\lambda})}{(r+2\overline{\lambda})(1-\overline{p})(r+\overline{\lambda})} - \frac{\overline{\lambda}\underline{\lambda}\phi L}{r+\overline{\lambda}} \\
\overline{s}_{3l} &= \overline{w} + \frac{r\,H}{1-\overline{p}} \qquad \text{and} \qquad \underline{s}_{3l} = \underline{w} \\
\overline{s}_{3s}(L) &= \frac{r+\overline{\lambda}}{r+2\overline{\lambda}}\left[\overline{w} + \frac{rH}{1-\overline{p}} + \overline{\lambda}\,(I - L(1-\phi))\right] > \overline{s}_{3l} \\
\underline{s}_{3s}(L) &= \underline{w} - \frac{\underline{\lambda}}{r+\overline{\lambda}}(\overline{s}_{3s} - \overline{s}_{3l}) = \underline{w} - \frac{\overline{\lambda}\underline{\lambda}}{(r+\overline{\lambda})}\left[\frac{r(H^{**}-H)}{(r+2\overline{\lambda})(1-\overline{p})} + \phi L\right]
\end{aligned}$$

The equityholders' and managers' claim values satisfy the following conditions:

$$\begin{aligned}
&\overline{E}_{3s} = I - D_{3s} > \overline{E}_{3l} = I - \overline{B}_{3l} \;\; \textit{and} \;\; \overline{M}_{3s} > \overline{M}_{3l} = \overline{W} + H \\
&\underline{E}_{3s} = \underline{E}_{3l} = 0 \qquad\qquad\qquad\quad \textit{and} \;\; \underline{M}_{3s} = \underline{M}_{3l} = \underline{o}_m = \underline{W} + \underline{\delta}H
\end{aligned}$$

ϕ^* is the root to $\overline{\pi}^{-1}\left[\overline{\pi}_3(I, L, H^{**}(\phi^*))\right] = \underline{\pi}^{-1}\left[\underline{\pi}_3(I, L, H^{**}(\phi^*))\right]$, or equivalently to:
$\overline{\pi}^{-1}\left[rI + (I-L)(r+2\overline{\lambda})\right] = \underline{\pi}^{-1}\left[\underline{\pi}^o - \frac{\overline{\lambda}\underline{\lambda}\phi^* L}{r+\overline{\lambda}}\right]$.

Proposition 8 states that if the required investment in human capital is low (i.e $H < H^{**}$) then all firms adopt a debt level that exceeds L in order to constrain managers in booms. Furthermore, firms that leave in recession adopt a higher debt level than survivors. High leverage prevents managers from capturing excess rents in booms, but causes these firms to incur bankruptcy costs in recession. Survivors, on the other hand, set their debt level sufficiently low so that in recession equityholders and managers are indifferent between staying and leaving. By doing so these firms are able to issue risk-free debt. However, by constraining the debt level, managers of these firms get excess rents in booms (i.e. $\bar{s}_{3s} > \bar{s}^o$).[15] These excess rents are partially clawed back in recession when managers of the surviving firms get paid below their reservation wage. Managers are willing to accept this cut because of the prospect of superior rents in future booms. While managers of firms that leave in recession break even in booms and recessions, managers of firms that stay have a positive NPV claim in booms (i.e. $\overline{M}_{3s} > \overline{W} + H$). This means that there is a first-mover advantage for managers that enter first into the industry and it explains why these managers do not leave in recession, despite being paid below the reservation wage. First movers have an interest to adopt a low debt level because by doing so these firms capture upon investment a positive NPV equal to:

$$\overline{E}_{3s} + \overline{M}_{3s} + D_{3s} - I - \overline{W} - H = \overline{M}_{3s} - \overline{W} - H > 0$$

Even though equityholders of all firms only break even, the market capitalization of first movers is larger (i.e. $\overline{E}_{3s} = I - D_{3s} > \overline{E}_{3l} = I - \overline{B}_{3l}$ since $\overline{B}_{3l} > D_{3s}$). In recession exit occurs up to the point where firms are indifferent between staying and going. As a result, $\underline{E}_{3s} = \underline{E}_{3l}$ and $\underline{M}_{3s} = \underline{M}_{3l} = \underline{o}_m$.

Undercollateralized debt leads to inefficiencies because it brings with it bankruptcy costs and debt overhang. The highly levered firms anticipate future bankruptcy costs and therefore require a higher equilibrium profit rate in booms to compensate. This creates underinvestment in booms. Firms that plan to stay in recession constrain their debt level such that both managers' and equityholders' outside option exactly binds in recession. This lower debt level gives managers excess rents in booms ($\bar{s}_{3s} > \bar{s}_{3l} = \overline{w}$). These excess rents increase with bankruptcy costs and allow managers' compensation to be cut in recession below the reservation wage by an amount $\frac{\lambda}{r+\lambda}(\bar{s}_{3s} - \bar{s}_{3l})$. This provides space for profits to be reduced in recession by an equal amount. Bankruptcy costs therefore create an overinvestment effect in recession. On the other hand, the profit rate $\underline{\pi}_3$ increases to the extent that D exceeds L. Since equityholders have limited liability, undercollateralized debt leads to the well known Myers (1977) underinvestment effect. For values of H just below H^{**} the bankruptcy cost effect dominates, resulting in overinvestment during recession, whereas for lower levels of H the debt level adopted is much higher and this causes the debt overhang effect to dominate in recession.

[15] The tradeoff between bankruptcy costs and managerial rent capture induces heterogeneity in capital structure in a similar fashion as in Maksimovic and Zechner (1991), where firms trade off the tax advantage of debt against the agency costs of debt. In equilibrium some firms issue low amounts of debt, forgoing debt-related tax shields but committing to the subsequent choice of the less risky project with higher pre-tax cash flows, whereas other firms adopt more debt, capturing large benefits but creating incentives to choose subsequently the riskier project.

Regime 3 is derived under the assumption that some firms leave in recession. However, as ϕ increases the equilibrium profit rate in booms (recessions) unambiguously rises (falls) (see proposition 8). Consequently, the industry output in booms (recessions) monotonically falls (rises) as ϕ increases. There exists therefore a level ϕ^* for which $\overline{Q}$ equals $\underline{Q}$, and firms no longer leave the market[16]: bankruptcy costs lower the firm's liquidation value and, as such, lead to more hysteresis. For sufficiently high levels of bankruptcy costs industry output therefore remains constant.

Proposition 9 *Assume that bankruptcy costs are high (i.e. $\phi > \phi^*$). Then an interval $[H''\ ,\ H']$ exists (with $H'' < H^{**} < H'$) for which no firm leaves the market, leading to underinvestment in booms and overinvestment in recession. Only as many firms enter in booms as the market can support in recession, and therefore (i) $\underline{E}_s = \underline{o}_e$ and (ii) $\underline{M}_s = \underline{o}_m$. The additional competitive entry condition for equityholders (iii) $\overline{E}_s = I - D_s$ and the bargaining condition (iv) $\overline{s}_s = \frac{\overline{\pi}_3 - rD_s}{2}$ complete the solution for $\tilde{Q}$, D, $\overline{s}$ and $\underline{s}$. The boundaries of this regime, H' and H'', are the solution to respectively $\overline{M}_s = \overline{W} + H'$ and $\underline{M}_s = \underline{W} + \underline{\delta}H''$.*

One can show that $\overline{Q}(H) - \underline{Q}(H)$ is minimized at H^{**}. Therefore a sufficient and necessary condition for no exit to occur for some H (under assumption 1) is that $\phi > \phi^*$, where ϕ^* is the root of $\overline{Q}(H^{**}(\phi^*)) = \underline{Q}(H^{**}(\phi^*))$, or equivalently the root of $\overline{\pi}^{-1}\left[\overline{\pi}_3(I, L, H^{**}(\phi^*))\right] = \underline{\pi}^{-1}\left[\underline{\pi}_3(I, L, H^{**}(\phi^*))\right]$.[17] A closed form solution for this system of equations does not exit.[18]

What is the intuition for the existence of this 'no-exit' regime (hereafter called regime 'EM' referring to the fact that both equityholders' and managers' outside option binds in recession)? With low bankruptcy costs the benefit of reducing managers' rents by adopting undercollateralized debt (i.e. $D > L$) outweighs bankruptcy costs for $H < H^{**}$. As bankruptcy costs increase, the interval $[H^{**}\ ,\ H^*]$ over which debt is kept at L widens. Within this interval the industry output in booms is independent of H. Consequently, the degree of underinvestment rises as H declines. This opens up another possibility for firms to avoid bankruptcy costs: instead of constraining the debt level to L, all firms adopt an identical debt level that exceeds L

[16] A value for ϕ^* always exists, but is not necessarily bounded by 1. Furthermore, H^{**} can be negative. As a result, a no-exit region $[H'', H']$ does not always exist for the set of parameter values that are economically relevant.

[17] It is easy to show that $\overline{\pi}$ (and therefore $\overline{Q}$) is continuous at H^{**} and that $\underline{\pi}$ displays a discrete downward jump at H^{**} if $\phi > 0$ (see figure 3 for an illustration). Since $\overline{Q} - \underline{Q}$ is constant in regime 2, it follows that $\overline{Q} - \underline{Q}$ is minimized at H^{**} for regime 3. Consequently, if some exit occurs for $H = H^{**}$, then exit occurs for all H satisfying assumption 1.

[18] Equivalently, H'' can be found as the root to $\overline{\pi}^{-1}\left[\overline{\pi}_3(I, L, H'')\right] = \underline{\pi}^{-1}\left[\underline{\pi}_3(I, L, H'')\right]$. In other words, H'' is the lowest value for H for which no exit occurs (i.e. $\overline{Q} = \underline{Q}$). See proof to proposition 8 for further details.

but still enables *all* firms to survive in recession. This reduces underinvestment in booms, but leads to overinvestment in recession. The reduction in the cost of underinvestment outweighs, however, the (present value) of the increased cost of future overinvestment in recession.

Within the interval $[H'', H']$ equityholders break even in booms, but managers earn excess rents (i.e. $\overline{M}_{EMs} \geq \overline{W} + H$). As a result the industry output does not depend on the investment cost H of the marginal manager. Managers' excess rents equal zero at H' and go up as $(H' - H)$ increases. That is why below H'' firms find it optimal to raise the debt level, and to switch to regime 3 in which some firms leave in recession. The number of firms that leave in recession increases continuously with $H'' - H$.

5 Equilibrium with cash holdings

Assumption 4 implies that only *net* debt matters because liquid assets grow at the riskfree rate and can be directly offset against the debt principal. While some liquid assets (such as bonds and stocks) earn the risk-free rate of return in equilibrium (remember that investors are risk neutral), cash typically earns a rate of interest ρ that is lower (i.e. $\rho \leq r$) to reflect the price of immediacy: firms want to hold cash to be able to respond *quickly* to adverse liquidity shocks.[19] Alternatively, firms may have a line of credit they can draw from to meet unexpected liquidity needs. We show below that cash is equivalent to a non-revocable credit line that charges a fee proportional to the size C of the credit line. Our exposition will be in terms of cash holdings but is analogous for lines of credit.[20]

In this section we analyze how costly cash holdings (or lines of credit) affect the firms' debt level, managerial compensation and equilibrium profits. We assume that firms wishing to survive need an amount of cash κ at the start of a recession in order to meet an adverse liquidity shock. A recession may impose, for example, restructuring costs (such as costs from laying off personnel). For firms that leave the industry, these costs (denoted by κ) merely reduce the liquidation proceeds to $L_c = L - \kappa$. However, surviving firms either need to

[19] Moral hazard is another possible reason for the wedge between r and ρ. For example, Myers and Rajan (1998) argue that highly liquid assets can reduce managers' ability credibly to commit to an investment strategy that protects investors. We do not model moral hazard in this paper.

[20] Note that, unlike cash holdings, lines of credit may not be a bullet proof hedge. Tirole (2006) notes that in practice lenders often prefer to keep discretion over the extension of credit by making the line revokable, or delivering promises such as "comfort or highly confident letters", which are legally hard to enforce and are only a moral promise to provide credit at the agreed terms. In the recent credit crunch many business have indeed seen their lines of credit withdrawn.

have cash (or a credit line) in place or raise the cash through an equity issue, as stated in the following assumption:

Assumption 5 *The start of a recession coincides with an adverse liquidity shock to each firm amounting to a cost* κ *(with* $\kappa \leq L$*). To stay in business, firms can cover the liquidity shock* κ *in two ways: 1) Firms can raise an amount cash* C *in booms at a cost* C *and store it till the arrival of a recession, with cash bearing an interest rate* ρ *(*$< r$*). 2) At the start of a recession, firms can raise any cash shortfall (*$\kappa - C$*) at a cost* $\theta\,(\kappa - C)$ *through an equity issue, where* $\theta \geq 1$*.*

We focus on systematic liquidity shocks that affect all firms (as opposed to idiosyncratic shocks that affect only individual firms). $(\theta - 1)$ captures the friction of raising cash at short notice in recession.

5.1 First-best liquidity policy

As before we first study the case where there is no separation between equity and human capital. We solve the problem backwards by deriving first the strategy of firms that leave the industry in recession. For these firms the liquidity shock κ in recession merely reduces the liquidation proceeds to $L_c \equiv L - \kappa$. There is no incentive to hold cash during booms because it is more efficient to pay the adverse liquidity shock κ out of the liquidation proceeds L. As a result the valuation of firms that leave the industry remains unaltered (except that L is replaced by L_c). The equilibrium profits $\overline{\pi}_c$ are the solution to $\overline{V}_l = I + \overline{W} + H$.

Consider next the liquidity policy of the survivors. These firms raise the optimal amount C (to be determined) of cash in booms at a cost C. This cashholding generates an interest flow ρC during booms which can be shared among the firm's stakeholders. At the start of the recession the cash holding C is depleted to cover the liquidity shock κ, and any shortfall $(\kappa - C)$ is raised from equityholders at a cost $\theta\,(\kappa - C)$. Note that holding cash is a proactive policy towards liquidity shocks, whereas an equity issue is a reactive policy. Let $\overline{V}_c$ and $\underline{V}_c$ denote the value of firms that remain in the industry in recession by adopting the above defined liquidity policy. It follows that in equilibrium:

$$\begin{aligned} r\,\overline{V}_c &= \overline{\pi}_c + \rho C + \overline{\lambda}\left[\underline{V}_c - \theta\,(\kappa - C) - \overline{V}_c\right] \\ r\,\underline{V}_c &= \underline{\pi}_c + \underline{\lambda}\left[\overline{V}_c - C - \underline{V}_c\right] \end{aligned} \tag{26}$$

Solving these 2 equations gives the expressions for $\overline{V}_c$ and $\underline{V}_c$ in the below proposition. In a competitive equilibrium it must be the case that $\overline{V}_c = I + C + \overline{W} + H$. The required investment now equals $I + C$ because owners raise not only an amount I to pay for the firm's physical assets, but also an amount C in cash. Given that $\overline{\pi}_c$ is known (and determined by the entry condition of firms that leave in recession), the above condition with respect to $\overline{V}_c$ can be solved for the remaining unknown $\underline{\pi}_c$, which determines the number of firms that remain active in recession. The following proposition summarizes the results and states the optimal liquidity policy.

Proposition 10 *Assume that all firms experience at the start of a recession an adverse liquidity shock as described in assumption 5 and that* $\overline{\pi}(\tilde{Q}) - \underline{\pi}(\tilde{Q}) > \Delta_c(H)$ *where* $\Delta_c(H) \equiv \Delta(I, L_c, H) - (r + \underline{\lambda})\, l(C)$ *and*

$$l(C) \;\equiv\; C + \frac{C\;(r - \rho)}{\overline{\lambda}} + \theta\;(\kappa - C)$$

Then the industry output in booms $(\overline{Q})$ *and recessions* $(\underline{Q})$ *are the solution to* $\overline{\pi}(\overline{Q}) = \overline{\pi}^o(L_c)$ *and* $\underline{\pi}(\underline{Q}) = \underline{\pi}^o(L_c) + (r + \underline{\lambda})\, l(C)$.

The values of firms that leave in recession is the same as in proposition 2, but with L *replaced by* L_c $(\equiv L - \kappa)$*. The value of surviving firms is given by:*

$$\overline{V}_c(\overline{Q}, \underline{Q}) = \frac{\left[\overline{\pi}(\overline{Q}) + \rho C - \overline{\lambda}\theta\,(\kappa - C)\right]}{r}(1 - \overline{p}) + \frac{\left[\underline{\pi}(\underline{Q}) - \underline{\lambda} C\right]}{r}\,\overline{p} = \overline{V}_s(\overline{Q}, \underline{Q}) - (r + \underline{\lambda})\frac{\overline{p}}{r} l(C) + C$$

$$\underline{V}_c(\overline{Q}, \underline{Q}) = \frac{\left[\underline{\pi}(\underline{Q}) - \underline{\lambda} C\right]}{r}(1 - \underline{p}) + \frac{\left[\overline{\pi}(\overline{Q}) + \rho C - \overline{\lambda}\theta\,(\kappa - C)\right]}{r}\,\underline{p} = \underline{V}_s(\overline{Q}, \underline{Q}) - \underline{\lambda}\frac{\overline{p}}{r} l(C)$$

where $\overline{V}_s(\overline{Q}, \underline{Q})$ *and* $\underline{V}_s(\overline{Q}, \underline{Q})$ *are defined in proposition 1. Firms that leave in recession do not hold cash while survivors store cash in booms if the expected cost of holding cash in booms is lower than the cost of raising funds in recessions, or equivalently:*

$$C^o = 0 \;\; (C^o = \kappa) \iff \theta - 1 \le (\ge)\, \frac{r - \rho}{\overline{\lambda}}$$

The optimal value for C *minimizes* $l(C)$ *and maximizes the industry output in recession.*

The proposition is a generalization of proposition 2 to allow for liquidity shocks and costly cash holdings. The equilibrium profit rate in booms is now given by $\overline{\pi}_c(\overline{Q}) = \overline{\pi}^o(L_c)$, which is the same profit rate as in the absence of liquidity shocks, except that the liquidation value L is now reduced by κ. As a result the presence of liquidity shocks unambiguously raises equilibrium profits and lowers industry output in booms. The profit rate in recessions is given by $\underline{\pi}_c = \underline{\pi}^o(L_c) + (r + \underline{\lambda})\, l(C) = \underline{\pi}^o(L) - \underline{\lambda}\kappa + (r + \underline{\lambda}) l(C)$. Liquidity shocks have a double effect on the equilibrium profit rate in recession. First, the liquidity shock reduces the liquidation proceeds by κ, which lowers (raises) the equilibrium profit rate (industry output) in recession. Second, the expected cost of raising or carrying cash increases the equilibrium profit rate and has a negative effect on industry output in recession. In equilibrium the expected cost of the liquidity policy is given by $(r + \underline{\lambda})\,\frac{\overline{p}}{r}\,l(C)$. It is easy to show that $(r + \underline{\lambda})\,l(C) \;-\; \underline{\lambda}\,\kappa \;>\; 0$ and consequently the latter effect dominates the former. As a result, the presence of liquidity shocks unambiguously raises the equilibrium profits in recessions and therefore reduces industry output. The result follows from the fact that firms incur the liquidity shock, whether they stay in the industry or not.

The optimal liquidity policy minimizes $l(C)$. Since $l(C)$ is linear in C we obtain a simple corner solution: either no cash is held in booms ($C = 0$) or either the cashholding provides a perfect hedge against a future liquidity shock (i.e. $C = \kappa$). The former (latter) policy prevails if and only if $\frac{\overline{\lambda}}{r+\overline{\lambda}}(\theta - 1)\kappa \;\leq\; (\geq)\; \frac{(r-\rho)\kappa}{r+\overline{\lambda}}$. The left hand is the cost $(\theta - 1)\kappa$ of issuing equity in recession multiplied by the discount factor ($\overline{\delta} \;=\; \frac{\overline{\lambda}}{r+\overline{\lambda}}$), whereas the right hand is the expected cost of holding an amount κ in cash for the duration of the boom. Simplifying gives the condition $\theta - 1 \;\leq\; (\geq)\; \frac{r-\rho}{\overline{\lambda}}$.

Substituting the solution for $\overline{\pi}_c$ and $\underline{\pi}_c$ into $\underline{V}_c$, one can easily verify that in equilibrium $\underline{V}_c$ satisfies the following condition:

$$\underline{V}_c(\overline{Q}, \underline{Q}) \;-\; \theta\,(\kappa - C) \;-\; \frac{C(r-\rho)}{\overline{\lambda}} \;\;=\;\; L \;-\; \kappa \;+\; \underline{W} \;+\; \underline{\delta} H \;+\; C \tag{27}$$

The left side of the equation represents the value $\underline{V}_c$, in recession, of an active firm after it has absorbed the liquidity shock minus the expected cost associated with the liquidity policy to absorb this shock. The right side equals the payoff from liquidating the firm at the start of the recession. The value $\underline{V}_c$ of a survivor in an economy with liquidity shocks can be reformulated in terms of the value $\underline{V}_s$ (see proposition 2) of a survivor in an economy without liquidity shocks (for given levels of industry output $\overline{Q}$ and $\underline{Q}$):

$$\underline{V}_c(\overline{Q}, \underline{Q}) \;=\; L_c \;+\; \underline{W} \;+\; \underline{\delta}\, H \;+\; l(C) \;\;=\;\; \underline{V}_s(\overline{Q}, \underline{Q}) \;-\; \frac{\underline{\lambda}}{r}\overline{p}\, l(C) \tag{28}$$

The following corollary summarizes the effect of an adverse liquidity shock κ on the industry output.

Corollary 1 *Increasing the size of the liquidity shock (but holding all else constant) reduces industry output in booms and in recessions (i.e.* $\frac{\partial \overline{Q}}{\partial \kappa} < 0$ *and* $\frac{\partial \underline{Q}}{\partial \kappa} < 0$*).*

Finally, we show that holding cash is equivalent to having a non-revocable line of credit charging a fee that is proportional to the size of the credit line. The cost of the line of credit over one business cycle is $\frac{\psi C}{r+\overline{\lambda}} \;+\; \overline{\delta}\left(\frac{rC}{r+\overline{\lambda}}\right) \;+\; \overline{\delta}\underline{\delta} C$, where the first term is the present value of the flow of fees (ψC) paid during booms, the second term is the present value of servicing interest (rC) on the debt during recession, and the third term is the present value of paying off the outstanding debt C at the start of the subsequent boom. On the other hand, the cost of holding cash over a business cycle is given by $C - \frac{\rho C}{r+\overline{\lambda}}$, which is the cost of setting up the cash holding in a boom, minus the present value of any interest earned on the cash during the boom. One can immediately verify that the cost associated with the line of credit equals the cost of holding cash if the fee rate ψ charged on the line of credit equals $\psi = r - \rho$. The cash policy can therefore be replicated by a short term debt policy, as short term cash withdrawals can be substituted by increases in short term debt through a non-revokable line of credit.

5.2 Liquidity policy with separation of human capital and equity capital

We now analyze the effect of liquidity shocks on the industry equilibrium when there is separation of equity and human capital. We know that liquidity shocks do not change the behavior of firms that leave the market in recession. The valuation of firms that leave, as well as the equilibrium profits in booms, remain therefore unaltered except that the liquidation value L is everywhere replaced by $L_c\,(\equiv\; L - \kappa)$. The drop in the liquidation value also affects the two thresholds for H that separate regimes 1, 2 and 3. These two thresholds are denoted by ${H_c}^*$ and ${H_c}^{**}$ and are a function of L_c instead of L, i.e. ${H_c}^* = H^*(L_c)$ and ${H_c}^{**} = H^{**}(L_c)$, where $H^*(L)$ and $H^{**}(L)$ are defined by equations (24) and (25), respectively.

Consider now firms that survive in recession. We know that in the absence of liquidity shocks equityholders' and managers' participation constraint is always binding in recession (i.e. $\underline{E}_s = \underline{o}_e$ and $\underline{M}_s = \underline{o}_m$), whereas managers sometimes enjoy slack during booms (i.e. $\overline{M}_s \geq \overline{W} + H$) because of the firm's inability to raise more debt. The presence of liquidity shocks can alter these outcomes quite dramatically. We showed in section 5.1 that owner-managed firms with positive cash holdings are strictly better off staying than leaving in recession (i.e. $\underline{V}_c - \theta(\kappa - C) > L_c + C + \underline{W} + \underline{\delta}H = \underline{o}_e + \underline{o}_m$) because these firms have cash available to meet the liquidity shock, which puts them at an advantage compared to firms that have to raise the cash in the market. The size of the available slack reflects the expected amount of interest lost on the cashholding over the period of an economic boom (i.e. $\frac{C(r-\rho)}{\overline{\lambda}}$). In an environment with separation of equity and human capital, cash holdings relax equityholders' participation constraint in recessions, because it is equityholders who have to raise the amount of cash $\kappa - C$, not managers. This slack has important implications for the debt and managerial compensation policy because it allows debt levels to be raised and managers' excess rents to be squeezed as shown in the following proposition:

Proposition 11 *Assume that bankruptcy costs are sufficiently low (i.e. $\phi \leq \phi_c{}^*$), and that in recession firms are subject to an adverse liquidity shock as described in assumption 5. Propositions 6, 7 and 8 describing the behavior of firms in the absence of liquidity shocks still apply, but with L is replaced everywhere by L_c. Firms that leave the industry do not hold cash. Equilibrium profits in booms are given by $\overline{\pi}_{jc} = \overline{\pi}_j(L_c)$ (for all regimes j). The following conditions apply for all regimes: $\overline{E}_{jc} = I + C - D_{jc}$ and $\underline{M}_{jc} = \underline{o}_m$, i.e. equityholders and managers always break even in booms and recessions, respectively.*

Firms that do not leave the industry store cash during booms when optimal to do so (see corollary 2 below). The debt and managerial compensation policy of surviving firms are characterized as follows:

If $H_c{}^* \leq H$ then we observe regime 1 with

$$\begin{array}{lll} \overline{s}_{1c} = \overline{s}_{1s} & \text{and} & \underline{s}_{1c} = \underline{s}_{1s} \\ D_{1c} = D_1(L_c) + \frac{\rho C}{r} & \text{and} & \underline{\pi}_{1c} = \underline{\pi}_1(L_c) + (r+\underline{\lambda})\, l(C) \\ \underline{E}_{1c} - \theta(\kappa - C) = (L_c - D_{1c} + C) + \frac{C(r-\rho)}{\overline{\lambda}} & \text{and} & \overline{M}_{1c} = \overline{W} + H \end{array}$$

If $\hat{H}_c^* < H < H_c{}^*$ then we observe regime 2' for which:

$$\begin{array}{lll} \overline{s}_{2'c} = \overline{w} + \frac{rH}{1-\overline{p}} - \frac{\overline{\lambda}\underline{\lambda}(H_c{}^* - H)}{r+\underline{\lambda}} & \text{and} & \underline{s}_{2'c} = \underline{w} + \underline{\lambda}(H_c{}^* - H) \\ D_{2'c} = L_c + \frac{\rho C}{r} + \frac{2(H_c{}^* - H)(r+\overline{\lambda})}{r} & \text{and} & \underline{\pi}_{2'c} = \underline{\pi}_2(L_c) + (r+\underline{\lambda})l(C) - \frac{r(H_c{}^* - H)}{\overline{p}} \\ \underline{E}_{2'c} - \theta(\kappa - C) > 0 & \text{and} & \overline{M}_{2'c} = \overline{W} + H \end{array}$$

If $H_c{}^{**} \leq H \leq \hat{H}_c^*$ then we observe regime 2 with:

$$\begin{array}{lll} \overline{s}_{2c} = \overline{s}_{2s}(L_c) - \frac{(r+\overline{\lambda})(r-\rho)C}{r+2\overline{\lambda}} & \text{and} & \underline{s}_{2c} = \underline{s}_{2s} + \frac{\underline{\lambda}(r-\rho)C}{r+2\overline{\lambda}} \\ D_{2c} = D_{2s}(L_c) + C + \frac{(r-\rho)C}{r+2\overline{\lambda}} & \text{and} & \underline{\pi}_{2c} = \underline{\pi}_2(L_c) + (r+\underline{\lambda})g(C) \\ \underline{E}_{2c} - \theta(\kappa - C) = 0 & \text{and} & \overline{M}_{2c} \geq \overline{W} + H \end{array}$$

If $\hat{H}_c^{**} < H < H_c{}^{**}$ then we observe regime 3' with:

$$\begin{array}{lll} \overline{s}_{3'c} = \overline{w} + \frac{rH}{1-\overline{p}} < \overline{s}_{3s}(L_c) & \text{and} & \underline{s}_{3'c} = \underline{w} > \underline{s}_{3s}(L_c) \\ D_{3'c} = D_{3l}(L_c) + \frac{\rho C}{r} & \text{and} & \underline{\pi}_{3'c} = \underline{\pi}^o(L_c) + (r+\underline{\lambda})\,(l(C) - \phi L_c) \\ \underline{E}_{3'c} - \theta(\kappa - C) > 0 & \text{and} & \overline{M}_{3'c} = \overline{W} + H \end{array}$$

If $H < \min[\hat{H}_c^{**}, H_c{}^{**}]$ then we observe regime 3 with:

$$\begin{array}{lll} \overline{s}_{3c} = \overline{s}_{3s}(L_c) - \frac{(r+\overline{\lambda})C(r-\rho)}{(r+2\overline{\lambda})} & \text{and} & \underline{s}_{3c} = \underline{s}_{3s}(L_c) + \frac{\underline{\lambda}C(r-\rho)}{(r+2\overline{\lambda})} \\ D_{3c} = D_{3s}(L_c) + C + \frac{C(r-\rho)}{r+2\overline{\lambda}} & \text{and} & \underline{\pi}_{3c} = \underline{\pi}_3(L_c) + (r+\underline{\lambda})g(C) \\ \underline{E}_{3c} - \theta(\kappa - C) = 0 & \text{and} & \overline{M}_{3c} > \overline{W} + H \end{array}$$

where $\hat{H}_c^* \equiv H_c{}^* - \frac{C(r-\rho)}{r+2\overline{\lambda}}$ *and* $\hat{H}_c^{**} = H_c{}^{**} - \frac{1-\overline{p}}{r}\left[\frac{(r+\overline{\lambda})(r-\rho)C}{\overline{\lambda}} - (r+2\overline{\lambda})\phi L_c\right]$ *and* $\phi_c{}^*$ *is the root to* $\overline{\pi}^{-1}\left[\overline{\pi}_{3c}(H_c{}^{**}(\phi_c{}^*))\right] = \underline{\pi}^{-1}\left[\underline{\pi}_{3c}(H_c{}^{**}(\phi_c{}^*))\right]\omega + \underline{\pi}^{-1}\left[\underline{\pi}_{3'c}(H_c{}^{**}(\phi_c{}^*))\right](1-\omega)$ *with* $\omega = 1$ *if* $(r+2\overline{\lambda})\phi L_c \geq \frac{(r+\overline{\lambda})(r-\rho)C}{\overline{\lambda}}$ *and* $\omega = 0$ *otherwise, and where:*

$$g(C) \equiv C + \frac{C(r-\rho)(r+2\underline{\lambda})}{(r+\underline{\lambda})(r+2\overline{\lambda})} + \theta(\kappa - C) \tag{29}$$

The proposition shows that depending on the required level of H, 5 different regimes are now possible. In each of these regimes equityholders' participation constraint binds in booms (i.e. $\overline{E}_{jc} = I + C - D_{jc}$) and managers' outside option binds in recession (i.e. $\underline{M}_{jc} = \underline{o}_m$), just as in an environment without liquidity shocks. The situation with respect to managers' participation constraint in booms and equityholders' constraint in recessions is quite different. Whereas in the absence of cash holdings, equityholders' outside option always binds in recession, with a positive amount of cash holdings this is no longer the case for regimes 1, 2' and 3'. Similarly, with cash holdings, managers' participation constraint binds in booms for regimes 2' and 3', whereas this would not be the case in the absence of cash holdings. Cash holdings therefore have an important disciplining effect for managers during economic booms, and they relax equityholders' constraint in recession. This result goes against the conventional wisdom that the availability of cash in good times boosts managerial rents. The intuition for our important result is simple: holding cash during booms imposes on firms an expected loss of interest with a present value equal to $(r+\underline{\lambda})\,\frac{\overline{p}}{r}\,\frac{C(r-\rho)}{\overline{\lambda}}$. This cost is shared between equityholders and managers because it reduces the free cash flow during booms. On the benefit side, cash holdings reduce the cost of covering the liquidity shock in recessions. Without cash holdings, funds have to be raised in recession through a dilutive equity issue. The cost of raising funds cannot be borne by managers because they are wealth constrained. Furthermore, once new equity has been issued and the liquidity shock been paid for, managers' stake cannot be compressed below its outside option value. Therefore, while cash holdings during booms force both equityholders and managers to internalize the cost of a future liquidity shock, equity issues in recession impose the full burden on equityholders. Cash holdings therefore have two important effects. First, they transfer value from booms to recessions. Second, this transfer can mitigate excess rents that accrue to managers in booms because the cost of holding cash accrues over the lifetime of the booms and as such reduces managers' compensation.[21]

We now discuss systematically the effect of cash holdings on debt and managerial com-

[21]Note that regimes 2' and 3' (where managers' participation constraint binds) of course only occur if holding cash is optimal in the first place. It is easy to see that the interval $[\hat{H}_c^*, H_c{}^*]$ over which regime 2' prevails shrinks to an empty set as $C \to 0$. Similarly, for regime 3' to exist the optimal cashholding C needs to exceed some minimum threshold.

pensation policy. For comparative purposes the expressions in proposition 11 are stated in terms of the incremental effect of liquidity shocks. A first conclusion is that cash holdings raise the optimal debt level of firms that hold cash (i.e. $D_{jc} > D_{js}(L_c)$ for all regimes j). For example, in regime 1 – where managers do not enjoy excess rents – the optimal debt level is now given by $D_{1c} = D_1(L_c) + \frac{\rho C}{r}$. If ρ equals r then the debt level is raised exactly by the amount of cash and net debt is unaffected. If holding cash is costly ($\rho < r$) then the net debt level is lower in the presence of cash, i.e. $D_{1c} - C = D_1(L_c) - \frac{C(r-\rho)}{r} < D_1(L_c)$. The decrease in net debt compensates managers for the cost of holding cash, and is necessary to make managers participate since managers just break even in regime 1. An increase in cash holdings increase debt in such a way that free cash flows (and therefore managers' claim) remain unaltered (i.e. $\rho\Delta C - r\Delta D = 0$). As a result, the marginal effect of the cash on debt is less than one (i.e. $\frac{\Delta D}{\Delta C} = \frac{\rho}{r} \leq 1$). In situations where previously debt levels had to be constrained because of bankruptcy costs (i.e. regimes 2 and 3) and where managers therefore enjoy excess rents, cash holdings can now lead to an increase in net debt levels. For example, $D_{jc} - C = D_{js}(L_c) + \frac{(r-\rho)C}{r+2\overline{\lambda}}$ (for $j = 2, 3$), and therefore net debt levels increase by an amount $\frac{(r-\rho)C}{r+2\overline{\lambda}}$. As a result, the marginal effect of the cash on debt exceeds one (i.e. $\frac{\Delta D}{\Delta C} > 1$). This leads to the empirical implication that the marginal effect of cash on the debt level is an indicator as to whether managers enjoy excess rents or not.

What is the effect of liquidity shocks and cash holdings on managerial compensation? For regime 1, there is no effect as managers still receive the efficient wage rates $\overline{w} + \frac{rH}{1-\overline{p}}$ and $\underline{w}$. One can show that in all other regimes positive cash holdings lead to smoothing in managers' salary rate, i.e. managers receive a lower salary in booms and a higher salary in recessions ($\overline{s}_{jc} < \overline{s}_{js}$, $\underline{s}_{jc} < \underline{s}_{js}$ for $j = 2, 2', 3', 3$). For example, $\overline{s}_{jc} = \overline{s}_{js}(L_c) - \frac{(r+\overline{\lambda})(r-\rho)C}{r+2\overline{\lambda}}$ and $\underline{s}_{jc} = \underline{s}_{js}(L_c) + \frac{\lambda(r-\rho)C}{r+2\overline{\lambda}}$ (for $j = 2, 3$). Managers receive less in booms, but (partially) make up for this in recessions.

Finally, the effects of cash holdings on industry output are as follows. Adverse liquidity shocks lower the proceeds from liquidation and as such increase the sunk cost of capital investment from $I - L$ to $I - L_c$. This raises equilibrium profits in booms (i.e. $\overline{\pi}_{jc} = \overline{\pi}_{js}(L_c) > \overline{\pi}_{js}(L)$) and therefore reduces industry output. Apart from this obvious effect, liquidity shocks have no other effect on output in booms. The effect of liquidity shocks on output in recessions is more interesting as there is not only a direct, but potentially also an indirect effect. The cost of holding cash in booms or raising funds in recessions has a direct, positive effect on the equilibrium profit rate in recessions. This direct effect equals $(r + \overline{\lambda})l(C)$ for regime 1, and unambiguously lowers industry output, leading to underinvestment ($\underline{\pi}_{1c} > \underline{\pi}_1(L_c) = \underline{\pi}^o(L_c)$). For regimes 2 and 3 the equilibrium profit rate in recession equals $\underline{\pi}_{jc} = \underline{\pi}_{js}(L_c) + (r+\underline{\lambda})g(C)$.

It is easy to show that $g(C) < l(C)$ for $C > 0$, and therefore the expected cost of the liquidity policy is lower than in regime 1. The reason is that in regimes 2 and 3 holding cash has a beneficial indirect effect because it constrains managers' rent extraction in booms, which in turn allows for a more efficient industry output in recessions. As this indirect effect is dominated by the direct cost effect associated with holding cash, there is still underinvestment in recession ($\underline{\pi}_{jc} > \underline{\pi}_j(L_c) = \underline{\pi}^o(L_c)$ for $j = 2, 3$)

The equilibrium profit rate for regime 2' equals $\underline{\pi}_{2'c} = \underline{\pi}_2(L_c) + (r + \underline{\lambda})l(C) - \frac{r(H_c{}^* - H)}{\overline{p}}$. The expected cost of the liquidity policy, $l(C)$, again raises the equilibrium profit rate, but this is partially offset by the beneficial effect that cash holdings have in terms of constraining managers' excess rents. Since the potential for managers earning excess rents increases for lower levels of investment in human capital, the beneficial disciplining effect increases with $H_c{}^* - H$. It is straightforward to show that the liquidity cost effect dominates the managerial disciplining effect such that overall there is underinvestment in recession (i.e. $\underline{\pi}_{2'c} > \underline{\pi}_{2s}(L_c) = \underline{\pi}^o(L_c)$).

Finally, we find that for regime 3' (i.e. when debt is undercollateralized) the equilibrium profit rate is given by $\underline{\pi}_{3'c} = \underline{\pi}^o(L_c) + (r + \underline{\lambda})l(C) - (r + \underline{\lambda})\phi L_c$. As before the expected liquidity costs $l(C)$ increase the equilibrium profit rate. Comparing with the solution in the absence of liquidity shocks, we still have a term related to the bankruptcy costs ϕL_c. This term reduces equilibrium profits and pushes towards higher industry output. As before bankruptcy costs discourage firms from leaving and increase inertia. However, comparing to the case without liquidity shocks, we no longer have a positive term that is proportional to $(H^{**} - H)$. Cash has a disciplining effect on managers in booms that positively affects industry output in recession. One can show that $l(C) > \phi L_c$, so that overall there is again underinvestment (i.e. $\underline{\pi}_{3'c} > \underline{\pi}^o(L_c)$).

We omit the case that describes the solution for the exit regime when bankruptcy costs are high because the results are similar to the ones described for regime 3 (i.e. net debt increases by $\frac{C(r-\rho)}{r+2\lambda}$, managerial compensation rates are reduced in booms and increased in recession, and the profit rates in recession are increased by $(r + \underline{\lambda})g(C)$).

The following corollary states the condition under which firms find it optimal to hold cash when there is separation of equity and human capital.

Corollary 2 *If bankruptcy costs are sufficiently low ($\phi < \phi_c{}^*$) then the optimal liquidity policy for regimes 1, 2' and 3' minimizes $l(C)$ and is the same as in the owner-manager case (see proposition 10) . For regimes 2 and 3, firms adopt the liquidity policy that minimizes*

$g(C)$ *(defined by equation 29), or equivalently:*

$$C = 0\,(C = \kappa) \iff \theta - 1 \leq (\geq) \frac{(r-\rho)(r+2\underline{\lambda})}{(r+\underline{\lambda})(r+2\overline{\lambda})}$$

Comparing the liquidity policy for regimes 2 and 3 with the first-best liquidity policy, one can immediately see that the former policy is more biased towards holding cash (i.e. $C = \kappa$). Holding cash is costly ($\rho < r$) and forces managers during booms to internalize the cost of a future adverse liquidity shock, and as such has a beneficial disciplining effect for regimes 2 and 3.

Figure 3 illustrates and summarizes the paper's main results. Panel A illustrates the negative relation between the debt principal, D, and the (sunk) cost of human capital investment, H. The relation between D and H is (weakly) decreasing. A higher investment in human capital requires in equilibrium higher managerial remuneration. Therefore, a lower (or even negative) level of debt is required for the bargaining game to lead to an outcome where both parties (equityholders and managers) break even. Moreover, due to the positive cash holding adopted by only a fraction of the firms, the debt level generally differs across the two groups. When the debt is fully collateralized, firms holding cash have a higher debt principal because cash is partly financed by debt. When the debt is undercollateralized ($H < H^{**}$) firms holding cash adopt a lower debt principal to avoid closure (and bankruptcy costs) in recession. The debt principal is not a strictly decreasing function of H. The flat segment is due to the presence of bankruptcy costs. For H falling into the interval $[H^{**}, H^{*}) = [38.4, 48)$ leavers adopt in equilibrium the second-best level of debt equal to $L - \kappa$. Such a policy allows shareholders to avoid bankruptcy costs but is associated with excess rents for managers. In the interval $[H^{**}, \hat{H}^{*}) = [38.4, 45.6)$ managers extract excess rents also in firms with cash. The debt level for these firms equals $L - \kappa + C + \frac{C(r-\rho)}{r+2\overline{\lambda}}$ or $L + \frac{\kappa(r-\rho)}{r+2\overline{\lambda}}$ (since $C = \kappa$), which slightly exceeds L. Still, shareholders of the firms with cash are able, by adopting higher debt, to fully depress managerial rents for $[\hat{H}^{*}, H^{*}) = [45.6, 48)$. For $H < H^{**} = 38.4$ the managerial disciplining effect dominates the expected bankruptcy costs: firms adopt risky debt that increases expected bankruptcy costs but allows for concessions from managers. The figure confirms that for $H < H^{**} = 38.4$ firms that are expected to leave the industry in recession (i.e. the second movers) adopt a risky debt level ($D > L$) that is substantially higher than their surviving counterparts (i.e. the first movers).[22]

[22]To ensure greater transparency, we select parameter values so that region $3'$ (see proposition 11) is degenerate. A non-degenerate region $3'$ would imply that firms holding cash prevent managers from extracting rents for a range of H for which the debt of leaving firms is risky.

The net debt ratio, NDR, is depicted in Panel B.[23] In booms, the $\overline{NDR}$ (as a function of the cost of human capital investment) follows closely the relation between the optimal debt principal and H. The net debt ratio differs across firms and is lower for firms that survive even when the debt is fully collateralized because the firm's cash holdings are netted out against the debt liabilities. In recessions, survivors can have a zero equity value and a market leverage of 100%. A zero equity value means that equityholders inject cash to cover managers' wages and interest repayments. The amount of cash equityholders are required to inject is such that they are exactly indifferent between staying or leaving. For highly human capital intensive firms the net debt ratio becomes negative. Negative net debt ratios are a frequent occurrence in practice. Bates, Kahle, and Stulz (2007) find that in 2004 the average (median) net debt ratio of US firms equals -1.5% (-0.3%). Net debt ratios have been declining substantially over the past decades as the average (median) net debt ratio in 1980 was still 16.5% (17.8%). We conjecture that the decline may be linked to the increased importance of human capital (among other factors).

Panel C plots the equilibrium profit rate. The size of the profit swing between booms and recessions tends to increase in absolute value with the sunk cost H. While the profit rate is (weakly) increasing with H in booms, this is not everywhere the case in recessions because the profit rate $\underline{\pi}$ decreases with H in regime 3 (and, in fact, strictly increases only in regime 2' where $H \in (\hat{H}_c^*, {H_c}^*))$. The implications for industry output are illustrated in Panel D. For $H \geq H^* = 48$ industry output is at the first-best level. In the absence of frictions debt is set optimally to equalize the rents of shareholders and managers. For levels of human capital investment where shareholders constrain the debt to be risk free (i.e. for $H \in [H^{**}, H^*[= [38.4, 48[)$, there is underinvestment in booms and overinvestment in recessions. The underinvestment results from the fact that in booms managers of the future leavers, extract a surplus because the level of debt is capped at $L - \kappa$. The overinvestment results from the disciplining effect that cash has on managers, which causes the actual cost ($g(C)$) of the liquidity policy to be lower than in the first-best scenario ($l(C)$), encouraging more firms to hold cash and therefore to stay in recession. In the absence of cash holdings we would observe the efficient output level in recessions. For very low levels of human capital investment (i.e for $H < H^{**}$) we observe underinvestment in booms because firms adopt a risky debt levels that leads to debt overhang and deadweight costs of bankruptcy. In recessions we observe over or underinvestment depending on whether bankruptcy costs or debt overhang costs dominate. Notice the jump in $\underline{Q}$ at $H^{**} = 38.4$ where the switch from risky to safe debt

[23]The net debt ratio (NDR) is defined as $(B - C)/(E + B - C)$ for $D \geq 0$ and $(D - C)/E$ otherwise. The NDR adjusts the more common leverage ratio for the presence of liquid assets by netting out cash against the firm's debt liabilities.

occurs. At this point overinvestment in recessions can be substantial even for modest levels of bankruptcy costs.[24]

Panel E shows that the managerial compensation rate (weakly) increases as a function of H and is equal to the fair rate of return on human capital investment as long as $H \geq H^* = 48$. For $H \in [H^{**}, H^*) = [38.4, 48)$ managers receive excess rents. For $H < 38.4$, managers are paid below the reservation wage $\underline{w}$ in recession, as stated in proposition 11. However, surviving managers are on average still better off because they enjoy large excess rents in booms.

Finally, panel F plots the payout rate in booms and recessions. A high debt level (corresponding to low values of H) can lead to a negative payout. Equityholders are willing to inject cash into the firm because of the possibility of an economic recovery. The payout rate (weakly) increases in H during booms, whereas in recessions this rate is decreasing in the region in which leavers adopt risky debt (i.e., for $H < H^{**}$) because of falling profits. A higher investment in human capital requires (all else equal) higher equilibrium profits and a lower debt level, both of which increase the free cash flows available for distribution during booms.

6 Empirical Implications

Our theory provides a number of new predictions and insights that are empirically testable:

1) We predict a target debt level D^o that is a linear function of (up to) 6 variables: investment in physical capital (I), the value of the firm's assets upon closure (L), investment in human capital (H), bankruptcy costs (ϕL), cash holdings (C^o) and the opportunity cost of human capital ($\overline{w}$) upon investment:

$$\begin{aligned} D^o \quad &= \quad \beta_{1j}\, I \;+\; \beta_{2j}\, L \;+\; \beta_{3j}\, \phi L \;+\; \beta_{4j}\, \overline{w} \;+\; \beta_{5j}\, H \;+\; \beta_{6j}\, C^o \\ \text{where} \quad j = s \quad & \text{for old firms (first movers or "survivors")} \\ j = l \quad & \text{for young firms (second movers or "leavers")} \end{aligned} \tag{30}$$

D^o is the target amount of outstanding debt on which interest is paid. The debt target is therefore expressed in book value rather than in market value terms. The coefficients β_{kj} are non-linear functions of the macro-economic factors: the risk-free rate of interest, and the hazard of booms and recessions (see propositions 6, 7 and 8). The optimal cashholding C^o is

[24]From proposition 9 we know that for high bankruptcy costs ($\phi \geq \phi^*$) it is even possible to have an interval $[H'', H']$ around H^{**} for which no firms leave the industry in recession and for which regime 'EM' prevails. ϕ^* equals 0.41 for our numerical example.

a function of these macro-factors and of the size of the liquidity shock, and is therefore not in itself an exogenous variable in our model, unlike the other 5 explanatory variables.[25]

The coefficients β_{kj} can vary according to whether the firm is "old" $(j = s)$ or "young" $(j = l)$. The former firm corresponds to first movers that survive recessions, whereas the latter firm relates to second movers that are expected to leave in recession. A dummy variable should be defined that distinguishes both groups so as to allow the regression coefficients to be different for each type of firm.

The model predicts that for human capital intensive industries debt policy is the same across all firms (i.e. $\beta_{ks} = \beta_{kl}$ for $k = 1..6$) and that the optimal debt level is fully collateralized (i.e. $D^o \leq L$). We predict that for physical intensive industries debt is undercollateralized ($D^o > L$) and that the debt policy is different for young and old firms ($\beta_{ks} \neq \beta_{kl}$). In particular, older firms have a lower debt level than younger firms. This difference is reflected in different coefficients for the explanatory variables, which means that the capital structure of each type of firm displays a different degree of sensitivity to explanatory variables such as tangibility and bankruptcy costs.

Importantly, the coefficient of human capital related variables is always negative: leverage declines as firms become more human capital intensive ($\beta_{5j} < 0$) and as the opportunity cost of human capital ($\beta_{4j} < 0$) increases. The coefficient of physical capital intensity and cash holdings is always positive ($\beta_{1j}, \beta_{6j} > 0$). The coefficients of the tangibility and bankruptcy variables depend on whether debt levels are being constrained and whether firms are susceptible to default.[26] Debt is overcollaterized for firms that are relatively more human capital intensive. Default is therefore not an issue, and higher tangibility means that equityholders get more of their capital investment back upon closure and, as a result, are willing to accept a lower debt level allowing more value to shift towards managers. Tangibility and leverage are therefore negatively related when debt is overcollateralized ($\beta_{2j} \leq 0$). When firms are relatively more physical capital intensive then they wish to adopt undercollateralized debt. Bankruptcy costs may, however, discourage firms from adopting a debt level as high as they would wish, causing tangibility to be positively related to leverage: higher tangibility means more collateral and allows firms to issue more debt without increasing the risk for bankruptcy

[25] The firm's cash holding can still be used as a predetermined variable in the equation explaining the debt level as the latter is set *after* the liquidity policy is optimally selected.

[26] The variable L is a proxy for the firm's tangible assets, and is hereafter loosely referred to as the "tangibility" variable. Since the dependent variable is the debt level and not leverage, L has not been scaled by the firm's assets. The current model for the target debt level could, however, be reformulated into a model for (book) leverage by scaling all variables by the firm's assets.

($\beta_{2j} \geq 0$). A positive coefficient for tangibility is therefore indirect evidence that the firm's self-imposed constraint on issuing debt is binding. Similarly, bankruptcy costs only matters for those firms that consider bankruptcy to be an issue in their capital structure decision. Decreased economic uncertainty reduces the sensitivity of leverage to tangibility and bankruptcy costs. In the limiting, deterministic case where booms are expected to last forever (i.e. $\overline{\lambda} = 0$), the tangibility and bankruptcy cost variables no longer influence the optimal leverage ratio (i.e. $\beta_{2j} = \beta_{3j} = 0$).

The coefficient of the cashholding variable is positive but less than one for firms where managers do not enjoy excess rents ($0 < \beta_{6j} \leq 1$). An increase in the equilibrium cash holdings increases debt in such a way that free cash flows (and therefore managers' claim) remain unaltered (ie. $\rho\Delta C - r\Delta D = 0$). The coefficient of the cash to asset ratio equals therefore $\frac{\rho}{r}$, the ratio of the interest on cash (ρ) to the risk-free rate (r) (i.e. $\beta_{6j} = \frac{\rho}{r} \leq 1$). For firms where managers enjoy excess rents, a dollar increase in cash increases debt by more than a dollar. The coefficient on the cash variable therefore exceeds 1 ($\beta_{6j} > 1$). Cash has a disciplining effect, because it forces managers to internalize an anticipated adverse liquidity shock.

2) The paper characterizes the equilibrium profit rate in a competitive industry and therefore provides a measure for the fair rate of return. We show how this return can be decomposed into the return on physical capital and the return on human capital. In booms the fair rate of return for physical capital is the opportunity cost on the capital invested (rI) plus a risk-premium for the sunk cost of investment ($\overline{\lambda}(I - L)$). This latter component can be quite sizeable in some industries and is relevant to competition regulators that determine the fair rate of return. The fair managerial salary rate equals the outside reservation wage ($\overline{w}$) plus a risk premium that is proportional to the amount invested in human capital. Furthermore, the shorter booms and the longer recessions are expected to last, the larger this risk premium to compensate managers for the fact that they may be laid off during recession. In recessions the fair rate of return on physical capital is the opportunity cost on the liquidation value (rL) minus a risk premium ($\underline{\lambda}(I - L)$). The fair managerial salary rate equals the outside reservation wage ($\underline{w}$). Higher sunk costs ($I - L$ and H) reduce output volatility but increase the volatility of dividends, managerial compensation and firm profits.

The model predicts how profits are actually shared between equityholders and managers. We find that the fair (efficient) dividend rate and managerial salary rate are adopted in industries that are highly capital intensive. In physical capital intensive industries bankruptcy costs can lead to firms being underlevered. This further increases volatility in managerial salary by increasing managers' pay in booms and (possibly) reducing managers' pay in recession. In recession managers of surviving firms can even end up being paid below reservation wage.[27]

3) By forcing managers to save for a future liquidity shock, cash holdings have a smoothing effect on managers' compensation. Cash holdings decrease managers compensation in booms, but increase managers' pay in recession.

We show that the optimal cash holdings makes a tradeoff between the costs and benefits of holding cash. Holding cash is costly because cash earns an interest that is below the risk-free rate. However, cash provides the firm with a cushion against a future adverse liquidity shock. Firms that do not have cash and want to survive the shock are forced to raise funds in the capital market. Shorter booms and increased volatility make it more attractive for firms to hold cash. Cash holdings force managers to internalize during good times a future liquidity shock. Holding cash becomes therefore relatively more attractive for firms where managers earn excess rents.

We show that a line of credit can be a substitute for cash holdings. This has implications for the way cash holdings ought to be measured in empirical studies.

4) The model predicts a constant level of long term debt (assuming investment policy is held constant). Since firm value is positively related to profits, it follows that more profitable firms have lower leverage. Market leverage is counter-cyclical: it falls in booms and rises in recessions.

5) Our model predicts that negative net debt occurs in volatile and, especially, human capital intensive industries. This prediction is confirmed by recent empirical evidence. Controlling for a fairly comprehensive list of traditional capital structure determinants, Qian (2003) finds a negative relation between financial leverage and human capital intensity. She shows that human capital intensity has explanatory power in addition to the collateral value of firm assets and the firm's growth opportunities. Bates, Kahle, and Stulz (2007) report that the average (median) net debt ratio for US firms has fallen from 16.5% (17.8%) in 1980 to -1.5% (-0.3%) in 2004. The negative trend is pretty much monotonic over time. They report that the average cash to asset ratio for US firms has increased by 129% over the same period. The paper finds that the rise in cash holdings is linked to an increasing trend in R&D and a decline in firms' net working capital (particularly inventories) and capital expenditures. The authors also show that the increase in cash flow risk has contributed to an increase in cash holdings, and conclude that the findings are "consistent with an explanation for the change in cash

[27]For example, GM's CEO Rick Wagoner and his counterpart at Ford, Alan Mulally, offered to accept salaries of $1 conditional on the implementation of the US federal government bailout plan, cf. "High price of a government lifeline to US carmakers", *Financial Times*, 12 Dec 2008.

holdings that relies on the precautionary motive and on changes in firm characteristics which affect the demand for cash by firms". We argue that over the past decades firms have become more reliant on transferable human capital and less on physical capital. This fundamental change in the nature of the firm has been reflected in firms' capital structure and liquidity policy.

7 Conclusions

This paper analyzes how corporate capital structure, liquidity, and managerial compensation are affected by economic uncertainty and the extent to which firms rely on human capital and physical capital. Equityholders provide the physical capital, which can be (partially) reclaimed by closing down the firm. Wealth constrained managers supply their human capital which is transferable to other firms within the industry. When the industry goes through a downturn, managers can leave the industry and earn an outside reservation wage. We consider a tax free industry where firms that are ex ante identical operate under perfect competition and are subject to industry-wide liquidity shocks and demand shocks.

The model and its results depend on two important differences between physical capital and human capital. First, while the firm owns the physical capital, it has no property rights over human capital. Human capital can at any time leave the firm to join another firm, hereby blurring the boundaries of the firm. Second, since physical capital can be sold, it can serve as collateral for the firm's corporate debt. The firm can, however, not borrow against its human capital. It is this asymmetry that can cause net debt to be negative in our model. Managers only invest in human capital if they expect to be compensated ex post. In human capital intensive industries equityholders therefore contribute a net surplus of liquid assets that throw off rents to be shared with the managers. This surplus of liquid assets (i.e. negative debt) ensures that managers get a fair return on their investment. If managers finance their investment in human capital by personal debt (instead of savings) then these rents serve to pay off the managers' debt. Negative net corporate debt therefore indirectly creates space for personal debt taken on by the firm's managers or employees. While the firm cannot borrow against human capital, its managers or employees can take out personal debt against the rents produced by the firm's assets. Transferable human capital is financed by the manager and not by the firm in order to overcome a hold-up problem: if a manager withdraws her human capital from the firm, then any financial liabilities associated with this key asset also leave the firm.

The paper provides a series of novel empirical predictions that are listed in the previous section. Our proposed linear regression model for the firm's debt target could, for example, form the basis of an empirical study. While the empirical model itself is simple, the main challenge for empiricists will be to construct suitable proxies for the human capital related variables. We refer to Qian (2003) for examples of possible proxies.

There are also theoretical extensions that remain to be explored. While financing policy is allowed to vary across firms, we hold investment and production policy constant, ignoring the effect of growth options or heterogeneity of productivity. The paper also assumes that investment in human capital can be financed efficiently. Credit rationing or frictions in the market for personal debt could lead to underinvestment in human capital and have effects on the industry equilibrium. Finally, the paper does not consider managers' incentives to invest in firm specific human capital or to exert effort. These incentives are particularly relevant for managers that leave the industry in recession.

8 Appendix

Proof of Proposition 1

The claim value $\overline{\delta}$ must satisfy the following equilibrium relationship: $r\overline{\delta} = \overline{\lambda}\left[1 - \overline{\delta}\right]$. Solving gives the expression for $\overline{\delta}$.
The value $\overline{\Pi}$ of a claim that pays $\overline{\pi}$ for as long as the current boom lasts must satisfy the following equation: $r\overline{\Pi} = \overline{\pi} + \overline{\lambda}\left[0 - \overline{\Pi}\right]$. Solving gives: $\overline{\Pi} = \frac{\overline{\pi}}{r+\overline{\lambda}}$.
In booms (recession) the value $\overline{V}$ ($\underline{V}$) of a perpetual claim that pays $\overline{\pi}$ during booms and $\underline{\pi}$ during recessions satisfies: $r\overline{V} = \overline{\pi} + \overline{\lambda}\left[\underline{V} - \overline{V}\right]$ and $r\underline{V} = \underline{\pi} + \underline{\lambda}\left[\overline{V} - \underline{V}\right]$. Solving this system of 2 equations gives the expressions for $\overline{V}$ and $\underline{V}$.

Proof of Proposition 2

The derivation is given in the main text.

Proof of Proposition 3

This is a two player game ($i = 1, 2$) between equityholders and managers where each player has an outside option and where there is a risk of breakdown during the negotiation (see Shaked and Sutton (1984) and Binmore, Rubinstein, and Wolinsky (1986)). Assume that equityholders are the first to make an offer at time $t = 0$. Managers can then make one of the following 3 possible decisions (1) accept the offer (2) reject the offer, but keep bargaining (3) reject the offer and stop bargaining by exercising their outside option. Decisions (1) and (3) lead to an immediate payoff at time $t = 0$, whereas decision (2) allows managers to make a counter offer at time $t = \tau$ provided that breakdown (i.e. recession) does not occur during the interval τ. If breakdown occurs during the interval τ then equityholders and managers receive the payoff $\underline{E}$ and $\underline{M}$, respectively, at time $t = \tau$.

The probability of a recession occurring at the end of interval τ is given by $\frac{\overline{\lambda}}{\overline{\lambda}+\underline{\lambda}}(1-e^{-(\overline{\lambda}+\underline{\lambda})\tau}) \equiv \overline{\beta}$. Therefore decision (2) leads with probability $1-\overline{\beta}$ to a counteroffer by managers at time $t = \tau$. Since both parties are risk-neutral the discount factor (per period) is the same and given by ρ, where $\rho \equiv\ e^{-r\,\tau}$.

Let a_e and A_e be the infimum and supremum of equilibrium payoffs to equityholders in the game where it is equityholders who makes the first offer. Let a_m and A_m be the infimum and supremum of equilibrium payoffs to managers in the companion game where it is managers' turn to make an offer.

Consider the subgame at $t = 2\tau$ that begins with an offer by equityholders. At $t = 2\tau$ (assuming a recession has not yet arrived) the subgame has the same structure as the original game that started at time $t = 0$ (all games that begin with an offer by equityholders are isomorphic). Therefore at $t = 2\tau$ the supremum of the share that equityholders can get must again equal A_e.

Consider next the offer made by managers in the preceding period at $t = \tau$. At $t = \tau$ the present value of shareholders' expected payoff equals at most $\max[o_e, (1-\overline{\beta})\rho A_e + \overline{\beta}\rho\underline{E}]$. Therefore, any offer by managers at $t = \tau$ which gives equityholders an amount $\max[o_e, (1-\overline{\beta})\rho A_e + \overline{\beta}\rho\underline{E}]$ will certainly be accepted. Consequently, the value that managers get in any perfect equilibrium cannot be less than $\overline{v} - \max[o_e, (1-\overline{\beta})\rho A_e + \overline{\beta}\rho\underline{E}]$. Therefore the infimum of managers' payoff in this subgame satisfies:

$$a_m \;\geq\; \overline{v} - \max[o_e, (1-\overline{\beta})\rho A_e + \overline{\beta}\rho\underline{E}] \tag{31}$$

Consider next equityholders' offer at $t = 0$. Any offer that gives managers less than $\max[o_m\,, (1-\overline{\beta})\rho\, a_m + \overline{\beta}\rho\underline{M}]$ will certainly be rejected. Hence at $t = 0$ equityholders receive at most $\overline{v} - \max[o_m\,, (1-\overline{\beta})\rho\, a_m + \overline{\beta}\rho\underline{M}]$. Consequently, the supremum of equityholders' payoff in any perfect equilibrium where they make the offer satisfies the following condition:

$$A_e \;\leq\; \overline{v} - \max[o_m\,, (1-\overline{\beta})\rho\, a_m + \overline{\beta}\rho\underline{M}] \tag{32}$$

The preceding argument can now be repeated to derive the following analogous conditions for A_m and a_e:

$$A_m \;\leq\; \overline{v} - \max[o_e\,, (1-\overline{\beta})\rho a_e + \overline{\beta}\rho\underline{E}] \tag{33}$$

$$a_e \;\geq\; \overline{v} - \max[o_m\,, (1-\overline{\beta})\rho A_m + \overline{\beta}\rho\underline{M}] \tag{34}$$

Depending on the values for $\overline{o}_e$ and $\overline{o}_m$ we can now distinguish the following cases.

<u>Case 1)</u> $\overline{o}_1 \;\leq\; (1-\overline{\beta})\rho a_e + \overline{\beta}\rho\underline{E}$ and $\overline{o}_2 \;\leq\; (1-\overline{\beta})\rho a_m + \overline{\beta}\rho\underline{M}$

Combining (31) and (32) and solving for A_e and a_m gives

$$A_e \;\le\; \frac{\overline{v}\left[1 - (1-\overline{\beta})\rho\right] - \overline{\beta}\,\rho\,\underline{M} + (1-\overline{\beta})\overline{\beta}\rho^2\underline{E}}{1-(1-\overline{\beta})^2\rho^2} \;\equiv\; \overline{P_e} \tag{35}$$

$$a_m \;\ge\; \frac{\overline{v}\left[1 - (1-\overline{\beta})\rho\right] - \overline{\beta}\,\rho\,\underline{E} + (1-\overline{\beta})\overline{\beta}\rho^2\underline{M}}{1-(1-\overline{\beta})^2\rho^2} \;\equiv\; \overline{P}_m \tag{36}$$

Analogously, solving (33) and (34) $a_e \ge \overline{P}_e$ and $A_m \le \overline{P}_m$. Using the fact that $a_i \le A_i$ (for $i = e, m$), it follows that there is a unique perfect equilibrium with $\overline{E} = A_e = a_e = \overline{P}_e$ and $\overline{M} = A_m = a_m = \overline{P}_m$. Substituting the solutions for a_e and a_m into the conditions for $\overline{o}_e$ and $\overline{o}_m$, we find that this case occurs for the parameter configurations that satisfy the following conditions:

$$o_e \;\le\; \frac{(1-\overline{\beta})\left[1-(1-\overline{\beta})\rho\right]\rho\overline{v} - (1-\overline{\beta})\overline{\beta}\rho^2\underline{M} + \overline{\beta}\rho\underline{E}}{1-(1-\overline{\beta})^2\,\rho^2} \;\equiv\; \overline{o}_e{}^*(\tau) \tag{37}$$

$$o_m \;\le\; \frac{(1-\overline{\beta})\left[1-(1-\overline{\beta})\rho\right]\rho\overline{v} - (1-\overline{\beta})\overline{\beta}\rho^2\underline{E} + \overline{\beta}\rho\underline{M}}{1-(1-\overline{\beta})^2\,\rho^2} \;\equiv\; \overline{o}_m{}^*(\tau) \tag{38}$$

Equityholders' offer will be accepted immediately (to prove this assume, by contradiction, that managers could do better by postponing till next period). Taking the limit for $\tau \to 0$ the solution simplifies to:

$$\begin{aligned}\overline{E} &= \lim_{\tau\to 0}\overline{P}_e(\tau) = \frac{\overline{v}}{2} + \frac{\overline{\delta}}{2}(\underline{E} - \underline{M}) = \lim_{\tau\to 0}\overline{o}_e{}^*(\tau) \equiv \overline{o}_e{}^* \\ \overline{M} &= \lim_{\tau\to 0}\overline{P}_m(\tau) = \frac{\overline{v}}{2} - \frac{\overline{\delta}}{2}(\underline{E} - \underline{M}) = \lim_{\tau\to 0}\overline{o}_m{}^*(\tau) \equiv \overline{o}_m{}^*\end{aligned} \tag{39}$$

The symmetry of the solution reveals immediately that a party's payoff does not depend on whether it makes or receives the offer. Case 1 occurs if $\overline{o}_e \le \overline{o}_e{}^*$ and $\overline{o}_m \le \overline{o}_m{}^*$

Case 2) $\overline{o}_e \ge (1-\overline{\beta})\rho A_e + \overline{\beta}\rho\underline{E}$ and $\overline{o}_m \le (1-\overline{\beta})\rho a_m + \overline{\beta}\rho\underline{M}$

Substituting the above inequalities into conditions (31) to (34) it follows immediately from $a_m \le A_m$ that $A_m = a_m = \overline{v} - o_e$. Substituting this result in the conditions for a_e and A_e gives $a_e = A_e = \overline{v} - (1-\overline{\beta})\rho(\overline{v} - o_e) - \overline{\beta}\rho\underline{M}$.

Substituting the solutions for A_e and a_m into the conditions for $\overline{o}_e$ and $\overline{o}_m$, we find that this case occurs for the parameter configurations that satisfy the following conditions: $o_e \ge o_e{}^*$ and $(1-\overline{\beta})\rho o_e + o_m \le (1-\overline{\beta})\rho\overline{v} + \overline{\beta}\rho\underline{M}$.

Taking the limit for $\tau \to 0$ we find that $\overline{E} = \overline{o}_e$ and $\overline{M} = \overline{v} - \overline{o}_e$, and that case 2 occurs if $\overline{o}_e \ge \overline{o}_e{}^*$ and $\overline{o}_e + \overline{o}_m \le \overline{v}$.

Case 3) $\overline{o}_e \le (1-\overline{\beta})\rho a_e + \overline{\beta}\rho\underline{E}$ and $\overline{o}_m \ge (1-\overline{\beta})\rho A_m + \overline{\beta}\rho\underline{M}$

This case is analogous as case 2 and gives as equilibrium for $\tau \to 0$: $\overline{M} = \overline{o}_m$ and $\overline{E} = \overline{v} - \overline{o}_m$. Case 3 occurs if $\overline{o}_m \ge \overline{o}_m{}^*$ and $\overline{o}_e + \overline{o}_m \le \overline{v}$.

Case 4) $\overline{o}_e \geq (1-\overline{\beta})\rho A_e + \overline{\beta}\rho \underline{E}$ and $\overline{o}_m \geq (1-\overline{\beta})\rho A_m + \overline{\beta}\rho \underline{M}$

Solving gives: $a_e = A_e = \overline{v} - o_m$ and $a_m = A_m = \overline{v} - o_e$. Substituting the solutions for A_e and A_m into the conditions for $\overline{o}_e$ and $\overline{o}_m$, we find that this case occurs for the parameter configurations that satisfy the following conditions: $\overline{o}_e + (1-\overline{\beta})\rho\overline{o}_m \geq (1-\overline{\beta})\rho\overline{v} + \overline{\beta}\rho\underline{E}$ and $(1-\overline{\beta})\rho\overline{o}_e + \overline{o}_m \geq (1-\overline{\beta})\rho\overline{v} + \overline{\beta}\rho\underline{M}$. Taking the limit for $\tau \to 0$ the condition simplifies to $\overline{o}_e + \overline{o}_m \geq \overline{v}$. Imposing the feasibility constraint $\overline{o}_e + \overline{o}_m \leq \overline{v}$, it follows that: $\overline{o}_e + \overline{o}_m = \overline{v}$. Consequently, $\overline{E} = \overline{o}_e$ and $\overline{M} = \overline{o}_m$.

Focussing on the result for $\tau = 0$, it follows that the regions covered by the above 4 cases cover all possible feasible combinations for $\overline{o}_e$ and $\overline{o}_2$ within the triangular space defined by the constraints $\overline{o}_e, \overline{o}_m \geq 0$ and $\overline{o}_e + \overline{o}_m \leq \overline{v}$.

Proof of Proposition 4

The proof is given in the main text.

Proof of Proposition 5

The proof is given in the main text. To see that it is never optimal for a manager not to return to a firm in a boom (and to receive outside wage $\overline{w}$ only), once can easily check that profit flows $\overline{\pi}$ in three basic regimes as described in propositions 6-8 are (weakly) greater than $\overline{\pi}^o$, which, in turn, is greater than $rI + \overline{\lambda}(I - (1-\phi\xi)L)$. Moreover, the output in region 'EM' (see proposition 9) is never greater than $\overline{\pi}^o$ either. Therefore, receiving in the boom managerial compensation $\overline{s_l}$ instead of reservation wage $\overline{w}$ will always be a dominant strategy for a manager exercising his outside option in recession.

Proof of Propositions 6, 7 and 8

We start off by assuming that it is optimal for some firms to leave in recession (i.e. $\overline{Q} > \underline{Q}$) and subsequently derive the condition under which this assumption is valid. We conclude the proof by deriving the equilibrium for the case where this condition is violated (and therefore no firms leave in recession).

We derive first the policies and claim value for firms that exit in recession, and subsequently derive the solution for firms that remain in the industry at all times. The equilibrium solution for the industry output in booms ($\overline{Q}$) and recessions ($\underline{Q}$) allows us subsequently to derive the condition under which $\overline{Q} > \underline{Q}$. We derive the proof assuming equityholders set debt policy, but show that equityholders' participation constraint is always binding in a competitive equity market and that managers' (equityholders') claim decreases (increases) in the debt level. As a result the same debt level is also constrained optimal from managers' viewpoint.

I. Policies and claim values for firms that exit in recession

Upon entry equityholders of firms that leave in recession solve the following optimization problem: $\max_{D_l} \left[\overline{E}_l - \left(I - \overline{B}_l\right)\right] \;\; s.t. \;\; \overline{M}_l \geq \overline{W} + H$. One can show that:

$$\overline{E}_l + \overline{B}_l \;=\; \frac{\overline{\pi} + r\,D + 2\overline{\lambda}L(1-\phi\xi)}{2(r+\overline{\lambda})} \; where\; \xi = 1 \; if \; D > L \; and \; zero \; otherwise$$

Equityholders' payoff $\overline{E}_l + \overline{B}_l - I$ is monotonically increasing in D, except at $D = L$ where there is a discrete downward jump because of the deadweight bankruptcy costs ϕL. It follows that it can only be optimal to adopt risky debt ($D > L$) rather than constrain the debt to the maximum safe debt level ($D = L$) if and only if:

$$\frac{\overline{\pi} + r\,D + 2\overline{\lambda}L(1-\phi)}{2(r+\overline{\lambda})} \;>\; \frac{\overline{\pi} + r\,L + 2\overline{\lambda}L}{2(r+\overline{\lambda})} \;\iff\; D \;>\; L + \frac{2\overline{\lambda}L\phi}{r} \tag{40}$$

Managers' participation constraint requires that:

$$\overline{M}_l \;=\; \frac{(\overline{\pi} - r\,D_l)}{2r}(1-\overline{p}) + \frac{w}{r}\overline{p} \;\geq\; \overline{W} + H \;\iff\; r\,D \;\leq\; \overline{\pi} - 2\left[\overline{w} + \frac{rH}{1-\overline{p}}\right] \tag{41}$$

To identify the maximum debt principal that can be issued, we first need to solve for $\overline{\pi}$. With competitive entry $\overline{\pi}$ is the solution to $\overline{E}_l = I - \overline{B}_l$, and is given by:

$$\overline{\pi} \;=\; 2\left[r\,I + \overline{\lambda}\,(I-L) + \phi\overline{\lambda}L\xi\right] - r\,D \tag{42}$$

Substituting (42) into (41), we find that the maximum debt level that satisfies managers' participation constraint equals:

$$r\,D_l \;=\; rI + \overline{\lambda}\,(I - L) + \overline{\lambda}\phi L\xi - \overline{w} - \frac{rH}{1-\overline{p}} \tag{43}$$

Using (40) and (43) it follows that equityholders choose risky debt if and only if:

$$r\,D_l \;=\; rI + \overline{\lambda}\,(I - L) + \overline{\lambda}\phi L - \overline{w} - \frac{rH}{1-\overline{p}} \;>\; r\,L + 2\overline{\lambda}\phi L \tag{44}$$

$$\iff \left(r+\overline{\lambda}\right)(I-L) - \overline{\lambda}\phi L \;>\; \overline{w} + \frac{r\,H}{1-\overline{p}} \;\iff\; H \;< H^{**} \tag{45}$$

where H^{**} is defined as: $\left(r+\overline{\lambda}\right)(I-L) - \overline{\lambda}\phi L \;=\; \overline{w} + \frac{r\,H^{**}}{1-\overline{p}}$. It follows immediately that:

$$\overline{B}_l \;>\; L \;\iff\; \frac{rD_l}{r+\overline{\lambda}} + \frac{\overline{\lambda}L(1-\phi)}{r+\overline{\lambda}} \;>\; L \;\iff\; (I-L)(r+\overline{\lambda}) > \overline{w} + \frac{rH}{1-\overline{p}} \tag{46}$$

which is satisfied since by assumption $H < H^{**}$.

Consider next the case where the optimal debt level is safe ($D_l \leq L$). From (43) (with $\xi = 0$) it follows that:

$$r\,D_l \;=\; rI + \overline{\lambda}\,(I - L) + \overline{\lambda}\phi L - \overline{w} - \frac{rH}{1-\overline{p}} \;<\; r\,L$$

$$\iff (r+\overline{\lambda})(I-L) \;<\; \overline{w} + \frac{rH}{1-\overline{p}} \;\iff\; H \;>\; H^{*} \tag{47}$$

where H^* is defined as:$(r+\overline{\lambda})(I-L) = \overline{w} + \frac{rH^*}{1-\overline{p}}$. Therefore, $D_l < L$ is optimal for $H^* < H < \tilde{H}$. The equilibrium profit rate for $H > H^*$ can be found by substituting back the optimal debt level into (42). The managerial compensation rate in booms is given by solving $\overline{s}_l = \frac{\overline{\pi} - r\,D_l}{2}$.

We know that $D_l > L$ for $H < H^{**}$ and that $D_l < L$ for $H > H^*$. What is the optimal debt level for the intermediate interval $[H^{**}, H^*]$? Since $H > H^{**}$ it follows that $D_l \leq L$. Therefore L is the highest debt level that equityholders wish to adopt. Managers' participation constraint requires that $\overline{M}_l \geq \overline{W} + H$ or equivalently:

$$rL \leq \overline{\pi} - 2\left[\overline{w} + \tfrac{rH}{1-\overline{p}}\right] = 2\left[rI + \overline{\lambda}(I-L)\right] - rL - 2\left[\overline{w} + \tfrac{rH}{1-\overline{p}}\right] \tag{48}$$

$$\iff \overline{w} + \tfrac{rH}{1-\overline{p}} \leq (r+\overline{\lambda})(I-L) \iff H \leq H^* \tag{49}$$

which is satisfied since $H \in [H^{**}, H^*]$. Consequently, for $H \in [H^{**}, H^*[$ the managers' participation constraint does not bind, i.e. $\overline{M}_l > \overline{W} + H$.

II. Policies and claim values for firms that do not exit in recession

Since the debt is risk-free it follows that $\overline{B}_s = \underline{B}_s = D_s$. The claim values for firms that do not exit are given by:

$$\begin{aligned}
\overline{E}_s &= \tfrac{\overline{\pi} - \overline{s}_s}{r}(1-\overline{p}) + \tfrac{\overline{\pi} - \underline{s}_s}{r}\,\overline{p} - D_s & \overline{M}_s &= \tfrac{\overline{s}_s}{r}(1-\overline{p}) + \tfrac{\underline{s}_s}{r}\,\overline{p} \\
\underline{E}_s &= \tfrac{\overline{\pi} - \underline{s}_s}{r}(1-\underline{p}) + \tfrac{\overline{\pi} - \overline{s}_s}{r}\,\underline{p} - D_s & \underline{M}_s &= \tfrac{\underline{s}_s}{r}(1-\underline{p}) + \tfrac{\overline{s}_s}{r}\,\underline{p}
\end{aligned}$$

In a competitive equilibrium with exit, the outside options of both equityholders and managers bind in recession, and therefore $\underline{E}_s = \underline{o}_e$ and $\underline{M}_s = \underline{o}_m$. We first derive explicit expressions for $\underline{o}_m$. Using our solution for $\overline{\pi}$ it follows that $\xi = 0$ for regimes 1 and 2 ($H^{**} \leq H$) and $\xi = 1$ for regime 3 ($H < H^{**}$), and that $\overline{\pi} > \overline{\pi}_w$. Substituting the solution for $\overline{\pi}$ into the expression for $\underline{o}_m$ (see proposition 5) gives:

$$\begin{aligned}
\underline{o}_m &= \tfrac{\underline{w}}{r}(1-\underline{p}) + \frac{\left(\overline{w} + \frac{rH}{1-\overline{p}}\right)}{r}\underline{p} &&= \underline{W} + \underline{\delta}\,H = \underline{M}_l \quad for\ H^* \leq H \\
\underline{o}_m &= \tfrac{\underline{w}}{r}(1-\underline{p}) + \frac{\left((r+\overline{\lambda})(I-L)\right)}{r}\underline{p} &&= \underline{M}_l > \underline{W} + \underline{\delta}\,H \quad for\ H^{**} \leq H < H^* \\
\underline{o}_m &= \tfrac{\underline{w}}{r}(1-\underline{p}) + \frac{\left(\overline{w} + \frac{rH}{1-\overline{p}}\right)}{r}\underline{p} &&= \underline{W} + \underline{\delta}\,H = \underline{M}_l \quad for\ H < H^{**}
\end{aligned}$$

Note that $\underline{M}_s = \underline{M}_l$, and firms are therefore indifferent between leaving or staying.
Equityholders solve the following constrained optimization problem:

$$\max_{D_s} \overline{E}_s - \left(I - \overline{B}_s\right) \ s.t.\ \underline{M}_s = \underline{o}_m \ and\ \overline{M}_s \geq \overline{W} + H$$

Consider first the effect of an increase in D_s on managers' participation constraint. We know that $\underline{M}_s = \underline{M}_l$ and consequently:

$$\underline{s}_s = \left[r\underline{M}_l - \frac{\overline{\pi}\underline{p}}{2} + \frac{rD_s\,\underline{p}}{2}\right]\frac{1}{1-\underline{p}} \tag{50}$$

Substituting (50) into $\overline{M}_s$ gives: $\overline{M}_s = \frac{\overline{\pi} - rD_s}{2(r+\overline{\lambda})} + \overline{\delta}\underline{M}_l$. Note that $\underline{M}_l$ is determined by $\overline{s}_l$ and $\underline{w}$, which are unaffected by the behavior of firms that do not exit (since $\overline{s}_l = \frac{\overline{\pi} - rD_l}{2}$ is determined by

the marginal entrant that leaves the market during recessions). Consequently, equityholders of firms that do not exit affect $\underline{M}_s$ only through D_s, and therefore $\frac{\partial \overline{M}_s}{\partial D_s} < 0$. Increasing D_s unambiguously lowers managers' claim, and the participation constraint therefore puts a cap on D_s.
Consider next the effect of an increase in D_s on the equityholders' payoff. Using the bargaining solution for $\overline{s}_s$ we get:

$$\overline{E}_s + D_s = \frac{\overline{\pi}}{r}(1-\overline{p}) - \frac{(\overline{\pi} - rD_s)}{2r}(1-\overline{p}) + \frac{(\pi - \underline{s}_s)}{r}\overline{p} \tag{51}$$

Using (50) we find that $\frac{\partial \underline{s}_s}{\partial D_s} = \frac{r\underline{p}}{2(1-\underline{p})}$. Substituting into the equityholders' payoff function it follows that:$\frac{\partial\left[\overline{E}_s + D_s - I\right]}{\partial D_s} > 0$. It follows that raising D_s unambiguously increases equityholders' payoff. Equityholders therefore want to adopt the highest debt level that satisfies managers' participation constraint. The equilibrium condition requires that $\underline{M}_s = \underline{M}_l$, which ensures that managers do not have an incentive to leave in recession. It remains to be shown that also $\overline{M}_s \geq \overline{W} + H$. We know that $\overline{M}_s = \frac{\overline{s}_s}{r+\overline{\lambda}} + \overline{\delta}\underline{M}_s \geq \overline{W} + H$. Since $\underline{M}_s = \underline{M}_l \geq \underline{W} + \underline{\delta}H$, a sufficient condition for the participation constraint to be satisfied is that:

$$\overline{s}_s \geq \left(r + \overline{\lambda}\right)\left[\overline{W} + H - \overline{\delta}\underline{W} - \overline{\delta}\underline{\delta}H\right] \iff \overline{s}_s \geq \overline{w} + \frac{rH}{1-\overline{p}} \tag{52}$$

We can subsequently verify that the equilibrium solution indeed satisfies this condition.

The equilibrium solution for $\underline{\pi}$, $\overline{s}_s$, $\underline{s}_s$ and D_s can now be derived as the solution to the system of equations: (i) $\underline{E}_s = \underline{o}_e$ (ii) $\underline{M}_s = \underline{o}_m$ (iii) $\overline{s}_s = \frac{\overline{\pi}-rD_s}{2}$ and (iv) $\overline{E}_s = I - D_s$. Condition (iv) reflects the fact that outside equity is supplied on a competitive basis.

Substituting the previously derived expressions for $\overline{\pi}$ and $\underline{o}_m$ into the system of equations, and solving, gives the expressions for $\underline{\pi}$, $\overline{s}_s$, $\underline{s}_s$ and D_s as in propositions 6, 7 and 8. One can immediately verify that $\overline{M}_s \geq \overline{M}_l \geq \overline{W} + H$, and therefore managers' participation constraint is satisfied.

We now need to verify whether $\overline{Q} > \underline{Q}$ as originally assumed. In regime 1, the solution for $\overline{\pi}$ and $\underline{\pi}$ coincides with the first-best outcome given in proposition 2. Since $H < \tilde{H}$ by assumption, it follows immediately that exit occurs for $H^* \leq H < \tilde{H}$ (i.e. regime 1). In regime 2, $\overline{\pi}$ and $\underline{\pi}$ are independent of H, and hence $\overline{Q}$ and $\underline{Q}$ are constant, and therefore also $\overline{Q} - \underline{Q}$. In regime 3, $\frac{\partial \overline{\pi}}{\partial H} > 0$ and $\frac{\partial \underline{\pi}}{\partial H} < 0$, which means that $\frac{\partial\left[\overline{Q}-\underline{Q}\right]}{\partial H} < 0$. Furthermore, one can show that $\overline{\pi}$ (and therefore $\overline{Q}$) is continuous at H^{**}, whereas $\underline{\pi}$ ($\underline{Q}$) displays a discrete downward (upward) jump at H^{**} if $\phi > 0$. It follows that if some firms leave in recession for $H = H^{**}$ (i.e. if $\overline{Q}(H^{**}) > \underline{Q}(H^{**})$) then exit occurs for all $H < \tilde{H}$. A sufficient and necessary condition for exit to occur for all $H < \tilde{H}$ is therefore that $\overline{Q}(H^{**}) > \underline{Q}(H^{**})$. Substituting the expression for H^{**} into $\overline{\pi}$ and $\underline{\pi}$ gives:

$$\overline{\pi}(H^{**}) = rI + \left(r + 2\overline{\lambda}\right)(I - L) \tag{53}$$

$$\underline{\pi}(H^{**}) = rL - \underline{\lambda}(I - L) + \underline{w} - \frac{\underline{\lambda}\overline{\lambda}\phi L}{r + \overline{\lambda}} \tag{54}$$

The condition for exit therefore becomes:

$$\overline{\pi}^{-1}\left[r\,I + \left(r + 2\overline{\lambda}\right)(I-L)\right] \;>\; \underline{\pi}^{-1}\left[r\,L - \underline{\lambda}\,(I-L) + \underline{w} - \frac{\underline{\lambda}\overline{\lambda}\phi L}{r+\overline{\lambda}}\right]$$

This condition is satisfied for $\phi = 0$ since in that case $H^{**} = H^{*}$ and we previously showed that exit occurs at H^{*}. However, since $\frac{\partial \underline{Q}(H^{**})}{\partial \phi} > 0$, it follows that for ϕ sufficiently large there will be a crossing point where $\overline{Q} = \underline{Q}$. Define ϕ^{*} as the root of the equation $\overline{Q}(H^{**}(\phi^{*})) = \underline{Q}(H^{**}(\phi^{*}))$. If $\phi < \phi^{*}$ then the exit condition is everywhere satisfied. If $\phi \geq \phi^{*}$ then the exit condition is violated at H^{**}. Therefore, if $\phi > \phi^{*}$ then there exists a region for which no firms leave the market in recession. Competitive pressure means that as many firms enter in booms as the market can support in recession. This means that in recession firms are indifferent between staying or leaving or equivalently: (i) $\underline{E}_s = \underline{o}_e$ and (ii) $\underline{M}_s = \underline{o}_m$. The bargaining solution, and the competitive supply of outside equity also imply that in this 'no-exit' equilibrium (iii) $\overline{s}_s = \frac{\overline{\pi} - r\,D_s}{2}$ and (iv) $\overline{E}_s = I - D_s$. This system allows us to solve for the (constant) equilibrium output $\tilde{Q}$, and for $\overline{s}_s$, $\underline{s}_s$ and D_s. We call this equilibrium 'regime EM' (see proposition 9).

Proof of Proposition 9

To determine the interval for H over which this regime 'EM' prevails, we first observe that, starting from the left, firms behave according to regime EM for as long as more firms can enter under regime EM than under regime 3. Equivalently, they do so for as long as:

$$\overline{\pi} \;\leq\; r\,I + \overline{\lambda}\,(I - (1-\phi)L) + \overline{w} + \tfrac{r\,H}{1-\overline{p}} \tag{55}$$

$$\Longleftrightarrow \quad \overline{\pi} - r\,I - \overline{\lambda}\,(I - (1-\phi)L) \leq \overline{w} + \tfrac{r\,H}{1-\overline{p}} \tag{56}$$

$$\Longleftrightarrow \quad \underline{o}_m \;\leq\; \underline{W} + \underline{\delta}\,H \tag{57}$$

Let H'' be part of regime EM. In particular, let H'' be the solution to the equation $\underline{o}_m = \underline{W} + \underline{\delta}\,H''$. It follows that for $H < H''$ regime 3 prevails, whereas for some interval $[H'', H']$ regime EM prevails. By construction $\overline{\pi}(\overline{Q}(H))$ is continuous at H'', and therefore also $\overline{Q}(H)$ is continuous. Furthermore, also $\underline{\pi}(\underline{Q}(H))$ is continuous at H'' because the conditions that determine $\underline{\pi}$ are exactly the same for both regimes EM and 3. Since there is no exit under regime EM, it follows that $\overline{Q}(H'') = \underline{Q}(H'')$. This implies that H'' is the solution to:

$$\lim_{\epsilon\to 0}\overline{\pi}^{-1}\left[r\,I + \overline{\lambda}\,(I-L) + \overline{\lambda}\phi L + \overline{w} + \frac{r\,(H'' + \epsilon)}{1-\overline{p}}\right] \tag{58}$$

$$= \lim_{\epsilon\to 0}\underline{\pi}^{-1}\left[(r+\underline{\lambda})D_s - \underline{\lambda}I + \underline{s}_s\right] \tag{59}$$

$$= \lim_{\epsilon\to 0}\underline{\pi}^{-1}\left[r\,I + \overline{\lambda}\,(I-L) + \overline{\lambda}\phi L + \underline{w} - \frac{\overline{w}}{1-\underline{p}} - \frac{r\,(H'' + \epsilon)}{(1-\underline{p})(1-\overline{p})}\right] \tag{60}$$

We now determine H', the point at which we switch from regime EM to regime 2. From conditions (i) to (iv), it follows that the equilibrium EM is independent of H. In other words, the industry output remains constant over the interval $[H'', H']$. The number of firms in regime EM exceeds the number of firms under regime 2 because:

$$\overline{\pi}_{EM} = r\,I + \overline{\lambda}(I - (1-\phi)L) + \overline{w} + \frac{rH}{1-\overline{p}} < r\,I + \left(r + 2\overline{\lambda}\right)(I - L) \text{ } (since\ H'' < H^{**})$$

Regime EM therefore dominates regime 2 for as long as regime EM is feasible, or equivalently for as long as the managers' participation constraint $\overline{M}_s \geq \overline{W} + H$ is satisfied. Increasing H tightens this constraint, and H' is the level of H for which $\overline{M}_s = \overline{W} + H'$. Therefore, for $H > H'$, we have $\overline{M}_s < \overline{W} + H$ and regime EM is not feasible, causing regime 2 to prevail.

Proof of Proposition 10

The proof is given in the main text.

Proof of Proposition 11

Since it is not optimal for leavers to hold cash, the value and policies of firms leaving the industry is unaffected (except for the fact that L is replaced by L_c). Managers' and equityholders' outside options bind in recession.

The value of survivors is determined by the following conditions: (i) $\overline{E}_c = I + C - D_c$, (ii) $\underline{M}_c = \underline{o}_m$ and (iii) $\overline{s}_c = \frac{\overline{\pi}_c + \rho C - rD_c}{2}$. As before, the fourth condition is determined by equityholders or managers' participation constraint. We know from proposition 10 that cash holdings create slack for equityholders in recession. Competition between firms will therefore push up debt levels until at least one of the following constraints are binding: $\underline{E}_c - \theta(\kappa - C) \geq (L_c - D + C)^+$ or $\overline{M}_c \geq \overline{W} + H$, respectively corresponding to equityholders' and managers' participation constraint.

Consider first regime 1 (${H_c}^* \leq H$). We conjecture that (iv) $\overline{M}_c = \overline{W} + H$ but $\underline{E}_c - \theta(\kappa - C) > (L_c - D + C)$, and subsequently verify this conjecture. Solving the system of 4 equations (i) to (iv) gives the expressions for regime 1 in the proposition. Substituting the solution into $\underline{E}_c$, one can verify that $\underline{E}_c - \theta(\kappa - C) = (L_c - D + C) + \frac{C(r-\rho)}{\overline{\lambda}} > L_c - D_c + C$, as conjectured.

Consider next the case for which ${H_c}^{**} \leq H < {H_c}^*$. From the solution in regime 1 we know that there is still slack for equityholders at a debt level $D = L_c$ at $H = {H_c}^*$. Therefore, for values of H just below ${H_c}^*$ firms raise the debt level up to the point where $\overline{M} = \overline{W} + H$. Complementing this equation with equations (i) to (iii) and solving gives the expressions for regime 2' in the proposition. For H sufficiently low, equityholders' constraint becomes binding in recession, i.e. $\underline{E}_c - \theta(\kappa - C) = (L_c - D + C)^+$. We first prove that if equityholders' constraint is binding, then it must be the case that $(L_c - D + C)^+ = 0$. Assume by contradiction that $(L_c - D + C) > 0$. Solve $\overline{E} = I + C - D$ and $\underline{E} - \theta(\kappa - C) = L_c - D + C$ for $\overline{s}$ and $\underline{s}$. Using the bargaining equation and solving for D gives: $rD = \overline{\pi} - 2\overline{s} + \rho C = r(L_c + C) + (r - \rho)C > r(L_c + C)$. Consequently, $L_c + C - D < 0$ and we have an inconsistency. It follows that $(L_c - D + C)^+ = 0$. Solving $\overline{E} = I + C - D$ and $\underline{E} - \theta(\kappa - C) = 0$ for $\overline{s}$ and $\underline{s}$, using the bargaining equation and solving for D gives: $D_{2c} = L_c + C + \frac{C(r-\rho)}{r+2\overline{\lambda}}$. Next, we verify managers' participation constraint. Substituting the solution into $\overline{M}_{2c}$ gives: $\overline{M}_{2c} =$

$\overline{W} + H_c{}^* - \frac{C(r-\rho)}{r+2\overline{\lambda}} \geq \overline{W} + H \iff H \leq H_c{}^* - \frac{C(r-\rho)}{r+2\overline{\lambda}} \equiv \hat{H}_c^*$. Solving (i), (ii), (iii) and (iv) $\underline{E}_{2c} = \theta(\kappa - C)$ gives the solution for regime 2 in the appendix.

Consider next $H < H_c{}^{**}$. Assume first that equityholders' constraint is binding in recession, i.e. (iv) $\underline{E}_c = \theta(\kappa - C)$ (we can show as before that $(L_c - D + C)^+ = 0$). Solving (i) to (iv) for D, $\overline{s}$, $\underline{s}$ and $\overline{\pi}$ gives the expressions for regime 3. For example: $D_{3c} = L_c + C + \frac{1}{r+2\overline{\lambda}}\left[\frac{r(H_c{}^{**} - H)}{1-\overline{p}} + (r-\rho)C\right]$, which confirms that $(L_c - D + C)^+ = 0$. Substituting into $\overline{M}_{3c}$ we find that $\overline{M}_{3c} \geq \overline{W} + H \iff \overline{s}_{3c} \geq \overline{w} + \frac{rH}{1-\overline{p}} \iff \frac{r(H_c{}^{**}-H)}{1-\overline{p}} + (r+2\overline{\lambda})\phi L_c \geq \frac{(r+\overline{\lambda})(r-\rho)C}{\overline{\lambda}}$. Define $\hat{H}_c^{**}$ as the value for H for which this inequality binds. It follows that managers' particpation constraint is violated for values of H that satisfy $\hat{H}_c^{**} < H < H_c{}^{**}$. In this interval the debt level therefore has to be constrained such that (iv) $\overline{M}_{3'c} = \overline{W} + H$. Solving (i) to (iv) gives the expressions for regime 3'.

A sufficient and necessary condition for regime 3' to exist is that $\hat{H}_c^{**} < H_c{}^{**}$ or equivalently, $\frac{(r+\overline{\lambda})(r-\rho)C}{\overline{\lambda}} > (r+2\overline{\lambda})\phi L_c$. Define the indicator function ω, where $\omega = 0$ if the previous inequality is satisfied and $\omega = 1$ otherwise. A sufficient and necessary condition for exit to occur for all $H < \tilde{H}_c$ is that $\overline{Q}(H_c{}^{**}) > \underline{Q}(H_c{}^{**})$. The critical value $\phi_c{}^*$ is therefore the solution to $\overline{Q}(H_c{}^{**}(\phi_c{}^*)) = \underline{Q}(H_c{}^{**}(\phi_c{}^*))$, where $\overline{Q}(H_c{}^{**}(\phi_c{}^*)) = \overline{\pi}^{-1}\left[rI + (r+2\overline{\lambda})(I - L_c)\right]$ and $\underline{Q}(H_c{}^{**}(\phi_c{}^*)) = \overline{\pi}^{-1}\left[\underline{\pi}_{3'c}(H_c{}^{**}(\phi_c{}^*))\right](1-\omega) + \omega\overline{\pi}^{-1}\left[\underline{\pi}_{3c}(H_c{}^{**}(\phi_c{}^*))\right]$.

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Supplementary Appendix 2: Analysis of the no-exit region ($H \geq \tilde{H}$)

This appendix provides an analysis of the no-exit region. This region corresponds to the special case for which no firms leave in recession. The analysis of this region is not covered in the main text, and included here in the interest of completeness.

Without separation of equity and human capital exit does not occur if and only if $\underline{V} \geq L + \underline{W} + \underline{\delta} H$ (or equivalently if $H \geq \tilde{H}$). A global optimizer therefore compares the *total* going concern value with the *total* value achieved from liquidation. With separation of human and equity capital additional constraints are being imposed because the providers of equity and human capital each compare their *individual* stake in the firm with the value of their outside option, ignoring bondholders' claim. Therefore, exit does not occur if and only if $\underline{E}_s \geq \underline{o}_e$ *and* $\underline{M}_s \geq \underline{o}_m$. A necessary condition for no exit to occur is therefore that $\underline{V}_s(\tilde{Q}) \geq D_s + \underline{o}_e + \underline{o}_m$. When D exceeds L (and therefore $\underline{o}_e = 0$) this condition may become more stringent than the global optimizer's condition. We show below that for some parameter configurations (eg. very high sunk costs $I - L$), exit may occur in some interval $[\tilde{H}, H''[$ (for $H'' > \tilde{H}$).

In addition to a possible exit regime, there are now 4 possible regimes that lead to a situation of no exit in recession: (1) neither the equityholders' nor the managers' outside option binds in recession (hereafter called 'regime N'), (2) only equityholders' outside option binds in recession (hereafter called 'regime E'), (3) only managers' outside option binds in recession (hereafter called 'regime M') and (4) both managers and equityholders outside options bind but such that no firm leaves. Case (4) corresponds to the previously described 'regime EM' where firms adopt a debt level $D > L$ and in recessions all firms are indifferent between staying or going. Which regime occurs depends primarily on the relative amount of sunk costs incurred by equityholders ($I - L$) and managers (H), as shown in the below proposition.

Proposition 12 *Assume that we are in the no-exit region (i.e. $H \geq \tilde{H}$) such that no industry exit occurs (in the first-best case) when there is no separation between equity capital and human capital. If there is separation between the providers of equity and human capital then we observe one of the following 5 regimes, depending on the value of H.*

(1) If $H < H''$ then we observe regime 3 in which some firms adopt a high debt level and some firms adopt a lower debt level. The former firms leave the industry in recession.

(2) If $H'' \leq H \leq H'$ then we observe regime EM in which no firm leaves the industry in recession. All firms adopt the same debt level, which exceeds L.

(3) If $H' < H \leq H_m$ then we observe regime M in which no firm leaves the industry in recession and only managers' outside option binds in recession. All firms adopt the same debt level (which can be above or below L).

(4) If $H_m < H < H_e$ then we observe regime N in which no firms leave the industry and neither

managers' nor equityholders' outside option binds in recession. All firms adopt the same debt level. (5) If $\max[H_m, H_e] \leq H$ *then we observe regime E in which no firm leaves the industry in recession and only equityholders' outside option binds in recession. All firms adopt the same debt level.*

Regimes 3 and EM are as derived in proposition 8. In regime N, M and E the industry output is the solution to: $\overline{V}_{js}(\tilde{Q}) = I + \overline{W} + H$ *and the following conditions are satisfied:* $\overline{E}_{js} = I - D_{js}$, $\overline{M}_{js} = \overline{W} + H$ *and* $\overline{s}_{js} = \frac{\overline{\pi}_j - r\,D_{js}}{2}$ *(for* $j = N, M, E$*).*

Regime N is characterized by the following conditions:

$$\begin{array}{lcllcl} D_{Ns} & = & I - \overline{W} - H & \textit{and} & \underline{s}_{Ns} & = \frac{\underline{\pi} - r\,D_s}{2} \\ \underline{E}_{Ns} & > & (L-D)^+ & \textit{and} & \underline{M}_{Ns} & > \underline{o}_m \end{array}$$

Regime E is characterized by the following conditions:

$$\begin{array}{lcllcl} rD_{Es} & = & \overline{\pi}_{Es}(\tilde{Q}) - 2\left[\overline{\pi}_{Es}(\tilde{Q}) - r\,I - \overline{\lambda}(I-L)\right] & \textit{and} & \underline{E}_{Es} & = L - D_{Es} \\ \underline{s}_{Es} & = & \underline{\pi}_{Es}(\tilde{Q}) - r\,L + \underline{\lambda}(I-L) & \textit{and} & \underline{M}_{Es} & > \underline{o}_m \end{array}$$

Regime M is characterized by the following conditions:

$$\begin{array}{lcllcl} r\,D_{Ms} & = & \overline{\pi}_M(\tilde{Q}) - 2\left[r\left(\overline{W} + H\right) + \overline{\lambda}\left[\overline{W} + H - \underline{o}_m\right]\right] & \textit{and} & \underline{E}_{Ms} & > (L - D_{Ms})^+ \\ \underline{s}_{Ms} & = & r\,\underline{o}_m - \underline{\lambda}\left[\overline{W} + H - \underline{o}_m\right] & \textit{and} & \underline{M}_{Ms} & = \underline{o}_m \end{array}$$

where H' *and* H'' *are as defined in the proof of proposition 8, and where* H_e *and* H_m *are the solution to respectively:*

$$\overline{\pi}\left(\tilde{Q}(H_e)\right) - \underline{\pi}(\tilde{Q}(H_e)) = 2\,(I - L)\left(r + \overline{\lambda} + \underline{\lambda}\right) \tag{61}$$

$$\overline{\pi}\left(\tilde{Q}(H_m)\right) - \underline{\pi}(\tilde{Q}(H_m)) = 2\left[\overline{W} + H_m - \underline{o}_m\left(\overline{\pi}\left(\tilde{Q}(H_m)\right)\right)\right]\left(r + \overline{\lambda} + \underline{\lambda}\right) \tag{62}$$

Proof of proposition 12

The proof is split up in 2 parts. In part A, we derive the equilibrium payoffs and debt level that corresponds to each scenario in which no exit occurs (i.e. regimes N, M, E and EM), assuming that such an equilibrium exists. In part B we then derive which of the above 4 no-exit regimes prevails for a particular set of parameters, and we also identify under what condition an exit regime occurs for values of H that exceed $\tilde{H}$.

Part A

(1) equilibrium payoffs if no outside options bind in recession (regime N)

If no outside options bind in recession then the equilibrium must satisfy the following boundary conditions:(i) $\overline{E} = I - D$, (ii) $\overline{M} = \overline{W} + H$, (iii) $\overline{E} = \frac{\overline{V} - D}{2}$, (iv) $\underline{E} = \frac{\underline{V} - D}{2}$.

Conditions (i) and (ii) imply that, respectively, equityholders and managers, break even in an equilibrium with competitive entry and exit. Conditions (iii) and (iv) follow from the bargaining solution (see prop 3).

Since there are no deadweight costs without exit, it follows that $\overline{E}+\overline{M}+D = \overline{V}$. Using (i) and (ii), it follows that $\tilde{Q}$ is the solution to $\overline{V}(\tilde{Q},\tilde{Q}) = I + \overline{W} + H$. Solving (iii) and (iv) for $\underline{s}$ and $\overline{s}$ gives: $\overline{s} = \frac{\overline{\pi}(\tilde{Q})-rD}{2}$ and $\underline{s} = \frac{\underline{\pi}(\tilde{Q})-rD}{2}$. Combining (i) and (iii) gives $2(I-D) = \overline{V}-D$. Using the solution for $\overline{V}$ and simplifying gives: $D = I - \overline{W} - H$.

(2) equilibrium payoffs if only equityholders' outside options bind in recession (regime E)

The equityholders' outside option equals $\underline{o}_e = (L-D)^+$. The boundary conditions now become: (i) $\overline{E} = I - D$ or equivalently $\overline{V}-\overline{M} = I$, (ii) $\overline{M} = \overline{W}+H$, (iii) $\overline{E} = \frac{\overline{V}-D}{2} + \frac{\overline{\delta}}{2}(\underline{E}-\underline{M})$, and (iv) $\underline{E} = (L-D)^+$ or equivalently $\underline{V}-\underline{M} = (L-D)^+ + D$.

We first prove that a situation where only equityholders' outside option binds and the optimal debt level D exceeds L (i.e. $D > L$) cannot arise. Assume that $D > L$, then $\underline{o}_e = 0$. Equityholders constraint is binding in recession if the payoff under the unconstrained bargaining solution is less than what equityholders can get by exercising their outside option, or equivalently if $\frac{\underline{V}-D}{2} \leq 0$, or $\underline{V} \leq D$. In the absence of deadweight costs this implies that $\underline{E}+\underline{M} \leq 0$. Since equityholders' claim cannot be strictly negative, it follows that managers' claim must be negative in recession. This is, however, inconsistent with our original assumption that managers' constraint is not binding, i.e. $\underline{M} = \frac{\underline{V}-D}{2} > \underline{o}_m > \underline{W} > 0$. Therefore our assumption that $D > L$ is false.

(i) and (ii) imply that $\tilde{Q}$ is again the solution to $\overline{V} = I+\overline{W}+H$. Solving (i) and (iv) for $D \leq L$ gives:

$$\overline{s} = \overline{\pi}(\tilde{Q}) - rI - \overline{\lambda}\,[I-L] \tag{63}$$

$$\underline{s} = \underline{\pi}(\tilde{Q}) - rL + \underline{\lambda}\,[I-L] \tag{64}$$

Finally, from condition (iii) it follows that $rD = \overline{\pi} - 2\overline{s}$.

(3) equilibrium payoffs if only managers' outside options bind in recession (regime M)

The boundary conditions now become: (i) $\overline{E} = I-D$, (ii) $\overline{M} = \overline{W}+H$, (iii) $\overline{E} = \frac{\overline{V}-D}{2}+\frac{\overline{\delta}}{2}(\underline{E}-\underline{M})$, and (iv) $\underline{M} = \underline{o}_m$, where $\underline{o}_m$ was previously derived in proposition 5. Solving this system of equation gives the expressions in proposition 12.

(4) equilibrium payoffs if managers' *and* equityholders' outside options bind in recession (regime EM)

This is regime EM in which no firm leaves the industry, and in recession all firms are indifferent between staying or going. The derivation of this regime is given in the proof of proposition 8.

Part B

Having derived all possible equilibrium payoffs that can arise, we now derive which equilibrium arises under what conditions, and we also derive the condition for no exit to occur in recession.

We first derive a few auxiliary results. Define $f(H) \equiv \overline{\pi}(\tilde{Q}(H)) - \underline{\pi}(\tilde{Q}(H))$. We first prove that $f'(H) > 0$. From $\overline{V}(\tilde{Q}(H)) = \overline{W} + H + I$ it follows that $\left[\frac{\overline{\pi}'(\tilde{Q})}{r}(1-\overline{p}) + \frac{\underline{\pi}'(\tilde{Q})}{r}\overline{p}\right] \frac{\partial \tilde{Q}}{\partial H} = 1$. Consequently,

$$f'(H) = \frac{\overline{\pi}'(\tilde{Q}) - \underline{\pi}'(\tilde{Q})}{\left[\frac{\overline{\pi}'(\tilde{Q})}{r}(1-\overline{p}) + \frac{\underline{\pi}'(\tilde{Q})}{r}\overline{p}\right]} > 0 \; since \; \overline{\pi}'(Q) < \underline{\pi}'(Q)$$

(1) If only equityholders' outside option binds in the no-exit region, then it follows that: $\frac{\underline{V}-D}{2} \leq (L-D)$, or equivalently $\underline{V} < 2L - I + \overline{W} + H$. Using the fact that $\overline{V}(\tilde{Q}) = I + \overline{W} + H$, it follows that $\overline{V} - \underline{V} \geq 2(I-L)$ or equivalently, $f(H) = \overline{\pi}(\tilde{Q}(H)) - \underline{\pi}(\tilde{Q}(H)) \geq 2(I-L)(r+\overline{\lambda}+\underline{\lambda}) \equiv e(H)$. Since $f'(H) > 0$ and $e'(H) = 0$, it follows that they only intersect once. In what follows, we define H_e as the unique value for which $f(H_e) = e(H_e)$.

(2) If only managers' outside option binds in the no-exit region, then it follows that:

$$\frac{\underline{V} - D}{2} \leq \underline{o}_m \iff \underline{V} \leq 2\underline{o}_m + I - \overline{W} - H \tag{65}$$

From $\overline{V} = I + \overline{W} + H$ it follows that managers' outside option binds iff

$$\overline{V} - \underline{V} \geq 2\left(\overline{W} + H - \underline{o}_m\right) \tag{66}$$

$$\iff f(H) = \overline{\pi}(\tilde{Q}(H)) - \underline{\pi}(\tilde{Q}(H)) \geq 2\left(\overline{W} + H - \underline{o}_m\right)\left(r + \overline{\lambda} + \underline{\lambda}\right) \equiv m(H)$$

Furthermore, $m'(H) > 0$. Indeed,

$$m'(H) > 0 \iff 1 - \frac{\overline{\pi}'}{r}\underline{p}\frac{\partial \tilde{Q}}{\partial H} > 0 \tag{67}$$

$$\iff \frac{\overline{\pi}'}{r}\underline{p} > \frac{1}{\frac{\partial \tilde{Q}}{\partial H}} = \frac{\overline{\pi}'}{r}(1-\overline{p}) + \frac{\underline{\pi}'}{r}\overline{p} \tag{68}$$

$$\iff 0 > \frac{\overline{\pi}'}{r}(1-\overline{p}-\underline{p}) + \frac{\underline{\pi}'}{r}\overline{p} \tag{69}$$

The last inequality is always satisfied because $\overline{p} + \underline{p} < 1$ and $\overline{\pi}'(Q)$, $\underline{\pi}'(Q) < 0$ for all Q.

$$f'(H) < m'(H) \iff \left[\frac{\overline{\pi}' - \underline{\pi}'}{r + \overline{\lambda} + \underline{\lambda}} + \frac{2\overline{\pi}'}{r}\underline{p}\right]\frac{\partial \tilde{Q}}{\partial H} < 2 \tag{70}$$

$$\iff \frac{(\overline{\pi}' - \underline{\pi}')}{r}(1-\overline{p}-\underline{p}) + \frac{2\overline{\pi}'}{r}\underline{p} > 2\left[\frac{\overline{\pi}'}{r}(1-\overline{p}) + \frac{\underline{\pi}'}{r}\overline{p}\right] \tag{71}$$

$$\iff \frac{\overline{\pi}'}{r}(\overline{p}+\underline{p}-1) - \frac{\underline{\pi}'}{r}(1+\overline{p}-\underline{p}) > 0 \tag{72}$$

The last inequality is satisfied because $\overline{p} + \underline{p} < 1$ and $\overline{\pi}'(Q)$, $\underline{\pi}'(Q) < 0$ for all Q. It follows that $f(H)$ and $m(H)$ intersect only once. We denote this intersection point by H_m (note that H_m is negative if $f(0) < m(0)$).

Let H_o be the unique solution to $m(H_o) = e(H_o)$. Since $f(H)$ and $m(H)$ are both monotonically increasing in H, and since $f'(H) < m'(H)$, it follows that:

$$H_m < (\geq) H_e \iff f(H_o) < (\geq) e(H_o)$$

By construction the solution at H_m satisfies the conditions: (i) $\overline{E}_s = I - D_s$, (ii) $\overline{M}_s = \overline{W} + H_m$, (iii) $\overline{s}_s = \frac{\overline{\pi}-rD_s}{2}$, (iv) $\underline{M}_s = \underline{o}_m$. The solution at H' satisfies the same conditions, but while at H_m the equityholders' outside option is not binding (i.e $\underline{E}_s > (L-D)^+$), at H' equityholders' outside option is binding (i.e. $\underline{E}_s = 0$). Consequently, $H' < H_m$. Therefore regime EM occurs for some values of H if $\tilde{H} < H'$. In particular, we know that regime EM then prevails over the interval $[H'', H']$. As a result, if $\tilde{H} < H''$ then it is optimal for some exit to occur in recessions over the interval $[\tilde{H}, H''[$. In particular, we have shown in proof to proposition 8 that regime 3 prevails for all $H < H''$. The subsequent analysis splits into two parts: (I) $H_m < H_e$ and (II) $H_m \geq H_e$.

(I) $f(H_o) < e(H_o)$ (or equivalently:$H_m < H_e$)

By construction, we know that *if* the equilibrium without exit prevails for a particular value of H then:
(1) if $H'' \leq H \leq H'$ then both equityholders' and managers' option bind in recession
(2) if $\max[\tilde{H}, H'] < H \leq H_m$ then only the managers' option binds in recession
(3) if $\max[\tilde{H}, H_m] < H < H_e$ then no outside options bind in recession
(4) if $\max[\tilde{H}, H_e] \leq H$ then only equityholders' outside option binds in recession.

(II) $f(H_o) \geq e(H_o)$ (or equivalently:$H_m \geq H_e$)

Assume that the equilibrium without exit prevails for H. By construction, it follows then that:
If $\max[\tilde{H}, H'] < H \leq H_e$ then only managers outside option binds
If $H_e < H < H_m$ then equityholders and managers' outside options bind.
If $H_m \leq H$ then only equityholders' outside option binds.

Since both equityholders' and managers' outside option strictly bind for $H_e < H < H_m$, this means that the no-exit equilibrium does not exist for this parameter set, and consequently $\tilde{H}$ must be to the right of H_m. Since $H_e \leq H_m < \tilde{H}$ and $H' < H_m < \tilde{H}$ it follows that only equityholders constraint binds for all $H > \tilde{H}$, and therefore regime E prevails for $H_m \geq H_e$.

Combining the above results gives the conditions in proposition 12.

Finally, we proof that a sufficient (but not necessary) condition for no exit to occur in $[\tilde{H}, +\infty[$ is that the optimal debt level does not exceed L. Assuming that $D \leq L$ then no exit occurs if

$$\underline{V}(\tilde{Q}) \geq \underline{o}_m(\overline{\pi}(\tilde{Q})) + (L-D) + D \iff \underline{\pi} \geq \underline{w} + r\,L - \underline{\lambda}(I-L) \tag{73}$$

where the equivalence is derived by substituting for $\underline{o}_m$. Since $\tilde{Q}$ is the solution to $\overline{V}(\tilde{Q}) = I + \overline{W} + H$, define $\tilde{H}'$ as the value for H for which the above inequality becomes binding, i.e. $\underline{\pi}(\tilde{Q}(H')) = \underline{w} + r\,L - \underline{\lambda}(I-L)$. Substituting $\underline{\pi}$ into $\overline{V}$ and solving for $\overline{\pi}$ gives: $\overline{\pi}(\tilde{Q}(\tilde{H}')) = rI + \overline{\lambda}(I-L) + \overline{w} + \frac{r\tilde{H}'}{1-\overline{p}}$. It follows that $\tilde{H}'$ is the solution to:

$$\overline{\pi}(\tilde{Q}(\tilde{H}')) - \underline{\pi}(\tilde{Q}(\tilde{H}')) = (I-L)\left(r + \overline{\lambda} + \underline{\lambda}\right) + \overline{w} - \underline{w} + \frac{r\tilde{H}'}{1-\overline{p}} = \Delta(I, L, \tilde{H}')$$

Consequently, $\tilde{H}' = \tilde{H}$ and no exit occurs for all $H \geq \tilde{H}$, if the optimal debt level is below L.

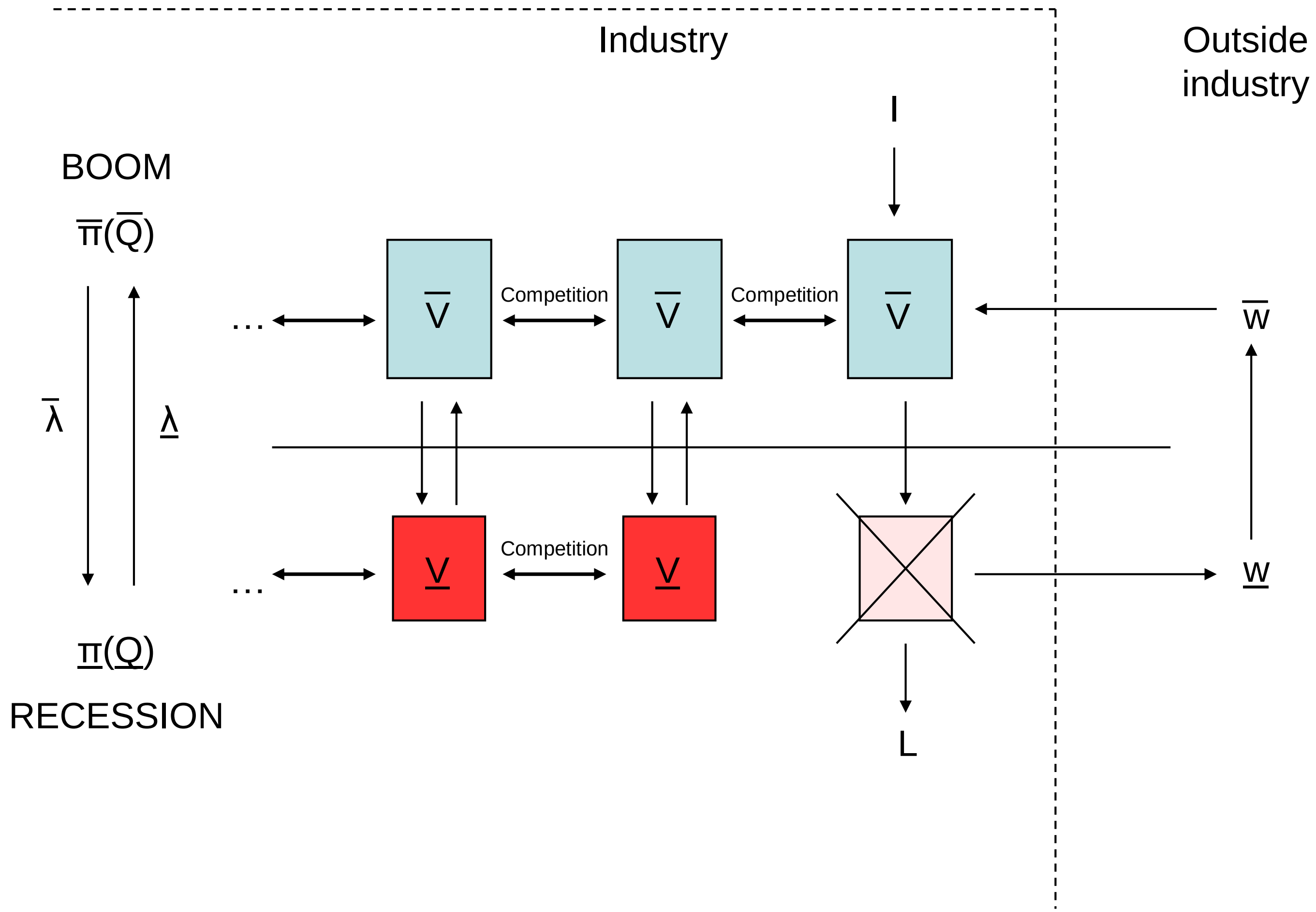

Figure 1: Industry dynamics. $\overline{V}$ and $\underline{V}$ denote the firm value in booms and recessions, respectively, I is the physical investment cost, L is the liquidation value, $\overline{w}$ and $\underline{w}$ ($\overline{\pi}(\overline{Q})$ and $\underline{\pi}(\underline{Q})$) represent outside wages (profits) in booms and recessions, respectively, and $\overline{\lambda}$ ($\underline{\lambda}$) denotes the hazard rate associated with the arrival of a recession (boom).

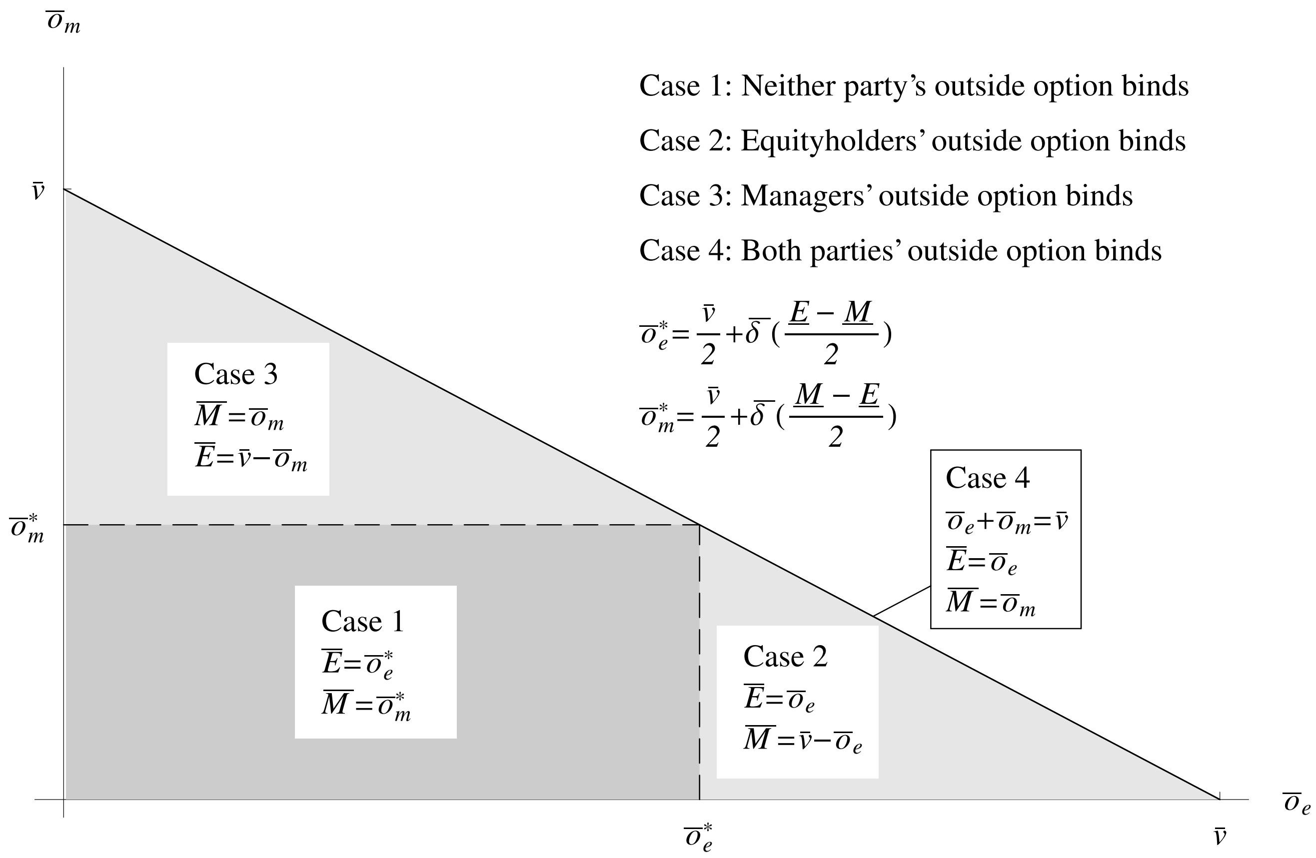

Figure 2: Geometry of the regions in the bargaining game. $\overline{E}$ and $\overline{M}$ denote the value of shareholders' and manager's claim, respectively, $\overline{o}_e$ and $\overline{o}_m$ are the corresponding outside options, whereas $\overline{v}$ is the value to be shared.

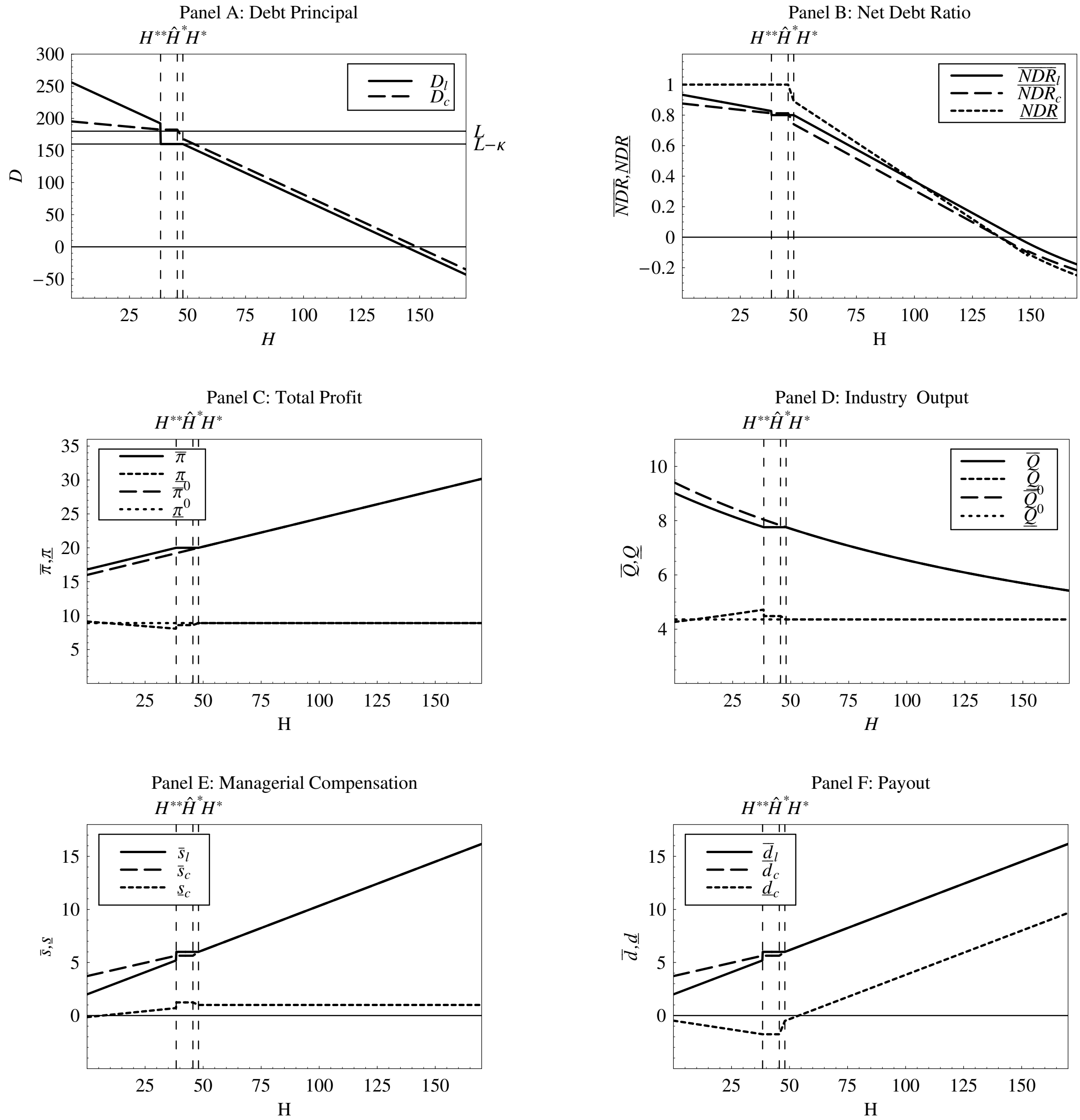

Figure 3: Comparative statics results for debt principal, net debt ratio, total profit, industry output, managerial compensation and payout to shareholders generated for the following parameter values: $\overline{\pi}(Q) = \overline{p}(Q) = \overline{a}Q^{-\epsilon} - \overline{b}$ and $\underline{\pi}(Q) = \underline{p}(Q) = \underline{a}Q^{-\epsilon} - \underline{b}$, where $\overline{a} = 200$ and $\underline{a} = 50$, $\overline{b} = \underline{b} = 1$ and $\epsilon = 1.1$. Furthermore, $\overline{\lambda} = \underline{\lambda} = 0.1$, $r = 0.05$, $I = 200$, $L = 180$, $\overline{w} = 2$, $\underline{w} = 1$, $\phi = 0.05$, $\kappa = 20$, $\rho = 0.02$, and $\theta = 1.2$.