Investment and Financing Options with Capital Constraints

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Abstract

We review and extend recent contingent claims models of capital structure. We focus on two models with analytic formulas in perpetual horizon- Leland (1994) and Mauer and Sarkar (2004). We implement the investment option in both models in finite horizon with a numerical lattice while maintaining the analytic structure for the capital structure decisions in the second stage by maintaining the perpetual horizon for debt. Incorporation of the investment option itself essentially extends and generalizes Leland's model. Using this framework we investigate two issues: the effect of capital financing constraints on both debt and new equity and the effect of managerial (equity-financed) options to enhance the value of the firm by R&D, advertising or marketing research efforts before the major investment decision takes place. In this general framework we are able to analyze when equity holders should invest to improve the investment option (control option), simply delay for more uncertainty resolution or invest early in a project that can be both equity and debt financed and may face constraints in the level of financing. Financing constraints may arise due to asymmetric information or moral hazard, affecting both optimal first stage control decisions and optimal financing, and in turn, the values of equity and debt.

1. Introduction

Since the initial contingent claims approach of valuing equity and debt was set by Merton (1973), several papers have emerged trying to generalize and extent this idea into new dimensions including coupon payments, the tax benefits of debt and bankruptcy costs. For example, Kane et al (1984, 1985) derive analytic solutions using the same finite maturity of debt setting including several realistic new elements like debt yield, the tax benefits of debt, and bankruptcy costs. The model, like in Merton uses a Miller-Modigliani world where the investment decision is taken as given when analyzing financing decisions.

Leland (1994) uses a perpetual horizon assumption and derives closed form expressions for the value of levered equity, debt and the firm in the presence of taxes and bankruptcy costs. Securities values are contingent on the uncertain unlevered value of the firm. He abstracts from the investment decision and focuses on two approaches to define the trigger point of bankruptcy 1) When equity holders optimally choose the trigger point of default (unprotected debt case). 2) When bankruptcy is triggered by a positive net worth covenant restriction (protected debt case). Leland and Toft (1996) extend Leland (1994) to allow the firm to choose the optimal maturity of the debt, and debt level. They still use a perpetual horizon for the firm's unlevered assets but now the debt does not have infinite maturity, but one that has to be optimized by equity holders in order to maximize current equity value. The tax advantage of debt is balanced with the bankruptcy and agency costs to determine the optimal maturity of debt and it is shown that short term debt reduces or eliminates asset substitution (agency costs). The effect on credit spreads, duration of bonds and bankruptcy rates and bond ratings in also explored.

The above mentioned papers analyze and incorporate realistic corporate finance features but they nevertheless do not incorporate equity holders investment option decisions. Brennan and Schwartz (1984) present a general model for the valuation of the levered firm when equity holders optimally choose *both* the investment and financial policy continuously over time. The model assumes a finite horizon with the debt maturing at the end of the horizon. Bankruptcy at the terminal point is triggered when the value of the firm is less than the face value of debt while in previous periods is triggered by bond covenant provisions. Mauer and Triantis (1994) analyze a more general framework of interactions of investment and financing decisions of the firm. They introduce the option to invest in a production facility, the flexibility to temporarily shut down and restart and the interaction of these decisions with financing decisions. The model allows for dynamic change in capital structure which is optimally set by the firm's equity holders after taking into consideration recapitalization costs, a positive net worth bond covenant restriction, the tax benefits of debt, and bankruptcy costs. Childs, Mauer and Ott (2000) present a numerical model that takes into account a stochastic value of a firm's growth option, tax benefits of debt, bankruptcy costs while at the same time allowing readjustments of the capital structure. While the conceptual framework that includes path-dependent actions is interesting, it is numerically intensive and does not allow for feedback between bondholders and equity holders optimal decisions. Schwartz and Moon (2000) present the valuation of internet companies that also includes the evolution of the cash flows and also attempts to tackle capital structure decisions.

Besides the standard option to invest and the operational flexibility to shut down and restart that has been modelled by other authors, another interesting aspect of managerial is the option to control (influence or enhance) the value of the firm before major investment decisions. Similarly to growth options, managerial control options may be targeted towards an increase in asset value but may have random outcome. Martzoukos (2000) (see also Martzoukos, 2003) has introduced managerial control actions in an all-equity financing framework and Koussis, Martzoukos and Trigeorgis (2004) have extended it to include path-dependency between actions. In this paper we explore the interaction between these managerial random control actions and financing options in a unified framework. This framework allows the investigation of whether R&D or other managerial actions like advertising and marketing research are beneficial prior to major investment decisions and how the values of the firm, equity and debt are influenced by these decisions.

Asymmetric information between the firm's different claimants and differences in their objectives may result in agency costs i.e. a reduction of firm value from the optimal level. Boyle and Guthrie (2003) analyze the effects on the timing of the option to investment under a liquidity constraint that comes from asymmetric information between current and new shareholders. Mauer and Ott (2000) analyze the interactions of investment and financing decisions of the firm confronted with a growth option. The model taking into account the benefits of debt and bankruptcy costs shows that equity holders delay investment (or underinvest) in the growth option compared to the first best strategy of optimizing total firm value. The difference in value is a measure of agency cost of underinvestment. Similarly issues of asymmetric information that relate to "asset substitution" may prevent the firm to issue debt at the optimal level. In particular Jensen and Meckling (1976) show that due to equity holders limited liability they would prefer to engage in risky projects at the expense of debt holders, and, debt holders rational response for a given stream of coupons would be to reduce the current level of debt financing. Mauer and Sarkar (2004) use a similar setting to Mauer and Ott (2000) but investigate the stockholder and bondholder conflicts that relate to the agency costs of debt. Their results show an incentive for equityholders to overinvest (invest early relative to the optimal time) and this causes a decrease in firm value and leverage and an increase in credit spreads relative to the optimal first-best strategy. Similarly, Leland (1998) investigates equityholders ability to engage in "asset substitution" i.e. engage in riskier strategies ex-post to debt agreement thus transferring wealth from bond holders to equity holders. In this model equityholders can switch between low risk and high risk strategies (without switching costs). Agency costs restrict leverage and debt maturity and increase yield spreads but their importance is minor (for the range of parameters considered).

Leland (1994) points out that one method to avoid asset substitution would be the issue of protected debt with a debt covenant restriction that forces bankruptcy whenever the asset value of the firm is not enough to cover the face value of debt. It is shown that in the case of protected debt equity may be a concave function of asset value and thus increased volatility may hurt equity holders instead of benefiting them. Leland notes that the difference in value between unprotected and protected debt may define a measure of agency costs of debt. In this paper we explore yet another possible way to measure agency costs of debt by assuming that due to moral hazard or asymmetric information debtholders may wish to set debt financing constraints that provide a cap to debt financing. We explore how these constraints affect equity and debt values and we define the difference in the optimal value of the firm with no financing constraints from the value with financing constraints as a potential measure of agency cost of debt.

In the next section, we make a more detailed review of the methodologies of Leland (1994), and Mauer and Sarkar (2004) and we then propose a more general theoretical framework that allows managerial (equityholders) actions with random outcome, financing constraints on debt and a measure of agency costs of debt. In section 3 we provide numerical results and discussion.

2. The proposed framework

In this section, we present and extend two models of the investment option with capital structure decisions to include shareholders option to enhance the value of real assets and capital constraints. In the first subsection, we present Leland's (1994) paper with the proposed extensions. In the second subsection we review Mauer and Sarkar (2004) that moves along the same lines with Leland (1994) but differs in that it models the firm cash flows (the uncertain variable being commodity price instead of the value of unlevered assets like in Leland). This model allows the incorporation of some additional characteristics like the option to abandon that is embedded in the value of unlevered assets. Figure 1 describes the two models with the enhanced features, in two panels. We provide more details about each one separately in the subsections that follow.

[Insert Figure 1]

2.1. Leland's (1994) model with an investment option and financing constraints

2.1.1. Assumptions and derivation of PDE of contingent claims

We assume that the firm's unlevered assets follow a Geometric Brownian Motion with optional control variable dq

$$\frac{dV}{V} = \mu dt + \sigma dZ + k dq \tag{1}$$

where μ denotes the capital gains of this asset, σ denotes its volatility, dZ is an increment of a standard Weiner process. Similarly to Martzoukos (2003) and Koussis et al (2004) the term dq is interpreted as a control counter that can be optionally activated ($dq = 1$) at a cost I_C and its effect on unlevered asset returns will have a random outcome k where[1]:

$$1 + k \sim \log N\left(\exp(\gamma), \exp(\gamma)\left(\exp(\sigma_C^2) - 1\right)^{0.5}\right) \tag{2}$$

We assume that these control actions represent shareholders option to engage in R&D or other related costly actions like advertisement or marketing that are targeted to improve the value of the investment option albeit have a random outcome. We also assume similarly to Leland (1994) that V is unaffected by the firm's capital structure: any coupon payments on debt are financed by new equity leaving the value of unlevered assets unaffected.

We assume that an equilibrium continuous-time CAPM (see Merton, 1973) holds and that controls have firm-specific risk which is uncorrelated with the market portfolio and is thus not priced. We assume that there are no dividend payouts to shareholders taken away from unlevered assets. Zero dividend payouts are not so unrealistic for growth firms which are the emphasis of the analysis in this paper (see for example Schwartz and Moon (2000) applications for internet companies and the discussion therein). We nevertheless include a dividend-like payout rate in the form of opportunity cost of waiting to invest δ that can also have the interpretation of a competitive erosion on the value of assets (e.g., Childs and Triantis (1999), Trigeorgis (1996) ch.9, and Trigeorgis (1991)).

Using either a replication argument of Black and Scholes-Merton or the risk-neutral valuation as established in Constantinides (1978) we know that any contingent claim f on V should satisfy the following PDE:

$$\frac{1}{2}\sigma^2 f_{VV} + (r - \delta)V f_V - f_t - rf + E[f(V(1+k), t) - f(V, t)]dq = 0 \tag{3}$$

where E denotes the expectation operator. The control counter $dq = 1$ only at the control activation else $dq = 0$.

[1] The assumption of lognormal effect of the control is given for simplicity and to exclude negative values of V (see also Koussis et al (2004)).

We describe shareholders position F as the following contingent claim: a (compound) option to invest at time zero I_C that will potentially enhance V (but has a random outcome) that gives the option to acquire a potentially levered position $E(V)$ by making an investment I. Shareholders debt financing has value $D(V)$. By defining shareholders choices with $\varphi_{I_C} =$ {activate control, do not activate control} at t = 0 and $\varphi_I =$ {Early development of investment, Delay investment} from t = 0 until t = T we can describe shareholders position as an optimal control problem of the following form:

$$rF = \max_{\varphi_{I_C},\varphi_I} \frac{1}{2}\sigma^2 F_{VV} + (r-\delta)VF_V - F_t + E[F(V(1+k),t) - F(V,t)]dq$$

s.t.

$$F(V) = \max(E(V) + D(V) - I, 0) \ \text{ at } t \in [0,T]$$

$$E(V) = V - (1-\tau)\frac{R}{r} + \left[(1-\tau)\frac{R}{r} - V_B\right]\left[\frac{V}{V_B}\right]^{\beta} \qquad (4)$$

$$D(V) = \frac{R}{r} + \left[(1-b)V_B - \frac{R}{r}\right]\left[\frac{V}{V_B}\right]^{\beta}$$

$$\beta = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} < 0$$

In contrast to Leland, the value of V in $F(V)$, $E(V)$ and $D(V)$ may either be after control has been activated, or in the case where the decision was to not activate the control. By I, τ, R, r we denote the investment cost, tax rate, coupon, and the risk free rate respectively. The term b denotes proportional (to V) bankruptcy costs and V_B the bankruptcy trigger point that will optimally be selected by equityholders (see discussion that follows). $E(V)$ describes equityholders levered position once investment is initiated and can be shown to equal V minus the value expected value given up in case of bankruptcy V_B, plus the expected value of tax shields until bankruptcy trigger point minus the expected value of coupon payments paid until bankruptcy. $D(V)$ describes the value of debt financing that equals the expected value of coupons received until bankruptcy triggering point plus the expected value of the firm, net of bankruptcy costs, received in case of bankruptcy. The derivations of the above formulas are described in Leland (1994).

Compared to Leland, our framework includes the investment decision option and the option to enhance the value of real assets before investment. We implement a binomial numerical lattice scheme that accommodates optimal timing of the investment option, and, like in Leland, investment is financed with new equity and perpetual debt and that equity holders have the option to default. With N lattice steps and total volatility v^2 the corresponding probabilities and up and down lattice parameters conditional on activation of managerial control to enhance is:

$$v^2 = \sigma^2 \frac{T}{N} + \frac{\sigma_C^2}{N}$$

$$u = \exp(v) \quad , d = 1/u$$

$$p_u = \frac{\exp\left((r-\delta)T + \frac{\gamma}{N}\right) - d}{u - d} \quad , p_d = 1 - p_u$$

Leland obtains the optimal default point by solving for the following smooth-pasting condition:

$$\frac{\partial E}{\partial V}\Big|_{V=V_B} = 0 \tag{5}$$

Condition (5), is equivalent to maximizing $E(V)$ by selecting V_B i.e. the condition $\frac{\partial E}{\partial V_B} = 0$ (see Leland, p. 1222). Using (5), the value of V_B is given by:

$$V_B = \frac{-\beta}{(1-\beta)}(1-\tau)\frac{R}{r} \tag{6}$$

Equation (6), compared to the one in Leland, includes dividend-like competitive erosion (included in term β). Since $\beta < 0$, this means that $V_B > 0$ for any positive level of coupons *R*. When *R* = 0, and equity holders get an unlevered position ($E(V) = V$) ($V_B = 0$ and thus default does not exist after the investment). Also note that V_B is independent of the value of unlevered assets *V*.

Besides the selection of the optimal default point, equity holders also optimally select the level of debt financing. For shareholder's selected level $D(V)$, debt holders will optimally set their coupon levels R to correspond to this value (according to the value function for debt given in (4)) and this will in turn affect the levered position $E(V)$ of equity holders. We solve for the equity holders search for optimal coupon payment for every level of V in the numerical lattice scheme. The search process for every V is described in figure 2.

[Insert Figure 2]

Mauer and Sarkar assume that $D(V)$ can be greater than the level of investment, with the assumption that any extra amount can be distributed to shareholders in the form of dividends.

We make the above framework more realistic by adding financing constraints that may exist, due for example to asymmetric information or moral hazard. Debt financing can have a cap $D^{\max}$, so $D(V) \leq D^{\max}$. We can also impose the constraint that $D^{\max} \leq I$ (combined constraint: $D(V) \leq D^{\max} \leq I$) thus forcing that some equity financing is needed for the investment of is launched.

2.1 Mauer and Sarkar (2004) with financing constraints

In Mauer and Sarkar (2004) the stochastic value driver is the commodity price that the firm sells. We maintain their Geometric Brownian Motion assumption by also incorporating the potential of managerial (shareholders) option to enhance the price with random outcome k.

$$\frac{dP}{P} = \mu dt + \sigma dZ + k dq \tag{7}$$

where μ is the growth rate in price and σ is the volatility. The commodity pays a continuous dividend yield (convenience yield) δ. This should not be confused with the competitive erosion opportunity cost that we defined in the previous section. We may of course assume that δ specified here is net of this erosion (but cannot turn negative).

Similarly to the previous section we define equity holders investment (and control) option decision with the following optimization (now contingent on the commodity price P)

$$rF = \max_{\varphi_{1C},\varphi_1} \frac{1}{2}\sigma^2 F_{PP} + (r-\delta)PF_P - F_t + E[F(P(1+k),t) - F(P,t)]dq$$

s.t. (8)

$$F(P) = \max(E(P) + D(P) - I, 0) \text{ at } t \in [0,T]$$

where $E(P)$ and $D(P)$ are defined below. For the implementation of the investment option problem we will use a standard binomial structure with parameters like the ones defined in equation (4) of the previous section.

The value of equity holders levered position and debt are contingent on P. If shareholders decide to invest, partly financing with debt, they enter a position where they receive $(P-(C+R))(1-\tau)$ continuously, where C a constant cost of selling the commodity and R is is the level of coupon payments. Their position that also includes the option to default is given (see Mauer and Sarkar) to be:

$$E(P) = \left(\frac{P}{\delta} - \frac{C+R}{r}\right)(1-\tau) - \left(\frac{P_D}{\delta} - \frac{C+R}{r}\right)(1-\tau)\left(\frac{P}{P_D}\right)^{\beta} \qquad (9)$$

where

$$P_D = \frac{-\beta}{1-\beta}\frac{\delta(C+R)}{r},$$

and β is the same as equation (4) given above. P_D is the optimally selected point of default by equityholders that maximizes $E(P)$ and is calculated using a smooth-pasting condition $\frac{\partial E(P)}{\partial P}\Big|_{P=P_D} = 0$.

To determine the value of debt, we remember that in case of default, debt holders receive the value of the firm's unlevered assets (at the default point) net of bankruptcy costs. The value of debt is thus given by:

$$D(P) = \frac{R}{r} + \left[(1-b)V(P_D) - \frac{R}{r}\right]\left(\frac{P}{P_D}\right)^{\beta} \qquad (10)$$

where the value of unlevered assets is:

$$V(P) = \left(\frac{P}{\delta} - \frac{C}{r}\right)(1-\tau) - \left(\frac{P_A}{\delta} - \frac{C}{r}\right)(1-\tau)\left(\frac{P}{P_A}\right)^{\beta} \qquad (11)$$

where

$$P_A = \frac{-\beta}{1-\beta}\frac{\delta C}{r}$$

The value of unlevered assets is calculated as the asset that pays $(P-C)(1-\tau)$ continuously and includes the option to abandon at the optimal commodity trigger point. Note that in equation (10), the value of unlevered assets is evaluated at the default trigger P_D .Besides the optimal timing of the investment option, shareholders will also optimally select the level of debt financing with a search procedure similar to the one described in the previous section (see figure 2).

The Mauer and Sarkar approach emphasizes the modeling of uncertain cash flows and thus goes down into greater detail allowing for the option to abandon embedded in the value of unleveled assets. We nevertheless note that compared to Leland they use similar assumptions and the models are consistent with each other.

Constraints on debt financing can be set similarly with the previous section. Their expected outcome should be to reduce shareholder value by reducing potential tax benefits of debt financing. The magnitude of these constraints is explored in the numerical section that follows.

3. Numerical results and discussion

This section is under development

4. Summary

We review and extend recent contingent claims models (Leland, 1994, and Mauer and Sarkar, 2004) of the capital structure. We implement in both models the investment option in a finite horizon with a numerical lattice while maintaining the analytic structure for the capital structure decisions in the second stage by maintaining the perpetual horizon for debt. We investigate two issues: the effect of capital financing constraints on both debt and new equity, and the effect of managerial (equity-financed) options to enhance the value of the firm by R&D, advertising or marketing research efforts before the major investment decision takes place.

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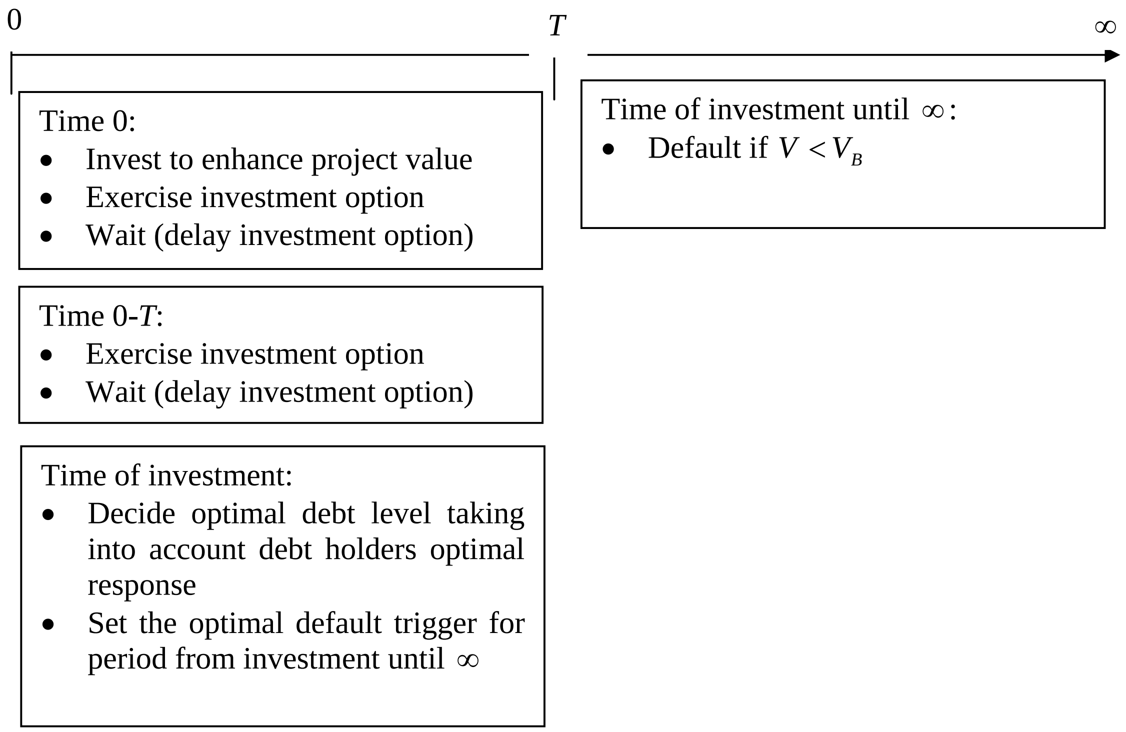
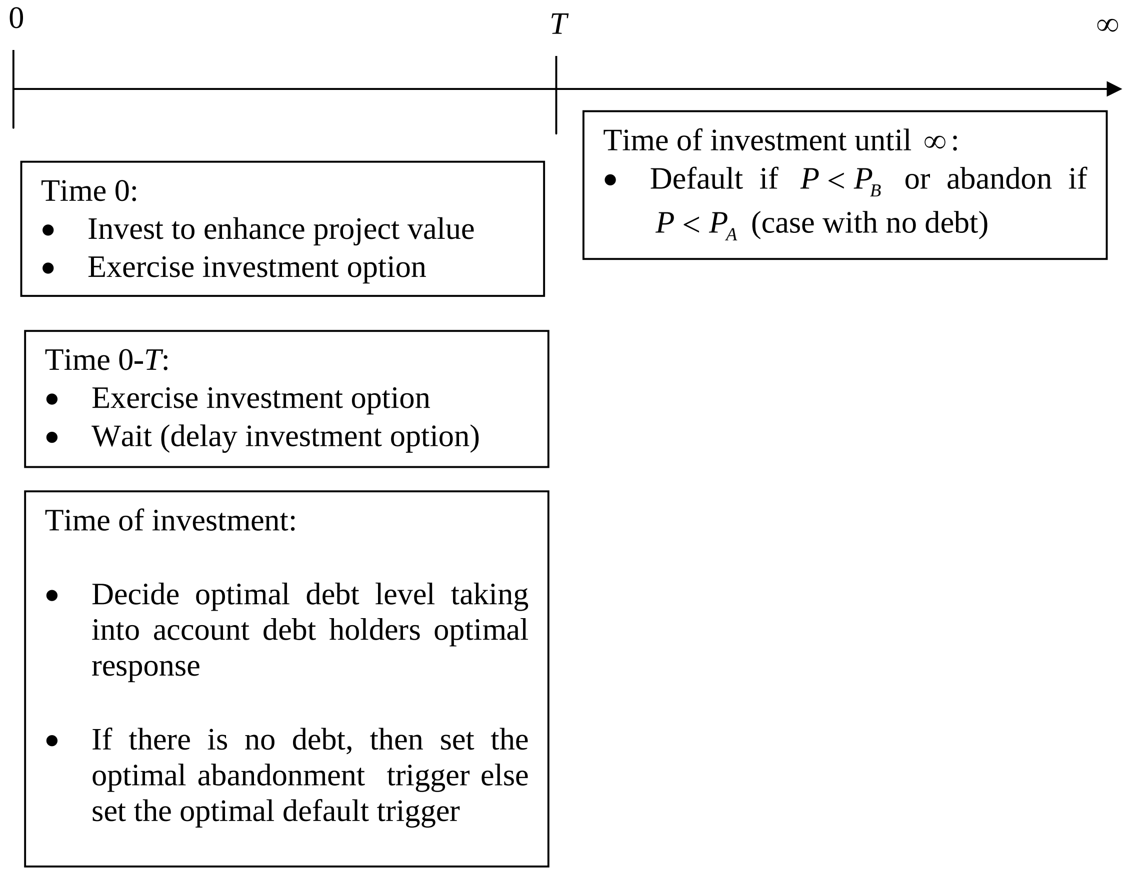
Figure 1. Two models of firm's investment options with financing decisions

Panel A. Leland's (1994) model with added: investment option, shareholder's option to enhance value, and debt financing constraints



Note: The equity holders should select the optimal timing of an investment option with maturity T on uncertain (unlevered) assets V by paying I. Before they decide to invest, they have the option to spend I_C (all-equity financed control action) to improve the value of assets but this has uncertain outcome on returns. At the time of investment the equity holders decide the optimal level of debt financing taking into account that debt holders will rationally set coupon payments according to the risk position they face (debt financing and equity financing constraints may exist). At this point the equity holders also decide on the optimal level of V, V_B, that will trigger bankruptcy.

Panel B. Mauer and Sarkar (2004) with added: shareholders control option to enhance value, and debt financing constraints



Note: The equity holders have the option to invest I that will give them $(P - C)(1 - \tau)$ continuously if they stay unlevered, and $(P - C - R)(1 - \tau)$ if they decide to take a levered position, where P is the price and C is the cost of the commodity, R is the level of coupons and τ is the tax rate (full-loss offset assumption is made). If they stay unlevered they can decide to optimally abandon the project at P_A. At time zero, before they decide to invest, they have the option to spend I_C (all-equity financed control action) to improve the selling price of the commodity (indirectly affecting the value of assets) but this has uncertain outcome on returns. At the time of investment the equity holders also decide the optimal level of debt financing taking into account that debt holders will rationally set coupon payments according to the risk position they face (debt financing and equity financing constraints may exist). At this point they also set the optimal level of P, P_D, that triggers bankruptcy.

Figure 2. Optimal debt level determination