Technological Investment Decisions: Implications for Real Options Logic

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Technological Investment Decisions: Implications for Real Options Logic

Abstract

When faced with technological opportunities over time, what is the optimal investment strategy for a firm seeking to invest in the very best technology? We discuss an alternative scenario of high uncertainty but limited flexibility than the one typically described in the real options literature, highlighting some shortcomings of traditional models. While real options modeling is quite informative in contexts where technological investment opportunities stem from the same technological trajectory, it falls short of guiding investment decisions regarding disruptive and/or architectural innovations. Under such circumstances, firms have little or no information about the distribution of expected returns from each investment option a priori. Moreover, they cannot revisit foregone opportunities due to intense rivalry. We propose a portfolio investment strategy for a firm seeking to invest in the very best technology. The analysis suggests that it is not optimal to spread out investments evenly over all options nor is it optimal to fully invest in options presented early on. Instead, the optimal strategy is for the investing firm to invest fully (all of its endowment) in the relatively best option after a threshold based on the number of observed options.
1 Introduction

A Puzzle...
Long ago, in a far away land, a sultan granted a commoner a chance to marry one of his hundred daughters. But there was a twist: the commoner was presented the daughters one at a time, and when a daughter was presented, the commoner was told the daughter’s dowry. The commoner had only one chance to accept or reject each daughter; he could not return to a previously rejected daughter. The sultan’s catch was that the commoner could only marry the daughter with the highest dowry. What was the commoner’s best strategy, assuming he knew nothing about the distribution of dowries?¹

In financial markets, portfolio selection models consider investments that are spread over a large set of stocks and assets. Real options theory applies this logic to explain how a firm can hedge against project risk in uncertain markets and expand. The option-based techniques are used to evaluate the value of managerial flexibility regarding portfolio selection models (Kester, 1984; Kogut, 1991; Mason and Merton, 1985; Myers, 1987; Trigeorgis, 1993; Trigeorgis and Mason, 1987). The convenient application of financial options logic to firms’ investment behavior has suggested that firms, like investors, should invest in multiple projects (and/or strategic partnerships) to hedge against risk. The heuristics provided by real options logic is intuitively appealing, yet the real options models have much less broader applicability. Although it is in fact convenient to consider risk reduction and investment management in terms of broadly defined real options logic, it has pitfalls for several reasons.

First, real options logic unrealistically assumes that the investor has full information

¹Similar problems appear in many statistics and mathematics books, such as Mathematical Plums, 1979 and Handbook of Sequential Analysis, 1991.
about the distribution of expected returns from each investment option \textit{a priori}. In most cases, prior to observing the investment option, the investor knows very little or nothing either about its return or the project’s relative profitability (rank) among all the possible investment opportunities that might arise in the future. Second, traditional real options logic implicitly assumes that investors observe all choices at time \(t\). This allows a complete \textit{a priori} ranking of the investment options before actually investing in the \textit{very best} one. However, in reality more often than not, investment opportunities present themselves over time.\footnote{Sequential real options logic recognizes that, especially large projects can be broken into discrete but dependent decision points over time. This is different than the investment scenario we are dealing with, since each discrete decision point is \textit{independent} from each other.} Firms face discrete but independent alternatives: a sequence of choices, each of which can be represented as sequential real options. Therefore, it is not hard to realize that, due to external forces such as competition, foregone investment options may not be revisited (or revisited with cost) once the investor realizes that that option was the \textit{very best} one (e.g. Due to intense competition Xerox cannot invest in its \textit{foregone} discovery of personal computers). Third, traditional real options logic cannot guide the investor’s project choice decision without knowledge of how returns are distributed, which in turn inhibits further valuation of cost/benefit or growth potential.

The shortcomings of the theory are emphasized in the context of guiding technological investment decisions regarding disruptive and/or architectural technologies. Generally, investing firms have little or no information about the distribution of expected returns associated with technologies that replace old technology trajectories. Such tech-
nologies create, at least temporarily, a winner-take-all market (e.g. Hill, 1997), where the innovator has a clear advantage over the other firms. When faced with many technological opportunities over time (a sequence of investment options), what is the optimal strategy for a firm seeking to invest in the very best option? This question provides an alternative scenario of high uncertainty, but limited flexibility, than the one typically described in the real options literature (Amram and Kulatilaka, 1999, Dixit and Pindyck, 1994), thereby extending investment decision models. Our puzzle mentioned above bears remarkable resemblance to such investment problems—particularly in industries where technology management is pivotal to the firms’ success. This paper provides an alternative to traditional real options logic and presents an optimal strategy for investing in such technologies.

First, we discuss the dynamics of this problem in the context of technology management, specifically in relation to underlying investment models. Second, we highlight the shortcomings of traditional real options models in valuing technological investments.

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3Winner-take-all component of the argument is not essential to the proposed investment model. However winner-take-all condition illustrates an extreme market condition that simplifies the model’s solution without changing the intuition. The proposed model can be extended into other types of markets with appropriate adjustments to the model assumptions.

4It is important to note that the model does not depend on monopolistic competition per se. On the contrary, the motivation to capture monopolistic rents in product markets are embedded in highly competitive factor markets. Aghion and Howitt argue that ‘product market competition is unambiguously bad for growth: the more competition, the lower the size of the monopoly rents that will be appropriated by successful innovators, and therefore the smaller the incentives to innovate’ (1998: 58). Otherwise in a monopolistic factor market setting, the motivation to innovate disruptive and/or architectural technology would not be there in the first place (e.g. Reinganum, 1983).

5Traditional real option valuation is applied to investments that share three characteristics: (i) investment is partially or completely irreversible; (ii) future payoffs are uncertain and (iii) there is some leeway about the timing of the investment (Brenman and Schwartz, 1985; Dixit and Pindyck, 1994; Trigeorgis, 1996). This approach contrasts the net present value (NPV) rule, which also considers an irreversible investment, in that it is a “now or never decision with no uncertainty,” ignoring investors ability to delay.
Third, we propose a model and an investment strategy for firms engaging in architectural and/or disruptive innovation. Finally, we discuss the implications of the theory and suggest further avenues of research.

2 Disruptive versus Incremental Technology

Henderson and Clark (1990) revisit the relationship between firm capabilities and the characteristics of technological change. Firms, they argue, can possess two types of technological capabilities: componentry and architecture. Componentry refers to knowledge regarding core design concepts (Clark, 1985) and the way in which they are implemented in a particular component. Architecture refers to the way the components interact within the design. They refine and make more translucent four general categories of innovation based on the two types of technological capabilities. Incremental innovation builds upon the firm’s expertise in both of these domains, and in turn reinforces the current capability set. Modular innovation introduces new component technology to the existing product architecture (e.g. auto focus enhancement in camera design). In contrast, architectural innovation requires a change in the existing product architecture, given the existing components. The last category, radical innovation, is one where both the current product architecture and the design components become obsolete due to the new innovation.

Technological trajectories define cumulative linear stochastic trends of innovative activity (e.g. Dosi 1982; Nelson and Winter, 1977). Building on the work of Hen-
derson and Clark (1990) and Christensen and Rosenbloom (1995), Figure 1 depicts the relationship between innovation categories and firm capabilities, along with corresponding investment models. Firms investing in incremental innovation must possess requisite component and architectural capabilities to advance on an existing trajectory (Levinthal, 1997). This investment strategy can be modeled as a linear stochastic trend easily represented by Brownian motion. Similarly, firms investing in modular innovation, such as a new entrant with a related patent (or an existing firm where the patent is ready to expire), must develop a variant of their existing capabilities to modify the existing trajectory where the underlying stochastic path registers a ‘discrete jump.’ This can be modeled as a Poisson process (Dixit and Pindyck, 1994: 85). Such linear stochastic paths and their variations (represented by the family of Brownian motion (e.g. Dixit, 1991b)) are the building blocks of real options logic.

Industry incumbents tend to invest in existing technological trajectories (e.g. Dosi, 1982; Reinganum 1989) that carry forward a shared technological paradigm. This process rarely results in the substitution of a new technological trajectory resulting from scientific breakthroughs (Zucker and Darby, 1996). The same logic is applicable to the emergence of dominant design (Tushman and Anderson, 1986) since alternative dominant designs can typically be traced back to the same technology trajectory or the architectural design. Although the developer of the dominant design emerges as the winner and creates a ‘lock in’ effect, the underlying investment model can be represented with real options logic with ‘discrete jump’ processes (e.g. Gaynor and Bradner, 2001).
Industries, however, can experience multiple technology trajectories over time (Christensen, 1997). In such cases, the investment decision is related to the questions of when will the management of an incumbent firm recognize a new dominant technology and whether they will then choose to transform the firm’s technological capabilities (Zucker and Darby, 1997) because this is the very best growth option. Generally, disruptive technologies that substitute old-technology trajectories create, at least temporarily, a winner-take-all market. Here, the innovator has a clear advantage over other firms that either were incumbents with old technology trajectories or firms with alternative disruptive technologies that were not as successful (or simply latecomers to the industry). Incumbent firms are expected to be displaced by entrants when there is externally generated, disruptive innovation that deems the incumbent’s existing set of skills obsolete. Real options logics falls short of guiding these types of investment decisions. Investing firms have little or no information about the distribution of expected returns associated with technologies that replace old technology trajectories. In this paper, we propose an investment strategy for such disruptive and/or architectural technologies.

2.1 The Analogy and Problem Setup

The puzzle is a good analogy to the investment dilemma a large pharmaceutical firm faces regarding which new product development project to invest, or which small biotechnology firm to form an alliance with to develop the next best product. In this
sense, most R&D projects inherit two dimensions of uncertainty: technical and input cost (Dixit and Pindyck, 1994). Technical uncertainty relates to the likely cost and probabilities of accomplishing technical success (Dixit and Pindyck, 1994; McGrath, 1997). This kind of uncertainty pressures the firm to invest since delays might expose the firm to competitive preemption and being locked out of the market. Therefore, in such high-tech industries, we assume, that an investment in the very best technology yields a lucrative, time-dependent rate of return, that uninvested capital keeps its risk-free value, whereas, “wrong” investments lose their value and do not generate economic rents.

We also assume that there are clear winners and losers, and innovations may be generated outside the large firm. We will describe the optimal strategy of a large high-tech firm (e.g. pharmaceutical company\(^6\)) seeking to form the most lucrative alliance with a small highly innovative firm (e.g. biotech start-up) among similar ones. To illustrate the decision process, the pharmaceutical industry provides a most useful and convenient context. For example, Pfizer has been developing a promising new drug of inhaled insulin, *Exubera*, with its partners-until, that is, concerns arose that it could impair lung functions\(^7\). However the model could be generalized into industries where there is a tension between the established technological trajectories and the emergence

\(^6\)In pharmaceutical industry, production costs are relatively low, and product innovation is the major expense item. High profits are plowed back into pharmaceutical firms to fund R&D. Patents are one of the major ways to help firms protect their profits and from a key patent, firms may generate considerable profits in their monopolistic markets. The pharmaceutical industry is full of examples where drug makers enjoy monopolistic market shares based on their patented products. Pfizer’s Viagra, Warner-Lambert’s Lipitor, Eli Lilly’s Zyprexa, Monsanto/Pfizer’s Celebrex are good examples of such market power.

\(^7\)Pfizer ended up not getting the FDA approval.
of disruptive technologies\(^8\).

### 2.2 Valuing Technological Investments with Real Options

Investment decisions incorporate three characteristics: irreversibility (sunk cost), uncertainty, and the choice of timing (Dixit and Pindyck, 1994). The interrelatedness of these characteristics have been ignored by the orthodox investment theory (Net Present Value). Real options logic deals with the irreversibility of investment under uncertainty and has been applied to the investment scenarios with delay in either monopolistic or perfect competition (Bertrand) (e.g. Dixit, 1989; 1991; McDonald and Siegel, 1986; Pindyck, 1988). Some of the applications of real options have been in managing investments in new capabilities (e.g. Bernardo and Chowdhry, 2002; Bowman and Hurry, 1993; Kogut and Kulatilaka, 2001; McGrath, 1997; Miller and Folta, 2002); and R&D and technology management (Gaynor and Bradner, 2001; Kogut and Kulatilaka, 1994; Kulatilaka and Perotti, 1998; McGrath, 1997; Pakes, 1986).

It is very hard to argue with the real options heuristic that firms should hedge against the downside risk of investments by taking into account the advantages of incremental investment and at each stage considering the economic value of further delay, partial or total abandonment of the project, or exercising the option of further or full commitment. Firms, like individuals would incline towards ‘keep(ing) (their) options open’ (Bowman and Hurry, 1993; Dixit and Pindyck, 1994; Hubbard, 1994). It makes sense. However, since real options logic is derived from the principles of financial

\(^8\)Please refer to Clayton M. Christensen, *Innovator’s Dilemma* (1997) for a great discussion.
derivatives, actual applications of valuations models are much more restrictive than the eagerly-adopted heuristic described above. “Heuristics permit faster solutions to real-time problems; they also suffer from the potential negative transfer to inappropriate applications” (Kogut and Kulatilaka, 2001: 744). This section is a brief illustration of the generic mathematical setup of real options models. The purpose is to disintegrate the real options logic as a useful heuristic from the real options logic as an investment valuation model.

The standard practice of modeling irreversibility of investment under uncertainty in real options assumes a Wiener process of gross project value, $x_t$, at time $t$ given by (1).

\[
dx/x = (\alpha - \delta)dt + \sigma dz
\]

where $\alpha$ is the instantaneous expected return on the nontraded asset (project), $\delta$ is the difference between the equilibrium rate of return on the similar-risk traded financial asset, $\alpha_e$, and the rate of return on the nontraded asset , $\alpha$ (McDonald and Siegel, 1984, 1985; Trigeorgis, 1991), and $dz$ is the increment of the Wiener process. The motion over time (discrete or continuous) is Brownian, and most importantly the prediction for period $t + 1$ is only dependent upon the current period, $t$ (Martingale).

Real options, by design are inseparable from their underlying asset’s state (gross project value of $x_t$). At each period $t$, available choices are represented by control variable $u_t$ (Dixit and Pindyck, 1994). The state and the control at time $t$ affect the
firm’s immediate profit flow, and the cumulative probability distribution function of the state next period conditional upon the current information is \((x_t, u_t): \Phi_t = (x_{t+1} | x_t, u_t)\).

It is useful to discuss two alternative investment decision scenarios. In the first scenario, Firm A is positioned on a technological trajectory therefore its investment decision can be represented by real options model as discussed above\(^{10}\). Such investment strategy is intended to generate mainly incremental and modular innovations, and rarely radical innovations (mostly as serendipitous occurrences). In the second scenario, Firm B *seeks* to pick among the possible disruptive technologies and/or architectural innovations to create competitive advantage. Real options modelling will not be helpful in this context because the distribution of choices cannot be represented by a Brownian motion. In our next section, we provide an investment model which guides firms in the second scenario.

\(^{10}\)It is possible to have radical innovation emerging from a series of incremental innovations (e.g. television industry). Authors acknowledge Steve Klepper for bringing this point to attention at the CCC, Boston 2002. However, it is hard to argue for such an investment strategy to realizing the deliberate and specific intentions of being able to generate radical innovation.
3 Forming a Technological Investment Portfolio

3.1 The Intuition

In its simplest form, the problem for an investing firm\(^{11}\) seeking to invest in disruptive or architectural innovations has the following features.\(^{12}\) The investing firm is faced with a sequence (each option is independent from each other) of \(n\) investment opportunities. For simplicity, at each stage the firm can invest in only one option.\(^{13}\) Investment opportunities are presented in a *random order*, where any order is equally likely. Each investment option’s nominal expected return \(V\) is revealed to the investing firm only when it is presented. It is assumed that the investing firm can rank these options from best to worst without ties. The decision to accept or reject an option must be based only on the relative ranks of those investment opportunities considered so far. An option passed (rejected) cannot later be recalled.\(^{14}\) The investing firm is very particular and will be satisfied with nothing but the *very best*. That is, the payoff (the real expected value of the *very best option*) is \(v\) if the firm chooses the best of the \(n\) opportunities and 0 otherwise. It is important to note that the *only* information the investing firm has up front is the total number of opportunities that will be presented.\(^{15}\) This type

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\(^{11}\)In this paper, in accordance with the strategy literature, the *firm* is viewed synonomiously with the *investor*.

\(^{12}\)For similar models representing ‘winner-take-all’ markets, refer to Ferguson and Gilstein, 1985; Ferguson, 1989; and Ferguson and Bruss, 1998.

\(^{13}\)We do this for simplicity, however, the results are robust to making investments in more than one option which is discussed in the following sections. A profit maximizing investor would want the very best and thus would want to invest all his wealth in the very best option.

\(^{14}\)In other words, the time value of the opportunity decays. If we allow for backward solicitation, we can modify the model by introducing cost or with probability \(q\) of the investment opportunity still being available.

\(^{15}\)This aspect of the model can be generalized to an unknown number of opportunities.
of problem can be classified as one of a sequential observation and selection where the
payoff depends on observing their relative ranks \( r \) (and not otherwise on their actual
values).

This maximization problem has a simple solution. The first class of solutions re-
stricts decision rules such that for some integer \( k \geq 1 \), the investor rejects the first \( k - 1 \)
choices, where \( k \leq n \). Then he chooses the next investment which is best in terms of
its relative ranking of the observed investment choices. For such a rule, the probability
\( \phi_n(k) \), of selecting the best investment is \( 1/n \) for \( r = 1 \), and, for \( r > 1 \),

\[
\phi_n(k) = \sum_{j=k}^{n} P(\text{\( j \)th opportunity is best and you select it})
\]

\[
= \sum_{j=k}^{n} \left( \frac{1}{n} \right) \left( \frac{k - 1}{j - 1} \right) = \left( \frac{k - 1}{n} \right) \sum_{j=k}^{n} \frac{1}{k - 1}
\]

The optimal \( k \) is the one that maximizes this probability or

\[
\max_k \phi_n(k) \text{ s.t. } k \leq n
\]

and solve \( \frac{\partial}{\partial k} (\left( \frac{k - 1}{n} \right) \sum_{j=k}^{n} \frac{1}{k - 1}) = 0 \) for \( k \).

For small values of \( n \) one can compute the optimal \( k \). For example, consider an
investing firm with 10 options, seeking to maximize the probability of picking the very
best option. Let us assume that the investing firm arbitrarily rejected the first 3 options.
The probability of the very best option being presented to the firm as, for example, the
8th choice and picking that choice would be \( \frac{1}{10} \times \frac{3}{7} = .043 \) or 4.3%. Of course, \textit{a priori}
the number of options worth rejecting \((k - 1)\) which would maximize the probability of choosing the very best option could be calculated.

This simple intuition suggests that it may be optimal for an investing firm to wait, observe a few initial relative rankings of returns (thereby passing up first \((k - 1)\) choices), and then pick the very best option that comes along. Of interest is also the approximate values of the optimal \(k\) for large \(n\). If we let \(n\) tend to infinity and write \(x\) as the limit of \(\frac{1}{n}\), then using \(t\) for \(j/n\) and \(dt\) for \(1/n\), the sum becomes a Reimann approximation to the integral

\[
\phi_n(k) = \frac{r - 1}{n} \sum_{j=k}^{n} \left( \frac{n}{j - 1} \right) \frac{1}{n} \to x \int_1^{1/x} \left( \frac{1}{t} \right) dt = -x \log(x) \tag{4}
\]

The value of \(x\) that maximizes this quantity is found by setting the derivative with respect to \(x\) equal to zero and then solving for \(x\).

### 3.2 The Basic Setup

In this setup, the investor firm can invest in more than one option and form a portfolio of sequential investments. In its quest to obtain the very best technology, a firm faces the decision to invest in a project and the selection of which new technology is a yes or no choice, and that the firm cannot hedge by selecting fraction of a new technology. Assume that the firm has \(w_0\) (wealth) to invest in technology alliances(options). The firm will observe a known number, \(n > 0\), of (rankable) projects presented to it by entrepreneurial firms. It examines the firms sequentially but in completely random
order. (For notational simplicity we assume that the stages are denoted by the project the firm is presented with).

In the first stage, the firm must decide whether to invest any amount \( i_1, 0 \leq i_1 \leq w_0 \) in that entrepreneurial firm, leaving \( w_1 = w_0 - i_1 \) for future investments. If after all projects are observed, this project is the very best overall, then the return on the investment is given by \( y_1 = R_1 i_1 \), where \( R_1 = (1 + r_1) \geq 1 \), a known rate of return available in stage \( k = 1 \). If it is not the very best overall, then the firm loses its investment, \( y_1 = 0 \). Similarly, in stage \( k = 2, 3, \ldots, n \), if the \( k \)th object is the relatively best and its remaining (uninvested) capital is \( w_{k-1} \), it may invest any amount \( i_k, 0 \leq i_k \leq w_{k-1} \), in the \( k \)th option. The return on this investment will be \( y_k = R_k i_k \) if the \( k \)th project is the very best overall, where \( R_k \geq 1 \). The firm seeks to choose a sequence of investments to maximize the expected value after the game is over. No interest accumulates on uninvested capital. More generally, assume there is a utility\(^{16} \) \( u_a \) (e.g. a return mapped into profits) defined on the firm’s investment decisions:

\[
\begin{align*}
  u_a(w) &= \begin{cases} \\
    \frac{w^a}{a} & \text{for } \alpha \neq 0 \\
    \log(w) & \text{when } \alpha = 0 
  \end{cases} 
\end{align*}
\]

(5)

This functions also gives us a natural setting to analyze how differing levels of risk affect

\(^{16}\) We assume that the decision maker’s utility is tied to the firm’s profit function with the right set of incentives, assuming away the potential agency conflicts. This assumption allows us to use utility function to describe preferences (‘..., but it should not be given any psychological interpretation. The only relevant feature of a utility function is its ordinal character. If \( u(x) \) will represents some preferences \( \geq \) and \( f : R \rightarrow R \) is a monotonic function, then \( f(u(x)) \) will represent exactly the same preferences since \( f(u(x)) \geq f(u(y)) \) if and only if \( u(x) \geq u(y) \). (Varian, 1992: 95)
3.3 The Recursive Formulation

We use Bellman Equations to describe the recursive behavior (Bellman, 1957; Sniedovich, 1992). The investing firm’s endowment is \( w_0 \) in the initial stage, \( k = 0 \). The state space corresponding to the amount of capital endowment available for investment is \( W = \{ w \mid w \geq 0 \} \). The action space, \( I \) corresponding to the investment decision of the firm at each stage \( k \) is \( I = \{ i \mid i \geq 0 \} \). In each stage \( k \), the investing firm cannot spend more than the maximum available amount \( w \) for that stage which defines the feasible action set, \( \Psi_k(w) = \{ i \mid 0 \leq i \leq w \} \), for \( k = 1, \ldots, n \). If the investing firm invests an amount \( i \) when there is total amount \( w \) available for investment, then the firm is left with \( w - i \), which corresponds to the transition function \( f_k(w, i) = w - i \), for \( k = 1, \ldots, n \).

Figure-2 represents this decision making process.

For simplicity, the investor is assumed to be risk neutral, \( \alpha = 1 \), with linear utility function, \( u_\alpha(w, y) = w + y \), where \( w + y > 0 \). The investing firm receives a reward at each stage that corresponds to the following reward function \( V_k(w, y) = u_\alpha(w, y) = w + y \); where \( y_k = R_ki_k \), and \( w_k = w_{k-1} - i_k \) for \( k = 1, \ldots, n - 1 \). It is important to notice that

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17 Two things are common to all the applications of Bellman Equations: (1) The solution to the maximization problem is found by starting from the very last stage \( k = n \) and then working backwards and (2) The maximization problem encountered at each stage \( k \) has a lower dimension, which is the total number of available choices (states) available at that stage; the actual outcome has at least the dimension \( n \), of a strategy incorporating each stage \( k = 1, \ldots, n \). These types of multistage (sequential) decision models are commonly discussed in dynamic programming. Sequential decision models represent real-world situations where a sequence of decisions, aimed at attaining a certain goal, is made. The most important feature of sequential decision models is that a decision made at any given time is affected by its predecessors and invariably affects its successors.
the utility function is increasing in \( i \) in a linear fashion.

We need to find for all \( k = 1, \ldots, n \) the value \( i \in \Psi_n(w) \) that maximizes the reward function \( V_n(w, y) \)--the first step in constructing the Bellman Equation. 

\[
\max_{0 \leq i_n \leq w_{n-1}} V_n(w - i, R_ki_k).
\]

Since the linear utility function is maximized when either \( i = 0 \) or \( i = w \), the maximization problem in stage \( k = n \) can be rewritten as:

\[
\max_{0 \leq i_n \leq w_{n-1}} V_n(w - i, R_ki_k) = \max \left\{ V_n(w, 0), V_n(0, R_nw) \right\}
\]

for \( k = n \)

Think of \( V_k(w, y) \) be the value function representing the investing firm’s expected utility from, for example, an alliance, using an optimal strategy, when at stage \( k \), \( 1 \leq k \leq n \), before the firm observes the \( k \)th innovative entrepreneurial firm, it has an amount \( w \) available for future investments and a preceding investment that will return \( y \) if the present, relatively best project is best overall. It is first important to recognize that the firm would invest in a project if it is of relative rank 1 because otherwise it cannot be best of all. We refer to this project as a record. In the last stage \( k = n \), the initial (backward) condition, which is based on (6), and recursive equations are provided in (7) and (8) respectively:
\[ V_n(w - i, y) = \frac{n - 1}{n} u_a(w, y) + \frac{1}{n} u_a(R_n i) \]
\[ = \frac{n - 1}{n} (w + y) + \frac{1}{n} R_n w \quad \text{for } k = n, \text{ and} \]

\[ V_k(w - i, y) = \begin{cases} 
\frac{k - 1}{k} V_{k+1}(w, y) + \frac{1}{k} \max_{0 \leq i \leq w} V_{k+1}(w - i, R_k i) 
\end{cases} \]

for \( k = 1, \ldots, n - 1 \)

where \( P(\text{\textsuperscript{k}th object is not relatively best}) = \frac{k - 1}{k} \)

\[ P(\text{\textsuperscript{k}th object is relatively best}) = \frac{1}{k} \]

In other words, if at the \( k \)th stage, the \( k \)th project is not relatively best (which occurs with probability \( \frac{k - 1}{k} \)), we proceed to the next stage with the same \( W \) and \( y \); if it is relatively best (which occurs with probability \( \frac{1}{k} \)), the firm loses \( y \), but now chooses to invest \( i \) in the \( k \)th project to maximize its utility. In the final stage, if the \( n \)th project is relatively best, it is certain to be best overall, so the investing firm invests all of its remaining capital.

In solving the maximization problem, we need to compute \( V_{n-1}(w, y) \) using (7) and (8). Thus, from (8), if we let \( k = n - 1 \) and substitute for \( V_n(w, y) \), then \( \max_{0 \leq i \leq w} V_n(w - i, R_{n-1} i) \) using (7) and (6), we get
\[ V_{n-1}(w, y) = \frac{n-2}{n-1} V_n(w, y) + \frac{1}{n-1} \max_{0 \leq i \leq w} V_n(w - i, R_{n-1} i) \]

\[
= \frac{n-2}{n-1} \left[ \frac{n-1}{n} (w + y) + \frac{1}{n} R_n w \right] + \frac{1}{n-1} \max \left\{ V_n(w, 0), V_n(0, R_{n-1} w) \right\} \\
= \frac{n-2}{n-1} \left[ \frac{n-1}{n} (w + y) + \frac{1}{n} R_n w \right] + \frac{1}{n-1} \max \left\{ \left( \frac{n-1}{n} w + \frac{1}{n} R_n w \right), \left( \frac{n-1}{n} R_{n-1} w \right) \right\} \\
= \frac{n-2}{n-1} \left[ \frac{n-1}{n} (w + y) + \frac{1}{n} R_n w \right] + \frac{w}{n} \max \left\{ \left( 1 + \frac{R_n}{n-1} \right), R_{n-1} \right\} \\
= \frac{n-2}{n} (w + y) + \frac{w}{n} \left[ \frac{n-2}{n-1} R_n + \max \left\{ \frac{1 + \frac{R_n}{n-1}}{n-1}, \frac{R_{n-1}}{n-1} \right\} \right] \\
\text{(9)}
\]

As mentioned above, \( R_k \geq 1 \). The optimal strategy of investing at stage \( k = n - 1 \) is determined based on the following inequality of \( R_{n-1} \geq 1 + \frac{R_n}{n-1} \). Therefore,

1. Invest everything (in the project, alliance partner, etc.) if \((n - 1) \geq \frac{R_n}{R_{n-1}}\)

2. Do not invest (in the project, alliance partner, etc.) if \((n - 1) < \frac{R_n}{R_{n-1}}\)

Basically, the overall return at stage \( n - 1 \) has the same form as it does at stage \( n \).

Compute \( V_n \) by using (9) by allowing \( k = n \). Also since this is the the end of the game \( R_{n+1} = 0 \), and \( i = w \).
\[ V_n(w, y) = \frac{n-1}{n} (w + y) + \frac{w}{n} \left[ \frac{n-1}{n} R_{n+1} + \max_{i=0} \left\{ 1 + \frac{R_{n+1}}{n} \right\} \right] \]

\[ = \frac{n-1}{n} (w + y) + \frac{w}{n} \left[ \max \{1, R_n\} \right] \]

\[ = \frac{n-1}{n} (w + y) + \frac{w}{n} d_n; \text{ where } d_n = R_n \quad (11) \]

We continue through the backward induction to solve over all stages since both \( V_n \) and \( V_{n-1} \) have the same form such that:

\[ V_{n-1} = \frac{n-2}{n} (w + y) + \frac{w}{n} \left[ \frac{n-2}{n-1} R_n + \max_{i=0} \left\{ 1 + \frac{R_n}{n-1} \right\} \right] \]

\[ = \frac{n-2}{n} (w + y) + \frac{w}{n} d_{n-1} \]

\[ V_{n-2} = \frac{n-3}{n-2} V_{n-1}(w, y) + \frac{1}{n-2} \max_{0 \leq i \leq w} V_{n-1}(w - i, R_{n-2} i) \]

\[ = \frac{n-3}{n-2} \left[ \frac{n-2}{n} (w + y) + \frac{w}{n} d_{n-1} \right] + \frac{1}{n-2} \max \{ V_{n-1}(w, 0), V_{n-1}(0, R_{n-2} w) \} \]

\[ = \frac{n-3}{n} (w + y) + \frac{w}{n} \left[ \frac{n-3}{n-2} d_{n-1} + \frac{1}{n} \max \{(n-2 + d_{n-1}), (n-2)R_{n-2}\} \right] \]

\[ = \frac{n-3}{n} (w + y) + \frac{w}{n} \left[ \frac{n-3}{n-2} d_{n-1} + \max \left\{ 1 + \frac{d_{n-1}}{n-2}, R_{n-2} \right\} \right] \]

\[ = \frac{n-3}{n} (w + y) + \frac{w}{n} d_{n-2} \]

\[ V_k = \frac{k-1}{n} (w + y) + \frac{w}{n} d_k, \text{ where } d_k = \left[ \frac{k-1}{k} d_{k+1} + \max_{0 \leq i \leq w} \left\{ 1 + \frac{d_{k+1}}{k}, R_{k} \right\} \right] \quad (13) \]

It appears that the value function has the form of (13) however our proof has to demonstrate that this function satisfies Bellman Equations for all stages \( k = 1, \ldots, n-1 \).
and states $W = \{ w \mid w \geq 0 \}$.

**Proof.** Assume that the proposed value function, (13), is also true for the stage $k+1$. We have shown it to be true for $V_n, V_{n-1}, \text{and } V_{n-2}$. Then

$$V_{k+1} = \frac{k}{n}(w + y) + \frac{w}{n}d_{k+1},$$

where $d_{k+1} = \frac{k}{k+1}d_{k+2} + \max \left\{ \left(1 + \frac{d_{k+1}}{k+1}\right), R_{k+1} \right\}$

for $k = 0, \ldots, n-2$

$$V_k = \frac{k-1}{k}V_{k+1}(w, y) + \frac{1}{k} \max_{0 \leq i \leq w} V_{k+1}(w - i, R_{k}i), \text{ for } k = 1, \ldots, n-1$$

$$= \frac{k-1}{k} \left[ \frac{k}{n}(w + y) + \frac{w}{n}d_{k+1} \right] + \frac{1}{k} \max \left\{ V_{k+1}(w, 0), V_{k+1}(0, R_{k}w) \right\}$$

$$= \frac{k-1}{k} \left[ \frac{k}{n}(w + y) + \frac{w}{n}d_{k+1} \right] + \frac{1}{k} \max \left\{ \left[ \frac{k}{n}w + \frac{w}{n}d_{k+1} \right], \left[ \frac{k}{n}R_{k}w \right] \right\}$$

$$= \frac{k-1}{n}(w + y) + \frac{k-1}{n}w \frac{w}{n}d_{k+1} + \frac{w}{n} \max \left\{ \left(1 + \frac{d_{k+1}}{k}\right), R_{k} \right\}$$

$$= \frac{k-1}{n}(w + y) + \frac{w}{n} \max \left\{ \left(1 + \frac{d_{k+1}}{k}\right), R_{k} \right\}$$

$$= \frac{k-1}{n}(w + y) + \frac{w}{n}d_k$$

Our theorem summarizes the value function for determining the optimal investment strategy for a firm wishing to invest in the very best opportunity (e.g. radical innovation that would end-up being the very best of all contenders) or, more generally, in markets where the winner takes all.
Theorem 1  In the case of $\alpha = 1$, risk neutrality,

\[
V_k(w, y) = \frac{k-1}{n}(w + y) + \frac{w}{n}d_k
\]

for $k = 2, \ldots, n$

\[
V_1(w, y) = \frac{w}{n} \max \{1 + R_2, R_1\}
\]

and for $k = 2, \ldots, n-1$

\[
d_k = \frac{k-1}{k}d_{k+1} + \max \left\{ \left( 1 + \frac{d_{k+1}}{k} \right), R_k \right\}, \quad \text{and}
\]

\[
d_n = R_n, \quad \text{for } k = n
\]

As shown by the $V_1(w, y)$, it is not optimal to invest all in the first stage, $k = 1$, because the value function does not depend on $y_1 = w_1R_1$, but depend on $w_1$. An Optimal Investment Policy is to invest everything in the first relatively best object, $k = 2, \ldots, n-1$, for which $R_k > 1 + \frac{d_{k+1}}{k}$. However, it is not optimal to invest in each project equal amount such that $i_k = \frac{w_0}{n}$ because of high opportunity costs associated with avoidable (e.g. not relatively best) but nonetheless wrong investment choices. In the last round it is optimal to invest whatever is left since uninvested capital does not earn interest (above risk-free rate). If the opportunity in the last stage is not the very best over all $k$, then $y_n = w_nR_n = 0$, which is equivalent to not investing $w_n$ at all at stage $k = n$. 
4 Discussion

Investment in technology is a convenient context to disintegrate real options logic as a useful heuristic from real options logic as an investment valuation model. Specifically, in traditional real options logic, there is an implicit assumption that the distribution of returns are \textit{a priori} known. For instance, the higher the volatility of expected returns to a project at hand, the higher the value of the option to invest in that project. The order of observing investment options are irrelevant to the analysis. However, in reality, investment opportunities tend to present themselves sequentially (multistage). Moreover, the order of investments (\textit{when} you observe them) matters because it affects the ranking of options over time. Also, most investment options surrounding disruptive or architectural technologies are mutually exclusive, in the sense that the investor is forced to choose one. This rephrases the decision criteria to one where the goal is to pick the \textit{very best of all} options. One can extend the simple analysis to a problem of constructing a portfolio of sequential real options with maximum expected returns, provided that the decision criteria is similar to the previous case for each decision node.

These kinds of project selection problems occur in winner-take-all markets, such as those characterized by disruptive or architectural innovations. Our investment strategy is useful for firms that have little or no information regarding the distribution of expected returns associated with new technologies that are likely to be mutually exclusive. For example, this could be a large pharmaceutical company presented with investment opportunities by small biotech firms seeking to develop proprietary products that cure
the same illness or seeking to pick the most profitable use of a particular drug. Pfizer will have spent $500 million between 1997 and 2004 on 300 different Lipitor studies to find out, among other things, if the drug can reduce heart-disease risk in diabetics.\textsuperscript{18}

What is Pfizer’s optimal strategy for picking the \textit{very best} option? In another investment problem, the same firm may wish to form a real-options portfolio that has the maximum expected return (e.g. portfolio of drugs). Again, the options are revealed sequentially, but they are not mutually exclusive, which allows the firm to allocate its wealth across choices, over time.

In our sequential investment model, the order in which projects are observed is assumed to be exogenous. However, the managers of large incumbent firms (for example) could endogenize the order with which they observe the projects and as a result make better investment decisions. For example, a large pharmaceutical company with a good track record of forming partnerships with innovative biotech companies will be more desirable for other similar small firms in the future. This increases the large pharmaceutical company’s likelihood of receiving a larger set of new choices. Moreover, such a firm would be more likely to be granted the ‘first right of refusal.’ Firms managing this process can develop their project selection capabilities, which in turn might lead to a sustainable competitive advantage.

Real options, although very appealing as a heuristic, has limited applicability as a technological investment model. Our findings suggest that real options logic is most useful for firms that seek to implement a technological investment strategy that is more

likely to yield incremental and modular innovations. In these cases one can model the underlying stochastic process as a Brownian motion and represent modular innovations as discrete jumps from Poisson distribution. However, real options logic is not applicable as an investment model when the stochastic process is unknown, which would be the case for radical and architectural innovations. Technological investment strategies that are implemented to generate radical and architectural innovations are better represented with the dynamic decision model we have proposed in this paper.

There are possible extensions to this framework. What is the optimal investment strategy when investing firms have varying levels of risk preference? What if investment choices increase or decrease the future options observed? For instance, in the case of choosing the best alliance partner, a pre-committment to an option early on might adversely affect the future choices due to potential leakages of intellectual property. In this specific extension we instead of independent options we have introduced an endogeneity into the project pool (thus the order).

In the specific case of choosing an alliance partner, one can also introduce the problem of adverse selection. It is fair to say that, assuming capital markets are efficient and there is borrowing and lending in financial markets, firms that are closer to finding a cure will not want to form an alliance to fully appropriate the monopolistic returns. Therefore, biotech firms that are willing to form alliances with the large firm are actually the ones who are less likely to succeed. If one were to model it as such, we would see a pooling equilibrium because the firms who are not close to finding a cure would also mimic those firms who are close to finding a cure by refusing to partner up. Thus,
transaction costs can be introduced to the project selection problem.

Since the selection criteria is formalized, it is also possible to introduce firm-specific factors that modify the value functions. For example, managerial capabilities, on the one hand, act as a constraint to limit the maximum number of options one can invest in, but on the other hand might increase the quality of the projects. In our model, we essentially considered a case of no-information with a known number of opportunities. The model can be extended to one where there is no-information with an unknown number of opportunities.
5 References


Figure 1: Investment models and underlying stochastic processes of innovation

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Figure 2: Dynamics of the decision-making process at a typical stage

Investment Decision
Ψ_i(w) = \{i \mid 0 \leq i \leq w\}

Stage k-1

... → Stage k

Available capital

W = \{w \mid w \geq 0\}

Transition
f_k(w, i) = w - i

Stage k+1

...