On the Real Option Value of Scientific Uncertainty for Public Policies

by

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Abstract. In this paper scientific uncertainty is defined as the impossibility to choose the correct stochastic process for the value of a public policy. The real option value of waiting under scientific uncertainty is derived using the difference between the geometric Brownian motion and the mean reverting process by applying contingent claim analysis. The results are compared with those generated by either using a geometric Brownian motion or a mean-reverting process only. The results show that scientific uncertainty is less important than one would expect at first hand. The small effect of scientific uncertainty adds confidence to the use of a geometric Brownian motion for the kind of public policy decisions discussed in this paper.

The paper contributes to the suggestion made by scientists to analyze the sensitivity public policy valuations, provides insights about the magnitude of error that can be made by choosing the wrong process, provides a solution to the problem and highlights the implication for public policy decision making.

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1. Introduction

Economists have proposed the real option theory for the ex-ante valuation of costs and benefits from public policies. Morel et al. (2003) and Wesseler (2003) suggested the use for the valuation of releasing transgenic crops, Pindyck (2000) for the timing of environmental policies and Leitzel and Weisman (1999) for policy reforms to name only a few.

One of the problems applying the real option approach is the correct identification of the process the stochastic decision variable follows. A solution to the problem would be simple if by using an appropriate econometric method time series data could be tested to decide which process to use. As Dixit and Pindyck (1994) and others (e.g. Gjolberg and Guttormsen, 2002) have pointed out the results are ambiguous. Depending on the time frame used, the tests either lead to a rejection or acceptance of a non-stationary process. They recommend choosing the process not based on time series analysis but based on theoretical grounds. This is a straightforward recommendation, if scientists can agree about the relevant theory. But as the case for releasing transgenic crops shows (Gilligan 2003), there is no scientific agreement about the stochastic benefits from transgenic crops. The same can be said for other cases, like opening of areas for resource exploitation or benefits from environmental policies.

Scientists are aware of the problem, but have no method available that tells them which model to choose. This is what in this context will be called scientific uncertainty.

From a decision makers point of view this may not be important if different processes do not lead to different recommendations. As Wesseler (2001) has pointed out, the choice of a stochastic process may but not necessarily will lead to different recommendations. Figure 1 shows a comparison between a geometric Brownian motion and a mean reverting process and a policy decision, where it is assumed that each process represents a scientific belief or view about the benefits. The situations depicted under quadrant I and quadrant IV lead to unequivocal decisions: either immediate implementation, quadrant I, or postponement, quadrant IV. On the other, hand the situations depicted in quadrant II and III are equivocal: depending on the stochastic processes either immediate implementation or postponement is economical. Specifically the situation in quadrant III is of importance as the geometric Brownian process dominates the mean reverting process by almost first degree of stochastic dominance (FSD).[1]

GB[1] MR[2]	I Implement Implement $V \geq V^*_{GB} \wedge V \geq V^*_{MR}$	II Postpone Implement $V \leq V^*_{GB} \wedge V \geq V^*_{MR}$
GB MR	III Implement Postpone $V \geq V^*_{GB} \wedge V \leq V^*_{MR}$	IV Postpone Postpone $V \leq V^*_{GB} \wedge V \leq V^*_{MR}$

[1]GB: geometric Brownian motion; [2]MR: mean reverting process.

[1] For a definition of almost first degree of stochastic dominance see Anderson et al. (1989).

Figure 1: Possible Combinations of Results Under Different Belief Systems (adopted from Wesseler, 2001).

These observations lead to an important question: *Do we have to choose between different processes or is it possible to combine the processes to also capture the uncertainty about the choice of the stochastic process?* This is what will be discussed in this paper.

In the following, the option value of waiting under scientific uncertainty as illustrated in quadrant III of figure 1 will be derived using the difference between the geometric Brownian motion and the mean reverting process by applying contingent claim analysis. The results will be compared with those generated by either using a geometric Brownian motion or a mean-reverting process only.

The paper contributes to the suggestion made by scientists to further analyze the sensitivity of policy valuation, provides insights about the magnitude of error that can be made by choosing the wrong process, provides a solution to the problem and highlights the implication for public policy decisions under uncertainty and irreversibility.

2. The Option Value Under Scientific Uncertainty

The full value of owning the right to implement a policy, $F(B,t)$, depends on the incremental net-benefits B of the policy. Exercising the option to implement provides a benefit stream $\pi(B,t)$ to the holder of the right[2] and produces not only irreversible costs but also irreversible benefits (e.g. Pindyck, 2000; Wesseler, 2003). The owner of the option, the decision maker, likes to know the value of the option and if to exercise immediately that is implementing the policy. Let's also assume the decision maker likes to implement the policy without bearing any economic risk. By replicating the uncertain returns with known values from the market e.g. will derive the riskless value of the option to implement the policy. This is one of the basic insights of real option theory.[3] As Fisher (2000) has demonstrated this is equivalent to the quasi-option approach in environmental economics by Arrow and Fisher (1974) and Henry (1974) and further developed by Fisher and Hanemann (1986) and Hanemann (1989).

If it is assumed that the incremental net-benefits *B* of the policy follow a mean reverting process, it should be implemented immediately if *B* is greater than the identified hurdle rate for a mean reverting process B^*_{MR}. As *B* may also follow a geometric Brownian motion that dominates the mean reverting process the additional uncertainty of the difference between the two stochastic processes, the scientific uncertainty, B^*_{SU}, can be added, resulting in a hurdle rate B^*_{MR+SU}:

$$B^*_{MR+SU} = B^*_{MR} \cdot B^*_{SU} \tag{1}$$

The hurdle for decision making under a mean-reverting process is already well-known. Now, the steps to derive the real option value under scientific uncertainty and hence the hurdle for scientific uncertainty will be presented. Following the contingent claim approach a portfolio can be constructed that replicates the risk of the policy which consists of *n* units of incremental net-benefits from the policy, *nB,* and one Euro invested in a riskless asset. If this portfolio will be hold over a short time interval, *dt*, the value of the portfolio will change depending on the rate of return, *r,* of the riskless asset and the change in value of *nB*. The change in value of *nB* may pay a

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[2] Think, e.g., of the EU-commission acting as the representative of EU citizens, similar to the manager of a private company acting on behalf of the stock owners.

[3] The seminal book by Dixit and Pindyck (1994) demonstrate the wide application possibilities of the real option approach. Nobel laureate Robert C. Merton (1998) provides an overview of the application

dividend, δ, from holding it over the short time interval $n\delta Bdt$ and an uncertain return ndB. dB follows a process which is the difference between a geometric Brownian process and a mean reverting process, where the geometric Brownian process dominates the mean reverting process by FSD:

$$dB = \alpha Bdt + \tilde{\sigma} Bdz - \eta\left(\overline{B} - B\right)Bdt - \overline{\sigma} Bdz \qquad (2)$$

with B: incremental net-benefits of policy,

α: growth rate of incremental benefits assuming geometric Brownian motion,

$\tilde{\sigma}$: variance rate of the geometric Brownian motion,

η: speed of reversion,

$\overline{\sigma}$: variance rate of the mean-reversion process,

$\overline{B}$: reversion level,

dz: Wiener process

The expected value of a percentage change in incremental net-benefits over a short time interval is $\alpha - \eta\left(\overline{B} - B\right)$ which is not constant as it depends on B which fluctuates stochastically. Therefore, as B has to provide an expected rate of return equal to the risk adjusted rare of return, μ, derived from the capital asset pricing model as otherwise it would be more economically to reallocate investments, the expected return of the investment has to equal $\mu = \alpha - \eta\left(\overline{B} - B\right) + \delta$ and hence δ depends on B, $\delta(B) = \mu - \alpha + \eta\left(\overline{B} - B\right)$ (McDonald and Siegel, 1986).

The return per Euro invested in the whole portfolio is:

$$\frac{r + n\sigma dB}{1 + nB} = \frac{r + n\delta Bdt + n\left(\alpha Bdt + \tilde{\sigma} Bdz - \eta\left(\overline{B} - B\right)Bdt - \overline{\sigma} Bdz\right)}{1 + nB}$$

This can be rearranged to provide:

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of option pricing theory outside financial economics. The book by Amram and Kulatilaka (1999) includes several case studies of real option pricing.

$$\frac{r + n\left(\alpha - \eta\left(\overline{B} - B\right) + \delta\right)B}{1 + nB}dt + \frac{nB\left(\tilde{\sigma} - \overline{\sigma}\right)}{1 + nB}dz \,. \tag{3}$$

The first part of equation 3 is certain while the second part is uncertain. To simplify the notation we write $\sigma Bdz = \tilde{\sigma}Bdz - \overline{\sigma}Bdz$. This portfolio can be compared with implementing the policy instead of buying the results from e.g. other implementing organizations. Implementing the policy means exercising the option and hence, costs *F(B,t).* Exercising the option provides immediate incremental net-benefits $\pi(B,t)dt$*.* At the time of release this benefits are known with certainty over the short time interval *dt*. Also, the value of the option to implement the policy changes over the time interval *dt*. This random change can be calculated by applying Ito's Lemma:

$$dF = \left[F_t + \left(\alpha - \eta\left(\overline{B} - B\right)\right)BF_B + \frac{1}{2}\sigma^2 B^2 F_{BB}\right]dt + \sigma BF_B dz \,.$$

The return per Euro invested than is:

$$\frac{\pi dt + dF}{F} = \frac{\pi + \left[F_t + \left(\alpha - \eta\left(\overline{B} - B\right)\right)BF_B + \frac{1}{2}\sigma^2 B^2 F_{BB}\right]}{F}dt + \frac{\sigma BF_B}{F}dz \,. \tag{4}$$

As the portfolio should replicate the risk of implementing the policy, the uncertain part of the portfolio has to be equal to the uncertain part of the returns from releasing them:

$$\frac{nB\sigma}{1 + nB}dz = \frac{\sigma BF_B}{F}dz \,. \tag{5}$$

The arbitrage pricing principle says that two assets in the market with the same risk have to have the same value. If the same line of thinking will be applied, than also the certain return of the portfolio and the certain return from the policy have to be the same:

$$\frac{r+n\left(\alpha-\eta\left(\overline{B}-B\right)+\delta\right)B}{1+nB}dt=\frac{\pi+F_t+\left(\alpha-\eta\left(\overline{B}-B\right)\right)BF_B+\frac{1}{2}\sigma^2B^2F_{BB}}{F}dt\,. \qquad (6)$$

If $nB/(1+nB)$ is substituted on the right-hand side by $\frac{BF_B}{F}dz$ from equation 4 and δ substituted by $\mu-\alpha+\eta\left(\overline{B}-B\right)$, equation 5 can be rearranged to provide:

$$\frac{1}{2}\sigma^2B^2F_{BB}+\left(r-\mu+\alpha-\eta\left(\overline{B}-B\right)\right)BF_B-rF+\pi=0\,. \qquad (7)$$

The term F_t dropped as an infinite stream of returns from the policy is assumed if once implemented. The boundary conditions for the differential equation 7 are the well-known 'value matching' (equation 9) and the 'smooth pasting' (equation 10) conditions and that the value of the option to implement the policy has no value if there are no incremental net-benefits (equation 8):

$$F(0)=0 \qquad (8)$$

$$F(B^*)=B^*-I+R \qquad (9)$$

$$F'(B^*)=B'^*\,. \qquad (10)$$

A solution to the differential equation 7 and hence, the value of the option, can be found by defining a function of the form:

$$F(B)=AB^\theta h(B), \qquad (11)$$

where A and θ are constants that have to be chosen to solve equation 7. Following the steps provided by Dixit and Pindyck (1994, 162-163), first equation 11 will be substituted in equation 6. After rearrangement:

$$\begin{aligned}&B^\theta h\left[\frac{1}{2}\sigma^2\theta(\theta-1)+\left(r-\mu+\alpha-\eta\overline{B}\right)\theta-r\right]+\\&B^{\theta+1}\left[\frac{1}{2}\sigma^2Bh_{BB}+\left(\sigma^2\theta+r-\mu+\alpha-\eta\overline{B}+\eta B\right)h_B+\eta\theta h\right]=0.\end{aligned} \qquad (12)$$

Second, the terms in brackets both have to be equal to zero. The first bracketed term is a quadratic equation. As one of the boundary conditions is *F(0) = 0,* only the positive solutions will be considered. Solving the quadratic equation provides the following solution for θ:

$$\theta = \frac{1}{2} + \frac{\left(\mu - r - \alpha + \eta\overline{B}\right)}{\sigma^2} + \sqrt{\left[\frac{r - \mu + \alpha - \eta\overline{B}}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} \tag{13}$$

Third, the second bracketed term can be transformed into a hypergeometric differential equation by the substitutions $x = \frac{-2\eta B}{\sigma^2}$, $h(B) = g(x)$, $h_B = \frac{-2\eta B}{\sigma^2} g_x$, $h_{BB} = \left(\frac{-2\eta B}{\sigma^2}\right)^2 g_{xx}$:

$$x g_{xx} + (b - x) g_x - \theta g = 0 \tag{14}$$

where

$$b = 2\theta + 2\left(r - \mu + \alpha - \eta\overline{B}\right) / \sigma^2$$

Fourth, the solution to equation 14 is the confluent hypergeometric function $H(x;\theta,b)$ (see Dixit and Pindyck 1994, p.163) which results in the following solution to equation 6[4]:

$$F(B) = AB^{\theta} H\left(\frac{-2\eta B}{\sigma^2};\theta,b\right) \tag{15}$$

The values for *A* and the critical value *B** where the release could be justified can be found numerically using the two remaining boundary conditions $F(B^*) = B^* - I + R$ and $F_B(B) = 1$.

3. Application of the model

[4] Note the difference to the result provided by Dixit and Pindyck for a mean-reverting process, where *x* is positive.

The paper will be completed by providing the results of a numerical analysis for reasonable parameter ranges.

4. Conclusion

In this paper we address the problem of scientific uncertainty defined as the problem of identifying the correct stochastic process of incremental net-benefits from a policy. Combining a mean reversion process and a geometric Brownian motion reduces the problem of scientific uncertainty. A numerical analysis indicates only a small impact of scientific uncertainty. Ignoring scientific uncertainty on the other hand may lead to a wrong decision.

The small impact of scientific uncertainty can be explained by the reduced variance rate of the combined two processes. The small effect of scientific uncertainty adds confidence to the use of a geometric Brownian motion for the kind of public policy decisions discussed in this paper.

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