

# Gas Fired Power Plants: Investment Timing, Operating Flexibility and Abandonment

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## **Abstract**

Many firms are considering investment in gas fired power plants. We consider a firm holding a license, i.e. an option, to build a gas fired power plant. The operating cash flows from the plant depend on the spark spread, defined as the difference between the unit price of electricity and cost of gas. The plant produces electricity when the spark spread exceeds emission costs, otherwise the plant is ramped down and held idle. The owner has also an option to abandon the plant and realize the salvage value of the equipment. We compute optimal entry and exit threshold values for the spark spread. Also the effects of emission costs on the value of installing CO<sub>2</sub> capture technology are analyzed.

**Key words:** Real options, spark spread, natural gas, power plant, CO<sub>2</sub> capture

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## 1 Introduction

The emergence of spot and derivative markets for electricity has facilitated the use of market based valuation methods for electricity production units. We analyze the problem of valuing a gas fired power plant. The plant's operating cash flows depend on the spark spread, defined as the difference between the price of electricity and the cost of fuel used for the generation of electricity. Spark spread based valuation of power plants has been studied in Deng, Johnson, and Sogomonian (2001). Deng (2003) extends the study to take into account jumps and spikes in price processes.

Investment valuation using the traditional discounted cash flow method ignores the asset holder's ability to react to changing market conditions. Real options theory captures these options inherent in investment opportunities. A thorough review of real options is given in Dixit and Pindyck (1994). Real options are usually analyzed under the risk-neutral probability measure, which can be inferred from forward prices (see e.g. Schwartz, 1997). We use electricity and gas forward prices.

An investment in a gas fired power plant contains both timing, operating flexibility and abandonment options. We consider an investor having a license to build a gas fired power plant. The license can be seen as an American call option on the plant value. Options to postpone an investment decision have been studied for example in McDonald and Siegel (1986).

Future cash flows from the plant depend on the spark spread between electricity and gas. If the spark spread is positive the plant produces electricity and the profits are given by the spark spread less nonfuel variable costs, emission costs and fixed costs. Once the spark spread becomes negative the plant is ramped down and only fixed costs remain. It is also possible to abandon the plant permanently and realize the salvage value of the plant. Options to alter operating scale have been studied for example in Brennan and Schwartz (1985). We compute optimal threshold values to build and abandon a gas fired power plant.

The paper is organized as follows. We present the mathematical model in Section 2, and in Section 3 the solution method is introduced. In Section 4 we apply our model to value a combined cycle gas fired power plant. In Section 5 we discuss the results of the application. Finally, Section 6 concludes the study.

## 2 Mathematical model

We assume that there exist spot and derivative markets for both electricity and gas. In these markets electricity, gas and their derivative instruments are traded continuously. In describing the probabilistic structure of the markets, we will refer to a probability space  $(\Omega, F, P)$ , where  $\Omega$  is a set,  $F$  is a  $\sigma$ -algebra of subsets of  $\Omega$ , and  $P$  is a probability measure on  $F$ . The following assumption characterizes our derivative markets.

ASSUMPTION 1. *There exist forward contracts on both electricity and gas. The derivative markets are complete and there are no arbitrages.*

Given the no arbitrage condition all the portfolios with the same future payoffs have the same current value. This is often called the law of one price. If all derivative instruments traded in the market can be replicated with some replicating portfolio the market is complete.

We denote by  $S_e(t, T)$  the  $T$ -maturity forward price on electricity at time  $t$ . Respectively, the  $T$ -maturity forward price on gas at time  $t$  is  $S_g(t, T)$ . By allowing  $T$  to vary from  $t$  to  $\tau$  we get forward curves  $S_e(t, \cdot) : [t, \tau] \rightarrow \mathbf{R}_+$  and  $S_g(t, \cdot) : [t, \tau] \rightarrow \mathbf{R}_+$  for electricity and gas. Seasonality of electricity and gas causes cycles and peaks in the forward curves.

The value of a gas fired power plant is determined by the spark spread. Spark spread is defined as the difference between the price of electricity and the cost of gas used for the generation of electricity.

Thus, the  $T$ -maturity forward price on spark spread is

$$S(t, T) = S_e(t, T) - K_H S_g(t, T), \quad (1)$$

where parameter  $K_H$  is the heat rate. Heat rate is the amount of gas required to generate 1 *MWh* of electricity. Heat rate measures the efficiency of the gas plant: the lower the heat rate, the more efficient the facility. The efficiency of a gas fired power plant does not vary much over time. Thus, the use of a constant heat rate is plausible. Note that the value of the spark spread can be negative as well as positive.

The absence of arbitrage assumption guarantees existence of an equivalent martingale measure  $Q$ . Under the martingale measure  $Q$  all expected rates of return equal the risk-free interest rate (see e.g. Schwartz, 1997). The value of a forward contract when initiated is defined to be zero. Thus the T-maturity forward price on spark spread at time  $t$  is

$$S(t, T) = E^Q[S(T) | F_t], \quad (2)$$

where  $S(T)$  is the spark spread at time  $T$ . Thus, the dynamics of the spark spread process under the pricing measure  $Q$  can be inferred from forward prices. When long-term commodity projects are valued, models with constant convenience yield give practically the same results as models using stochastic convenience yield (see e.g. Schwartz, 1998). Motivated by this, we ignore the seasonality in the spark spread and use the long term average as a price process. The following assumption gives the dynamics of the spark spread process.

*ASSUMPTION 2. The spark spread has following dynamics*

$$dS(t) = \alpha dt + \sigma dB(t) \quad (3)$$

where  $\alpha$  and  $\sigma$  are non-negative constants and  $B(\cdot)$  is a standard Brownian motion on the probability space  $(\Omega, F, Q)$ , along with standard filtration  $\{F_t : t \in [0, T]\}$ .

Assumption 2 states that changes in spark spread are normally distributed with mean  $\alpha dt$  and variance  $\sigma^2 dt$ . As the spark spread values are normally distributed the values are not bounded. Thus spark spread can be negative as well as positive. Commodity forward prices tend to have time

structured volatility reflecting the mean reverting nature of spot prices (see e.g. Schwartz, 1997). For simplicity we ignore the time structure in the volatility and use the long term average volatility. The estimation of drift and volatility from forward prices will be considered in Section 4.

The following assumptions give the operational characteristics of the gas plant.

*ASSUMPTION 3. Ramp ups and ramp downs of the plant can be done immediately. The costs associated with starting up and shutting down can be amortized into fixed costs. The lifetime of the power plant is assumed to be infinite.*

In a gas fired power plant the operation and maintenance costs do not vary much over time and the response times are in the order of several hours. Usually, the life time of a gas fired power plant is assumed to be around 25 years. Upgrading and reconstructions can increase the lifetime considerably. Thus, we assume that the plant is more flexible than it really is, but for efficient plants such as this one the error will be small, judging by the results in Deng and Oren (2003).

We consider an investor having a license to build a gas plant. In order to keep the license alive the firm faces constant payments  $W$  due to salaries etc. The investor can build the plant at any time, thus the license can be seen as a perpetual American call option. Perpetual options have a constant threshold value  $S_H$  under which exercising is not optimal. Once the spark spread exceeds the threshold value  $S_H$ , building becomes optimal. The option to invest  $F_0$  must satisfy the following Bellman equation

$$rF_0 dt = E^Q[dF_0] - Wdt \quad \text{when } S \leq S_H, \quad (4)$$

where  $E^Q$  is the expectation operator under the pricing measure  $Q$ . For simplicity risk-free interest  $r$  is assumed to be constant. Itô's lemma gives following differential equation for the option value

$$\frac{1}{2} \sigma^2 \frac{\partial^2 F_0}{\partial S^2} + \alpha \frac{\partial F_0}{\partial S} - rF_0 - W = 0 \quad \text{when } S \leq S_H. \quad (5)$$

Once the option to build has been exercised the investor has a power plant, which will be ramped down whenever emission costs  $E$  exceed spark spread (i.e.  $E > S(t)$ ). Moreover, the investor can always abandon the plant and realize the salvage value of the plant. As the lifetime of the plant was assumed to be infinite, there is a constant threshold value  $S_L$  for the abandonment. The value of the plant  $F_1$  consists of three terms: the present value of operating cash flows, the value of the option to ramp down, and the value of the option to abandon. The value  $F_1$  must satisfy the following Bellman equation

$$rF_1 dt = E^{\mathcal{Q}}[dF_1] + \left( \bar{C}(S(t) - E)^+ - G \right) dt \quad \text{when } S(t) \geq S_L, \quad (6)$$

where  $(S(t) - E)^+$  denotes  $\max(S(t) - E, 0)$ .  $\bar{C}$  is the capacity of the plant. For simplicity, we assume that the emission and fixed costs  $G$  are constant. Uncertainty in emission costs  $E$  will be considered in Section 4. By using Itô's lemma we get that the value of the plant  $F_1$  must satisfy following differential equations

$$\frac{1}{2} \sigma^2 \frac{\partial^2 F_1}{\partial S^2} + \alpha \frac{\partial F_1}{\partial S} - rF_1 + \bar{C}(S - E) - G = 0 \quad \text{when } S \geq E. \quad (7)$$

$$\frac{1}{2} \sigma^2 \frac{\partial^2 F_1}{\partial S^2} + \alpha \frac{\partial F_1}{\partial S} - rF_1 - G = 0 \quad \text{when } E \geq S \geq S_L. \quad (8)$$

In addition to differential equation (5) the option to build  $F_0$  must satisfy following boundary conditions

$$\lim_{S \rightarrow -\infty} F_0(S) = 0 \quad (9)$$

$$F_0(S_H) = F_1(S_H) - I \quad (10)$$

$$\frac{\partial F_0(S_H)}{\partial S} = \frac{\partial F_1(S_H)}{\partial S}. \quad (11)$$

Equation (9) arises from the observation that as the value of the spark spread decreases the option to build should become valueless. The value-matching equation (10) follows from the fact that when the option is exercised the values lost should be equal to value gained. The investment costs are denoted by  $I$ . The smooth-pasting condition (11) states that also the derivatives of the values must match when the option is exercised. For an intuitive proof of smooth-pasting condition see Dixit and Pindyck (1994) and for a rigorous derivation see Samuelson (1965).

In addition to differential equations (7) and (8) the value of the plant  $F_1$  has following boundary conditions

$$\lim_{S \rightarrow \infty} F_1(S) = \lim_{S \rightarrow \infty} \int_0^{\infty} e^{-rt} \left( \bar{C}(S + \alpha t - E)^+ - G \right) dt \quad (12)$$

$$F_1(S_L) = F_0(S_L) + D \quad (13)$$

$$\lim_{S \downarrow E} F_1(E) = \lim_{S \uparrow E} F_1(E) \quad (14)$$

$$\frac{\partial F_1(S_L)}{\partial S} = \frac{\partial F_0(S_L)}{\partial S} \quad (15)$$

$$\lim_{S \downarrow E} \frac{\partial F_1(E)}{\partial S} = \lim_{S \uparrow E} \frac{\partial F_0(E)}{\partial S}. \quad (16)$$

Equation (12) states that as the value of the spark spread increases the value of the plant should approach the net present value of the plant. When spark spread is large it is very unlikely that the spark spread will decrease to a level where it is optimal to ramp down or abandon the plant. The value-matching condition must hold when it is optimal to exit the market, as well as when it is optimal to ramp up or down. In equation (13) the salvage value of the plant is  $D$ . Equations (15) and (16) are the smooth-pasting conditions.

To summarize: we have derived three partial differential equations (equations (5), (7), and (8)) and eight boundary conditions (equations from (9) to (16)) for the values  $F_0$  and  $F_1$ .

### 3 Solution method

Generally a solution to a partial differential equation is a linear combination of two independent solutions plus any particular solution. Let us start with the option to build  $F_0$ . The solution of the partial differential equation (5) is of the form

$$F_0(S) = A_1 \exp(\beta_1 S) + A_2 \exp(\beta_2 S) - \frac{W}{r}, \quad (17)$$

where  $A_1$  and  $A_2$  are unknown parameters.  $\beta_1$  and  $\beta_2$  are the roots of the fundamental quadratic equation

$$\frac{1}{2} \sigma^2 \beta^2 + \alpha \beta - r = 0, \quad (18)$$

which are

$$\beta_1 = \frac{-\alpha + \sqrt{\alpha^2 + 2\sigma^2 r}}{\sigma^2} > 0 \quad (19)$$

$$\beta_2 = \frac{-\alpha - \sqrt{\alpha^2 + 2\sigma^2 r}}{\sigma^2} < 0. \quad (20)$$

In order to satisfy boundary condition (9) the parameter  $A_2$  must be zero. Thus, the value of the option to build is

$$F_0(S) = A_1 \exp(\beta_1 S) - \frac{W}{r}. \quad (21)$$

Similarly, we get for the value of the plant

$$F_1(S) = B_1 \exp(\beta_1 S) + B_2 \exp(\beta_2 S) + \frac{\bar{C}}{r} S - \frac{\bar{C} E + G}{r} + \frac{\bar{C} \alpha}{r^2} \quad \text{when } S \geq E \quad (22)$$

$$F_1(S) = K_1 \exp(\beta_1 S) + K_2 \exp(\beta_2 S) - \frac{G}{r} \quad \text{when } E \geq S \geq S_L. \quad (23)$$



The particular solution in equation (22) is equal to the net present value of an operating gas plant, i.e.

$$\int_0^{\infty} e^{-rt} \left( \bar{C}(S + \alpha t - E)^+ - G \right) dt = \frac{\bar{C}}{r} S - \frac{\bar{C}E + G}{r} + \frac{\bar{C}\alpha}{r^2} \quad \text{when } S \geq E. \quad (24)$$

Thus, due to the boundary condition (12) the general solution in equation (22) must approach zero as  $S$  increases (i.e.  $B_1 = 0$ ). We get for the value of the plant

$$F_1(S) = \begin{cases} B_2 \exp(\beta_2 S) + \frac{\bar{C}}{r} S - \frac{\bar{C}E + G}{r} + \frac{\bar{C}\alpha}{r^2} & \text{when } S \geq E \\ K_1 \exp(\beta_1 S) + K_2 \exp(\beta_2 S) - \frac{G}{r} & \text{when } E \geq S \end{cases} \quad (25)$$

We assume that the investment and variable costs satisfy following natural conditions

$$D < I \quad (26)$$

$$W < G. \quad (27)$$

When the investment and variable costs are realistic (satisfy conditions (26) and (27)) it is never optimal to build a plant if it is not optimal to use it. Thus the threshold to build the plant must exceed emission costs, i.e.

$$E \leq S_H. \quad (28)$$

Moreover, the threshold value to build the plant must be bigger than the threshold value to abandon, i.e.

$$S_L \leq S_H. \quad (29)$$

The inequalities (28) and (29) state that either

$$S_L \leq E \leq S_H \quad (30)$$

or

$$E \leq S_L \leq S_H. \quad (31)$$

In the case of inequality (30) the emission costs are greater than the threshold value to abandon. Thus, if the investor has built the plant and the spark spread is between emission costs and threshold value to abandon it is optimal to hold the plant idle. In the case of inequality (31) the threshold value to abandon is greater than emission costs. Thus, the option to ramp down the plant is valueless. In other words, it is optimal to abandon before it is optimal to ramp down the plant. The value-matching and smooth-pasting conditions (10), (11) and (13)-(16) give following six equations for six unknown parameters ( $S_H$ ,  $S_L$ ,  $B_2$ ,  $A_1$ ,  $K_1$ ,  $K_2$ ) when inequality (30) holds

$$A_1 \exp(\beta_1 S_H) - \frac{W}{r} = B_2 \exp(\beta_2 S_H) + \frac{\bar{C}}{r} S_H - \frac{\bar{C} E + G}{r} + \frac{\bar{C} \alpha}{r^2} - I \quad (32)$$

$$A_1 \beta_1 \exp(\beta_1 S_H) = B_2 \beta_2 \exp(\beta_2 S_H) + \frac{\bar{C}}{r} \quad (33)$$

$$K_1 \exp(\beta_1 S_L) + K_2 \exp(\beta_2 S_L) - \frac{G}{r} = A_1 \exp(\beta_1 S_L) - \frac{W}{r} + D \quad (34)$$

$$K_1 \beta_1 \exp(\beta_1 S_L) + K_2 \beta_2 \exp(\beta_2 S_L) = A_1 \beta_1 \exp(\beta_1 S_L) \quad (35)$$

$$B_2 \exp(\beta_2 E) + \frac{\bar{C} \alpha}{r^2} = K_1 \exp(\beta_1 E) + K_2 \exp(\beta_2 E) \quad (36)$$

$$B_2 \beta_2 \exp(\beta_2 E) + \frac{\bar{C}}{r} = K_1 \beta_1 \exp(\beta_1 E) + K_2 \beta_2 \exp(\beta_2 E). \quad (37)$$

By eliminating  $B_2$ ,  $A_1$  and  $K_2$  from equations (32)-(37) we get following two equations for the threshold values

$$K_1 + M_1 \beta_2 \exp(-\beta_1 S_L) = \exp(-\beta_1 S_H) \frac{\beta_2 \left( M_2 - \frac{\bar{C}}{r} S_H \right) + \frac{\bar{C}}{r}}{(\beta_1 - \beta_2)} \quad (38)$$

$$\exp(-\beta_2 E) \left( K_1 \exp(\beta_1 E) - \frac{\bar{C} \alpha}{r^2} \right) = \exp(-\beta_2 S_H) \frac{\beta_1 \left( M_2 - \frac{\bar{C}}{r} S_H \right) + \frac{\bar{C}}{r}}{(\beta_1 - \beta_2)} - M_1 \beta_1 \exp(-\beta_2 S_L), \quad (39)$$

where

$$K_1 = \exp(-\beta_1 E) \left( \frac{\bar{C}(r - \alpha\beta_2)}{r^2(\beta_1 - \beta_2)} \right). \quad (40)$$

$$M_1 = \frac{(G - W + Dr)}{r(\beta_1 - \beta_2)} \quad (41)$$

$$M_2 = \frac{\bar{C}E + G - W}{r} - \frac{\bar{C}\alpha}{r^2} + I. \quad (42)$$

The value-matching and smooth-pasting conditions (10), (11) and (13) - (16) give following four equations for four unknown parameters ( $S_H$ ,  $S_L$ ,  $B_2$ ,  $A_1$ ) when the option to ramp down is not needed, i.e. inequality (31) holds

$$A_1 \exp(\beta_1 S_H) - \frac{W}{r} = B_2 \exp(\beta_2 S_H) + \frac{\bar{C}}{r} S_H - \frac{\bar{C}E + G}{r} + \frac{\bar{C}\alpha}{r^2} - I \quad (43)$$

$$A_1 \beta_1 \exp(\beta_1 S_H) = B_2 \beta_2 \exp(\beta_2 S_H) + \frac{\bar{C}}{r} \quad (44)$$

$$B_2 \exp(\beta_2 S_L) + \frac{\bar{C}}{r} S_L - \frac{\bar{C}E + G}{r} + \frac{\bar{C}\alpha}{r^2} = A_1 \exp(\beta_1 S_L) - \frac{W}{r} + D \quad (45)$$

$$B_2 \beta_2 \exp(\beta_2 S_L) + \frac{\bar{C}}{r} = A_1 \beta_1 \exp(\beta_1 S_L). \quad (46)$$

By eliminating  $A_1$  and  $B_2$  from the equations (43) - (46) we get following two equations for the threshold values.

$$\exp(-\beta_1 S_L) \left( \beta_2 \left( -D + L_1 + \frac{\bar{C}}{r} S_L \right) - \frac{\bar{C}}{r} \right) = \exp(-\beta_1 S_H) \left( \beta_2 \left( L_1 - I + \frac{\bar{C}}{r} S_H \right) - \frac{\bar{C}}{r} \right) \quad (47)$$

$$\exp(-\beta_2 S_L) \left( \beta_1 \left( -D + L_1 + \frac{\bar{C}}{r} S_L \right) - \frac{\bar{C}}{r} \right) = \exp(-\beta_2 S_H) \left( \beta_1 \left( L_1 - I + \frac{\bar{C}}{r} S_H \right) - \frac{\bar{C}}{r} \right), \quad (48)$$

where

$$L_1 = \frac{\bar{C} \alpha}{r^2} - \frac{\bar{C} E + G - W}{r}. \quad (49)$$

To summarize; the threshold values are found either by equations (38), (39) and inequality (30) or by equations (47), (48) and inequality (31). In the case that the solution is found by equations (38) and (39) it is optimal to ramp down the plant before it is abandoned. If the solution is given by equations (47) and (48), the option to ramp down is not needed. Neither of the cases can be solved analytically, but a numerical solution can easily be attained.

#### 4 Application

Norwegian energy and environmental authorities have given three licenses to build a gas fired power plant. All potential plants are situated along the western coast of southern Norway. In this section we illustrate our framework by taking the view of an investor having one of these licenses.

The example consists of four parts. First, we introduce the data used for the valuation including methods to estimate the parameters from the data. Second, we calculate threshold values to build and abandon the plant. The threshold values are compared to the threshold values calculated with discounted cash flows. The sensitivity of the threshold values to some key parameters are illustrated in part three. In the final part we study the effects of carbon emission costs to the installation of CO<sub>2</sub> capture technology. We assume that a plant with CO<sub>2</sub> capture technology does not face emission costs.

The costs of building and running a combined cycle gas plant in Norway are estimated by Undrum, Bolland, Aarebrot (2000). We use an exchange rate of 7 *NOK/\$*. Table 1 contains a summary of the parameters needed for our model.

*Table 1: The gas plant parameters*

<b>Parameter</b>	$W$	$\bar{C}$	$G$	$I$	$D$
<b>Unit</b>	<i>MNOK/year</i>	<i>MMwh/year</i>	<i>MNOK/year</i>	<i>MNOK</i>	<i>MNOK</i>
<b>Value</b>	2.5	3.27	50	1620	570

Let us make few comments on the parameters. Undrum, Bolland, Aarebrot (2000) estimate that building a natural gas fired combined cycle power plant in Norway costs approximately 1620 *MNOK*, and that the maintenance costs  $G$  are approximately 50 *MNOK/year*. We estimate that the costs of holding the license  $W$  are 5% of the fixed costs of a running a plant. In Undrum, Bolland, Aarebrot (2000) approximately 35% of the investment costs are used for capital equipment. We assume that if the plant is abandoned, all the capital equipment can be realized on second hand market, thus we assume that  $D$  is 570 *MNOK*. The estimated parameters are for a gas plant whose maximum capacity is 415 *MW*. We assume that the capacity factor of the plant is 90%. Thus the production capacity is 3.27 *MMWh/year*.

The drift parameter  $\alpha$  in the spark spread process is estimated with linear regression from electricity and gas forward prices. For electricity, long term prices from Nord Pool and 10-year contracts traded bilaterally are used. For natural gas we use data from International Petroleum Exchange (IPE). Gas prices are adjusted by the heat rate so that a unit of gas corresponds to 1 *MWh* of electricity generated. The efficiency of a combined cycle gas fired turbine is estimated to be 58.1% (see Undrum, Bolland, Aarebrot, 2000).

For the estimation of the volatility parameter  $\sigma$  the correlation between the spot price of electricity and gas in the period 1998-2001 was estimated at 0.53. The estimation of electricity and gas forward volatilities is based on data from Nord Pool and IPE from 1997 to 2001. The one year average forward volatilities were 0.1 for electricity and 0.2 for gas.

Table 2 contains estimates for the spark spread process parameters and the risk-free interest rate. The interest rate is quite high, but the long term average interest rate in Norway is approximately 6%. Commodity forward prices under stochastic interest rate have been studied for example in Schwartz (1997).

*Table 2: Spark spread process parameters*

<b>Parameter</b>	$\alpha$	$\sigma$	$r$
<b>Value</b>	0.16	28	6%

The long term average spark spread was estimated to be approximately 19 *NOK/MWh*. The expected spark spread values with 68% confidence level are illustrated in Figure 1.

[Figure 1 about here]

Table 3 contains threshold values  $S_H$  and  $S_L$  computed when emission costs are assumed to be zero (i.e.  $E = 0$ ). The solution is found with equations (38), (39) and with inequality (30). Thus, with the given parameters it is optimal to ramp down the plant before abandoning. Threshold values  $S_H^{DCF}$  and  $S_L^{DCF}$  are calculated with discounted cash flows. The upper threshold value  $S_H^{DCF}$  is calculated by setting the difference of expected cash flows before and after investment equal to investment costs. The lower threshold value  $S_L^{DCF}$  is calculated by setting the difference of expected cash flows before and after abandonment equal to salvage value.

Table 3: Threshold values

Variable	$S_L$	$S_H$	$S_L^{DCF}$	$S_H^{DCF}$
Value	-39 NOK/MWh	95 NOK/MWh	22 NOK/MWh	42 NOK/MWh

The optimal threshold values calculated using real options approach differ considerably from values calculated using traditional discounted cash flow method. Uncertainty makes waiting more favorable (see e.g. Dixit and Pindyck, 1994). Thus, uncertainty increases threshold value to build the plant and decreases threshold value to abandon the plant. Deng, Johnson, and Sogomonian (2001) find that the value of a gas power plant calculated with a simple spark spread valuation method is over six times the value calculated with a discounted cash flow method. Note that the discounted cash flow approach suggests that it is optimal to abandon the plant with current spark spread. With real options method the current value of spark spread is almost equally distant from the threshold values.

The upper picture in Figure 2 illustrates the values  $F_0$  and  $F_1$  as a function of spark spread. Also the threshold values are shown. Note that  $F_1(S_L)$  exceeds  $F_0(S_L)$  by the resale value of the equipment ( $D = 570 \text{ MNOK}$ ) and  $F_0(S_H)$  is 1620 MNOK below  $F_1(S_H)$ , corresponding to the investment costs. Incremental values for building a plant (i.e.  $F_1 - F_0$ ) are presented in the lower picture of Figure 2. Incremental values illustrate how much more valuable an investor with a plant is compared to a firm with just the license to build. Note the tangency to the horizontal lines  $I$  and  $D$  caused by the smooth pasting conditions.

[Figure 2 about here]

Next we study how the threshold values change as a function of some key parameters. In Figure 3 the threshold values as a function of volatility  $\sigma$  are illustrated.

[Figure 3 about here]

The gray parts in the threshold lines are given with equations (38), (39) and the black parts with equations (47), (48). When the lines are gray the option to ramp down the plant has no value, while for the black parts the ramp down option has value and may thus be exercised. If volatility is less than 11 the option to ramp down the plant is valueless. In Figure 3 additional uncertainty increases the threshold value to build a plant, but at the same time the threshold value to abandon the plant decreases, i.e. uncertainty makes waiting more favorable (see e.g. Dixit and Pindyck, 1994). Note that when the uncertainty in spark spread process approaches zero the threshold values converge to the DCF values in Table 3.

Figure 4 illustrates threshold values as a function of emission costs. In Figure 4 the unit of emission costs is *NOK/MWh*. Usually the unit for emission costs is *\$/ton*. The CO<sub>2</sub> production of a gas fired power plant is 363 *kg/MWh*, thus with an exchange rate of 7 *NOK/\$* an emission cost of 10 *NOK/MWh* corresponds 3.94 *\$/ton*.

[Figure 4 about here]

In Figure 4 threshold values increase linearly as a function of emission costs. Moreover, the slope is one. If the emission costs are increased by one *NOK/MWh* both threshold values are also increased by one *NOK/MWh*. This is a consequence of a normally distributed spread process. Change in emission costs can be seen as a change in initial value of the spread process.

Even though we have used constant emission costs, there is great amount of uncertainty in emission costs. The uncertainty in emission costs can be seen as an additional uncertainty in the spread process. Thus, uncertainty in emission costs increases the threshold value to build the plant and decreases the threshold value to abandon.



Undrum, Bolland, Aarebrot (2000) evaluate different alternatives to capture CO<sub>2</sub> from gas turbine power cycles. They estimate that costs to install equipment to capture CO<sub>2</sub> from exhaust gas using absorption by amine solutions are 2140 *MNOK*. Thus, the costs of a gas power plant with CO<sub>2</sub> capture technology are 3760 *MNOK*. Figure 5 illustrates threshold values as a function of investment costs when the salvage value is 35% of the investment costs (i.e.  $D = 0.35I$ ). Thus, the resale value of a plant with CO<sub>2</sub> capture technology is 1316 *MNOK*. We have ignored the reduced efficiency of the plant when the greenhouse gas capture equipment is in place.

[Figure 5 about here]

In Figure 5 the threshold value to build a gas turbine with CO<sub>2</sub> capture equipment is 146 *NOK/MWh*. Figure 4 indicates that once the emission costs are 51 *NOK/MWh* the upper threshold value for a plant without CO<sub>2</sub> capture equipment is 146 *NOK/MWh*. By assuming that all emission costs are caused by CO<sub>2</sub> we get that it is optimal to install the CO<sub>2</sub> capture equipment when emission costs are larger than 20.1 *\$/ton* (i.e. 51 *NOK/MWh*).

The current estimate is that emission costs will be somewhere between 5*\$/ton* and 20*\$/ton*, where the lower range is most likely. When emission costs are 8 *\$/ton* threshold value to build a plant without CO<sub>2</sub> capture equipment is 115 *NOK/MWh*. As is calculated earlier the upper threshold value for the plant with CO<sub>2</sub> capture equipment is 146 *NOK/MWh*. The upper threshold value for a plant with CO<sub>2</sub> capture equipment is 115 *NOK/MWh* if the investment costs are lowered to 2550 *MNOK*. Thus, if companies building a gas plant with CO<sub>2</sub> capture equipment are subsidized with 1210 *MNOK* it is optimal to build gas plants with such equipment.

## 5 Discussion

Our results indicate that even with zero emission costs it is not optimal to exercise the option to build a gas fired power plant. Regardless, the reality may be different. Some of the three firms holding a

license to build gas fired power plant have stated publicly that they are willing to invest if the government relieves them of emission costs. There are several possible explanations why our results differ from the apparent policies of the actual investors.

First, we have ignored the effects of competition between the firms holding the license. The preemptive effect of early investment gives the license holders an incentive to build the plant (see e.g. Smets, 1991).

Secondly, we have used the UK market as a reference for gas. There is lot of natural gas available in the Norwegian continental shelf. Due to the physical distance from the Norwegian coastline to the UK, the gas price at a Norwegian terminal will be equal to the UK price less the transportation costs. By using price quotas from IPE we overestimate the gas price for delivery at a Norwegian terminal.

There is also a tax issue that has not been considered. Oil and gas companies operating on the Norwegian shelf have a 78% tax rate, while onshore activities are taxed at 28%. If an oil company invested in a gas power plant, it could sell the gas at a loss with offshore taxation, and buy the same gas as a power plant owner with onshore taxation.

We have also calculated the value of a CO<sub>2</sub> capture plant attached to the gas fired power plant. We found that if the emission costs are over 20.1  $\$/ton$  it is optimal to install such an equipment. When emission costs were assumed to be 8 $\$/ton$  building of a gas fired power plants with CO<sub>2</sub> capture equipment is optimal if companies building such a plant are subsidized with 1210  $MNOK$ .

## **6 Conclusions**

We use real options theory to analyze the investment in a gas fired power plant. Our valuation is based on electricity and gas forward prices. We have derived a method to compute threshold values for building and abandoning a gas fired power plant when the plant can be ramped down if it turns to be

unprofitable. Normally it is optimal to ramp down the plant before abandoning, while sometimes it is optimal to abandon a running plant directly.

In our example we take a view of an investor having a license to build a gas fired power plant in Norway. The example is based on forward prices from Nord Pool and International Petroleum Exchange (IPE). Our results indicate that the investors should wait and hope the electricity prices go up or gas prices go down before they commence the power plant project. Our numerical results also indicate that with current estimates of emission costs it is not optimal to attach a CO<sub>2</sub> capture plant to a gas plant.

## References

- Brennan, M., Schwartz, E.S. (1985) Evaluating natural resource investments, *Journal of Business* 58 (2), pp. 135-157
- Deng, S.J. (2003) Valuation of Investment and Opportunity to Invest in Power Generation Assets with Spikes in Power Prices, Working paper, Georgia Institute of Technology
- Deng, S.J., Johnson B., Sogomonian A. (2001) Exotic electricity options and the valuation of electricity generation and transmission assets, *Decision Support Systems*, (30) 3, pp. 383-392
- Deng, S.J., Oren S.S. (2003) Valuation of Electricity Generation Assets with Operational Characteristics, *Probability in the Engineering and Informational Sciences*, forthcoming
- Dixit, A.K., Pindyck, R.S. (1994) *Investment under Uncertainty*, Princeton University Press
- McDonald, R., and Siegel D. (1986) The value of waiting to invest, *Quarterly Journal of Economics* 101 (4), pp. 707-727
- Samuelson, P.A. (1965) Rational theory of warrant pricing, *Industrial Management Review* 6, pp. 13-31
- Schwartz, E.S. (1997) The Stochastic Behavior of Commodity Prices: Implications for Valuation and Hedging, *The Journal of Finance*, 52 (3), pp. 923-973
- Schwartz, E.S. (1998) Valuing Long-Term Commodity Assets, *Financial Management*, 27 (1), pp. 57-66
- Smets, F. (1991) Exporting versus FDI: The Effect of Uncertainty, Irreversibilities and Strategic Interactions" Working paper, Yale University.
- Undrum, H., Bolland, O., and Aarebrot, E. (2000) Economical assessment of natural gas fired combined cycle power plant with CO<sub>2</sub> capture and sequestration, presented at the Fifth International Conference on Greenhouse Gas Control Technologies, Cairns, Australia

## Figures

Figure 1: Spread process

Figure 2: Option values

Figure 3: Threshold values as a function of volatility

Figure 4: Threshold values as a function of emission costs

Figure 5: Threshold values as a function of investment costs

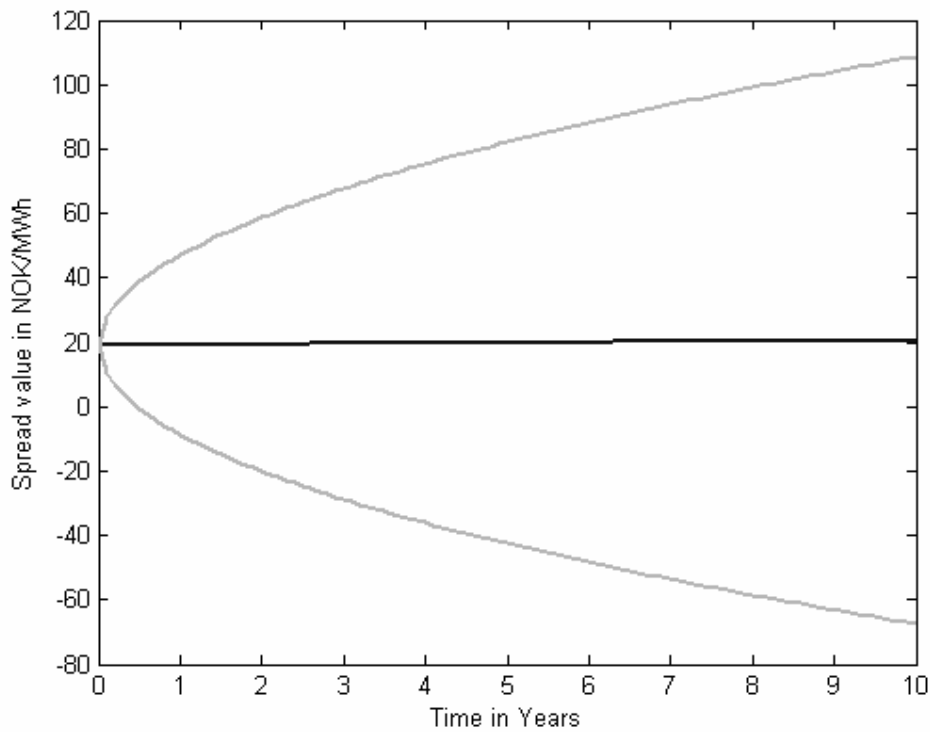


Figure 1

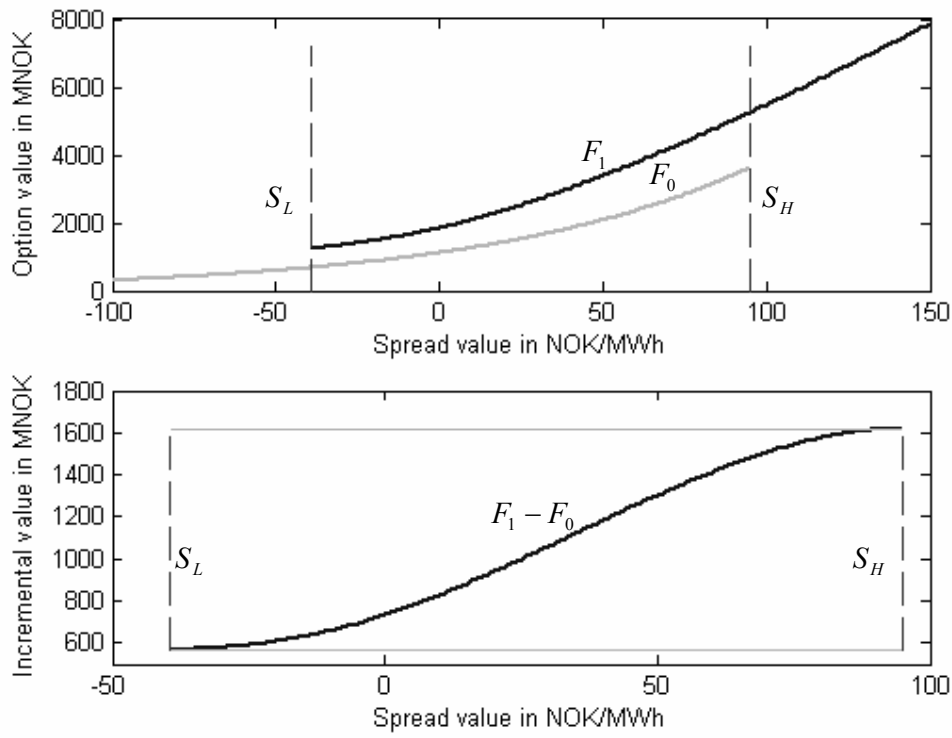


Figure 2

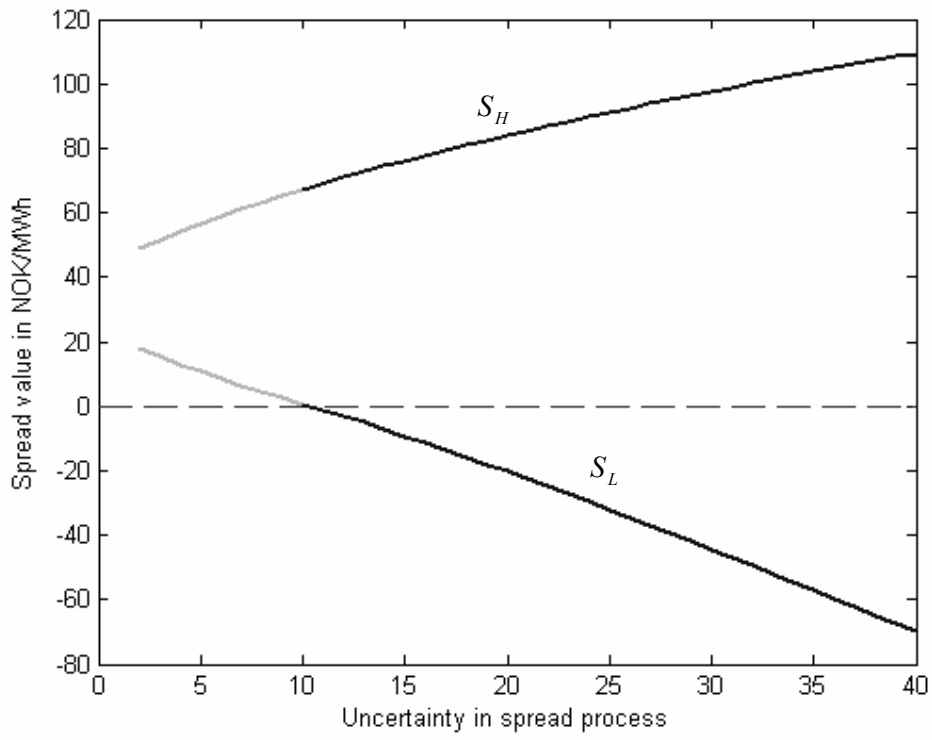


Figure 3

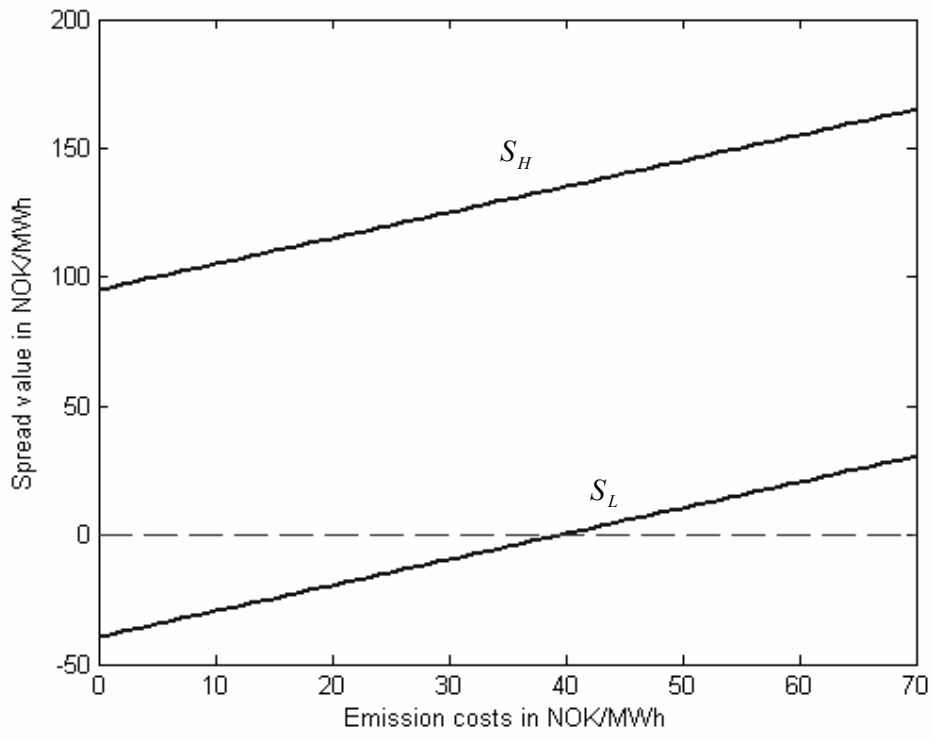


Figure 4



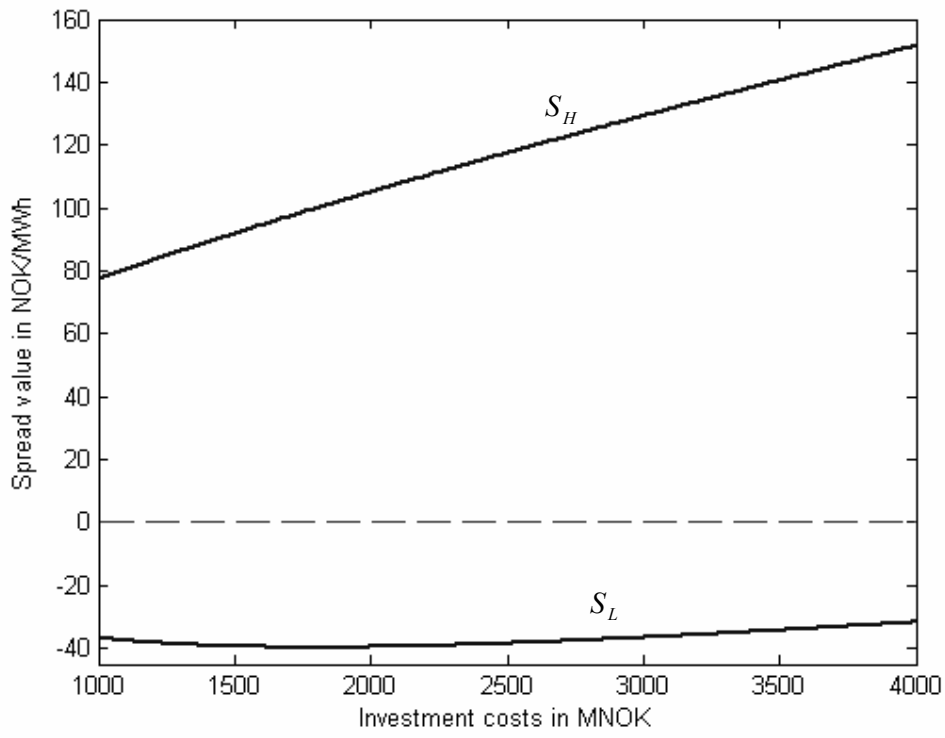


Figure 5

$S_L$