An Exact solution for the investment and value of a firm facing uncertainty, adjustment costs, information costs and irreversibility

Mondher BELLALAH [1]

Following the analysis in Abel and Eberly (1997), I derive closed-form solutions for the investment and value of a competitive firm with a constant-returns-to-scale production function and convex costs of adjustment. The analysis concerns reversible and irreversible investments under incomplete information. Optimal investment seems to be a non-decreasing function of q, the shadow value of capital. This depends also on the magnitude of shadow costs of incomplete information.

Key words : Investment, Irreversibility, shadow costs, incomplete information
JEL classification : G12, G30, G31 G19, G39

[1] Professor of Financial Economics, THEMA, university of cergy and CEREG, university of Paris-Dauphine.

Introduction

Capital investment decisions by firms under uncertainty have focused on irreversibility or on convex costs of adjustment. When the investment cost function incorporates convex costs of adjustment and irreversibility, investment is a non-decreasing function of the shadow price of capital, denoted by q.

This paper presents a parametric example of a firm facing convex costs of adjustment and irreversibility when there are shadow costs of incomplete information. These costs are defined in the vein of Merton's (1987) simple model of capital market equilibrium with incomplete information. Following the methodolgy in Bellalah (2001 a, b) in the treatment of shadow costs of incomplete information, I provide closed-form solutions for the investment and firm's value in this context.

Agents can spend time and resources to gather information about the stock market, firms and the economic activity. For example, they may read newspapers, participate in seminars, subscribe to newsletters, join investment clubs, etc. Information in financial economics can be viewed as a commodity purchased in the market or produced in the household using both time and money as inputs.

An important question in financial economics is how frictions affect equilibrium in capital markets since in a world of costly information, some investors will have incomplete information.

Merton (1987) advanced the investor recognition hypothesis in a mean-variance model. The investor recognition hypothesis (IRH) in Merton's context states that investors buy and hold only those securities about which they have enough information. Merton (1987) adapts the rational framework of the static CAPM to account for incomplete information.

Shapiro (2000) examines equilibrium in a dynamic pure-exchange economy under a generalization of Merton's (1987) investor recognition hypothesis (IRH). In his model, a class of investors is assumed to have incomplete information which suffices to implement only a particular strategy because of information costs. For a further analysis and justifications of the information costs in asset pricing, the reader can refer to Bellalah (2001 a, b).

We proceed by first analyzing the investment and value of a competitive firm that faces convex adjustment costs. The motivation for starting with the case of reversible investment is based on expositional considerations. First, the model of reversible investment here is richer than existing models that lead closed-form solutions.
Second, much of the analytic apparatus for irreversible investment is similar to the case of reversible investment.
Third, the case of reversible investment represents a benchmark against which to compare the effects of irreversibility on the fundamental value of the firm. The value of q is unaffected by irreversibility. For high values of q in the reversible case and positive investment, the optimal rate of investment is unaffected by irreversibility. For low values of q in the reversible case and negative investment, the optimal investment is zero in the irreversible case.

The invariance of q to irreversibility arises because the value of the firm is linear in the capital stock, the marginal value of an additional unit of capital is also independent of the stock of capital. It is also independent of restrictions on the accumulation or decumulation of capital. Irreversibility does not modify q but reduces the firm's value.

This paper is organized as follows.
Section 1 presents the optimization problem for a competitive firm under incomplete information. Section 2 studies the case of a reversible investment under incomplete information. Section 3 analyzes the case of irreversible investments in the same context.

1. The optimization problem of the competitive firm under incomplete information

1.1. The price process

Consider a competitive firm selling its output at a price p_t with the following dynamics :

$$\frac{dp_t}{p_t} = \mu dt + \sigma dz_t, \qquad p_0 \succ 0 \quad (1)$$

where :

- μ: the instantaneous drift,

- σ: the instantaneous standard deviation
- dz_t: an increment to a standard Wiener process.

At this level, we recall that in the context of incomplete information, the use of Merton's (1987) model is equivalent to applying an additional discount rate to the future cash flows. Hence, it appears that taking into account the effect of incomplete information on the equilibrium price of an asset or an investment opportunity is similar to the application of an additional discount rate that reflects the shadow cost of incomplete information .
As it appears in Merton (1987), the model gives a general method for discounting future cash flows under uncertainty. In this model, assets with higher idiosyncratic risk are rationally priced to earn a higher expected return. [2]

The expected value of p^γ for various values of γ denoted by ${}_t[p^\gamma_{t+s}]$ grows at a constant rate $\gamma\mu + \frac{1}{2}\sigma^2\gamma(\gamma-1)$ when s increases for a given t. Hence, in the context of Merton (1987) analysis, the present value of ${}_t[p^\gamma_{t+s}]$ discounted to time t at the rate $(R+\lambda)$ is

$$e^{-Rs}E_t\{p^\gamma_{t+s}\} = p^\gamma_t e^{-(R+\lambda)s} e^{\left[\gamma\mu+(1/2)\sigma^2\gamma(\gamma-1)\right]s} = p^\gamma_t e^{-f(\gamma;(R+\lambda))s}$$

where

$$f(\gamma;R) \equiv (R+\lambda) - \gamma\mu - \frac{1}{2}\sigma^2\gamma(\gamma-1) \quad (3)$$

corresponds to the growth-rate-adjusted discount rate, GRADR. It is equal to the discount rate $(R+\lambda)$ in the context of incomplete information minus the growth rate of ${}_t[p^\gamma_{t+s}]$, $\gamma\mu + \frac{1}{2}\sigma^2\gamma(\gamma-1)$.

[2] Merton's model may be stated as follows :

$$\bar{R}_X - R = \beta_X[\bar{R}_m - R] + \lambda_X - \beta_X\lambda_m$$

where :
$\bar{R}_X$: the equilibrium expected return on an asset X,
$\bar{R}_m$: the equilibrium expected return on the market portfolio,
R: 1 + the riskless rate of interest,
$\beta_X = \frac{\mathrm{cov}(\tilde{R}_X/\tilde{R}_m)}{\mathrm{var}(\tilde{R}_m)}$,
λ_X: the equilibrium aggregate "shadow cost" for the asset X. It is of the same dimension as the expected rate of return on this asset X,
λ_m: the weighted average shadow cost of incomplete information over all assets.

The rate GRADR $f(\gamma, (R+\lambda)$ is a concave quadratic function in γ and the equation $f(\gamma, (R+\lambda)) = 0$ has two distinct roots (one positive and one negative) for a positive R.

Define $PV_t[p^\gamma, R+\lambda]$ as the present value discounted at $(R+\lambda)$ of expected p^γ from time t or

$$PV_t\left[p^\gamma; (R+\lambda)\right] \equiv \int_0^\infty E_t\{p_{t+s}^\gamma\}e^{-(R+\lambda)s}ds = p_t^\gamma \int_0^\infty e^{-f(\gamma;(R+\lambda))s}ds = \frac{p_t^\gamma}{f(\gamma;(R+\lambda))} \quad (4)$$

1.2. Investment cost functions and the operating profit

The output Y_t is produced using capital K_t and labor L_t where the following Cobb-Douglas production function is used :

$$Y_t = L_t^\alpha K_t^{(1-\alpha)}$$

The firm's operating profit is the revenue minus wages :

$$p_t L_t^\alpha K_t^{(1-\alpha)} - wL_t$$

where w stands for the fixed wage. The firm chooses L_t to maximize the instantaneous operating profit $\imath(K_t, p_t)$

$$\pi(K_t, p_t) \equiv hp_t^\theta K_t \quad (5)$$

where

$$\theta \equiv \frac{1}{1-\alpha} > 1$$

and

$$h \equiv \theta^{-\theta}(\theta-1)^{\theta-1}W^{1-\theta} > 0$$

where hp_t^θ corresponds to the marginal revenue product of capital at time t. The change in the capital stock is

$$dK_t = (I_t - \delta K_t)dt \quad (6)$$

where I_t is the gross investment and $\delta \geq 0$ is the rate of depreciation. We denote by $c(I_t)$ the total cost of investing at rate I_t and assume that $c(I_t)$ is convex.

1.3. The Bellman equation

The firm maximizes the expected present value of its cash flows and uses a discount rate $(r+\lambda)>0$. We denote by $V(K_t,p_t)$ the fundamental value of the firm at time t with

$$V(K_t,p_t)=\max_{\{I_{t+s}\}}E_t\{\int_0^{\infty}\left[hp_{t+s}^{\theta}K_{t+s}-c(I_{t+s})\right]e^{-(r+\lambda_v)s}ds\} \quad (7)$$

The fundamental value of the firm satisfies the Bellman equation

$$(r+\lambda_v)V(K,p)=\max_I\left[hp^{\theta}K-c(I)+\frac{E\{dV\}}{dt}\right] \quad (8)$$

The right-hand side of equation (8) corresponds to the net cash flow $hp^{\theta}K-c(I)$ and the expected capital gain $\frac{E(dV)}{dt}$. This equation shows that the sum of these terms must be equal to the required return $(r+\lambda_v)V(K,p)$. Using Ito's lemma and equations (6) and (1), the expected capital gain is

$$\frac{E\{dV\}}{dt}=(I-\delta K)V_K+\mu pV_p+\frac{1}{2}\sigma^2p^2Vpp \quad (9)$$

where V_K refers to the marginal valuation of a unit of installed capital. Using the q theory of investment, we define $q=V_K$, which represents the shadow value of installed capital.
Substituting q for V_K in equation (9) and replacing equation (9) in equation (8) gives

$$(r+\lambda_v)V(K,p)=\max_I\left[hp^{\theta}K-c(I)+(I-\delta K)q+\mu pV_p+\frac{1}{2}\sigma^2p^2Vpp\right] \quad (10)$$

By 'maximizig out' the rate of investment, equation (10) can be written as

$$(r+\lambda_v)V(K,p)=hp^{\theta}K+\phi-\delta Kq+\mu pV_p+\frac{1}{2}\sigma^2p^2Vpp \quad (11)$$

where

$$\phi\equiv\max_I\left[Iq-c(I)\right] \quad (12)$$

Φ corresponds to the maximized value of rents accruing to the technology from investing at rate I. If the firm invests at rate I over an interval dt, it obtains Idt units of capital. Since q represents the shadow price of this capital, then the firm acquires capital worth $qIdt$ and pays $c(I)dt$ to increase the capital stock by Idt. The excess of additional value over costs, $qI - c(I)$ corresponds to the value of the rents accruing per unit time to the firm for investing at rate I.

2. Reversible investment and the fundamental value of the firm

This section specifies an investment cost function for which the optimal level of investment may be negative. Expressions of the optimal rate of investment as a function of q and the fundamental value of the firm are given.

Consider the total cost of investing $c(I_t)$ at time t

$$c(I_t) = bI_t + \gamma I_t^{n/(n-1)} \quad (13)$$

where $b \geq 0, \gamma > 0$ and $n \in [2, 4, 6, , ...]$.

The term bI_t is the cost of buying new capital at a fixed price b per unit. When the gross investment is negative, $bI_t \leq 0$, corresponds to the proceeds to the firm of selling capital at a price b per unit.
The term $\gamma I_t^{\overline{(n-1)}^{n}}$ corresponds to the convex cost of adjustment. When $n = 2$, optimal investment is a linear function of q and b. When n differs from 2, investment may be a nonlinear function of its fundamental determinants.

When the function $f(n\theta, r) > 0$, there is a fundamental value of the firm. Using the investment cost in (13), equation (12) can be written as

$$\phi = \max_I \left[(q-b)I - \gamma I^{n/(n-1)} \right] \quad (14)$$

When equation (14) is differentiated with respect to I, and the derivative is set to zero, this gives the optimal rate of investment $\hat{I}$ as

$$\hat{I} = \left[\frac{n-1}{n\gamma} \right]^{n-1} (q-b)^{n-1} \quad (15)$$

When the shadow price of capital q is higher than b, investment is positive, otherwise ($q < b$) investment is negative.
When equation (15) is substituted in equation (14), this gives the value

$$\phi = (q - b)^n \Gamma \quad (16)$$

where

$$\Gamma \equiv (n-1)^{n-1} n^{-n} \gamma^{1-n} > 0$$

Since the investment cost function is convex, the firm earns rents on infra-marginal units of investments when q is different from b.
This analysis shows the optimal rate of investment as a function of q, the marginal value of installed capital.

2.1. The fundamental value of the firm and information uncertainty

To obtain the fundamental value of the firm $V(K, p)$, we first determine q as a function of p. Consider the following linear solution

$$V(K, p) = q(p)K + G(p) \quad (17)$$

where $q(p)$ and $G(p)$ are functions to be determined.

When (17) is substituted into (11) and the expression for Φ in (16) is used, this gives

$$(r+\lambda_g)qK+(r+\lambda_g)G = hp^\theta K+(q-b)^n\Gamma-\delta Kq+\mu pq_pK+\mu pG_p+\frac{1}{2}\sigma^2p^2q_{pp}K+\frac{1}{2}\sigma^2p^2G_{pp} \quad (18)$$

This differential equation must hold for all values of K and the terms multiplying K on both sides must be equal. Besides, the other terms (without K) on both sides must be equal. This gives

$$(r+\lambda_g)q = hp^\theta - \delta q + \mu pq_p + \frac{1}{2}\sigma^2p^2q_{pp} \quad (19)$$

$$(r+\lambda_g)G = (q-b)^n\Gamma + \mu pG_p + \frac{1}{2}\sigma^2p^2G_{pp} \quad (20)$$

These equations with a recursive structure can be solved using (19) for $q(p)$ and then (20) for $G(p)$.

2.2. The marginal value of installed capital

Equation (19) is solved to obtain the marginal value of installed capital. The general solution is of the form

$$q(p) = Bp^{\theta} + A_1 p^{n_1} + A_2 p^{n_2} \tag{1}$$

where

$$B \equiv \frac{h}{f(\theta; r+\delta)} > 0$$

and
$\eta_1 > \eta_2$ are the roots of the quadratic equation $f(\eta, r+\delta) = 0$ with the condition $\eta_1 > n\theta \geq 2\theta > 0 > \eta_2$.

The term Bp^{θ} corresponds to the present value of expected marginal revenue products of capital hp^{θ} accruing to the undepreciation portion of a unit of currently installed capital. The other terms $A_1 p^{\eta_1}$ and $A_2 p^{\eta_2}$ are solutions to (19) and their expected growth rates are $(r+\delta)$ in the presence of shadow costs of incomplete information.
If these terms are viewed as bubbles and we restrict the analysis to the fundamentals (cash flows), then $A - 1 = A_2 = 0$ and

$$q(p) = Bp^{\theta} \quad (22)$$

The value of q is the present value of expected marginal revenue products and q is independent of the specification of the adjustment cost function. Using the definitions of B in (21) and of the growth-rate-adjusted discount rate in (3), the fact that $\theta > 1$, implies that B increases with the variance σ^2. Since investment increases with q, investment increases with σ^2 for a given value of p. This result is similar to that in Hartman (1972), Abel (1983) and Caballero (1991).

2.3. The value of the adjustment technology

The intercept term $G(p)$ in the fundamental value of the firm equals the present value of the expected rents ϕ accruing to the adjustment technology. This is represented by the convex cost of adjustment function. Each firm has access to only one adjustment technology characterized by $c(I)$. The function $G(p)$ can be determined using the differential equation (20). It can be verified by direct substitution that $GP(p)$ is a particular solution.

$$GP(p) = PV_t\left[(q-b)^n\Gamma;(r+\lambda_g)\right] \quad (23)$$

Since $\phi = (q-b)^n\Gamma$, the particular solution corresponds to the present value of the expected rents ϕ.

The fundamental value of the firm is composed of two parts. First, the value of existing capital qK which corresponds to the expected value of the returns to the existing capital.
Second, the value of the adjustment technology $PV_t[\phi,(r+\lambda_g)]$ which corresponds to the present value of the expected rents accruing to investment through the adjustment technology.

3. The analysis for the case of irreversible investment

3.1. The modified investment cost function and the optimal rate of investment

Assume the following cost function $c(I)$:

$$c(I_t) = \begin{cases} bI_t + \gamma I_t^{n/(n-1)} & \text{for } \mathrm{I_t} \succeq 0 \\ g(I_t) > 0 & \text{for } \mathrm{I_t} < 0 \end{cases} \quad (24)$$

with $b \geq 0, \gamma > 0$ and n is an even positive integer. For negative gross investment, the investment cost function is different from that in equation (13). For all negative values of gross investment, the cost function $c(I_t)$ is positive. This implies that negative gross investment will never be optimal.

When $q \geq b$, the optimal rate of investment is positive and identical to the case of reversible investment. Since the optimal rate of investment can not be negative, the use of the cost function in (24) corresponds to a case of irreversible investment. For an irreversible investment, we have

$$\hat{I} = \left\{ 0, \left[\frac{n-1}{n\gamma} \right]^{n-1} (q-b)^{n-1} \right\} \quad (25)$$

and

$$\phi = (max\,[0, q-b])^n \Gamma \quad (26)$$

This equation gives the optimal rate of investment as a function of the shadow price of capital q.

3.2. The fundamental value of the firm

The value of q must be determined as a function of the price of the output p. Assume the existence of two regimes : regime H applies when $q \geq b$; regime L applies for $q \leq b$. Hence

$$V^{(i)}(K,p) = q^{(i)}(p)K + G^{(i)}(p), \quad i = \begin{cases} L & \text{for q} \preceq \text{b} \\ H & \text{for q} \succeq \text{b} \end{cases} \quad (27)$$

The function $q^i(p)$ and $G^i(p)$ can be determined by replacing this last equation into equation (11) and using the expresion for ϕ in (26). As in the reversible case, by gathering the terms involving K (and not), we obtain the following equations

$$(r+\lambda_g)q^{(i)} = hp^\theta - \delta q^{(i)} + \mu p q_p^{(i)} + \frac{1}{2}\sigma^2 p^2 q_{pp}^{(i)} \quad i = L, H \quad (28)$$

$$(r+\lambda_g)G^{(i)} = (max\,[0, q-b])^n \Gamma + \mu p G_p^{(i)} + \frac{1}{2}\sigma^2 p^2 G_{pp}^{(i)} \quad i = L, H \quad (29)$$

The recursive structure of these equations can be used to solve (28) for $q^i(p)$ and (29) for $G^i(p)$.

3.3. The solution for q and the optimal rate of investment in the same context

A general solution to equation (28) is

$$q^{(i)}(p) = Bp^\theta + A_1^{(i)} p^{n_1} + A_2^{(i)} p^{n_2} \quad i = L, H \quad (30)$$

where the values of B, η_1 and η_2 are similar to their values in the reversible investment case (equation 21). The difference with the reversible case is that the coefficients A_1^i and A_1^i can differ across the two regimes L and H. The two regimes are equivalent when $q = b$. The reversible case and the irreversible case give the same value of q. The only difference appears when $q < b$. This corresponds to a negative investment in the reversible case and a zero investment in the irreversible case.

4.3. The value of the adjusting technology

The rents to the adjustment technology, represented by the intercept $G(P)$ are different in each case (reversible and irreversible). The function $G(p)$ is determined using equation (29) for the two regimes. For the regime L, the term $(max[0, q - b]^n \Gamma = 0)$ and (29) becomes

$$(r + \lambda_g)G^{(L)} = \mu p G_p^{(L)} + \frac{1}{2}\sigma^2 p^2 G_{pp}^{(L)} \quad (31)$$

The general solution to equation (31) is

$$G^{(L)}(p) = C_1^{(L)} p^{\omega_1} + C_2^{(L)} p^{\omega_2} \quad (32)$$

where $\omega_1 > \omega_2$ are the roots of $f(\omega, r) = 0$. By ruling out bubbles, C_2^L must be zero and the fundamental value of the firm with no capital in regime L when $q < b$ is

$$G^{(L)}(p) = C_1^{(L)} p^{\omega_1} \quad (33)$$

In this situation, a firm with no capital and not undertaking gross investment, has a positive value because of the prospect that one day $q > b$ and it will be profitable to invest.

In the regime H, $q \geq b$, and $(max[0, q - b]^n \Gamma = (q - b)^n \Gamma)$ and equation (29) becomes

$$(r + \lambda_g)G^{(H)} = (q - b)^n \Gamma + \mu p G_p^{(H)} + \frac{1}{2}\sigma^2 p^2 G_{pp}^{(H)} \quad (34)$$

This equation is similar to equation (20) in the reversible case.
The particular solution $GP(p)$ in (23) is also a particular solution to (34). We can obtain a general solution to (34) by adding the particular solution in (23) and the solution to the homogeneous part of (34).

The solution to the homogeneous part of (34) is

$$C_1^H p^{\omega_1} + C_2^H p^{\omega_2}$$

Following the analysis in Abel and Eberly (1997), ruling out bubbles, $C_1^H = 0$ and the solutions for G^L and G^H are :

$$G^{(L)}(q) = \left[(\omega_1 - \omega_2) \prod_j^n (\omega_1 - j(\theta)) \right]^{-1} \frac{2\Gamma b^n \theta^n n!}{\sigma^2} (\frac{q}{b})^{\omega_1/\theta} \quad (2)$$

$$G^{(H)}(q) = PV_t\left[(q-b)^n; (r+\lambda_g)\right]\Gamma + (\left[(\omega_1 - \omega_2) \prod_j^n (\omega_2 - j(\theta)) \right]^{-1} \frac{2\Gamma b^n \theta^n n!}{\sigma^2} (\frac{q}{b})^{\omega_2/\theta} \quad (36)$$

Equation (35) implies that $G^L(q) > 0$ and when it is not currently profitable to undertake positive gross investment ($q < b$), the prospect that q will be higher than b means that the present value of the rents accruing to the adjustment technology is positive.

The term $PV_t[(q-b)^n; (r+\lambda_g)]\Gamma = G(p)$ in the reversible case.The difference between $G(p)$ in the irreversible and reversible cases correspond to the second term in equation (36). Since the second term in equation (36) is negative, G^H in the irreversible case is smaller than $PV_t[(q-b)^n; (r+\lambda_g)]\Gamma$.

Summary

Information plays a central role in investment decisions and in the process of firm valuation. This paper develops closed-form solutions for the optimal investment and fundamental value of a competitive firm under uncertainty and information costs in a reversible and an irreversible investment case. Optimal investment is a nondecreasing function of the shadow value of capital. The effect on investment behavior is to set investment to zero when it would otherwise be negative. Irreversibility implies a costly technology to disinvesting and the value of the firm will reflect this cost disadvantage and will be reduced by this irreversibility. This finding implies that the average q or ratio of the firm's value to its capital stock $\frac{V(K,p)}{bK}$ is reduced by irreversibility even the (marginal) q is unaffected.
Hence, irreversibility reduces the difference between marginal and average q-.

The average q is higher than the marginal q by an amount equal to the ratio of the present value of rents to the capital stock $\frac{G(p)}{bK}$.
The invariance of q to the imposition of irreversibility is in contrast with the results in the standard literature of irreversible investment by McDonald and Siegel (1986), Dixit (1989), Bertola (1987) and Pindyck (1988) where imposition of a nonnegative constraint on investment reduces the marginal value of capital as a consequence of 'the option value of waiting'. In our analysis, q does not depend on the capital stock. When the operating profit function is linear in capital, the marginal operating profit of capital is invariant to the capital stock.

REFERENCES

- Abel A.B. (1983), "Optimal investment under uncertainty", American Economic Review, 73, pp 228-233
- Abel A.B. and J.C. Eberly. (1997), "An Exact solution for the investment and value of a firm facing uncertainty, adjustment costs, and irreversibility", Journal of Economic Dynamics and Control , 21, pp 831-852
- McDonald R. and D. Siegel (1986), "The value of waiting to invest" Quarterly Journal of Economics, 101, pp 707-728
- Dixit JA. (1989), "Entry and exit decisions under uncertainty" Journal of Political Economy, 97, pp 620-638
- Bellalah M., (2001 a)," Irreversibility, Sunk costs and Investment Under Uncertainty", R&D Management,
- Bellalah M., (2001 b)," A Reexamination of Corporate Risks Under Incomplete Information", International Journal of Finance and Economics,
- Bertola G. (1987), " Irreversible Investment" Unpublished Manuscript, MIT, Cambridge MA.
- Caballero R.J. (1991), "On the sign of the investment -uncertainty relationship", American Economic Review, 81, pp 279-288

- Hartman R. (1972), " The effect of price and cost uncertainty on investment", Journal of Economic Theory, 5, pp 258-266
- McDonald, R. and Siegel, D. (1986) The Value of Waiting to Invest. Quarterly Journal of Economics, 101, 707-728.
- Merton, Robert., (1987), "A Simple Model of capital market equilibrium with Incomplete Information.", Journal of Finance 42, 483-510.
- Shapiro. A, (2000),"The investor recognition hypothesis in a Dynamic General Equilibrium : Theory and Evidence", Working Paper, Stern School of Business, New York University.
- Pindyck JR.S. (1988), "Irreversible investment, capital choice and the value of the firm", American Economic Review 78, 969-985