Investment in Information in Petroleum, Real Options and Revelation

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Abstract:

A firm owns the investment rights over one undeveloped oilfield with technical uncertainties on the size and quality of the reserve. In addition, the long run expected oil price follows a stochastic process. The firm needs to select the best alternative of investment in information with different costs and different *revelation powers*. The modeling of technical uncertainty uses the practical concept of *revelation distribution*, which works directly with the possible *new expectations* after the information revelation caused by an investment in information. Expectations drive the valuation of the development option exercise. With a *partial revelation* of uncertainty of a technical parameter, is necessary to know only the initial uncertainty (prior distribution) and the expected percentage of variance reduction induced by the investment in information. After the information revelation, the development threshold decision depends on the value of the project normalized by the development cost. This normalized threshold is the same for any technical scenario revealed by the new information when the oil price follows a geometric Brownian motion. In addition, there is a time to expiration of the rights for the option to develop, so that the normalized threshold is a *free boundary* obtained from the *optimal stochastic control theory*. The model includes a penalty factor for the lack of information, which causes sub-optimal development, and this factor is introduced into the dynamic real options model. The model outputs are the real options value with and without the technical uncertainty, with and without the information, and the *dynamic net value of information*.

Keywords: value of information, dynamic value of information, real options, investment in information, information revelation, revelation distribution, Monte Carlo simulation, optimization under uncertainty, investment under uncertainty, valuation of projects.

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1 - Introduction

Technical uncertainties and learning processes have been frequent issues in the literature of real options in petroleum[3]. The investment in additional information before the development of petroleum reserves is a very important alternative for both the earlier oilfield development and the waiting for better market conditions. For the cases without technical uncertainty, the choice is reduced between immediate investment and the "wait and see" policy, that is, the traditional real options model. There are at least two important sources of uncertainties, *market uncertainty* represented mainly by the oil prices, and *technical uncertainty* about the size and the quality of the reserve[4]. The aim of this paper is to build a model that evaluates the investment in additional information in a *dynamic way* (considering the factor *time*), taking into account different sources of uncertainties and the *revelation power* for each alternative of investment in information.

The selection of the best alternative for the investment in information (including not investing in information) is very important. But it presents complex practical challenges, even more in a dynamic framework considering the expiration of the option to start the oilfield development, the time to learn (the gathering of data and the processing takes time) and interaction with market uncertainties. This paper presents a practical way to simplify this job, keeping a sound theoretical foundation.

The information revelation is modeled in one or more discrete-time points (event-driven process) rather in continuous-time as adopted in other papers[5]. The reason is that the development plan is revised only if there is new (relevant) technical information, and after the processed information to become knowledge or wisdom about the reserve properties. See Chen & Conover & Kensinger (2001) for an in-depth discussion of a real options model of information gathering, storage and

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[3] See Chorn & Carr (1997), Chorn & Croft (2000), Dias (2001a), Galli & Armstrong & Jehl (1999), Whiteside & Drown & Levy (2001) among others. For a discussion of real options models, see Dixit & Pindyck (1994) and Trigeorgis (1996). The term "real options" was first used by Myers (1977) and the real options in petroleum started with Tourinho (1979).

[4] Quality of the reserve includes permeability/porosity properties of the reservoir-rock, oil and/or gas properties, reservoir inflow mechanism and other issues. For deepwater oilfields, uncertainty on the reserve quality includes the uncertainty about the cost and performance of new technology. The quality of the reserve later will be more precisely defined.

[5] Martzoukos & Trigeorgis (2001) is one exception. They model the costly learning (investment in information) as a jump of random size *activated by the management* (so, it’s an event-driven process like here). In their setting, the learning is related only with the underlying asset value (here is included the effect on the exercise price of the option) and their focuses are the timing of learning and the multistage learning. Here the focuses are the distribution of expectations (jump size) after the investment in information and the selection of the best project to invest in information.

processing. In other words, the new expectation about reserve size and quality revealed by the investment in information is a good reason for a revision of the development plan. Hence, the new expectations for the technical uncertainties are *event-driven* process[6] (the event is the new knowledge after the investment in information) rather *time-driven* process as in the case of market uncertainties[7].

The technical uncertainty is modeled into a dynamic framework of real options with the concept of *revelation distribution*[8]. The main contribution of this paper are not the proofs of the propositions[9], but the recognition of the *practical value for the distribution of expectations* and its insertion into a dynamic real options model to evaluate investments in information.

After the revelation, the threshold for the optimal development exercise is the same even changing the exercise price of the option, since the model uses a normalized threshold curve that is the same after any revelation. This approach saves computational time because the threshold is calculated once. Without the normalization, each iteration in the Monte Carlo simulation requires a new threshold value estimate, because the exercise price of the real option to develop changes with the revelation (we will see this in section 3, equation 4).

The paper is divided as follow. In the second section is presented the technical uncertainty modeling using the concept of revelation distribution and three properties. In the third one is presented the payoff function for the real options exercise (the NPV function), how the uncertainties can be inserted into the model, and the effect of technical uncertainty on the NPV function (penalty function). The fourth section presents the real options model, including the risk-neutral simulation equation for the oil prices, the normalized threshold curve and how the revelation distribution is placed into the real options simulation. The fifth section presents some case studies with numerical

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[6] Lawrence (1999, p.156) argues that it is a time-driven process with *different chronological length* because the events are successive along the time. It doesn't work here because the events are optional, activated by the firm, and in parallel there is another process (oil prices) which the information arrive continuously along the time, which is not optional. The complexity of these two superposed processes (revelation process and market process) requires a special approach.

[7] Even for a low explored basin, with many different firms with tracts in this basin, new information from a drilling or from new processed seismic data can take many months. This issue can be modeled with a Poisson process (discrete-time process). In other words, the model of *technical* information arriving continuously along the time is not appropriate for petroleum exploration and production investments at firm level.

[8] The revelation distribution helps to solve the apparent *real options paradox* (Why learn?) described in the interesting paper of Martzoukos & Trigeorgis (2001).

[9] Even because the propositions are based in some selected known theorems from the conditional expectations literature.

results. The sixth section presents some extensions, such us the timing of investment in information, the sequential investment in information case and the (event-driven) *martingale property* for the revelation distribution; and a non-linear case for the NPV function that occurs with the petroleum fiscal regime of *production sharing*. The last section concludes the paper. In the appendix are presented the proofs for the propositions and some others conditional expectation properties.

2 - Investment in Information and the Revelation Distribution

Assumption (Investment in Information Goal): the primary goal of any investment in information is to *reduce the uncertainty* on one or more parameters. In other words, the main benefit of an investment in information is the reduction of uncertainty, which can be conveniently expressed as the *percentage of variance reduction*. All the other benefits from an investment in information (if any) are *extra benefits*, which need to be quantified in monetary terms and in present value, if relevant.

After a new information, the manager decision is driven by the *new expectation* about the value of the variable and the uncertainty around this new expectation. So, the decision *after* the information depends of the properties of the *distribution of expectations*. The distribution of *conditional expectations*[10] here is named the ***revelation distribution***. This denomination[11] emphasizes the change of expectations of the manager with the revealed new scenario and the *process of learning* or *discovery process* towards the *true* value of the variable[12].

The concept of revelation here is different of the famous *principle of revelation*, from the *literature of asymmetric information* (or more specifically of the *theory of mechanism design*) and in *Bayesian games*. Our setting means the revelation of the true state of the technical parameter (true state of the

[10] The use of conditional expectation as basis for decision has strong theoretical basis. Imagine a variable with technical uncertainty X and the new information I, random variable defined in the same probability space (Ω, Σ, P). We want to estimate X by observing I, using a function g(I). The most frequent measure of quality of a predictor g is its *mean square error* defined by $\text{MSE}(g) = E[X - g(I)]^2$. The choice of g* that minimizes MSE(g) is exactly the conditional expectation $E[X \mid I]$. This is a very known property used in econometrics, see for example Gallant (1997, pp.64-65).

[11] Denomination of revealed variance and related concepts has been used in papers on value of information in a dynamic framework. For example, Childs & Ott & Riddiough (2001, p.46) names "*revealed variance*" the variance of conditional expected values. Here this variance is named *variance of revelation distribution*.

[12] The author has been using this concept since 1998, and it appeared in Dias (2001a). However, here it is presented in a more formal framework and also the nice properties of the revelation distribution.

nature of one parameter), whereas the mechanism design concept is when the revelation of the true type of one agent is optimal (a direct mechanism that is optimal to say the truth).

The highest efficiency for one investment in information is when reduce to zero the variance of the posterior distribution, resulting in a *full revelation* (reveal the truth on the technical parameter). What are the possible scenarios after this very efficient investment in information? Of course all the scenarios from the previous total uncertainty (a priori distribution) are possible. With this reasoning, let us consider the first proposition for the revelation distribution[13].

Proposition 1 (full revelation): For the full revelation case, the revelation distribution is equal to the unconditional (prior) distribution.

This proposition is obvious and draws directly from the definition of prior distribution, because the prior distribution represents the total technical uncertainty about one single parameter, so that represents all the possible values that the parameter can assume. In case of full revelation, one value from this distribution will be revealed, and the probability for this value to be revealed must be the same from the prior distribution in order to preserve the consistency.

Hence, for the full revelation case is trivial to obtain the revelation distribution. However, in real life typically we obtain only a *partial revelation* with the investment in information. The concept of *partial revelation* is related with the concept of "imperfect information", whereas the concept of full revelation is related with "perfect information"[14] one from *decision analysis* literature. As in this literature, *the value of information with partial revelation cannot exceed the value of information with full revelation* (see the equivalent in decision analysis in the book of Pratt & Raiffa & Schlaifer, 1995, p.252). However, in this paper the concept of partial revelation is introduced into a more dynamic framework, putting the revelation distributions into the real options model.

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[13] These propositions are valid *almost surely*, that is, are valid except in a set with probability measure equal to zero, and use other regular assumptions (e.g., X is integrable). See the appendix for technical details.

[14] Lawrence (1999, p.69) states: "*Perfect information occurs when the information structure provides categorical direct messages that identify precisely and unequivocally the state that occurs*". Childs & Ott & Riddiough (2001, p.47) use the terminology "full information" with the same meaning. Here is used *full revelation*.

How to proceed in the partial revelation case? Fortunately, the revelation distribution has some nice probabilistic properties that help us to model dynamically the value of information. The expected value and the variance from the revelation distribution are given below[15].

Definition: Let X be the variable with technical uncertainty (e.g., the reserve size B), and the investment in information reveals the information I = i. ***Revelation distribution*** **is defined as the distribution of $R_X = E[X \mid I]$**. The revelation distribution properties such us the mean and variance, are presented as propositions.

Proposition 2: The expected value for the revelation distribution is equal the expected value of the original (a priori) technical parameter distribution (proof: see the appendix)[16].

$$\mathbf{E[R_X] \ = \ E[X]} \qquad \textbf{(1)}$$

Hence, the weighted average of the conditional expected value of X given that I = i (that is, R_X) being each term $R_X(i) = E[X \mid I = i]$ weighted by the probability of the event on which it is conditioned, is simply the original (unconditional) expected value of X.

Proposition 3: the variance of the revelation distribution is equal to the expected reduction of variance induced by the new information. This result is not obvious (proof: see the appendix).

$$\mathbf{Var[R_X] \ = \ Var[X] - E[Var\{X \mid I\}]} \qquad \textbf{(2)}$$

This is an outstanding issue that makes the revelation distribution very useful for practical purposes. By knowing only the original variance and the percentage of reduction of variance, we can find the variance of revelation distribution. Note that the right side is just the difference between the variance before the information (unconditional) and the *expected* remaining variance after the information.

For the *full revelation* case (imagine a very efficient investment in information revealing all the uncertainty), the residual variance is zero, and hence $Var[R_X] = Var\{X\}$ as required for the consistency between Propositions 1 and 3.

[15] See the book of Sheldon Ross (1998) for a good introduction to conditional expectation without measure theory.

[16] This can be viewed as an application of a property known as *law of iterated expectations*, from conditional expectation literature.

The above formulas allow a practical way to ask the technical expert on the *power of revelation* of a specific investment in information. It is necessary to ask him/her for the following information:

- What is the total uncertainty of a particular parameter (e.g., the reserve size B)? The specialist answer needs to specify the distribution of technical uncertainty, that is, mean, variance, and the class of distribution (Triangular, LogNormal, Uniform, etc.).

- What is the expected percentage of reduction of uncertainty (read *reduction of variance*) with this new information?

With these two answers from the experts we can specify the mean and the variance of the revelation distribution, which is used in our dynamic framework for the value of information. The best way to understand the concept of revelation distribution is by using a simple example.

Simple Numerical Illustration of Revelation Distribution

Consider the following stylized oilfield with technical uncertainty on the reserve volume B (in million barrels, displayed as MM bbl), illustrated in the Figure 1.

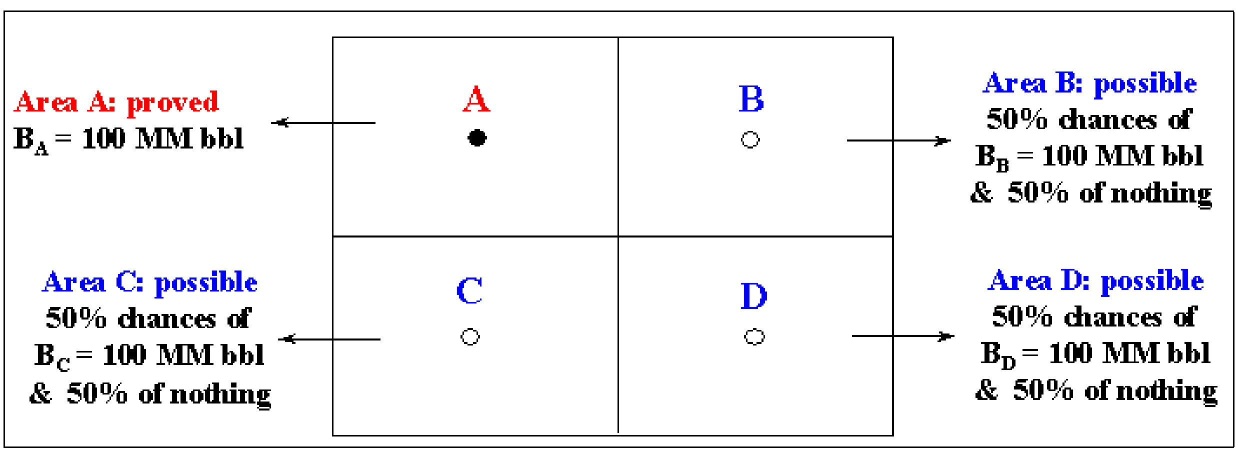

Figure 1 - Stylized Oilfield with Technical Uncertainty

Assume that there are three alternatives[17] of investments in information, besides the *alternative zero* of not investing in information. The alternative 1 drills one appraisal well in the area B and revel all

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[17] In reality in these alternatives are not mutually exclusive and can be performed sequentially. However, here we consider as three different alternatives in terms of cost of information and power of revelation, in order to develop the intuition for the revelation distributions for alternatives with different powers of revelation. Later we examine the case of sequential investment in information.

about *that area*, but nothing about the remaining areas C and D (partial revelation). The alternative 2 have a higher revelation power because drills two appraisal wells (e.g., in the areas B and C). The alternative 3 have the highest power revelation by drilling three appraisal wells, and in this example this means the condition of *full revelation*.

First note that the initial uncertainty (*unconditional* distribution or *prior* distribution) is represented by the following discrete scenarios distribution:

- 100 million bbl with 12.5 % chances;
- 200 million bbl with 37,5 % chances;
- 300 million bbl with 37,5 % chances; and
- 400 million bbl with 12,5 % chances.

It is easy to see that the expected value for the unconditional distribution is E[B] = 250 million bbl and the variance is Var[B] = 7500 (million bbl)^2. Let us see what happen with the different investments in information, in both the posterior (or conditional) distribution and the revelation distribution.

The revelation distribution obtained with one alternative is the *distribution of expectations after the information revelation* for each alternative. What are the new possible scenarios of expectation after the appraisal drilling in the area B (Alternative 1)?

Alternative 1 generate two possible scenarios, because the well B result can be success proving more 100 million bbl (positive revelation with 50% chances) or dry (negative revelation with 50% chances). These two scenarios of new expectations revealed with one appraisal well, form the discrete *revelation distribution for Alternative 1*. This revelation distribution is presented below.

$E_1(B|A_1)$ = 100 + 100 + (0.5 x 100) + (0.5 x 100) = 300 million bbl …… with 50% chances

$E_2(B|A_1)$ = 100 + 0 + (0.5 x 100) + (0.5 x 100) = 200 million bbl ……. with 50% chances

Where $E_1(.)$ is the expectation in case of positive revelation (good news) and $E_2(.)$ is the expectation for negative revelation (bad news). Note that, <u>with the Alternative 1, it is *impossible* to reach more extreme scenarios</u> of revelation such us 100 million bbl or 400 million bbl. This is because the *revelation power* of Alternative 1 is not sufficient to change the expectation of the *entire* reserve so much to reach extreme cases.

Alternative 1 reaches only a partial revelation, so that the uncertainty remains and the posterior distribution $B|A_1$ has variance nonzero. What is the expected variance for the posterior distribution with the Alternative 1?

In case of positive revelation, the *posterior distribution* is {200 million bbl with 25 % chances; 300 million bbl with 50 % chances; and 400 million bbl with 25 % chances}. For the negative revelation scenario, the posterior distribution is {100 million bbl with 25% chances; 200 million bbl with 50%; and 300 million bbl with 25%}.

The reader can calculate that the remaining variance (variance of posterior distribution) in both scenarios of revelation are 5000 (million bbl)^2, and so the expected variance of posterior distribution is also 5000 (million bbl)^2. So, Alternative 1 reduces the variance of B in 33% (from 7500 to 5000).

Let us check the Propositions 2 and 3. The expected value of the revelation distribution for the Alternative 1 is:

$$\mathbf{E_{A1}[R_B] \;=\; 50\%\ x\ E_1(B|A_1) \;+\; 50\%\ x\ E_2(B|A_1) = 250\ million\ bbl}$$

As expected by the Proposition 2. The variance of the revelation distribution for Alternative 1 is:

$$\mathbf{Var_{A1}[R_B] \;=\; 50\%\ x\ (300 - 250)^2 \;+\; 50\%\ x\ (200 - 250)^2 \;=\; 2500\ (million\ bbl)^2}$$

As expected by the Proposition 3, the variance of revelation distribution is equal to the expected reduction of variance with the investment in information (7500 – 5000).

The reader can check the Propositions 2 and 3 for the Alternatives 2 and 3, and the Proposition 1 for Alternative 3 (full revelation). The reduction of variance of Alternative 2 is 66% (from 7500 to 2500), whereas the reduction of variance for Alternative 3 is 100% (from 7500 to zero).

Figure 2 shows the revelation distributions for the three alternatives (MM = million). Note that, as higher is the *revelation power* as higher is the variance of revelation distribution.

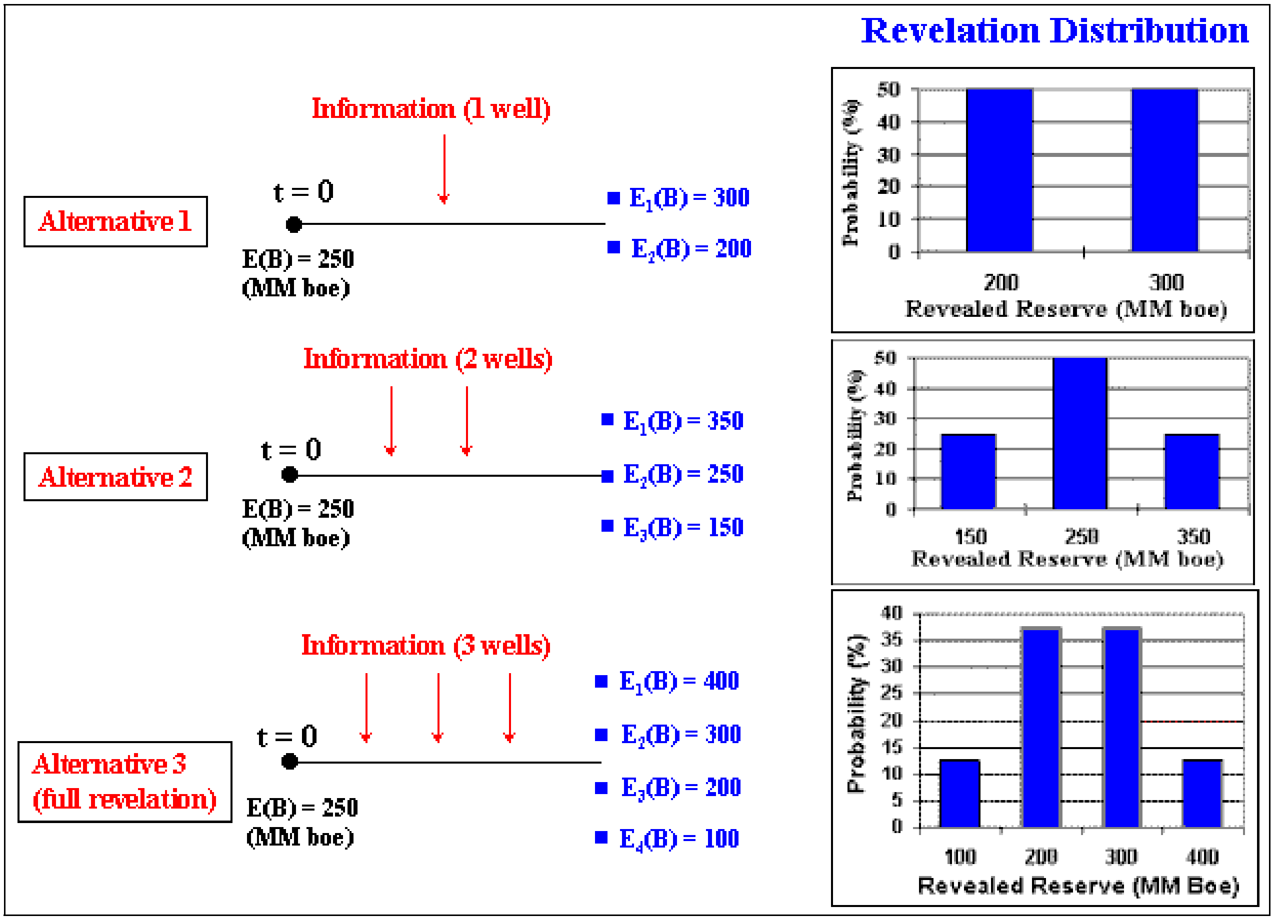

Figure 2 - Alternatives of Investment in Information and Revelation Distributions

The remaining issue: How about the *class* of the revelation distribution (in case of partial revelation)? In general this depends of the distribution of the outcomes from the new set of information, that is, the conditioning distribution (e.g., for Alternative 1, a discrete distribution with 50% for success and 50% for dry well).

Although texts on conditional expectation are very common, the study of the *distribution* of conditional expectations (revelation distribution) when the conditioning is discrete is hard to find. One exception is Lee & Glynn (1999), which uses Monte Carlo methods plus some theorems to estimate this distribution. Of course we can use this more sophisticated setting, but there is an additional cost of complexity. The setting below is more simplified, aimed to practitioners.

For the limiting case (full revelation), Proposition 1 tells that revelation distribution and the prior distribution are of the same class (in reality are equal). Even the partial revelation distributions having different shapes (see the last picture to see this), as the variance grows the *tendency* for the shape of these distributions is *to evolve towards* the prior distribution class. Simply we assume that the class of distribution for partial revelation is the same of the prior distribution of technical parameter, but with variance given by the Proposition 3. For example, if the reservoir engineer expert uses a triangular distribution for the prior distribution of the reserve size B, the partial revelation distributions will be also triangular but with the variance given by Proposition 3. This is absolutely correct for the full revelation case and a convenient simplification for the partial revelation case.

Figure 3 illustrates this example and compares the effect of *expected variance reduction* of the posterior distribution[18] over the revelation distribution, for different levels of variance reduction. The displayed variances are exact, but the distribution shapes of partial revelation are approximated.

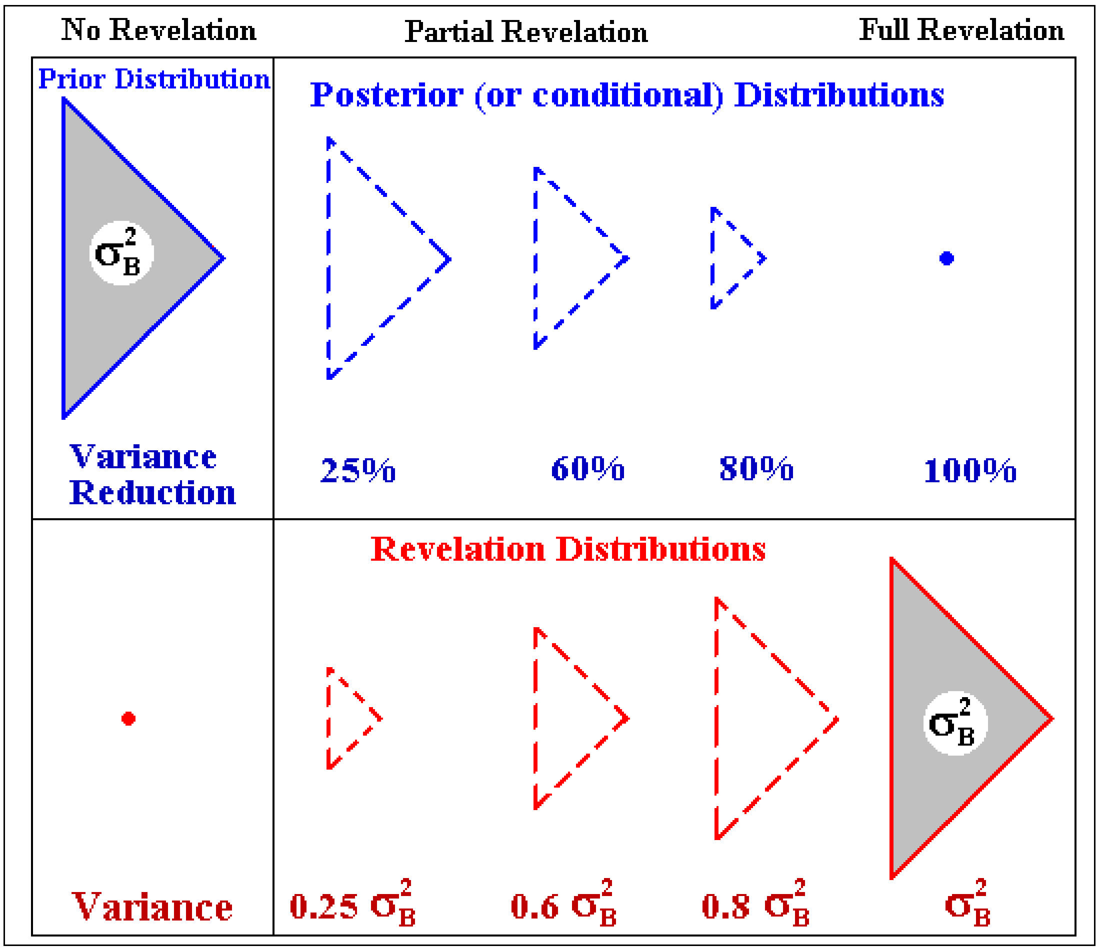

Figure 3 - The Variance of Posterior and Revelation Distributions

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[18] In our setting doesn't matter the posterior distribution class, only its variance is necessary to estimate the variance of the revelation distribution and the penalty factor for the NPV function. In general the posterior distribution doesn't need to have the same type of the prior distribution. However, this occurs for the *conjugate distributions*, e.g., the exponential families (exponential, gamma, normal), see for example Bedford & Cooke (2001, pp.67-70) or Jammernegg (1988, p.10).

3 - The Payoff for the Exercise of the Option to Develop: the NPV Function

3.1 - NPV Function for Monte Carlo Simulation and Related Topics

The exercise of the development option provides the project Net Present Value (NPV)[19] given by the difference of the value of the developed reserve V with the development cost D.

$$\mathbf{NPV} \quad = \quad \mathbf{V} \; - \; \mathbf{D} \qquad \mathbf{(3)}$$

The value of the developed reserve V is given by market value from developed reserves transactions or most commonly by the discounted cash flow (DCF) approach. With the DCF approach, V is the present value of the revenue net of operational costs and taxes, whereas the investment D is the present value of the flow of investment[20] net of tax benefits.

The challenge is how to change the NPV function when performing a Monte Carlo simulation of the key factors with technical and economic uncertainties.

There are at least three alternatives to consider both technical and economic uncertainties into the NPV function. First using a model as simple as possible but considering the main uncertainties, which are parameterized from the DCF model. This model and its associated reserve business vision will be detailed later.

The second alternative is by working directly with the cash-flows, for example an integral with revenues and costs explicitly written as function of variables with uncertainty[21]. This can be done also by putting formulas and correlation among cells in the spreadsheet linked to the sources of uncertainties, because the Monte Carlo simulation needs to change every cell in the appropriated way. Although this is possible, the formulas can be complex to link the uncertainties on the reserve

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[19] Bjerksund & Ekern (1990) showed that for initial oilfield development purposes, in general is possible to ignore both temporary stopping and abandonment options in the presence of the option to delay the investment (the abandon is very far in time to weight in the development decision). In some case (short-duration projects, projects with option to expand) can be necessary to consider other options when exercising the development option. See Trigeorgis (1993) and Dixit & Pindyck (2000) for discussion of interactions of different real options.

[20] The cost of abandon can be considered as investment, and its present value (net of tax benefit) is included in D.

[21] This second way was used in the PUC-Petrobras research project to model an option to expand the production through new wells. This case was easier than the general case because the technical parameter was set at *well* level outcome.

size and productivity of wells, complicating the interpretation and with a much higher computational cost than the first alternative.

The third alternative for a Monte Carlo simulation of the NPV function is by using more complex models and tools in tandem. The uncertainties are introduced into the *reservoir simulator* software, generating the distribution of production profile with its associated values for V and D (and so the NPV = V − D) in the NPV spreadsheet. The problem is that the reservoir simulator is called for every sample used in the Monte Carlo simulation, and the reservoir simulator (that solves a *system* of partial differential equations) is not fast enough, so that the computation is very slow[22].

The first way is used in this paper for the NPV function simulation. It is necessary to think about the main sources of uncertainties, which have important impact in the NPV. Let us use also the business intuition of market value of a developed reserve.

The developed reserve value V, in both DCF valuation and reserve market valuation, is an *increasing function* of some important factors:

- the reserve size B (expressed in million of equivalent oil barrels[23]);
- the technical quality of this reserve, that is, the *fluids quality* and the *permo-porosity quality* of the reservoir, which can be represented by a normalized productivity index (PI/h) for the wells in this reserve;
- the (long-run) oil prices P (US$/bbl); and
- the financial-economic quality of the reserve, which depends of the location, factors like the *discount rate* (function at least of the basic interest rate, the risk-premium for the E&P business, and the *country risk-premium*), the country's fiscal regime, and operational costs (e.g., reserves in deepwaters have higher maintenance cost for the wells).

The development investment D is function of the reserve size B. The function D(B) depends of the reserve location (shallow water x deepwater; near to petroleum industry infrastructure or remote area; etc.) and can be estimate using historical data or estimating the cost of systems for different reserve sizes. Empirical data has been pointing the linear equation with fixed and variable factors, as a good approximation.

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[22] In the future, it will become the preferable one because uses the revelation distributions in a more realistic way.

[23] The associate gas reserve in this oilfield problem is incorporated into the reserve value B by using an economic equivalence relation between the gas and the oil. This relation depends on the local gas market price and demand.

$$\mathbf{D = fixed\ cost\ factor + (variable\ cost\ factor\ x\ B)} \qquad \mathbf{(4)}$$

The value of a developed reserve V is an increasing function of B, for a positive[24] value of one barrel of developed reserve v. Let us assume that for oilfield development projects, the NPV is also an increasing function of reserves volume B. The gain of scale with the reserve size B is a widely known feature for the NPV function of E&P projects. In addition, the value of reserve V is increasing with its *qualities* in terms of reservoir properties, fluid properties, extraction cost environment, taxes, and so on. It is possible to think these qualities linked with a market value or some estimate using a discounted cash flow. Let us explore this point now.

In order to get a simple and adequate equation for the NPV, think about the market value of one barrel of reserve **v** (that is, v is the price of the barrel of reserve). If this reserve price v is directly related with the long-run oil prices, let be **q** the factor of proportionality[25] so that **v = q P**. For developed reserve transactions, as higher is the price per barrel of a specific reserve, as higher is the economic quality for that reserve. For a fixed reserve size and fixed oil price, as higher is the factor q as higher is the value of this reserve. So, let call the factor **q** as ***the economic quality of the reserve***[26]. By using this insight, the value of a reserve V is the price of the barrel of reserve v times the size of this reserve B, that is, V = v B. The equation for the developed reserve value V is:

$$\mathbf{V = q\ P\ B} \qquad \mathbf{(5)}$$

This is the easiest way to work with the three most relevant variables to access the value of a developed reserve, using *business thinking*, which is very adequate for market valuation. The value q can be assessed either by *market transactions* in markets like USA (see Adelman & Koehn & Silva, 1989; and Adelman & Watkins, 1996) or by using the *discounted cash flow* approach (see below).

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[24] For *geriatric* reserves (reserves near of the abandonment time) are possible to get developed reserve assets with negative value because there is the *abandon cost* (environmental recovery cost). This can be ruled out by incorporating this cost into the development cost account D instead V (abandon cost is not operational cost). Even if by convenience the abandon cost is accounted in V, our problem is the investment decision on the oilfield development, that is, an option to get *young* reserves and not the geriatric one. So, here the value of the barrel of reserve always is positive.

[25] Paddock & Siegel & Smith (1988) claims that the *one-third rule* (q = 1/3) for the US reserves is a good first estimate.

[26] We can think that q has a technical component $q^T \in [0, 1]$ representing the fluid quality and the productivity (for example the average normalized well productivity index), and another component q^M representing market characteristics. The expectation of q^T changes only when a new investment in information is performed, whereas the market component q^M evolves continuously with time, starting with $q^M(t = 0) = 1$. This is left for a future extension of the model.

The three factors in the equation above are assumed random in this paper. These factors can be considered the three <u>*basic sources of uncertainties*</u> for the reserve value V, which are assumed mutually independent[27].

For the *fiscal regime of concessions* (USA, UK, Brazil, and others), the linear equation for the NPV with the oil prices is at least a very good approximation. The NPV equation with the oil prices P, considering the factor q is given below.

$$\mathbf{NPV = q\,P\,B - D} \qquad \mathbf{(6)}$$

Without a good market value for q, by using the discounted cash flow analysis we can assess the quality parameter. Most cash flow spreadsheets present a chart with the sensitivity analysis of NPV with the (long-run) oil prices P. Figure 4 presents the link between this chart and the above equation.

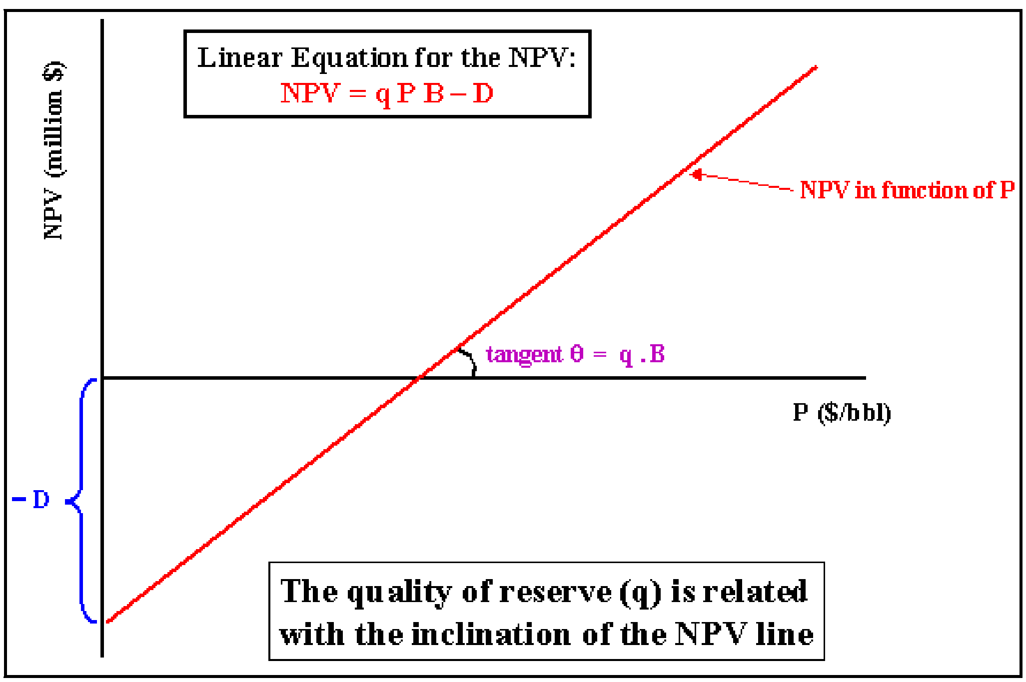

Figure 4 - The NPV x P Function for Petroleum Concessions

[27] This assumption simplifies the model but it is not necessary. We could think q and B with a small positive correlation. Using a positive correlation between q and B enhances the value of information, but with some costs in computational time, input design and output interpretation. By considering the reserve quality q linked to a *normalized* productivity index (dividing by the thickness of reservoir, the *net pay*) instead the productivity index itself, is easier to agree with the simplifying independence assumption. In addition, in a more rigorous setting B is not independent of P, because higher P permits to extend the reserve life. The *volume of oil and gas in-place* are the true pure technical parameters independent

3.2 - The Effect of Technical Uncertainty on the NPV function

The theory of finance tells that the technical uncertainty (like the uncertainty on reserve volume B) has correlation zero with the market portfolio, so that it doesn't demand risk-premium considering that the stockholders of the firm are diversified investors[28]. However, the optimal management of technical uncertainty is appreciated by the investors because can leverage value either by optimal management of the opportunities of investment in information, as by the more valuable exercise of option to develop the oilfield using an optimized alternative of development. The exercise of the option to develop without know the correct volume of reserve B and the technical factors affecting the quality of the reserve, almost surely will conduct to sub-optimal development (e.g., see cases reported in Demirmen, 2001). Let us quantify the losses with sub-optimal development.

The net present value equation $NPV = V - D = q\ P\ B - D(B)$ is based on expected value for the cash-flows, and it is necessary answer the following question: Is the NPV the same if we have no technical uncertainty on q and B and the case with technical uncertainty and using E[q] and E[B]? The answer is negative and the reason is given below.

First, with information on the reserve size B we can fit the optimal investment D to its size B. It is partially performed in this model because the investment cost D is a function of the reserve B and the optional nature of development. For example, knowing that B is *lower than expected*, it is possible reduce according the investment D. So, it is possible to exercise option with positive NPV in some scenarios of B, which could be negative NPV if using investment D higher than the necessary.

There are some *sources of value* for the investment in information. The best fit of D with B and the asymmetry caused by the optimal option exercise is the one source[29]. However this is not all, there are other losses in NPV function due the lack of information. The value of V depends on q, and it

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of P. However, *for development decisions*, most people from oil industry model B and P as independent variables. Of course is possible to set correlation between q, B, P, at cost of complexity, using concepts of revelation and real options.

[28] If we forget that the rational stockholders of the firm are diversified investors, rejecting investment opportunities with positive NPV because the risk-aversion of managers for these kind of uncertainties (as argued by traditional *decision analysis*), we could be destroying value of the stockholders. Agency conflicts between managers and stockholders are largely reported in finance literature, and they can have very different risk preferences, see for example Byrd & Parrino & Oritsch (1998) for the problems of "differential risk preference" and "different horizon for investment results".

[29] In other words, the knowledge that the reserve size is lower than we estimate before permits to reduce the investment D, exercising options with positive NPV (that could be negative with over-investment).

depends of technical factors. The quality q is related with the present value of net revenues, that is, with the *speed that the reserves are extracted* and sold in the market.

If the capital in place is under-dimensioned for the reserve size (B is revealed higher than expected), then the capacity limitation[30] reduces the associated value of q. This occurs because even if all the reserve volume B can be extracted with the under-dimensioned capital D, the present value of reserve (the product q P B) is lower due the slower extraction schedule. This limitation suggests a *penalty factor* for the quality q for the cases where the reserve size reveals higher than expected[31]. The same reasoning is possible if the average well productivity is higher than expected (the revealed higher productivity cannot be plenty transformed in value because the under-dimensioned capacity in place). Let us call this penalty factor of $\boldsymbol{\gamma_{up}}$, which is defined in the interval (0, 1]. This factor penalizes the value of the developed reserve V if $\mathbf{q\ B > E[q\ B]}$.

We could define another penalty factor $\boldsymbol{\gamma_{down}}$ for the cases where the capital in place D reveals over-dimensioned for the reserve size and/or for the average well productivity. However, some empirical studies have been showing this value is near one, and in some special cases can be even higher than one. This occurs because if the reserve B is lower than expected, then an over-investment in capacity can permit a little bit higher production-peak when compared with the optimized process plant[32]. This can makes the value of V a little bit higher, although the NPV remains lower than the optimized case (with full information) because the cost D is higher than the necessary (an apparent positive NPV can reveal an ex-post negative NPV). However, even in this case the factor γ_{down} can be lower than 1 because the location of the wells could not be optimal and even with more wells than necessary the speed of extraction can be slightly lower than the optimal wells grid with full information. These offsetting effects explain why empirical studies points $\gamma_{down} \approx 1$.

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[30] Limitation of processing facilities in the platform, limitation of the pipeline capacity, limitation of the number and position of the wells, limitation of water injection system, etc.

[31] This penalty factor is not derived from "risk aversion of managers on technical uncertainty " or "manager's utility " as used in traditional decision analysis literature. The penalty factors will be derived from discounted cash flow analysis of value lost due the reserves production with limited capacity system and sub-optimal location for the wells. The method presented in this paper is simple, but more sophisticated methods can be used for the penalty factors.

[32] It is optimal to make some limitation in the capacity, by design a *plateau of production* peak for 2 to 4 years instead designing a more expensive higher capacity to meet a maximum production for a single year. See for example Ekern (1988) for the typical production profile in this kind of projects.

In general, the NPV function with remaining technical uncertainties can be *estimate* with a Monte Carlo simulation of the distributions of q and B, by using the following equations[33]:

$$\mathbf{NPV = q\,P\,B - D(B)} \qquad \text{if} \quad \mathbf{q\,B = E[q\,B]} \qquad \mathbf{(7a)}$$

$$\mathbf{NPV = q\,P\,B\,\gamma_{up} - D(B)} \qquad \text{if} \quad \mathbf{q\,B > E[q\,B]} \qquad \mathbf{(7b)}$$

$$\mathbf{NPV = q\,P\,B\,\gamma_{down} - D(B)} \qquad \text{if} \quad \mathbf{q\,B < E[q\,B]} \qquad \mathbf{(7c)}$$

To apply these equations, the remaining issue is how to estimate the value of the penalty factors. It can be done using discounted cash flow analysis by fixing the investment (capacity) and calculating the present value of the net revenues (that is, calculating V) for different scenarios for q and B.

A practical way to estimate γ is by performing this analysis for a few representative scenarios of the technical distribution and assuming that the penalty factors changes only with the variance of the technical uncertainty. For the partial revelation case (even with the information there is a remaining technical uncertainty) the penalty factor must be updated to a value between the previous value of γ (without information) and the penalty factor for the full revelation case (for the full revelation, $\gamma = 1$). If we build some rule of update for γ such us "the difference between γ and 1 is proportional to the remaining variance of technical uncertainty", it is easy to update the value of γ without the necessity of running additional DCF analysis[34] for different levels of partial revelation.

By using the above practical updating procedure for γ, we need only to estimate the penalty factors for the initial technical uncertainty (before the investment in information). One suggestion is to reduce the technical uncertainty distribution into three scenarios: *upside*, *expected*, and *downside*[35].

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[33] Of course, considering the independence between q and B, it is possible to simplify writing E[q B] = E[q] E[B].

[34] But is possible if a better precision is required in the model. The simplification reduces the number of DCF rounds.

[35] The upside scenario can be a subjective representative optimistic scenario or, even better, the *expected value of the upside distribution* obtained by truncating the original distribution at the original expected value.

By running the DCF analysis for these scenarios with and without full information, with D projected for the expected case, we have typically the values[36] of V displayed in the Figure 5.

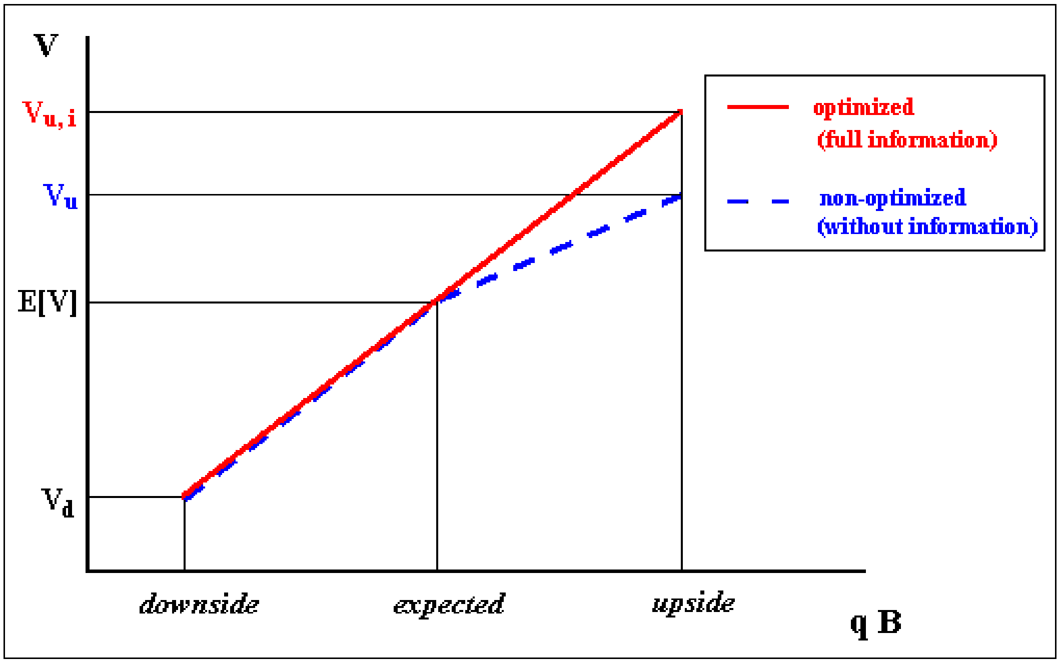

Figure 5 - Lack of Optimization Analysis to Set the Penalty Factor

The picture above illustrates a case with $\gamma_{down} = 1$ (the value V_d is the same for the cases with and without information[37]). The factor γ_{up} is given simply by:

$$\gamma_{up} = V_u / V_{u,\,i} \tag{8}$$

In words, the factor γ_{up} is the relation of the reserve value without information with the reserve value with full information for the upside scenario. This factor depends also on the flexibility embedded into the development plan. In case of using a development system with an option to expand, the value γ_{up} can be designed nearer of 1 than in case of a production system without flexibility (see Dias, 2001b, for the option to expand case).

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[36] The value V in each scenario s is simply $V_s = NPV_s - D$. Remember that D is in present value and the NPV_s are obtained by running three times the DCF spreadsheet, using different production profiles (constrained in the upside scenario) and operational costs, but the same investment.

[37] However, the NPV with information generally is higher than the NPV without information for the downside scenario because the revelation of a lower value for B permits that the investment D be reduced.

4 - The Real Options Framework with Monte Carlo Simulation

First, assume that the long-run expected oil price follows the popular Geometric Brownian Motion (GBM)[38]. The *risk-neutralized* format for this stochastic process is obtained using the risk-neutral drift $(r - \delta)$ instead the real drift α.

$$\mathbf{dP = (r - \delta)\, P\, dt + \sigma\, P\, dz} \qquad \textbf{(9)}$$

Where:

r = interest rate, assumed to be 6% p.a.;

δ = convenience yield of the oil, assumed to be 6% p.a., too;

σ = volatility of the long-run oil price, assumed to be 20% p.a.; and

dz = Wiener increment = $\boldsymbol{\varepsilon \sqrt{dt}}$, where $\varepsilon \sim N(0, 1)$

The equation necessary to perform the Monte Carlo simulation of petroleum price sample paths is:

$$\mathbf{P_t = P_{t-1} \exp\{ (r - \delta - 0.5\, \sigma^2)\, \Delta t + \sigma\, N(0, 1) \sqrt{\Delta t}\, \}} \qquad \textbf{(10)}$$

With the simulated oil price P(t) is possible to estimate the value of a developed reserve given by V(t) = q B P(t). The simulation equation above is exact (doesn't need small time step).

The development threshold gives the *decision rule for the optimal development*. In order to ease the model, is better to work with normalized value of the reserve V/D. For V/D = 1, the NPV is zero. The use of normalized value of the developed reserve V/D, instead for example the oil price, for decision rule curve permits to combine the technical and market, that is values of P, q, B, D(B), into the same threshold curve. Note that without the normalization, after a revelation of the reserve size B, the exercise price of the development option changes because D is function of B and so the threshold curve making much more time consuming the evaluation of the Monte Carlo simulation.

This normalization is possible because the real option value F is homogeneous of degree one[39] in V and D, that is, F(V/D, D/D) = (1/D) F(V, D), and it permits to use V/D in the maximization problem.

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[38] Analyzing the case of petroleum real assets, Pindyck (1999) concludes that for applications like real options "*the GBM assumption is unlikely to lead to large errors in the optimal investment rule*". This conclusion is reaffirmed in his more recent study (Pindyck, 2001). See Dias (2001b) for a discussion of different stochastic processes for oil prices.

[39] A function is homogeneous of degree n in **x** if $F(t\mathbf{x}) = t^n F(\mathbf{x})$ for all $t > 0$, $n \in \mathbf{Z}$, and **x** is a vector of variables.

The threshold (V/D)* is homogeneous of degree zero in V and D, see McDonald & Siegel (1986, p.713) and Dixit & Pindyck (1994, pp.207-211). This trick was proved only for the Geometric Brownian Motion case, and extensions to other stochastic processes are a research matter.

The normalized threshold is the critical (V/D)* level that makes optimal the immediate investment to develop the oilfield. It is the decision rule to exercise the option (exercise at or above the threshold), which maximizes the real options value. The optimal exercise curve (V/D)* of real option is function of the time and, for the GBM, depends only of σ, r, and δ. The threshold is obtained from the stochastic optimal control literature, the earlier exercise curve for American options (*free boundary*).

Figure 6 shows the how the risk-neutral simulation of oil prices is combined with the revelation distributions for the technical uncertainties. This occurs at the "revelation time", when the information becomes knowledge on the reservoir properties, changing the expectation on the reserve size and quality. So, the model considers the issue of "*time to learn*", penalizing the alternatives that takes much time to reveal the information/knowledge. Figure 6 shows also the normalized threshold curve considering two years for the expiration of the development rights, and how the simulated sample paths are evaluated in this real options model.

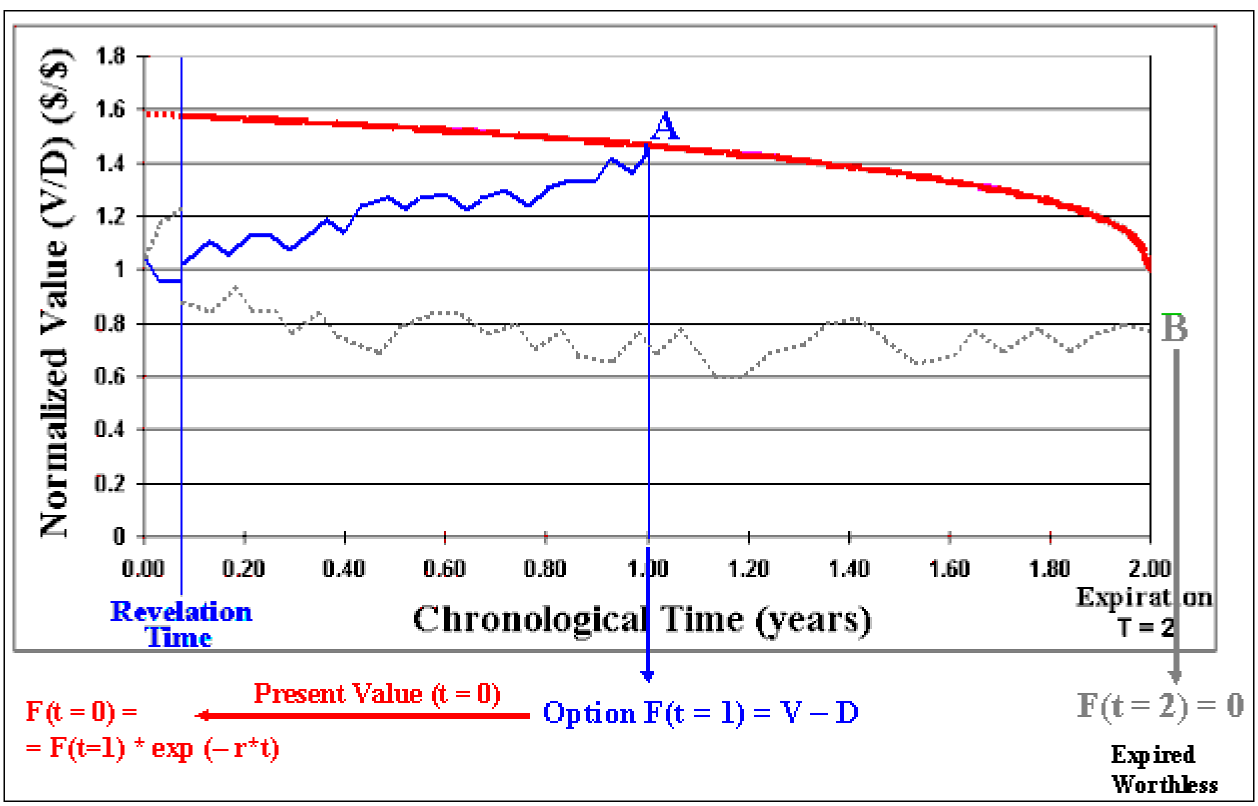

Figure 6 - The Sample Paths, Revelation Jumps, and Optimal Development Threshold

There are two sample paths in the picture. In the first (blue, almost continuous line) the normalized value V/D evolves randomly due the risk-neutral simulation of the oil prices component. At the revelation time are drawn one sample from each revelation distribution (q and B) and the normalized value of reserve jumps (in this case jumps-up) almost surely, reflecting the new expectation on these technical parameters in this path. The value V/D continues its random trajectory, now due only the (long-run) oil price oscillations, and in this case reaches the threshold curve at the point A (see the picture). The option is optimally exercised and multiplied by the risk-free discount rate factor.

The second path (gray dashed line) evolves similarly but after suffering a jump-down from the samples of revelation distributions, the path evolves until the expiration without reaching the threshold line and expires worthless (point B). The value of option for this path is zero (or negative, considering the cost of information). After thousands of simulated paths, we get an estimate of the real options value by summing the value of the options F from the sample paths and dividing by the number of simulations.

The ***dynamic net value of information*** is the difference between two real options value. The real option value with investment in information (using the revelation distribution) and the real options without investing in additional information. Let us see some case studies.

5 - Case Studies and Numerical Results

Before presenting the examples, let us see the model outputs that we are interested:

- NPV without Technical Uncertainty: This is the NPV calculated with the Equation 6 and using the current expectations on q, B and P;
- Real Options without Technical Uncertainty: This is the traditional real options value considering only the uncertainty on the oil prices and using the current expectations on q and B;
- Simulated NPV with Technical Uncertainty (with γ_{up}): The NPV function is calculated by simulation considering the penalty factor γ_{up} (Equations 7a, 7b, 7c), and so it is lower than the NPV without technical uncertainty;
- Simulated Real Options without Information: The real options payoff is penalized by the factor γ_{up}, and so its is lower than the traditional real options value (without technical uncertainty);

- Simulated Real Options with Technical Uncertainty and with Information Revelation: The real options model considers one alternative of investment in information. This value is different for each alternative of investment in information. This value is net of the information cost, and considers the *time to learn*. This value or the next select the best alternative (the higher one); and
- Dynamic Net Value of Information: It is the difference between the real options with and without the investment in information. This value is different for each alternative of investment in information. This value (or the previous) selects the best alternative (the higher one)[40].

Let us consider two oilfields cases, with two alternatives of investment in information for each case. What is the better alternative in each case? Is the investment in information better than the not investing in information?

Consider first the **Oilfield 1**. The technical uncertainties for q (in %) and B (in million barrels) are modeled with triangular distributions (minimum; most likely; maximum):

B ~ Triang (300; 600; 900)
q ~ Triang (8%; 15%; 22%)
The gamma factor (γ_{up}) is 75%.

Alternative 1 is the less expensive one and consists in drilling one vertical well. The learning cost is US$ 10 million and takes 45 days to get the information and knowledge on q and B. The revelation power (percentage of reduction of variance) for B is 50%, whereas for q is 40%.

Alternative 2 consists in drilling one *horizontal* well. The cost is US$ 15 million and takes 60 days to get the information (knowledge on q and B). The revelation power (percentage of reduction of variance) for B is 75%, whereas for q is 60%.

The development cost function is: $D(B) = 310 + (2.1 \times B)$

The results for the Oilfield 1 are given in the Table 1[41].

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[40] Because it is the difference of two simulated values, the error can be larger than the previous indicator.

[41] Used 10,000 iterations for Alternative 1 and 100,000 for Alternative 2, with a hybrid quasi-Monte Carlo simulation (it is more efficient than traditional Monte Carlo) see http://www.puc-rio.br/marco.ind/quasi_mc.html. The estimated errors of the simulation were lower than – 0.3% for both alternatives. The computational time using Pentium III, 1 GHz, were less than 2 minutes for 10,000 iterations and about 16 minutes for 100,000 iterations, with Excel spreadsheet.

Table 1 - Real Options Results for Oilfield 1

Alternatives	**Alternative 1**	**Alternative 2**
(1) NPV without Technical Uncertainty	230	230
(2) Real Options without Technical Uncertainty	302.1	302.1
(3) Simulated NPV with Technical Uncertainty (with γ_{up})	178.6	178.9
(4) Simulated Real Options without Information	267.9	263.3
(5) Simulated Real Options with Technical Uncertainty and with Information Revelation from Alternatives	298.4	307.0
(6) Dynamic Net Value of Information [(5) – (4)]	30.4	43.7

There are some values in the table that are different only due the simulation error (rows 3 and 4). The last two rows in the table present values net of cost of information. By looking the row 5 or 6, we conclude that the Alternative 2 is the best even being 50% more expensive. It is recommended to increase the number of simulations in case of smaller differences between the alternatives.

Now consider the **Oilfield 2**. This second case study was presented in Souza Jr. & Dias & Maciel (2002). The development cost function is the same of Oilfield 1 given by D(B) = 310 + (2.1 x B). The technical uncertainties for q (in %) and B (in million barrels) are also modeled with triangular distributions (minimum; most likely; maximum):

B ~ Triang (145; 320; 560)
q ~ Triang (6%; 15%; 25%)
The gamma factor (γ_{up}) is 65%.

<u>Alternative 1</u> is the less expensive one and consists in drilling one vertical well, but without performing production test. It costs US$ 6 million and takes 35 days to get the information and knowledge on q and B. The revelation power (percentage of reduction of variance) for B is 75%, whereas for q is 60%.

<u>Alternative 2</u> consists in drilling one vertical well, but this time performing a production test. It costs US$ 12 million and takes 65 days to get the information and knowledge on q and B. The revelation power (percentage of reduction of variance) for B is 80%, whereas for q is 70%.

The results for the Oilfield 2 are given in the Table 2.

Table 2 - Real Options Results for Oilfield 2

Alternatives	**Alternative 1**	**Alternative 2**
(1) NPV without Technical Uncertainty	20.3	20.3
(2) Real Options without Technical Uncertainty	116.2	116.2
(3) Simulated NPV with Technical Uncertainty (with γ_{up})	– 32.5	– 33.1
(4) Simulated Real Options without Information	87.8	86.6
(5) Simulated Real Options with Technical Uncertainty and with Information Revelation from Alternatives	128.3	126.6
(6) Dynamic Net Value of Information [(5) – (4)]	40.5	39.9

In the case of Oilfield 2, the less expensive Alternative 1 is the best one. However, the difference is too small, so that is recommended another simulation with a higher number of iterations. In addition, we can revaluate the production test just after the drilling. But it requires quick evaluation of the new expectations of reserve size and productivity, because the deepwater rig is very expensive.

6 - Extensions

6.1 - Timing Issue for the Investment in Information

Investment in information is expensive (e.g., drilling offshore well demands an amount ranging from US$ 4 million to US$ 20 million) and reveals only partial information on the size and the quality of the reserve. However, the investment cost for the oilfield development in general is much higher than the cost to acquire additional information (an offshore development typically requires more than US$ 1 billion). So, the development cost is typically about 100 times the cost of learning!

Hence, for the petroleum case, the issue of optimal timing of learning is not so relevant as the issue of optimal timing of development. The postponement of the investment in information has the benefit to delay a cost, but it has the disadvantage of delay the exercise of possible "deep-in-the-money" projects (in case of revelation of good news)[42]. Even a neutral revelation (imagine the expectations remain the same after the revelation), the real options value is improved with the revelation because the penalty factor on the NPV function is lower after the new information.

[42] In addition, the incremental value of appraisal is higher for out-of-money projects than for in-the-money projects (see Whiteside & Drown & Levy, 2001, p.6).

Let us reexamine the case study presented before, if we delay six months and one year the investment in information. The present value of investment in information is reduced with the discount factor[43], but the development option exercise is allowed only after this delay (6 months or one year) plus the time to learn. The results for the case of oilfield 1 are given in the Table 3.

Table 3 - Simulated Real Options with Technical Uncertainty and with Information Revelation for Oilfield 1 - Immediate Learning versus Learning Delay

	Alternative 1	**Alternative 2**
Real Option without Learning	267.9 (a)	263.3 (b)
Real Option with Immediate Learning	298.4 (a)	307.0 (b)
Real Option with Learning Delay of 6 Months	293.9 (c)	305.9 (d)
Real Option with Learning Delay of 1 Year	291.2 (e)	299.7 (f)

Order of simulation errors[44]: (a) – 0.26% (b) – 0.29% (c) – 0.03% (d) – 0.16% (e) – 0.36% (f) – 0.43%.

The immediate learning is better for the Alternative 1 and slightly better for Alternative 2. Remember that Alternative 2 is more expensive than Alternative 1. The option to delay learning can be of some importance for cases of high cost of learning, lower power of revelation, lower penalization in the NPV function (γ_{up} near 1), and real options "out-of-money".

6.2 - Valuation of Sequential Investment in Information

The valuation of an entire appraisal plan, with two or more *appraisal wells* (for offshore oilfields, generally one to three sequential drilling), is a typical case encountered in the upstream oil industry. The economic value for the last wells are less obvious. Let us return to the stylized example from Section 2, the rectangular oilfield where each well can be success or dry hole. What happens if after drilling the first well "B", the drilling of the second well "C" is conditional to the success scenario for the first well? This *path-dependence* can be important in the investment in information analysis. When comparing the parallel drilling of the wells "B" and "C" (Alternative 2 from Section 2) with the sequential drilling, both have the same revelation distribution after the two wells, see Figure 7.

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[43] The discount factor is e^{-rt}. For r = 6%, the discount factors are 0.970 and 0.942, respectively for 6 months and 1 year.

[44] These values are for 10,000 simulations, except items (b) and (d), which were used 100,000 simulations due the close values. The error is only an estimate because compares the *traditional* real options value resulted from the simulation with the efficient analytic *approximation* for American call option of Bjerksund & Stensland (see Haug, 1998, pp.26-29).

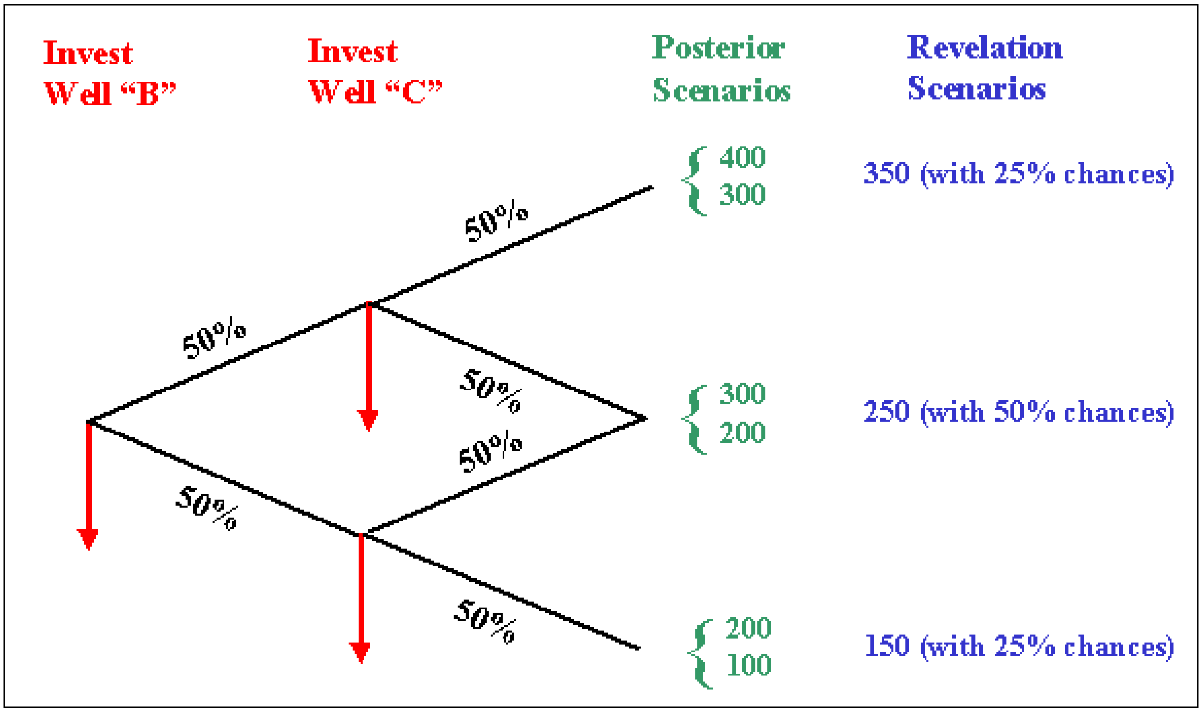

Figure 7 - Two Sequential Investments in Information and the Resultant Revelation Distribution

However, the sequential drilling alternative has an additional option of not drilling the second well in case of first bad news, that is, the option to abandon the appraisal plan. So, the sequential drilling is more valuable than the parallel drilling[45]. The parallel drilling is calculated as a single shot investment in information, the alternative 2 estimated in section 2. So, that alternative 2 value is a lower bound for the value of the alternative of sequential investment in information.

In order to simulate adequately the path-dependence nature of sequential investment in information, the revealed scenario by the first well becomes the new expectation for the second revelation distribution. By definition, the revealed scenario is the new expectation for the technical parameter (reserve size), so by Proposition 2 the revelation distribution at this point has mean equal to the current revealed scenario. This insight is useful for the Monte Carlo simulation of sequential investment in information, in order to consider the path-dependence of sequential investment with options that can be exercised along the path. The forward-looking path needs to be evaluated backwards in order to consider the possibility of options exercise between the revelations (option to abandon the appraisal plan, option to delay the second investment in information, and even the

[45] See Dias (1997) for a numerical example of value added with the option to abandon the sequential appraisal plan.

option to develop the oilfield without gather more information). These more complex issues will be object of another paper, but note that after this paper approach is valid to evaluate the *last* revelation.

In addition, when no option is exercised after the first revelation (so that there is a new revelation in *every* scenario of the first revelation), the distribution of revelation after two sequential investments in information has the same expected value of the first revelation distribution. See in the last picture the revelation distribution after two sequential investments that this expectation is 250, that is the same of the revelation distribution for the Alternative 1 (single revelation, see section 2). So, the ex-ante expected value of revelation distributions for sequential investments in information, are all the same. This is a very general result and led us to the following proposition.

Proposition 4: The sequential revelation distributions $\{R_{X,1}, R_{X,2}, R_{X,3}, \ldots\}$ are (event-driven) martingales[46] (proof: see appendix). In short, ex-ante these random variables have the same mean[47].

You can think martingale as a "fair" game. Let K_n be capital of a gambler after the bet n and if all bets are fair in the sense that result in zero expected gain, then for $n \geq 0$, K_n are martingales. Think bets as investments in information and K_n as the (conditional) expectation of a technical parameter after the n "bets". This property is useful for sequential investment in information because there is a well-developed theory of *optimal stopping for martingales*. A bit more complex case is when we allow the earlier development option exercise *between* two revelations[48] (or two planned investments in information). In other words, consider the option to abandon the sequential investment in information because is optimal the exercise of the option to develop the oilfield with the current accumulated *level of revelation*. In this case there is a free-boundary threshold for optimal development *between two revelations*. This more complex case was studied in a real case for the fiscal regime of production sharing (see next sub-section) and will be presented in another paper.

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[46] For a discussion on martingales, see the didactic book of Williams (1991).

[47] Although in the Monte Carlo simulation the second revelation distribution uses as mean the scenario revealed by the first revelation (path-dependence), the combined distribution resultant from the two revelations has the mean equal the mean of the first revelation distribution.

[48] For example, the first revelation occurs after the drilling of the wells from a Pilot Production System (small system) and the second revelation occurs after two years of pilot production when we can learn about the aquifer performance as primary reservoir inflow mechanisms. Imagine the oilfield become "deep-in-the-money" oilfield after the first revelation.

6.3 - The Fiscal Regime of Production Sharing

The main fiscal regimes in petroleum upstream are the concessions system and the production sharing one. The production-sharing regime has two main phases. The first one is named *cost recovering*, so that the revenues net of operational cost from the first years are destined to the oil companies, in order to recover the amount invested in the petroleum field (in general considering an interest like Libor plus x%). In this phase, the *Government Take* (GT) is inexistent or very small. The second phase, named *profit phase*, the revenues net of operational cost are destined to both Govern (larger part) and oil companies. It seems like two different regimes, if the project (ex-post) has no profit, the GT is zero or very small[49], whereas for ex-post profitable projects the GT is significant.

Figure 8, the typical chart NPV x P for Production Sharing Regime, illustrates these two different phases. Remember that for the fiscal regime of concession, this chart has a straight line.

Figure 8 - The NPV x P Function for the Fiscal Regime of Production Sharing

[49] For the concessions regime, profit or non-profit project is an oil company problem. Even with negative NPV, the oil companies pay royalties, income tax (if the company is profitable, doesn't matter the project), and other taxes.

If the oil price is under \$ 15, the project has lower fiscal charge because it stays in the cost recovering phase[50], but for higher prices the fiscal charge is heavier because the project reaches the profit phase. This case is a bit more complex to simulate (the simple equation 6 is not useful here).

7 - Conclusions

The paper presents a dynamic approach to combine the technical uncertainties with market uncertainties using a Monte Carlo simulation. The contributions of the paper are the concept of revelation distribution to work with technical uncertainties and its insertion in the real options model. As the volatility in traditional real models, the revelation distribution variance adds value to the real option. As higher is the revelation power of one alternative of investment in information, as higher is the revelation distribution variance.

The approach presented simplifies the implementation of real options model considering costly learning because the technical expert has to estimate only the initial (prior) distribution and, for each relevant alternative of investment in information, the expected percentage of reduction of variance. By using a simple parameterized NPV function, it is possible to get faster results using a simple spreadsheet with Monte Carlo simulation facility. The simple equation adopted is based in a business vision on the quality of reserve, and considers other uncertain key parameters like the reserve volume and the oil price. In addition, the exercise price of the option (the development investment) changes with the revealed size of reserve, including this realistic aspect of the investment decision.

The case studies presented illustrated the applicability of the methodology in practical problems, including a factor due the sub-optimal development due the incomplete information. The methodology permits to select the best alternative of investment in information from a relevant set, because it considers the cost of learning, the time to learn, and the revelation power, for each alternative of investment in information. Some extensions were briefly analyzed, such us the timing of investment in information, and the sequential (optional) investment in information. An in-depth analysis of these extensions is left to a future work.

[50] This change of regimes is not at the NPV = 0 level because the discount rate for NPV is different of the discount rate used by the National Agency to reward the investment for cost recovery rule purposes.

APPENDIX

Definition of Conditional (on a σ-Algebra) Expectation

Some definitions are possible depending if the conditioning is on an event, discrete random variable or an arbitrary random variable. Is presented the more general case. Let X be an *integrable*[51] random variable mapping the probability space (Ω, Σ, P) into a measurable space, where Ω is the sample space (set of all possible outcomes ω), Σ is the *sigma-algebra*[52] and P the probability measure. Let Ψ be a *sub-sigma-algebra* of Σ (that is, $\Psi \subseteq \Sigma$)[53]. The conditional expectation of X given Ψ, $\mathbf{E[X \mid \Psi]}$ is a Ψ-measurable function[54] that satisfies the equation below for every $Y \in \Psi$:

$$\int_Y E[X \mid \Psi](\omega)\, dP(\omega) = \int_Y X(\omega)\, dP(\omega)$$

Definition of Conditional (on a σ-Algebra) Variance:

Let the random variable X have a conditional expectation $E[X \mid \Psi]$ with respect to the *sigma-algebra* Ψ of subsets of the sample space Ω. The *conditional variance of X* is a random variable defined by[55]:

$$\mathbf{Var(X \mid \Psi) = E\{(X - E[X \mid \Psi])^2 \mid \Psi\}}$$

Where Ψ is a sub-sigma-algebra of Σ, the sigma-algebra from the (unconditional) probability space, the triple (Ω, Σ, P).

Conditional Expectation Properties & Miscellaneous

Let $\mathbf{X_1}$ and $\mathbf{X_2}$ random variables, and ψ the possible outcomes from the information revelation[56].

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[51] A function f that is μ-integrable is written $f \in \boldsymbol{L}^1(\Omega, \Sigma, \mu)$. So, it is assumed that $X \in \boldsymbol{L}^1$ (where *L* is from Lebesgue).

[52] Sigma-algebra Σ on Ω is a family of events E (subsets of Ω), including the empty set, complements of sets that belong to Σ and countable union of sequence of sets $E_n \in \Sigma$.

[53] See Shyraiev (1996, p.212) or Williams (1991, p.84).

[54] See for example Williams (1991, p.29-30) for the definition of Ψ-measurable functions in a measurable space (Ω, Ψ).

[55] See for example Shyraiev, 1996, p.214.

Linearity property: $\mathbf{E[(a\, X_1 \; + \; b\, X_2) \mid \psi\,] \quad = \quad a\, E[\, X_1 \mid \psi\,] \; + \; b\, E[\, X_2 \mid \psi\,]}$

Jensen Inequality: If **g(.)** is a convex function, $\mathbf{g\{\, E[\, X \mid \psi\,]\,\} \quad \leq \quad E[\, g(X) \mid \psi\,]}$

Joint Probability Density: $\mathbf{f(x, \psi) \quad = \quad f_{\Psi}(\psi)\;\; f(x|\psi)}$

In the petroleum problem, $f_{\Psi}(\psi)$ is the density of the outcomes from the investment in information (e.g., drilling reveals data on net-pay h, area A and productivity index PI), whereas $f(x|\psi)$ is the density of X (e.g., X is the size of the reserve) conditional on the information revelation.

Existence of Expectation for Revelation Distribution and the Proof of Proposition 2

Assume that X has finite expectation (is "integrable"), and I is the conditioning new information both in the probability space (Ω, Σ, P). Hence the conditional expectations $R_X(i) = E[X \mid I = i]$ exists and is finite almost surely (as). This is a consequence of Radon-Nikodým Theorem, see for example James (1996, p.176) and Kolmogorov (1933, p.53). See Kolmogorov (1933, p.40) to understand why $E[\,|X|\,] < \infty$ is the necessary and sufficient condition for the existence of E[X].

Let X be a random variable with $E[\,|X|\,] < \infty$. Let Ψ be a sub-sigma-algebra of Σ. Then exists a random variable R_X such that R_X is Ψ measurable and its expectation also exists[57]. The proofs that if $E[\,|X|\,] < \infty \Rightarrow E[\,|\, R_X \,|\,] < \infty$ (and hence the existence of revelation distribution expectation) and R_X is Ψ measurable, are given in Williams (1991, pp.85-86).

The Proposition 2 can be formulated as: if R_X is any *version*[58] of $E[X \mid \Psi]$ then $E[R_X] = E[X]$. For a more general proof, see Williams (1991, p.89). The proof below uses a simpler approach without concepts from measure theory. First let us prove for the case when the parameter with technical uncertainty X and the conditioning information I are discrete random variables (the proof follows Ross, 1998, p.338). In this discrete case the Proposition 2 becomes (P{.} means probability):

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[56] See for example James (1996, p. 177) or Williams (1991, p.88) for the next two properties.

[57] R_X is also the Hilbert space projection of X on the closed linear subspace $\boldsymbol{L}^2(\Omega, \Psi, P)$ of $\boldsymbol{L}^2(\Omega, \Sigma, P)$ and hence the conditional expectation does exist (see Jacod & Protter, 2000, p.196).

[58] See Williams (1991, p.84) for the definition of version. If R^*_X is a version of R_X, we have $R^*_X = R_X$ almost surely.

$$\sum_i R_X(i)\,P\{I=i\} = E[X]$$

The left side is the expectation of R_X by definition. By definition of R_X the left side can be written:

$$E[R_X] = \sum_i \mathbf{R_X(\mathit{i})\,P\{I=i\}} = \sum_i \sum_x x\,P\{X=x\,|\,I=i\}\,P\{I=i\} =$$

$$E[R_X] = \sum_i \sum_x x \frac{P\{X=x,\ I=i\}}{P\{I=i\}}\,P\{I=i\} = \sum_i \sum_x x\,P\{X=x,\ I=i\} =$$

$$\mathbf{E[R_X] = \sum_x x \sum_i P\{X=x,\ I=i\} = \sum_x x\,P\{X=x\} = E[X]} \quad \text{(and Proposition 2 is proved)}$$

Now the proof for the continuous case (following James, 1996, p.176), when X and I have joint probability density f(x, i) and the conditional density is $f(x|i) = f(x, i)/f_I(i)$, being $f_I(i) > 0$.

$$R_X(i) = \int x\,dF_X(x\,|\,I=i) = \int_{-\infty}^{\infty} x\,f(x\,|\,i)\,dx = \int_{-\infty}^{\infty} x\,\frac{f(x,i)}{f_I(i)}\,dx \quad \text{, so if X is integrable follows:}$$

$$E[R_X] = \int R_X(i)\,dF_I(i) = \int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty} x\,\frac{f(x,i)}{f_I(i)}\,dx\right) f_I(i)\,di = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\,f(x,i)\,dx\,di =$$

$$\mathbf{E[R_X] = \int_{-\infty}^{\infty} x\left(\int_{-\infty}^{\infty} f(x,i)\,di\right)dx = \int_{-\infty}^{\infty} x\,f_X(x)\,dx = E[X]} \quad \text{(and Proposition 2 is proved)}$$

Proof of Equation of the Variance of Revelation Distribution (Proposition 3)[59]

Let $R_X = E[X \mid I]$ be the random variable with probability distribution named revelation distribution. We know that the conditional variance of X given the information I = i, is defined by:

$$\mathrm{Var}(X\,|\,I) = E[(X - E[X\,|\,I])^2\,|\,I\,]$$

A very known equation for the variance of a random variable Y is $Var[Y] = E[Y^2] - (E[Y])^2$. So[60]:

[59] The proof is also the solution of the problem 2, Shyraiev, 1996, p.83, and is similar to Ross (1998, p.348).

$\mathrm{Var}(R_X) = \mathrm{Var}(E[X \mid I]) = E[(E[X \mid I])^2] - (E[E[X \mid I]])^2 = E[(E[X \mid I])^2] - (E[X])^2$ (*)

By using the same known equation for $\mathrm{Var}(X \mid I)$, we obtain:

$\mathrm{Var}(X \mid I) = E[X^2 \mid I] - (E[X \mid I])^2$. By taking the expectations, we have:

$E[\mathrm{Var}(X \mid I)] = E[E[X^2 \mid I]] - E[(E[X \mid I])^2] = E[X^2] - E[(E[X \mid I])^2]$ (**)

By summing (*) and (**) and rearranging, we complete the proof:

$\mathrm{Var}(E[X \mid I]) = \mathbf{Var(R_X)} = \mathbf{Var(X) - E[Var(X \mid I)]}$

Proof for the Proposition 4 (Sequential Revelation Variables Are Martingales)

The sequential revelation distributions $\{R_{X,1}, R_{X,2}, R_{X,3}, \ldots\}$ are (event-driven) martingales[61].

Proof[62]: A martingale must meet three conditions (see Williams, 1991, p.94). The first condition is a consequence of the assumption that the technical parameter with uncertain, X, is integrable and from the Radon-Nikodým Theorem we must have $E[\,|R_X|\,] < \infty$ almost surely (existence of revelation distribution expectation, mentioned before). The second condition, the revelation process is adapted to a filtration $\{\Im_n: n \geq 0\}$, that is, an increasing family of sub-sigma-algebras of Ψ that is, ($\Im_0 \subseteq \Im_1 \subseteq \Im_2 \subseteq \ldots \subseteq \Psi$). The third condition is that $E[R_{X,n} \mid \Im_{n-1}] = R_{X,n-1}$ almost surely. In order to proof this, let us first set the *Tower Property* (see Williams, 1991, p.88). If Υ is a sub-sigma-algebra of Ψ, then almost surely we have $E[R_X \mid \Upsilon]$ (= $E[E[X \mid \Psi] \mid \Upsilon]$) = $E[X \mid \Upsilon]$. This property is immediate from the definition of conditional expectation (Williams, 1991, p.90)[63]. Now, we follow the example of martingale from Williams (1991, p.96) called "accumulating data about a random variable". Let the variable $\xi \in \boldsymbol{L}^1(\Omega, \Psi, P)$ and define $R_n = E[\xi \mid \Im_n]$. By Tower Property we have almost surely:

$E[R_n \mid \Im_{n-1}] = E[E[\xi \mid \Im_n] \mid \Im_{n-1}] = E[\xi \mid \Im_{n-1}] = R_{n-1}$. Hence, $R_{X,n}$ are martingales.

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[60] Assume that X is square-integrable, that is, $X \in L^2$.

[61] The sequence of revelation random variables $R_{X,n}$ are called *Doob type martingale* (see Ross, 1996, p.297).

[62] For a nonmeasure theoretic proof, see Ross (1996, p.297). For any $n > 0$, $E[R_n \mid I_1, I_2, \ldots I_{n-1}] = R_{n-1}$.

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[63] Brzezniak & Zastawniak (1999, p.30) presents this "immediate" result into 4 lines.

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