Understanding the Economic Value of Legal Covenants in Investment Contracts: A Real-Options Approach to Venture Equity Contracts

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Comments Welcome

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Abstract

Valuing early-stage high-technology growth-oriented companies is a challenge to current valuation methodologies. This inability to come up with robust point estimates of value should not and does not lead to a breakdown of market liquidity: instead, efforts are redirected towards the design of investment contracts which materially skew the distribution of payoffs in favor of the venture investors. In effect, limitations in valuation abilities are addressed by designing the investment contracts as baskets of real options instead of linear payoff functions. This paper investigates four common features (covenants) of venture capital investment contracts from a real option perspective, using both analytical solutions and numerical analysis to draw inferences for a better understanding of contract features. The impact of the concept for pricing issues, valuation negotiation and for contract design are considered. It is shown for example how "contingent pre-contracting" for follow-up rounds is theoretically a better proposition than the simple "rights of first refusal" commonly found in many contracts.

1 Introduction

Valuing early-stage high-technology growth-oriented companies is a challenge to current valuation methodologies. This inability to come up with robust point estimates of value could potentially lead to a breakdown of market liquidity. This is not what is witnessed: indeed, billions of dollars of early stage venture capital has been poured onto promising start-ups in Europe and the United States over the last few years. So, how do venture capitalists cope with the valuation uncertainties?

The pioneering work of Sahlman (1990) pointed the way to the solution, or at least the "coping" mechanism used: instead of expending useless amounts of time and effort coming up with a better estimate of an inherently uncertain future, efforts are redirected towards (1) the design of investment contracts which materially skew the distribution of payoffs in favor of the venture investors, and (2) an active involvement in the development process of the invested company. In effect, limitations in valuation abilities are addressed by designing the investment contracts as baskets of real options instead of linear payoff functions and by directly intervening on the underlying processes. Appendix A summarizes the key items of a standard venture capital term sheet for an early stage deal, as presented by Linde et Al (2000).

The key items outlined by Sahlman (1990) in the relationship between venture capitalists and entrepreneurial ventures include (1) the staging of the commitment of capital; (2) the use of convertible securities instead of straight equity investments and the presence of liquidation preference for the VC; and (3) antidilution provisions to secure the investor's equity position in the company. While the recent empirical literature has explored further the presence of these features in VC contracts and analyzed theoretical arguments for their use (see Kaplan and Stromberg (2000)), no paper to date has systematically valued them, both in isolation and as a whole. We exploit the real options literature to obtain the economic value of the classical legal features of VC contracts.

Some authors have mentionned the optionality of VC contract features without developing systematically the analysis that we do here. Notice that obtaining a fair understanding of the economic value of the legal features of investment contracts should lead to a better understanding of their optimality as well. While our model includes many realistic features of current VC markets, the analysis remains very much a partial equilisbrium type of answer. A richer model would incorporate the agency issues, the adverse selection or asymmetry of information issues that lead to these contract features. The model would thus be extended to combine the continuous time analysis we provide with a complex game theoretical approach. While this has been attempted in the credit risk literature (eg Anderson and Sundaresan (1996)), this is beyond the scope of this paper where the combination of multiple complex optional features presents in itself a challenging problem. This problem is but a first step to a good and applicable economic understanding of VC contractual features.

The features we specifically address in this paper are the liquidation preference, the convertibility, the ex-ante staging and the antidilution provisions.

All these features have been shown to be common in venture capital contracts (with staging remain in general unformal rather than contractual: we analyze the strong difference this leads to) (see Sahlman (1990), Kaplan and Stromberg (2000/2002)).

We present closed-form solutions under restrictive assumptions, thus allowing for analytical experimentation. The paper also provides numerical analyses (using finite difference methods) of more general cases. We obtain results from the overall analysis. It is shown in particular how "contingent pre-contracting" for follow-up rounds(or "ex-ante staging" as described in Kaplan and Stromberg (2002)) is theoretically a better proposition than the simple "rights of first refusal" commonly found in many contracts.

The paper is organized as follows. Section 2 locates this paper at the intersection of different strands of literature on venture capital and real options. Section 3 presents the general framework and discusses the underlying assumptions to the ongoing model. The following sections 4 & 5 focus on the main contractual features such as the liquidation preference and staging. We show that the liquidation preference itself is a package of liquidation right and automatic conversion. It is important to note that the analytical and numerical solutions do take account of the complete interdependence of the features. Whereas liquidation preference sets up the framework for the VC end payoff and the conditions (event) under which it is exercised, staging aims not only at maximizing the final payoff by optimally distributing the investment amount over discrete stages but also in minimizing the downside risk given the early exit option. This subsection continues with an analysis of staging as a contractual feature (a phenomenon well known amongst practitioners and recognized in the literature). We also provide intuitive closed form solutions as well as a complete numerical analysis of this feature.

Section 6 bears on the most common antidilution feature of venture capital contracts. It carefully analyses the impact of the issuing price on the VC ownership stake. One of the main conclusions of this section regards the concave shape of the VC ownership stake. The section carries also a sensitivity analysis of this feature to some of the model parameters.

Section 7 integrates these features in a contract valuation and carries a large sensitivity analysis. The complete model is extended to include for possibility of an IPO with underpricing, long term effects and different lock up periods. The last section concludes.

2 Literature Review

This paper lies at the crossroad of two literatures, the venture capital literature and the real options literature in order to break new ground and bring them together in a way not attempted before for private equity investment contracts.

The current literature on venture capital can be subdivided into three main classes, depending on whether it addresses the theoretical optimality of contracts, empirically analyses the existing contracts used in practice or values the investments underlying the contracts (notably with real-options techniques).

A rich, mostly game-theoretic literature addresses the agency and moral hazard problems arising in a multi-stage financial contracting environment with information asymmetries (Dessi (2001), Casamatta (2001), Aghion, Bolton and Tirole (2000), Bergmann and Hege (1998), Bergmann and Hege (2000), Admati and Pfleiderer (1994), Baldwin (1982), Noldeke and Schmidt(1998), Bascha and Walz (2001), Bascha (2001)). While this literature would have much to bring to this paper in terms of optimality (and vice versa), we have simplified the issue and taken actual contracts as given. Note that this first step should be a necessary condition for a full development of optimality argument in the future, although it faces large analytical hurdles. In other words, having a valid valuation framework for contract features should lead to new optimality results (notably as far as interaction of the features go).

The second class of research includes empirical studies that analyze the specificities of venture backed projects. (See for example Cochrane (2001), Gompers (1995), Kaplan and Stromberg (2002), Lerner (1994), Gompers and Lerner (1996), Seppa and Laamanen (2000), Manigart et al. (1999), etc.). The closest papers to our concern study specifically the contractual feature we investigate the value of and both illustrate their omnipresence as well as their relative fit to theoretical concerns.

The third class of research is closest in methodology to what we develop here, although its goal is different. The real options literature has indeed participated to a better understanding of the venture capital literature by helping the actual valuation of investments, and notably high-tech investments (Berk, Naik and Green (1998), Schwartz and Moon (2000), Schwartz and Gorostiza (2000)). Pricing American option type in terms of valuation and optimal exercise is one of the most challenging problems in derivatives finance, since one has to account for the early exercise feature. Consequently, many papers applying option models to real assets resort to the Black and Scholes approximation methodology [Benaroch and Kauffman (1999)][1], or approximate the option to a European one [Trigeorgis and Panayi (1998)]. Others refer to Magrabe's formula when modeling the investment opportunity as an exchange option with uncertain cash flows [Kumar (1996, 1999)]. Perlitz et al. (1999) applied the and Kemna's (1993) adjustment of the Geske approximation in an R&D project valuation. Jagle (1999) use a binomial model as its pricing numerical technique[2]. Schwartz and

[1] Hull 1997, pg. 252

[2] A more extended summary of these papers is provided in Schwartz and Gorostiza (2000),

Cortazar (1998) use the Barraquand and Martineau (1995) methodology to price the underdeveloped oil field as a two dimension American Option. Schwartz and Moon (2000) used the Least Square Monte Carlo approach to price the internet companies with uncertainty in expected return and growth rate of return. Lastly Schwartz and Gorostiza (2000) implement the successive over relaxation and alternating direction implicit methodology in pricing the information technology acquisition and development with stochastic stream of cash flows.[3] Beyond classical investment valuation, the real-options literature has also provided interesting strategic results on innovation management (with Grenadier and Weiss (1997) and more recently Bernardo and Chowdry (2002), two papers that have some technical similitude to ours). Up to this paper though, we are not aware that the literature has addressed the systematic valuation of the contract features themselves (even though some papers have mentionned the idea before). Notice that our approach relies on a somewhat different view of the world from the real options literature applied to investment contracts. The latter literature tries to resolve the uncertainty of the project value through complex valuation techniques (and thus tries to reach a one point estimate of project values). We (along with practitioners) assume a strong uncertainty to the project value itself (assumed to follow a stochastic process) and try and determine the impact of this uncertainty on the VC contract and its different features.

While equity investment contracts have not been analyzed systematically in this way, there is a strong literature that applies similar techniques to debt contracts. Indeed, understanding the value of legal contracts using real options methodologies is now classical (and highly successful in practice) for credit risk. Since the seminal work of Merton (1974) which valued a simple zero-coupon straight debt contract, much research has concentrated in moving the base analysis along to more and more complex features such as convertibility (Ingersoll (1977)), safety covenants (Black and Cox (1976)), agency issues (Anderson and Sundaresan). While debt contracts have now been thoroughly analyzed (see Cossin and Pirotte (2000) for an overall review of that literature), private equity contracts such as the considered VC contracts have not been addressed in the same way. Issues such as lack of market completeness (which we address here by using an alternative technology) are not enough to justify the lack of results on equity contracts as debt contracts face the same difficulties (the assets of a firm are typically not traded). We show that such an analysis, when brought to equity contracts, provides for rich and meaningful results and should drive further research on contract optimality and contract design.

The aim of our paper is to provide the first systematic economic analysis and an overall good understanding of the economic value of the legal covenants classically used in regulating venture contracts. This paper builds up an intuitive framework to evaluate these main covenants both separately and together. The framework is flexible enough to accommodate any possible interactions between

pg.2.

[3] Recently Fourier transformation of the characteristic function is used to calculate the respective probabilities, achieving a closed form solution, for high dimension state - varibles european option.

different covenants. Thus the paper offers another dimension to the corresponding real-option related literature. Mathematically speaking, we show that the pricing of a VC contract is similar to that of a complex package of financial options. Each feature separately is an option or a bundle of options. And bringing all the features together in one contract (as often done) drives an interaction between these options that affect these values (as analyzed in a different context in Trigeorgis (1997) and as mentionned as a specific limitation of current optimal contract theory in Kaplan and Stromberg (2002)). Because all options considered tend to be of the American type (where early exercise is possible), we also address the issue of timing, a problem well-known by practitioners and to which we provide an analytical understanding. For example, we can address the complex impact of the length of the rounds of financing.

3 General Framework

This paper investigates specifically four common features (covenants) of venture capital investment contracts (staging, liquidation preference, convertibility and anti-dilution provisions).

1.The preferred stock investment format provides the venture capitalist with preferences in liquidation, a feature reinforced by the accruing dividends. The latter are not meant to be paid unless the exit mechanism is not rich enough to provide adequate returns to the investors. In such circumstances, for example liquidation, accruing dividends (often 20% per annum) guarantee a disproportionate share of the assets to the venture capital investors. The net effect of the preferred equity format with accruing dividend is to skew the payoff distribution in case of liquidation.

2. Venture capitalists rarely, if ever, invest all the external capital that a company will require to accomplish its business plan: instead, they stagger (stage) the investments over time in synch with distinct stages or milestones in their development. Each company is provided with just enough capital to reach the next stage, at which point the venture capitalist reserves the right to release the next round of capital or to abandon the project funding.

Staging of the investment is said to be a mutually beneficial arrangement. It gives the venture capitalist the option to reinvest or abandon the project. It also provides the investee with gradually cheaper funding, as the sources of uncertainty are progressively removed.

3. To capture the upside potential of the investment, venture capitalists reserve the right to convert the preferred equity or bond into common stock at a predefined conversion rate. Most conversions tend to be automatic conversions. Black and Gilson (1998) and Kaplan and Stromberg (2002) argue that the effect of the conversion feature is to be a further incentive to the entrepreneur to perform as it is a way for the venture capitalists to give up their superior control rights if performance is good enough, a proposal that we modelize here.

4. The conversion feature of the preferred equity or bond investment is often contingent on the pricing of future rounds of funding, providing a level of pro-

tection for the venture investors. The intensity of these anti-dilutions provisions varies greatly from "top-off" provisions which guarantee the maintenance of the equity percentage of the initial investor in future rounds, "full ratchets" which reset the conversion price of all pre-existing series of funding to the lowest price of follow-up rounds, maintaining the value of the investor's stake if not its actual equity percentage in the investee company, or "weighted average ratchets", which provide for a readjustment of the conversion price of prior series to a weighted average of succeeding rounds of fund raising.

Notation

$(i+1)$	number of stages for $i = 0, 1$
I_i	investment commitment per stage in time 0 value
L_i	liquidation preference level
C_i	conversion level
ρ	discount rate
r	risk free interest rate
α	"natural" drift of the firm value process, before VC value-added
δ	VC value added (includes VC non-financial contribution)
ν	capital cost of the VC beyond $\alpha + \delta$, the drift of the process
V	value of the firm or of the project
x_i	VC ownership stake in each stage
$\Phi_i(V)$	VC contract value at the beginning of each stage

3.1 Discount Rate

As described in Sahlman (1990): "In theory, the required rate of return on an entrepreneurial investment reflects the risk-free interest rate in the economy, the systematic risk of the particular asset and the market risk premium, the liquidity of the asset, and compensation for the value added by the supplier of capital. This last adjustment is required to compensate the VC for monitoring and playing an active role in management". Hence it exists a ρ where ρ is the discount rate used by the VC for decision making purposes (this may differ from the discount rate presented by the VCs for gaming purposes).

A crucial but realistic assumption concerns the stochastic changes in the project value which cannot be "spanned" or replicated from the existing assets in the economy. The underlying state variable in this model is a non traded asset (as often in the real options literature, and frequently in the credit risk literature as well). Therefore the riskless portfolio cannot be constructed making use of the discount rate per stage ρ which accounts for the decision maker's subjective valuation of risk[4]. Furthermore, the risk free rate can not cover the VC cost in providing value added. Consequently, instead of using replication arguments that rely on complete markets (and lead to risk-neutral pricing), we make use of the more realistic dynamic programing argument as developed in Dixit and Pindyck (1996) that will use the discount rate ρ.

Empirical results (Gompers (1995)) and research studies (Berk, Green and

[4] Dixit & Pyndick, 1996, "Investment under Uncertainty", pg. 121.

Naik (1998)) have shown that the systematic risk as well as the volatility levels are highest early in its life and decrease as the project approaches completion. As per Myers and Howe (1997) cited in Berk, Green and Naik (1998), the cost of capital should thus decrease through the life of the project, due to the higher "leverage" of the project early in its life. In order to study the impact of these factors in the contracting value process, we consider stepwise parameters such as volatility, drift rates and cost of capital.

3.2 Project or firm value as a diffusion process

Let V_t denote to the firm (or project) value at date t. The dynamics for V_t under the follows the diffusion process defined as

$$dV_t = \begin{cases} V_t\left(\alpha+\delta\right)dt + V_t\sigma dW_t & \text{if} \quad t \neq t_i \\ I_i & \text{if} \quad t = t_i \end{cases} \tag{1}$$

where $(\alpha + \delta)$ is the drift parameter as affected by the VC's effort in the firm, $V_t^2\sigma^2 > 0$, is the instantaneous variance and $dW_t \sim N\left(0, dt\right)$ is a classical Wiener process. Notice that we consider that the investment I_i affects the value of the firm one to one as done in practice by venture capitalists. Because VCs provide value to the firm beyond the financial investment, as well documented in the literature (either by the services they provide directly or by the VC's reputation or relationships: see Lerner (1995)), the drift of the process (and indirectly its probability of success) is increased by the VC from α to $(\alpha + \delta)$. The role of the VC value added (his/her supportive non-financial contribution) affects the probability of success. The drift of the venture project is defined as the sum of a drift of a similar project α plus the VC value added δ given its involvement in the project. The larger the VC contribution the higher the project's drift. Notice that δ could become an argument for a game theory model based on our valuation (with δ capturing the effort put in by the VC). We leave this open for further research as the valuation taking δ deterministic and constant at each stage is quite complex already, as shown underneath.

3.3 VC ownership stake calculation

The VC non financial contribution is a crucial component in valuing the venture project contract, since it captures the VC dual role of his/her financing the project as well as offering his/her non-financial contribution which not only affect the drift of the firm value, but also entitles him/her to an additional reward in terms of a higher ownership stake on the final venture project's payoff. Although the VC value added is beneficial for the firm value, i.e. increases the probability of success of the venture project, the VC does occur some additional costs by participating to a specific project. This will lead the VC to a premature exit from the venture backed project, if no additional reward (in terms of a higher ownership stake) is accounted for. The existence of the VC value added leads to major implications, which make the private equity investment different from an outside equity financing. Indeed, as these actions are costly for the VC, s/he is entitled to an additional reward. The VC thus uses a higher discount rate, as expressed by Sahlman (1990) quoted above. Because of the effort put in to increase the chances of success of the firm, but also because of liquidity effects (preference for liquidity) as well as the opportunity cost of focusing on this particular company rather than another one (while it is well documented that the number of firms a venture capitalist can oversee is limited), we assume that the discount rate ρ is larger than $\alpha + \delta$ the drift of the process when the VC participates. We set $\upsilon = \rho - (\alpha + \delta) > 0$. This represents also a stylilzed modelization of the high discount rates usually asked for by VCs. The role of the VC non-financial contribution is evident in the presence of the discounted[5] future projections $V_{t_i}(DP)$:

$$V_{t_0}(DP) = V_{t_0} \exp^{-(\rho-(\alpha+\delta))T} \quad (2)$$

$$V_{t_0}(DP) = V_{t_0} \exp^{-\upsilon T} \quad (3)$$

At the beginning of each stage i, the VC calculates the ownership stake of the firm value that s/he is entitled to, given his/her financial and non-financial contribution.

Denote $V_{t_0}^{+} = V_{t_0}^{-} + I_0$ for $V_{t_0}^{-}$ $\left(V_{t_0}^{+}\right)$ the firm value before (after) investment takes place (what is known in venture capital terms as pre-(post-) money valuation). The firm value at time $t = t_0$ is increased by the amount of investment at I_0.The drift of the process is equal to $(\alpha + \delta)$. Therefore the higher ownership stake $\dfrac{I_0}{V_{t_0}^{+} \exp^{-\upsilon(T-t_0)}} > \dfrac{I_0}{V_{t_0}^{+}}$ rewards the VC for his/her value added. Thus, at $t = t_0$,the ownership stake x_{t_0} is given by

$$x_{t_0} = \frac{I_0}{V_{t_0}^{+} \exp^{-\upsilon(T-t_0)}} \quad (4)$$

At the contractual time t_0, the VC expects the firm value to be driven by the drift rate $(\alpha + \delta)$ and discounted by ρ. Assuming that I_1 is committed at time $t = t_0$, by the time that investment takes place $t = t_1$, it will be $I_1 * \exp^{r(t_1-t_0)}$. Moreover since $I_1 * e^{r(t_1-t_0)}$ it will be part of the firm value at t_1 it will be discounted by the VC cost $e^{-\upsilon(T-t_1)}$. The ownership stake x_{t_1} is given by

$$x_{t_1}{}^{6} = \frac{I_1 * \exp^{r(t_1-t_0)}}{V_{t_1}^{+} * \exp^{-\upsilon(T-t_1)}} \quad (5)$$

for $V_{t_1}^{+} = V_{t_0}^{+} \exp^{(\alpha+\delta)(t_1-t_0)} + I_1 \exp^{r(t_1-t_0)}$. Hence the VC ownership stake x_{t_1} is determined from the growth on the firm value during $(t_1 - t_0)$ and the level

[5] Schwartz and Gorostiza (2000, pg 14) follow a similar approach in deriving the present value of the asset at time=0, in presence of a term η, risk-premium for being a non-traded asset. The two approaches are equivalent in the sense that: $\rho - (\alpha + \delta) = \rho - ((\alpha^* + \eta) + \delta) = (\rho - \eta) - (\alpha^* + \delta) = r_f - (\alpha^* + \delta) = \upsilon$, for r_f as the risk free rate, α^* adjusted growth rate for the project and δ the VC effort.

[6] The expected firm value at time of investment t_1 is given from the following expression $V_{t_1}(DP) = \exp^{-\rho(T-t_1)} \left((V_{t_1} + I_{t_1}) \exp^{(\alpha+\delta)(T-t_1)}\right) = V_{t_1}^{+} \exp^{-\upsilon(T-t_1)}$

of value added that the VC will provide during the second period. Therefore the VC ownership stake is an endogenously defined variable:

$$x_{t_i} = x\left(\delta, \alpha, \rho, t, I_i, V_t\right) \tag{6}$$

At the exit date, the VC ownership stake is a linear combination of each individual stake:

$$x_{tot} = x_{t_0} + x_{t_1} \tag{7}$$

The marginal reward for the VC additional costs for each stage is as follows:

$$\triangle x_{t_0} = \frac{I_0}{V_{t_0}^{+}}\left(\exp^{\upsilon(T-t_0)} - 1\right) \tag{8}$$

and

$$\triangle x_{t_1} = \frac{I_1 * \exp^{r(t_1-t_0)}}{V_{t_1}^{+}}\left(* \exp^{\upsilon(T-t_1)} - 1\right) \tag{9}$$

In a two stage environment reaching the next stage means deciding whether to further invest and be entitled to the accumulated ownership stake x_{tot} or to convert at a x_{t_0} stake exercising his/her liquidation preference (liquidation right+automatic conversion). S/he will continue investing in the project only if the expected discounted payoff from this strategy brings higher value than exercising the liquidation preference at that moment.

Proposition 1 *The effects on the VC ownership stake level of the changes in different parameters are as follows:*

$$\frac{\partial x}{\partial \upsilon} > 0 \textit{ and } \frac{\partial^2 x}{\partial \upsilon^2} > 0 \tag{10}$$

implying that the VC ownership stake is a convex increasing function of the VC additional costs. The importance of the investment timing is captured from the first and the second derivative of the VC stake with respect to the investment time, while having a fixed final exit date T.

$$\frac{\partial x}{\partial t_i} < 0 \textit{ and } \frac{\partial^2 x}{\partial t_i^2} > 0 \tag{11}$$

The later the VC decide to invest, the lower will be his/her stake. In other words the VC ownership stake is a decreasing convex function of the investment time. Given

$$\frac{\partial x}{\partial I_i} > 0 \textit{ and } \frac{\partial^2 x}{\partial I_i^2} < 0 \tag{12}$$

the VC ownership stake increases at a decreasing rate for increasing investment levels. Lastly

$$\frac{\partial x}{\partial V_t} < 0 \textit{ and } \frac{\partial^2 x}{\partial V_t^2} > 0 \tag{13}$$

says that the VC claim on project value is lower if the project value at that moment is higher.

These simple results correspond well to classical practice.

4 The Liquidation Preference and Convertibility

Liquidation preferences appear in venture contracts in different formats. Most often, the liquidation preference is constructed as a participating feature in the convertible preferred securities design used to finance a majority of the venture investments, whereby the first tranche of capital obtained in an exit is entirely committed to the investor group and any residual is then distributed pro-rata to the actual equity ownership. The participating feature usually disappears once the exit valuations are sufficiently high to guarantee the outside investors a solid return on their initial investments. Two examples are presented below from actual venture capital term sheets that closed in 2001:

Preference: In case of merger, reorganization or transfer of control of edocs, first pay cost of Preferred Stock. Participating goes away on valuation that corresponds to $50 million. Thereafter Preferred and Common share on as-converted basis. [a series A first round funding of $4 million at a pre-money valuation of $6 million]

Liquidation Preference: Upon the occurrence of a Liquidation Event, the Series A Preferred Stock investors shall be entitled to receive in preference to the existing Preferred Stock investors and the Common Stock investors an amount equal to the initial purchase price per share of Series A Preferred Stock plus all accrued but unpaid dividends. Thereafter, and after payment of the purchase price in respect of the other series of Preferred Stock, any remaining proceeds shall be allocated pro rata among the holders of the Common Stock and the Series A Preferred Stock, treating the investors in the Series A Preferred Stock on an as converted basis, until the Series A Preferred Stock holders shall receive an aggregate on their initial investment of 150%. A merger, consolidation, dissolution, winding up, change of control or sale of the assets shall be deemed to be a liquidation event ("Liquidation Event"). [a series A first round funding of $6.5 million at a pre-money valuation of $3 million]

The participating preferred debentures or preferred stock investment format provides the venture capitalist with preferences in liquidation, a feature reinforced by the accruing dividends, as shown in the second example above. The accruing cash flows (dividends or coupons) are not meant to be paid unless the exit mechanism is not rich enough to provide adequate returns to the investors. In such circumstances, for example liquidation, accruing dividends (often 10-20% per annum) guarantee a disproportionate share of the assets to the venture capital investors. Very high, out of the money liquidation preference features have been known to be implemented in order to skew values even more towards the venture capitalist (or a specific member of a syndicate). The net effect of the preferred security format with accruing dividend is to skew the payoff distribution in case of liquidation.

Notice that the convertibility feature of VC contracts is somewhat different from the convertibility of traded convertible bonds as analyzed by Ingersoll (1977). The most common security used by venture capitalists are indeed convertible preferred stocks and debentures (Kaplan and Stromberg (2000, Table 1) and Sahlman (1990). There exists an extensive literature concerning the prevalent use of the convertible securities in venture capital finance. Schmidt (2001) gives a short summary on papers dealing with the issue of the optimal contract design for an inside investor.

The conversion feature can be understood as a reallocation of control rights in case of success of the project from the venture capitalist to the entrepreneur. It is an incentive to perform to the entrepreneur beyond the direct financial incentives offered by the venture capitalist. The conversion feature can thus be understood as barrier that transforms the considered features (such as liquidation preference) in barrier options of the up-and-out type. In other words, if a certain milestone is achieved (i.e. a certain level of value for the project), the previous options (here the liquidation preference) are cancelled, thus reducing the differences in rights between the venture capitalist and the entrepreneur. The convertibility is thus as much a redistribution of rights towards the entrepreneur (or passive investors) as it is an upside potential for the VC.

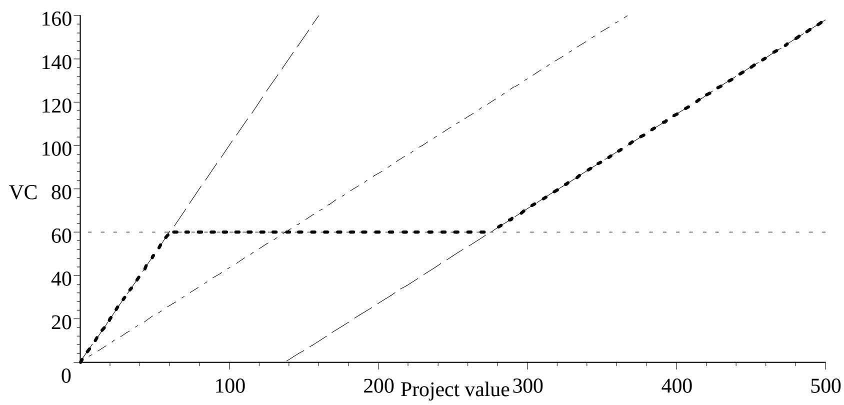

Figure 1: The bold line stands for the VC payoff value for any realisation of firm value V_T. The dashed line on the right is the upside potential coming from the conversion optionality $\max(xV_T - C_0, 0)$. The automatic conversion option takes a positive value only when the upside potential (dashed line on the right) crosses the liquidation right structure $\min(V_t, L_0)$. This is due to the fact that VC has to relinquish his liquidation right to gain the automatic conversion option. Consequently the respective critical firm values are higher than in the case of a simple convertible security. If the project value $V_T \in [0, V_T^*]$ the VC payoff is defined in the following interval $[0, 60]$, otherwise for $V_T > V_T^* = 275.2$ the VC converts and its payoff is larger than 60. Note that the parameters are: $\upsilon = 0.05, V_0^+ = 160, I_0 = 60, V_T^* = 275.2, x_{t_0} = 43.6\%, L_0 = 60, C_0 = 60, T = 3yr$.

From the VC's perspective, s/he would like to maximize the value of his/her holdings in case the project is successful or recoup the maximum possible value in case of project liquidation. Usually the liquidation right entitles the VC to collect L_0 (often equal but not necessarily to the original investment I_0), if there is enough money in the company, or the remaining value V_T after the creditors and before all the other shareholders' claims. Thus the liquidation value, if liquidation is decided, is of $\min(V_T, L_0)$. In case of good performance, nonetheless, the VC may decide to convert and obtain conversion value $x_{conv}V_T - C_0$ where C_0 is the conversion level and $x_{conv}$the number of shares obtained of the company after conversion.

It is important to note at this point that although the liquidation preference may look as a package of the upside potential from conversion and liquidation right, it is less valuable than the linear sum of the option components given that the VC has to give up the liquidation right $\min(V_T, L_0)$ to buy the conversion option (hence the realignment of rights described above between the venture capitalist and the entrepreneur). Note that x_{conv} can be either the absolute upside sharing of the VC (in the case of convertible debentures) or the relative gain in upside sharing by going from convertible preferred to common stock. In the extreme case where $x_{conv} = 0$, the VC will convert only when forced (by the arrival of an IPO for example). We treat here a stylized case that corresponds more exactly to the situation of convertible debenture (a form of financing that is widely used in VC contracts, notably in Europe). We address more specifically the determination of x_{conv} and the impact of antidilution in a later section.

The VC payoff at time T is given as:

$$\varphi(V_T) = \max(x_{conv}V_T - C_0, \min(V_T, L_0)) \tag{14}$$

where $x_{conv} = x_{t_0}$, the conversion price is C_0 and the liquidation value is L_0.

Figure 1 shows that the VC will convert his/her holdings only if the upside potential from converting is high enough to pay for the loss of the downside protection linked to the liquidation right. This expression can be simplified to

$$\begin{aligned}
\varphi(V_T) &= \max(x_{t_0}V_T - C_0 - \min(V_T, L_0), 0) + \min(V_T, L_0) && (15)\\
\varphi(V_T) &= \max(x_{t_0}V_T - C_0 + \max(-V_T, -L_0), 0) + \min(V_T, L_0)\\
\varphi(V_T) &= \max(\max(x_{t_0}V_T - C_0 - V_T, x_{t_0}V_T - C_0 - L_0), 0) + \min(V_T, L_0)\\
\varphi(V_T) &= V_T + \max(\max((x_{t_0} - 1)V_T - C_0, x_{t_0}V_T - (C_0 + L_0)), 0) +\\
&\quad - \max(V_T - L_0, 0) && (16)
\end{aligned}$$

Given that $x_{t_0} \in [0, 1]$, the expression $(x_{t_0} - 1)V_T - L_0 < 0$ with positive probability almost as sure. Hence the expression can be further simplified as follows

$$\varphi(V_T) = V_T + \max(x_{t_0}V_T - (C_0 + L_0), 0) - \max(V_T - L_0, 0) \tag{17}$$

Equation (17) is a package of the following options:

$$\varphi_1(V_T) = \max(x_{t_0}V_T - (C_0 + L_0), 0) \tag{18}$$

and

$$\varphi_2(V_T) = \max(V_T - L_0, 0) \quad (19)$$

the VC payoff is defined as:

$$\varphi(V_T) = V_T - \varphi_1(V_T) - \varphi_2(V_T) \quad (20)$$

- **Proposition 2** *The VC final payoff at $t = T$ can be written as:*

$$\varphi(V_T) = \begin{cases} V_T & if \quad V_T \leq L_0 \\ V_T - (V_T - L_0)^+ & if \quad L_0 < V_T \leq (C_0 + L_0) \\ V_T + (x_{t_0} V_T - (C_0 + L_0))^+ - (V_T - L_0)^+ & if \quad V_T > (C_0 + L_0) \end{cases}$$

Proof. For any $V_T \leq L_0$

$$V_T + (x_{t_0} V_T - (C_0 + L_0))^+ - (V_T - L_0)^+ = V_T \quad (21)$$

for

$$(x_{t_0} V_T - (C_0 + L_0))^+ = 0 \quad (22)$$

The only possibility for which $(x_{t_0} V_T - (C_0 + L_0))^+ > 0$ requires $1 > x_{t_0} > \frac{(C_0+L_0)}{V_T}$. To put it differently $(x_{t_0} V_T - (C_0 + L_0))^+ > 0$ only for $V_T > (C_0 + L_0)$, otherwise $(x_{t_0} V_T - (C_0 + L_0))^+ = 0$. ■

4.1 Infinite Horizon Case / Endogenous threshold exit level

Pushing the analysis further, one can solve for the VC payoff process in an infinite horizon case. Closed form solution are available. The timing of exercising the liquidation preference becomes then endogenous to the model, and is determined by a smooth pasting condition. The optimal stopping times are then given by

$$\tau_1 = \{\inf t \geq 0 \mid V_t \geq V_B^*\} \quad (23)$$

in case of a conversion and

$$\tau_2 = \{\inf t \geq 0 \mid V_t \leq V_A^*\} \quad (24)$$

in case of a liquidation. The critical values for V_A^* and V_B^* are given below. The pricing process uses the dynamic programming argument. The idea is to start from the end of the project life and folds recursively to the present. In a perpetual framework it reduces to the finding of the optimal thresholds to exit. Once the boundary conditions are known, we work backward toward the initial contracting time $t = 0$.

The VC contract whose terminal payoff is a piecewise linear function of the terminal price of the underlying asset is similar to a package option, that is a combination of standard options and underlying asset.

The equation (17) defines the VC contract payoff for $t = \tau_i$ where $i = 1, 2$ as

$$\varphi(V_{\tau_i}) = V_{\tau_i} + \max\left(x_{t_0} V_{\tau_i} - (C_0 + L_0), 0\right) - \max\left(V_{\tau_i} - L_0, 0\right) \tag{25}$$

If rewritten as

$$\varphi_1(V_{\tau_i}) = \max\left(x_{t_0} V_{\tau_i} - (C_0 + L_0), 0\right) \tag{26}$$

and

$$\varphi_2(V_{\tau_i}) = \max\left(V\tau_i - L_0, 0\right) \tag{27}$$

the VC payoff is defined as:

$$\varphi(V_{\tau_i}) = V_{\tau_i} + \varphi_1(V_{\tau_i}) - \varphi_2(V_{\tau_i}) \tag{28}$$

Proposition 3 *In an infinite horizon case, the VC contractual payoff at* $t \in [0, \tau]$, *given the liquidation right and automatic conversion value, is as follows:*

$$\varphi(V_t) = \begin{cases} V_t & V_t < L_0 \\ V_t - \dfrac{L_0}{u-1}\left(\dfrac{V_t}{V_A^*}\right)^u & L_0 \leq V_t < (C_0 + L_0) \\ V_t - \dfrac{L_0}{u-1}\left(\dfrac{V_t}{V_A^*}\right)^u + \dfrac{(C_0 + L_0)}{u-1}\left(\dfrac{V_t}{V_B^*}\right)^u & V_t \geq (C_0 + L_0) \end{cases} \tag{29}$$

for option components $\varphi_i(V_t)$ *equal to*

$$\varphi_1(V_t) = \frac{(C_0 + L_0)}{u-1}\left(\frac{V_t}{V_B^*}\right)^u \tag{30}$$

$$\varphi_2(V_t) = \frac{L_0}{u-1}\left(\frac{V_t}{V_A^*}\right)^u \tag{31}$$

V_A^* *would be the lower threshold level to liquidate the project. At this level the VC is entitled only to the liquidation right and has zero (if* $V_A^* < (C_0 + L_0)$*) or very low (if* $V_A^* \geq (C_0 + L_0)$*) upside potential from conversion:*

$$V_A^* = \left(\frac{u}{u-1}\right) L_0$$

V_B^* *represents the upper optimal threshold level to exit the project:*

$$V_B^* = \left(\frac{u}{u-1}\right)\frac{(C_0 + L_0)}{x_{t_0}} \tag{32}$$

This is equivalent to say that for any $V_t \leq V_A^*$ *liquidation right* $\min\left(V_T, L_0\right)$ *is given by*

$$\varphi(V_{\tau_1}) = L_0$$

For any $V_A^* < V_t < V_B^*$ *the VC does not exit. Lastly for* $V_t \geq V_B^*$ *the final payoff at the exit* $\max\left(x_{t_0} V_T - C_0, \min\left(V_T, L_0\right)\right)$ *is*

$$\varphi(V_{\tau_2}) = L_0 + \left(x_{t_0} V_{\tau_2} - (C_0 + L_0)\right) = \left(x_{t_0} V_{\tau_2} - C_0\right) \tag{33}$$

Proof in Appendix B.

The pricing is carried out under two crucial implicit assumptions:

- The VC optimizes its position, by choosing the upper threshold level such as to maximize its payoff at the exit of the venture, once the project value is known. To put in other words the VC has all the bargaining power in deciding when and how to end the venture project. In reality exit options in a venture project are limited. The board of directors is the only entity to have the power to initiate exit. The VC will be entitled to a certain degree of control rights over exit only towards the end of the venture project life. Kaplan & Stromberg (2000) provide empirical evidence of this phenomena.

- When the VC provides some value added in terms of reducing the level of asymmetric information, co.-managing the project etc. s/he is at the same time occurring some additional cost apart from investing in the project. Consequently s/he is induced to have an early exit from the venture. Obviously the VC will like to minimize this cost, by equalizing it to zero and to provide no value added. The reality shows that the VC does have a dual role and we do account for that.

  The derivation provides some useful insights in understanding the value of this important contractual feature.

Figure 2 consider a large versus a small stake holder. The larger the ownership stake of the VC in the project, the larger is its payoff at the exit. Liquidation preference entitles the VC to large payoff if the venture project performs well reducing the downside loses, if the project performs poorly. It is important to note that in both VC contracts, the payoff for the liquidation right is similar. Nonetheless the payoffs are quite different for the upside part, since a lower stake on the company will convert at lower levels and than a higher ownership stake given the lower investment cost. Moreover this does not implies that a lower stake holder exit have higher chances of early exit, given that the initial firm value depend on the investment amount.

Figure 3 is a simple realization of the VC contract value ($x_{t_0} = 43.6\%$) at any time t, for different firm values. The VC holds a convertible preferred stock which allows him to a fraction of the upside potential (=automatic conversion) as well as to the liquidation right. Whereas for low performance his/her contract value is equal or almost equal to the firm value, the gap in between is large in case of a good performance. One may think of it as a reward (punishment) for the entrepreneur if the project is successful (failure).

Figure 4 represents the VC contract value as the sum of the liquidation right $\min(V_t, L_0)$, plus the opportunity to convert $\max(x_{t_0}V_t - (C_0 + L_0), 0)$. The VC payoff at the exit is equal to the firm value or initial investment in case of poor performance or to the converted value for good performance.

Moreover the derivation sheds light on the relative importance of each component (automatic conversion & liquidation right) of the liquidation preference for different project performance levels.

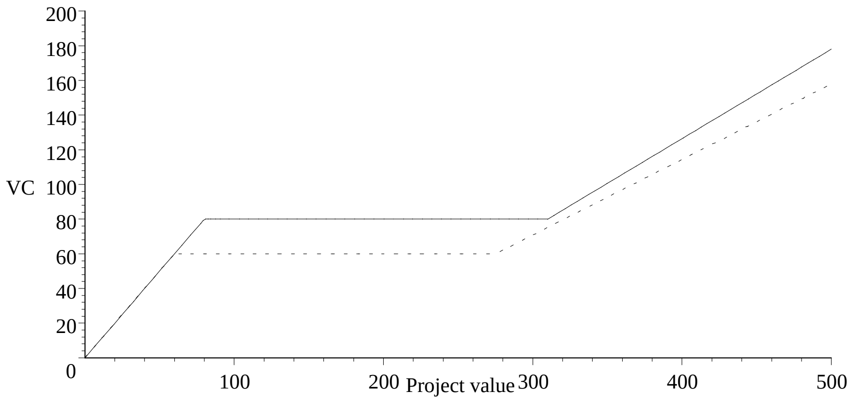

Figure 2: Large (solid line) versus small (dashed line) VC ownership stake. Small stakeholder case: VC invest 60 in a 100 worth company hence is entitled to $x_{1,t_0} = 43.6\%$ stake. Large stakeholder case: VC invests 80 equivalent to a $x_{2,t_0} = 51.6\%$ stake. The VC converts his holdings therefore exit only if firm value reaches the $V_{1,T}^{*} = 275.2$ for an inital investment of 60 and a critical project value of $V_{2,T}^{*} = 310.1$ for the 80 unit of investment case. Whereas the downside part is quite similar in both cases, upside part tells us that the large stakeholder is entitled to a higher project value at T. Note that the parameters are: $\upsilon = 0.05, V_{1,0}^{+} = 160, V_{2,0}^{+} = 180, V_{1,T}^{*} = 275.2, V_{2,T}^{*} = 310.1, x_{1,t_0} = 43.6\%, x_{2,t_0} = 51.6\%, L_0(x_{1,t_0}) = 60, C_0(x_{1,t_0}) = 60, L_0(x_{2,t_0}) = 80, C_0(x_{2,t_0}) = 80, T = 3yr.$

4.2 Liquidation in Finite Horizon Case

In order to derive the closed-form solution equation 29, we attributed no role to the entrepreneur in deciding the conditions under which the VC may liquidate his/her position. Furthermore we assumed that the VC additional capital cost had no impact on the VC ownership stake. This section releases these rigid restriction in the model by assuming exogenously predefined project life as well as the duration of each stage. To put it differently the VC is allowed to exit only at some preset discrete events, which may correspond to the end of each financing stage. Moreover the model accounts for an additional reward for the VC value added.

As already mentioned the liquidation payoff has two main components, the upside potential coming from the conversion possibility and the liquidation right $\min(V_t, L_0)$. The upside potential is implied by the automatic conversion. The VC would convert his/her holdings in common stock against a cost equal to the initial investment amount, only if the conversion value covers the investment cost.

The way to proceed is to use dynamic programing and to work recursively. This model considers an up front financing. At $t = 0$, the VC meets the entre-

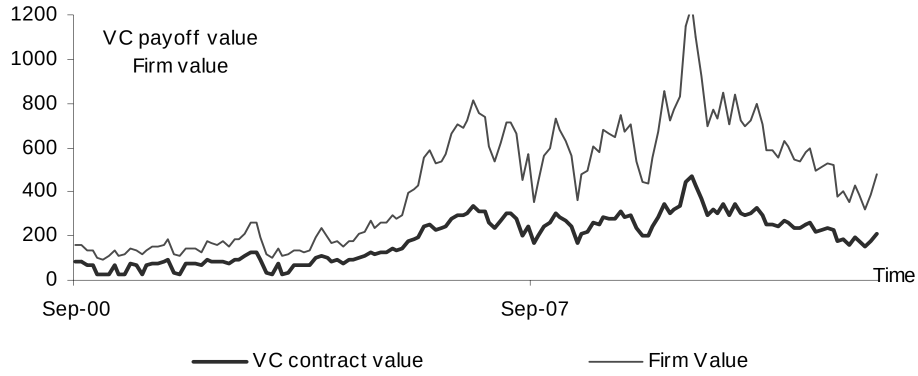

Figure 3: VC payoff versus firm value. Note that the parameters used are as follows: $\alpha + \delta = 0.35, \upsilon = 0.05, \sigma = 0.6, V_{t_0}^{+} = 100, V_0^{+} = 160, C_0 = 60, L_0 = 60$.

preneur and both enter in a contractual agreement which will last up to time $T = 3yr$, the VC exit point. If the project has been successful until the end of the contractual period the VC exercises his/her liquidation preference at $t = T$. The VC payoff value, in case of an acquisition or merger exit event, is defined as

$$\Phi(V_T, T) = \max\left(x_{t_0} V_T - C_0, \min\left(V_T, L_0\right)\right) \tag{34}$$

In case of a forced conversion at T (as would occur for example in an initial public offering), then the VC payoff at the exit is as follows:

$$\Phi^{'}(V_T, T) = \max\left(x_{t_0} V_T - C_0, 0\right) \tag{35}$$

4.2.1 Numerical Results

Table 1 studies the importance of each component of the liquidation preference. The liquidation preference is a package of liquidation right (3) plus the automatic conversion (1). The value of the automatic conversion (1) is lower than the value of the upside potential (2), since it incoporates the liquidation right in the strike price.

The result of table 1 are compatible with proposition 1. Table 2 indicates that the VC expected contract value at the end of the venture is very sensitive to the financing level. The more the VC disburses in the project the larger is the ownership stake in a successful project. The amount invested is positively related to the VC ownership stake and to the after money project value (higher cash inflow). Furthermore the liquidation right limits the VC downside losses in case of poor performance, in other words limits the risk of occurring large looses. The initial project value plays an ambiguous role, first it reduces the

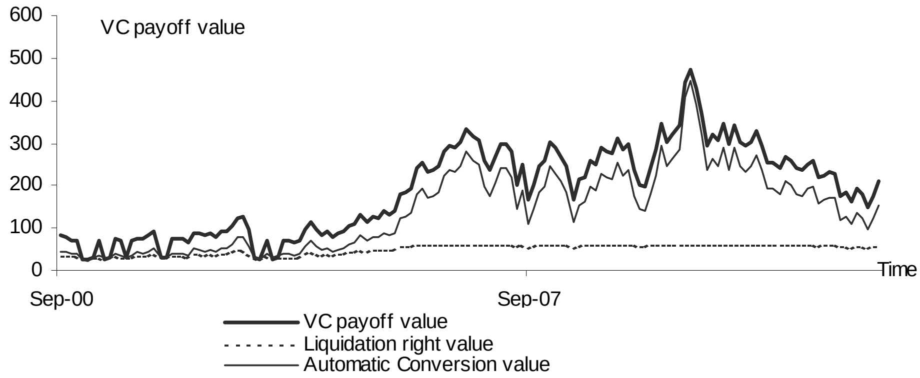

Figure 4: The VC contract value is the sum of the liquidation right $\min(V_t, L_0)$, plus the opportunity to convert $\max(x_{t_0}V_t - (C_0 + L_0), 0)$. Note that the parameters are: $\alpha + \delta = 0.35, \upsilon = 0.05, \sigma = 0.6, V_0^+ = 160, C_0 = 60, L_0 = 60$.

0	T
VC disburse I_0	Exit 1.
	Exit 2

$$1.\Phi(V_T, T) = \max(x_{t_0}V_T - C_0, \min(V_T, L_0))$$
$$2.\Phi^{'}(V_T, T) = \max(x_{t_0}V_T - C_0, 0)$$

Figure 5: Time line for a venture backed project.

VC ownership stake, since his/her financing amount is relatively less important for a higher firm value and secondly it implies larger expected firm value at time T. Whereas the first plays a negative role on the VC ownership stake x_{t_0} the second one has a positive impact on project value V_T. Any increase in uncertainty implies high project value and a negligible impact on the downside part. The marginal increase in the upside more than compensates the drop in liquidation right which serves as a cushion for any potential losses. A larger project drift $(\alpha_1 + \delta_1)$ means larger future firm value hence both components of the liquidation preference will experience an increase in their respective values. For high additional costs υ_i, the VC is compensated with a higher stake on the firm value. Lastly, the timing of the exit T $(1yr \rightarrow 3yr)$ determines not only the value of the liquidation preference but also its decomposition in terms of the importance of upside value $(36.057 \rightarrow 171.471)$ versus liquidation right $(59.687 \rightarrow 58.66)$. Obviously the longer the project life the higher the probability that the project value will hit the upper threshold which makes possible the convertibility feature. Furthermore the liquidation right becomes relatively less important as the expected project duration increases.

5 Staged financing

Sequential investment is strongly related to the existence of great uncertainty concerning high tech, R&D and generally VC-financed projects, a phenomenon specifically analyzed in Gompers (1995) and well known in practice. Staged capital infusions allow venture capitalists to periodically gather information and monitor the progress of the project (and notably the evolution of the risk of the project, or project volatility, as it matures from an early stage to a later stage). Staging can be organized formally or unformally. Kaplan and Stromberg (2002) for example analyze ex-ante staging in which VCs commit at initiation of the investment to future investments with contingencies on milestones being met. They also mention that "most VC financings are at least implicitly staged, in the sense that even when all the funding in the initial round is released immediately, it is understood that future financing rounds will be needed to support the firm until the IPO." We modelize here both formal and unformal staging.

We consider also both the case of endogenous staging and the case of deterministic times for sequential investments. In the case of endogenous staging, the staging is considered to be similar to a compound call option. The next financing round can take place as soon as the project value reaches the optimal project value endogenously determined from the model. The venture capitalist has first rights to further investments in the company. We consider the possibility that these rights are formal (ex-ante staging), i.e. that the VC commits to further financing of a certain amount at a certain valuation if some milestone is achieved. We will compare this to informal rights (linked notably to competitive issues, in which the VC has a first right to participate but where the pricing is set competitively at the time of the financing) and to the situation in which no rights (formal or informal) are acquired by the VC, so that the VC's investment corresponds to a short term investment.

This section proposes a simple model to analyses stage financing. The model allows to gain a better understanding of the factors influencing the staging process. Gompers (1995) observes that the duration of a particular round, the size of each investment, the total financing provided and the number of rounds are important measures of staged investment structure[7]. The model allows for a more precise investigation of these issues. It also allows for an analysis of the venture capitalist contracting value as a function of the factors directly influencing the unpredictably of the project value.

5.1 Infinite Horizon Case/ Endogenous Staging Times

Initially we define the procedure to follow in order to determine the value of the VC contract including the option to invest in the later stages once invested in one of the stages. The model allows to compute the critical project value at which the VC invests, buying also the option for the next stage. Intuitively the Venture Capitalist would like to know the value of the option to invest in the

[7] Gompers (1995), pg. 1462.

later stage and how much the project is worth at the end of each stage, before stipulating the contract with the entrepreneur.

The VC takes the decision to invest or not in the project at the beginning of the stage 1. Such a funding is necessary for the project to start. At the end of the stage one, the VC gets $(x_{t_0}V_t - C_0)$ plus the option to invest in the following stage, where the value of C_i depends on I_i. This option value is not a simple difference of the project value of that stage minus investment cost, but it incorporates also the additional value of investing in the following stage and so on. Our simplified model considers a two stage project. At contractual time $t = 0$ the VC agrees in financing the project in two rounds. The VC is rewarded with $(x_{t_i}V_t - C_i)$, if the firm is successful or zero otherwise. Each round ends when a certain milestone, where the optimal threshold level is reached. At this point the VC decides to further invest.

Pushing the analysis further, one can model a special case of the staging process as a compound call option for infinite horizon. The timing of the investment becomes then endogenous to the model, and is determined by a smooth pasting condition. The optimal investing time τ_i is then given by

$$\tau_i = \{\inf t \geq 0 \mid V_t \geq V_i^*\} \tag{36}$$

The pricing process uses the dynamic programming argument. In a perpetual framework it reduces to the finding of the optimal threshold V^* to invest. Once the boundary conditions are known, we work backward toward the initial contracting time $t = 0$. More specifically, the value of the formal contractual right for a two stage model given the perpetuity assumption is defined as

$$\varphi(V_{\tau_1}) = \max\left(\varphi_1(V_{\tau_1}) + x_{t_0}V_{\tau_1} - C_0, 0\right) \tag{37}$$

for $\varphi_1(V_{\tau_1})$ incorporated in the VC payoff as the payoff that he is getting at $t = \tau_2$

$$\varphi_1(V_{\tau_2}) = \max\left(x_{t_1}V_{\tau_2} - C_1, 0\right) \tag{38}$$

Proposition 4 *The VC contract value for any $t < \tau_1$ is equal to*

$$\varphi(V_t) = \left(\frac{x_{t_1}}{q_1}V_B^{*1-q_2} + \frac{x_{t_0}}{q_1}V_A^{*1-q_1}\right)V_t^{q_1} \tag{39}$$

For $t = \tau_1$ end of first stage the VC is entitled to

$$\varphi(V_{\tau_1}) = (x_{t_0}V_{\tau_1} - C_0) + \frac{x_{t_1}}{q_1}V_B^{*1-q_2}V_{\tau_1}^{q_2} \tag{40}$$

Lastly for $t = \tau_2$ the VC receives the following payoff.

$$\varphi(V_{\tau_{2)}}) = (x_{t_0} + x_{t_1})\,V_{\tau_2} - (C_1 + C_0) \tag{41}$$

Proof in appendix C.

The infinite horizon case gives some useful insights. Figure 6 shows the relation of a formal VC contract versus an informal contract. A formal contract brings always additional value to the VC, compared to the informal contract. The gap is the option value that a formal contract gives to the VC to further invest or exit from the venture. Hence entering in a competitive basis at each stage has lower value than committing from the very beginning of the project.

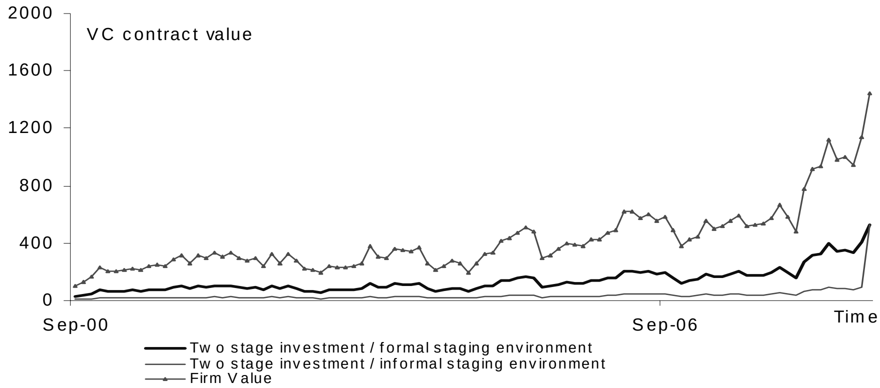

Figure 6: A formal contract brings always additional value to the VC, compare to the informal contract. The gap in between constitutes the option value to further invest in the project. Formal and informal contract have the same payoff for firm value higher than the critical value of the first stage $V_t > V_A^* = 1320$. Note that the parameters are: $\alpha + \delta = 0.35, \upsilon = 0.05, V_0 = 100, x_{1,t_0} = 9\%, x_{2,t_0} = 32\%, C_0(x_{t_0}) = 10, C_1(x_{t_1}) = 50$.

If a VC invests at the very beginning of the project life, s/he is rewarded by a larger stake since s/he is taking on more risks than a later VC. Consequently if the project does well s/he is rewarded by a larger payoff than the VC who stages his/her investment. On the other hand if the stake taken in the project is large, the VC is taking too much risk and obviously may occur large losses.

Furthermore, the VC who chooses a staged financing in reality has a larger payoff than the one represented in the figure, since s/he will be given a larger ownership stake because of the value added which is positively related to the number of stages (see also Gompers 1995). Overall the staged financing turns out to be more beneficial than no stage financing. This issue is carefully analyzed in the following section, where we endogenize the VC value added and additional costs in terms of a higher expected reward.

5.2 Staging in Finite Horizon

The closed form solution solves for the endogenous optimal level at which VCs would proceed with project financing. The purpose of staged financing is two fold: by staging, the VC aims at lowering the level of the project's risk and most importantly allows for early exit from the venture project. To treat a finite horizon case, we assume a predefined number and duration of stages. This allows us to incorporate liquidity preference and convertibility in more details.

We release most of the assumptions made previously. First, any financing strategy will significantly affect the value of the venture backed project and introduce jumps in the project value process for a given investment amount. Second, we account explicitly for the additional reward in the VC ownership stake. Third, the following model allows for more realistic cash flows upon the predefined contractual provisions. A further specification of the model allows for a more flexible model, to be solved numerically.

At $t_0 = 0$ the VC meets the entrepreneur and both enter in a contractual agreement which will last up to time $T = 3yr$. The VC has two exit possibility: Liquidate at $t_1 = 1yr$ before making any additional investment, otherwise liquidate at T. If the project has been successful until the end of the contractual period, the VC exercises his/her conversion right. At the end of project life, the VC is entitled to the accumulative ownership stake x_{tot} defined in equation 7 in the firm value since s/he has been investing in all stages. The conversion price is usually set to the total of the original investment amount for a conversion ratio equal to one. Nonetheless for the sake of generalization we set the conversion price C_{tot} and the liquidation value L_{tot} multiples of I_{tot}. Whereas the conversion price C_i and the liquidation value L_i follow the notation of I_i.

At the end of the project life, the VC contract value is defined as

$$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right) \tag{42}$$

Note that the max () operator implements the decision to liquidate the position in the venture project at a specific time/event. This event may corresponds to a merger, buy-out etc. In case an initial public offering takes place at date T, the VC payoff can be written as follows (as forced conversion eliminates the liquidation rights):

$$\Phi^{'}(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, 0\right) \tag{43}$$

The way to proceed up to the initial time is quite the same as below. At t_1 the VC the max() operator reflects the decision to invest if the expected value of the project in future stages is higher than liquidating at t_1.

$$\Phi(V_{t_1}, t_1) = \max\left(\frac{1}{(1+\rho)^{T-t_1}}\tilde{E}_{t_1}\left[\Phi(V_T, T)\right], \max\left(x_{t_0}V_{t_1} - C_0, \min\left(V_{t_1}, L_0\right)\right)\right) \tag{44}$$

At the time of writing $t_0 = 0$ the contract the value of the VC future investment process is given by:

$$\Phi(V_{t_0}, t_0) = \frac{1}{(1+\rho)^{t_1-t_0}}\tilde{E}_{t_0}\left[\Phi(V_{t_1}, t_1)\right] \tag{45}$$

The model implicitly defines the probability that the VC will continue investing in the project:

$$P = \Pr\left(continue > liquidate\right) \tag{46}$$

Applying this concept to date $t = t_1$ the probability of continuing is given by:

$$P = \Pr\left(\frac{1}{(1+\rho)^{T-t_1}}\tilde{E}_{t_1}\left[\Phi(V_T,T)\right] > \max\left(x_{t_0}V_{t_1} - I_0, \min\left(V_{t_1}, I_0\right)\right)\right) \tag{47}$$

Two Stage Venture Project

Stage 1	Stage 2	
0	$\frac{T}{2}\ (\frac{T}{3})$	T
Invest I_0	Invest I_1	Exit[1]
	or Liquidate	Exit[2]

Three Stage Venture Project

Stage 1.	Stage 2.	Stage 3.	
0	$\frac{T}{3}$	$\frac{T}{2}$	T
Invest I_0	Invest I_1	Invest I_2	Exit[1]
	or Liquidate	or Liquidate	Exit[2]

Exit1.$\Phi(V_T, T) = \max(x_{tot}V - C_{tot}, \min(V, L_{tot}))$
Exit 2.$\Phi^{'}(V_T, T) = \max(x_{tot}V - C_{tot}, 0)$

Figure 7: Time line for a two and three stage financing project. Note that conversion price and the liquidation level are different for each case.

Does the number & duration of stages matter? Table 3 looks at the VC contract value for different types of contract for a three year project. In order to study the difference between these different possibilities, we kept the same volatility, growth rate and VC value-added among stages. The following observations can be drawn: Investing all the money at time zero (one stage contract) may lead to a lower value than a two or a three stage contract since the later has the option to revise the investment policy. Entering near the end of the project leads to a lower value than entering at the beginning. The senior VC would require more than a junior VC given the larger risk and uncertainty concerning the project at the very early stages. There is an optimal distribution of investment for a two and three stage contract. In other words the VC's contract value is a concave function of the amount invested in first stages with respect to the later ones. Lastly it is important to note that while increasing the number of stages increases the value given the earlier exit possibility, it gradually reduces the VC ownership stake over stages. This may be a reason for the concavity. The shape of the concavity greatly depends on the level of parameters such as drift, volatility and VC value added and its additional costs.

A three stage contract performs better than a two or one stage contract, though the marginal impact is much lower. Furthermore the performance of these contracts greatly depends on the duration of the very first stages. Consequently we would expect the VC to set up a multiple stage contract with short early stages duration. These results are compatible with the empirical findings in Gompers (1995) concerning the number and duration of the stages in successful venture projects.

Design Considerations for Staging Table 4 studies the impact of investment timing. The results show that investing at the beginning of the project is rewarded by a larger payoff than entering towards the end. This is due to the fact that the VC who enters at the beginning takes more risks, given the high degree of uncertainty characterizing the start up projects. Nonetheless the best investment policy requires a certain distribution of total investment amount amongst stages. In other words the VC contract value behave as a concave function of investment amount at first and second stage. Investing early on leads to a larger payoff. This result makes sense since the early investors have to be rewarded by more than the later ones.

An important observation concerns the value of automatic conversion and liquidation rights. The automatic conversion embeds some of the characteristics of a warrant payoff $(x_{t_i} V_t - C_i)$. The idea is to look at the two components of the payoff at the exit as if they were independent of each other. Postponing investment toward the end leads to a lower ownership stake in the firm value, given the positive drift $V_{t_0} \exp^{\left(\alpha+\delta-\frac{\sigma^2}{2}\right)(t-t_0)}$. As automatic conversion value is driven from the ownership stake of VC holdings, the liquidation right behavior is more complex. The entrepreneur may not be able to pay back the original investment amount of the VC. In terms of the evaluation procedure the VC contract value is less than the linear sum of upside potential and liquidation right, since the VC has to give up his/her liquidation right to gain the automatic conversion right. The investment amount of the very first stage is negatively related to liquidation right value and positively related to the upside potential value. Furthermore the VC payoff decreases as the investment takes place in the later stages rather than in the earlier ones. To conclude the investment policy chosen from the VC has a stronger impact on the upside potential value rather than on the downside protection of liquidation right.

5.3 Conclusions

- The option to reinvest brings higher value to the VC contract. True, formal staging thus dominates the informal staging frequently seen in current contracts. It also dominates the absence of staging or a full commitment to future investments[8].

[8] Note that this idea is already in Trigeorgis (1997).

- Investing all at once is not the optimal decision. Determining the optimal investment level requires the full analysis to be drawn.
- Investing without further options to invest leads to a lower VC contract value.
- The number of stages matters, and so does the timing of the stages. The critical price is an increasing function of the number of stages. The marginal impact seems to be decreasing with the number of stages though.
- A full economic analysis as the one proposed here helps determine the relative value of the different contractual features of the staging. It shows how these features affect the distribution of the outcomes. It thus helps determine the optimal contract. A similar analysis would thus have important consequences for the valuation of the contracts, but even more importantly their design and their management.

6 Antidilution

The anti-dilution feature appears in different forms in venture agreements. The purpose of this provision consists in protecting the earlier investors from dilution. Stock splits, stock dividends, any new financing at a lower price per share than the previous one, all drive the value of the initial investors down.

The way that anti-dilution provision protects the VC from the dilution effect depends on the methods or the provision written in the contractual agreement. In general this can be done by:

- Decreasing the conversion price to the price level of the new financing issue (full ratchets). The founders, the management have to issue as much as necessary free share to the VC, to bring the old conversion price down to the lowest price level.
- Both parties agree on a new conversion price, which would be the weighted average of the old conversion rate and the price level of the last round (weighted average formula).

We will look at the second method since it is more common in venture agreements as documented in Kaplan and Stromberg (2002) . The anti-dilution feature is similar to a long put option, which guarantees to the VC an additional value in case the share price drops below the old conversion rate. The additional value that the VC receives consists in free shares to compensate for the dilution effect of the negative change in project value.

The conversion price will be reduced on a weighted average formula basis. The formula is defined as:

$$p = \frac{p_1 q_1 + p_2 q_2}{q_1 + q_2} \qquad (48)$$

where

p	conversion price per share of preferred stocks following new issuance
p_1	conversion price per share of preferred stocks prior to new issuance
p_2	price per share of the new issue
q_1	number of equivalent shares of common stocks before the new issue
q_2	effective number of equivalent shares of common stocks newly issued
$k = \frac{q_2}{q_1+q_2}$	fraction of newly released shares
q^+	free number of shares offered to the VC after the financing round
q_t	total number of outstanding shares before the new financing round
$x_{t_0} = \frac{q_1}{q_t}$	VC ownership stake before the new issue
x_1^*	fully diluted VC ownership stake after the new issue
x_2^*	partially diluted VC ownership stake after the new issue
I_0	VC total investment in the project
I_t^+	new entry, additional source of financing

The anti-dilution right is modeled as the difference between the old and the new conversion rate[9]. The dilution protection has a reward $f(.)$, and maturity T, leads to a stochastic payoff $f(V,t)$,with $t \in [0,T]$,where $\{G_t\}$ is the filtration on information up to date t. The dilution protection is valuable only if the price per share of the new issue is lower than the conversion price prior the issuance. The entrepreneur may need additional investment amount I_t^+ at a certain point in time.

$$I_t^+ = q_2 p_2 \tag{49}$$

equivalent to

$$I_t^+ = \left(\frac{k}{1-k}\right) q_1 p_2 \tag{50}$$

The ownership stake of the VC holdings is determined from the equivalent number of effective stock of the venture capitalist relative to the total number of outstanding common shares. Hence for q_1 number of shares the VC holds

$$x_{t_0} = \frac{q_1}{q_t} = \frac{I_0}{V_{t_0}^+}$$

before the new issuance.

Note that after money valuation is equal to $V_{t_0}^+ = \begin{cases} V_{t_0} & \text{if} \quad t < t_0 \\ V_{t_0} + I_0 & \text{if} \quad t = t_0 \end{cases}$

The new conversion price p is defined from the weighted average formula

[9] For example if the VC holds 200 shares with the conversion (conversion rate 1:1) price of 5$ each, his investment is equal to 1000$. The management and founders altogether have 200 shares evaluated at 5$. The company needs further financing, hence issue additional shares at 2$ price per share for 200 more shares. The new conversion price, calculated from the weighted average formula, is equal to $\dfrac{200*5+200*2}{200+200} = 3.25$$ per share. The VC will call for $(200*5)/3.25 - 200 = 107.7$ free shares. Therefore the difference $200*(5-3.25) = 3.25*(307.7-200) = 350$$ extra in value, is the payoff that the anti-dilution option guarantees to the VC.

$$p = \frac{q_1 p_1}{q_1 + q_2} + \frac{p_2 q_2}{q_1 + q_2} \tag{51}$$

or

$$p = (1 - k)\, p_1 + k p_2 \tag{52}$$

6.1 No Dilution Protection

Consider first the case of a VC with no dilution protection. At time T the entrepreneur decide to issue more shares in order to collect more financing for the project. q_2 is the effective number of equivalent shares of common stock newly issued. Define x_1^* as the ownership stake of the VC holdings after the event.

$$x_1^* = \frac{q_1}{q_t + q_2} \tag{53}$$

The VC diluted ownership stake is defined as

$$x_1^* = x_{t_0} \left(\frac{p_2}{p_2 + I^+} \right) \tag{54}$$

if we normalize for $q_t = 1$.

Proposition 5 *For $q_1 = 1$ and assuming that the new issue is done at the firm value $p_2 = V_t$, the VC diluted ownership stake is equal*

$$x_1^* = x_{t_0} \left(\frac{V_t}{V_t^+} \right) \tag{55}$$

for $V_t^+ = V_t + I_0$.

Proof. Rearranging terms

$$x_1^* = \frac{q_t x_{t_0}}{q_t + \left(\frac{k}{1-k}\right) q_t x_{t_0}} \tag{56}$$

$$x_1^* = \frac{(1 - k)\, x_{t_0}}{1 - k + x_{t_0} k} \tag{57}$$

Using the following expression

$$I_t^+ = \left(\frac{k}{1 - k} \right) q_t x_{t_0} p_2 \tag{58}$$

$$x_1^* = x_{t_0} \frac{\frac{k q_t x_{t_0} p_2}{I_t^+}}{\frac{k q_t x_{t_0} p_2}{I_t^+} + x_{t_0} k} \tag{59}$$

$$x_1^* = x_{t_0} \left(\frac{p_2}{p_2 + I_t^+} \right) \tag{60}$$

■

Furthermore $x_1^* < x_{t_0}$ since $\frac{V_t}{V_t^+} < 1$. The factors driving the VC ownership stake are the additional investment amount I_t^+ and the price per share of the new issue p_2.

6.2 Dilution Protection

The dilution protection aims at reducing the dilution impact on the VC holdings in case of a new financing round. In other words the VC is entitled to a certain amount of free shares q_1^+ to compensate for the loss in value of its holdings[10].

$$x_2^* = \frac{q_1 + q_1^+}{q_t + q_2 + q_1^+} \tag{61}$$

where

$$q_1^+ = \frac{q_1 * p_1}{p} - q_1 \tag{62}$$

according to the weighted average formula.

Proposition 6 *For any new issue at a price level lower than the old conversion price $V_t \leq p_1$ the dilution protection offers to the VC a larger share $x_2^* > x_1^*$, defined as*

$$x_2^* = \frac{p_1 x_1^*}{x_1^* p_1 + (1 - x_1^*)\, p} \tag{63}$$

Otherwise for any issue at $V_t > p_1$ the VC ownership stake lends at $x_2^* = x_1^*$. In both cases the VC ownership stake is diluted $x_{t_0} > x_2^*$ and $x_{t_0} \geq x_1^*$.

Proof. Replacing back q_{VC}^+ the VC ownership stake takes the following form

$$x_2^* = \frac{\frac{q_1 * p_1}{p}}{q_t + q_2 + \frac{q_1 * p_1}{p} - q_1} \tag{64}$$

$$x_2^* = \frac{p_1}{p}\left(\frac{q_1}{q_t + q_2 + \frac{q_1 * p_1}{p} - q_1}\right) \tag{65}$$

Since in the previous section we derived

$$x_1^* = \frac{q_1}{q_t + q_2} \tag{66}$$

the protected after ownership stake is a function of not protected after ownership stake.

$$x_2^* = \frac{p_1}{p}\left(\frac{q_1}{\frac{q_1}{x_1^*} + \frac{q_1 * p_1}{p} - q_1}\right) \tag{67}$$

[10] Note that these shares are given to the VC for free.

Simplifying for q_1 the expression is equal to

$$x_2^* = \frac{p_1 x_1^*}{p + x_1^* (p_1 - p)} \tag{68}$$

Another implication concerns $x_2^* > x_1^*$ given that

$$p_1(\text{old conversion}) > p \ (\text{new conversion}) \tag{69}$$

and

$$p + x_1^* (p_1 - p) = x_1^* p_1 + (1 - x_1^*) p > x_1^* p_1 + (1 - x_1^*) p_1 \tag{70}$$

■

The VC stake after the cash inflow, is driven from the new investment amount I^+, the previous conversion price p_1, the issuing price set at firm value $p_2 = V_t$ and lastly the existing VC ownership stake before the new round x_{t_0}. The antidilution contractual feature sets the VC ownership stake at a higher level than if no dilution protection is in place.

Proposition 7 *An important consequence can be drawn so far in terms of relation between the two ownership stake:*

$$\textit{VC ownership stake}\,(x^*) = \begin{cases} x_2^* & x_{t_0} > x_2^* > x_1^* \quad \textit{if} \quad p_1 \geq V_t = p_2 \\ x_1^* & x_{t_0} > x_2^* = x_1^* \quad \textit{if} \quad p_1 < V_t = p_2 \end{cases} \tag{71}$$

$x_2^* > x_1^*$ *since* $\frac{p_1}{p} > 1$ $(q_t = 1)$ *where* x_{t_0} *is the original VC ownership stake.*

6.3 Antidilution feature value

The antidilution feature would be the dilution protection offered to the VC in case of new financing round.

Proposition 8 *The antidilution feature at time t is modeled as the difference between the value of the VC ownership stake with dilution protection versus the respective ownership stake in case of no dilution protection multiplied by the after money valuation.*

$$f(V_t, t) = \max\left((x_2^* - x_1^*)\, V_t^+, 0\right) \tag{72}$$

This expression is equivalent to

$$f(V_t, t) = \max\left(\left(\frac{x_1^* (1 - x_1^*) (p_1 - p)}{p + x_1^* (p_1 - p)}\right) V_t^+, 0\right) \tag{73}$$

The antidilution feature is similar to a put option. At the contractual time t_0, it is difficult to predict p_2, the price at which new shares are released. For $p_1 = \frac{I_0}{q_t x_{t_0}}$ prevailing conversion price per share of preferred stocks prior to new issuances and for $p = (1 - k) p_1 + k p_2$,and $k = \frac{q_2}{q_1 + q_2} = \frac{I_t^+}{V_T x_{t_0} + I^+}$, $p_2 = \frac{V_t}{q_t}$ we have

$$p = \frac{1}{q_t}\left(\frac{V_t}{V_t x_{t_0} + I_t^+}\right)\left(I_0 + I_t^+\right) \quad (74)$$

If the issuing price is set at $V_t = V_{t_0}$ there is no protection $f(V,t) = 0$, given $p_1 = p$[11].

It is particularly difficult to evaluate such an option since it V_t, x_2^*, x_1^* are all stochastic variables. In order to derive some results we set the prevailing conversion price of preferred stocks equal to the initial one p_1. Hence

$$p_1 q_1 = p_1 q_t x_{t_0} = I_0 \quad (75)$$

Once more if we normalize $q_t = 1$ then $p_1 = \frac{I_0}{x_{t_0}}$. The next step is to check for the antidilution feature once the price of shares of the new issue is lower than the preset conversion price level. Table 5 illustrates the impact of an additional issue at lower or higher price level than the preset conversion price. Consider a two stage financing process, and a new entry at year three $t = 3yr$ of the project life.

It is interesting to see that for a fixed additional amount of money $I_t^+ = I_0^+ * \exp^{rt}$ for $I_0^+ = 60$ commited at time zero, the ownership stake after dilution is subject to the new issuing price set at the firm value level. In other words the VC ownership stake is subject not only to the flow of the amount of money that the entrepreneur is getting, but more importantly to the level of the issuing price at the financing round. Let us consider a particular case: The VC has invested at the beginning of the project and holds $x_{t_0} = x_{tot} = 45.9\%$ of the firm value equivalent to $I_i = \{40, 20\}$ and $I_{tot} = I_0 + I_1 * \exp^{rt_0}$. The new financing round $I_t^+ = I_0^+ * \exp^{rt}$ takes place after 3 years at $V_t = 65.4147\$$. The preset conversion price $p_1 = \frac{I_{tot}}{x_{tot}}$ for $q_t = 1$ is equal to 132.6. After the new issue the VC's x_{t_0} ownership stake is diluted leading to a lower level of $x_1^* = 22.5\%$ of the firm value. The antidilution protection entitles VC to a certain amount of free shares q_1^+ up to the initial investment cost of I_{tot}. That bids up the VC ownership stake at $x_2^* = 30.9\%$ of the firm value Hence the antidilution protection is the difference between $(x_2^* - x_1^*)$ multiplied by the after money firm value $V_t^+ = V_t + I_t^+$, equal $(30.9\% - 22.5\%) * (65.4147 + 67.6498) = 11.20$ protection value.

The following conclusions can be drawn so far: New financing rounds dilute the VC holdings, no matter the price of the new issue. The ownership stake of the VC is partially protected for lower price financing rounds. Third, the lower the price of the financing round, the higher the protection offered to the VC. The higher the price of the financing round the lower the dilution impact on the VC holdings. The VC exercises his/her antidilution protection only if the barrier (old conversion price) is reached. The non protected VC ownership stake is positively related to the issuing price. The ownership stake has a complex concave V-shape. For a low issuing price the entrepreneur transfers to the VC a

[11] $\left(\frac{V_t}{V_t x + I^+}\right)\left(I_0 + I^+\right) = V_t$ for $V_t = V_{t_0}$ we have $I_0 = V_t x$.

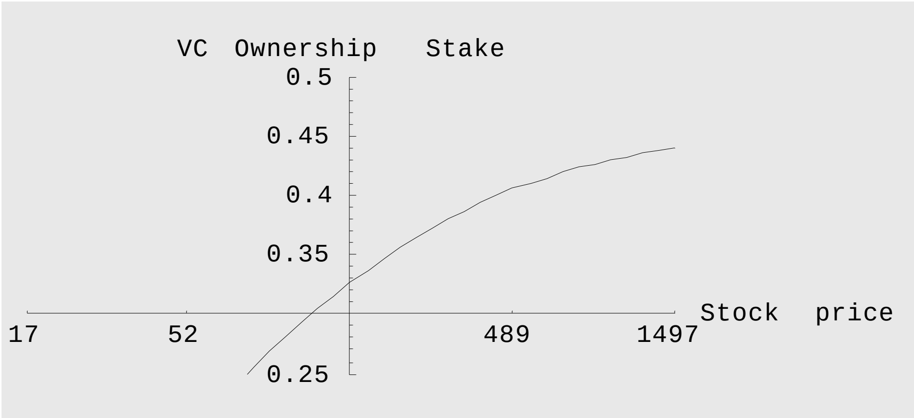

Figure 8: VC ownership stake x_1^* with no operative dilution protection. VC diluted stake is an increasing concave function of the issuing price. For very large issuing price the VC stake would converge to the ownership stake before the new issue.

larger number of free shares, the ownership stake goes up by more to compensate for the low price level. If the issuing price is around the old conversion price level, no protection is needed. And lastly, if the issuing price is high or little money is coming into the project, the dilution level is negligible ($\frac{V_t}{V_t+I_t^+} \simeq 1$). An important feature is captured by figure 8 which shows that for high (low) issuing price, the VC ownership stake increases at a decreasing rate.

Next we consider the sensitivity analysis of the VC ownership stake to factors driving its value. Given the direct dependency of the VC ownership stake to the issuing price at the time of the new financing round, the expected future ownership stake is given from the average of all possible ownership stakes. In other words we report

$$x^* = \frac{1}{2M+1} \sum_{k=1}^{2M+1} x_k^* \tag{76}$$

for $k = 1, 2...2M+1$, for M number of space steps in the grid.

Table 6 shows that a high initial firm value V_{t_0} implies a lower stake on the project payoff, but a higher preset conversion rate (strike price for the antidilution feature goes from $132.6 \rightarrow 151.1$). The more the VC is involved in occuring additional costs υ the lower would the strike price be for the antidilution feature to come to existence, since the reward s/he is getting for his/her non-financial contribution (higher stake) serves as a cushion for this feature. Higher υ_i means higher x_{t_0},hence lower p_1. For large investments $I_i = \{60, 20\}$, the VC requires a higher strike price for his/her protective option $p_1 = 145.6$. Lastly, the level of dilution $x^* = 33.0\%$ depends positively on the amount of money injected in future financing rounds. Nonetheless it has no impact on the strike price level

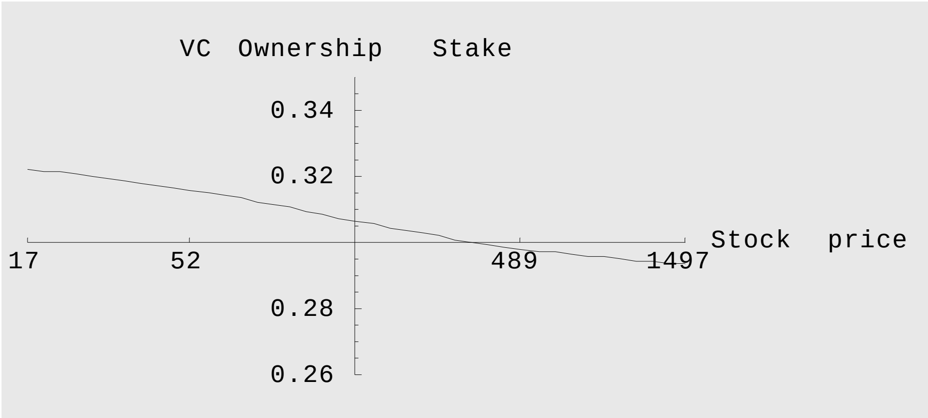

Figure 9: The lower is the issuing price the higher would be the VC stake x_2^*, given the anti_dilution protection.

$p_1 = 132.6$ at which the antidilution feature comes to existence.

6.4 Conclusions

Our analysis has thus shown the impact on the contract value of the antidilution feature and has shown how it transforms the underlying's distribution. This contractual feature can now be balanced economically in a contractual negotiation versus other features. The analysis could be pushed further as well. For example, we could extend it in order to compare the value impact of different antidilution formulae (such as the "top off" that guarantees equity percentage, or the "full ratchets" that maintains value).

7 Full Contract Valuation with Features Interaction

This section looks at the staging process with a more complete payoff at any staging date, thus integrating the liquidation feature and the antidilution protection in the contract valuation. The methodology, although it does not allow for analytic solutions, let us investigate the interaction between the different features described above. It also allows for the more realistic feature of liquidation consideration at each refinancing point. We also consider the strong valuation impact of a public offering, taking here the analogy of a jump in valuation, as well as the impact of the lockup the VC is classically tied by (VCs are typically not allowed to sell their shares at the time of the IPO, sometimes by regulation, as in the US). We also integrate the underpricing of IPOs and the fact that the literature has documented a relationship between underpricing and long term

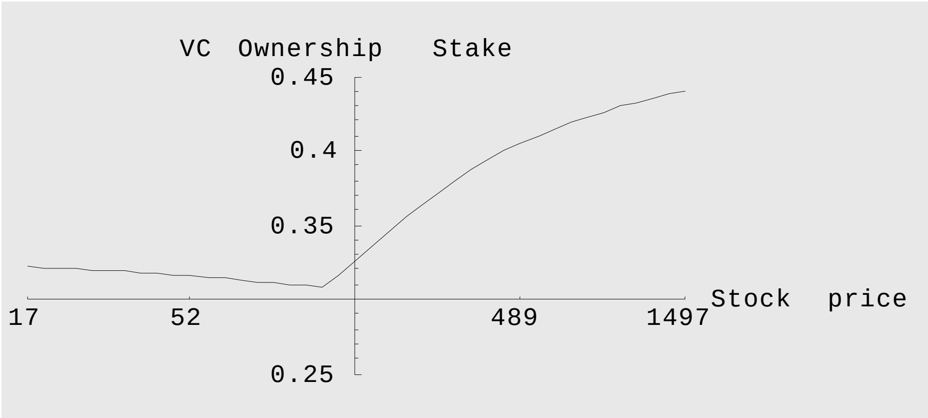

Figure 10: VC ownership stake x^* behaviour including anti-dilution downside protection. The VC stake reaches the lowest level for price levels near the old conversion price (srike price), and is higher in extrems. Note that x^* is defined as in equation (71).

performance as well as between shares sold by the VC and the lockup expiration price effect (Field and Hanka (2000), Bradley and alii (1999), Ofek and alii (2000)). The VC decision to continue or to drop the project includes the following features: staging, the upside potential, converting his/her holdings in common shares, the liquidation right at any liquidation date, the antidilution protection as well as the possibility of a public offering.

While the VC may commit to invest the necessary amount, staging the investment rewards him with the option to liquidate the project if certain predefined milestones are not reached. Logically the option to abandon adds value to his/her contract. Staging alone is beneficial not only for the VC in limiting any downside losses, revaluing the project and repeatedly updating his/her filtration based on information up to actual time t, limiting the asymmetric information problem, monitoring the project, but also facilitating the financing problem for the entrepreneur by committing to a total investment, as well as reducing window-dressing concerns.

This section is in line with the previous sections since it incorporates all the specific assumptions made for each contractual feature. Furthermore it outlines a framework which considers contemporaneously all these features altogether. It allows for jumps in the firm value if the VC decides to further invest given the available information at that time. The VC contract value is conceptualized as the sum of the automatic conversion and liquidation right. Secondly, the VC ownership stake is an endogenous variable depending on the investment amount, initial firm value and more importantly on the VC value added. The more effort VC puts in the project, the more he has to be rewarded for. Moreover the VC value added (the VC non-financial contribution) being a cost for the VC, may

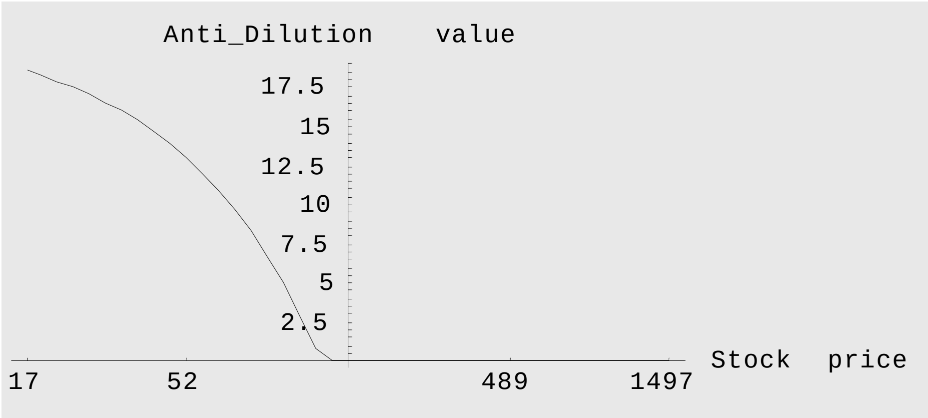

Figure 11: Anti_dilution value as a function of issuing price.

induce him to exit at early stages if the project performs badly. The valuation is carried on taking care of the firm value before and after money valuation. As before we make use of the finite difference method in valuing the contractual features altogether.

7.1 Project characteristics

Table 7 shows that future uncertainty is positively related to the automatic conversion value and negatively related to the liquidation right value. The marginal impact of the project's uncertainty is larger for the automatic conversion value due to the increase in the probability level of reaching upper states of the firm value. Table 8 looks at the VC additional capital costs. The more the VC is involved in the project, the higher the stake that s/he is entitled to in case conversion takes place at the exit. There is no impact though on the downside value of the liquidation right. Table 9 studies the impact of project's drift on the VC payoff. The results show that both components of the liquidation preference will increase if the drift rate increases, although automatic conversion has a higher marginal impact on VC contract value than liquidation rights. Lastly, table 10, a long term contract is more valuable than a short term one. Longer maturity involves higher uncertainty for the future, therefore a higher contract value. The automatic conversion drives the VC contract value for different maturity values. The maturity has a very low effect on downside value. This is due to the VC commitment to help the management for a longer (shorter) period, hence requiring a larger (reduced) ownership stake.

7.2 Staging, liquidation, convertibility and antidilution

Any additional investment from another party will dilute the VC claims on the project value. If the need for alternative resource funding is high, the VC will be diluted in terms of ownership stake. For low issue price the antidilution feature becomes more valuable.

At time t_0, assume that both parties know the additional amount of money to be raised from another source and when the event will take place. Nonetheless they are unable to predict the firm value of the new issue, since the project value process is unpredictable. The only way to deal with this problem is to create an expectation of all possible ownership stakes that the original VC will end up with after the financing round. Therefore the empirical counterpart of the expectation is defined in the equation (76).

The last updated conversion price as reference in calculating the dilution effect on the original VC contract value of a new entry in the venture project, is equal to that firm value that can guarantee the original investment back. The only differences with the base case model are: the VC ownership stake at the final date x_{tot} is replaced by the average of all possible diluted stakes x^*, and any new cash inflow will increase the firm value.

The following tables focus on the impact of the after money firm value and newly diluted ownership stake in different scenarios. Table 11 looks at an increase in the firm value after the financing round assuming that the original VC ownership stake has not been affected. We have a very large effect on the upside part of the contract, nonetheless the results can be considered to be wrong since the assumption of no dilution is not correct.

The following two tables 12 & 13 consider not only a jump in the firm value but also a diluted ownership stake of the VC. The level of this dilution is different if protective measures such as dilution protection are in place. It is clear the protection covers partially losses from dilution (p.ex. $(188.888 - 149.069)$=39.819$ instead of $(188.888 - 129.086) = 59.820$$). Nevertheless this is an overall measure since we are working with an average of all possible ownership stakes at year three.

7.3 Alternative Exit Provision/ IPO with lock-up period

Assume now that the VC has the option to choose at time T, at the end of a three year project, between liquidation preferences (possibly applied in merger or acquisition exits) or demand registration rights (applied in initial public offerings). In case of an IPO, the VC will be required to lock his/her shares for 3-6 month, and only after that date the VC will be able to exit from the venture. Hence at time T the VC has to decide whether a liquidation preference (M&A case) or demand registration right (IPO) would maximize his/her terminal payoff.

We model a case with a 6 month lock up period. The model is flexible enough to incorporate any lock up period. Furthermore we extend the model including different IPO coefficients like mid-term price effect, any jump in the first transaction at the IPO date, volume of the shares released at the lock-up date, as well as any underpricing possibility, widely used especially in the VC environment. The coefficients β_2 and γ capture respectively the mid term positive impact of the underpricing coefficient β_1 and the jump of the project value at the time of public offering $t = T$. The VC ownership stake x^* is the empirical counetrpart of the expected diluted ownership stake as in equation (75).

The timing is as described in the subsequent framework:

Stage 1	Stage 2	Lock-Up	
0	$\frac{T}{3}$	T	$T + \Delta$
Invest I_0	Invest I_1or Liquidate	Exit[2]	Exit[1]

1. $\Phi(V, T + \Delta) = x^* V_{T+\Delta} - C_{tot}$
2. $\max \left(\frac{E_T[\Phi(V,T+\Delta)]}{(1+\rho)^{\Delta}}, \max \left(x^* V_{T+\Delta} - C_{tot}, \min \left(V_{T+\Delta}, L_{tot} \right) \right) \right)$

Figure 12: Time line for MA/IPO with a lock up period.

The VC contract value at the lock up date is as follows:

$$\Phi(V, T + \Delta) = x^* V_{T+\Delta} - C_{tot} \tag{77}$$

The VC will go for an initial public offering, hence exercise his/her demand rights at the end of the third year T only if the expected value of his/her shares at expiration of the lock up $T + \Delta$ (after six months) is higher than the value of his/her shares for a merger or acquisition. Consequently,

$$\Phi(V, T) = \max \left(\frac{\tilde{E}_T \left[\Phi(V, T + \Delta) \right]}{(1 + \rho)^{\Delta}}, \max \left(x^* V_T - C_{tot}, \min \left(V_T, L_{tot} \right) \right) \right) \tag{78}$$

In order to find the value of the contract at the contractual time, once again we work recursively up to time $t_0 = 0$ in the same fashion as for the basic case. Furthermore we are able to determine the level of the probability that the VC at time T will prefer an initial public offering versus a merger/acquisition.

$$\Pr_{IPO} = \Pr \left(\frac{\tilde{E}_T \left[\Phi(V, T + \Delta) \right]}{(1 + \rho)^{\Delta}} > \max \left(x^* V_T - C_{tot}, \min \left(V_T, L_{tot} \right) \right) \right) \tag{79}$$

The way to proceed is the following. Initially set all the coefficients equal to zero, so that there is no external impact on the choice of the exit strategy. Calculate the probability that an IPO is preferable to a merger or an acquisition exit. The following step is to look at the impact that the above mentioned coefficient may have not only in the final payoff, but also in determining the probability of an IPO.

Table 15 shows that the main factors driving the probability of an IPO are the VC value-added as well as the initial firm value before investing in the project. Furthermore any increase on the level of the following parameters; volatility σ_i, drift rate $(\alpha_i + \delta_i)$, VC additional cost υ_i as well as investment level I_i increases the VC payoff at the lock up date. An increase in V_{t_0} has a negative impact, which may be related to a lower firm value at the end of the project as well as a lower ownership stake.

We also model a classical IPO process with underpricing, lock up and price impacts at medium term of the underpricing (as per the Allen and Faulhaber (19) model for example) as well as the price impact of the lock up expiration. Our model can thus be used in complete reproduction of market conditions, from seed financing to exit. It encompasses a good number of the many contractual options observed in venture capital, from early investment contractual features including choice of securities, liquidation preference, antidilution to the lock up period at the time of exit.

The higher are the coefficients related to the underpricing, positive mid-term impact and jump in value coming from the market the more valuable is the VC contract value at the time zero. The increase in value may be mitigated by a negative market reaction to the quantity of the VC shares released at the lock up date.

8 Conclusion and Remarks

We analyze in terms of real options four classical features of venture capital contracts: staging, liquidation preference, convertibility and antidilution. It is shown what effect these covenants have not only on the value of the contracts but also on the distribution of the final outcome. This work could be the basis for negotiation around the features of the contracts or for overall redesign for optimal contracting.

The high non linearities of the features analyzed here should have convinced the reader early on that linear analysis of the IRR or NPV type would not work. But beyond valuation issues, the issue of contract design lies ahead. Better design (as shown on the staging feature of contracts) could improve current contract, and contract negotiation. We present results that deal with the interaction of the different features in a complete set-up and thus provide the basis for the design of equity investment contracts.

As outlined by Kaplan and Stromberg (2002), venture capital contracts are best seen as flexible contractual mechanisms for the contingent reallocation of control, ownership and cash flow rights. But for this complex package of real options to attain its goal of optimally balancing incentives, risk protections, and the sharing of the upside potential, we need to be able to look at the basket of options as a basket, not a collection of individually priced options. A first attempt at this can be found here but this is but a first glimpse at the complex universe of venture capital investment contracts. The contribution from the real options approach should not be underestimated though, and it holds the promise of significant advances in the understanding of the true value-added of contractual features which have often been part of "boilerplate" contracts for decades with little understanding of their effect on the investors' wealth nor behavior.

Much further research is needed in the area. The analysis could be extended in many ways. More contractual features could be studied (notably for antidilution). The analysis itself could be refined many ways, for example by allowing for more stochastically state variables (and thus allowing for the well-known leptokurtosis of actual returns). It would be nice also to introduce asymmetric information issues as well as agency issues (may be in a set up a la Anderson and Sundaresan (1996)), although the challenges of combining continuous time pricing to complex games remain hard to solve. Finally, new contracting solutions should be explored, bringing the fields of law, finance and microeconomics together for a search for an optimal design.

9 Appendix A. Illustrative Term Sheet for VC Investment

Company A

A California Corporation

Terms of Series C Preferred Stock Financing

Issuer: Company A (the "Company").

Security: Series C Preferred Stock.

Amount of Series C Financing: 5,000,000 shares of Series C Preferred.

Price: $1.00 per share of Series C Preferred Stock, for an aggregate purchase price of up to $5,000,000.

Dividends: The holder of the Series C Preferred shall be entitled to receive dividends at a rate of 8% of the Original Purchase Price per annum when and if legally declared by the Board of Directors. The holder of Series C, Series B and Series A shall receive dividends in preference to any dividends on Common Stock.

- Liquidation Preference: The holders of the Series C, Series B and Series A Preferred will be entitled to receive in preference to the holders of the Common Stock n amount equal to their Original Purchase Price per share plus all declared but unpaid dividends (if any). Series B and C Preferred will be participating so that after the ayment of the Original Purchase Price (plus unpaid dividends, if any) to the holders of Series B and C Preferred, the remaining assets shall be distributed pro rata to holders of Series B and C Preferred and Common on a common equivalent basis until the holders of Series B and C Preferred have received an aggregate of two (2) times their Original Purchase Price (including the preference amount set forth in the preceding paragraph). Thereafter, all remaining assets shall be distributed pro rata to the holders of Common Stock.

- Conversion: A holder of the Series C Preferred shall have the right to convert the Series C Preferred at the option of the holder, at any time, into shares of Common Stock. The total number of Common Shares into which the Series C Preferred may be converted initially will be determined by dividing the Original Purchase Price by the "Conversion Price". The initial Conversion Price shall be the Original Purchase Price. Initially, each share of Series C Preferred will convert into one share of Common Stock.

- Automatic Conversion: All Preferred shares automatically will be converted into Common upon (1) the closing of an underwritten public offering of shares of ommon Stock of the Company at a public offering price per share (prior to underwriting commissions and expenses) in an offering of not less than $10 million, before deduction of underwriting discounts and registration expenses or (2) approval of a majority of the Preferred.

- Antidilution Protection: The Conversion Price of the Series C Preferred shall be proportionately adjusted in case of subdivisions, stock dividends, combinations, consolidations or reclassifications of Common Stock. The Conversion Price shall be subject to full ratchet antidilution protection till July 31, 2000 and broad based weighted average dilution adjustment thereafter.

- Voting Rights: The Series C Preferred will vote together with the Series A and Series B Preferred Stock and with the Common Stock and not as a separate class. Each of Series C Preferred shall have a number of votes equal to the number of shares of Common Stock then issuable upon conversion of such shares of Series C Preferred.

- Protective Provisions: For so long as any shares of Preferred remain outstanding, consent of the holders of at least 50% of the Preferred then outstanding, voting as a single class, shall be required for any amendment of the Articles of Incorporation, bylaws, or other charter documents of the Company which (1) adversely alters or changes the rights, preferences, or privileges of any Preferred Stock, or (2) increases or decrease the authorized number of shares of Preferred Stock.

- Restrictions or Transfers: The Company will have the right of first refusal to purchase an investor's shares if the investor seeks to sell or otherwise transfer investor's Series C Preferred or the Common Stock into which such Series C Preferred is converted to a competitor of the Company. This provision shall terminate upon a registered public offering of the Company's Common Stock.

- Registration Rights: Demand Rights: If investors holding at least 10% of the Preferred (or Common issued upon conversion of the Preferred or a combination of such Common and Preferred) request that the Company file a Registration Statement for at least 20% of the their shares, the

Company will use its best efforts to cause such shares to be registered; provided, however, that the Company shall not be obligated to effect any such registration prior to the earlier of (1) December 31, 2000 and, (2) Within six months following the effective date of the Company's initial public offering. The Company shall not be obligated to effect more than two registrations under these demanded right provisions. Company Registration: The holders of the Preferred shall be entitled To "piggy-back" registration rights on registration of the Company's Shares or on demand registrations of any later round investor subject to the right, however, of the Company and its underwriters to reduce the number of shares proposed to be registered pro rata in view of market conditions. S3 Rights: The holders of the Preferred shall be entitled to two demand registrations on Form S-3 (if available to the Company) so long as such registration offerings are in the excess of $3,000,000. Expenses: The Company shall bear registration expenses (exclusive of underwriting discounts and commissions and special counsel of the selling shareholders) of all demands, piggy-backs, and S-3 registrations. The expenses of any special audit required in connection with a demand registration shall be borne pro rata by the selling shareholders. Transfer of Rights: The registration rights may be transferred Provided that the Company is given written notice thereof and provided that the transferee receives at least 100,000 shares. Other Provisions: Other provisions shall be contained in the Series C Preferred Stock Purchase Agreement, the Exchange Agreement, and such other documents acceptable to the Company and the holders of the Preferred, with respect to registration rights as are reasonable, including cross-indemnification, the period of time in which the Registration Statement shall be kept effective, standard standoff provisions, underwriting arrangements and the ability of the Company to delay demand registrations for up to 90 days (S-3 Registrations for up to 60 days).

- Purchase Agreement: The investment shall be made pursuant to a Series C Preferred Stock Agreement and other related documents reasonably acceptable to the Company and the investors. Such agreements shall contain, among other things, appropriate representations and warranties of the Company and covenants of the Company reflecting the provisions set forth herein and appropriate conditions of closing, which will include, among other things, qualification of the shares under applicable Blue Sky laws and filing of Amended and Restated Articles of Incorporation. Unless expressly excluded by this Term Sheet, such documents shall contain such other provisions as are customary in connection with venture financings. Purchasers who purchase in the aggregate more than 300,000 shares may elect to purchase a portion of their shares in equal monthly installments provided that the first installment is purchased by July 15, 1998 and all of the remaining shares are purchased by December 31, 1998. The installment purchase arrangement shall accelerate with the closing of a Series D financing, or, at the decision of the Board, if the Board reasonably believes that the undelivered portion is necessary to execute its Plan.

10 Appendix B.

Proof. Define the upside potential to convert his/her holdings in common stocks and downside protection to exercise his/her liquidation right as follows:

$$\begin{aligned}\varphi_1(V_t) &= (x_{t_0}V_t - (C_0 + L_0))^+ \\ \varphi_2(V_\tau) &= (V_\tau - L_0)^+\end{aligned}$$

Hence the partial differential equation for each of the options are:

$$\frac{1}{2}\sigma^2 V_t^2 \varphi_{ivv}(V_t) + (\alpha + \delta)\, V_t \varphi_{iv}(V_t) - \rho\varphi_i(V_t) = 0 \tag{80}$$

Identify the general solution as $\varphi_2(V_t) = BV_t^u$ and $\varphi_1(V_t) = AV_t^u$.It is important to note that $u > 1$. The polynomial root u is computed as:

$$u = \frac{-(\alpha + \delta - \frac{\sigma^2}{2}) + \sqrt{(\alpha + \delta - \frac{\sigma^2}{2})^2 + 2\sigma^2\rho}}{\sigma^2}, \tag{81}$$

The boundary conditions of value matching (vm) and smooth pasting (sp) for the upside potential are as follows:

$$\begin{cases} AV_T^u = V_T - L_0 & vm \\ BV_T^u = x_{t_0}V_T - (C_0 + L_0) & vm \\ AV_T^{u-1}u = 1 & sp \\ BV_T^{u-1}u = x_{t_0} & sp \end{cases}$$

These four equations determine the four unknowns, critical values of the project that trigger liquidation :

$$\begin{cases} A = \frac{1}{u}V_A^{*1-u} \\ B = \frac{x_{t_0}}{u}V_B^{*1-u} \\ V_A^* = \left(\frac{u}{u-1}\right)L_0 \\ V_B^* = \left(\frac{u}{u-1}\right)\frac{(C_0 + L_0)}{x_{t_0}} \end{cases}$$

For $V_t < V_A^*$ the VC payoff is determined from the liquidation right, for $V_A^* \leq V_t < V_B^*$ is as if the VC would hold a riskless bond plus the option to convert for high firm value, whereas for $V_t \geq V_B^*$, the VC converts his/her holdings and exits the venture project. ■

11 Appendix C.

Proof. The VC contract value for the each stage must satisfy the differential equation

$$(\alpha_i + \delta_i)\, V_t \varphi_{iV}(V_t) + \frac{1}{2} V_t^2 \sigma_i^2 \varphi_{iVV}(V_t) - \rho_i \varphi_i(V_t) = 0 \tag{82}$$

The VC payoff at the end of first round / beginning of second round $t = \tau_1$ is

$$\varphi(V_{\tau_1}) = \max\left(\varphi_1(V_{\tau_1}) + x_{t_0} V_{\tau_1} - C_0, 0\right) \tag{83}$$

First we solve for $t = \tau_2$ at the final boundary

$$\varphi_1(V_{\tau_2}) = \max\left(x_{t_1} V_{\tau_2} - C_1, 0\right) \tag{84}$$

given the boundary conditions:

$$x_{t_1} V_B^* - C_1 = B V_B^{*q_1} \tag{85}$$

$$x_{t_1} = B q_1 V_B^{*q_1 - 1} \tag{86}$$

$$\varphi(0) = 0 \tag{87}$$

The coefficients for both stages take the following values:

$$q_i = \frac{\left(-\left(\alpha_i + \delta_i\right) + \frac{\sigma_i^2}{2}\right) + \sqrt{\left(\left(\alpha_i + \delta_i\right) - \frac{\sigma_i^2}{2}\right)^2 + 2\sigma_i^2 \rho_i}}{\sigma_i^2} \tag{88}$$

Thus the general solution have the well known form: $\varphi_1(V_t) = B V_t^{q_2}$ where $q_2 > 1$. The unknown coefficient and the critical project value V_B^* are identified as

$$B = \frac{x_{t_1}}{q_1} V_B^{*1-q_2} \tag{89}$$

$$V_B^* = \frac{C_1}{x_{t_1}} \left(\frac{q_2}{q_2 - 1}\right) \tag{90}$$

Given the $\varphi_1(V_t)$ and V_B^* we solve for $\varphi(V_t)$. The procedure to solve for the optimal threshold V_A^* and the VC contract value during the first stage $\varphi(V_t)$ is the same. The VC contract payoff admits the general solution for $\varphi(V_t) = A V_t^{q_1}$ where $q_1 > 1$ computed above. The VC contract value for the first stage satisfy the differential equation and the following boundary conditions:

$$B V_A^{*q_2} + x_{t_0} V_A^* - C_0 = A V_A^{*q_1} \tag{91}$$

$$q_2 B V_A^{*q_2 - 1} + x_{t_0} = A q_1 V_A^{*q_1 - 1} \tag{92}$$

$$\varphi(0) = 0 \tag{93}$$

The unknown coefficient A and V_A^* are also determined from the boundary conditions and identified as

$$A = \frac{q_2}{q_1} B V_A^{*q_2-q_1} + \frac{x_{t_0}}{q_1} V_A^{*1-q_1} \tag{94}$$

$$C_0 = B V_A^{*q_2}\left(1-\frac{q_2}{q_1}\right) + x_{t_0} V_A^*\left(\frac{q_1-1}{q_1}\right) \tag{95}$$

V_A^* is the solution of the above polynomial and it stands for the critical price of further investing. For $q_1 = q_2$, constant parameters the above expression are simplified as follows:

$$A = B + \frac{x_{t_0}}{q_1} V_A^{*1-q_1} \tag{96}$$

$$V_A^* = \frac{C_0}{x_{t_0}}\left(\frac{q_1}{q_1-1}\right) \tag{97}$$

For the case that $q_1 = q_2$, the VC contract value is then determined in three regions:

$$\varphi(V_t) = \begin{cases} \left(\frac{x_{t_1}}{q_1} V_B^{*1-q_1} + \frac{x_{t_0}}{q_1} V_A^{*1-q_1}\right) V_t^{q_1} & if \quad t < \tau_1 \\ \frac{x_{t_1}}{q_1} V_B^{*1-q_1} V_t^{q_1} + x_{t_0} V_t - C_0 & if \quad \tau_1 \leq t < \tau_2 \\ \left(x_{t_1} + x_{t_0}\right) V_{\tau_2} - (C_1 + C_0) & if \quad t = \tau_2 \end{cases} \tag{98}$$

For the general case, the optimal level of investing in the next stage V_A^* (or V_B^*) depends on the conversion level C_i, on the VC ownership stake x_{t_i} on the project value as well as on the level of the uncertainty σ_i, project's growth rate $(\alpha_i + \delta_i)$ and VC additional cost υ_i. ■

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VC Contract Value $T = 3yr$				
Investm. I_0	VC Contract Value[1]	Liquidation Right[2]	Automatic Conversion[3]	Upside Potential[4]
40	141.661	39.572	102.148	126.241
60	193.926	58.660	135.333	71.472
80	245.83	77.387	168.518	216.703
100	297.471	95.851	201.704	261.935

1.$\Phi(V_T, T) = \max(x_{t_0} V_T - C_0, \min(V_T, L_0))$
2.$V_T - \varphi_2(V_T) = \min(V_T, L_0) = V_T - \max(V_T - L_0, 0)$
3.$\varphi_1(V_T) = \max(x_{t_0} V_T - (C_0 + L_0), 0)$
4.$\Phi'(V_T, T) = \max(x_{t_0} V_T - C_0, 0)$

$V_{t_0} = 100, I_0 = 60, C_0 = 60, L_0 = 60, \alpha_1 + \delta_1 = 0.35, \upsilon_1 = 0.05, \sigma_1 = 0.6, r = 4\%$

Table 1: The VC contract value at time t=0 as the linear combination of the liquidation right (2) and automatic conversion (3). The table shows that VC contract value (1), determined from the (liquidation preference), (1)=(2) +(3), hence the closed form pricing formula is correct. Furthemore one would overvalue the VC contract value if it does not account for the fact that converting it also implies giving up the downside protection.

VC Contract Value					
Parameters		Ownership Stake x_{t_0}	VC Contract Value[1]	Liquidation Right[2]	Upside Potential[3]
I_0	40	0.332	141.661	39.572	126.241
	60	**0.436**	**193.926**	**58.66**	**171.472**
	80	0.516	245.83	77.387	216.703
V_{t_0}	80	0.497	190.987	58.363	168.895
	100	**0.436**	**193.926**	**58.66**	**171.472**
	120	0.387	193.318	58.906	170.583
σ_1	0.4	0.436	159.885	59.95	147.205
	0.6	**0.436**	**193.926**	**58.66**	**171.472**
	0.8	0.436	225.709	56.083	198.157
$(\alpha_1+\delta_1)$	0.2	0.436	120.295	56.954	89.592
	0.35	**0.436**	**193.926**	**58.66**	**171.472**
	0.4	0.436	229.312	59.017	209.694
υ_1	0	0.375	175.409	58.66	149.579
	0.05	**0.436**	**193.926**	**58.66**	**171.472**
	0.1	0.506	216.609	58.66	197.332
T	1	0.394	71.976	59.687	36.057
	3	**0.436**	**193.926**	**58.66**	**171.472**
	4	0.458	312.237	58.568	293.307
1.$\Phi(V_T,T) = \max(x_{t_0}V_T - C_0, \min(V_T, L_0))$					
2.$V_T - \varphi_2(V_T) = \min(V_T, L_0) = V_T - \max(V_T - L_0, 0)$					
3.$\Phi'(V_T,T) = \max(x_{t_0}V_T - C_0, 0)$					
$V_{t_0} = 100, I_0 = 60, C_0 = 60, L_0 = 60, \alpha_1+\delta_1 = 0.35, \upsilon_1 = 0.05, \sigma_1 = 0.6, r = 4\%$					

Table 2: The table shows how the VC contract value, estimated at time zero, reacts to changes in parameters. In bold is the base case.

Investment			VC ownership stake	**VC Contract Value**	
I_0	I_1	I_2	$x_{tot}=\sum_{i=1}^{2} x_{t_i}$	Exit 1	Exit 2
Two stage financing (short duration first stage); $\left[0, \frac{T}{3}\right]$ and $\left[\frac{T}{3}, T\right]$					
60	0		0.435	159.885	147.205
50	10		0.438	160.933	148.176
40	20		0.437	160.015	146.920
20	40		0.411	150.288	134.978
0	60		0.337	125.07	103.253
Two stage financing (long duration first stage): $\left[0, \frac{T}{2}\right]$ and $\left[\frac{T}{2}, T\right]$					
60		0	0.435	159.892	147.205
50		10	0.431	157.770	144.432
40		20	0.421	154.016	139.607
20		40	0.380	139.329	121.185
0		60	0.295	112.088	84.621
Three stage financing: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, \frac{T}{2}\right]$ and $\left[\frac{T}{2}, T\right]$					
60	0	0	0.435	159.892	147.205
50	10	0	0.438	160.94	148.176
40	10	10	0.431	157.839	144.248
20	20	20	0.406	148.453	132.558
0	0	60	0.295	112.089	84.6217
Exit 1. $\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$					
Exit 2. $\Phi^{'}(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, 0\right)$					
$V_{t_0} = 100, C_i = L_i = I_i, \alpha_i + \delta_i = 0.35, \upsilon_i = 0.05, \sigma_i = 0.6, r = 4\%, T = 3yr$					

Table 3: Listed are the VC contract value for different types of contractual agreements between the entrepreneur and the VC. The first three columns represents the investment amount commited at the time zero. More financing rounds, implies a more valuable contract. Slicing the investment amount over stages reduces the downside losses and implying the possibility of an early exercise. Moreover it has also a negative impact on the VC ownership stake which reduces as investment is postponed in time, due to the positive drift characterising the project process. Note that in order to highlight the role of the number of stages and the respective durations, we kept the parameters constant over stages.

Staged Financing				
Two stage investment: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, T\right]$ where $T = 3yr$				
Investment I_0	I_1	VC Contract Value[1]	Liquidation Right[2]	Upside Potential[3]
60 (0.46)	0 (0.0)	135.178	59.7605	113.243
40 (0.35)	**20** (0.11)	**135.247**	**60.1839**	**112.802**
20 (0.20)	40 (0.23)	127.766	60.9648	102.834
0 (0.0)	60 (0.35)	108.98	61.7457	77.3436

1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$
2.$\min\left(V_T, L_{tot}\right)$
3.$\Phi^{'}(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, 0\right)$
$V_{t_0} = 100, C_i = L_i = I_i, \alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$

Table 4: The investment strategy may have a concave shape, nonetheless these results strongly depends on the level of the parameters. VC stake drives the value of the downside protection, which tend to increase as the money is invested in stages. Also investing earlier has a higher upside potential than if invested later on. The term in brakets is the VC ownership stake.

Price at the new issue	VC ownership stake Before	After		Antidilution at time $t = 3yr$
$p_2 = V_t$	x_{t_0}	x_2^*	x_1^*	$f(V_t, t)$
17.1005	45.9%	**31.7%**	9.5%	19.0454
37.4022	45.9%	**31.3%**	16.3%	15.7836
65.4147	45.9%	**30.9%**	22.5%	11.2024
102.305	45.9%	**30.6%**	27.6%	5.0788
143.075	45.9%	**-**	**31.1%**	0
250.232	45.9%	-	**36.1%**	0
855.961	45.9%		**42.5%**	0

$f(V_t, t) = \max\left(\left(x_2^* - x_1^*\right) V_t^+, 0\right)$

$I_i = \{40, 20\}, I_0^+ = 60, (\alpha_i + \delta_i) = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 4\%$

Table 5: The result shows that the VC fraction is diluted as long as addittional investment are required from other sources . The new fraction is subject to the level of new investment I+=60. In bold is the VC stake after the new financing round or any given issuing price. Note that V0=100, additional I=60 and the initial VC invested I=40+20exp(rt).

	Parameters		VC Ownership Stake		Strike Price
	Base Case	Perturb.	Before	After	$p_1 = \frac{I_{tot}}{q_t x_{tot}}$
			$x_{t_0} = \mathbf{45.9}\%$	$x^* = \mathbf{35.5}\%$	**132.6**
V_{t_0}	100	120	40.2%	32.9%	151.1
ν_i	(0.1,0.05)	(0.15,0.1)	52.6%	39.3%	115.5
I_i	(40,20)	(60,20)	55.5%	42.8%	145.6
I_t^+	60	80	45.9%	33.0%	132.6
Strike price is the old conversion price.p_1					

Table 6: The table analysis the VC ownership stake sensitivity to the project characteristics. Moreover it studies the impact of these parameters on the inital purchasing price. While carried on this analysis the rest of the parameters is kept the same as before.

Project uncertainty on VC Contract Value				
Two stage investment: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, T\right]$ where $T = 3yr$				
Volatility		VC Contract	Liquidation	Upside
σ_1	σ_2	Value[1]	Right[2]	Potential[3]
0.4	0.2	114.13	60.8142	103.099
0.6	**0.4**	**135.247**	**60.1839**	**112.802**
0.9	0.6	167.2	56.8301	138.31
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$				
2.$\min\left(V_T, L_{tot}\right)$				
3.$\Phi'\left(V_T, T\right) = \max\left(x_{tot}V_T - C_{tot}, 0\right)$				
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}$				
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$				

Table 7: Any increase in the volatility level leads to a higher contract value for the VC. The impact is significantly larger for the upside potential than for the liquidation right value.

VC additional capital cost on VC Contract Value				
Two stage investment: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, T\right]$ where $T = 3yr$				
VC addit. cost		VC Contract	Liquidation	Upside
υ_1	υ_2	Value[1]	Right[2]	Potential[3]
0.05	0.00	120.258	60.1839	93.9973
(0.30)	(0.09)			
0.1	**0.05**	**135.247**	**60.1839**	**112.802**
(0.35)	**(0.11)**			
0.2	0.1	159.461	60.1839	141.733
(0.43)	(0.12)			
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$				
2.$\min\left(V_T, L_{tot}\right)$				
3.$\Phi^{'}(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, 0\right)$				
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}$				
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$				

Table 8: For high additional capital costs VC requires higher stake since larger costs are involved. Consequently large value added implies higher upside potential in terms of converting possibilities and no impact on the downside protection. The term in brakets stand for the VC ownership stake on the project.

Project's drift on VC Contract Value				
Two stage investment: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, T\right]$ where $T = 3yr$				
Drift rate		VC Contract	Liquidation	Upside
$\alpha_1 + \delta_1$	$\alpha_2 + \delta_2$	Value	Right	Potential
0.25	0.2	116.844	59.8521	90.8565
(0.35)	0.114)			
0.3	**0.25**	**135.247**	**60.1839**	**112.802**
(0.35)	**(0.11)**			
0.35	0.3	158.098	60.4138	139.127
(0.35)	(0.10)			
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$				
2.$\min\left(V_T, L_{tot}\right)$				
3.$\Phi^{'}(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, 0\right)$				
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}$				
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$				

Table 9: Any increase in the project's drift, which incorporates the VC effort on the project, leads to a higher project value in the future. The drift (growth rate) is positively related to the upside and downside part of liquidation preference. The term in brakets stand for the VC ownership stake on the project.

Duration effect on VC Contract Value			
Two stage investment: $\left[0, \frac{T}{3}\right], \left[\frac{T}{3}, T\right]$ where $T = 3yr$			
Project Duration	VC Contract Value[1]	Liquidation Right[2]	Upside Potential[3]
1 year	67.8378	60.1953	32.5525
3 year	**135.247**	**60.1839**	**112.802**
6 year	400.661	60.8023	386.827
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$			
2.$\min\left(V_T, L_{tot}\right)$			
3.$\Phi'(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, 0\right)$			
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}$			
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$			

Table 10: For long life projects the VC would ask more than otherwise, since more uncertainty is involved during the process.

VC Contract Value if No Dilution Impact is assumed			
Two stage investment: $\left[0, \frac{T}{3}\right], \left[\frac{T}{3}, T\right]$ where $T = 3yr$			
Investments		VC ownership	VC Contract
I_0	I_1	stake x^*	Value[1]
60	0	(0.46)	**188.880**
40	20	(0.46)	**188.888**
20	40	(0.43)	**176.963**
0	60	(0.35)	**146.281**
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$			
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}, I_t^+ = 60$			
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$			

Table 11: The table shows the consequences of another VC steping in. The overall VC contract will increase if no dilution considered.

VC Contract Value with Protective Measures against Dilution			
Two stage investment: $\left[0, \frac{T}{3}\right], \left[\frac{T}{3}, T\right]$ where $T = 3yr$			
Investments		VC ownership	VC Contract
I_0	I_1	stake x^*	Value[1]
60	0	(0.36)	**148.740**
40	20	(0.36)	**149.069**
20	40	(0.34)	**143.265**
0	60	(0.29)	**127.938**
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$			
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}, I_t^+ = 60$			
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$			

Table 12: The table shows the consequences of another VC steping in. The overall VC contract will increase if no dilution considered.

VC Contract Value without Protective Measures against Dilution			
Two stage investment: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, T\right]$ where $T = 3yr$			
Investments		VC ownership	VC contract
I_0	I_1	stake x^*	Value
60	0	(0.30)	**128.843**
40	20	(0.30)	**129.086**
20	40	(0.28)	**122.783**
0	60	(0.22)	**107.051**
1.$\Phi(V_T, T) = \max\left(x_{tot}V_T - C_{tot}, \min\left(V_T, L_{tot}\right)\right)$			
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}, I_t^+ = 60$			
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$			

Table 13: The table shows the consequences of another VC steping in. The VC stake will be diluted by more if no protection is offered to the VC.

Change in Param.		**VC contract value**	Pr_{IPO}
6- months		**160.619**	**41.5%**
υ_i	+50%	176.227	43.9%
$\alpha_i+\delta_i$	+50%	266.054	43.9%
σ_i	+50%	194.591	41.5%
I_i	+50%	239.620	43.9%
V_{t_0}	+50%	153.744	42.8%
2- months		**142.795**	**41.0%**
υ_i	+50%	155.303	43.6%
$\alpha_i+\delta_i$	+50%	224.024	41.0%
σ_i	+50%	175.493	41.0%
I_i	+50%	214.273	41.0%
V_{t_0}	+50%	136.303	42.6%
1- months		**138.343**	**39.5%**
υ_i	+50%	150.429	41.8%
$\alpha_i+\delta_i$	+50%	213.040	39.6%
σ_i	+50%	170.349	39.5%
I_i	+50%	208.607	39.6%
V_{t_0}	+50%	133.874	40.8%
$V_{t_0} = 100, I_i = \{40, 20\}, C_i = \{40, 20\}, L_i = \{40, 20\}, I_t^+ = 60$			
$\alpha_i + \delta_i = \{0.3, 0.25\}, \upsilon_i = \{0.1, 0.05\}, \sigma_i = \{0.6, 0.4\}, r = 0.04$			

Table 14: Any increase on the level of the following parameters; volatility, drift rate, VC value added as well as investment level, increases the VC contract value. Whereas an increase in V(0) has a negative impact on the VC contract value which may be related to a lower firm value at the end of the project as well as a lower ownership stake. A 50 percent increase in all parameters leads to a higher probability for an IPO vs MA.

Alternative Exit possibility/IPO 6- mnth Lock -up				
Two stage investment: $\left[0, \frac{T}{3}\right]$, $\left[\frac{T}{3}, T\right]$ where $T^{+} = 3yr$				
Underpr. Coeff.	Medium term Price Impact	Market Jump	VC Contract Value	$\Pr._{IPO}$
β_1	β_2	γ		
0.00	0.00	0.00	160.619	41.5%
0.05	0.00	0.20	181.27	43.9%
0.15	0.05	0.30	185.885	46.3%
$V_{t_0}= 100, I_i= \{40, 20\}, C_i= \{40, 20\}, L_i= \{40, 20\}, I_t^{+}= 60$				
$\alpha_i+\delta_i= \{0.3, 0.25\}, \upsilon_i= \{0.1, 0.05\}, \sigma_i= \{0.6, 0.4\}, r = 0.04$				

Table 15: The table studies the impact that underpricing coefficient, medium term price impact or market jump, may have not only in the VC contract value, but also in determining the probability of IPO versus a MA. The coefficients beta2 and gamma capture respectively the mid term positive impact of the underpricing coefficient beta1and the jump of the project value at the time of public offering t=T.