Extended DCF analysis and real options analysis within Information uncertainty : applications for project valuation and R&D

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Abstract

This paper presents a survey of some results regarding the standard discounted cash flow techniques, the economic value added and real options. Since the standard literature ignores the role of market frictions and the effect of incomplete information, we rely on Merton's (1987) model of capital market equilibrium with incomplete information to introduce information costs in the pricing of real assets. Using this model instead of the standard CAPM allows a new definition of the weighted average cost of capital and offers some new tools to compute the value of the firm and its assets in the presence of information uncertainty. Using the methodology in Bellalah (2001 a,b) for the pricing of real options, we propose some new results by extending the standard models to account for shadow costs of incomplete information. The extended models can be used for the valuation of R&D projects as well as projects with several stages like joint ventures.

Key words : EVA, DCF analysis, real options, information costs, joint venture

JEL Classification : G12, G20, G31

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The standard literature on capital budgeting techniques uses the net present value as a reference criteria in investment decisions. The analysis is mainly based on the use of the cost of capital in the discounting of future cash flows. A project is accepted if its extended Net Present Value, NPV is positive, otherwise it is rejected. The extended NPV corresponds to the standard NPV plus the flexibility in investment decisions. The standard technique for calculating the NPV has not changed much since Fisher (1907) by discounting the expected cash flow at an appropriate discount rate. The research in this area is based on the specification and estimation of the discount rate.

Over the last two decades, a body of academic research takes the methodology used in financial option pricing and applies it to real options in what is well known as real options theory. This approach recognizes the importance of flexibility in business activities. Today, options are worth more than ever because of the new realities of the actual economy : information intensity, instantaneous communications, high volatility, etc.[2]

The literature on real options and discounted cash flow techniques ignores the role of information uncertainty. However, these costs play a central role in financial markets and capital budgeting decisions. Financial models based on complete information might be inadequate to capture the complexity of rationality in action. Some factors and constraints, like entry into a business are not costless and may influence the short run behavior of asset prices. The treatment of information and its associated costs play a central role in capital markets. If an investor does not know about a trading opportunity, he will not act to implement an appropriate strategy to benefit from it. However, the investor must determine if potential gains are sufficient to warrant the costs of implementing the strategy. These costs include time and expenses required to create data base to support the strategy, to build models and to get informed about the technology. This argument applies in varying degrees to the adoption in practice of new structural models of evaluation.

This reasoning holds not only for individual investors but also for professional managers who spend resources and time in the same spirit. It is also valid for the elaboration and implementation of option pricing models. Hence, recognition of information costs might be important in asset valuation and has the potential to explain empirical biases exhibited by prices computed from complete information models. As shown in Merton (1987), the "true" discounting rate for future risky cash flows must be coherent with his simple model of capital market equilibrium with incomplete information.

[2]For a survey of important results in the literature, the reader can refer to Brealey and Myers (1985), Copeland and Weston (1988), Lee (1988), Agmon (1991), Smith and Nau (1994) and Bellalah (1998) among others.

This model can be used in the valuation of real assets. Nowadays, a rich set of criteria is used to recognize the companies real options. Consultants look beyond traditional financial analysis techniques to get reasonable guidelines in investment practices. Actual decision making in firms resort to real options. The value of the firm can have two components : the value of the existing projects and the value of the options hold by the firm to do other things. The use of standard option valuation techniques in the valuation of real assets is based on some important assumptions.[3] Managers are interested not only in real options, but also in the latest outgrowth in DCF analysis; the Economic Value Added. EVA simply means that the company is earning more than its cost of capital on its projects. [4]

The structure of the paper is as follows.
Section 1 presents a simple framework for the valuation of the firm and its assets in the presence of information costs. Using Merton's (1987) model of capital market equilibrium with incomplete information, we are able to extend the analysis in Modigliani-Miller (1958, 1963) to account for the effects of incomplete information in the computation of the firm value. This setting allows us to extend the concepts of economic value added and standard DCF analysis to account for information costs. An application is proposed for the valuation of a biotechnology firm.
Section 2 uses the main results in the real option literature to make the standard analogy between financial and real options. This allows the presentation of the main applications of the real option pricing theory. It presents some simple examples and proposes the main results in the literature regarding the analysis and the valuation of real options.
Section 3 develops a simple context for the pricing of real options in a continuous-time setting. We develop some simple analytic formulas for the pricing of standard and complex European and American commodity options in the presence of information costs. Then, we extend the results in some real option pricing models to account for information costs. This allows us to study the investment timing and the pricing of real assets using standard and complex options. We also extend the results in Lint and Pennings (1998) for the pricing of the option on market introduction.

[3]For a survey of the important results in the standard literature regarding option pricing, the reader can refer to Black and Scholes (1973), Merton (1973, 1992), Cox, Ross and Rubinstein (1979), Cox and Ross (1976), Cox and Rubinstein (1985), Hull (2000), Briys-Bellalah et al. (1998) among others.

[4]EVA is powerful in focusing senior management attention on shareholder value. Its main message concerns whether the company is earning more than the cost of capital. It says nothing about the future and on the way the companies can capitalize on different contingencies. Hence, a useful criterion must account for both the DCF analysis and real options. The NPV and the EP (economic profit) ignore the complex decision process in capital investment. In fact, business decisions are in general implemented through deferral, abandonment, expansion or in series of stages.

Section 4 develops some simple models for the pricing of real options in a discrete time setting by accounting for the role of shadow costs of incomplete information. We first extend the Cox, Ross and Rubinstein (1979) model to account for information costs in the valuation of managerial flexibility and the option to abandon. Then, we use the generalization in Trigeorgis (1990) for the pricing of several complex investment opportunities with embedded real options to account for the effects of information costs. Most of the models presented in this paper can be applied to the valuation of biotechnology projects and investments with several stages.

1. The cost of capital, the value of the firm and its investment opportunities in the presence of shadow costs of incomplete information

The standard analysis in corporate investments needs the projection of the project's cash flows and then to perform an NPV analysis. The discount rate is set with regard to the risk of the project. The riskier the project, the higher the manager sets the discount rate. [5] This standard approach ignores the presence of information costs. However, information plays a central role in the valuation of financial assets and must be accounted for in the valuation process. Merton (1987) presents a simple context to account for information costs. Before applying the main implications of Merton's model, we remind first this model and the definition of the shadow costs of incomplete information.

1.1. Merton's model

Merton's model is a two period model of capital market equilibrium in an economy where each investor has information about only a subset of the available securities. The main assumption in the Merton's model is that an investor includes an asset S in his portfolio only if he has some information about the first and second moment of the distribution of its returns. In this model, information costs have two components : the costs of gathering and processing data for the analysis and the valuation of the firm and its assets, and the costs of information transmission from an economic agent to an other. Merton's model may be stated as follows :

$$\bar{R}_S - r = \beta_S[\bar{R}_m - r] + \lambda_S - \beta_S\lambda_m \tag{1}$$

where :

- $\bar{R}_S$: the equilibrium expected return on security S,
- $\bar{R}_m$: the equilibrium expected return on the market portfolio,

[5] This approach leads to a real bias toward projects that produce return in the short run. In fact, the more distant the payoff horizon, the more uncertainty enters the game so that even huge pay back opportunities, if long term, tend to be discounted away. The NPV analysis obliges managers to compute present values of their investments as if they have engaged all the costs.

- R: one plus the riskless rate of interest, r,
- $\beta_S = \frac{\text{cov}(\tilde{R}_S/\tilde{R}_m)}{\text{var}(\tilde{R}_m)}$: the beta of security S,
- λ_S: the equilibrium aggregate " shadow cost" for the security S,
- λ_m: the weighted average shadow cost of incomplete information over all securities in the market place.

The CAPM of Merton (1987), referred to as the CAPMI is an extension of the standard CAPM to a context of incomplete information. Note that when $\lambda_m = \lambda_S = 0$, this model reduces to the standard CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966).

1.2. The cost of capital, the firm's value and Information costs

The cost of capital or the weighted average cost of capital, (WACC), is a central concept in corporate finance. It is used in the computation of the Net present value, NPV, and in the discounting of future risky streams. The standard analysis in Modigliani-Miller (1958, 1963) ignores the presence of market frictions and assumes that information is costless. Or, as it is well known in practice, information costs represent a significant component in the determination of returns from investments in financial and real assets. Merton (1987) provides a simple context to account for these costs by discounting future risky cash flows at a rate that accounts for these costs. In this context, the cost of capital and the firm's value can be computed in an economy similar to that in Merton (1987).

We denote respectively by :
D: the face value of debt ,
B: the market value of debt,
S: the market value of equity,
O: perpetual operating earnings ,
τ: the corporate tax rate,
V_u: the value of the unlevered firm,
V: the value of the levered firm,
k_d: the cost of debt,
k_b: the current market yield on the debt,
k_e: the cost of equity or the required return for levered equity,
k_o: the market value-weighted of these components known as the WACC,
ρ: the market cost of equity for an unlevered firm in the presence of incomplete information.

Using the main results in the Modigliani and Miller analysis and Merton's λ, it is clear that discounting factors must account for the shadow cost of information regarding the firm and its assets. By adding Merton's λ in the analysis of Modigliani Miller in the discounting of the different streams of cash-flows for levered and unlevered firms, similar very simple formulas can be derived in an extended Modigliani-Miller-Merton context. The formulas follow directly from the analysis in Modigliani-Miller and the fact that future risky streams must be discounted at a rate that accounts for Merton's λ. The following Table presents the main results regarding the components of the costs of capital and the values of the levered and unlevered firms with information costs.

Table 1 : Summary of the main results regarding the components of the costs of capital and the values of the levered and unlevered firms with information uncertainty

No tax	corporate tax
$\rho = \frac{O}{S_u} + \lambda_u$	$\rho = (\frac{O}{S_u} + \lambda_u)(1-\tau)$
$B = D\frac{k_d'}{k_b'}$	$B = D\frac{k_d'}{k_b'}$
$\mathrm{k}_e = \frac{[O-k_d' D]}{S}$ $\mathrm{k}_e = \rho + \frac{B}{S}(\rho - k_b')$	$\mathrm{k}_e = \frac{[(O-k_d' D)(1-\tau)]}{S}$ $[\rho + \frac{B}{S}(\rho - k_b')](1-\tau)$
$\mathrm{V}_u = \frac{O}{\rho} = S_u$	$\mathrm{V}_u = (1-\tau)\frac{O}{\rho}$
$\mathrm{V} = \mathrm{V}_u$	$\mathrm{V} = \mathrm{V}_u + \tau B$
$\mathrm{k}_o = k_e\frac{S}{V} + k_b'\frac{B}{V}$ $\mathrm{k}_o = \frac{O}{V}$ $\mathrm{k}_o = \rho$	$\mathrm{k}_0 = k_e\frac{S}{V} + k_b'(1-\tau)\frac{B}{V}$ $\mathrm{k}_0 = \frac{O}{V}(1-\tau)$ $\mathrm{k}_o = \rho(1-\tau\frac{B}{V})$

with $k_b' = k_b + \lambda_d$ and $k_d' = k_d + \lambda_d$.
The term λ_d indicates the information cost for the debt and the term λ_u corresponds to the information cost for the unlevered firm.

These results show the components of cost of capital and the values of the firms in the presence of information costs. When these costs are equal to zero, this Table is equivalent to the results in the Modigliani-Miller analysis.
The results show how to calculate the firm's value, the weighted average cost of capital, and the Net present value of future risky cash flows in the presence of information costs.
The above formulas are simulated for an illustrative purpose using : $O = 2000, D = 10000, B = 10000, S = 10000, V = 20000$, $\tau = 40\%, \rho = 10\%$ and $k_d = 5\%$, $\lambda_u = 0\%, \lambda_d = 0\%$. These figures represent the standard benchmark case. The simulations allow to appreciate the impact of information costs on the computation of the different values of the levered and unlevered firm and the costs of capital with corporate taxes.

Table 2 : Summary of the main results regarding the components of the costs of capital and the values of the levered and unlevered firms with information costs : the standard case

$O = 2000, D = 10000, B = 10000, S = 10000, V = 20000,\ \tau = 40\%, \rho = 10\%$ and $k_d = 5\%,\ \lambda_u = 0\%, \lambda_d = 0\%$.

No tax	Corporate tax
$\rho = 10\%$	$\rho = 10\%$
$B = 10000$	$B = 10000$
$k_e = 15\%$	$k_e = 15\%$
$k_e = 15\%$	$k_e = 15\%$
$V_u = 20000$	$V_u = 12000$
$V = 20000$	$V = 16000$
$S = 10000$	$S = 6000$
$k_o = 10\%$	$k_o = 7.5\%$
$k_o = 10\%$	$k_o = 7.50\%$
$k_o = 10\%$	$k_o = 7.5\%$

The fact that k_e is equal to 15 % in this case is consistent with the MM assumptions. The effect of incomplete information on the firm value and the cost of capital is simulated using the following data : $O = 2000, D = 10000, B = 10000, S = 10000, V = 20000,\ \tau = 40\%, \rho = 10\%, k_d = 5\%,\ \lambda_u = 3\%, \lambda_d = 1\%$.

Table 3 : The main results for the cost of capital and the values of the levered and unlevered firms with information costs

$O = 2000, D = 10000, B = 10000, S = 10000, V = 20000,\ \tau = 40\%, \rho = 10\%, k_d = 5\%,\ \lambda_u = 3\%, \lambda_d = 1\%$.

No tax	Corporate tax
$\rho = 13\%$	$\rho = 13\%$
$B = 10000$	$B = 10000$
$k_e = 26\%$ $k_e = 26\%$	$k_e = 26\%$ $k_e = 26\%$
$V_u = 15384.62$ $V = 15384.62$ $S = 5384.62$	$V_u = 9230.77$ $V = 13230.77$ $S = 3230.77$
$k_o = 13\%$ $k_o = 13\%$ $k_o = 13\%$	$k_o = 9.07\%$ $k_o = 9.07\%$ $k_o = 9.07\%$

The value of k_e is equal to 26 % in this case. Every scenario is consistent with the Modigliani-Miller assumptions and the Merton's shadow cost (λ). When compared to the benchmark case with no information costs, we see that information costs increase significantly k_e. These shadow costs reduce the value of the firm in the two cases : with no tax and with corporate tax.

1.3. Application to a biotechnology firm

For a biotechnology firm, the development of a drug needs several stages : discovery, pre-clinical, Phase I clinical trials, Phase II clinical trials, Phase III clinical trials, submission for review and post approval. We show how to apply Merton's (1987) model of capital market equilibrium with incomplete information for the computation of the cost of capital, the expected net present value (ENPV) in the decision tree method. Following the analysis in Kellogg and Charnes (1999), we will generalize their decision-tree method and the application of the binomial model to account for shadow costs of incomplete information. A model is constructed to compute the expected net present value (ENPV) without accounting for growth options. The (ENPV) can be computed in the presence of information costs. In the decision tree method, the ENPV is computed as :

$$ENPV = \Sigma_{i=1}^{7}\rho_i\Sigma_{t=1}^{T}\frac{DCF_{it}}{(1+r_d)^t} + \rho_7\Sigma_{j=1}^{5}\frac{q_jCCF_{j,t}}{(1+r_c)^t}$$

where :
$i = 1, , 7$: an index of the 7 stages in the project,
ρ_i : the probability that stage i is the end stage for product i,

T : the time at which all future cash flows become zero,
DCF_{it} : the expected development stage cash flow at time t given that stage i is the end stage,
r_d : the discount rate for development cash flows,
$j = 1$ to 5 : an index of quality for the product,
q_j : the probability that the product is of quality j,
r_c: the discount rate for commercialization cash flows.

The discounting rates r_d and r_c can be estimated using Merton's CAPMI as in Bellalah (2000 b, 2001 b). This method is easy to implement and accounts for the effects of information costs in project valuation.

1.4. Economic Value Added, EVA, and Information costs

In standard financial theory, every company's ultimate aim is to maximize shareholders' wealth. The maximization of value is equivalent to the maximization of long-term yield on shareholders' investment. Currently, EVA is the most popular Value based measure.
A manager accepts a projet with positive NPV; i.e; for which the internal rate of return IRR is higher than the cost of capital. With practical performance measuring, the rate of return to capital is used because the IRR can not be measured. However, the accounting rate of return is not an accurate estimate of the true rate of return. As shown in several studies, ROI underestimates the IRR in the beginning of the period and overestimates it at the end. This phenomenon is known as wrong periodizing.
The EVA valuation technique provides the true value of the firm regardless of how the accounting is done. The EVA is a simply a modified version of the standard DCF analysis in a context where all of the adjustments in the EVA to the DCF must result net to zero.
EVA can be superior to accounting profits in the measurement of value creation. In fact, EVA recognizes the cost of capital and, the riskiness of the company. Maximizing EVA can be set as a target while maximizing an accounting profit or accounting rate of return can lead to an undesired outcome.
The weighted average cost of capital, WACC, is computed using Merton's (1987) model of capital market equilibrium with incomplete information for the cost of equity component. The WACC is computed as in Table 1.

Stewart (1990) defines the EVA as the difference between the Net operating profit after taxes (NOPAT) and the cost of capital. EVA gives the same results as the Discounted cash flow techniques or the Net present value (NPV). It can be described by one of the three equivalent formulas :
EVA = NOPAT - Cost of capital x(Capital employed)
or
EVA = Rate of return - Cost of capital x(capital employed)

or
EVA = (ROI - WACC) Capital employed

with
Rate of return = NOPAT/Capital,
Capital = Total balance sheet - non-interest bearing debt at the beginning of the year.
ROI = the return on investment after taxes, i.e; an accounting rate of return.

The cost of capital is the WACC as in the Modigliani-Miller analysis where the cost of equity is defined with respect to the CAPM of Sharpe (1964), Lintner (1965) and Mossin (1966). In the presence of information costs, the cost of capital can be determined in the context of Merton's model of capital market equilibrium as described above. In this case, the above formulas must be used. Hence, the analysis in Stewart (1990) can be extended using the CAPMI of Merton (1987) rather than the standard CAPM in the computation of EVA.
In the presence of taxes, EVA can also be calculated as :
EVA = [NOP -((NOP - Excess depreciation - Other increase in reserves)x(Tax rate))] - WACC x (Capital)
where NOP is the Net operating profit.
Stewart (1990) defines the Market Value Added, MVA, as the difference between a company's market and book values :
MVA = Total market asset value - Capital invested

When the book and the market values of debt are equal, MVA can be written as :
MVA = Market value of equity - Book value of equity
The MVA can also be defined as :
MVA = the present value of all future EVA.

Using the above definitions, it is evident that :
Market value of equity = Book value of equity + Present value of all future EVA.

In this context, this formula is always equivalent to Discounted cash flow and Net present value. Again, the cost of capital with information costs represents an appropriate rate for the discounting of all the future EVA. Hence, the main concepts in Stewart (1990) can be extended without difficulties to account for the shadow costs of incomplete information in the spirit of Merton's model.

2. From financial options to real options : some standard applications

Managers recognize that the NPV analysis is incomplete and shortsighted. This analysis ensures in theory perpetual profitability for a company. The NPV

fails because it assumes the decision to invest in a project is all or nothing. Hence, it ignores the presence of many incremental points in a project where the option exists to go forward or abort.[6] Realistic view of the capital budgeting process portrays projects as a sequence of options. [7]
Real option valuation maps out the possibilities available to a company, including those not readily apparent in the decision tree. By varying the discount rate through the tree, it accounts for the relative level of risk for different cash flows. Real option valuation can also identify the optimal course of the company at each stage in the process.

2.1. The standard analogy between financial and real options

There is a well established analogy between financial options and corporate investments that lead to future opportunities. It is evident for a manager why investing today in research and development or in a new marketing program can lead to a possibility of new markets in the future. Dixit (1992, 1995 a, b) and Dixit and Pindyck (1994) suggest that option theory provides helpful explanations since the goal of the investments is to reveal information about technological possibilities, production costs or market potential.

Consider for example a generic investment opportunity or a capital budgeting project to see the analogy with financial options. The difficult task lies in mapping a project onto an option. A corporate investment opportunity looks like a call because the firm has the right but not the obligation to acquire a given underlying, (the operating assets of a project or a new business). If the manager finds a call option in the market similar to the investment opportunity, then the value of that option can give him information about the value of the investment opportunity. Using this analogy between financial options and real options allows to know more about the project. This approach is more interesting than the standard discounting cash flow techniques DCT. [8]
The option implicit in the project (the real option) and the NPV without the option are easily compared when the project can no longer be delayed. [9]

[6]For a survey of the literature on real options, the reader can refer to Trigeorgis (1990, 1993 a, b, c, 1995, 1996), Pindyck (1991), Padock, Siegel and Smith (1988), Newton (1996), Myers (1984), McDonald and Siegel (1984, 1986), Myers and Majd (1990) among others.

[7]For a review of the main results in this literature , the reader can refer to Luehrman (1997, 1998), Baghay et al. (1996), Carr (1988), etc.

[8]For a review of the main results in this literature, the reader can refer to Baldwin and Ruback (1986), Dentskevich and Salkin (1991), Ingersoll and Ross (1992), etc.

[9]Opportunities may be thought of as possible future operations. When a manager decides how much to spend on R&D, or on which kind of research and development R&D, he is valuing real opportunities. The crucial decision to invest or not will be made in general after some uncertainty is resolved or when time runs out. An opportunity is analogous to an option. Option pricing models contain parameters to capture information about cash, time, and risk. The theory handles simple contingencies better than standard DCF models. The reader can refer to Kogut (1991), Kogut and Kulatilaka (1994 a, b) and Mac Donald and Siegel (1984,

A real option confers flexibilities to its holder and can be economically important. Paddock, Siegel and Smith (1988) and Berger et al (1996) show that the value of a firm is the combined value of the assets already in use and the present value of the future investment opportunities.
There are several situations that lead to real options in different sectors in the economy. Most of these options appear in Dentskevich and Salkin (1991), Dixit (1992, 1995), Faulkner (1996) among others.

2.2. Standard and complex real options and their applications : some examples

If you consider the example of high-tech start-up companies, these firms are valued mainly for their real options rather than their existing projects. The market recognizes today the value of these options. While standard options are easily identified, it is more difficult to identify compound and learning options. Compound options generate other options among exercise. These options involve sequenced or staged investments. When a manager makes an initial investment, he has the right to make a second investment, which in turn gives the right to make a third investment, and so on.
Learning options allow the manager to pay to learn about an uncertain technology or system. Staged investments give managers the right to abandon or scale up projects, to expand into new geographic areas and investing in research and development.[10]

A first example of compound options can be found in a staged investment, which may be assimilated to a sequence of stages where each stage is contingent on the completion of its predecessor. This is the case for a company seeking to expand in foreign markets. The firm might start in a single territory. It can then learn and modify the specific features of its product. The first experience enables the firm to expand into similar overseas markets. However, the manager must weigh the value of the option to expand cautiously against the potential costs of coming second in some or all of these markets. This situation corresponds also to joint ventures and the valuation of joint ventures and

1986) among others.

[10]The main element in the determination of profitability in certain cyclical activities is the ability of timing a business cycle to build for example a new factory. The manager does not have to commit himself outright to a new factory. He has the option of staging the investment over a given period by paying a certain amount up front for design, an other amount in a period for pre-construction work and an other outlay to complete construction at the end of the year. This gives him the flexibility to walk away if profit projections fall below a given level or to abandon at the end of the initial construction phase and save a given additional outlay. The factory is designed to convert an input into an output and its profitability would be a function of the spread between these prices. The manager can invest in new factories only when the input/output spread is higher than its long-term average. The NPV assumes that the factory is built and operated, ignoring the flexibility offered to managers.

biotechnology products where each stage is contingent on the subsequent stages.

A second example is given in the market for corporate control and acquisitions. A sequence of acquisitions represents a staged series of investments and can be assimilated to compound options. Real options can be used in this context to value all possible contingencies. In this case, the literature regarding exotic options can be applied to value the different real options.

A third example corresponds to mining companies.
Mining companies must often give an answer to the following question : when to develop the properties they own and how much to bid for the right to implement additional properties. These decisions refer to a combination of options: the option to learn about the quantity of ore and the option to defer the development waiting for favorable prices.[11]

A fourth example is given for the development of a natural gas field (compound rainbow options)
Combinations of learning options and rainbow options can arise for some firms. [12]

A fifth example is given by R&D in pharmaceuticals (Rainbow options)
Projects in R&D combine learning and compound options. R&D projects contain both technological and product uncertainties. [13]

3. The valuation of real options with information costs in a continuous-time setting

Several models in financial economics are proposed to deal with the abil-

[11] In general, learning options appear when a company has the possibility to speed up the arrival of information by making an investment. Real option theory can be used to determine the optimal time to exercise the option. When the company does not know the quantity of ore in its mine, it has a learning option : to pay money to find out. Here also, the main models for the pricing of exotic options can be applied.

[12] Consider a company deciding on how much production capacity to install in an undeveloped natural gas field. The company can create a decision tree for a real option valuation model (ROV) to weigh up the various decisions in view of the uncertainty regarding the price and quantity. Using the information regarding the volatility of gas prices and quantity, the ROV model can estimate the total value of the different courses open to the company. The reader can refer to the work of Brennan (1991), Brennan and Schwartz (1985), Pickles and Smith (1993), etc.

[13] Consider a pharmaceutical company ranking different RD projects in order of priority. The real option approach handles both uncertainties. R&D projects can be classified as compound rainbow options, each contingent on the preceding options and on multiple sources of uncertainty (rainbow options and multi factor options). In this context, the models for the pricing of exotic options can be applied. For the general approach regarding the pricing of these options, the reader can refer to Bellalah (2002).

ity to delay an irreversible investment expenditure. [14] Before presenting some models for the valuation of real options in a continuous time setting, we present the general context for the valuation of financial options with information costs. We first present the valuation of simple options then a formula for the valuation of compound options.

3.1. The valuation of simple European and American Commodity options with information costs

Following Black (1976), we assume that all the parameters of the Merton's (1987) CAPMI are constant through time. Under these assumptions, the value of the commodity option, $C(S,t)$, can be written as a function of the underlying price and time. The spot price is described by the following equation :

$$\Delta S/S = \mu \Delta t + \sigma \Delta z \tag{2}$$

where μ and σ refer to the instantaneous rate of return and the standard deviation of the underlying asset, and z is an increment to a Brownian motion.
The relationship between a commodity option's beta and its underlying security's beta is given by :

$$\beta_C = S(\frac{C_S}{C})\beta_S \tag{3}$$

where β_c and β_S refer respectively to the betas of the commodity option and its underlying commodity contract.
The expected return on a security in the context of Merton's model is:

$$\bar{R}_S - r = \beta_S[\bar{R}_m - r] + \lambda_S - \beta_S \lambda_m$$

This equation can be written for the expected return on a commodity contract in the presence of a carrying cost over a small interval of time as :

$$E(\frac{\Delta S}{S}) = [b + \beta_S(\bar{R}_m - r) + \lambda_S - \beta_S \lambda_m]\Delta t \tag{4}$$

where b is the cost of carrying the commodity.
Using Merton's model, the expected return on a commodity call option must be :

$$E(\frac{\Delta C}{C}) = [r + \beta_C(\bar{R}_m - r) + \lambda_C - \beta_C \lambda_m]\Delta t \tag{5}$$

where an information cost λ_C enters the option's expected return.
When the expected returns on the commodity option and its underlying contract are multiplied by C and S, this gives :

$$E(\Delta S) = [bS + S\beta_S(\bar{R}_m - r) + \lambda_S S - S\beta_S \lambda_m]\Delta t \tag{6}$$

[14] These models undermine the theoretical foundation of standard neoclassical investment models and invalidate the net present value criteria in investment choice under uncertainty. For a survey of this literature, the reader can refer to Pindyck (1991) and the references in that paper.

$$E(\Delta C) = [rC + C\beta_C(\bar{R}_m - r) + \lambda_C C - C\beta_C\lambda_m]\Delta t \tag{7}$$

Substituting for the option's elasticity from equation (3), equation (7) can be written as :

$$E(\Delta C) = [rC + SC_S\beta_S(\bar{R}_m - r) + \lambda_C C - SC_S\beta_S\lambda_m]\Delta t \tag{8}$$

When a hedged position is constructed and "continuously" rebalanced, using limiting arguments allows to write ΔC as dC :

$$dC = \frac{1}{2}C_{SS}dS^2 + C_S dS + C_t dt \tag{9}$$

When the expectation is applied to (9) and dS is replaced by its value, this gives :

$$E(dC) = \frac{1}{2}\sigma^2 S^2 C_{SS}dt + C_S[bS + S\beta_S(\bar{R}_m - r) + \lambda_S S - S\beta_S\lambda_m]dt + C_t dt \tag{10}$$

Combining the expected values for the call and rearranging yields :

$$\frac{1}{2}\sigma^2 S^2 C_{SS} + (b + \lambda_S)SC_S - (r + \lambda_C)C + C_t = 0 \tag{11}$$

This equation appears in Bellalah (1999) for the pricing of commodity options. When λ_S and λ_C are set equal to zero, this equation collapses to that in Barone-Adesi and Whaley (1987). The value of a European commodity call is:

$$C(S,T) = Se^{((b-r-(\lambda_C-\lambda_S))T)}N(d_1) - Ke^{-(r+\lambda_C)T}N(d_2) \tag{12}$$

with:

$$\begin{aligned} d_1 &= [\ln(\frac{S}{K}) + (b + \frac{1}{2}\sigma^2 + \lambda_S)T]/\sigma\sqrt{T} \\ d_2 &= d_1 - \sigma\sqrt{T} \end{aligned}$$

and where $N(.)$ is the cumulative normal density function.

When λ_S and λ_C are equal to zero and $b = r$, this formula is the same as that in Black and Scholes. A direct application of the approach in Barone-Adesi and Whaley (1987), allows to write down immediately the formulas for American commodity options with information costs. In this context, the American commodity option value $C_A(S,T)$ is given by :

$C_A(S,T) = C(S,T) + A_2(S/S^*)^{q_2}$ when $S < S^*$
$C_A(S,T) = S - K$ when $S \geq S^*$
with :

$$A_2 = \frac{S^*}{q_2}(1 - e^{(b+\lambda_S-r-\lambda_C)T}N(d_1(S^*))) \tag{13}$$

$q_2 = \frac{1}{2}(-(N-1) + \sqrt{(N-1)^2 + 4\frac{M}{k}})$
$N = \frac{2(r+\lambda_C)}{\sigma^2}$,
$M = \frac{2(b+\lambda_S)}{\sigma^2}$
$k = 1 - e^{-(r+\lambda_C)T}$.
The critical underlying commodity price is given by an iterative procedure from the following equation :

$$S^* - K = C(S^*, T) + \frac{S^*(1 - e^{(b+\lambda_S - r - \lambda_C)T} N(d_1(S^*))}{q_2}) \tag{14}$$

In the same context, the American commodity option put value $P_A(S, T)$ is given by :
$P_A(S, T) = P(S, T) + A_1(S/S^*)^{q_1}$ when $S > S^{**}$
$P_A(S, T) = K - S$ when $S \leq S^{**}$
with :

$$A_1 = \frac{-S^{**}}{q_1}(1 - e^{(b+\lambda_S - r - \lambda_C)T} N(-d_1(S^{**}))) \tag{15}$$

$q_1 = \frac{1}{2}(-(N-1) - \sqrt{(N-1)^2 + 4\frac{M}{k}})$,
$N = \frac{2(r+\lambda_C)}{\sigma^2}$,
$M = \frac{2(b+\lambda_S)}{\sigma^2}$,
$k = 1 - e^{-(r+\lambda_C)T}$.

The critical underlying commodity price is given by an iterative procedure from the following equation :

$$K - S^* = P(S^*, T) - \frac{S^*(1 - e^{(b+\lambda_S - r - \lambda_C)T} N(-d_1(S^*)))}{q_1} \tag{16}$$

A similar algorithm as the one developed in Barone-Adesi and Whaley (1987) can be used to determine the critical underlying asset price. The above formulas can be applied to the valuation of several real options embedded in project valuation. In particular, the formulas can be applied for the pricing of European and American call and put options in the presence of a continuous dividend stream. The advantages of these formulas over many formulas for American options is the speed of computation since this analytic approximation is faster than numerical methods and the lattice approaches. These formulas can be used in the valuation of complex projects as those described in Trigeorgis (1991).

3.2. The valuation of compound options within information costs

Several projects are often valued using the concept of compound options introduced by Geske (1979). For example, the development process for a new product requires several stages where the manager resorts to the new information revealed up to that point to decide whether to abandon or to continue the project. This is particularly the case for a biotechnology firm for which the development of a drug needs several stages. The idea is that engaging in the development phase is equivalent to buying a call on the value of a subsequent product. Hence, there is the initial option and the growth option. In the presence of only two stages a formula for a call on a call can be used. We show how to value compound options in the presence of information costs. For the sake of simplicity, we use the general context proposed by Geske (1979).

If the stock is considered as an option on the value of the firm, V, then the value of the call as a compound option can be expressed as a function of the firm's value. This analysis follows from the setting in Geske (1979). Following Geske (1979), consider a levered firm for which the debt corresponds to pure discount bonds maturing in T years with a face value M. Under the standard assumptions of liquidating the firm in T years, paying off the bondholders and giving the residual value (if any) to stockholders, the bondholders have given the stockholders the option to buy back the assets of the firm at the debt's maturity date. In this context, a call on the firm's stock is a compound option, $C(S,t) = f(g(V,t),t)$ where t stands for the current time. Using the standard dynamics, the return on the firm's assets follows the stochastic differential equation :

$$dV/V = \alpha_v dt + \sigma_v dz_v \tag{17}$$

where α_v and σ_v refer to the instantaneous rate of return and the standard deviation of the return of the firm per unit time, and dz_v is a Brownian motion.

Using the definition of the call $C(V,t)$, its return can be described by the following differential equation :

$$dC/C = \alpha_c dt + \sigma_c dz_c \tag{18}$$

where α_c and σ_c refer to the instantaneous rate of return and the standard deviation of the return on the call per unit time, and dz_c is a Brownian motion. Using It's lemma as before, the dynamics of the call can be expressed as :

$$dC = \frac{1}{2} C_{vv} \sigma_v^2 V^2 dt + C_v dV + C_t dt \tag{19}$$

It is possible to create a riskless hedge with two securities, in this case, between the firm and a call to get the following partial differential equation :

$$\frac{1}{2}\sigma_v^2 V^2 C_{vv} + (r+\lambda_v) V C_v - (r+\lambda_C) C + C_t = 0 \tag{20}$$

where λ_v in an information cost relative to the firm's or the project's value. At the option's maturity date, the value of the call option on the firm's stock must satisfy the following condition :

$$C_t = max[S_t - K, 0]$$

where K stands for the strike price.
Investors suffer sunk costs to get informed about the equity and the assets of the firm. The costs regarding the equity and the firm's cash-flows reflect the agency costs and the asymmetric information costs. These costs characterize also joint ventures. In this situation, the formula is given by :

$$\begin{aligned} C_0 = V_0 e^{-(\lambda_c-\lambda_v)T} N_2(h+\sigma_v\sqrt{t}, k+\sigma_v\sqrt{T}, \sqrt{\frac{t}{T}}) \\ -Me^{-(r+\lambda_c)T} N_2(h, k, \sqrt{\frac{t}{T}}) - Ke^{-(r+\lambda_c)t} N(h) \end{aligned} \quad (21)$$

The value $\bar{V}$ is determined by the following equation :

$$S_t - K \quad = \quad \bar{V} e^{-(\lambda_c-\lambda_v)(T-t)} N(k+\sigma_v\sqrt{T-t}) - Me^{-(r+\lambda_c)(T-t)} N(k) - K = 0$$

with :
$h = [\ln(\frac{V}{\bar{V}}) + (r + \lambda_v - \frac{1}{2}\sigma_v^2)t]/\sigma_v\sqrt{t}$
$k = [\ln(\frac{V}{M}) + (r + \lambda_v - \frac{1}{2}\sigma_v^2)T]/\sigma_v\sqrt{T}$
If the information cost is zero, this compound option pricing formula becomes that in Geske (1979). This formula is also useful for the valuation of real options in the presence of information costs.

Table $-$ 4

Simulation of equity values as compound options in the presence of information costs using our model for the following parameters
The following parameters are used :

$$K = 20,\ M = 100,\ r = 0.08,\ T = 0.25, t = 0.125,\ \sigma_v = 0.4$$

C_0	$\lambda_c = 0\%$ $\lambda_v = 0\%$	$\lambda_c = 2\%$ $\lambda_v = 2\%$	$\lambda_c = 1\%$ $\lambda_v = 2\%$	$\lambda_c = 1\%$ $\lambda_v = 2\%$
110	6.82	7.13	7.16	7.14
120	15.17	15.65	15.70	15.67
130	26.52	27.16	27.25	27.20

Table 4 provides the simulation results for the compound option formula with information costs and the Geske's compound call formula using the following parameters : $K = 20$, $M = 100$, $r = 0.08$, $T = 0.25$, $t = 0.125$, $\sigma_v = 0.4$. The parameters used for information costs are:
case a : $(\lambda_c = \lambda_v = 0\%)$,

case b : $(\lambda_c = \lambda_v = 2\%)$,
case c : $(\lambda_c = 1\%, \lambda_v = 2\%)$,
case d : $(\lambda_c = 1\%, \lambda_v = 2\%)$.
In case (a), we have exactly the same values as those generated by the formula in Geske [1979]. The table shows that the compound option price is an increasing function of the firm's or the project's assets. This result is independent of the values attributed to information costs. The compound option price is an increasing function of the information costs regarding the firm's assets, λ_v. When λ_v is fixed, this allows the study of the effects of the other information costs on the option value. In this case, the option price seems to be a decreasing function of the information cost λ_c. We intend to test this model on real data.

3.3 The investment timing and the pricing of real assets within information uncertainty

The investment opportunity is analogous to a call option on a common stock since it gives the right to make an investment expenditure at the strike price and to receive the project. The firm's option to invest refers to the possibility to pay a sunk cost I and to receive a project which is worth, V. [15] Irreversibility is an important component of the investment process. [16]

In a different context, Roberts and Weitzman (1981) developed a model of sequential investment that puts the stress on the role of information gathering during the investment process. Each stage of investment yields information which plays a significant role in reducing the uncertainty over the value of the completed project.[17] The model shows that, contrary to the findings in Pindyck (1991), the use of a simple net present value can reject projects that should be undertaken. [18]
The dynamics of the project's value can be described by the following equation :

$$dV/V = \alpha dt + \sigma dz \tag{22}$$

where α and σ refer to the instantaneous rate of return and the standard deviation of the project, and dz is a geometric Brownian motion.
This equation shows that the current project value is known, whereas its future

[15] Unlike standard options, this call is perpetual and has no expiration date. This result is used in McDonald and Siegel (1986) and Pindyck (1991). In this context, the investment opportunity is equivalent to a perpetual call. The decision regarding the timing of the investment is equivalent to the choice of the exercise time of this option.

[16] Pindyck's (1991) presents a survey of some applications of this theory to a variety of investment problems.

[17] The crucial assumption is that prices and costs do not evolve stochastically. The value of the completed project may not be known at least until the early stages are acheived. However, there is no gain from waiting and no cost to investing now.

[18] This result is the opposite to that in Pindyck (1991), i.e. a simple NPV rule can accept projects that should be rejected.

values are log-normally distributed. Following Bellalah (2001 a), we denote by X the price of an asset perfectly correlated with V. The dynamics of X are represented by :

$$dX/X = \mu dt + \sigma dz \tag{23}$$

where μ stands for the expected return from owning a completed project.
We denote by $\delta = \mu - \alpha$. If V were the price of a share, δ would be the dividend rate on the stock. In this context, δ represents an opportunity cost of delaying investment. If δ is zero, then there is no opportunity cost to keeping the option alive.
Let $C(V)$ be the value of the firm's option to invest. Using Merton's (1987) model, Bellalah (2001 a) obtain option prices in the context of incomplete information.
Consider the return on the following portfolio P : hold an option which is worth $C(V)$ and go short C_V units of the project where the subscript V refers to the partial derivative with respect to V :

$$P = C - C_V V \tag{24}$$

The total return for this portfolio over a short interval of time dt is :

$$dC - C_V dV - \delta V C_V dt \tag{25}$$

Since there are information costs supported on the option and on its underlying assets, the return must be equal to $(r + \lambda_V)$ for the project and $(r + \lambda_C)$ for the option where λ_V and λ_C refer respectively to the information costs on the project and the option. In this context, we have :

$$dC - C_V dV - \delta V C_V dt = (r + \lambda_c) C dt + (r + \lambda_V) V C_V dt \tag{26}$$

or:

$$\frac{1}{2}\sigma^2 V^2 C_{VV} + (r + \lambda_V - \delta) V C_V - (r + \lambda_C) C = 0 \tag{27}$$

This equation for the value of $C(V)$ must satisfy the following conditions :

$$C(0) = 0 \tag{28}$$

$$C(V^*) = V^* - I \tag{29}$$

$$C_V(V) = 1 \tag{30}$$

The value V^* is the price at which it is optimal to invest. At that time, the firm receives the difference $V^* - I$. The solution to the differential equation under the above conditions gives the value of C(V). The solution under the first condition is :

$$C(V) = aV^\beta \tag{31}$$

where a is a constant and :

$$\beta = \frac{1}{2} - \frac{(r-\delta+\lambda_V)}{\sigma^2} + \sqrt{[\frac{(r-\delta+\lambda_V)}{\sigma^2} - \frac{1}{2}]^2 + \frac{2(r+\lambda_c)}{\sigma^2}}$$

The value of the constant a and the critical value V^* are :

$$V^* = \frac{\beta I}{\beta - I}, a = \frac{(V^* - I)}{(V^*)^\beta}$$

In Myers and Majd (1985), the sunk costs are related to the decision to exit or abandon a project for different reasons including severance pay for workers, and land reclamation for the case of a mine. In the Brennan and Schwartz (1985) model, the decision to invest contains the sunk cost of land reclamation. Several other models are proposed to deal with the ability to delay an investment expenditure and to study the behavior of firms toward the adoption of innovations. Some firms adopt new technologies when they are first available. Other firms delay the adoption until the technology is proved. The factors that drive the differences in behavior are analyzed by several authors.[19] Following the work of Myers (1984), Kester (1984) and Grenadier and Weiss (1997), the option-pricing theory can be applied to real-investment decisions as well as to strategies. The innovation investment strategy can be viewed as a link in a chain of future investment options. Grenadier and Weiss (1997) identify four potential strategies. [20].

3.4. Investment timing, project valuation and the pricing of real assets with compound options within information uncertainty

The timing option gives the right to the manager to choose the most advantageous moment to implement the investment project and allows him to pull out of the project when the economic environment turns out to be unfavorable. Several standard models are proposed in the literature for the pricing of these options.

Lee (1988) proposes a model for the valuation of the timing option arising from the uncertainty of the project value and for the detection of the optimal timing. He considers three cases: the optimal timing of plant and equipment replacement, the real estate development and the marketing of a new product.

The investment project is interpreted as the replacement of a capital asset, the inauguration of a new product and the development of real estate. The manager has the option to implement the project in the time interval $[0, T]$ where T is the option's maturity. The possibility to implement an investment project in

[19] For a survey of this literature, the reader can refer to Siegel, Smith and Paddock (1987), Pindyck (1991), Grenadier and Weiss (1997) and the references therein.

[20] For an extension of their model to account for the effects of incomplete information, see Bellalah (2000 a,b)

$[0, T]$ can be seen as an American call option on a security with no dividend payments. In the presence of information costs, our formula (13) for the valuation of American options can be used to price options in this context.
Let us denote by :
V: the present value of the project implemented,
S: the present value of the project not yet implemented,
I : the cost of the project,
D : a known anticipated jump in the project's value,
$C(S, 0, T, I)$: an American call without dividend where 0 refers to the starting time,
$c(S, 0, T, I)$: a European call option,
$P_{Ti}(0, T)$: the value of timing option.

The value of $P_{Ti}(0, T)$ corresponds to the difference between the value of the deferrable investment opportunity when the timing option is "alive" and when the timing option is "dead". The project's value if it is implemented now is :

$$C(S, 0, 0, I) = Max[V - I, 0]$$

where the NPV of the implemented investment opportunity is $(V - I)$.
In this case, the timing option value is given by:

$$P_{Ti}(0, T) = C(S, 0, T, I) - C(S, 0, 0, I)$$

$$P_{Ti}(0, T) = min[C(S, 0, T, I), C(S, 0, T, I) - (V - I)] \geq 0 \qquad (32)$$

This equation shows that it is profitable to implement the project now ($V - I > 0$) when the value of the timing option is equal to the value of the deferrable investment opportunity minus NPV. The cost of waiting D can be seen as a dividend in the pricing of American call options. It is possible to study three different specifications.

Specification 1 :
(i) : the present value changes of the not-yet-implemented project is :

$$dS/S = \mu dt + \sigma dz$$

(ii) If the project is implemented before t^*, it generates an extra cash-flow at t^* :

$$V_{t*} = S_{t^*} + D \qquad (33)$$

This specification corresponds for example to the real estate development. In fact, leaving property vacant can be seen as holding a timing option on the real estate development. The cost of development is I.

Specification 2:
Same as (i) of specification 1.

The cost of the project increases by D when implemented after t^*:
$I_{t*+h} = I$ for all $h > 0$

$$X_{t^*-h} = I - D, for, h > 0 \tag{34}$$

It is possible to use the formula in Whaley (1981) to compute the value of the optimal timing option and the optimal timing of project implementation. It is possible to show that the value of an American call in the presence of a cash discrete dividend and information costs is given by :

$$C = S[e^{((b-r-(\lambda_C-\lambda_S))t^*)}N(b_1)+e^{((b-r-(\lambda_C-\lambda_S))t^*)}N_2(a_1,-b_1,-\sqrt{\frac{t^*}{T}})]-I[e^{-(r+\lambda_C)t^*}N(b_2)+$$

$$e^{-(r+\lambda_C)t^*}N_2(a_2,-b_2,-\sqrt{\frac{t^*}{T}})] + De^{-(r+\lambda_C)t^*}N(b_2) \tag{35}$$

with :
$a_1 = [\ln(S/I) + (b + \frac{1}{2}\sigma^2 + \lambda_S)t^*]/\sigma\sqrt{t^*}$
$a_2 = a_1 - \sigma\sqrt{t^*}$
$b_1 = [\ln(S/S_{cr,t^*}) + (b + \frac{1}{2}\sigma^2 + \lambda_S)t_i]/\sigma\sqrt{t^*}$
$b_2 = b_1 - \sigma\sqrt{t^*}$
where S_{cr,t^*} corresponds to the trigger point present value, $N(.)$ stands for the cumulative normal distribution and $N_2(.,.,)$ is the bivariate cumulative normal density function with upper integral limits a and b and a correlation coefficient ρ.[21]
The "trigger point" for specification 1 is given by :

$$P_{Ti}(t^*, T) = c(S_{cr,t^*}, t^*, T, I) - (S_{cr,t^*} + D - I) = 0 \tag{36}$$

The trigger-point value for specification 2 is given by a formula identical to equation (36). This case fits well with the replacement of plant and equipment.

[21] The formula can be derived using a similar context as that in Roll (1977), Geske (1979), Whaley (1981) and Bellalah (1999). The valuation by duplication technique can be implemented. Consider the following portfolio of options :
a/ the purchase of a European call c_a having a strike price I and a maturity date T,
b/ the purchase of a European call c_b with a strike price S_{cr,t^*} and a maturity date $(t^* - \epsilon)$,
c/ the sale of a European call option c_c on the option defined in a/ with a strike price $(S_{cr,t^*} + D - I)$ and a maturity date $(t^* - \epsilon)$.
The contingent payoff of this portfolio of options is identical to that of an American call. In a perfect market, the absence of costless arbitrage opportunities ensures that the American call value is identical to that of this portfolio. The American call value must be equal to the sum of the three options in the portfolio.
The option c_a, can be valued using an extension of the Merton's (1973) commodity option formula or the model in Bellalah (1999). The option c_b can be priced using Bellalah (1999) formula for which the strike price is S_{cr,t^*}. The option c_c can be priced using an extension of the compound option formula proposed in Geske (1979). Since the value of the American call is equivalent to the algebraic sum of the three options in the portfolio, we have :
$C = c_a + c_b - c_c$

If we denote by S, I and T the present value, the cost of replacement and the remaining life, then a firm keeping the equipment in operation will face expenditures at time t^* of amount D. In this case, formula (35) can be applied to compute the value of the timing option and trigger point present value. These two specifications allow a single occurrence of discrete cash flow at time t^*. It is possible to generalize the results using specification 3.
Formula (35) is simulated in the following Tables 5, 6 and 7.

The parameters are $S = 175$, $D = 1.5$, $r = 0.1$ and the constant "carrying cost" is 0.6. We use different values for the information costs λ_S and λ_C. The option has a maturity date of one month. The volatility is $\sigma = 0.32$ and the "dividend" is paid in 24 days.

Table 5 uses these parameters with no information costs. It gives the computation of the American call value referred to as Call, the option c_a, the option c_b, the option c_c, the algebraic sum of the three options $(c_a + c_b - c_c)$ and the critical underlying asset price. The results are given for different "strike prices" varying from 100 to 240. Table 5 shows that the algebraic sum of the three options is equal to the American call price. The "critical asset price" corresponding to an early exercise is an increasing function of the strike price.
Table 6 uses the same data except for information costs. Information costs are set equal to $\lambda_S = 0.01$ and $\lambda_C = 0.001$. The reader can check that the algebraic sum of the three options is exactly equal to the American call price. With these costs, the call price is slightly higher than in Table 5.
Table 7 uses the same parameters except for the information costs which are set equal to $\lambda_S = 0.1$ and $\lambda_C = 0.05$.

Table 5 : Simulations of option values for the continuous-time model using the following parameters :

$S = 175, r = 0,1, D = 1,5, T = 30, t = 24, \sigma = 0,32, \lambda_c = 0, \lambda_s = 0.$

Strike	Call	c_a	c_b	c_c	$c_a + c_b - c_c$	S^*
100	76.03	74.42	74.25	72.65	76.03	100.02
105	71.06	69.46	69.11	67.51	71.06	105.00
110	66.09	64.51	63.96	62.37	66.09	110.00
115	61.13	59.55	59.07	57.49	61.13	115.00
120	56.16	54.59	53.65	52.07	556.16	120.00
125	51.19	49.63	48.48	46.92	51.19	125.00
130	46.22	44.67	43.32	41.76	46.23	130.00
135	41.26	39.72	38.49	36.94	41.26	135.00
140	36.30	34.79	32.99	31.47	36.30	140.00
145	31.37	29.90	27.87	26.39	31.37	145.00
150	26.50	25.11	22.85	21.46	26.50	150.00
155	21.78	20.52	18.05	16.80	21.78	154.99
160	17.31	16.23	13.64	12.56	17.31	159.99
165	13.25	12.38	9.78	8.92	13.25	164.99
170	9.72	9.07	6.63	5.98	9.72	169.99
175	6.82	6.37	4.22	3.77	6.82	174.99
180	4.56	4.27	2.52	2.23	4.56	179.99
185	2.91	2.74	1.41	1.24	2.91	184.99
190	1.77	1.68	0.74	0.64	1.77	189.99
195	1.03	0.98	0.36	0.31	1.03	194.99
200	0.57	0.55	0.16	0.14	0.57	200.00
240	0.00	0.00	0.00	0.00	0.00	240.00

Table 6 : Simulations of option values for the continuous-time model using the following parameters :

$S = 175, r = 0,1, D = 1,5, T = 30, t = 24, \sigma = 0,32, \lambda_c = 0,001, \lambda_s = 0,01.$

Strike	Call	c_a	c_b	c_c	$c_a + c_b - c_c$	S^*
100	76.14	74.56	74.34	72.76	76.14	100.02
120	56.427	54.73	53.73	52.19	56.27	120.00
140	36.41	34.93	33.06	31.57	36.41	140.01
145	31.48	30.04	27.94	26.49	31.48	145.00
150	26.61	25.25	22.92	21.55	26.61	150.00
155	21.89	20.65	18.11	16.87	21.89	154.99
160	17.42	16.35	13.67	12.61	17.42	159.99
165	13.34	12.49	9.83	8.97	13.34	164.99
170	9.80	9.16	6.66	6.02	9.80	169.99
175	6.89	6.44	4.24	3.80	6.89	174.99
180	4.62	4.33	2.53	2.25	4.62	179.99
185	2.95	2.78	1.42	1.25	2.95	184.99
190	1.80	1.71	0.74	0.65	1.80	189.99
195	1.05	1.00	0.36	0.32	1.05	194.99
200	0.58	0.56	0.17	0.14	0.58	200.00
240	0.00	0.00	0.00	0.00	0.00	240.00

Table 7 : Simulations of option values for the continuous-time model using the following parameters :

$S = 175, r = 0,1, D = 1,5, T = 30, t = 24, \sigma = 0,32, \lambda_c = 0,05, \lambda_s = 0,1.$

Strike	Call	c_a	c_b	c_c	$c_a + c_b - c_c$	S^*
100	76.92	75.55	74.95	73.58	76.92	100.02
120	57.12	55.79	54.31	52.98	57.12	120.00
140	37.32	36.06	33.59	32.33	37.32	140.01
145	32.40	31.18	28.45	27.23	32.40	145.00
150	27.54	26.38	23.40	22.25	27.54	150.00
155	22.80	21.75	18.55	17.51	22.80	154.99
160	18.29	17.39	14.08	13.18	18.29	159.99
165	14.15	13.43	10.15	9.42	14.15	164.99
170	10.52	9.97	6.90	6.36	10.52	169.99
175	7.49	7.11	4.41	4.04	7.49	174.99
180	5.09	4.85	2.65	2.41	5.09	179.99
185	3.31	3.16	1.49	1.34	3.31	184.99
190	2.05	1.97	0.78	0.70	2.05	189.99
195	1.22	1.18	0.38	0.34	1.22	194.99
200	0.69	0.67	0.18	0.16	0.69	200.00
240	0.00	0.00	0.00	0.00	0.00	240.00

Specification 3 :

(i) The present value of the implemented project V follows the equation :

$dV/V = \mu dt + \sigma dz$
(ii) If the project is not implemented immediately, its value will fall by a known amount D_i at time t_i where $i = 1, 2, ..., n$.
(iii) If the project is implemented at time t_k, its present value is given by :
$S_k = V_0 - \sum_{i=1}^{k-1} D_i e^{-(r+\lambda_s)t_i}, 0 \leq t_i < t_k \leq T$
In this expression, S_k corresponds to the present value at time 0 for the project to be implemented at t_k. V_0 corresponds to the present value of the project to be implemented now. The cost of waiting is given by the difference between the two present values. In this case, an extended version of the Black's (1975) approximation with information costs can be used :

$$C(S, 0, T, I) = max[c(S_k, 0, t_k, I) \mid k = 1, 2, ..., n] \tag{37}$$

At each instant t_h, just before the known present value decline, D_h, it is possible to compute the trigger point project value, $V_{cr,h}$ as in Lee (1988) using the following equation :

$$S_k = V_{cr,h} - \sum_{i=1}^{k} D_i e^{-(r+\lambda_s)(t_i - t_h)}, k = h, h+1, ..., n \tag{38}$$

where k^* is the argument of k at which $[c(S_k, t_h, I) \mid n \geq k \geq h]$ is a maximum and :

$$c(S_{k*}, t_h, t_{k^*}, I) = V_{cr,h} - I \tag{39}$$

In this expression :
t_{k^*}: the planned optimal timing when the manager decides to wait,
S_{k^*}: the present value at t_k of the project when it is implemented at the optimal planned time.
A firm has a timing option on the introduction of a product with a cost I for a time horizon T. If a new product is introduced at time 0, its present value V can be described by the above dynamics. Before a given firm introduces the product, the introduction by the competitor at time t_k can reduce the value of a given firm new product by D_k. Each episode of innovation at time i can reduce the value of the new planned product line by D_i. This fits with specification 3. In this case, equations (38, 39) can be used for an optimal timing decision.

3.5. Research and development and the option on market introduction in the presence of information costs

Several companies face the difficulty of selecting an optimal portfolio of research projects. As it appears in the analysis of Lint and Pennings (1998), the standard DCF techniques for capital budgeting can distort the process of selecting a portfolio of research projects. When managers have the option to abandon a project, it is possible to think of the cost of R&D as an option on major follow-on investments. Newton and Pearson (1994) provide an option pricing framework for R&D investments. Lint and Pennings (1998) report the application of an option pricing model for setting the budget of R&D projects. Their model captures a discontinuous arrival of new information that affects the project's value. R&D options can be viewed as European when two conditions hold. [22]

In the Lint and Pennings's (1998) model, the variance of the underlying value σ^2 is given by the product of a parameter representing the number of annual business shifts η and a parameter γ for the expected absolute change in the underlying value at every business shift : $\sigma^2 = \eta\gamma^2$.
Applying asymptotic theory, the option value can be approximated with the Black and Scholes (1973) formula where σ^2 is replaced by $\eta\gamma^2$, or :

$$C(S,T) = S(t)N(d + \sqrt{\eta(T-t)}\gamma) - Ie^{-r(T-t)}N(d) \tag{40}$$

$$d = [\ln(\frac{S(t)}{I}) + (r - \frac{1}{2}\eta\gamma^2)(T-t)]/\sigma\sqrt{\eta(T-t)}\gamma$$

where $S(t)$, $I, r, T-t$ stand respectively for the underlying value at present, the costs for market introduction, the risk free rate and the option's time to maturity.

Lint and Pennings (1998) use their model in Philips and show that the option value is largely determined by the opportunity to make a final decision on market introduction with more technological and market information. They show that the option value must compensate the R&D costs necessary to create the option. Their estimation of the option value of the potential benefits to market new products based on R&D goes beyond myopic use of DCF analysis. In the conclusion of their paper, they suggest to classify a variety of past and current R&D projects into sets of similar risks and returns. This can allow the estimation of the value of future idiosyncratic R&D projects by option analysis as in Newton and Pearson (1994). This line of research imposes an information cost in the spirit of the costs in Merton's (1987) model of capital market equilibrium with incomplete information.[23] It is possible to use the methodology in Lint and Pennings (1998) and in Bellalah (1999) to account for the role of information costs. In this case, the option value is given by :

$$C(S,T) = S(t)e^{-(\lambda_C - \lambda_S)(T-t)}N(d + \sqrt{\eta(T-t)}\gamma) - Ie^{-(r+\lambda_C)(T-t)}N(d) \tag{41}$$

[22] Lint and Pennings (1998) assume that the costs associated with the irreversible investment, required for market introduction, and the time for completing R&D are given with reasonable accuracy. By ignoring dividends, they propose a simple model which is an extension for R&D option pricing in practice. The approach in Lint and Pennings (1998) is based on a discontinuous arrival of information affecting the project.

[23] These costs appear also in the models of Bellalah and Jacquillat (1995) and Bellalah (1999) for the pricing of financial options in the presence of incomplete information.

$$d = [\ln(\frac{S(t)}{I}) + (r + \lambda_S - \frac{1}{2}\eta\gamma^2)(T-t)]/\sigma\sqrt{\eta(T-t)}\gamma$$

where λ_S and λ_c denote respectively the information costs relative to S and C.

4. The valuation of real options and R&D projects within information costs in a discrete-time setting

The majority of the papers concerned with the pricing of real assets in a discrete time setting derive from the models for financial options pioneered by Cox, Ross and Rubinstein (1979).

4.1 The valuation of real assets in a simple discrete-time framework

Salkin (1991) extends the basic binomial option pricing methodology to derive a consistent technique for the pricing of real hydrocarbon reserves. We extend this analysis to account for the effect of information costs.
In the classic binomial model of Cox, Ross and Rubinstein (1979), the price of the underlying asset goes up (u) or down (d) with a probability p and $(1-p)$. The use of this model is based on the presence of a "twin security" which exactly mimics the structure of the project.

Consider an investor who can either trade a commodity or invest in a project which supplies the commodity. The use of the dynamics of prices of the commodity must provide a good foundation for the examination of the structure of the cash flows of the project.
By introducing information costs, the probability of an upward movement in the underlying asset price can be shown to be equal to :

$$p = \frac{r + \lambda_c - d}{u - d}$$

The price uncertainty is described by a lattice : $S_{i,t} = S_{0,0}u^i d^{i-t}$
where $S_{0,0}$ is the price of the underlying commodity.

Let us denote by :
P_t : the production of a commodity at time t,
F_t : the fixed costs of production at time t,
V_t : the variable costs of production per unit of commodity at time t,
τ : corporation tax rate on positive cash flows at time t.

These profiles can be used to construct gross revenue, net revenue and post-tax cash flows. Using a lattice of post-tax cash flows, it is possible to calculate the Expected NPV of the project (ENPV). The lattice gross revenue $G_{i,t}$ corresponds to the spot lattice $S_{i,t}$ times the production profile P_t for all time and states t.

$$G_{i,t} = S_{i,t} P_t$$

The net revenue lattice $N_{i,t}$ pre-taxation corresponds to the gross revenue less the cost profiles F_t and V_t:

$$N_{i,t} = G_{i,t} - F_t - P_t V_t$$

The application of a taxation rate to all positive cash flows, gives a lattice that describes the cash flows of the project :
$\Phi_{i,t} = N_{i,t} \geq 0, N_{i,t}(1-\tau)$
$\Phi_{i,t} = N_{i,t} < 0, N_{i,t}$
The resulting lattice describes the post tax cash flows of the project. The added value to the project resulting from the ability to implement any decision contingent on the cash flows, $\Phi_{i,t}$.
In general, a decision rule is used to decide on the abandonment of a project, the contraction of its scale, the expansion of its scale ,or capacity, etc. For example, the decision to abandon is taken when both the post tax cash flows in the current period are negative, and the expected future post cash flows from the current time t and state i is negative.

The expected value of all future post tax cash flows from current time t can be calculated by beginning at the end for $T = N$. If we denote by $\Psi_{i,t}$ the expected value of all future post tax cash flows for the current time t and state i, then :

$$\Psi_{i,t} = \frac{1}{R + \lambda_c}[p(\Psi_{i+1,t+1} + \Phi_{i+1,t+1}) + (1-p)(\Psi_{i,t+1} + \Phi_{i,t+1})] \tag{42}$$

where R refers to one plus the riskless rate of interest. Now, it is possible to get a structure of cash flows that accounts for the abandonment decision :

$$\Pi_{i,t} = Max[\Phi_{i,t}; \Psi_{i,t}]$$

Repeating this procedure for all states at each period gives the project's value $\Pi_{0,0}$ with the embedded option to abandon the production. The process by which $\Pi_{0,0}$ is calculated is denoted by :

$$\Pi = F_n(P_t, F_t, V_t, \tau, \sigma, r, \lambda_s, \lambda_c, S_{0,0})$$

4.2. The valuation of a biotechnology firm using a discrete-time framework within information costs

Following the analysis in Kellogg and Charnes (1999), the value of the firm can be found also using the binomial lattice with the addition of a growth option. The growth option is represented by a second binomial lattice for a research phase. The current value of the asset S (or $S_{0,0}$ is computed using the discounted value of the expected commercialization cash flows to time zero as :

$$S_{0,0} = S = \Sigma_{j=1}^{5} q_j \Sigma_{t=1}^{T} \frac{}{CCF_{jt}} (1 + r_c)^t$$

where the discount rate is estimated using Merton's CAPMI. The number of stages can be arbitrarily any number.

It is possible to construct an n-period binomial lattice of asset values. The value of the underlying asset S goes up by u or down by d. This multiplicative process is continued for n period until the nth lattice. Kellogg and Charnes (1999) use the fact that : $u = e^{\sigma}$ and $d = e^{-\sigma}$ and impose that
$h = Sul = Se^{\sigma l}$ where l corresponds to a given number of years. They used an example in which the periods are supposed to have a length of one year.

The next step is to add in the value of the growth option. The idea is that engaging in the development phase is equivalent to buying a call on the value of a subsequent product. Hence, there is the initial option and the growth option. The value of the growth option at the time of the launch of the first product is added to each of the E_k values of the first NME.

Once the binomial tree of asset values is completed, it is possible to compute the possible payoffs and roll back the values using the risk neutral probabilities. The different payoffs are computed as :

$$P_k = max[E_k(\theta_t) - DCF_t, 0]$$

where :
θ_t : the probability of continuation to the next year in t,
DCF_t: the R&D payment in year t.
The P_k values are rolled back by multiplying the adjacent values, such as P_1 and P_2 (denoted by $V_{t+1,k}$ and $V_{t+1,k+1)}$ by the risk neutral probabilities p and $(1 - p)$, the probability of continuation to the next year and a discount factor to obtain $V_{t,k}$.

The risk neutral probabilities are calculated as :

$$p = \frac{e^{(r+\lambda_S)\Delta t} - d}{(u - d)}$$

As the option values are rolled back, they are adjusted for the probability of success at that phase of development and for the cost of development that year. The option values can be obtained at each node as :

$$V_{t,k} = max[(pV_{t+1,k} + (1 - p)V_{t+1,k+1})\theta_t e^{-(r+\lambda_V)\sqrt{\Delta t}} - DCF_t, 0]$$

4.3. The generalization of discrete time models for the pricing of projects and real assets within information uncertainty

Trigeorgis (1991) proposed a Log-transformed binomial model for the pricing of several complex investment opportunities with embedded real options. The model can be extended to account for information costs. The value of the expected cash flows or the underlying asset V satisfies the following dynamics :

$$\frac{dV}{V} = \alpha dt + \sigma dz$$

.

Consider the variable $X = \log V$ and $K = \sigma^2 dt$. If we divide the project's life T into N discrete intervals of length τ, then K can be approximated from $\sigma^2 \frac{T}{N}$.
Within each interval, X moves up by an amount $\Delta X = H$ with probability π or down by the same amount $\Delta X = -H$ with probability $(1-\pi)$. The mean of the process is $E(dX) = \mu K$; and its variance is $Var(dX) = K$ with $\mu = \frac{(r+\lambda_S)}{\sigma^2} - \frac{1}{2}$.
The mean and the variance of the discrete process are :
$E(\Delta X) = 2\pi H - H$ and $Var(\Delta X) = H^2 - [E(\Delta X)]^2$.

The discrete time process is consistent with the continuous diffusion process when :
$2\pi H - H = \mu K$
with $\mu = \frac{(r+\lambda_s)}{\sigma^2} - \frac{1}{2}$ so
$\pi = \frac{1}{2}(1 + \frac{\mu K}{H})$
and $H^2 - (\mu K)^2 = K$
so that $H = \sqrt{K + (\mu K)^2}$.

The model can be implemented in four steps.
In the first step, the cash flows CF are specified.
In the second step, the model determines the following key variables :
the time-step :
$K$from $\frac{\sigma^2 T}{N}$,
the drift μ from $\frac{(r+\lambda_S)}{\sigma^2 - \frac{1}{2}}$,
the state-step H from $\sqrt{K + (\mu K)^2}$
and the probability π from $\frac{1}{2}(1 + \frac{\mu K}{H})$.

Let "j" be the integer of time steps (each of length K), i the integer index for the state variable X (for the net number of ups less downs).
Let $R(i)$ be the total investment opportunity value (the project plus its embedded options).
In the third step, for each state i, the project's values are $V(i) = e^{(X_0 + iH)}$.
The total investment opportunity values are given by the terminal condition

$R(i) = max[V(i), 0]$.

The fourth step follows an iterative procedure. Between two periods, the value of the opportunity in the earlier period j at state i, $R'(i)$ is given by :

$$R'(i) = e^{-(r+\lambda_c)(\frac{K}{\sigma^2})}[\pi R(i+1) + (1-\pi)R(i-1)]$$

In this setting, the values of the different real options can be calculated by specifying their payoffs.
The payoff of the option to switch or abandon for salvage value S is :
$R' = \max(R, S)$.
The payoff of the option to expand by e by investing an amount I_4 is :
$R^{'} = R + \max(eV - I_4, 0)$.
The payoff of the option to contract the project scale by c saving an amount I_3' is :
$R' = R + \max(I_3' - cV, 0)$.

The payoff of the option to abandon by defaulting on investment I_2 is :
$R' = \max(R - I_2, 0)$.
The payoff of the option to defer (until next period) is :
$R' = \max(e^{-(r+\lambda_c)T} E(R_{j+1}), R_j)$.

When a real option is encountered in the backward procedure, then the total opportunity value is revised to reflect the asymmetry introduced by that flexibility or real option. This general procedure can be applied for the valuation of several projects and firms in the presence of information costs.

Summary
This paper provides a survey of the main results in the literature regarding the valuation of the firm and its assets using the real option theory when we account for the effects of information uncertainty.

We propose some simple models for the analysis of the investment decision under uncertainty, irreversibility and sunk costs. First, we use Merton (1987) model of capital market equilibrium with incomplete information to determine the appropriate rate for the discounting of future risky cash flows under incomplete information. The use of this model allows the computation of the weighted average cost of capital under incomplete information. This cost can be used to reformulate the Modigliani-Miller (1958, 1963). It allows the extension of the standard DCF analysis, the EVA and the theory of firm valuation under incomplete information.

Second, we review the main possible and potential applications of option pricing theory to the valuation of simple and complex real options.

Third, we develop some simple models for the pricing of European and American commodity options in the presence of information costs. We propose also simple analytic formulas for the pricing of compound options in the presence of information costs. These formulas are useful in the study of the main results in the literature regarding the investment timing and the pricing of real assets using standard and complex options in the presence of incomplete information. The analysis is extended to the valuation of research and development and the option on market introduction. It is also applied to the valuation of flexibility as a compound option in the same context.

Fourth, a general context is proposed for the valuation of real options and the pricing of real assets in a discrete-time setting. Salkin (1991) shows how to apply the Cox, Ross and Rubinstein (1979) model for the valuation of complex capital budgeting decisions. The methodology is applied to a hypothetical case of a marginal natural resource project. The real benefit of this technique arises in its ability to value more realistically situations in which traditional techniques attributed little or no worth. Following the analysis in Salkin (1991), we develop a simple context for the valuation of real options using option pricing techniques in the presence of information costs. Then, using the Trigeorgis (1991) general Log-transformed binomial model for the pricing of complex investment opportunities, we provide a context for the valuation of these options under incomplete information. Trigeorgis (1991) proposed a Log-transformed binomial model for the pricing of several complex real options. We use that generalization to account for information costs in the pricing of complex investment opportunities.

Our approach can be extended to price most well-known real options in the presence of information costs. While the estimation of the magnitude of these costs is done in Bellalah and Jacquillat (1995) for financial options, it is possible to look for a convenient approach to estimate these costs for real options. We let this point for a future research.
The analogy between standard and exotic financial options facilitates considerably the valuation of real options. It is possible to use the main results in exotic options to value different real options. However, it is important to note that real options can be sometimes more difficult to value in the presence of information costs and a dependency between different real options in the same project.

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