Interactions of Corporate Financing and Investment Decisions: The Effect of Growth Options to Exchange or Expand

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Abstract

We construct a contingent claims model of the firm to examine interactions between investment and financing decisions in a setting where equityholders make self-interested investment decisions. The key feature of the model is the separate, but possibly correlated stochastic processes for the firm's assets-in-place and the asset underlying a growth option. Importantly, the firm has the flexibility to exercise and subsequently reverse the exercise of the growth option at any time. We examine the impact on firm value and the credit spread of risky debt when equityholders have an incentive to either overinvest or underinvest in the growth option. We show that these suboptimal investment incentives significantly reduce firm value and optimal leverage, and increase the credit spread of debt. We further illustrate how these investment incentives and the costs they engender depend on the characteristics of the firm and it asset structure.

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I. Introduction

This paper uses a real options framework to examine interactions between financing and investment decisions. We construct a model where the firm has assets-in-place and a growth option to invest in another set of assets. Separate, but possibly correlated diffusion processes describe both the assets-in-place and the asset underlying the growth option. In a finite horizon setting, the firm may exercise the growth option at any time, and once exercised, may reverse the exercise decision at any time. However, both the exercise and reversal decisions are costly.

The most interesting feature of the growth option exercise decision is that we model both asset substitution and asset expansion. In particular, when the growth option is exercised we assume that a fraction of assets-in-place is given up in exchange for the asset underlying the growth option. Thus, the case of pure asset substitution occurs if all of the assets-in-place are given up in exchange for the asset underlying the growth option. On the other hand, the case of pure asset expansion holds when none of the assets-in-place is exchanged for the asset underlying the growth option. Assets-in-place are fully restored if the firm later reverses the growth option exercise decision.

Within this dynamic real option setting, we examine the influence of risky debt financing on the optimal exercise policy of the growth option. To motivate debt financing, we assume that the interest payment of debt is tax deductible. However, the tax advantage of debt is offset by the cost of bankruptcy (triggered by the continuation value of equity falling below zero). The investment incentive effects of debt financing are examined by comparing growth option exercise policies and resulting firm and securityholder values under a first-best firm-value-maximizing exercise policy and a second-best equity-value-maximizing exercise policy.

The type of asset structure rearrangement that results when the growth option is exercised determines whether equityholders underinvest or overinvest in the growth option relative to the firm-value-maximizing exercise policy.[1] In the pure expansion case, equityholders underinvest in the growth option because they pay the full cost of the investment but risky debt receives a portion of the benefits. By contrast, in the pure exchange (substitution) case, equityholders overinvest in the growth option given sufficiently risky debt and assuming that the asset underlying the growth option is riskier than assets-in-place. Of course, there are intermediate cases between pure expansion and pure exchange that engender either underinvestment or overinvestment. In the analysis of the model we examine the determinants of over- and underinvestment incentives, including the effects of asset correlation, relative asset volatility and firm characteristics such as the tax rate and bankruptcy costs.

The cost of these suboptimal investment incentives is computed by comparing overall firm values under first- and second-best growth option exercise strategies. We find that both over- and underinvestment incentives significantly reduce firm value. An important component of this analysis is the identification of the agency cost component of the credit spread of risky debt. We compute this alternative measure of the cost of suboptimal investment incentives by comparing first- and second-best credit spreads holding leverage fixed. Importantly, we find a significant agency cost component of the credit spread. We further illustrate how the agency cost component of the credit spread and the overall loss in firm value vary with the characteristics of the firm and its asset structure.

[1] Equityholders underinvest in the growth option if they exercise the option at an underlying asset value that is greater than that chosen under a firm-value-maximizing strategy. One may interpret this as delaying investment rather than underinvesting. However, by waiting for a higher asset value, the probability that investment takes place by any future date decreases. It is in this sense that equityholders underinvest relative to a first-best value-maximizing growth option exercise strategy. Using analogous logic, equityholders overinvest in the growth option if they choose to exercise the option at a smaller underlying asset value.

Finally, we examine the influence of the growth option investment policy on optimal capital structure. We find that both over- and underinvestment incentives drive the firm to choose a significantly lower level of debt, and we examine how this choice is influenced by the rich set of parameters in the model. Future extensions of the model (discussed below) will allow for an examination of the interaction between growth option exercise policy and overall financial policy, including optimal initial debt level, maturity structure and intertemporal recapitalization policy.

There are a number of recent papers that examine interactions between financing and investment decisions in a dynamic real options framework. These papers include, Mello and Parsons (1992), Mauer and Triantis (1994), Mello, Parsons and Triantis (1995), Fries, Miller and Perraudin (1997), and Mauer and Ott (2000). However, these papers employ fairly simplistic models with only one source of uncertainty (e.g., the output price) and typically assume infinite maturity debt.[2] By comparison, our model structure is much more realistic, having possibly correlated stochastic processes for assets-in-place and the asset underlying the growth option, finite horizon debt allowing for an analysis of debt maturity structure, and a flexible model structure allowing for both underinvestment and overinvestment. Our much richer model structure allows for a deeper understanding of the interactions between financing and investment decisions, and a more complete analysis of the determinants of the agency costs of debt.

The next section provides a detailed description of the model and the procedure that we use to solve the model. Section III discusses the results of numerical solutions of the model. Section IV offers conclusions and discusses extensions.

II. The Model

Consider a firm that has assets-in-place and a growth option. The evolution of assets-in-place (A) and the asset underlying the growth option (G) are described by geometric Brownian motion:

$$dA = \alpha_A A dt + \sigma_A A dZ_A \quad \text{and} \quad dG = \alpha_G G dt + \sigma_G G dZ_G, \tag{1}$$

where the drift rates (α_A and α_G) and volatility rates (σ_A and σ_G) are constants, and dZ_A and dZ_G are the increments of standard Wiener processes. The Wiener processes are correlated such that $dZ_A dZ_G = \rho dt$. The convenience yields of the assets-in-place and the asset underlying the growth option, δ_A and δ_G, are assumed to be constant proportions of A and G respectively, and the riskless security is assumed to yield a constant instantaneous rate of r per year.

The firm may exercise the growth option at any time by giving up the fraction γ ($0 \le \gamma \le 1$) of assets-in-place and by paying a fixed switching cost of K_I. If $\gamma = 0$ exercise of the growth option leads to a pure expansion of assets-in-place by the asset underlying the growth option. On the other hand, if $\gamma = 1$ exercise of the growth option leads to a pure substitution of assets-in-place by the asset underlying the growth option. Thus, by varying γ we can examine asset expansion or asset substitution and the resulting incentives of managers acting to maximize either total firm value or equity value with respect to the growth option exercise strategy.

Once the firm has exercised the growth option, we allow the firm to reverse the decision at any time (and to subsequently re-exercise the growth option at a later date) by paying a fixed switching cost of K_O. Depending on the value of γ, reversing the growth option exercise

[2] Mauer and Triantis (1994) study interactions between investment and financing decisions in a finite horizon model with finite maturity debt. However, they assume that managers always maximize total firm value and thereby ignore agency costs of debt financing.

decision will re-establish the amount γA of assets-in-place. However, as $K_O \to \infty$ the growth option exercise decision is completely irreversible. Finally, the model has a finite horizon of T years. As such, the growth option also has a maturity of T years.

The firm is financed with both equity and debt. We model debt having a face value of F and a coupon rate (periodic interest payment) of c (cFdt). Initially we assume that the maturity of debt is T years. Subsequent extensions of the model will allow for debt having any maturity less than or equal to T years. To motivate debt financing, we assume that interest payments of debt are tax-deductible at the constant corporate rate τ.[3] While taxes provide an incentive to issue debt in the model, the debt level is limited by requiring that equity value must always be greater than or equal to zero. If this condition is violated, i.e., if the continuation value of equity is negative, bondholders assume ownership of the firm and deadweight costs are incurred in the process. These costs are equal to the fixed amount B plus the proportion b of the value of the firm if it were unlevered, where $B \geq 0$ and $0 \leq b \leq 1$. The bondholders may then choose a growth option exercise strategy that maximizes the value of their unlevered assets, i.e., the model does not currently allow the bondholders to choose a new capital structure.

Our objective is to determine the growth option exercise policy (i.e., values of G at which the firm either exercises the growth option or reverses the exercise decision) over time that maximizes levered equity value and that *separately* maximizes total levered firm value. We call the firm-value-maximizing exercise strategy the first-best growth option exercise strategy and the levered-equity-maximizing strategy the second-best growth option exercise strategy.

In order to value the firm and its claims, we assume that an equilibrium model such as a single- or multi-factor CAPM holds and that the firm's claims do not alter the equilibrium-

[3] Note that τ can be interpreted as the net tax advantage of debt relative to equity after adjustment for personal taxes.

pricing kernel. An equivalent martingale probability measure is used to value the claims by risk-adjusting the drift rate of the firm's assets-in-place and the asset underlying the growth option. Thus, the drift rate of A, α_A, is shifted to $r-\delta_A$, and the drift rate of G, α_G, is shifted to $r-\delta_G$. It is clear that, given the complexity of the problem we are considering, it will not be possible to obtain a closed-form solution. Thus, a computational technique will be necessary. Rather than present a complex set of partial differential equations that we will not use directly, we instead describe a computational approach that can be used to value the firm and its claims and determine the first- and second-best growth option exercise strategies.

We use a two stochastic variable, trinomial tree structure to obtain numerical solutions of the model. The steps are as follows: (i) transform the stochastic state variables to driftless, constant variance normal variables, (ii) determine grid spacing and step probabilities for each variable's individual tree (i.e., construct marginal distributions for the two variables), (iii) take inverse transformations to determine the values of the original variables at all nodes, and (iv) cross the two trees to generate a two-variable tree, using adjustments to the product of the marginal probabilities to induce correlation.[4] We briefly describe each of these steps below.

The transformation of a lognormal variable to a driftless constant variance normal variable is accomplished by taking the natural log of the original variable and subtracting the expected growth rate. Thus, let

$$W_A \equiv \Phi_A(t) = \ln A(t) - \left(r - \delta_A - 0.5\sigma_A^2\right)(t), \tag{2}$$

and

$$W_G \equiv \Phi_G(t) = \ln G(t) - \left(r - \delta_G - 0.5\sigma_G^2\right)(t). \tag{3}$$

[4] See Childs and Triantis (2000) for a similar numerical procedure involving two correlated variables.

A straightforward application of Ito's Lemma confirms that $W_A = \sigma_A dZ_A$ and $W_G = \sigma_G dZ_G$, i.e., W_A and W_G are driftless, constant variance normal variables.

Next, we build a tree for each variable. Both trees will have the same time step, Δt, the same periods per year, PPY = 1/Δt, and the same total number of periods, N = T/Δt, where T is the time horizon in years. Consider the tree for W_A first. Let the grid spacing, ΔW_A, be[5]

$$\Delta W_A = \sigma_A \sqrt{1.5\Delta t}\,. \tag{4}$$

Hull and White (1990) show that the probabilities of moving up to $W_A + \Delta W_A$, over to W_A, or down to $W_A - \Delta W_A$ in a time step, Δt, are

$$p_A^u = \frac{1}{2}\sigma_A^2 \frac{\Delta t}{(\Delta W_A)^2}, \quad p_A^o = 1 - \sigma_A^2 \frac{\Delta t}{(\Delta W_A)^2} \text{ and } p_A^d = \frac{1}{2}\sigma_A^2 \frac{\Delta t}{(\Delta W_A)^2}, \tag{5}$$

respectively. Combining equations (4) and (5) yields the probabilities $p_A^u = p_A^o = p_A^d = \frac{1}{3}$ and stability is assured.

The tree for W_A is completely defined. If i is the number of nodes above the center of the lattice and j is the number of time steps since time 0, then $W_A(i,j) = W_A(0,0) + i\Delta W_A$, where $W_A(0,0)$ is the value of W_A at the initial or center node of the lattice at time 0. Figure 1 displays the branching for the simple, one variable tree. There are actually 2n + 1 initial values corresponding to the n nodes above and below the initial node (where n = 4 in the figure). This contrasts with a traditional lattice framework where the lattice would start from a single initial node (the current state). In our setting, we need an initial grid that contains the current state and enough nodes above and below the current state to also contain the initial exercise and reversal boundaries for the growth option. Given the node values for each transformed variable, $W_A(i, j)$, the value of the original variable is easily obtained by inverting the transformation in equation (2):[6]

$$A(i, j) = e^{W_A(i,j)+(r-\delta_A-0.5\sigma_A^2)j\Delta t} = A_0 e^{i\Delta W_A+(r-\delta_A-0.5\sigma_A^2)j\Delta t}. \tag{6}$$

The tree for G is developed in a similar way.

Given the two trees for the individual variables, a two-asset tree can be built using the characteristics of the one variable trees and adjusting the probability structure to induce correlation between the variables. In the two asset tree, nine branches emanate from each node (i.e., all combinations of up, over and down for the two variables) and the time step, grid spacing and node values have exactly the same values as in the one asset trees. If the variables are uncorrelated, the joint step probabilities are simply the product of the marginal probabilities from the individual trees. For non-zero correlation, Hull and White (1990) provide the joint probabilities (using the marginal probabilities) so that the two variable tree has the appropriate correlation. Define $\eta \equiv |\rho|/36$. If $\rho > 0$ the joint probabilities for the two-asset tree are[7]

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[5] Hull and White (1990) suggest using $\Delta W_A = \sigma_A\sqrt{3\Delta t}$, whereas in a similar setting Boyle (1986) suggests the spacing in (4). We choose to use Boyle's spacing because it produces tighter grid spacing which helps mitigate potential valuation problems caused by discontinuities at the default boundary.

[6] Due to Jensen's inequality, even if the mean and variance of the transformed variable are correct, the mean and variance of the original variable (and subsequently the correlation between variables) may not be exact. For our stochastic processes and parameter choices, these differences are very small.

		W_A		
		move down	move over	move up
	move up	$p_A^d p_G^u - \eta$	$p_A^o p_G^u - 4\eta$	$p_A^u p_G^u + 5\eta$
W_G	mover over	$p_A^d p_G^o - 4\eta$	$p_A^o p_G^o + 8\eta$	$p_A^u p_G^o - 4\eta$
	move down	$p_A^d p_G^d + 5\eta$	$p_A^o p_G^d - 4\eta$	$p_A^u p_G^d - \eta$

For computational accuracy, we also employ finer grid spacing in the middle section of the lattice using an extension of the adaptive mesh technique of Figlewski and Gao (1999). We use this technique to further mitigate the problems of finding an optimal capital structure caused by discontinuities at the default boundary. Thus, in the middle section of the lattice we break each coarse grid step into two or more fine grid steps depending on the degree of accuracy desired. Below we briefly describe the technique for a one-variable tree when the finer middle region is twice as dense (in the asset dimension) as the coarse region. A more detailed description of this technique is available on request.

Figure 2 depicts the branching for a tree with finer grid spacing in the middle. The first of the three panels highlights the coarse nodes on the outer section of the tree. This section uses the same node values and probabilities of an otherwise identical tree that has all coarse nodes. If the asset fine grid spacing is one-half the coarse grid spacing and the fine time step is one-quarter of the coarse time step, then the probabilities in the fine section are the same as in the coarse

[7] When $\rho < 0$, the joint probabilities in the four corners of the matrix are different. Starting in the upper left-hand corner of the matrix and moving clockwise, the joint probabilities when $\rho < 0$ are $p_A^d p_G^u + 5\eta$, $p_A^u p_G^u - \eta$, $p_A^u p_G^d + 5\eta$, and $p_A^d p_G^d - \eta$, respectively.

section.[8] The third panel of Figure 2 highlights the fine grid spacing in the middle of the tree. For the line between the coarse node and fine node sections, the tree branches to the nearest coarse node and equations (4) and (5) are used to determine the appropriate probabilities. The branching for this crossover region is highlighted in the second panel of Figure 2.

Once the lattice is constructed, the valuation procedure requires starting at the horizon T and working backward through time in the standard dynamic programming fashion. For purposes of this discussion, *implemented* means that the growth option has been exercised and *unimplemented* means that the growth option has not been exercised (or the exercise decision has been reversed).

Consider first the valuation and growth option exercise policy for an unlevered firm. At time T the growth option expires and the value of the firm if it is currently unimplemented, $V_u^U(T)$, is equal to

$$V_u^U(T) = A(T) + \max\left(G(T) - \gamma A(T) - K_I, 0\right). \quad (7)$$

Notice in (7) that the cost of exercising the growth option is the fraction γ of assets in place A(T) plus the fixed cost K_I. Alternatively, if the firm is currently implemented at T, firm value, $V_i^U(T)$, is equal to

$$V_i^U(T) = (1-\gamma)A(T) + G(T) + \max\left(\gamma A(T) - G(T) - K_O, 0\right). \quad (8)$$

Again, notice in (8) that if the firm decides to reverse the growth option exercise decision it exchanges G(T) for γA(T) and incurs the fixed cost K_O.

[8] This result is quickly obtained using equations (4) and (5).

At earlier nodes, say at time t, we calculate (1) the discounted expected value of remaining implemented (over the time step t to t + Δt) if the firm is currently implemented, (2) the discounted expected value of remaining unimplemented (over the time step t to t + Δt) if the firm is currently unimplemented, (3) the value of exercising the growth option if the firm is currently unimplemented, and (4) the value of reversing the growth option exercise decision if the firm is currently implemented.[9] If the firm arrives at a time t node implemented, its value is the maximum of (1) and (4). If the firm arrives at a time t node unimplemented, its value is the maximum of (2) and (3). Once the dynamic programming procedure is completed, the value of the unlevered firm is $V_0^U(A(0),G(0))$, i.e., the value at the initial node.[10]

Consider next the valuation of equity and debt. Time T equity, E(T), and debt, D(T), values are computed as follows.

If the firm is implemented and remains implemented:

$$\begin{array}{lll} E_{ii}(T) = (1-\gamma)A(T) + G(T) - (F + cF(1-\tau)\Delta t) \text{ and } D_{ii}(T) = F + cF\Delta t & \text{if } E_{ii}(T) \geq 0 \\ E_{ii}(T) = 0 \text{ and } D_{ii}(T) = (1-b)V_i^U(T) - B & \text{otherwise} \end{array} \tag{9}$$

Note in (9) that bankruptcy is triggered when the value of equity is negative. In bankruptcy bondholders receive the implemented value of the unlevered firm net of variable (b) and fixed (B) bankruptcy costs. Also note that time T debt value outside of bankruptcy is equal to the final principal repayment, F, plus the final coupon payment over the time step T - Δt to T, $cF\Delta t$.

If the firm is implemented and switches to unimplemented:

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[9] Note that expected values are based on the nine branches emanating from the node at t, and the associated probabilities shown in the matrix above. We use the risk-free rate to discount over the time step t to t + Δt given our risk-neutral (equivalent martingale) valuation setting, i.e., the discount factor for a time step is $e^{-r\Delta t}$.

[10] Note that there actually are 2n + 1 initial values corresponding to the n nodes above and below the initial node at t = 0, i.e., the node at n = 0 and m = 0.

$$E_{iu}(T) = A(T) - K_O - (F + cF(1-\tau)\Delta t) \text{ and } D_{iu}(T) = F + cF\Delta t \qquad \text{if } E_{iu}(T) \geq 0$$
$$E_{iu}(T) = 0 \text{ and } D_{iu}(T) = (1-b)V_u^U(T) - B \qquad \text{otherwise} \tag{10}$$

Note in (10) that in bankruptcy bondholders receive the unimplemented unlevered firm value net of bankruptcy costs.

If the firm is unimplemented and remains unimplemented:

$$E_{uu}(T) = A(T) - (F + cF(1-\tau)\Delta t) \text{ and } D_{uu}(T) = F + cF\Delta t \qquad \text{if } E_{uu}(T) \geq 0$$
$$E_{uu}(T) = 0 \text{ and } D_{uu}(T) = (1-b)V_u^U(T) - B \qquad \text{otherwise} \tag{11}$$

Finally, if the firm switches from unimplemented to implemented:

$$E_{ui}(T) = (1-\gamma)A(T) + G(T) - K_I - (F + cF(1-\tau)\Delta t) \text{ and } D_{ui}(T) = F + cF\Delta t \quad \text{if } E_{ui}(T) \geq 0$$
$$E_{ui}(T) = 0 \text{ and } D_{ui}(T) = (1-b)V_i^U(T) - B \qquad \text{otherwise} \tag{12}$$

We can now specify levered firm value at time T under the first- and second-best growth option exercise strategies. Under the first-best strategy, the firm chooses the growth option exercise strategy to maximize the sum of equity and debt value. Thus, first-best levered firm value is specified as

$$V_i^{LF}(T) = \max\left(E_{ii}(T) + D_{ii}(T), E_{iu}(T) + D_{iu}(T)\right) \qquad \text{if implemented}$$
$$V_u^{LF}(T) = \max\left(E_{uu}(T) + D_{uu}(T), E_{ui}(T) + D_{ui}(T)\right) \qquad \text{if unimplemented} \tag{13}$$

Under the second-best strategy, the firm chooses the growth option exercise strategy to maximize equity value alone. If the firm is implemented, the second-best levered firm value is specified as

$$V_i^{LS}(T) = \begin{cases} E_{ii}(T) + D_{ii}(T) & \text{if } E_{ii} \geq E_{iu} \\ E_{iu}(T) + D_{iu}(T) & \text{otherwise} \end{cases} \tag{14}$$

Alternatively, if the firm is unimplemented, the second-best levered firm value is specified as

$$V_u^{LS}(T) = \begin{cases} E_{uu}(T) + D_{uu}(T) & \text{if } E_{uu}(T) \geq E_{ui}(T) \\ E_{ui}(T) + D_{ui}(T) & \text{otherwise} \end{cases} \tag{15}$$

At earlier nodes, the values of equity, debt and levered firm value are computed using the following procedure. If the firm arrives at a node implemented, we first compute the discounted expected value of equity assuming (1) the firm remains implemented over the next time step, and (2) the firm switches to the unimplemented state. If the discounted expected value of equity is non-negative under (1) and/or (2), the value of debt is simply the discounted expected value of debt over the next time step.[11] However, if the discounted expected value of equity is negative under (1) and (2), the value of debt is equal to the appropriate unlevered firm value net of bankruptcy costs and equity value is set equal to zero.[12] Under the first-best policy, levered firm value is set equal to the maximum of debt and equity values under (1) or (2). Alternatively, under the second-best policy, levered firm value is set equal to the sum of debt and equity values for the policy ((1) or (2)) that maximizes equity value.

A similar procedure applies if the firm arrives at a node unimplemented. However, in this case we first calculate the values of equity and debt for the cases where the firm (1) remains unimplemented over the next time step and (2) switches to the implemented state. First- and second-best levered firm values are then computed in an analogous fashion using these equity

[11] Note that if the firm switches states at a node (e.g.., from implemented to unimplemented in (2)), the value of debt is equal to the discounted expected value of debt over future nodes corresponding to the switched state.
[12] If equity defaults at a node, debtholders receive the implemented unlevered firm value at that node.

and debt values.

Once the dynamic programming procedure is completed, we have the first- and second-best values of equity, debt and the levered firm at the initial node (i.e., at node $\{A(0), G(0)\}$) and at the n nodes above the initial node and the n nodes below the initial node, i.e., a total of $2n + 1$ nodes at time $t = 0$. For each of these nodes we also have the value of G at which the firm (under either the first- or second-best exercise strategy) would implement the growth option (assuming the option is not implemented at time $t = 0$), and the value of G at which the firm would reverse the growth option exercise decision (assuming the option is implemented at time $t = 0$). Of course, the analysis also allows for the determination of the optimal face value (or coupon rate) of debt that maximizes firm value under each one of these cases.

III. Numerical Results

This section examines numerical solutions to the valuation problem detailed above. We separate the discussion by whether equityholders have an incentive to overinvest or underinvest in the growth option. In each subsection, we first discuss the choice of parameter inputs. This is followed by an analysis of solutions of the model holding leverage constant, which in turn is followed by an analysis of solutions of the model allowing for the determination of optimal capital structure.

A. Agency Cost of Overinvestment

If the asset underlying the growth option is riskier than assets-in-place and if debt is sufficiently risky, then equityholders should have an incentive to overinvest in the growth option relative to a firm-value-maximizing growth option exercise strategy. At any point in time, the growth option exercise strategy is characterized by two unique values for G at which the firm will either exercise the growth option if it has not already done so or reverse the exercise decision if it was exercised at an earlier point in time. Given the finite time horizon of the model, these exercise trigger values will not be stationary, but rather will be time-dependent. Furthermore, within this framework, overinvestment is characterized by equityholders choosing to exercise the growth option at a smaller value of G than that chosen by a manager concerned with maximizing overall firm value (i.e., the sum of equity and debt).[13] Of course, equityholders' incentive to overinvest is driven by the residual nature of their claim in the presence of debt, and the resulting incentive to take actions that shift wealth from bondholders to themselves.

We choose a set of base case parameter inputs that allow us to focus the analysis on overinvestment incentives and the consequent agency cost of debt. We assume that the initial values of assets in place, A_0, and the asset underlying the growth option, G_0, are 1.0.[14] The base case volatility rate, σ_A, and convenience yield, δ_A, of assets in place are 0.15 and 0.02, respectively. By contrast, the asset underlying the growth option has a base case volatility rate, σ_G, and convenience yield, δ_G, of 0.40 and 0.05, respectively. As we will see, the higher volatility rate underlying the growth option encourages equityholders to shift investment from assets in place to the growth option, which in turn drives their incentive to overinvest. Although not necessary, the higher convenience yield for the asset underlying the growth option encourages earlier exercise of the growth option for both first-best (firm-value-maximizing) and second-best (equity-value-maximizing) decision-makers. Following Trigeorgis (1991), the

[13] Although it is much more natural to view overinvestment incentives from the perspective of exercising the growth option, one might also be inclined to view overinvestment as equityholders reversing the growth option exercise decision later than the reversal policy under a firm-value-maximizing policy. Nevertheless, our analysis will focus on the exercise policy rather than the reversal policy.

higher convenience (dividend) yield may be viewed as a competition factor that encourages the firm to exercise the growth option before its value is dissipated by the entry of competitors. Finally, we assume that the stochastic evolutions of the assets in place and the asset underlying the growth option have a correlation, ρ, of 0.50.[15]

For the base case, we assume that the firm must give up 75 percent of assets in place when it exercises the growth option, i.e., $\gamma = 0.75$. In addition, the firm must pay a fixed exercise cost, K_I, of 0.03. However, to focus only on the growth option exercise decision, we assume that once the growth option is exercised it is infinitely costly to reverse the exercise decision, i.e., $K_O = \infty$. We further assume a base case coupon rate of debt, c, of 0.07 per year, a corporate tax rate, τ, of 0.30, a risk-free rate, r, of 0.06 per year, and variable and fixed bankruptcy costs, b and B, of 0.50 and 0, respectively. Finally, we assume a twenty-year time horizon with four periods per year (each asset value node is three months apart), allowing for two additional fine nodes within each period in the middle of the lattice.

Table 1 reports numerical solutions for the base case and for variations of base case parameter values, holding leverage constant at a face value of debt of F = 0.90. Although the choice of 0.90 for the fixed face value is somewhat arbitrary, we show later that it is reasonable given the optimal face values for the cases presented in Table 1. Note that we initially hold leverage constant across the cases in Table 1 to examine the pure effect of input parameter variation on the solution. For each set of parameters, the table reports first-best (firm-value-

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[14] Recall that the solution technique that we employ generates firm and securityholder values for this initial time zero node (i.e., $\{A_0, G_0\}$) and for n time zero nodes above and below the initial node. We use a value for n of 12.

[15] At first blush, it may seem more reasonable to assume that the two assets are completely unrelated or even negatively correlated. After all, the standard textbook risk-shifting story is usually one where a low risk business shifts assets into a high risk investment project such as drilling for oil in the North Sea. However, we feel that it is much more realistic to model a growth option that is related to the existing business (although imperfectly) but having substantially more uncertain prospects. Nevertheless, we examine the full range of possible correlations in our analysis.

maximizing) and second-best (equity-value-maximizing) growth option exercise policies, G_{FB}^{*} and G_{SB}^{*}, levered firm values (sum of debt and equity value), V_{FB}^{L} and V_{SB}^{L}, leverage ratios (debt value divided by firm value), LEV_{FB} and LEV_{SB}, and credit spreads (in basis points), CS_{FB} and CS_{SB}.[16] The table also reports the agency cost of debt, AC, and the agency cost component of the credit spread, CS_{AC}, where

$$AC = \frac{V_{FB}^{L} - V_{SB}^{L}}{V_{SB}^{L}} \quad \text{and} \quad CS_{AC} = CS_{SB} - CS_{FB}.$$

For the base case set of parameters (and at the initial time zero values $A_0 = G_0 = 1.0$), the first-best policy is to exercise the growth option when G rises to 1.942. By comparison, equityholders choose a much more aggressive exercise policy as indicated by a second-best policy of exercising the growth option when G rises to 1.389. Equityholders' incentive to overinvest in the growth option induces an agency cost of 1.44 percent, and an agency cost component of the credit spread of approximately 35 basis points.

As expected, the market value of equity is larger and the market value of debt is smaller under the second-best policy than under the first-best policy. For the base case parameters, the first- and second-best equity value comparison is 0.611 versus 0.623, and the first- and second-best debt value comparison is 0.904 versus 0.871. Notice that equityholders' gain from pursuing a more aggressive growth option exercise strategy of 0.012 (0.623 – 0.611) is smaller than the loss in debt value of 0.033 (0.904 – 0.871), which results in an agency cost of overinvestment of 0.021 or, as noted above about 1.44% of (second-best) levered firm value.

[16] The yield of debt is simply computed as the discount rate that sets the present value of debt coupon payments and final face payment equal to the value of debt. The credit spread is then computed as the difference between this

Figure 3 displays graphs of first- and second-best growth option exercise policies (i.e., G^*_{FB} and G^*_{SB}) as a function of the initial level of assets in place.[17] Observe that both policies are increasing in the initial level of assets in place. The reason is that the firm must give up the proportion γ of assets in place when it exercises the option, and therefore a larger initial value of assets in place encourages both sets of policy makers to wait for a larger value of G before exercising the option. More importantly, observe that the largest difference between first- and second-best policies occurs at low values of assets in place, with the difference narrowing as assets in place increase in value. The reason is that at large values of assets in place debt becomes virtually riskless and equityholders' gain from aggressively exercising the growth option vanishes.

Note that the base case parameter values produce somewhat high leverage ratios (in the high fifty percent range) but reasonable credit spreads. For example, in a large sample of corporate bonds over the period 1988 to 1997, Collin-Dufresne, Goldstein and Martin (1999) report average credit spreads (in basis points) of 124, 153 and 178 for long-term noncallable debt when a firm's leverage ratio is in the range of 35-45%, 45-55%, and > 55%, respectively. In comparison, the base case parameter values produce a first-best credit spread of 96.2 basis points and a second-best credit spread of 130.9 basis points. Although the first-best (ideal) credit spread is not surprisingly low in comparison to the empirical distribution of credit spreads, the second-best credit spread is consistent with the empirical distribution. This suggests that a fairly large component of observed credit spreads is attributable to the agency cost of debt.

The remaining rows in Table 1 report results for various input parameter changes holding

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yield and the risk-free rate of interest.

[17] Notice that the figure contains the base-case policies reported in Table 1 for the case where assets in place equal 1.0.

constant all other parameters at base case values. As γ increases, the cost of exercising the growth option increases because a larger proportion of assets in place must be given up in exchange for the asset underlying the growth option. As a result, both first- and second-best growth option trigger prices are increasing in γ. Despite the desire to wait, agency costs increase as γ increases. The reason is that as γ approaches 1.0 the exercise decision converges to a pure substitution of low-risk assets in place for the high-risk asset underlying the growth option. As such, the divergence between the first- and second-best exercise policies widens as γ increases, and agency costs increase. By contrast, agency costs are decreasing in the fixed cost component, K_I, of exercising the growth option. Holding γ constant, a higher value of K_I encourages both first- and second-best policy makers to wait for a higher value of G before exercising the growth option. The higher value of G tends to offset the negative effect on debt value of the shift in risk when the growth option is exercised, and so agency costs fall as K_I increases.

As expected, agency costs increase as bankruptcy costs increase.[18] However, it is interesting to note that agency costs are decreasing in τ. As τ increases the tax shield of debt is more valuable. This raises overall levered firm value, which in turn reduces the default risk of debt. Thus, as τ increases equityholders have less of an incentive to aggressively exercise the growth option and the difference between the first- and second-best growth option exercise policies narrows, and agency costs sharply decrease.

The convenience yields of assets in place, δ_A, and the asset underlying the growth option, δ_G, have opposite effects on the growth option exercise policy and agency costs. As δ_A increases agency costs decrease. Holding the risk-adjusted discount rate for assets in place constant, a larger value of δ_A means a smaller rate of appreciation, α_A.[19] This influences the growth option exercise policy because the proportion γ of assets in place must be given up when the option is exercised. Thus, as δ_A increases the expected cost of exercising the growth option decreases, which encourages both first- and second-best policy makers to delay exercise of the option and agency costs decrease. In sharp contrast, as δ_G increases both types of policy makers are encouraged to more aggressively exercise the growth option and agency costs sharply increase. Again, holding the risk-adjusted discount rate for the asset underlying the growth option constant, a larger value of δ_G means a smaller rate of appreciation (α_G) and therefore a smaller value of waiting to invest.[20] This encourages equityholders to exercise the risky growth option sooner, making debt considerably more risky at higher values of δ_G. The implication is that agency costs sharply increase as δ_G increases. An analogous effect is observed for the risk-free rate, r. As r decreases the risk-adjusted drift rate of the asset underlying the growth option decreases, which encourages a more aggressive growth option exercise policy and sharply increases agency costs.

The remainder of Table 1 highlights the influence of the correlation, ρ, between assets in place and the asset underlying the growth option and these assets' respective volatility rates, σ_A and σ_G. Despite the much higher base volatility rate of G, the smaller the correlation, the greater the diversification potential of combining A and G. This *co-insurance* effect should enhance the value of debt and serve to mitigate the incentive of equityholders to use the growth

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[18] Note that a larger b increases G^*_{FB} but has no affect on G^*_{SB}. Holding leverage fixed, bankruptcy costs have no influence on the market value of equity and so equityholders ignore bankruptcy costs when choosing the (second-best) growth option exercise policy.

[19] Alternatively, as δ_A increases the risk-adjusted drift rate of A, $r-\delta_A$, decreases.

option exercise policy to shift wealth from debtholders to themselves. Consistent with this intuition, the disparity between the first- and second-best growth option exercise policies narrows as ρ decreases and agency costs are reduced.

The effects of σ_A and σ_G on the agency cost of debt are interesting. First, agency costs sharply increase as σ_A decreases. Holding σ_G constant, as σ_A decreases the asset underlying the growth option becomes relatively more risky and the difference between the first- and second-best growth option exercise policies widens. As such, agency costs are magnified as σ_A decreases. However, σ_G has a non-monotonic effect on the agency cost of debt. Agency costs first increase and then decrease as σ_G increases. There are two countervailing effects. First, as σ_G increases equityholders have more of an incentive to risk-shift. However, as σ_G increases both first- and second-best policy makers are encouraged to delay exercise of the growth option, which reduces the riskiness of debt and therefore the incentive of equity to risk-shift. At low levels of σ_G the first effect dominates and agency costs are increasing in σ_G. At high levels of σ_G the second effect dominates and agency costs are decreasing in σ_G.

Table 2 reports numerical solutions for parameter input cases parallel to those in Table 1 at the optimal capital structure. Starting with the base case (and for each parameter variation thereafter) we report the face value of debt that maximizes overall firm value for both the first- and second-best growth option exercise policies. These values, denoted F^*_{FB} and F^*_{SB}, are reported in the second and third columns of Table 2.

Consider first the solution for the base case parameters. As expected, the agency cost of debt induces a smaller optimal face value under the second-best (equity-value-maximizing)

[20] Viewing δ_G as a dividend yield or cash flow rate on the underlying asset, as δ_G increases the opportunity cost of

growth option exercise policy. The first-best optimal face value is 0.84 and the second-best optimal face value is 0.76. This lower overall level of leverage results in an agency cost of about one percent of (second-best) firm value at the optimal capital structure. Interestingly, the credit spreads under the first- and second-best policies are roughly equal at about 80 basis points.

The remaining comparative static results reported in the table are qualitatively similar to those in Table 1, except that the overall magnitude of agency costs is typically lower because the firm can now optimally adjust leverage to mitigate equityholder risk-shifting incentives. However, there are some notable exceptions. First, observe that the optimal face value falls as bankruptcy costs increase. As a result, and in sharp contrast to the results in Table 1 where leverage is held constant, agency costs decrease as bankruptcy costs increase. A similar but opposite effect is observed for the corporate tax rate. When leverage is held constant (Table 1), agency costs fall as the tax rate increases. By contrast, optimal leverage sharply increases with the corporate tax rate in Table 2, and therefore agency costs increase. Finally, note that in sharp contrast to the results in Table 1, for negative values of ρ the agency cost of debt increases as ρ decreases. Observe that both first- and second-best decision-makers choose a larger face value of debt as ρ decreases. Apparently, the increased debt capacity resulting from imperfectly correlated assets results in a debt tax shield that is more valuable than the resulting higher agency costs.

B. Agency Cost of Underinvestment

Equityholders have an incentive to underinvest in the growth option when the asset underlying the option closely mimics assets in place and the exercise decision represents an expansion rather

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waiting to invest in the growth option increases, thereby encouraging earlier exercise of the option.

than a substitution investment decision. As originally analyzed by Myers (1977), equityholders may underinvest in this type of growth option when they pay the full cost of making the investment but share the benefits with debtholders. Debtholders may benefit because the larger asset base reduces default risk and thereby enhances the value of their claim. In the context of this model, the underinvestment incentive would be manifested by equityholders waiting for a higher value of G (than that chosen under a firm-value-maximizing strategy) before exercising the growth option.[21] The resulting agency cost of underinvestment can be computed as the difference between first- and second-best levered firm values, and by a comparison of credit spreads under the two exercise policies.

Consistent with the notion that the growth option constitutes a decision to expand the asset base, we assume that assets-in-place and the asset underlying the growth option are identical. Thus, base case volatilities (σ_A and σ_G) are equal to 0.20, convenience yields (δ_A and δ_G) are equal to 0.06, and the correlation between the two asset processes (ρ) is 1.00. The initial value of assets in place (A_0) is 1.00 and the initial value of the asset underlying the growth option (G_0) is 1.25. We further assume that exercise of the growth option constitutes a pure expansion decision by assuming that the fraction of assets-in-place that must be given up in exchange for the asset underlying the growth option (γ) is 0. Nevertheless, the fixed cost of exercising the growth option (K_I) is 1.00, and in keeping with our focus on the investment decision, we continue to assume that it is infinitely costly to reverse the growth option exercise decision, i.e., $K_O = \infty$. Finally, as before we assume the coupon rate of debt (c) is 0.07 per year, the risk-free rate (r) is 0.06 per year, the corporate tax rate (τ) is 0.30, and variable and fixed

[21] Note that the larger is G the smaller is the default risk of debt at the point of exercise. As such, equityholders wait for a larger G value before exercising the growth option to minimize the default-risk-reducing benefits accruing to

bankruptcy costs (b and B) are 0.50 and 0, respectively. The time horizon is 20 years with 4 periods per year, and the number of fine nodes between each course node in the middle part of the lattice is 2.

Table 3 reports numerical solutions holding the face value of debt fixed at 1.00. For the base case parameters and at the initial values of assets in place and the asset underlying the growth option, the firm-valuing maximizing growth option exercise strategy is to exercise the option when G rises to 1.420. By comparison, equityholders significantly underinvest in the growth option by waiting for G to rise to 1.889. As expected, equity value is larger and debt value is smaller under the second-best policy; first- and second-best equity values are 0.554 and 0.588, and first- and second-best debt values are 0.917 and 0.870. The resulting agency cost of debt is 0.861 percent, and the agency cost component of the credit spread is a more than 50 basis points.

Figure 4 displays graphs of the first- and second-best growth option trigger values (i.e., G^*_{FB} and G^*_{SB}) as a function of the initial value of assets in place. The disparity between the two policies is greatest at low initial values of assets in place and narrows as assets in place increase in value. At low values, where default risk is greatest, equityholders have a strong incentive to delay exercise of the growth option to minimize the benefits of asset expansion accruing to bondholders. However, this incentive naturally dissipates as the initial value of assets in place increases because the default-risk-reducing benefits of exercising the growth option are reduced and the lion's share of the benefits accrue to equityholders. Indeed, as seen in the figure, the two policies converge at high initial values of assets in place.

The comparative static effects of γ and K_I are similar. As the cost of exercising the

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debtholders.

growth option increases, the value of the growth option decreases, and both first- and second-best policy makers delay exercise of the growth option by waiting for a higher value of G. Since the option is exercised at a higher underlying asset value, there is less of a potential wealth transfer to debtholders on exercise. Thus, regardless of the divergence between first- and second-best exercise policies, agency costs decrease as either γ or K_I increase.

The agency cost of underinvestment increases as bankruptcy costs increase. Since expected bankruptcy costs increase as b increases, the first-best growth option investment policy is to expand the firm's asset base sooner by exercising the growth option at a lower value of G. However, equityholders have no such incentive, and therefore the second-best exercise policy is invariant to b. Hence the disparity between the first- and second-best policies is magnified and agency costs increase as b increases.

As the corporate tax rate decreases the tax shield of debt and therefore levered firm value decreases. Equityholders respond by (slightly) delaying exercise of the growth option. By contrast, the benevolent firm-value-maximizing policy maker exercises the growth option sooner. The net effect is that first- and second-best growth option exercise policies diverge, and agency costs increase as the corporate tax rate decreases.

The convenience yield of assets in place (δ_A) has a relatively negligible effect on agency costs. However, agency costs sharply decrease as the convenience yield of the asset underlying the growth option (δ_G) decreases. Holding the risk-adjusted discount rate of the asset underlying the growth option constant, a smaller convenience yield implies a larger rate of price appreciation (α_G) and therefore a larger firm and growth option value. This reduces the default risk of debt and thereby mitigates the potential enhancement of debt value when the growth option is exercised. The implication is that agency costs decrease as δ_G decreases. Similarly, agency costs sharply decrease as r increases. A larger value of r causes the risk-adjusted drift rate of both assets to increase, which increases the firm and growth option values and reduces the default risk of debt.

Finally, variations of ρ or σ_A appear to have negligible effect on the agency cost of debt. However, agency costs decrease sharply as σ_G increases. Note that the value of the firm's growth option increases as σ_G increases, as does the value of G at which the firm exercises that option.[22] Because the option and the firm are more valuable as σ_G increases, the probability of default actually decreases as σ_G increases; therefore, equityholders are less concerned about a wealth transfer to debtholders when the growth option is exercised. As a result, first- and second-best growth option exercise policies tend to converge and agency costs decrease as σ_G increases.

Table 4 reports numerical solutions for parameter input cases parallel to those in Table 3 at the optimal capital structure. Starting with the base case (and for each parameter variation thereafter) we report the face value of debt that maximizes overall firm value for both the first- and second-best growth option exercise policies. These values, denoted F^*_{FB} and F^*_{SB}, are reported in the second and third columns of Table 4.

The agency cost of underinvestment could be eliminated by choosing a low enough level of debt such that the probability of default is negligible. By choosing such a debt level, equityholders would no longer have an incentive to delay the exercise of the growth option because any wealth transfer to debtholders would be minimal, if not completely eliminated. However, an examination of the base case in Table 4 clearly shows that agency costs are not driven to zero at the optimal face value of debt. Although the second-best optimal face value is less than the first-best optimal face value, agency costs remain significant at about 0.8 percent of firm value. These agency costs are driven by the fact that there is still a considerable wedge between first- and second-best growth option exercise policies.

The effects of varying the various parameters on the growth option exercise policy and resulting agency cost of debt are generally similar to those reported in Table 3 where leverage is held fixed. However, there are a few interesting differences. First, observe in Table 4 that bankruptcy costs (b) and the corporate tax rate (τ) have opposite effects on agency costs than those found in Table 3. When the firm is allowed to optimally adjust leverage, it chooses a smaller face value as bankruptcy costs increase and a larger face value as the corporate tax rate increases. As a result of these debt level adjustments, agency costs actually decrease as bankruptcy costs increase, and as also seen in Table 4, agency costs increase as the corporate tax rate increases.

Finally, observe that agency costs tend to increase as the correlation (ρ) between assets-in-place and the asset underlying the growth option decreases. As the correlation decreases (from a base case value of 1.0), the growth option provides diversification benefits that reduce the probability of bankruptcy and increase overall debt capacity. The impact of these effects can be seen by observing that the optimal face value of debt increases, the credit spread of debt decreases, and the disparity between the first- and second-best growth option exercise policies narrows as ρ decreases. However, this narrowing of the difference in the growth option exercise strategies masks an underlying divergence of investment incentives; G^*_{FB} increases and G^*_{SB}

[22] This reflects the standard result from the real options literature that greater uncertainty increases the value of the firm's investment options and discourages investment by increasing the value of waiting to invest (see, e.g., McDonald and Siegel (1986)).

decreases as ρ decreases. The net effect is that agency costs tend to increase as the correlation decreases.

IV. Conclusions and Extensions

We construct a contingent claims model of the firm to examine interactions between investment and financing decisions in a setting where equityholders make self-interested investment decisions. The firm has assets-in-place and a growth option whose underlying asset may be correlated with assets-in-place. The analysis of the model examines the impact on firm value and the pricing of risky debt when equityholders choose the dynamic growth option investment policy to maximize their value instead of overall firm value. The model allows for an examination of the agency costs of overinvestment and underinvestment as a function of the relationship between assets-in-place and the asset underlying the growth option and of the cost structure of exercising the growth option.

Equityholders aggressively overinvest in the growth option when the asset underlying the option is riskier than assets-in-place and when equityholders are allowed to exchange assets-in-place for the asset underlying the growth option upon exercise. This risk substitution enhances the value of equity by transferring wealth from bondholders, and the resulting suboptimal growth option exercise policy significantly reduces firm value and increases the credit spread of debt. We find that this agency cost of overinvestment is quite sensitive to the characteristics of the firm and of the stochastic processes of assets-in-place and the asset underlying the growth option. We also demonstrate that the agency cost of overinvestment has a significant impact on optimal leverage; the optimal amount of debt is decreasing in the agency cost of overinvestment.

By contrast, equityholders underinvest in the growth option when the investment decision involves a pure expansion investment. In this case, the asset underlying the growth option is closely related to assets-in-place and the exercise decision constitutes an expansion of the existing asset base rather than an exchange of assets. As such, equityholders are reluctant to exercise the growth option when the firm has risky debt outstanding because they pay the full cost of the expansion but share the benefits with bondholders. We document that this suboptimal exercise decision results in a significant reduction in overall firm value and significantly increases the credit spread of debt. We further show that under a variety of parameter environments this agency cost of underinvestment significantly reduces optimal leverage.

The current version of the paper solely focuses on the growth option exercise decision by assuming that it is infinitely costly to reverse the investment once it is made. However, future revisions will include an analysis of the effects of reversing the exercise decision on equityholders' incentives to initially over- and underinvest in the growth option. In addition, we expect that both over- and underinvestment incentives will have a significant impact on the firm's overall financial policies. Accordingly, we plan to model debt maturity structure and to allow for possible debt level adjustments (i.e., recapitalization) in the model. Given the model's finite time horizon and resulting finite maturity of debt, debt maturity structure choice can be incorporated in the model by allowing the firm to initially choose both the level and rollover time of the debt. For example, given an initial twenty-year horizon, the firm may choose a five-year rollover time, i.e., four debt issues with each having a five-year original maturity. By the same token, we can model debt level adjustments by allowing the firm to increase or decrease the level of debt subject to fixed and/or variable recapitalization costs.

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Table 1 The Agency Cost of Debt Holding the Face Value of Debt Fixed When Equityholders have an Incentive to Overinvest in the Growth Option

	Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)		
	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}	CS_{AC}
Base Case	1.942	1.389	1.515	1.493	1.442	0.597	0.583	96.2	130.9	34.7
$\gamma = 0.50$	1.344	1.017	1.634	1.616	1.157	0.553	0.540	96.6	129.2	32.6
$\gamma = 1.00$	2.443	1.646	1.445	1.422	1.623	0.624	0.610	97.9	134.5	36.6
$K_I = 0.01$	1.886	1.328	1.520	1.493	1.810	0.594	0.577	97.4	140.5	43.1
$K_I = 0.05$	1.992	1.438	1.510	1.493	1.172	0.599	0.588	94.9	123.4	28.5
$b = 0.25$	1.878	1.389	1.538	1.520	1.186	0.601	0.590	74.5	102.9	28.4
$b = 0.75$	1.995	1.389	1.492	1.467	1.728	0.592	0.575	118.1	160.2	42.1
$\tau = 0.15$	1.884	1.303	1.388	1.359	2.074	0.629	0.612	128.2	175.0	46.8
$\tau = 0.25$	1.927	1.365	1.472	1.449	1.655	0.607	0.592	106.0	144.8	38.8
$\tau = 0.35$	1.953	1.409	1.557	1.536	1.326	0.586	0.574	87.6	119.9	32.3
$\delta_A = 0.01$	1.877	1.315	1.509	1.481	1.853	0.608	0.592	81.7	123.8	42.1
$\delta_A = 0.03$	2.003	1.468	1.522	1.505	1.140	0.583	0.572	112.6	141.1	28.5
$\delta_G = 0.04$	2.208	1.619	1.554	1.542	0.780	0.588	0.580	85.9	105.1	19.2
$\delta_G = 0.06$	1.760	1.226	1.482	1.446	2.476	0.603	0.580	106.9	165.4	58.5
$r = 0.05$	1.911	1.258	1.488	1.448	2.721	0.638	0.615	151.1	209.1	58.0
$r = 0.07$	1.939	1.486	1.527	1.515	0.766	0.555	0.548	60.4	81.4	21.0

Table 1 continued

	Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)		
	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}	CS_{AC}
$\rho = -1.00$	2.870	2.822	1.706	1.694	0.714	0.574	0.566	23.9	43.3	19.4
$\rho = -0.75$	2.897	2.543	1.687	1.674	0.774	0.577	0.569	28.6	48.7	20.1
$\rho = -0.25$	2.570	2.062	1.630	1.617	0.839	0.582	0.573	51.6	72.9	21.3
$\rho = 0.25$	2.165	1.610	1.558	1.541	1.084	0.591	0.580	80.0	106.8	26.8
$\rho = 0.75$	1.690	1.147	1.465	1.427	2.693	0.606	0.582	113.9	174.7	60.8
$\rho = 1.00$	1.408	0.892	1.407	1.288	9.255	0.618	0.545	131.0	342.3	211.3
$\sigma_A = 0.10$	2.084	1.391	1.554	1.527	1.817	0.616	0.600	43.9	83.5	39.6
$\sigma_A = 0.20$	1.839	1.426	1.493	1.479	0.945	0.574	0.564	145.9	171.0	25.1
$\sigma_G = 0.20$	1.135	1.013	1.394	1.383	0.770	0.664	0.656	74.6	93.0	18.4
$\sigma_G = 0.30$	1.473	1.171	1.445	1.426	1.367	0.631	0.617	89.1	120.9	31.8
$\sigma_G = 0.50$	2.511	1.654	1.583	1.562	1.359	0.568	0.554	101.4	135.8	34.4
$\sigma_G = 0.60$	3.262	1.969	1.643	1.623	1.252	0.544	0.532	106.0	139.0	33.0

Notes. The face value of debt is fixed at 0.90. The base case parameter values are as follows: the initial value of assets in place (A_0) is 1.0; the volatility of assets in place (σ_A) is 0.15; the convenience yield of assets in place (δ_A) is 0.02; the initial value of the asset underlying the growth option (G_0) is 1.0; the volatility of the asset underling the growth option (σ_G) is 0.40; the convenience yield of the asset underlying the growth option (δ_G) is 0.05; the correlation between assets in place and the asset underlying the growth option (ρ) is 0.50; the fixed cost of exercising the growth option (K_I) is 0.03; the fixed cost of reversing the growth option exercise decision (K_O) is infinity; the fraction of assets in place exchanged for the asset underlying the growth option (γ) is 0.75; the coupon rate of debt (c) is 0.07 per year; the variable and fixed costs of bankruptcy (b and B) are 0.50 and 0; the corporate tax rate (τ) is 0.30; the risk-free rate of interest (r) is 0.06 per year; and the time horizon (T) is 20 years.

Table 2 The Agency Cost of Debt at the Optimal Face Value of Debt When Equityholders have an Incentive to Overinvest in the Growth Option

	Optimal Face Value		Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)	
	F^*_{FB}	F^*_{SB}	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}
Base Case	0.84	0.76	1.937	1.490	1.516	1.501	0.966	0.567	0.518	78.6	79.8
$\gamma = 0.50$	0.88	0.78	1.339	1.074	1.635	1.622	0.763	0.544	0.489	91.0	85.3
$\gamma = 1.00$	0.82	0.78	2.447	1.775	1.446	1.430	1.121	0.583	0.552	74.3	89.9
$K_I = 0.01$	0.84	0.76	1.879	1.428	1.521	1.504	1.120	0.565	0.513	79.8	85.3
$K_I = 0.05$	0.84	0.79	1.987	1.530	1.511	1.498	0.834	0.570	0.537	77.3	84.3
$b = 0.25$	0.99	0.84	1.883	1.427	1.541	1.521	1.325	0.644	0.562	98.2	83.3
$b = 0.75$	0.76	0.70	1.975	1.534	1.499	1.488	0.780	0.522	0.483	72.3	77.1
$\tau = 0.15$	0.61	0.56	1.855	1.586	1.418	1.413	0.311	0.460	0.423	39.7	40.2
$\tau = 0.25$	0.76	0.69	1.913	1.526	1.478	1.468	0.707	0.535	0.488	64.2	64.9
$\tau = 0.35$	0.92	0.84	1.957	1.443	1.558	1.538	1.250	0.595	0.548	92.7	97.8
$\delta_A = 0.01$	0.88	0.76	1.875	1.420	1.509	1.492	1.149	0.599	0.526	75.8	70.4
$\delta_A = 0.03$	0.86	0.79	2.001	1.553	1.523	1.511	0.778	0.565	0.523	100.1	100.0
$\delta_G = 0.04$	0.88	0.84	2.207	1.671	1.555	1.545	0.624	0.578	0.553	80.1	85.1
$\delta_G = 0.06$	0.84	0.73	1.756	1.346	1.484	1.465	1.306	0.574	0.504	88.1	89.5
$r = 0.05$	0.74	0.64	1.921	1.487	1.501	1.487	0.942	0.558	0.492	88.9	81.1
$r = 0.07$	1.00	0.91	1.957	1.478	1.530	1.515	0.948	0.601	0.557	83.3	84.0

Table 2 continued

	Optimal Face Value		Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)	
	F^*_{FB}	F^*_{SB}	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}
$\rho = -1.00$	1.51	1.10	3.750	2.633	1.795	1.712	4.865	0.869	0.661	70.1	74.8
$\rho = -0.75$	1.21	1.03	3.235	2.443	1.718	1.680	2.278	0.730	0.630	67.3	74.6
$\rho = -0.25$	1.01	0.92	2.628	2.049	1.636	1.617	1.195	0.636	0.583	72.4	77.8
$\rho = 0.25$	0.92	0.82	2.172	1.673	1.558	1.543	0.971	0.600	0.543	85.0	81.7
$\rho = 0.75$	0.80	0.69	1.706	1.302	1.469	1.453	1.102	0.557	0.488	80.0	74.7
$\rho = 1.00$	0.73	0.58	1.475	1.129	1.420	1.397	1.616	0.533	0.435	66.5	57.5
$\sigma_A = 0.10$	0.93	0.81	2.091	1.482	1.555	1.532	1.483	0.632	0.557	50.3	53.3
$\sigma_A = 0.20$	0.84	0.78	1.835	1.494	1.494	1.483	0.700	0.546	0.511	126.7	127.1
$\sigma_G = 0.20$	0.91	0.83	1.137	1.028	1.394	1.384	0.674	0.669	0.619	77.1	71.7
$\sigma_G = 0.30$	0.89	0.79	1.472	1.214	1.445	1.431	0.990	0.626	0.563	86.2	82.2
$\sigma_G = 0.50$	0.84	0.76	2.509	1.807	1.585	1.571	0.871	0.540	0.493	82.8	82.8
$\sigma_G = 0.60$	0.84	0.75	3.269	2.210	1.646	1.633	0.765	0.518	0.469	86.0	81.2

Notes. The base case parameter values are as follows: the initial value of assets in place (A_0) is 1.0; the volatility of assets in place (σ_A) is 0.15; the convenience yield of assets in place (δ_A) is 0.02; the initial value of the asset underlying the growth option (G_0) is 1.0; the volatility of the asset underling the growth option (σ_G) is 0.40; the convenience yield of the asset underlying the growth option (δ_G) is 0.05; the correlation between assets in place and the asset underlying the growth option (ρ) is 0.50; the fixed cost of exercising the growth option (K_I) is 0.03; the fixed cost of reversing the growth option exercise decision (K_O) is infinity; the fraction of assets in place exchanged for the asset underlying the growth option (γ) is 0.75; the coupon rate of debt (c) is 0.07 per year; the variable and fixed costs of bankruptcy (b and B) are 0.50 and 0; the corporate tax rate (τ) is 0.30; the risk-free rate of interest (r) is 0.06 per year; and the time horizon (T) is 20 years.

Table 3 The Agency Cost of Debt Holding the Face Value of Debt Fixed When Equityholders have an Incentive to Underinvest in the Growth Option

	Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)		
	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}	CS_{AC}
Base Case	1.420	1.889	1.471	1.458	0.861	0.623	0.597	181.5	231.7	50.2
$\gamma = 0.15$	1.573	2.034	1.362	1.357	0.401	0.638	0.622	233.9	262.0	28.1
$\gamma = 0.25$	1.674	2.140	1.300	1.296	0.292	0.650	0.639	261.7	281.5	19.8
$K_I = 0.98$	1.387	1.859	1.483	1.469	0.952	0.623	0.595	174.8	227.8	53.0
$K_I = 1.10$	1.589	2.060	1.417	1.409	0.534	0.623	0.606	219.0	250.4	31.4
$b = 0.25$	1.485	1.889	1.505	1.494	0.737	0.623	0.607	159.7	192.6	32.9
$b = 0.75$	1.308	1.889	1.435	1.421	0.985	0.632	0.587	192.5	273.2	80.7
$\tau = 0.15$	1.316	1.945	1.338	1.324	1.057	0.667	0.630	207.5	271.2	63.7
$\tau = 0.25$	1.405	1.903	1.426	1.413	0.920	0.638	0.608	189.3	244.4	55.1
$\tau = 0.325$	1.426	1.883	1.492	1.481	0.798	0.616	0.592	178.6	225.1	46.5
$\delta_A = 0.05$	1.401	1.879	1.469	1.457	0.810	0.633	0.607	167.9	216.7	48.8
$\delta_A = 0.07$	1.428	1.899	1.473	1.459	0.936	0.616	0.588	191.9	246.5	54.6
$\delta_G = 0.05$	1.569	2.117	1.512	1.508	0.279	0.599	0.585	193.1	218.3	25.2
$\delta_G = 0.065$	1.352	1.798	1.455	1.439	1.136	0.637	0.602	171.5	235.9	64.4
$r = 0.055$	1.338	1.878	1.454	1.436	1.266	0.661	0.617	137.3	214.0	76.7
$r = 0.07$	1.582	1.925	1.500	1.495	0.323	0.569	0.558	156.8	178.5	21.7

Table 3 continued

	Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)		
	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}	CS_{AC}
$\rho = -0.75$	1.586	1.698	1.541	1.526	0.996	0.691	0.676	43.5	72.4	28.9
$\rho = -0.25$	1.534	1.762	1.513	1.497	1.076	0.675	0.650	81.0	125.0	44.0
$\rho = 0.25$	1.481	1.804	1.492	1.478	0.923	0.659	0.629	115.9	168.3	52.4
$\rho = 0.75$	1.423	1.865	1.474	1.461	0.890	0.640	0.608	154.6	210.0	55.4
$\sigma_A = 0.175$	1.424	1.868	1.474	1.462	0.814	0.634	0.609	162.9	209.4	46.5
$\sigma_A = 0.25$	1.416	1.933	1.463	1.450	0.897	0.603	0.573	218.8	277.1	58.3
$\sigma_G = 0.175$	1.288	1.793	1.452	1.435	1.152	0.655	0.611	146.5	225.2	78.7
$\sigma_G = 0.25$	1.672	2.097	1.509	1.503	0.398	0.583	0.572	220.9	244.6	23.7

Notes. The face value of debt is fixed at 1.00. The base case parameter values are as follows: the initial value of assets in place (A_0) is 1.0; the volatility of assets in place (σ_A) is 0.20; the convenience yield of assets in place (δ_A) is 0.06; the initial value of the asset underlying the growth option (G_0) is 1.25; the volatility of the asset underling the growth option (σ_G) is 0.20; the convenience yield of the asset underlying the growth option (δ_G) is 0.06; the correlation between assets in place and the asset underlying the growth option (ρ) is 1.00; the fixed cost of exercising the growth option (K_I) is 1.00; the fixed cost of reversing the growth option exercise decision (K_O) is infinity; the fraction of assets in place exchanged for the asset underlying the growth option (γ) is 0; the coupon rate of debt (c) is 0.07 per year; the variable and fixed costs of bankruptcy (b and B) are 0.50 and 0; the corporate tax rate (τ) is 0.30; the risk-free rate of interest (r) is 0.06 per year; and the time horizon (T) is 20 years.

Table 4 The Agency Cost of Debt at the Optimal Face Value of Debt When Equityholders have an Incentive to Underinvest in the Growth Option

	Optimal Face Value		Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)	
	F^*_{FB}	F^*_{SB}	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}
Base Case	0.96	0.91	1.447	1.860	1.471	1.460	0.767	0.602	0.560	176.1	201.3
$\gamma = 0.15$	0.89	0.90	1.640	2.005	1.365	1.360	0.329	0.587	0.580	199.2	224.5
$\gamma = 0.25$	0.84	0.79	1.760	2.066	1.307	1.304	0.244	0.576	0.546	204.1	197.7
$K_I = 0.98$	0.97	0.91	1.403	1.821	1.483	1.472	0.769	0.609	0.558	166.4	196.4
$K_I = 1.10$	0.91	0.91	1.642	2.023	1.418	1.412	0.432	0.581	0.569	194.1	218.2
$b = 0.25$	1.20	1.13	1.366	1.940	1.511	1.496	0.969	0.719	0.658	193.2	231.1
$b = 0.75$	0.85	0.74	1.454	1.793	1.443	1.435	0.573	0.547	0.475	169.6	177.8
$\tau = 0.15$	0.54	0.48	1.586	1.731	1.372	1.371	0.126	0.395	0.355	97.6	87.2
$\tau = 0.25$	0.85	0.80	1.495	1.829	1.431	1.424	0.541	0.558	0.519	158.7	175.2
$\tau = 0.325$	1.02	1.02	1.409	1.889	1.493	1.481	0.852	0.627	0.600	180.0	231.2
$\delta_A = 0.05$	0.94	0.91	1.443	1.846	1.469	1.460	0.661	0.602	0.569	157.1	186.5
$\delta_A = 0.07$	1.00	0.94	1.428	1.881	1.473	1.461	0.794	0.616	0.564	191.9	225.4
$\delta_G = 0.05$	0.95	1.00	1.646	2.117	1.512	1.508	0.300	0.578	0.585	178.6	218.3
$\delta_G = 0.065$	1.03	0.91	1.333	1.771	1.457	1.442	1.034	0.654	0.565	173.2	204.5
$r = 0.055$	0.88	0.82	1.411	1.797	1.456	1.442	0.943	0.590	0.538	167.1	196.2
$r = 0.07$	1.09	1.03	1.528	1.937	1.501	1.496	0.327	0.607	0.570	272.9	280.9

Table 4 continued

	Optimal Face Value		Growth Option Exercise Policy		Firm Value		Agency Cost (%)	Leverage Ratio		Credit Spread (Basis Points)	
	F^*_{FB}	F^*_{SB}	G^*_{FB}	G^*_{SB}	V^L_{FB}	V^L_{SB}	AC	LEV_{FB}	LEV_{SB}	CS_{FB}	CS_{SB}
$\rho = -0.75$	1.19	1.01	1.524	1.700	1.550	1.526	1.566	0.800	0.681	63.0	74.6
$\rho = -0.25$	1.07	0.95	1.502	1.751	1.516	1.498	1.170	0.714	0.627	89.9	110.6
$\rho = 0.25$	1.05	0.89	1.457	1.778	1.493	1.480	0.824	0.687	0.579	122.5	134.7
$\rho = 0.75$	0.95	0.89	1.459	1.822	1.475	1.466	0.567	0.611	0.561	148.8	173.7
$\sigma_A = 0.175$	0.98	0.95	1.438	1.848	1.475	1.464	0.725	0.622	0.586	157.4	190.1
$\sigma_A = 0.25$	0.96	0.90	1.439	1.895	1.463	1.452	0.786	0.583	0.534	212.5	242.7
$\sigma_G = 0.175$	0.94	0.86	1.327	1.740	1.454	1.439	1.058	0.623	0.550	134.7	177.5
$\sigma_G = 0.25$	0.98	0.98	1.678	2.091	1.510	1.504	0.397	0.575	0.564	215.5	237.9

Notes. The base case parameter values are as follows: the initial value of assets in place (A_0) is 1.0; the volatility of assets in place (σ_A) is 0.20; the convenience yield of assets in place (δ_A) is 0.06; the initial value of the asset underlying the growth option (G_0) is 1.25; the volatility of the asset underling the growth option (σ_G) is 0.20; the convenience yield of the asset underlying the growth option (δ_G) is 0.06; the correlation between assets in place and the asset underlying the growth option (ρ) is 1.00; the fixed cost of exercising the growth option (K_I) is 1.00; the fixed cost of reversing the growth option exercise decision (K_O) is infinity; the fraction of assets in place exchanged for the asset underlying the growth option (γ) is 0; the coupon rate of debt (c) is 0.07 per year; the variable and fixed costs of bankruptcy (b and B) are 0.50 and 0; the corporate tax rate (τ) is 0.30; the risk-free rate of interest (r) is 0.06 per year; and the time horizon (T) is 20 years.

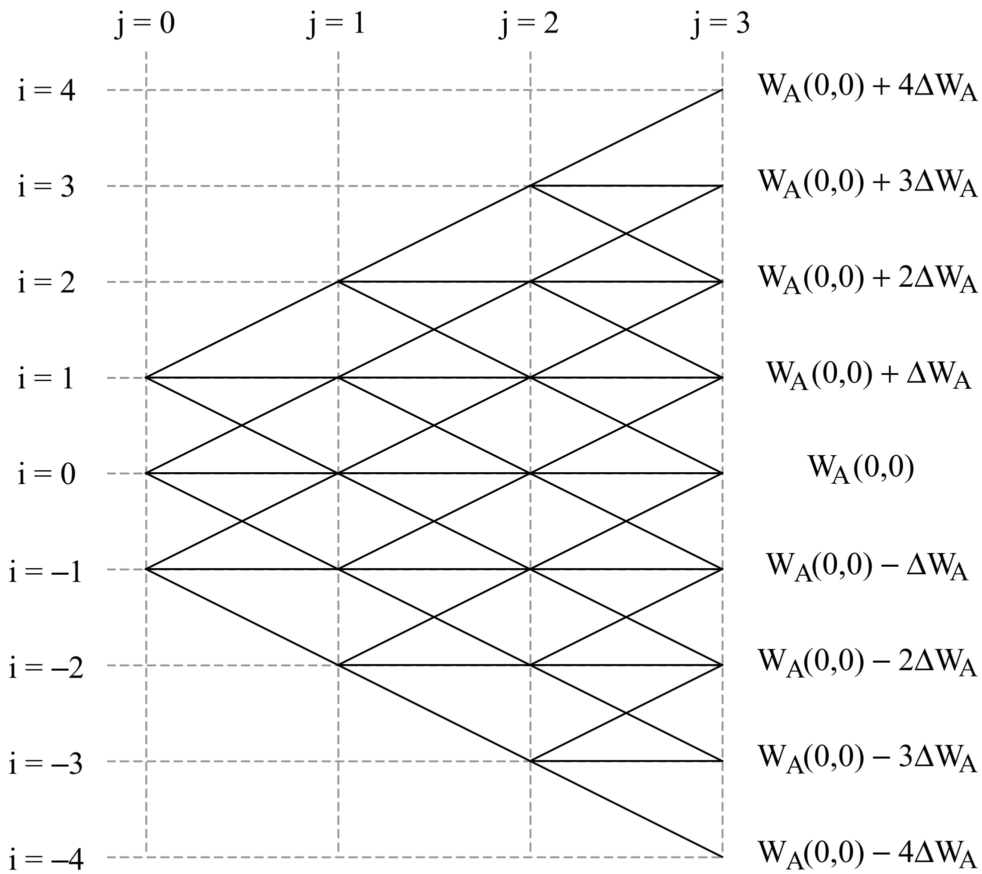

Figure 1. The One Variable Tree

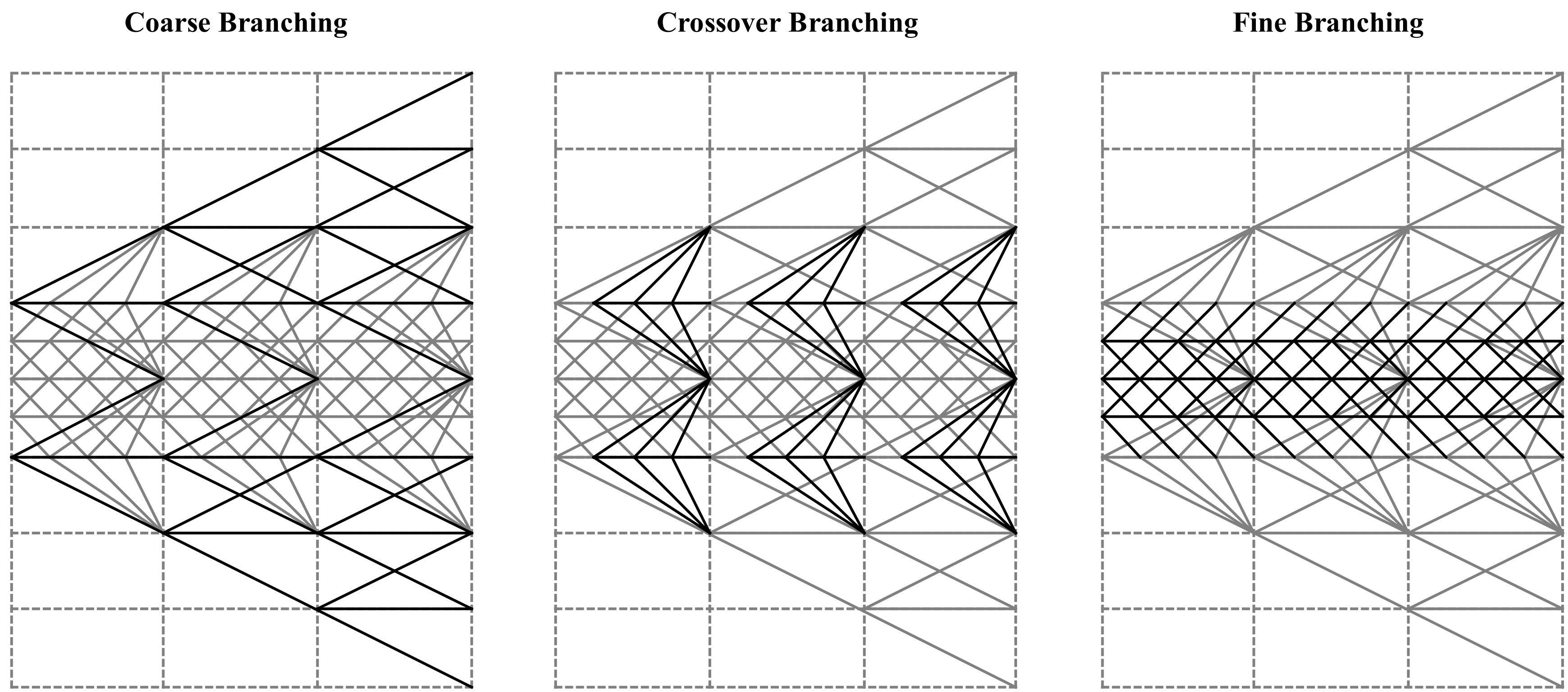

Figure 2. A One Variable Tree with a Fine Middle Section

Figure 3. First-Best and Second-Best Growth Option Exercise Policies When Equityholders have an Incentive to Overinvest

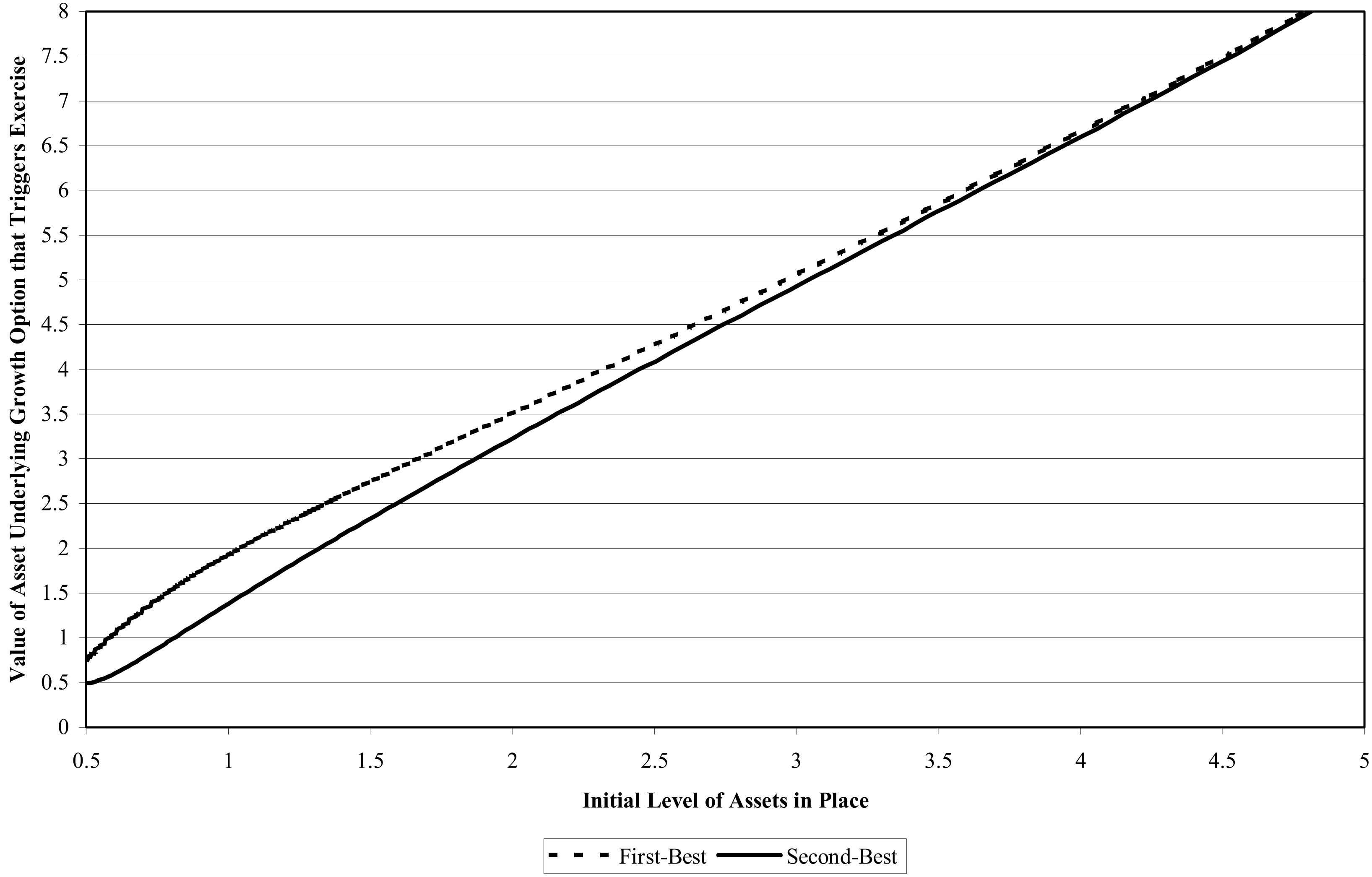

Figure 4. First-Best and Second-Best Growth Option Exercise Policies When Equityholders have an Incentive to Underinvest