# The Value of Manufacturing Flexibility: Real Options in Practice 

Author:<br>Jens Bengtsson, PhD-student<br>Department of Production Economics<br>Linköping Institute of Technology<br>S-581 83 Linköping<br>International Graduate School of Management and Industrial Engineering


#### Abstract

This paper treats manufacturing flexibility, ways to achieve it and ways to evaluate flexibility using option pricing. Different ways to increase flexibility are considered from the point of view of two companies. Different types of flexibility are interpreted and modelled as options and ways to evaluate them are considered with respect to amongst other set-up cost and capacity constraints. A numerical example shows the value of having the opportunity to hire personnel on short contracts.


## 1. Introduction

Manufacturing flexibility has during the last decades become a very important aspect on the competitive arena where production oriented companies work. Many researchers, e.g. Fine \& Hax (1985), Hill (1995), and Hayes \& Upton (1998) also rank manufacturing flexibility as a competitive priority together with e.g. cost and quality. Although many researchers has identified the benefits and the importance of flexibility there are still problems to evaluate and measure flexibility in an appropriate way. Some ways to measure flexibility can be seen in Sethi \& Sethi (1990). Many of these measures are onedimensional, e.g. measuring the number of parts that can be produced in a system. It is hard for a company, based on these measures, to get an idea of the value of flexibility because there is no relationship between the companies' actual need for flexibility and the measures. If a company faces no uncertainty at all, there will be no value of holding flexibility to cope with uncertainties, and vice versa, and this is a relationship that a measure should take into respect.

Another way to measure flexibility is to use capital budgeting and estimate the value of flexibility. This approach gives a relationship between the actual need, requiring a valuation of flexibility, and the cost of acquiring it. The major problem with this approach is to find the value of flexibility in an appropriate way. However, flexibility can be seen as options, or real options to separate them from the more familiar financial options. In e.g. Trigeorgis (1996) projects with embedded flexibility are treated as options and evaluated with the theories founded by Black \& Scholes (1973) and Merton (1973). The major benefits of using option pricing to evaluate projects with embedded flexibility are among other that the problem to find the risk-adjusted rates using e.g. the Capital Asset Pricing Model (CAPM) are avoided. A more extensive treatment of these problems concerning the use of techniques based on models such as CAPM can be found in e.g. Trigeorgis (1996).

This paper is partly based on experiences from two Swedish midsize companies and considers relevant types of flexibility, and the value of flexibility. To do this, ways to increase and achieve flexibility are looked at and connected to the type of uncertainties that the companies face. Relevant types of options are identified and modelled. The identified options will then restrict which type of valuation methods that can be used. Thus, the aim of this paper is to use theories from finance and production management and connect these with needs for flexibility in real life situations and present relevant models and ways to evaluate the flexibility using option pricing.

This paper is organised as follows: First, a brief literature review showing some of what are done in the field of real option but also giving some flexibility aspects from the production/operations management point of view. Second, the companies are briefly described from a view, highlighting the aspects important to flexibility and identifying real options. Third, company specific questions are highlighted and ways to evaluate the different kinds of flexibility that the companies have or strive to achieve are shown. Here, we use existing valuation models with some minor changes. Aspects such as interpreting flexibility into different kinds of options, source of uncertainty and relating these to the financial market are also treated here. Finally, some numerical, stylised examples are presented.

## 2. Literature Review and Flexibility in Production

### 2.1. Option pricing and flexibility

First, to avoid any confusion about the different types of flexibility that will be used in this paper the definitions from Sethi \& Sethi (1990) will be used. Sethi \& Sethi define volume flexibility as the ability to operate profitably at different output levels. Process flexibility according to Sethi \& Sethi relates to the set of parts that can be produced without a major set-up. The latter is sometimes referred to as product-mix flexibility in the literature.

Valuing managerial flexibility with option pricing has been done for almost two decades and during this time different kinds of real options has been treated. In this paper, however, we will focus on the flexibility related to manufacturing on a more operational level and on applications of real options and real option thinking. As input we have two different companies and their needs of flexibility, ways to increase and achieve flexibility and finally, ways to evaluate the desired flexibility. Numerous authors have earlier valued "operational" flexibility in manufacturing. Triantis \& Hodder (1990) evaluate process flexibility in a given, fixed capacity equipment as a complex option. The profit margins of different products are assumed to be stochastic and dependent on produced quantity. The latter effect is a result of allowing for downward sloping demand curves. In their model there are no switching costs. Capacity constraints are considered and the model allows the firm to temporarily shut down and restart operation. Kamrad \& Ernst (1995) models multi-product manufacturing with stochastic input price, output yield uncertainty and capacity constraints to value multi-product production agreements. During one period, only one product type is produced with respect to the inventory available. Tannous (1996) carries out capital budgeting for volume flexible equipment and
compares a non-flexible to a flexible system in a case based on a real company. In his model demand is uncertain, dependent on price (downward-sloping demand curve) and a stochastic factor. In Tannous' model the effect of having inventory available is also considered. Andreou (1990) evaluates process flexibility in different configurations of dedicated and flexible equipment when demand of two products is uncertain. Kulatilaka (1988) uses a stochastic dynamic programming model to evaluate the options in a flexible production process and incorporates the effects of switching costs. He \& Pindyck (1992) examine investments in flexible production capacity. Here, the capacity choice problem is considered, i.e. whether to buy flexible or non-flexible equipment and how much capacity with respect to the fact that investment is irreversible. As in Tannous (1996) demand is uncertain but in this case differs, via a demand shift parameter depending on whether market is perfectly competitive or not.

The option pricing theory has also been used to evaluate projects with real options and flexibility on a more strategic level. Some of them are Brennan \& Schwartz (1985) where decisions concern questions as whether to open, close, reopen or abandon a mine depending on the evolution of a stochastic commodity price. McDonald \& Siegel (1985) consider the valuation problem of the option to temporarily shut down production when output price is stochastic. McDonald \& Siegel (1986) and Majd \& Pindyck (1987) evaluate the option to wait to invest when the investment is irreversible.

### 2.2. Flexibility and production management

There are many approaches to increase flexibility, see e.g. Olhager (1993), Chambers (1992), Slack (1988). Four central ways are:

- Reductions of set-up time at installed equipment.
- Multipurpose stations (FMS).
- Parallel assembly lines.
- Flexible work force.

The first three approaches are dependent on production equipment and the last on personnel. Olhager (1993) also mentions control system and suppliers as important aspects to achieve and increase flexibility but we do not treat issues these in this paper. All approaches above require some kind of initial investment. Each approach is highlighted below to point at different important aspects, which affect flexibility and costs.

Reduction of set-up time at equipment in place requires often some kind of additional investment in equipment. The result from the investment is an increase in process and volume flexibility due to shorter set-up time and that more capacity is made available. The effect on volume flexibility is marginal, but the effect on process flexibility is substantial. This is a consideration especially relevant to equipment based assembly systems, where set-up is a time consuming and costly activity. Consideration to this approach should thereby be given to ABB Robotics.

Multi-purpose stations are often built as flexible manufacturing system (FMS) where one machine performs a lot of operations with a minor or no set-up time at all. The multipurpose stations are characterised by high flexibility both in process and volume and taking care of most of the operations. The desired degree of flexibility can often be builtin with different modules when the machine is bought and a higher degree of flexibility is associated with higher investment costs. This is an interesting approach to ABB Motors by the same reason as mention above. ABB Motors can do this, for example, by adding more wire magazines and feeders to a new fully automatic machine and thereby reducing set-up cost to almost zero.

Parallel stations increase flexibility because different products can be assembled in different stations. The flexibility of the whole system will depend on the capability and flexibility of the parallel machines. This approach can be carried out with more or less flexible machines, dedicated machines, human worker or a mix of these and is thereby of interest approach to both companies although this might be an expensive way to achieve flexibility.

Personnel with high skills are of great importance to companies using machines as well as they who do not. Well-trained and educated personnel leads to process flexibility but flexible personnel can also be used in another way as in two of the companies. These companies have the opportunity to hire personnel on short contracts, typically lasting for three or six months, which increases the volume flexibility.

## 3. Brief company descriptions

## ABB Motors

ABB Motors is located in Västerås, Sweden and produces medium power electrical motors and $70 \%$ of the orders are made-to-stock and the remaining $30 \%$ are made-toorder. The lead-time of a made-to-order motor is 4 weeks while a made-to-stock motor has to be sent within 24 hours from a placed order. Even though some orders are considered large, these orders are often divided into sub-orders, which may differ substantially. Thereby, the assembly stations face uncertainty both in total volume and product-mix. The system at ABB Motors is an assembly line, where the bottleneck operation is wiring, consisting of one full-automatic wiring station and three halfautomatic stations in a line. Most of the features and characteristics of the system are set by the machines and equipment. The wiring stations constrain flexibility of the whole assembly line. Because the wiring stations have to cope with almost eight different motors a flexible assembly line is required to produce them profitably. The flexibility of the line today is perceived as being too low and thereby it is of interest to examine different ways to increase flexibility and to evaluate them.

## ABB Robotics

ABB Robotics is also located in Västerås, Sweden and is producing industrial robots on a make-to-order basis. Today ABB Robotics is assembling their robots in two different lines, where one line is assembling three robot types, IRB 1400, 2400 and 4400 and the other line is assembling IRB 6400. In addition to these lines, there is also a third line,
which assembles the control boxes belonging to each robot. Almost all assembling are manual. The work is quite easy, requiring only a short introduction. Today there are no exchanges of workers between the lines. Since the work is easy to learn and that assembling is quite the same for all products the process flexibility in the company is high. ABB Robotics faces high uncertainty in demand making it hard to estimate the required capacity needed in the future periods. Thus ABB Robotics must be volume flexible.

## 4. Why Search for a Value of Flexibility

Flexibility can thus be acquired in different ways and each of these ways is associated with costs when acquiring them. Therefore, it is interesting to evaluate the benefits given by flexibility. Set-up time reduction investments can be applied to equipment in place and proactively for planned equipment. In both cases it is interesting to know if the value of the flexibility increase exceeds the cost of acquiring it and if the investment thereby should be carried out. It might in some cases be enough to do a smaller set-up time reduction than was thought from the beginning if this requires a smaller investment but might give substantial effects to the flexibility of the company. If the set-up time reduction investment is done for equipment in place it might be enough to evaluate this reduction investment alone, but if the investment concerns brand new equipment other aspects such as new capacity constraints has to be dealt with. In the latter case it could therefore be better to do an evaluation of the whole system.

Multi-purpose stations are often very expensive to acquire and it is thereby interesting to find out if the value of the benefits, given in form of flexibility by these stations, exceeds the cost of them. As in the case with the set-up time reduction, there might be a point where investment in more flexibility is not profitable any longer. Thereby, it can be interesting to find the point, if it exists, where investment in more flexibility is unprofitable and telling management that it is of no use to invest more.

Parallel stations gives flexibility as described above but requires substantial investments in capacity. The parallel stations can be set-up in different ways e.g. two dedicated lines producing two types of products or two flexible lines where both line are able to produce both products. The flexible lines are more expensive but give more flexibility when temporary demand peaks of one product can be produced in both lines if capacity is available. More parallel lines give even more flexibility but for a given uncertainty it might not be optimal to buy only flexible machines but mix dedicated with flexible machines, which might give a higher value. It might thereby be of great interest to evaluate different machine configurations and compare these to each other to find a tradeoff between acquired flexibility and the cost of acquiring it.

Flexible personnel give a way to handle fluctuations in demand. This flexibility is achieved at the companies by the possibility to hire and is of course worth something to the companies but the question is how much? Related to this way to achieve flexibility a couple of questions are interesting: i) What is the opportunity to hire a person for three months worth given demand today, uncertainty in demand, costs of hiring etc. This can, for example, be interesting if extra workers have to get some education before hiring or
that the company has to pay for this opportunity in some other way. ii) How many people can be employed on short time contracts until the present value of the marginal worker is null or negative? iii) If a cost is associated with holding the pool of workers from where people are taken into production, how many workers should be connected th this pool?

In summary, flexibility has a value and that an estimated value of flexibility might serve as an important input parameter in decision making resulting in better decision in favour to the company and its shareholder.

## 5. The Value of Flexibility

All flexibility investments described above result in options and the first step is to identify the options that the companies already have and get when they invest in one or some of the approaches above. We follow the structured approach of Amram \& Kulatilaka (1999) in principle.

### 5.1 Dependent decision and triggers

The dependent decision available to ABB Motors is which product to produce and in which line. This decision is taken with respect to capacity constraints, set up times, etc. and the decision increases in complexity with the number of lines and products. The basic decision is that production will take place if the revenues from an order exceed the cost of producing it, but if the demand exceeds available capacity, the most profitable products have to be produced first to maximise profit. The production decisions at ABB Motors are taken every four weeks, two weeks ahead of that point in time where it affects production. The production decisions concern capacity and production planning.

The major decision concerning flexibility at ABB Robotics is how many workers to hire on short contract to each line. No exchange of workers between the three lines takes place. ABB Robotics hires if demand is too high to be produced by the available personnel within the company. The hiring decisions are made every four weeks at the same time as capacity planning and the contracts last for three month. Today, ABB Robotics knows the demand for the next two months, wherefore it is uncertain whether the extra worker will be used during the last month of the contract or not.

### 5.2 Source of uncertainty

The biggest source of uncertainty to all companies is uncertainty in demand. However, due to differences in their way to assemble the uncertainties are of different importance to the companies. In the ABB Robotics case, it is uncertainty in individual product volume and in aggregated volume that affects the company. Uncertainty in product-mix is easily handled since robots are similar. At ABB Motors the situation is different since productmix uncertainty affects the company more than uncertainty in aggregated demand.

In many real options articles price is modelled as the source of uncertainty. All of the companies in this paper work in highly competitive markets and can be considered as price takers, i.e. that they can not affect the market price by producing more or less.

The form and evolution of demand is an important input to valuation based on option pricing theory. We assume that the demand of a product $i$ is stochastic and follows a geometric Brownian motion and this assumption can also be found in Pindyck (1988), He \& Pindyck (1992), Tannous (1996) and Chung (1990). The demand process is thereby written as:

$$
\begin{equation*}
d D_{i}=\alpha_{i} D_{i} d t+\sigma_{i} D_{i} d z_{i} \tag{1}
\end{equation*}
$$

where

$$
\begin{aligned}
& d z_{i}=\varepsilon_{i}(t) \sqrt{d t} \\
& \varepsilon_{i}(t) \sim N(0,1)
\end{aligned}
$$

$\alpha_{i}$ and $\sigma_{i}$ are the instantaneous drift and volatility. In the general case, these parameters can also be dependent on time $t$ and demand $D_{i} . d z_{i}$ is the increment of a Wiener process where $\varepsilon_{i}(t)$ is a serially uncorrelated and normally distributed random variable. From equation (1) follows that demand is log-normally distributed with a variance that grows with the time horizon. Between the increments of product $i$ and $j$, $d z_{i}$ and $d z_{j}$ respectively, there is also a pair-wise correlation coefficient $\rho_{i j}$ which is defined as $\rho_{i j} d t=d z_{i} d z_{j}$.

Although the demand is modelled as a continuous process this might be an unrealistic assumption in some cases, e.g. when only discrete number of products can be produced. The approximation should not be too rough if it is a high volume producer, but might be less suitable if it is a low volume producer. The latter could also be the case if the production is make-to-stock and no production below the economic lot size is carried out.

For valuation purposes it is also necessary with one more assumption. We assume, as in Pindyck (1988) that existing assets span stochastic changes in demand i.e. there exists an asset or a portfolio of assets whose price is perfectly correlated with $D_{i}$. This is the same as assuming that markets are sufficiently complete in that the firm's decisions to invest or produce do not affect the opportunity sets available to investors. One should note that if the spanning assumption does not hold the CAPM would not hold either. Thus there exists an asset or a dynamic portfolio of assets with price $A_{i}$ that are perfectly correlated with $D_{i}$ and thereby also follows a Brownian motion:

$$
\begin{equation*}
d A_{i}=\mu_{i} A_{i} d t+\sigma_{i} A_{i} d z_{i} \tag{2}
\end{equation*}
$$

The drift rate of demand $\alpha_{i}$ might differ from the drift rate $\mu_{i}$, which is the equilibrium rate of return, although they have the same covariance with the market portfolio. Therefore the drift rate of the asset or the dynamic portfolio has to be adjusted with a factor, often called convenience yield, $\delta_{i}=\mu_{i}-\alpha_{i}$ to get the correct value of the option, see also McDonald and Siegel (1985).

### 5.3 Decision rules and pay-off

Basic production option in a single assembly line
If we study one production option, which expires at time $T$, and gives us the option to produce if benefits exceed production cost the value of the option at time $T$ can be written as:

$$
\begin{equation*}
V(T)=\operatorname{Max}[(P-V) D-S, 0] \tag{3}
\end{equation*}
$$

where
$P=$ price
$V=$ variable cost
$S=$ set-up cost and other costs related to exchange of products.
$D=$ demanded quantity
In its simplest form, without any capacity constraints this option can be seen as an ordinary European call option where $(P-V) D$ is the total value of the underlying asset. In this paper it is assumed that the companies are price takers, i.e. that the companies can not affect the market price by producing more or less. The set-up cost $S$, can be seen as the exercise price. Production will then take place if $V(T)>0$.

Options from set-up time reduction and FMS in a multi-product single assembly line.
Acquiring a flexible manufacturing system can be seen as acquiring a portfolio of options. Each time a production decision is made a portfolio of options is exercised, one option for each product that will be produced. For example, if two products are demanded they will be produced if demand is high enough to cover the associated costs and if available capacity is sufficient. The pay-off from a production option, which gives the right to produce product $i$ and expires a time $T$ without any capacity constraints is then

$$
\begin{equation*}
V_{i}(T)=\max \left[\left(P_{i}-V_{i}\right) D_{i}-S_{i}, 0\right] \tag{4}
\end{equation*}
$$

When calculating the net present value $V$ of an investment opportunity, at time $t$, where one decision is made about the proportions of the $N$ different products that will be produced in one machine, expression (4) has to be extended to include fixed cost $F$ and investment cost $I$.

If production decisions can be made more than once we also have to sum over the number of times $K$ when decisions can be made. However, to do this the options with expiration days at different points in time have to be independent of each other. This is achieved either through assuming that no set-up time is needed or assuming that all kinds of production requires set-ups, irrespective of the fact that we produced the same product just before in the earlier period. We choose to assume the latter because set-up time is an essential parameter in our model and can not be excluded. The NPV, at time $t(t<T)$, of this investment opportunity can thereby be expressed as:

$$
\begin{equation*}
N P V(t)=\sum_{k=1}^{K} \sum_{i=1}^{N} V_{i k}(t)-F-I \tag{5}
\end{equation*}
$$

Options in multi-product line with limited capacity.
When including capacity constraints the production options might not be seen as individual. When the demand is higher than actual capacity a selection between orders takes place where an optimal production scheme is estimated which maximise the value of production. In the single line case the pay-off from production option $k$ at expiration time $T_{k}$ can be written as

$$
\begin{equation*}
V_{k}\left(T_{k}\right)=\operatorname{Max}\left\{\sum_{i=1}^{N} \operatorname{Max}\left[\left(P_{i}-V_{i}\right) Q_{i}-S_{i}, 0\right]\right\} \tag{6}
\end{equation*}
$$

s.t

$$
\begin{aligned}
& Q_{i} \leq D_{i} \\
& \sum_{i=1}^{N} Q_{i} \leq C_{\max }
\end{aligned}
$$

where
$C_{\max }=\max$ capacity (units, hours, etc.)
$Q_{i}=$ quantity produced of product $i$. (units, hours, etc.)

However, if the capacity constraint is not binding optimisation is not necessary and the pay-off is the sum of $N$ individual production options. When several lines are available, expression (6) has to be extended to include that production can take place in $L$ lines, which is written as

$$
\begin{equation*}
V_{k}\left(T_{k}\right)=\operatorname{Max}\left\{\sum_{l=1}^{L} \sum_{i=1}^{N} \operatorname{Max}\left[\left(P_{i}-V_{i l}\right) Q_{i l}-S_{i l}, 0\right]\right\} \tag{7}
\end{equation*}
$$

s.t.

$$
\begin{aligned}
& \sum_{l=1}^{L} Q_{i l} \leq D_{i} \\
& \sum_{i=1}^{N} Q_{i l} \leq C_{\max }^{l}
\end{aligned}
$$

where
$Q_{i l}=$ quantity produced of product $i$ in line $l$.
$V_{i l}=$ variable cost of product $i$ produced in line $l$.
$S_{i l}=$ set-up cost of product $i$ in line $l$.
$C_{\max }^{l}=\max$ capacity in line $l$.
This expression can be used even though some product-line combinations are impossible if the set-up cost in these cases is set to an infinite or a large number.

## Valuing the options

The value of the production options in (6) and (7) is dependent on multivariate underlying assets represented by the demand of each product that can be produced in the system. The value $V_{k}$ of an option to produce during the time $\tau$, between decision $k$ and $k+1$, must satisfy the following partial differential equation.

$$
\begin{equation*}
\frac{d V_{k}}{d t}+\sum_{i=1}^{N}\left(r-\delta_{i}\right) \frac{d V_{k}}{d D_{i}}+\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \rho_{i j} \sigma_{i} \sigma_{j} \frac{d V_{k}}{d D_{i} d D_{j}}-r V_{k}=0 \tag{8}
\end{equation*}
$$

The option value must also satisfy the terminal boundary conditions given in expression (6), in the one line case, and in expression (7) when production can take place in more than one line. A similar approach to the two-step approach used in Triantis \& Hodder (1990) can thereby be used. However, the optimisation problem in Triantis \& Hodder is non-linear and they use Lagrange optimisation to solve it.

Even though analytical solutions to expression (6), (7) and (8) could be found they are often complex and hard to find already in the case of two underlying variables. Another way to solve the problem, which also allows for American options, is to use a lattice approach. Different ways to carry out risk-neutral valuation in lattices for multivariate underlying assets are described in Boyle (1988), for two assets, and in Boyle et al (1989) for several underlying assets. Other variants and extensions are found in Kamrad \& Ritchken (1991), allowing for horizontal jumps and Ekvall (1996), who use equal and non-negative jump probabilities. Monte Carlo simulation can also be used but this approach is restricted to European options.

If a lattice approach is used in the case of $n$ products, demand at each end node at time $T$ together with prices, variable costs, set-up cost and capacity constraints can be used to find an optimal production schedule at each end node. The schedule, stating which product that should be produced and in what line is used as input when calculating the value of the production option at expiration. Then, using the risk-neutral probabilities and the risk-free rate, the value of the option at the valuation date can be estimated by working backward in the lattice.

## Options to hire extra personnel

Options with different characteristics can be identified depending upon when the company faces uncertainty. Delivery time, length of the contracts, number of production decisions during the contract etc. will affect what kind of uncertainty that the company faces and the options that they have. All options are here considered as "just-in-time" options: When all production is made-to-order, demand that can not be produced immediately or during the current period is considered as lost.

If the aggregated demand in a line is stochastic and the demand for the next period is known at the time for deciding whether to hire or not, management, at decision time, will know if the worker will be utilised during the next period or not. The option to hire can
be seen as European option, and is exercised when an extra worker is hired, and has the following pay-off at the expiration day $T$

$$
V(T)=\left\{\begin{array}{l}
\max \left[(P-V)\left(D-C_{\max }\right)-F, 0\right] \quad 0 \leq D \leq C_{\max }^{+}  \tag{9}\\
\max \left[(P-V)\left(C_{\max }^{+}-C_{\max }\right)-F, 0\right] \quad C_{\max }^{+} \leq D
\end{array}\right.
$$

where
$C_{\max }^{+}=$max capacity, extra worker included.
$C_{\max }=\max$ capacity, extra worker not included.
$F=$ fixed cost from exercising the option (wage to extra person, etc.)
The pay-off is equal to a bullish call spread and can be valued using the Black \& Scholes formula, eventually adjusted for convenience yield, on a long call with exercise price $(P-V) C_{\max }+F$ and a short call with exercise price $(P-V) C_{\max }^{+}+F$. Note also that each marginal worker will raise the max capacity, resulting in a new exercise price.

Assume now that demand is uncertain during a part or the whole time of the contract. The characteristic of the option to hire and its value will now be different from the one described above. The option in this case is a so-called compound option where the option to hire, if exercised, gives one or numerous production options which can be exercised if demand is sufficiently high. The pay-off from the production option at expiration day $T$ can be written as

$$
V(T)=\left\{\begin{array}{l}
\max \left[(P-V)\left(D-C_{\max }\right)-F,-F\right] \quad 0 \leq D \leq C_{\max }^{+}  \tag{10}\\
\max \left[(P-V)\left(C_{\max }^{+}-C_{\max }\right)-F,-F\right] \quad C_{\max }^{+} \leq D
\end{array}\right.
$$

Note that in this case we have a fixed cost $F$, which is always present. The number of production options, which the option to hire gives, is dependent on the length of the contracts and how frequently the company decides on what and how to produce.

## Solutions to the options

Due to the compound structure, i.e. option on options, of the latter option to hire, there is no simple analytical formula to apply on this problem. Geske (1979) presents an analytical expression to value compound options but this formula can not be used when there are more than one option in the second stage and when capacity constraints are incorporated. Again, a numerical approach such as using a lattice is required to solve this problem.

## 6. Numerical Examples

To generate some insight in some of the options described above we will present some numerical examples. Let us also clarify that numbers used are not directly taken from the
companies described in the paper. The problems are also stylised to get more descriptive problems.

Assume that a company has the opportunity to set up a job-pool, from which people can be hired for three months. After three months a new decision is made whether to hire the person again for three months or not. If one consider this as an investment opportunity, that is legally restricted to last for twelve months, the company would have four opportunities when the options to hire can be exercised. We also assume that aggregated demand is stochastic, following a Brownian motion, and that demand for the next month is known. When making the decision to hire or not, the demand in the first month is known with certainty, but uncertain for the second and the third month. If production decisions are made every month, then, during a three month contract, there will be three production options: the first expires at the same moment as the option to hire is exercised, the second one month later, and the third two months later. The option to hire will be exercised if the value of the three production options exceeds the "exercise price" of hiring a person for three months. Therefore a person could be hired although demand in the first month is too low to utilise the extra worker.

Assume that we want to know the value of the option to hire one month before it expires, to find out if the company should search for and educate appropriate personnel. The actual demand over the next month is 480 units and that actual max capacity is 500 units. If one worker is added the max capacity will increase 20 units. The price of a unit is 150000 SEK, the variable cost is 125000 SEK and the cost of hiring a person for three months is 76800 SEK. In table 1 below values of the option to hire are presented for different standard deviations, $\sigma$, and for different convenience yields, $\delta$. A continuously compounded risk-free rate $r$ of $4.88 \%$ is used in all calculations

| Convenience yield | Standard deviation |  |  |
| :---: | :---: | :---: | :---: |
|  | $10 \%$ | $20 \%$ | $40 \%$ |
| $0 \%$ | 93 | 285 | 422 |
| $5 \%$ | 58 | 243 | 396 |
| $10 \%$ | 34 | 206 | 371 |

Table 1. The value of an option to hire one person for three month, one month before it expires (in thousands of SEK). $D=480$, $\max C=500, r=4.88 \%$.

Since the company has the opportunity to hire four times during a year, i.e. the first expires after one month from the valuation date (as above), the second after four months, the third after seven months and the fourth after 10 months, the corresponding options have to be evaluated to find the total value. In figure 1 below, the total value of having these four options is shown for each marginal worker. It is assumed that the first option to hire is exercised, thereby increasing the max capacity and the exercise price, before the second option is evaluated.


Figure 1. Total value of the option to hire four times during a year for each marginal worker when $D=480$, $\max C=500, \sigma=30 \%, \delta=0$

As seen in figure 1 the total value of the option to hire a marginal worker decreases rapidly from almost 2 MSEK for the first person down to 0.25 MSEK for the tenth person and to almost zero for the twentieth person. Therefore, it is not profitable to a company to search for and educate too many workers. Searching and education should only take place as long as the value of an additional option to hire exceeds the cost of the search and the education.

It could also be interesting to compare the value of the option to hire for three months to the value of the option to hire for twelve months. The latter has only one hiring decision but has twelve production options and the exercise price will also be almost four times higher than the exercise price of the first option. In figure 2 below, the value of the option to hire for twelve month is compared to the total value of the option to hire for three months four times during a year. Here, we have summed up the value of each individual option to find the aggregated value of having numerous workers available to hire on three month contracts and twelve month contracts respectively. As can be seen in figure 2, the value of having the opportunity to hire for three months is higher than the opportunity to hire on twelve months, which can be explained by higher flexibility.


Figure 2. The aggregated values of the options to hire when $D=480$, max $C=500$ and $\sigma=30 \%$. The thinner line represents the aggregated value of the option to hire for three months during a year. The thicker line represents the aggregated value of the option to hire for twelve months.

## 7. Summary

There are different ways available to a company to increase and achieve desired flexibility but these ways are often associated with costs and it is therefore interesting to evaluate the flexibility. The need for flexibility is dependent on the company and the uncertainties that it faces. In this paper which is based partly on experiences from two Swedish midsize companies we have studied different kinds of flexibility that are relevant to them, proposed some model that can be used to enable an appropriate valuation. The major source of uncertainty to both companies is demand and the uncertainty is in both product-mix and volume.

Two types of option are identified and modelled and ways to evaluate them are suggested. The first type considers options resulting from investment in set-up time reductions, multipurpose stations and parallel assembly lines. Depending on if one or several products are produced in a line and if there is capacity constraint, the options have to be modelled and evaluated in different ways. The second type considers the option to hire personnel on short contract, which in some cases result in compound options.

The numerical examples shows that the opportunity to hire extra personnel can be valuable but the value decreases quite fast when more workers are added. It is also showed that the value of the options to hire for three month at a time is lot more worth than the option to hire for twelve months at a time.

## Acknowledgement

The research is supported by grants from the Swedish Foundation for Strategic Research and Swedish National Board for Industrial and Technical Development.

## References

Amram B. \& N. Kulatilaka, (1999); Real Options - Managing Strategic Investment in an Uncertain World, Harvard Business School Press.
Andreou, S.A., (1990); A Capital Budgeting Model for Product-Mix Flexibility, Journal of Manufacturing and Operations Management, 3, pp. 5-23.

Black F. \& M. Scholes (1973); The Pricing of Option and Corporate Liabilities, Journal of Political Economy, 81, pp. 637-654.
Boyle, P.P., (1988); A Lattice Framework for Option Pricing with Two State Variables, Journal of Financial and Quantitative Analysis, 23, No. 1, pp. 1-12.
Boyle, P.P., J. Evnine and S. Gibbs, (1989); Numerical Evaluation of Multivariate Contingent Claim, The Review of Financial Studies, 2, pp. 241-250
Brennan, M. and E. Schwartz, (1985); Evaluating Natural Resource Investments, Journal of Business 58, No. 2, pp. 135-157.
Chambers, S (1992); Flexibility in the Context of Manufacturing Strategy, In Manufacturing Strategy: Process and Content, Chapman \& Hall, London, pp. 283-295. ed. C. A. Voss.
Chung, K., (1990); Output Decision under Demand Uncertainty with Stochastic Production Function: A Contingent Claims Approach, Management Science, 36, No. 11, pp. 1311-1328.
Ekvall, N., (1996); A Lattice Approach for Pricing of Multivariate Contingent Claims, European Journal of Operational Research, 91, pp. 214-228.
Fine, C.H. and A.C. Hax, (1985); Manufacturing Strategy: A Methodology and an Illustration, Interfaces, 15, No. 6, pp. 28-46.
Geske, R., (1979); The Valuation of Compound Options, Journal of Financial Economics, 7, pp. 63-81.
Hayes, R.H. and D.M. Upton, (1998); Operations-Based Strategy, California Management Review, 40, No. 4, pp. 8-25.

He, H. and R. S. Pindyck, (1992); Investment in Flexible Production Capacity, Journal of Dynamics and Control, 16, pp. 575-599.
Hill, T. (1995); Manufacturing Strategy: Text and Cases, MacMillan, London
Kamrad, B. and R. Ernst, (1995); Multiproduct manufacturing with stochastic input prices and output yield uncertainty, In Real options in Capital Investment: Models, Strategies, and Application. ed. L. Trigeorgis. Praeger.
Kamrad, B. and P. Ritchken, (1991); Multinomial Approximating Models for Options with $k$ State Variables; Management Science, 37, No. 12, pp. 1640-1652.
Kulatilaka, N. (1988); Valuing the Flexibility of Flexible Manufacturing System, IEEE Transaction in Engineering Management, Vol. 35, No. 4, pp. 250-257.
Majd, S. and R.S. Pindyck, (1987); Time to Build, Option Value, and Investment Decisions, Journal of Financial Economics, 18, No. 1, pp. 7-27

McDonald, R. and D. Siegel, (1985); Investment and the Valuation of Firms When There Is An Option to Shut Down, International Economic Review, Vol. 26, No. 2, pp. 331-349. McDonald, R. and D. Siegel, (1986); The Value of Waiting to Invest, Quarterly Journal of Economics, 101, pp. 707-727.
Merton, R.C., (1973); Theory of Rational Option Pricing, Bell Journal of Economic and Management Science, 4, pp. 141-183.
Olhager, J., (1993); Manufacturing Flexibility and Profitability, International Journal of Production Economics, 30-31, pp. 67-78.

Pindyck, R. S. (1988); Irreversible Investment, Capacity Choice, and the Value of the Firm, The American Economic Review, Vol. 78, No. 5, pp. 969-985.
Sethi, A.K. and S.P. Sethi, (1990); Flexibility in Manufacturing: A Survey, The International Journal of Flexible Manufacturing Systems, 2, pp. 289-328.
Slack, N (1988) Manufacturing Systems Flexibility - An Assessment Procedure, Computer-Integrated Manufacturing Systems, 1, No. 1, pp. 25-31.
Tannous, G.F., (1996); Capital Budgeting for Volume Flexible Equipment, Decision Sciences, 27, No. 2. pp. 157-184.

Triantis, A and J. Hodder, (1990); Valuing Flexibility as a Complex Option, The Journal of Finance, Vol. 45, No. 2, pp. 549-565.

Trigeorgis, L. (1996); Real Options: Managerial Flexibility and Strategy in Resource Allocation. The MIT Press

