Real Option Valuation of Staged Manufacturing - Extended Abstract

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1 Introduction

Real option analysis (ROA) is recognized as a superior method to quantify the value of real-world investment opportunities where managerial flexibility can influence their worth, as compared to standard discounted cash-flow methods typically used in industry. ROA stems from the work of Black and Scholes (1973) on financial option valuation. Myers (1977) recognized that both financial options and project decisions are exercised after uncertainties are resolved. Early techniques therefore applied the Black-Scholes equation directly to value put and call options on tangible assets (see, for example, Brennan and Schwartz (1985)). Since then, ROA has gained significant attention in academic and business publications, as well as textbooks (Copeland and Tufano (2004), Trigeorgis (1996)).

The focus of this research is to develop an analytical model in a staged manufacturing situation. We take the viewpoint of a firm contemplating to enter a new market in a two-stage process. An initial investment would allow the firm to serve a market whose demand is significantly uncertain. The idea is that this initial investment will enable the firm to create future growth opportunities that may come to fruition should the market and technology conditions develop favourably. In the case of a positive outcome, the firm will make a large strategic move in the direction of becoming a leading industry player, or niche leader, involving a large investment and business expansion. The initial investment will enable the firm to develop market knowledge, technology know-how and internal capabilities that places it in a position to take a leading industry position. Two aspects of our approach are unique: 1) our model is analytical, and 2) it utilizes managerial estimates. In some ways, our work is an expansion of the news vendor problem (Arrow, Harris, and Marshak 1951).

2 Relevant Literature

This section is under development.

3 Theory

As discussed above, our model is based on the premise that a manufacturing firm is entering a new market, likely utilizing new technology. As there is significant uncertainty regarding the market itself, the firm can stage their investment – specifically, two stages in this case. First stage entry may have lower profit margins, but, at the same time, may provide the firm with first mover advantage.

Table 1: Managerial Supplied Variables

Variable	**Description**
τ	Time when it is expected that there could be a market disruption / further adoption where the company has an opportunity to expand production and sales
T	Terminal time horizon
X_0	Initial demand for product
r	Risk adjusted rate of return for the analysis
The following parameters are utilized in the timeframe $0 \leq t < \tau$	
$\hat{x}_\tau$	Expected demand at $t = \tau$
$\hat{x}_{\sigma,\tau}$	Uncertainty in expected demand at the 95% confidence interval (i.e. ± two standard deviations) at $t = \tau$
A_0, B_0	Variable and fixed investment costs as a function of size of system (Q_0), where the initial investment cost is given by $I_0 = A_0 Q_0 + B_0$
α_0	Profit margin associated with sales
The following parameters are utilized in the timeframe $\tau \leq t < T$	
$\hat{x}_T$	Expected demand at $t = T$
$\hat{x}_{\sigma,T}$	Uncertainty in expected demand at the 95% confidence interval (i.e. ± two standard deviations) at $t = T$
A_1, B_1	Variable and fixed investment costs as a function of size of system (Q_τ), where the investment cost at $t = \tau$ is given by $I_\tau = A_1 Q_\tau + B_1$
α_1	Profit margin associated with sales
β	Portion of initial capacity available for the second period ($0 \leq \beta \leq 1$)

The managerial inputs are summarize in Table 3. The methodology consists of assuming a stochastic process, X_t, such that the number of units sold at any time $0 \leq t < \tau$ is given by an average value, i.e.,

$$X_{AVG_0} = \frac{X_\tau + X_0}{2} \tag{1}$$

and for $\tau \leq t \leq T$ as,

$$X_{AVG_\tau} = \frac{X_\tau + X_T}{2}. \tag{2}$$

We assume that firm can buy initial production capacity, in the first stage, Q_0 for the investment cost $I_0 = A_0 Q_0 + B_0$. At time $t = \tau$, depending on the value of X_τ there exists an optimal capacity to which the firm can add capacity, Q_τ^*, by investing $I_\tau = A_1 Q_\tau^* + B_1$. Thus, during the second stage, $\tau \leq t \leq T$, the firm's production capacity would be $\beta Q_0 + Q_\tau^*$, where β would typically be in the range $[0, 1]$ and represents the amount of initial capacity that is available during the second stage.

The value of the project can be written as

$$V_0 = \alpha_0 \int_0^\tau e^{-rs} \mathbb{E}\left[\min\left(X_{AVG_0}, Q_0\right)\right] ds + \mathbb{E}\left[V_\tau\right] - I_0, \tag{3}$$

where the discounted value at time τ is given by

$$V_\tau = \alpha_1 \int_\tau^T e^{-rs}\mathbb{E}_\tau\left[\min\left(X_{AVG_\tau}, \beta Q_0 + Q_\tau\right)\right] ds - I_\tau e^{-r\tau}. \tag{4}$$

By taking the derivative of equation (4) with respect to Q_τ and setting it equal to zero, after some manipulation, the optimal Q_τ^* can be determined as

$$Q_\tau^* = \frac{1}{2}\left(F_{X_T|X_\tau}^{-1}\left(1 - \frac{A_1 e^{r\tau}}{\alpha_1 \gamma}\right) + X_\tau - 2\beta Q_0\right), \tag{5}$$

where $F_{X_T|X_\tau}^{-1}(\cdot)$ is the inverse distribution of X_T given X_τ at $t = \tau$ and $\gamma = \frac{e^{-rt} - e^{-rT}}{r}$. Substituting equation (5) into equation (4), substituting appropriately into equation (3), taking the derivative with respect to Q_0, setting the expression equal to zero, after some manipulation, the optimal Q_0^* can be determined as

$$Q_0^* = \frac{1}{2}\left(F_{X_\tau}^{-1}\left(1 - \frac{\left(A_0 - \beta A_1 e^{-r\tau}\right) r}{\alpha_0\left(1 - e^{-r\tau}\right)}\right) + X_0\right), \tag{6}$$

where $F_{X_\tau}^{-1}(\cdot)$ is the inverse distribution of X_τ. Substituting equation (5 into equation (3) leads to a complicated non-analytical expression. However, if we assume Brownian motion for X_t an analytical expression can be determined.

We proceed by assuming that for $0 \le t < \tau$ we have

$$dX_t = \mu_0 dt + \sigma_0 dW_t \tag{7}$$

and for $\tau \le t \le T$ we have

$$dX_t = \mu_1 dt + \sigma_1 dW_t. \tag{8}$$

Utilizing the managerial estimates we have,

$$\begin{aligned} \mu_0 &= \frac{\hat{x}_\tau - X_0}{\tau} \\ \sigma_0 &= \frac{\hat{x}_{\sigma,\tau}}{2\sqrt{\tau}} \\ \mu_1 &= \frac{\hat{x}_T - \hat{x}_\tau}{T - \tau} \\ \sigma_1 &= \sqrt{\frac{\frac{\hat{x}_{\sigma,T}^2}{4} - \sigma_0^2\tau}{T - \tau}}. \end{aligned}$$

The analytical expression for V_0 can now be determined as

$$\begin{aligned} V_0 = \frac{1}{2r}&\left(\alpha_0\sigma_0\sqrt{\tau}\left(\mathrm{e}^{-r\tau} - 1\right)\phi\left(\Phi^{-1}\left(\frac{\mathrm{e}^{-r\tau}\alpha_0 - \alpha_0 + (-\beta\, A_1 + A_0)\, r}{\alpha_0\left(\mathrm{e}^{-r\tau} - 1\right)}\right)\right)\right. \\ &+ \left(\left(\left((\beta - 1)\,\mu_0 + \mu_1\right) A_1 - A_0\mu_0\right)\tau - A_1 T\mu_1 + \left((2\,\beta - 1)\, X_0 - X_\tau\right) A_1 - 2\,A_0 X_0 - 2\,B_0 - 2\,B_1\right) r \\ &+ \left(-\left(\mathrm{e}^{-rT} - \mathrm{e}^{-r\tau}\right)(\mu_0 - \mu_1)\,\alpha_1 - \left(\mathrm{e}^{-r\tau} - 1\right)\alpha_0\mu_0\right)\tau + \left(\mathrm{e}^{-rT} - \mathrm{e}^{-r\tau}\right) \\ &\left.\left(\sqrt{T - \tau}\;\phi\left(\Phi^{-1}\left(\frac{A_1 r + \alpha_1\mathrm{e}^{-rT} - \alpha_1\mathrm{e}^{-r\tau}}{\left(\mathrm{e}^{-rT} - \mathrm{e}^{-r\tau}\right)\alpha_1}\right)\right)\sigma_1 - \mu_1 T - X_\tau - X_0\right)\alpha_1 - 2\,\left(\mathrm{e}^{-r\tau} - 1\right)\alpha_0 X_0\right), \end{aligned}$$

where $\phi(\cdot)$ is the standard normal density and $\Phi^{-1}(\cdot)$ is the inverse of the standard normal distribution.

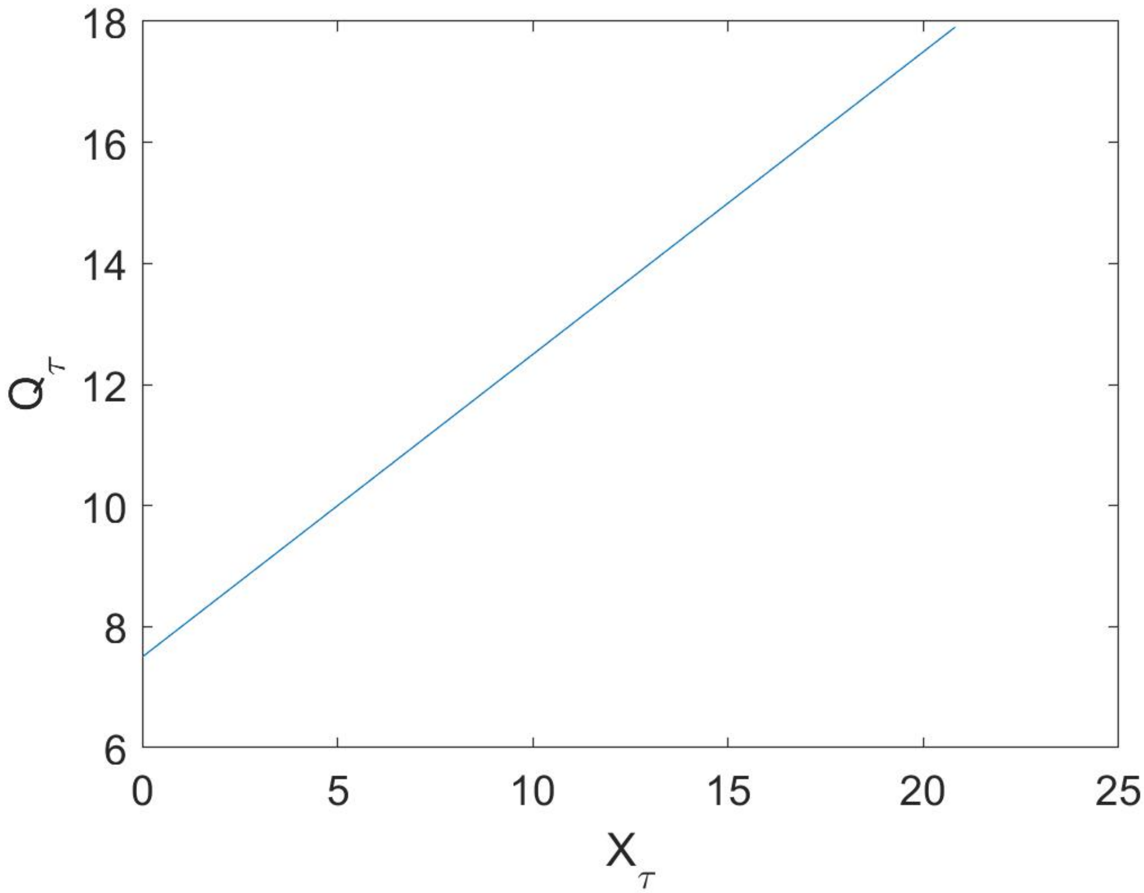

Figure 1: Optimal Q_τ as a function of X_τ.

4 Results

We plant to present more results. However, for a set of inputs we have calculated $V_0 = 18.9$, $Q_0^* = 12.8$ and Q_τ^* as a function of X_τ as presented in figure (1).

5 Conclusions

The focus of this research was to present of a real options valuation methodology to value a staged investment based on managerial estimates with an analytical expression. While our final formulation has many terms, it can be easily utilized within a spreadsheet.

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