Get Out or Get Down: Competitive Strategies in a Declining Market

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Abstract

We propose solutions for a multi-factor real option duopoly game model which determines the optimal time to divest in an incumbent technology or to switch to a new small-scale technology, under uncertainty about the output price and the output quantity is declining over time. We use two formulations, one in which the option to divest and the option to switch are treated separately and another in which these two options are mutually-exclusive. For the technology switch, there is a second-mover revenue market share advantage, whereas for the divestment there is a first-mover salvage value advantage. We find that curiously the leader should exit earlier as quantity declines faster, if there are no other options, yet, if there is an alternative technology for lower operating costs, it should adopt it sooner than the follower, but both are motivated to adopt earlier as the quantity decline rate increases, so look at the choices and decisions of your competitors before exiting or downsizing with lower cost technology. Moreover, the market share advantage of the follower, while operating alone with the incumbent technology, plays an important role in the timing of the technology switch, delaying the switch of both firms.

JEL Classification: D81, D92, O33.

Key Words: Investment Under Uncertainty, Revenue Decline, Real Option Game, Technology Adoptions, Downsizing.

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1. Introduction

In a duopoly, when should a leader divest, if there is output price uncertainty and the quantity declines over time? What is the real option value and threshold investment that justifies immediate divestment for a leader/follower with an investment opportunity to invest in an alternative technology that enables lower operating cost? What should be the follower's behaviour after the adoption of that technology by the leader?

We focus on a set of firms or industry or countries facing stagnation or revenue decline, due to natural factors in petroleum production, or economic/structural factors, where possible new alternative technologies may validate delaying exit-abandonment, or switching to lower cost production. Due to both pollution concerns and competition from natural gas, coal almost everywhere is being shut down, possibly awaiting cheaper emission control. Book shops and shopping malls in the US (Borders vs Barnes & Noble) are being closed, or converted to alternative uses (cafes and reading rooms, rather than book selling). Ceramics, textiles and other zombie industries in Portugal face closure or downsizing. Taxis, accommodation, and universities are experiencing competition from mobile-digital technologies. Thus, the three initial questions are of concern for several economic contexts.

Décamps et al. (2006) study irreversible investments in alternative projects and show that when firms hold the option to switch from a smaller scale to a larger scale project, a hysteresis region between the investment region can persist even if the uncertainty of the output price increases. Bobtcheff and Villeneuve (2010) examine investments in two mutually exclusive projects with two sources of uncertainty, and conclude that when these uncertainties hold simultaneously, the project payoffs are not sufficient criteria for deciding on the investment timing. Kwon (2010) looks at a declining profit stream following an arithmetic Brownion motion process, so the exit threshold decreases as volatility increases. Adkins and Paxson (2011) investigate optimal capital replacement and abandonment decisions considering that both revenues and costs are uncertain and their value declines over time.

Chronopoulos and Siddiqui (2015) study the timing of the replacement of an incumbent technology, assuming that there is technological uncertainty, and the ex-post revenues which the adoption of the new technology generate are uncertain. This investment analysis is examined under three different strategies, compulsive, laggard, and leapfrog. Their results reveal that, under the compulsive strategy, technological uncertainty has a non-monotonic impact on the optimal investment decision. Hagspiel et al. (2016) look at investment decisions in a new technology under uncertainty in profit declining markets, where firms hold the option to invest in a new technology with which they produce a new product, holding the option to exit the market and considering that the firms also decide on the capacity size. Among other findings, they show that a higher potential profitability of the new product market, to be produced with the adopted technology, accelerates the investment, but the capacity choice can alter this result, reversing the above intuitive result, if the choice of the investment capacity is smaller.

Støre et al. (2018) study an irreversible switch from oil to gas production, with both oil and gas production declining over time. They provide analytical solutions for the switching threshold and the real option value of the switching opportunity. Huberts et al. (2019) show that entry may be deterred, possibly in a war of attrition or pre-emption, following interesting strategies. Adkins and Paxson (2019), study the appropriate rescaling from an incumbent large-scale technology considering that market revenue declining. They also consider the case of abandonment and treat the two investment both jointly and separately, showing different implications for government policies.

Several authors focus on the uncertainty of new technologies, which should provide interesting extension of our current approach. Farzin et al. (1998) assume both the speed of arrival and degree of improvement of future technologies are uncertain. Doraszelski (2004) allows for future technolgies with improvements. Hagspiel et al. (2015) also consider changing arrival rates for new technologies.

The paper is organized as follows. Section 2 introduces the duopoly investment game, describes the assumptions underlying the model for scenario 1 (when should the leader divest when there are no other options), and also considers a second scenario, where the only option is adopting the lower cost technology. It ends with a sensitivity analysis for the effect of changes in the base case models' parameters on the investment thresholds and the coefficients of the real options. Section 3 develops a model for a third scenario where, after adoption of the new technology, there is also the option of eventual divestment, so the initial divest or switch technology are mutually exclusive choices. It also shows a sensitivity analysis. Section 4 concludes the work and provides some suggestions for further research.

2. The Base Model

In this section, we extend Adkins and Paxson (2019) model to a duopoly market. We neglect however the option to divest for the region where the two firms operate with the small-scale technology (tech 2). In the Appendix A, we present a comparable model for a monopoly market.

2.1 Duopoly

Let us assume that there are two active firms (the leader and the follower) in a duopoly market where there is output price, $P(t)$, uncertainty, and the output quantity, $q(t)$, declines over time. Each firm holds the option of either continuing operations with the incumbent technology (tech 1) or abandoning production and receiving a salvage value. Furthermore, the firm that exits the market first receives a λ percent higher salvage value.[2]

For the leader and the follower, we study both the optimal time to divest in tech 1, which is a large-scale technology, and the optimal time to switch to tech 2, which is a small-scale technology. We use two distinct modelling settings for the derivation of the divestment and switching thresholds. One

[2] We assume that the market for second-hand assets is illiquid, so the divestment value for the follower is lower than that of the leader, i.e., there is a first-mover divestment value advantage.

setting in which the option to divest in tech 1 and the option to invest in tech 2 are treated independently and, another, in which these two options co-exist. For the latter model setting, we follow Décamps et al. (2006). Adkins and Paxson (2019) show the differences between the investment thresholds for these two modelling settings applied to a monopoly market.

Moreover, let us assume that the output price follows a geometric Brownian motion (gBm) process given by:

$$dP = \mu_P P dt + \sigma_P P dz \tag{1}$$

where, μ_P is the instantaneous conditional expected percentage changes in P per unit of time, σ_p is the instantaneous conditional standard deviation of P per unit of time, and dz is the increment of a standard Wiener process. For convergence of the solution $r - \mu_P > 0$, where r is the riskless interest rate. For simplicity of notation we assume the asset or convenience yield $\delta_p = r - \mu_P$. The output quantity flow q declines over time according to Equation (2):

$$dq = -\theta q dt \tag{2}$$

where $\theta > 0$ denotes a known constant depletion rate. The residual reserve volume is given by $q_R = q/\theta$, so for any given q_R a flow increase (decrease) has to be compensated by a commensurate increase (decrease) in the depletion rate.

Therefore, firm i's revenue flow, if operating with technology k, is given by:

$$P(t)q(t). D_{k_i k_j} \tag{3}$$

where $D_{k_i k_j}$ is a deterministic competition factor that represents the percentage of the firm i's market revenue share for a given investment scenario, with $i, j = \{L, F\}$, where L means "leader" and F

"follower" and $k = \{0,1,2\}$, where "0" indicates inactive, "1" technology 1 (tech 1), and "2" means technology 2 (tech 2).[3]

For the leader and the follower, respectively, inequalities 4a and 4b hold for the "divestment game", and inequalities 4c and 4d hold for the "switching game":

$$D_{1_L 1_F} > D_{0_L 1_F} \tag{4a}$$

$$D_{1_F 0_L} > D_{1_F 1_L} \tag{4b}$$

$$D_{1_L 1_F} = D_{2_L 2_F} > D_{2_L 1_F} \tag{4c}$$

$$D_{1_F 2_L} > D_{1_F 1_L} = D_{2_F 2_L} \tag{4d}$$

For the divestment case, $D_{1_F 0_L} = 1.0$ and $D_{1_F 1_L} = 0.5$, therefore, we assume that the follower gets 100 percent of the market share when it is active alone and each firm gets 50% of the market share when they are both active with tech 1 or with tech 2. We note that, the market competition is embedded in our model through two factors: the firms' market share, which are governed by the inequalities 4a, 4b, 4c, and 4d above, and the parameter λ which ensures that there is first-mover divestment value advantage, i.e. Z_1^L is the leader's divestment value, so $Z_1^L(1-\lambda)$ is the follower's divestment value. Because tech 1 is a large-scale technology and tech 2 is a small-scale technology, the former operates with higher operating costs, thus we use w_k, with $k \epsilon \{1,2\}$, as the operating cost ($w_1 > w_2$). Notice also that the by switching to tech 2 first the leader reduces its operating costs from

[3] As an illustration about how these competition factors work: $D_{1_F 0_L}$ represents the follower's market share when the follower operates with tech 1 alone (after the leader has left the market) and $D_{1_F 1_F}$represents the follower's market share for when both firms are active operating with tech 1. A similar rationale applies to the competition factors related to the case where the two firms switch from the incumbent technology (tech 1) to a new small-scale technology (tech 2), where only the notation ("1") changes (to "2"). For instance, $D_{2_L 2_F}$ represents the leader's market share when both firms are active operating with tech 2, whereas $D_{2_L 1_F}$ represents the leader's market share for when the leader is active with tech 2 and the follower is active with tech 1. Notice that $D_{k_F k_L} + D_{k_L k_F} = 1$ (the full market), therefore, it possible to express the leader's competition factors as a function of the follower's (and vice-versa): $D_{k_L k_F} = 1 - D_{k_F k_L}$.

w_1 to w_2 but it also reduces its revenue market share, because after the technology switch it operates with a small-scale production technology. For instance, in our base case scenario, we use $D_{2_L 1_F} = 0.35$ so $D_{1_F 2_L} = 0.65$. Obviously, the leader will switch to tech 2 only if the operating cost saving offsets the revenue market share loss.

Using a risk-neutral framework and Ito's Lemma, we find that the value of an active follower with the option to divest and operating costs w_k satisfies the following differential equation:

$$\frac{1}{2}\sigma^2 p^2 \frac{\partial^2 F_k^{i,j}}{\partial p^2} + (r-\delta)p\frac{\partial F_k^{i,j}}{\partial p} - \theta q \frac{\partial F_k^{i,j}}{\partial q} + pqD_{1_F 1_L} - w_k - rF_k^{i,j} = 0 \quad (5)$$

where $F_k^{i,j}$ denotes the option value of firms i and j when they are active with technology k, $\delta = r - u_p > 0$ is the asset convenience yield, and r is the risk-free rate.

Based on the American perpetuity solution, the valuation function $f_k^{i,j}(p,q)$ satisfying (5) takes the form:[4]

$$f_k^{i,j}(p,q) = A_k^{i,j} p^{\beta_1} q^{\gamma_1} + B_k^{i,j} p^{\beta_2} q^{\gamma_2} + \frac{pqD_{1_F 1_L}}{\delta+\theta_k} - \frac{w_k}{r} \quad (6)$$

where A_k and B_k are two non-negative coefficients to be determined, with β_1 and β_2, and γ_1 and γ_2 are related through the following characteristic equation:

$$Q(\beta^k, \gamma^k) = \frac{1}{2}\sigma^2 \beta^k(\beta^k - 1) + (r-\delta)\beta^k - \theta\gamma^k - r = 0 \quad (7)$$

By examining the respective smooth-pasting conditions, we conclude that the principle of similarity can be applied. It implies that $\gamma_1^k = \beta_1^k$ and $\gamma_2^k = \beta_2^k$ (see, Paxson and Pinto, 2005). Consequently:

[4] See Adkins and Paxson (2011) and Adkins and Paxson (2017).

$$\beta_{1(2)}^{k} = \left(\frac{1}{2} - \frac{r-\delta-\theta_k}{\sigma^2}\right) + (-)\sqrt{\left(\frac{1}{2} - \frac{r-\delta-\theta_k}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \quad (8)$$

where $\beta_1^k > 1$ and $\beta_2^k < 0$.

Notice that both β_1^k and β_2^k vary with the depletion rate θ_k, and might be different for the two technologies. Thus, for θ_k the power parameters are expressed as β_1^1 and β_1^2. If $\theta_1 > \theta_2$, then $\beta_1^1 > \beta_1^2$ and $\beta_1^1 > \beta_2^2$. Additionally, because the similarity principle holds, the analysis can be framed in terms of a single variable, using the following variable change: $\varphi = p.q$. Thus, Equation (6) becomes:

$$f_k^{i,j}(\varphi_k) = A_k\, {\varphi_k}^{\beta_1^k} + B_k\, {\varphi_k}^{\beta_2^k} + \frac{\varphi_k D_{1_{F}1_L}}{\delta+\theta} - \frac{w_k}{r} \quad (9)$$

2.1.1 A Separate Duopoly Formulation - *à la Smets* (1993)

2.1.1.1 Divestment

In this section, we derive the leader's and the follower's investment thresholds for a duopoly divestment game, assuming that the follower gets 100 percent of the market revenue while active alone, but also a λ percent lower (eventually nil) divestment value. The motivation for being the leader (divesting first) is because the output is declining over time and there is high output price uncertainty, so, the business can become unprofitable soon. Furthermore, being the first to close down production also ensures a higher salvage value. Following well-established backward-induction procedures, we determine first the divestment threshold for the follower.

2.1.1.1.1 The Follower

The value function is given by:

$$f_1^F(\varphi_1)\begin{cases} Z_1^L(1-\lambda) & if\ \ \varphi_1 \leq \varphi_1^F \\ \frac{\varphi_1 D_{1_F 0_L}}{\delta+\theta_1} - \frac{w_1}{r} + B_1^F\ \varphi_1{}^{\beta_2^1} & if\ \ \varphi_1 \in]\varphi_1^F, \varphi_1^L[\\ \frac{\varphi_1 D_{1_F 1_L}}{\delta+\theta_1} - \frac{w_1}{r} + B_1^F\ \varphi_1{}^{\beta_2^1} & if\ \varphi_1\ \geq \varphi_1^L \end{cases} \tag{10}$$

The economic interpretation for (10) is the following: the term in the first row is the follower's divestment value; in the second row, the first two terms represent the present value of the revenue stream less the operating costs for the region where the follower is alone in the market, and the second term is the option value to divest; in the third row, the first two terms represent the present value of the revenue stream less the operating costs for the region where both firms are active with tech 1.

The value-matching and smooth-pasting conditions are given by Equations (11) and (12), respectively:

$$\frac{\varphi_1^F D_{1_F 0_L}}{\delta+\theta_1} - \frac{w_1}{r} + B_1^F\ \varphi_1^{F^{\beta_2^1}} = Z_1^L(1-\lambda) \tag{11}$$

$$\beta_2^1 B_1^F\ \varphi_1^{F^{(\beta_2^1-1)}} + \frac{D_{1_F 0_L}}{\delta+\theta_1} = 0 \tag{12}$$

Using Equations (11) and (12), we determine the constant B_1^F and the divestment threshold φ_1^F:

$$B_1^F = \frac{-D_{1_F 0_L}}{(\delta+\theta_1)\beta_2^1 \varphi_1^{F^{(\beta_2^1-1)}}} \tag{13}$$

$$\varphi_1^F = \frac{\left(\frac{w_1}{r}+Z_1^L(1-\lambda)\right)(\delta+\theta_1)\beta_2^1}{D_{1_F 0_L}(\beta_2^1-1)} \tag{14}$$

2.1.1.1.2 The Leader

The value function is given by:

$$f_1^L(\varphi_1) = \begin{cases} Z_1^L & if \quad \varphi_1 \leq \varphi_1^F \\ Z_1^L & if \ \ \varphi_1 \in]\varphi_1^F, \varphi_1^L[\\ \frac{\varphi_1 D_{1_L 1_F}}{\delta+\theta_1} - \frac{w_1}{r} + B_1^L \, \varphi_1{}^{\beta_2^1} & if \ \varphi_1 \geq \varphi_1^L \end{cases} \tag{15}$$

The economic interpretation of (15) is the following: the term in the first two rows represents the leader's divestment value; and the first two terms in the third row represent the revenue stream less the operating costs for the region where the two firms operate with tech 1, and the third term represents the option value to divest.

The value-matching and smooth-pasting conditions are given by Equations (16) and (17), respectively:

$$\frac{\varphi_1^L D_{1_L 1_F}}{\delta+\theta_1} - \frac{w_1}{r} + B_1^L \, \varphi_1^{L^{\beta_2^1}} = Z_1^L \tag{16}$$

$$\frac{D_{1_L 1_F}}{\delta+\theta_1} + \beta_2^1 B_1^L \, \varphi_1^{L^{(\beta_2^1 - 1)}} = 0 \tag{17}$$

Using Equations (16) and (17), we determine the constant B_1^L and the divestment threshold φ_1^L:

$$B_1^L = \frac{-D_{1_L 1_F}}{(\delta+\theta_1)\beta_2^1 \varphi_1^{L^{(\beta_2^1-1)}}} \tag{18}$$

$$\varphi_1^L = \frac{\left(\frac{w_1}{r} + Z_1\right)(\delta+\theta_1)\beta_2^1}{D_{1_L 1_F}(\beta_2^1 - 1)} \tag{19}$$

2.1.1.2 Switching

In this section, we evaluate a policy where the two firms have the option to switch to a small-scale production technology (tech 2), see Dixit (1993), for which they have to invest I_2, benefiting thereafter from a lower operating cost, w_2. However, because tech 2 is a small-scale technology, there is a η percent downward jump in output production (q) from tech 2. For the sake of simplicity, we neglect the option to divest in tech 2.

Thus, let us assume that the two firms are active with the incumbent (tech 1) and the leader switches to tech 2 first. Therefore, there is a time period during which the leader operates with a small-scale technology (tech 2) and the follower operates with a large-scale technology. During this time period, the follower gets a higher revenue market share ($D_{1_F2_L} > D_{1_F1_L}$).

Notice that the decision to switch to tech 2 is also driven by low revenues, therefore, if we denote the switching thresholds for the leader and the follower as φ_{SW}^L and φ_{SW}^F, respectively, the firm continues to extract output with tech 1 if $\varphi_2 > \varphi_{SW}^F$ and switches to tech 2 if $\varphi_2 < \varphi_{SW}^F$.

2.1.1.2.1 The Follower

The value function is given by:[5]

$$f_2^F(\varphi_2) = \begin{cases} \frac{\varphi_2/(1-\eta D_{1_F1_L}}{\delta+\theta_1} - \frac{w_1}{r} & if\ \varphi_2 > \varphi_{2sw}^L \\ \frac{\varphi_2/(1-\eta)D_{1_F2_L}}{\delta+\theta_2} - \frac{w_1}{r} + B_{2sw}^F\ \varphi_2{}^{\beta_2^2} & if\ \ \varphi_2 \in [\varphi_{2sw}^L, \varphi_{2sw}^F) \\ \frac{\varphi_2 D_{2_F2_L}}{\delta+\theta_2} - \frac{w_2}{r} + Z_L(1-\lambda) - I_2 & if\ \ \varphi_2 \leq \varphi_{2sw}^F \end{cases} \tag{20}$$

The economic interpretation of (20) is the following: the first two terms in the first row represents the present value of the revenue stream less the operating costs for the region where both firms operate with tech 1; the first two terms in the second row represents the present value of the revenue stream less the operating costs for the region where the follower operates with tech 1 and the leader operates with tech 2, and the third term is the option to switch to tech 2; the first two terms in the third row represent the present value of the follower's revenue stream less the operating costs for the region where both firms operate with tech 2, and the third and fourth terms are the divestment value and the investment cost in tech 2, respectively.

[5] Notice that there is a revenue flow (η percent) drop when the follower switches to tech 2. In (20) we can express the follower's value function while active with tech 1 as a function of φ_2 using $\varphi_1 = \varphi_2/(1-\eta)$.

The value-matching and smooth-pasting conditions at $\varphi_2 = \varphi_{2sw}^F$ are given by Equations (21) and (22), respectively.

$$\frac{\varphi_{2sw}^F D_{1_F 2_L}}{(\delta+\theta_1)(1-\eta)} - \frac{w_1}{r} + B_{2sw}^F \, {\varphi_{2sw}^F}^{\beta_2^2} = \frac{\varphi_{2sw}^F D_{2_F 2_L}}{\delta+\theta_2} - \frac{w_2}{r} + Z_L(1-\lambda) - I_2 \tag{21}$$

$$\frac{D_{1_F 2_L}}{(1-\eta)(\delta+\theta_1)} + \beta_2^2 B_{2sw}^F \, {\varphi_{2sw}^F}^{(\beta_2^2-1)} - \frac{D_{2_F 2_L}}{\delta+\theta_2} = 0 \tag{22}$$

Using Equations (21) and (22), we determine the constant B_{2sw}^F and the switching threshold, φ_{2sw}^F:

$$B_{2sw}^F = \frac{D_{2_F 2_L}(1-\eta)(\delta+\theta_1) - D_{1_F 2_L}(\delta+\theta_2)}{\beta_2^2 \varphi_2^{F(\beta_2^2-1)}(1-\eta)(\delta+\theta_1)(\delta+\theta_2)} \tag{23}$$

$$\varphi_{2sw}^F = \frac{[w_1 - w_2 - r(I_2 - Z_L(1-\lambda))]\beta_2^2(1-\eta)(\delta+\theta_1)(\delta+\theta_2)}{r\left[D_{2_F 2_L}(1-\eta)(\delta+\theta_1) - D_{1_F 2_L}(\delta+\theta_2) + \beta_2^2\left[D_{1_F 2_L}(\delta+\theta_2) - D_{2_F 2_L}(1-\eta)(\delta+\theta_1)\right]\right]} \tag{24}$$

2.1.1.2.2 The Leader

The value function is given by:

$$f_2^L(\varphi_2)\begin{cases} \frac{\varphi_2 D_{1_L 1_F}}{(\delta+\theta_1)(1-\eta)} - \frac{w_1}{r} + B_2^L \, {\varphi_2}^{\beta_2^2} & if \;\; \varphi_2 > \varphi_{2sw}^L \\ \frac{\varphi_2 D_{2_L 1_F}}{\delta+\theta_2} - \frac{w_2}{r} + (Z_L - I_2) + \frac{\varphi_2 (D_{2_L 2_F} - D_{2_L 1_F})}{\delta+\theta_2}\left(\frac{\varphi_2}{\varphi_{2sw}^F}\right)^{\beta_2^2} & if \;\; \varphi_2 \in [\varphi_{2sw}^L, \varphi_{2sw}^F) \\ \frac{\varphi_2 D_{2_L 2_F}}{\delta+\theta_2} - \frac{w_2}{r} & if \;\; \varphi_2 \le \varphi_{2sw}^F \end{cases} \tag{25}$$

The economic interpretation of (25) is the following: the first two terms in the first row represents the present value of the leader's revenue stream less the operating costs for the region where the two firms operate with tech 1, the third term is the option to switch to tech 2; the first two terms in the second row represents the present value of the leader's revenue stream less the operating costs for the region where the leader operates with tech 2 and the follower operates with tech 1, the third term is the difference between the divestment value and the investment cost in tech 2, and the fourth term is the present value of the revenue stream gains for the leader due to the fact that the follower will also switch to tech 2 when φ_2 reaches φ_{2sw}^F.[6]

The value-matching and smooth-pasting conditions at $\varphi_2 = \varphi_{2sw}^L$ are given by Equations (26) and (27), respectively:

$$\frac{\varphi_{2sw}^L D_{1_L 1_F}}{(\delta+\theta_1)(1-\eta)} - \frac{w_1}{r} + B_{2sw}^L\, {\varphi_{2sw}^L}^{\beta_2^2} = \frac{\varphi_{2sw}^L D_{2_L 1_F}}{\delta+\theta_2} - \frac{w_2}{r} + (Z_L - I_2) + \frac{\varphi_{2sw}^L (D_{2_L 2_F} - D_{2_L 1_F})}{\delta+\theta_2}\left(\frac{\varphi_{2sw}^L}{\varphi_{2sw}^F}\right)^{\beta_2^2} \quad (26)$$

$$\frac{D_{1_L 1_F}}{(1-\eta)(\delta+\theta_1)} + \beta_2^2 B_{2sw}^L\, {\varphi_{2sw}^L}^{(\beta_2^2-1)} - \frac{D_{2_L 1_F}}{\delta+\theta_2} - \frac{(\beta_2^2+1)\,(D_{2_L 2_F} - D_{2_L 1_F})\, {\varphi_{2sw}^L}^{\beta_2^2}}{(\delta+\theta_2){\varphi_{2sw}^F}^{\beta_2^2}} = 0 \quad (27)$$

Using Equations (27), we determine the constant B_2^L:

$$B_{2sw}^L = \frac{(1-\eta)(\delta+\theta_1){\varphi_{2sw}^F}^{\beta_2^2} D_{2_L 1_F} + (1-\eta)(\delta+\theta_1)(\beta_2^2+1)(D_{2_L 2_F} - D_{2_L 1_F})\, {\varphi_2^L}^{\beta_2^2} - (\delta+\theta_2){\varphi_{2sw}^F}^{\beta_2^2} D_{1_L 1_F}}{\beta_2^2 (1-\eta)(\delta+\theta_1)(\delta+\theta_2){\varphi_{2sw}^F}^{\beta_2^2} {\varphi_{2sw}^L}^{(\beta_2^2-1)}} \quad (28)$$

Rearranging Equation (26), we obtain (29) through which we can get a numerical solution for φ_{2sw}^L:

$$\frac{\varphi_{2sw}^L D_{1_L 1_F}}{(1-\eta)(\delta+\theta_1)} + B_{2sw}^L\, {\varphi_{2sw}^L}^{\beta_2^2} + \frac{w_2 - w_1}{r} + (Z_L - I_2) - \frac{\varphi_{2sw}^L D_{2_L 1_F}}{\delta+\theta_2} - \frac{\varphi_{2sw}^L (D_{2_L 2_F} - D_{2_L 1_F})}{\delta+\theta_2}\left(\frac{\varphi_{2sw}^L}{\varphi_{2sw}^F}\right)^{\beta_2^2} = 0 \quad (29)$$

2.1.3 Results

[6] In the second row of expression (25), the term $(\varphi_2/\varphi_{2sw}^F)^{\beta_2^2}$ can be interpreted as a stochastic discount factor which is equal to the present value of \$1 received when the variable φ_2 hits φ_2^L (see Pawlina and Kort, 2006, p. 8). The term $\varphi_2(D_{2_L 2_F} - D_{2_L 1_F})/(\delta+\theta_2)(\varphi_2/\varphi_{2sw}^F)^{\beta_2^2}$ accounts for the effect of competition on the leader's behaviour. More specifically, $(\varphi_2/\varphi_{2sw}^F)^{\beta_2^2}$ can be interpreted as a stochastic discount factor which is equal to the present value of \$1 received when the variable φ_2 hits φ_2^L (see Pawlina and Kort, 2006, p. 8). It is positive or negative depending on whether the switching option game is modelled as a second-mover market share advantage or as a first-mover market share advantage, respectively. In our case, it is negative because $D_{2_L 2_F} < D_{2_L 1_F}$ (see inequalities 4c and 4d), because tech 1 is a large-scale technology whereas tech 2 is a small-scale technology and, therefore, there is a market revenue share advantage for the follower when it operates with tech 1 and the leader operates with tech 2.

In this section, we show a sensitivity analysis for the effect of the changes in the model parameters on the investment thresholds of the leader and the follower and the option coefficients. Table 1 shows the base case parameter values:

Table 1: Base case model input values

Notation	***Definition***	***Value***
I_2	Switching investment cost to tech 2	6,000
r	Risk-free rate	0.05
μ_P	Output price drift	0.02
δ	Convenience yield	0.03
σ_P	Output price volatility	0.30
θ_1	Depletion rate while operating with tech 1	0.05
θ_2	Depletion rate while operating with tech 2	0.03
w_1	Periodic operating costs for tech 1	1600
w_2	Periodic operating costs for tech 2	300
Z_L	Leader's divestment value for tech 1	2,500
λ	% drop in divestment value for the follower	0.30
η	% drop in output production if switch to tech 2	0.30
	Competition Factors:	
D_{1F1L}	Follower's market share while active with the leader, both firms with tech 1	0.50
D_{1L1F}	Leader's market share while active with the follower, both firms with tech 1	0.50
D_{1F0L}	Follower's market share while active alone with tech 1	1.00
D_{1F2L}	Follower's market share while active with tech 1 and the leader is active with tech 2	0.65
D_{2L1F}	Leader's market share while active with tech 2 and the follower is active with tech 1	0.35
D_{2F2L}	Follower's market share while active with the leader, both firms with tech 2	0.50
D_{2L2F}	Leader's market share while active with the follower, both firms with tech 2	0.50

Note: In the figures of the sensitivity analyses, for the output price volatility (σ_P) and the output price drift (μ_P), we drop the subscript "P".

Table 2 shows the sensitivity the divestment and switching thresholds to changes in the model parameter values, respectively.

Table 2: this table shows a sensitivity analysis on the effect of changes in our model parameters on the leader's and the follower's option coefficients and switching thresholds for the separate formulation. In Panels A and B, third rows, it is shown the results for the base cases, for the "divest" and the "switch" cases, respectively, whereas in the remaining rows it is shown our results for the cases where each of the models' base case parameters are increased by 10%.

		Monopoly		*Leader*		*Follower*	
	Divest	B_{1D}^{M}	φ_{1D}^{M}	B_{1D}^{L}	φ_{1D}^{L}	B_{1D}^{F}	φ_{1D}^{F}
Panel A	Base case	0.000208	932.64	1066705	1865.28	0.000216	912.37
Notation	Increase by 10%						
r	0.055	0.000155	954.06	1308805	1908.11	0.000161	931.41
μ_P	0.022	0.000210	919.36	1122896	1838.72	0.000217	899.37
σ_P	0.33	0.000300	866.03	716373	1732.06	0.000309	847.20
θ_1	0.055	0.000206	964.33	942540	1928.66	0.000213	943.37
w_1	1,760	0.000183	1019.15	1219628	2038.29	0.000188	998.87
Z_M, Z_L	2,750	0.000206	939.40	1078401	1878.80	0.000214	917.10
λ	0.33	N/A	N/A	1066705	1865.28	0.000216	910.37
D_{1F1L}	0.55	N/A	N/A	1016057	1695.71	0.000216	912.37
	Switch	B_{2sw}^{M}	φ_{2sw}^{M}	B_{2sw}^{L}	φ_{2sw}^{L}	B_{2sw}^{F}	φ_{2sw}^{F}
Panel B	Base case	115134	1135.37	1623920	1487.93	592750	673.53
Notation	Increase by 10%						
I_2	6,600	110239	1105.09	733179	1117.62	274134	484.05
r	0.055	156808	1060.50	788166	1097.93	319140	513.09
μ_P	0.022	108258	1122.15	736888	1055.26	296519	482.02
σ_P	0.33	76771	1120.26	492299	1026.76	207651	464.76
θ_1	0.055	17270	1320.63	1706380	1605.22	619365	724.05
θ_2	0.033	158271	1060.61	1401699	1406.28	521350	637.48
w_1	1,760	142570	1296.84	1924570	1656.63	768508	791.63
w_2	330	110239	1105.09	1569697	1456.31	561741	651.39
Z_M, Z_L	2,750	117197	1147.98	1547787	1443.58	552816	644.93
η	0.33	156496	1025.83	1488675	1342.18	535722	602.46
λ	0.33	N/A	N/A	1540420	1371.60	569033	625.50
D_{1F2L}	0.72	N/A	N/A	1552322	1356.24	535679	570.12
D_{2F2L}	0.55	N/A	N/A	1677492	1536.59	565239	622.84

Panel A shows that, if we want to discourage divestment (lower thresholds) for the leader, the depletion rate, fixed operating costs, interest rate, and salvage value should be decreased, and the output price drift and the output price volatility should be increased. If we want to discourage divestment (lower thresholds) for the follower, the depletion rate, fixed operating costs, salvage value and interest rate should be decreased and the output drift, output volatility, and the discount in the salvage value should be increased. The effect on the real option value for the leader and the follower may not be however in the same direction as the effect on the investment thresholds. In Appendix A, we provide additional results on the sensitivity analysis.

Panel B shows that, if we want to encourage new small-scale investments (raise thresholds) for both the leader and the follower, the initial depletion rate and the initial fixed operating costs should be increased, and the salvage value, output price drift, subsequent depletion rate, downward jump in revenue, subsequent operating costs, new investment costs, interest rate, and output price volatility should be decreased. The effect on the real option value for the leader and the follower may not be in the same direction. Comparing the results for the monopolistic firm with those for the follower for the divestment, we conclude that both the thresholds and the option coefficients are very similar (the follower divest slightly later than the monopolistic firm), and the leader's threshold much earlier (higher threshold), as it is expected.

3. A Joint Duopoly Formulation: *à la Smets* (1993)

In this section, we extend the Décamps et al. (2006) model to a duopoly market, following the Smets (1993) framework. We assume that, ex-ante, two firms hold two mutually-exclusive options: the option to divest from a large-scale technology (tech 1) and the option to switch to a small-scale technology (tech 2). Thus, when exercising one option, a firm gives up the other option, so the value matching conditions are more complex than in the separate formation case. For a monopolist there is an inactive revenue region following Décamps et al. (2006) in which the firm should not exercise either the option to divest or the option to switch, but wait instead. In this case, the upper boundary represents a threshold which, if crossed from below, triggers the switching to tech 2, whereas the lower boundary represents a threshold which, if crossed from above, triggers the divestment. For convenience, we ignore the option to divest in tech 2.[7]

Figure 1 shows a timeline which illustrates the sequence of the investment thresholds for the leader and the follower, and Table 2 provides the model notations. Notice that, despite both the divestment in tech 1 and the switching to tech 2 involve divestment, the leader's and the follower's switching option values increase with φ_k because, for $\varphi_k \epsilon (\varphi_{1D}^L, \varphi_{2SW}^L)$ and $\varphi_k \epsilon (\varphi_{1D}^F, \varphi_{2SW}^F)$ respectively, an increase in φ_k is likely to trigger the switching to tech 2.

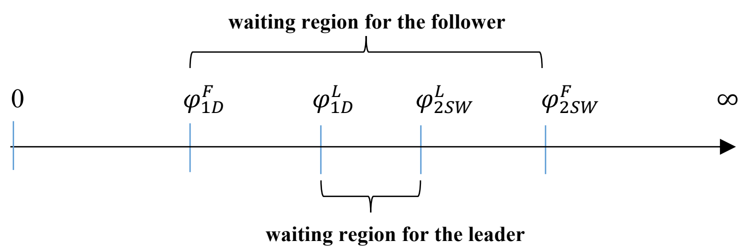

Figure 1 – Timeline: this figure illustrates the investment thresholds for the leader and the follower, where φ_{1D}^F and φ_{2SW}^F are the follower's thresholds to divest tech 1 and to switch to tech 2, respectively, and φ_{1D}^L and φ_{2SW}^L are the leader's thresholds to divest in tech 1 and to switch to tech 2, respectively.

[7] Figure A1 in the Appendix provides a comparative illustration of investment thresholds for a monopoly and a duopoly market for both a separate and a joint model setting.

Table 3: This tables shows the notation used in the derivations below.

Notation	*Definition*
$f_{JL}(\varphi_2)$	Joint formulation, leader's value function
$f_{JF}(\varphi_2)$	Joint formulation, follower's value function
φ_{1D}^F	Follower's threshold to divest tech 1
φ_{2SW}^F	Follower's threshold to switch to tech 2
φ_{1D}^L	Leader's threshold to divest tech 1
φ_{2SW}^L	Leader's threshold to switch to tech 2
	Joint formulation:
B_{1D}^F	Option coefficient follower's option to divest tech 1
B_{SW}^F	Option coefficient follower's option to switch to tech 2
B_{1D}^L	Option coefficient leader's option to divest tech 1
B_{SW}^L	Option coefficient leader's option to switch to tech 2
	Competition Factors:
$D_{1_F0_L}$	Market share of the follower when alone in the market operating with tech 1
$D_{1_F1_L}$	Market share of the follower when active with the leader, both firms with tech 1
$D_{1_F2_L}$	Market share of the follower when active with tech 1 and the leader is active with tech 2
$D_{2_F2_L}$	Follower's market share when active with the leader, both firms with tech 2
	Betas:
β_2^1<0	Beta of the coefficient of the option to divest tech 1
β_1^2>0	Beta of the coefficient of the option to switch to tech 2

3.1 The Follower

The value function is given by:

$$f^F(\varphi_{1,2}) = \begin{cases} Z_1^L(1-\lambda) & if \;\; \varphi_1 \leq \varphi_{1D}^F \\ \frac{\varphi_1 D_{1_F0_L}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^F\, \varphi_1{}^{\beta_2^1} + B_{SW}^F\, \varphi_2{}^{\beta_1^2} & if \;\; \varphi_1 \,\epsilon (\varphi_{1D}^F, \varphi_{1D}^L) \\ \frac{\varphi_1 D_{1_F1_L}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^F\, \varphi_1{}^{\beta_2^1} + B_{SW}^F \varphi_2{}^{\beta_1^2} & if \;\; \varphi_{1,2} \,\epsilon [\varphi_{1D}^L, \varphi_{2SW}^L) \\ \frac{\varphi_2 D_{1_F2_L}}{\delta+\theta_2} - \frac{w_1}{r} + B_{1D}^F\, \varphi_1{}^{\beta_2^1} + B_{SW}^F \varphi_2{}^{\beta_1^2} & if \;\; \varphi_2 \,\epsilon [\varphi_{2SW}^L, \varphi_{2SW}^F) \\ \frac{\varphi_2 D_{2_F2_L}}{\delta+\theta_2} - \frac{w_2}{r} - I_2 + Z_1^L(1-\lambda) & if \;\; \varphi_2 \geq \varphi_{2SW}^F \end{cases} \tag{30}$$

The economic interpretation of (30) is the following: the term in the first row represents the divestment value; the first two terms in the second row represent the present value of the revenue stream less the operating costs for the region where the follower operates with tech 1 alone, the third term is the option value to divest, and the fourth term is the option value to switch to tech 2; the first two terms in the third row represent the present value of the revenue stream less the operating costs for the region where both firms operate with tech 1; the first two terms of the fourth row represent the present value of the revenue stream less the operating costs for the region where the follower operates with tech 1 and the leader operates with tech 2; and the fifth row represents the present value of the revenue stream less both the operating costs and the investment cost for the region where both firms operate with tech 2 plus the divestment value.

For the follower, the lower and upper thresholds are obtained from the two respective value-matching relationships evaluated at the lower (φ_{1D}^{F}) and upper (φ_{SW}^{F}) thresholds, given by Equations (31) and (32), where the first equation represents the optimal conditions prevailing when the divestment option is exercised and the second when the technology switching option is exercised.

$$Z_1^L(1-\lambda) = \frac{\varphi_{1D}^F D_{1_F 0_L}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^F\, {\varphi_{1D}^F}^{\beta_2^1} + B_{sw}^F\, {\varphi_{1D}^F}^{\beta_1^2} \tag{31}$$

$$\frac{\varphi_{SW}^F D_{1_F 2_L}}{\delta+\theta_2} - \frac{w_1}{r} + B_{1D}^F\, {\varphi_{SW}^F}^{\beta_2^1} + B_{SW}^F {\varphi_{SW}^F}^{\beta_1^2} = \frac{\varphi_{SW}^F D_{2_F 2_L}}{\delta+\theta_2} - \frac{w_2}{r} - I_2 \tag{32}$$

From (31) and (32), we obtain B_{1D}^F and B_{SW}^F, respectively:

$$B_{1D}^F = \frac{\varphi_{1D}^F D_{1_F 0_L} - w_1(\delta+\theta_1)r - Z_1^L(1-\lambda)(\delta+\theta_1)r + B_{sw}^F\, {\varphi_{1D}^F}^{\beta_1^2}}{(\delta+\theta_1)r\, {\varphi_{1D}^F}^{\beta_2^1}} \tag{33}$$

$$B_{sw}^F = \frac{w_1(\delta+\theta_2) - r\left(D_{1_F 2_L} - D_{2_F 2_L}\right)\varphi_{sw}^F - (\delta+\theta_2)(w_2 - rI_2) - B_{1D}^F {\varphi_{sw}^F}^{\beta_2^1}}{r(\delta+\theta_2)\, {\varphi_{sw}^F}^{\beta_1^2}} \tag{34}$$

The first derivative of (31) with respect to φ_{1D}^F is given by (35), whereas the first derivative of (32) with respect to φ_{sw}^F is given by (36), and solutions to the two investment thresholds are obtainable from these two associated smooth-pasting conditions.

$$0 = \frac{D_{1_F 0_L}}{\delta+\theta_1} + \beta_2^1 B_{1D}^F\, {\varphi_{1D}^F}^{(\beta_2^1-1)} + \beta_1^2 B_{sw}^F\, {\varphi_{1D}^F}^{(\beta_1^2-1)} \tag{35}$$

$$\frac{D_{1_F2_L}}{\delta+\theta_2} + \beta_2^1 B_{1D}^F \, {\varphi_{SW}^F}^{(\beta_2^1-1)} + \beta_1^2 B_{SW}^F {\varphi_{SW}^F}^{(\beta_1^2-1)} - \frac{D_{2_F2_L}}{\delta+\theta_2} = 0 \qquad (36)$$

3.2 The Leader

The value function for the leader is given by:

$$f^L(\varphi_{1,2}) = \begin{cases} Z_1^L & if\ \ \varphi_1 \le \varphi_{1D}^L \\ \frac{\varphi_1 D_{1_L1_F}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^L \, {\varphi_1}^{\beta_2^1} + B_{SW}^L \, {\varphi_2}^{\beta_1^2} & if\ \varphi_1 \in (\varphi_{1D}^L, \varphi_{2SW}^L) \\ \frac{\varphi_2 D_{2_L1_F}}{\delta+\theta_2} - \frac{w_2}{r} - I_2 + Z_1^L + \frac{\varphi_2 (D_{2_L2_F} - D_{2_L1_F})}{\delta+\theta_2} \left(\frac{\varphi_2}{\varphi_{2SW}^F}\right)^{\beta_1^2} & if\ \varphi_2 \in [\varphi_{2SW}^L, \varphi_{2SW}^F) \\ \frac{\varphi_2 D_{2_L2_F}}{\delta+\theta_2} - \frac{w_2}{r} & if\ \ \varphi_2 \ge \varphi_{2SW}^F \end{cases} \qquad (37)$$

The economic interpretation of (37) is the following: the term in the first row represents the divestment value; the first two terms in the second row represent the present value of the revenue stream less the operating costs for the region where both firms operate with tech 1, the third term is the value of the option to divest, and the fourth term is the option value to switch to tech 2; the first three terms in the third row represent the present value of the revenue stream less both the operating costs and the net investment cost (less the tech 1 salvage value) for the region where the leader operates with tech 2 and the follower operates with tech 1, and the fourth term represents the gains in the leader's value due to the fact that the follower also switches to tech 2 at the moment φ_2 reaches φ_{2SW}^F;[8] the fourth row represents the revenue stream less the operating costs for the region where both firms operate with tech 2.

For the leader, the lower and the upper thresholds are obtained from the two respective value-matching relationships, given by Equations (38) and (39).

[8] Notice that the expression $\varphi_2(D_{2_L2_F} - D_{2_L1_F})/(\delta+\theta_2)(\varphi_2/\varphi_2^F)^{\beta_2^2}$ becomes $\varphi_2(D_{2_L2_F} - D_{2_L1_F})/(\delta+\theta_2)$ at the moment φ_2 reaches φ_{2SW}^F and this latter expression is positive because, according to Inequality (4c), $D_{2_L2_F} > D_{2_L1_F}$. The economic rationale underlying this inequality assumption is that tech 2 is a small-scale technology, therefore, by switching to tech 2 first, the leader faces a decline in its revenue market share.

$$Z_1^L = \frac{\varphi_{1D}^L D_{1_{L}1_{F}}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^L\, {\varphi_{1D}^L}^{\beta_2^1} + B_{SW}^L\, {\varphi_{1D}^L}^{\beta_1^2} \tag{38}$$

$$\frac{\varphi_{SW}^L D_{1_{L}1_{F}}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^L\, {\varphi_{SW}^L}^{\beta_2^1} + B_{SW}^L\, {\varphi_{SW}^L}^{\beta_1^2} = \frac{\varphi_{SW}^L D_{2_{L}1_{F}}}{\delta+\theta_2} - \frac{w_2}{r} + Z_1^L - I_2 + \frac{\varphi_{SW}^L (D_{2_{L}1_{F}} - D_{2_{L}2_{F}})}{\delta+\theta_2}\left(\frac{\varphi_{SW}^L}{\varphi_{SW}^F}\right)^{\beta_1^2} \tag{39}$$

From (38) and (39), we obtain B_{1D}^L and B_{SW}^L, respectively:

$$B_{1D}^L = \frac{r(\delta+\theta_1)Z_1^L - rD_{1_{L}1_{F}}\varphi_{1D}^L + w_1(\delta+\theta_1) - (\delta+\theta_1)rB_{SW}^L\, {\varphi_{1D}^L}^{\beta_1^2}}{r(\delta+\theta_1)\, {\varphi_{1D}^L}^{\beta_2^1}} \tag{40}$$

$$B_{SW}^L = \frac{\varphi_{SW}^L D_{2_{L}1_{F}} - (\delta+\theta_2)w_2 - (\delta+\theta_2)r(I_2 - Z_1^L) + \mathrm{r}\varphi_{SW}^L\left(D_{2_{L}1_{F}} - D_{2_{L}2_{F}}\right)\left(\frac{\varphi_{SW}^L}{\varphi_{SW}^F}\right)^{\beta_1^2} - \mathrm{r}D_{1_{L}1_{F}}\varphi_{SW}^L - (\delta+\theta_1)w_1}{r(\delta+\theta_1)\, {\varphi_{1D}^L}^{\beta_2^1}} \tag{41}$$

The first derivative of (38) with respect to φ_{1D}^L is given by (42), whereas the first derivative of (39) with respect to φ_{sw}^L is given by (43). Solutions to the two investment thresholds are obtainable from these two associated smooth-pasting conditions.

$$\frac{\varphi_{1D}^L}{\delta+\theta_1} + \beta_2^1 B_{1D}^L\, {\varphi_{1D}^L}^{(\beta_2^1-1)} + \beta_1^2 B_{SW}^L\, {\varphi_{1D}^L}^{(\beta_1^2-1)} = 0 \tag{42}$$

$$\frac{\varphi_{SW}^L}{\delta+\theta_1} + \beta_2^1 B_{1D}^L\, {\varphi_{1D}^L}^{(\beta_2^1-1)} + \beta_1^2 B_{SW}^L\, {\varphi_{1D}^L}^{(\beta_1^2-1)} - \frac{1}{\delta+\theta_2} - \frac{(\beta_1^2+1)(D_{2_{L}1_{F}} - D_{2_{L}2_{F}})}{(\delta+\theta_2)}\left(\frac{\varphi_{SW}^L}{\varphi_{SW}^F}\right)^{\beta_1^2} = 0 \tag{43}$$

3.3. Results

To be completed….

4. Conclusions

Regarding the switching to a small-scale technology, our model setting assumes that at the moment the firm switches from one technology to another, there is an output product production (revenue) drop, affecting its market share, a decrease in its fixed operating costs, and a salvage value advantage. The firm which switches first (the leader) has to balance between the gains (lower fixed operating cost, a higher salvage value, a lower market revenue share). Our findings show that in (a first-mover salvage value advantage) duopoly competition, the leader divests much earlier than it would be the case if it was in a monopoly market, and the follower divest more or less at the same time a monopolist would do (slightly later than a monopolist). However, regarding the switching to a small-scale technology, the above behaviours do not hold: a monopolist switches much later than the leader does (as expected) but much earlier than the follower. In a duopoly market, the follower's behaviour tends to mimic that of a monopolist because, after the leader has invested, it is in a monopoly-like.

The reason why the follower, for the switching scenario, does not behave like a monopolist might be because the leader switches to a small-scale technology and so there is a drop in the leader's market share (50% to 35% in our base case), so the follower gets an additional (50% to 65%) while alone operating with tech 1, so it is an incentive the delay the switch to the new small-scale technology (after which its market share drops to 50%). This behaviour of the follower happens despite the existence of a salvage value disadvantage (the follower's divestment value is lower than that of the leader), which means that the additional market share that the follower gets while operating alone with the large-scale technology more than offsets the salvage value disadvantage. The follower's market share advantage, while operating alone with the large-scale technology, plays an important role in the leader's and the follower's behaviour regarding the timing of the switch: an increase delays the technology switch for the two firms.

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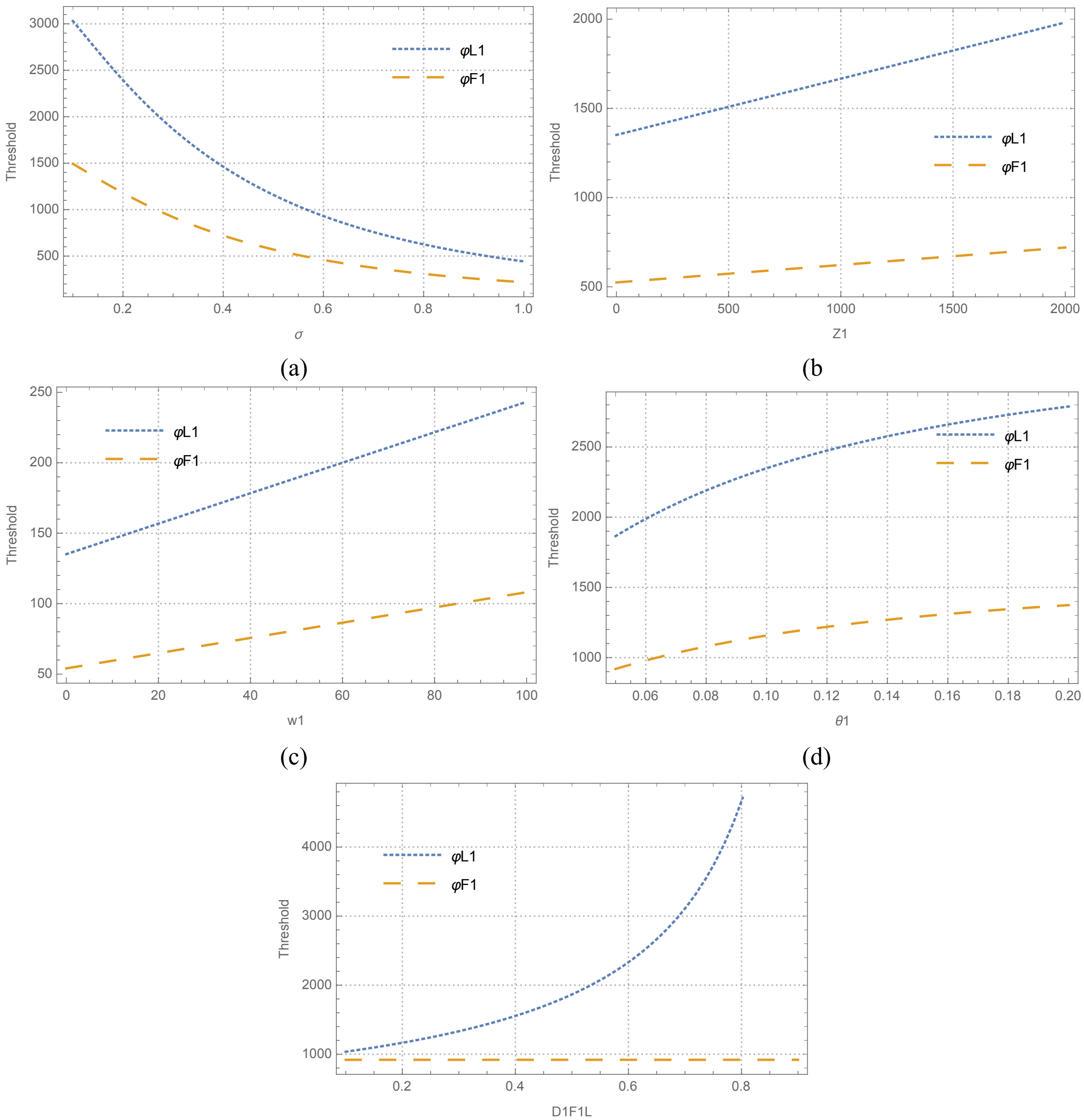
Appendix A Separate Formulation

Figure A1: this figure shows a sensitivity analysis for the effect of changes in the parameter values on the firms' divestment thresholds, assuming that the option to switch to tech 2 is not available.



(a) (b) (c) (d) (e)

The leader's threshold is more sensitive to changes in the model parameters, in particular to changes in the ex-ante follower's market share and output price uncertainty. The lower sensitivity of the follower to changes in the model parameters is because it divests after the leader, therefore, once the leader has divested it operates alone in the market and has a monopoly-like behaviour. Notice that, the divestment behaviour of the follower does not depend on the its market share, while active with the leader, because, after the divestment of the leader, it gets 100% of the market (1e). The higher the ex-ante market share of the follower, the sooner the leader divests. The thresholds of two firms increase linearly with the divestment value and with the operating costs (1b and 1c). Also relevant is the effect of output quantity rate of decrease which speeds up the investment of the two firms if it increases (1d).

Appendix B

1. A Monopoly Model

In this section we derive a comparable monopoly real options model for both the divesting and the switching cases.

1.1 Divest

The value-matching and smooth-pasting conditions are given by Equations (A1) and (A2), respectively:

$$\frac{\varphi_{1D}^{M}}{\delta+\theta_1} - \frac{w_1}{r} + B_{1D}^{M}\, {\varphi_{1D}^{M}}^{\beta_2^1} = Z_M \tag{A3}$$

$$\frac{1}{\delta+\theta_1} + \beta_2^1 B_{1D}^{M}\, {\varphi_{1D}^{M}}^{(\beta_2^1-1)} = 0 \tag{A4}$$

Using Equations (A3) and (A4), we determine the constant B_{1D}^{M} and the divestment threshold φ_{1D}^{M}:

$$B_{1D}^{M} = \frac{-1}{(\delta+\theta_1)\beta_2^1 {\varphi_{1D}^{M}}^{(\beta_2^1-1)}} \tag{A5}$$

$$\varphi_{1D}^{M} = \frac{(\frac{w_1}{r}+Z_M)(\delta+\theta_1)\beta_2^1}{\beta_2^1-1} \tag{A6}$$

1.2 Switch

In this section, in order to make easier the comparison of our monopoly results with those for the duopoly, we depart from the derivations of Adkins and Paxson by neglecting the option value to divest in tech 2. Therefore, using the adequate value-matching and smooth-pasting conditions, we obtain the following option coefficient and investment threshold:

The value-matching and smooth-pasting conditions are given by Equations (A7) and (A8), respectively.

$$\frac{\varphi_{2sw}^{M}/(1-\eta)}{\delta+\theta_1} - \frac{w_1}{r} + B_{2sw}^{M}\, {\varphi_{2sw}^{M}}^{\beta_2^2} = \frac{\varphi_{2sw}^{M}}{\delta+\theta_2} - \frac{w_2}{r} - I_2 + Z_M \tag{A7}$$

$$\frac{1}{(1-\eta)(\delta+\theta_1)} + \beta_2^2 B_2^F\, {\varphi_2^F}^{(\beta_2^2-1)} - \frac{1}{\delta+\theta_2} = 0 \tag{A8}$$

Using Equations (A7) and (A8), we determine the constant B_{2sw}^{M} and the switching threshold φ_{2sw}^{M}:

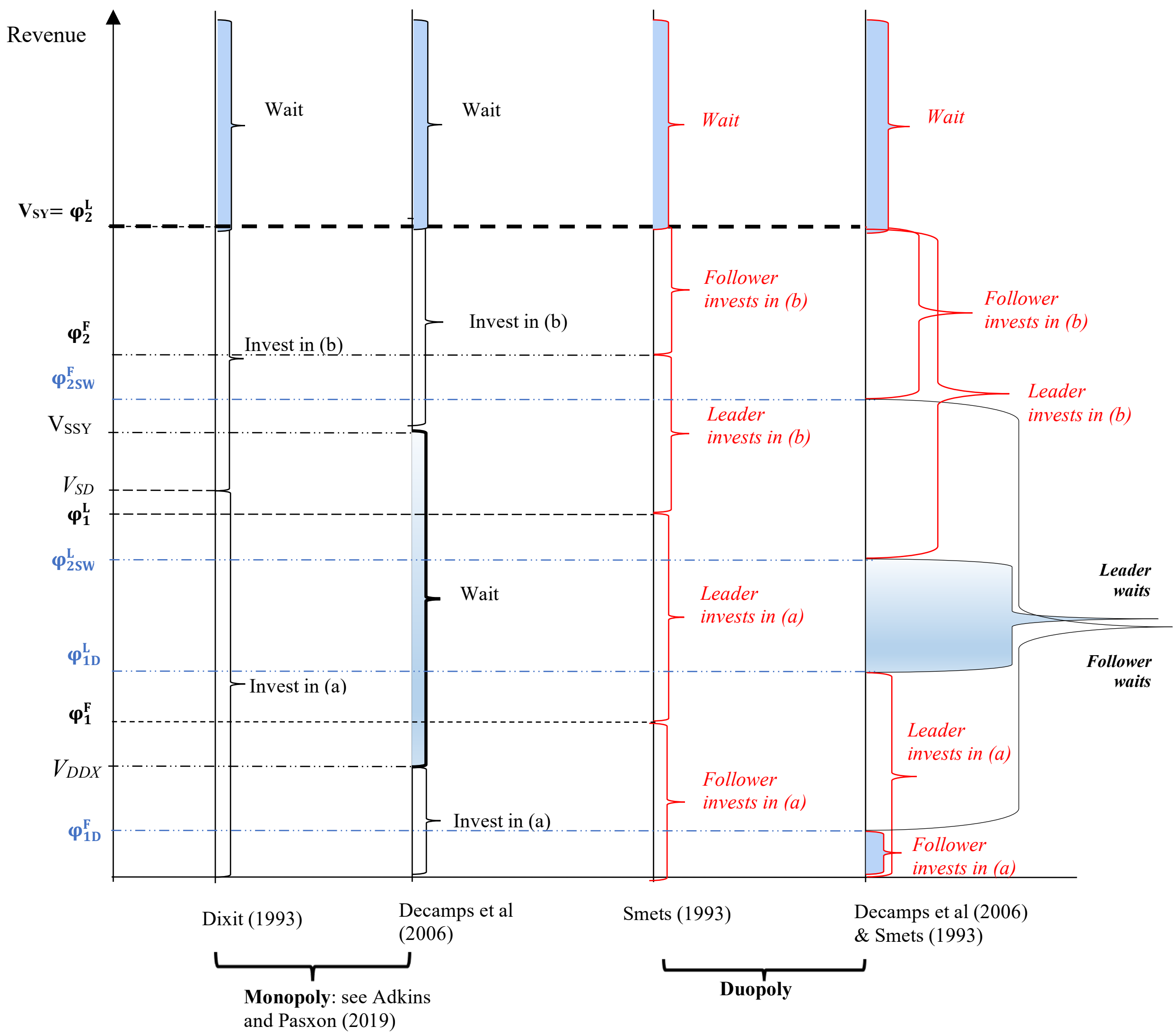
$$B_{2sw}^{M} = \frac{(1-\eta)(\delta+\theta_1)-(\delta+\theta_2)}{\beta_2^2 {\varphi_{2sw}^{M}}^{(\beta_2^2-1)}(1-\eta)(\delta+\theta_1)(\delta+\theta_2)} \tag{A9}$$

$$\varphi_{2sw}^{M} = \frac{[w_1-w_2-(I_2-Z_M)r](1-\eta)(\delta+\theta_1)(\delta+\theta_2)\beta_2^2}{r\left[(1-\eta)(\delta+\theta_1)-(\delta+\theta_2)+(\delta+\theta_2)\beta_2^2\right]} \tag{A10}$$

Appendix C

(To be updated…)

Figure C2: this figure illustrates the relationship amongst the investment thresholds for a monopoly and a leader-follower duopoly market considering two alternative investment policies: (a) divesting the associated assets and terminating oil extraction, and (b) divesting the associated assets but switching to a small-scale extraction technology, considering both a Separate (Dixit, 1993 & Smets, 1993) and a Joint (Decamps et al., 2006 & Smets (1993)) formulation.



Note 1:

a: follower's investment threshold in policy (a) for the joint formulation
b: follower's investment threshold in policy (a) for the separate formulation
c: leader's investment threshold in policy (a) for the joint formulation
d: follower's investment threshold in policy (b) for the joint formulation
e: follower's investment threshold in policy (b) for the separate formulation

Note 2: in the inactive region the follower has a much larger waiting region than the leader because it waits longer to divest and waits longer to switch to the small-scale technology.