Effects of Creative Destruction on the Size and Timing of an Investment

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Abstract

In the current innovation-driven economy project lives are short due to the economic phenomenon *creative destruction*. This paper investigates its implications for optimal firm investments. We find that if the firm is a monopolist, a reduced length of the project life does not affect the size of the investment but the firm waits longer for better market conditions before it invests. If, in addition, the option to invest could also expire in finite time, the firm invests earlier and less. Besides a monopoly setting we also investigate a duopoly. An entry deterring incumbent invests in the same way as the monopolist except that the incumbent will invest earlier in a scenario where the investment option never expires.

When, initially, firms are both potential entrants, the project life being finite reduces the incentive to preempt the investment of the opponent. Finally, we show that considerable value losses will be achieved when the project life being finite is mistakenly not taken into account in taking the investment decision. This value loss is enlarged by the preemption effect just mentioned.

Keywords: finite project life, capacity choice, real options, duopoly

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1 Introduction

In today's innovation driven economy (McGrath (2019), Geelen et al. (2019)) creative destruction considerably reduces product lives. A prominent example is the mobile phone industry where the innovative success of Apple and Samsung caused the end of the economic lifetime of Nokia phones. For this reason it is more and more important to take account of the finiteness of product lifetime in capital investment decisions of firms. The present article focuses on this topic within a monopoly as well as a duopoly framework. In particular, we consider a scenario where the occurrence of one event can end market activities related to the current product. An obvious example is thus the arrival of a drastic innovation that makes selling this product obsolete.

The article considers capital investment opportunities of firms, where investing implies that the particular firm acquires a production plant. The firms have to decide when and how much to invest. Taking into account the uncertain economic environment, we deal with a real option problem. The real options literature took off with the seminal works of Dixit and Pindyck (1994) and Trigeorgis (1996), mainly considering the optimal timing of investment where project life is infinite. Contributions where also the investment size is determined are, e.g., Capozza and Li (1994), Dangl (1999), and Bar-Ilan and Strange (1999). Bensoussan and Chevalier-Roignant (2019) allows for sequential capacity decisions. A finite project life, but then for investment projects with exogenously given size, is considered in Dixit and Pindyck (1994, Chapter 6), Gryglewicz et al. (2008), and Wang et al. (2019). The latter paper focuses on the effects of extreme climate events.

Competition is of prime importance for the profitability of today's investment projects. For investment projects of exogenously given size, frameworks including competition have elaborately been taken into account within the theory of real options, as emphasized in survey contributions by Grenadier (2000), Chevalier-Roignant et al. (2011), and Azevedo and Paxson (2014). Hellmann and Thijssen (2018) extend this literature by allowing for ambiguity. Competition is for sure an important element when determining the size of an investment project. Recently, a research line started in which competing firms have to simultaneously determine timing and size of their investments (see Huisman and Kort (2015) and also the survey article by Huberts et al. (2015)).[1] The present paper belongs to this strand of research, where our extension lies in the consideration of finite time projects. To our knowledge, the combination of competing firms that can decide about their investment size and finite time projects has not been studied yet within one model.

A paper that comes close to our work is Chevalier-Roignant et al. (2019). That paper focuses

[1] These works form a theoretical basis to underpin the empirical analysis of entry deterrence in the casino industry by Cookson (2018).

on an oligopolistic industry, where firms develop product innovations that arrive according to a Poisson process with an endogenously determined Poisson parameter. This so-called development stage is not modeled by us. Instead, we let the firms decide about the timing and the size of the investment, which is not considered in Chevalier-Roignant et al. (2019).

Kouvelis and Tian (2014) introduce a three-stage model of a single firm that initially decides about a flexible capacity level, then at a later point in time decides on how much capacity to allocate to the production process of two different products, and finally decisions are taken about quantities and prices. The timing of the different decisions is pre-determined, where Kouvelis and Tian (2014) focus on demand forecast and product mix. The latter is not considered by us. Instead, we develop the optimal investment times, take into account that project life is finite, and consider strategic interactions between firms.

We study a framework where future demand is subject to stochastic shocks, which admit a geometric Brownian motion process. Firms are not active initially but have an opportunity to enter the market. They have to determine the optimal time to do so and at the same time decide about the size of the production capacity. The product market is considered to be homogenous and firms produce up to capacity. The characteristics we focus on are that, first, the project has a finite life. At some unknown point in the future an event takes place due to which the firms have to stop selling the product. Such an event could be, for instance, the launch of an innovative product by an outside firm that destroys demand of the current product. The moment this occurs is uncertain and cannot be influenced by the firms under consideration. We assume that its timing satisfies a Poisson process. Second, we focus on the fact that also an option to invest can have a finite life. This option expires at a point in time determined by the same Poisson process. This can be motivated by the fact that in our example the launch of an innovative product does not only end the life of the project after market entry, but also before the investment takes place it destroys the value of the option to invest in a plant producing that particular product.

We start off with a monopoly framework in which the project has a finite life whereas the investment option exists forever as long as the firm has not exercised it. We find that the optimal size of the investment is not affected by the probability that the project will end in finite time. However, at the same time the investment threshold goes up, implying that it is optimal for the firm to invest later. If, in addition, the life of the option to invest is also finite, this will lead to an earlier investment. This is because as long as the firm does not invest, it runs the risk of not being able to invest at all due to the possibility that the option will vanish. The firm speeding up investment implies that it will invest at the moment that the output price is smaller. For this reason the firm decides to invest less when the option could expire in finite time.

When we turn to a duopoly framework, we first distinguish between the first investor (or leader), who becomes the incumbent upon investing, and the second investor (or follower), who becomes the potential entrant once the other firm has invested. We first again consider the scenario where the investment option is infinite for the incumbent. For the potential entrant we impose that, as long as it has not invested yet while the incumbent has already done so, the life of its investment option ends at the moment that the project of the incumbent comes to an end. The motivation is that when this event takes place due to which selling this product stops, also the option to invest becomes worthless. Second, we analyze the situation in which also for the incumbent the investment option life is finite. Hence, we impose that when the unforeseen event takes place, for both firms this implies that the life of the project as well as the investment option life comes to an end.

When analyzing such a situation, we get that, when the investment option life is infinite, the project life being finite induces the incumbent to invest the same amount but later compared to a situation with the project life being infinite. In addition, the leader invests earlier than the monopolist without entry threat, facing a project with the same length.

As soon as the entrant will invest, the first investor is already in and has taken its decisions, namely when to invest and how much. Therefore, the entrant in fact faces the same decision as the monopolist in the case of a finite project and option life, while taking into account that the market size has reduced because the incumbent has already taken part of it. So in such a situation, as for the monopolist it also holds for the entrant that when the probability that the project and the option life stops increases, the entrant will invest earlier and less.

If, in addition, also the investment option for the incumbent will expire with the same probability as that the project will stop, the incumbent reacts by investing earlier and less. As in the monopoly case without entry threat, the incumbent wants to preempt the event that the investment option will expire. For the entrant, the fact that the incumbent invests less implies that for a given quantity the output price is larger, which makes investing more attractive. As a result, the entrant will invest earlier and more compared to the situation where the incumbent's option life had infinite length.

If in the duopoly investment game firm roles are endogenous, both firms are entitled to invest first. We know already from Fudenberg and Tirole (1985) that there will be a preemption equilibrium with an early investment of the first investor. Since the investment payoff is lower if the project life is shorter, firms are less inclined to become the first investor. Therefore, a finite project life mitigates the preemption effect. Finally, we check what will happen if, in taking their investment decisions, firms make the mistake to ignore that project lives are finite. We show that the reduction in value is huge, and even loss-making strategies can be expected in case of sufficiently short project lives. This value loss will be considerably enlarged by the preemption effect just mentioned.

The paper is organized as follows. Section 2 presents the model, whereas the monopoly problem is studied in Section 3. In Section 4 we extend the analysis towards a duopoly. The duopoly investment game with endogenous firm roles, and the value losses resulting from mistakenly not taking into account that the project life is finite, are analyzed in Section 5. Section 6 concludes this paper.

2 Model

Consider a homogenous product market where the output price is given by

$$P_t = X_t f(Q_t), \tag{1}$$

in which Q_t is the market quantity and $f'(Q_t) < 0$. Future demand is uncertain, which is modeled by letting X_t follow the geometric Brownian motion process

$$dX_t = \mu X_t dt + \sigma X_t dz_t, \tag{2}$$

where μ is a parameter reflecting the trend, σ is the uncertainty parameter, and dz_t is the increment of a Wiener process.

There are two firms having an option to invest in production capacity denoted by K_L for the first investor and K_F for the second investor. The subscript L stands for the first investor being the leader in the investment game, and the subscript F indicates that the second investor is the follower. Both K_L and K_F are endogenous, i.e. both the leader and the follower can decide on their investment size.

For both firms the investment cost is sunk and proportional to the acquired capacity. If the capacity size is K, investment costs are equal to δK, with δ being the unit investment cost. The firms are risk neutral and discount with a fixed rate r. As usual (see, e.g., Dixit and Pindyck (1994)), we impose that $r > \mu$.

For both firms it holds that their project has a finite life. To do so we introduce a parameter $\lambda > 0$, and assume that during the next time instant dt the probability that the project stops is λdt. This is different from, e.g., Huisman and Kort (2015) in which investment project lives are infinite.

Concerning the option to invest we distinguish between scenarios where the investment option exists forever and where for both firms the investment option ceases to exist over the next time instant dt with probability λdt. This is the same parameter λ as the one that governs the project life. The idea is that with probability λdt some exogenous event happens that makes the current market obsolete. Here we can think of a product innovation resulting in a new product that takes all demand away from the current product. This ends the project life when the firm has already invested, and makes the option to invest in this project worthless in case the firm has not invested yet. Following this interpretation the parameter λ can be interpreted as the "creative destruction parameter".

We assume that firms always produce up to capacity, i.e. the market quantity Q_t is equal to the total available capacity acquired by the firms that have invested by time t. This simplifies the analysis considerably, especially in a duopoly framework. On the other hand, based on the analysis developed in, e.g., Hagspiel et al. (2016), we expect it will not change the qualitative aspects of our results when we relax this constraint. In reality it both can happen that firms always produce up to capacity, due to e.g. fixed costs associated to labor, commitment to suppliers, and production ramp-up (Goyal and Netessine (2007)), or that they can leave some capacity idle in case of a downturn. A typical example where firms produce up to capacity is the semiconductor industry. When demand falls, a semiconductor fabrication plant keeps producing at its maximum. Running costs are low, because these plants are highly automated with few staff. Therefore, a policy of producing up to capacity makes sense but also results in decreasing prices. This happened through most of the year 2019[2].

3 Monopoly

Let us first consider the scenario where the project life is finite and the option to invest always remains existent as long as the firm has not invested yet. The firm has to determine the optimal time to invest and the optimal size of the investment. In other words, after denoting the value of the firm by V, a profit-maximizing firm faces the following maximization problem:

$$V(X) = \max_{T\geq 0, K\geq 0} E\left(\int_T^{\infty} e^{-(r+\lambda)t} X_t f(K) K dt - e^{-rT}\delta K \mid X(0) = X\right), \tag{3}$$

in which T is the time of the investment, and K is the capacity level that the firm acquires at time T. The expectation sign is there because future cash flows are uncertain. This is due to the fact that the output price $X_t f(K)$ depends on the geometric Brownian motion process X_t, and that with probability λdt the project will stop during the next time instant dt.

As explained in, e.g., Dixit and Pindyck (1994), we treat this problem as an optimal stopping problem. To do so we distinguish between a continuation region, where X, and thus also the output price $Xf(K)$, is too low for an investment to be optimal, and a stopping region where X is large enough for the firm to invest. In between the two regions we have the boundary X^* being the threshold value triggering investment. At the moment of the investment the firm has to determine the investment level K^*. To determine X^* and K^* we use the following conditions.

[2] The Economist, Jan 16th 2020 edition.

First, for a given X we maximize the stopping value with respect to K. Then, to determine X^* we value match and smooth paste the value functions of the stopping and the continuation region at the "free boundary" X^*. Below we give expressions for the value functions in both regions. For a more elaborate mathematical derivation we refer to the proof of Proposition 1 presented below.

Consider first the continuation region and denote by $F(X)$ the value of the option to invest. Application of Ito's lemma gives

$$E(dF(X)) = \left(\mu X F'(X) + \frac{1}{2}\sigma^2 X^2 F''(X)\right) dt. \tag{4}$$

Combining this with the Bellman equation results in the following differential equation that $F(X)$ has to satisfy:

$$rF(X) = \frac{1}{2}\sigma^2 X^2 F''(X) + \mu X F'(X). \tag{5}$$

Solving the differential equation, while taking into account that $F(0) = 0$, gives

$$F(X) = AX^{\beta}, \tag{6}$$

in which A is an unknown constant and β is the positive root (larger than one) of the quadratic equation

$$\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - r = 0. \tag{7}$$

In the stopping region the value of the firm $V(X)$ satisfies the following differential equation:

$$rV(X) = Kf(K)X + \frac{1}{2}\sigma^2 V''(X) + \mu V'(X) - \lambda V(X), \tag{8}$$

where the last term on the right-hand side results from the fact that with probability λdt the project ends and thus its value jumps down from $V(X)$ to zero. Taking into account that $V(0) = 0$ and that we abstract away from speculative bubbles, solving the differential equation gives

$$V(X) = \frac{Kf(K)X}{r + \lambda - \mu} - \delta K. \tag{9}$$

So, it is clearly seen that the larger the probability the project ends, the lower the value of the firm after investment, as reflected by the presence of λ in the denominator.

The optimal investment decision of the monopolist is presented in the next proposition. The proofs of all propositions can be found in the appendix.

Proposition 1. *The optimal investment threshold equals*

$$X^*(K) = \frac{\beta}{\beta - 1} \frac{\delta\left(r + \lambda - \mu\right)}{f(K)}, \tag{10}$$

whereas the investment size implicitly satisfies

$$\beta \frac{K^* f'(K^*)}{f(K^*)} + 1 = 0. \tag{11}$$

An important conclusion we can draw from the expression for the investment size (11) is that capacity size K^* does not depend on λ. Hence, this holds despite of the fact that, as we have obtained from (9), the project value negatively depends on λ. Still, the creative destruction parameter λ has an effect on the investment decision, because expression (10) learns that the investment threshold goes up with λ, which implies that the monopolist will invest later, provided that the initial level of the process X, which is X_0, falls below the threshold level $X^*(K)$. This means that the reduced project value causes that the firm invests later in the same capacity size.

An explicit expression of the capacity size can be obtained once we specify $f(Q_t)$. In the duopoly model that we analyze in the next section, we take inverse demand to be linear in the quantity:

$$P_t = X_t\left(1 - \eta Q_t\right), \tag{12}$$

which, due to the fact that the firm produces up to capacity, implies that after investment of the monopolist the output price is given by

$$P_t = X_t\left(1 - \eta K\right). \tag{13}$$

Using the result of Proposition 1, we obtain that

$$X^*(K) = \frac{\beta}{\beta - 1} \frac{\delta\left(r + \lambda - \mu\right)}{1 - \eta K}, \tag{14}$$

and

$$K^* = \frac{1}{\eta\left(\beta + 1\right)}. \tag{15}$$

This straightforwardly leads to the result specified in the next corollary.

Corollary 1. *If the inverse demand function is given by (12), the optimal investment threshold satisfies*

$$X^* = \frac{\beta + 1}{\beta - 1} \delta\left(r + \lambda - \mu\right), \tag{16}$$

and the corresponding capacity level is given by

$$K^* = \frac{1}{\eta\left(\beta+1\right)}. \tag{17}$$

3.1 Finite life of the option to invest

We now consider the setting that the "creative destruction" parameter λ not only relates to the project life, but also to the investment option. The idea is that with probability λdt an outside product innovation makes the current product obsolete. This ends not only the project life when the firm has already invested, but also makes the option to invest in this project worthless in case the firm has not invested yet. To analyze this problem we again distinguish between the stopping and the continuation region. In the stopping region nothing has changed so that the project value is still represented by expression (9). However, in the continuation region we have to take into account that the investment option can become worthless with probability λdt. To that end expression (4) has to be replaced by

$$E\left(dF\left(X\right)\right) = \left(\mu X F'\left(X\right) + \frac{1}{2}\sigma^2 X^2 F''\left(X\right) - \lambda F\left(X\right)\right) dt. \tag{18}$$

Inserting this in the Bellman equation results in the differential equation

$$\left(r+\lambda\right) F\left(X\right) = \frac{1}{2}\sigma^2 X^2 F''\left(X\right) + \mu X F'\left(X\right), \tag{19}$$

and the solution

$$F\left(X\right) = AX^{\beta_\lambda}, \tag{20}$$

where β_λ is the positive root of the quadratic equation

$$\frac{1}{2}\sigma^2\beta_\lambda^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta_\lambda - \left(r+\lambda\right) = 0. \tag{21}$$

Analogous to Proposition 1 we get the following result.

Proposition 2. *The optimal investment threshold satisfies*

$$X^*\left(K\right) = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{\delta\left(r+\lambda-\mu\right)}{f\left(K\right)}, \tag{22}$$

and the corresponding capacity level is given by

$$\beta_\lambda \frac{K^* f'\left(K^*\right)}{f\left(K^*\right)} + 1 = 0. \tag{23}$$

The difference with the optimal investment size for the infinite option case, as given by (11), is that from (23) we can conclude that now the creative destruction parameter λ does influence the size of the investment.

For the linear inverse demand case this translates into the result presented in the following corollary.

Corollary 2. *If the inverse demand function is given by (12), the optimal investment threshold satisfies*

$$X^* = \frac{\beta_\lambda + 1}{\beta_\lambda - 1}\delta\left(r + \lambda - \mu\right), \tag{24}$$

and the corresponding capacity level is given by

$$K^* = \frac{1}{\eta\left(\beta_\lambda + 1\right)}. \tag{25}$$

From the two quadratic polynomials (7) and (21) it is obtained that

$$\beta_\lambda > \beta. \tag{26}$$

Comparing, for the linear inverse demand function, the investment decision for the finite time option, (24)-(25), with the one where the option life is infinite, (16)-(17), we conclude that for the finite time option the investment threshold is lower, indicating an earlier investment, and that the size of the investment is smaller. The reason is that the monopolist wants to preempt the event that the option to invest is not available anymore. Investing earlier implies that at the moment of the investment the output price is lower, given the capacity size. Therefore, the firm will spend less on investing so that it acquires a smaller capacity.

4 Duopoly

We consider a scenario with two firms competing for a market share. Both have to decide on when to invest, i.e. to enter the market, and on how much to invest, which determines the size of the production capacity. Throughout this section we impose that the inverse demand function is the one of expression (12).

One firm is assigned to be the leader, which has the right to invest first. The other firm is the follower, which has the choice to invest at the same time as the leader or to invest later. In a situation where the leader has invested and the follower still waits, we in fact have an incumbent-entrant situation. For this reason we sometimes call the leader the incumbent and the follower the (potential) entrant.

We first treat the scenario where the project life is finite but the investment option for the leader is infinite, followed by an analysis of the case where also the investment option has a finite life.

4.1 Infinite life of the investment option for leader and finite for follower

We consider a situation in which the product market has a finite life from the moment the leader has invested. This situation could arise when the launch of the current product triggers outside firms to start developing a new project that makes the current product obsolete. The probability that these firms obtain a breakthrough in their innovation process is assumed to be equal to λdt. The implication is that the lifetime of the option to invest is infinite for the leader, but after the leader has invested, due to creative destruction the project will end with probability λdt. For the follower this implies that once the leader has invested, the lifetime of the follower's investment option becomes finite. This is because, once the outside firms accomplish the breakthrough that makes the current product obsolete, the opportunity to invest in the current product market also becomes worthless.

We first consider the follower's decision in a situation that the leader already has invested and acquired a capacity level K_L. As we just stated, then both the follower's project and investment option life is finite. After determining the follower's optimal investment decision we turn to the leader's problem. We derive the leader's optimal investment decision, taking into account how the follower will react. The next proposition specifies the optimal investment decision of the follower.

Proposition 3. *Given the capacity level K_L of the leader, the optimal investment threshold of the follower is given by*

$$X_F^*(K_L) = \frac{\beta_\lambda + 1}{\beta_\lambda - 1}\frac{\delta\left(r + \lambda - \mu\right)}{1 - \eta K_L}, \tag{27}$$

whereas the capacity level equals

$$K_F^*(K_L) = \frac{1 - \eta K_L}{\eta\left(\beta_\lambda + 1\right)}. \tag{28}$$

The decision of the follower is qualitatively similar as the one of the monopolist when the option life is finite. The only change is that due to the investment of the leader, the reservation price has reduced by a factor ηK_L. This explains that, apart from this factor ηK_L, the investment decision is the same as the monopolist's investment decision expressed in (24)-(25). For the leader the follower's investment decision (27)-(28) provides important information in the sense that increasing its capacity K_L not only reduces the follower's capacity but also let the follower invest later, so that the leader enjoys a longer period during which it is the monopolist in the market. Hence, as already concluded by Huisman and Kort (2015), the leader has two reasons to overinvest.

Turning to the leader's problem we first have to remark that, based on the fact that the follower can invest at the same time or later than the leader, we have to distinguish between an *entry deterrence* and an *entry accommodation* strategy. With an entry deterrence strategy the leader invests so much that it will generate a monopoly period for itself due to the fact that the follower will invest later. From (27) we obtain that the acquired capacity level should then be such that

$$K_L > \hat{K}(X) = \frac{1}{\eta}\left(1 - \frac{\beta_\lambda + 1}{\beta_\lambda - 1}\frac{\delta\left(r + \lambda - \mu\right)}{X}\right). \tag{29}$$

If the capacity size does not satisfy this constraint, the follower will invest at the same time as the leader and then we are in the entry accommodation scenario.

Let us first consider the *entry deterrence* strategy. The value of the leader in the continuation region is qualitatively similar to the one of the monopoly in (6). In the stopping region the leader value is given by

$$V_L^{\text{det}}(X, K_L) = \frac{K_L\left(1 - \eta K_L\right)}{r + \lambda - \mu}X - \delta K_L - \left(\frac{X}{X_F^*(K_L)}\right)^{\beta_\lambda}\frac{K_L \eta K_F^*(K_L)}{r + \lambda - \mu}X_F^*(K_L). \tag{30}$$

The first two terms represent the leader value if the leader were a monopolist until the end of the project, and is therefore similar to the monopolist expression (9). The last term is a negative correction for the fact that the follower will enter as soon as X reaches the follower threshold X_F^*. Then the output price will be reduced by an amount $\eta K_F^* X_F^*$, leading to an instantaneous revenue reduction of $K_L \eta K_F^* X_F^*$. This negative correction needs to be properly discounted, because it follows from the entry of the follower taking place at a later point in time. In fact, this discounting is achieved by the term $\left(\frac{X}{X_F^*}\right)^{\beta_\lambda}$ denoting the stochastic discount factor, that is, it holds that $\left(\frac{X}{X_F^*}\right)^{\beta_\lambda} = E\left(e^{-(r+\lambda)(t-T_F)}\right)$, where T_F is the (stochastic) entry time of the follower, taking place as soon as the stochastic process X reaches the follower threshold X_F^* for the first time. Comparing this to the analogous term in expression (29) of Huisman and Kort (2015), we see that in their paper "our" β_λ is replaced by β. This implies that, because $\beta_\lambda > \beta$ (see (26)) and $X < X_F^*$ (which holds in the region considered here), the negative correction due to the follower's entry is smaller in our case. The finiteness of the project works here: it may be possible that the project is stopped already before the follower enters. This mitigates the strategic effect in the sense that the negative correction of the leader value due to the possible follower entry is lower.

Application of the value matching and the smooth pasting conditions, and maximizing V_L^{det} with respect to K_L, where we have to take into account that X_F^* and K_F^* depend on K_L as obtained from (27)-(28), gives the following result.

Proposition 4. *The leader will consider the entry deterrence strategy whenever X is in the interval $\left(X_1^{\text{det}}, X_2^{\text{det}}\right)$, where X_1^{det} is implicitly given by*

$$\frac{X_1^{\text{det}}}{r+\lambda-\mu} - \delta + \left(\frac{X_1^{\text{det}}}{X_F^*(0)}\right)^{\beta_\lambda} \frac{\delta}{\beta_\lambda - 1} = 0, \tag{31}$$

and

$$X_2^{\text{det}} = \frac{\beta_\lambda + 1}{\beta_\lambda - 1} 2\delta\left(r+\lambda-\mu\right). \tag{32}$$

The leader's entry deterrence investment size equals

$$K_L^{\text{det}} = \frac{1}{\eta\left(\beta+1\right)}, \tag{33}$$

and the investment threshold X_L^{det} is implicitly determined by

$$\left(X_L^{\text{det}}\right)^{\beta_\lambda} \frac{\left(\beta_\lambda - \beta\right)\delta}{\beta\left(\beta_\lambda - 1\right)} \left(\frac{\beta\left(\beta_\lambda - 1\right)}{\left(1+\beta\right)\left(1+\beta_\lambda\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda} + X_L^{\text{det}} \frac{\left(\beta-1\right)}{\left(r+\lambda-\mu\right)\left(\beta+1\right)} - \delta = 0. \tag{34}$$

Concerning the entry deterrence investment strategy we have an explicit expression for the investment size, whereas the investment threshold is implicitly determined. As in the monopoly case with infinite option life (Corollary 1), also here the investment size (33) is not influenced by the creative destruction parameter λ. The capacity K_L^{det} is in fact the Stackelberg leader quantity level, because due to the production-up-to-capacity assumption, the leader, or incumbent, is committed to produce the quantity K_L^{det} at any time after the investment. The follower, or (future) entrant, will adjust its capacity level accordingly (see (28)).

Although we do not know X_L^{det} explicitly, from the implicit expression (34) we can derive that the threshold is between certain bounds, as the following corollary shows.

Corollary 3. *Under an entry deterrence strategy the leader's investment threshold X_L^{det}, which is implicitly defined by* (34)*, is in between the following values*

$$\frac{\beta_\lambda + 1}{\beta_\lambda - 1}\delta\left(r+\lambda-\mu\right) < X_L^{\text{det}} < \frac{\beta+1}{\beta-1}\delta\left(r+\lambda-\mu\right). \tag{35}$$

From the upperbound of X_L^{det} in (35), and expression (16) we obtain that the leader invests earlier than the monopolist without entry threat. This is due to the fact that the investment option of the follower will expire in finite time, so that the follower will invest earlier and thus less. The follower investing earlier reduces the monopoly period of the leader and makes the leader's investment less profitable. However, that the follower invests less enhances the leader's investment payoff. Since the leader invests relatively soon, we conclude that the latter is the dominant factor here[3].

The lowerbound of X_L^{det} in (35), where $\frac{\beta_\lambda+1}{\beta_\lambda-1}\delta\left(r+\lambda-\mu\right)$ is the investment threshold of the leader if its investment option has finite life (see (43) in the next section), shows that the leader will invest later if the investment option will not expire in finite time. It makes sense that, as in the monopoly case, if the investment option will expire in finite time, this will accelerate the investment decision of the leader.

Turning to the *entry accommodation* strategy, it holds that the investment capacity size $K_L^{\text{acc}}(X)$ is such that $K_L^{\text{acc}}(X) \le \hat{K}(X)$ where $\hat{K}(X)$ is given by (29). We then are in a situation that the leader invests at the same time as the follower. However, despite the fact that leader and follower invest at the same time, it is still the case that the leader acts first, where the follower has the choice to invest later or to follow suit, where it chooses for the latter option in the entry accommodation case. As a result we obtain the Stackelberg equilibrium in investment quantities. In this situation, one reason for overinvestment, namely that the follower will invest later if the leader quantity is higher, does not exist. However, we still have the mechanism that a larger value of K_L will result in a smaller capacity of the follower, as confirmed by expression (28). The following proposition contains the leader's entry accommodation strategy.

Proposition 5. *The leader will consider the entry accommodation strategy whenever* $X \ge \max\left(X_1^{\text{acc}}, X_L^{\text{acc}}\right)$, *where*

$$X_1^{\text{acc}} = \frac{3+\beta_\lambda}{\beta_\lambda-1}\delta\left(r+\lambda-\mu\right), \tag{36}$$

$$X_L^{\text{acc}} = \frac{1+\beta}{\beta-1}\delta\left(r+\lambda-\mu\right), \tag{37}$$

and the corresponding capacity size is equal to

$$K_L^{\text{acc}}(X) = \frac{1}{2\eta}\left(1-\frac{\delta\left(r+\lambda-\mu\right)}{X}\right). \tag{38}$$

The optimal accommodation threshold from the perspective of the leader is X_L^{acc}, whereas X_1^{acc} is the minimum level of X for which the follower is willing to invest at the same time as the leader. The conclusion is that only when $X \ge \max\left(X_1^{\text{acc}}, X_L^{\text{acc}}\right)$, both firms are willing to invest at the same time.[4]

[3]Note that in the next section where also the leader's option life is finite, leader and monopoly threshold are the same.

[4]Note that in case the project has infinite life, i.e. $\lambda = 0$, it always holds that $X_L^{\text{acc}} < X_1^{\text{acc}}$, implying that for $X > X_1^{\text{acc}}$ the leader accommodates entry in the sense that the leader and the follower will invest at the same time.

If the probability that the project will end goes up, the effect on X_1^{acc} is positive because $\frac{\partial X_1^{\text{acc}}}{\partial \lambda} > 0$ (see Appendix A.12 where $a = 3$). A shorter expected project life reduces the investment's net present value (NPV), due to which the follower wants to invest later. However, the follower also has an incentive to invest earlier because with probability λdt it can happen that the option disappears within the next time period of length dt. Apparently, the NPV effect dominates here. Furthermore, X_L^{acc} increases with λ, because due to the NPV effect simultaneously investing with the follower is less profitable. Further, we see that an increase of λ will result in the leader to invest less because of the obvious reason that the project is expected to last during a shorter period of time.

4.2 Finite life of the investment option

As in the previous section we consider a situation in which the product market has a finite life from the moment the leader has invested. However, where in the previous section the launch of the current product triggers outside firms to start developing a new project that makes the current product obsolete, here from the beginning onwards, thus also before the current product is launched, the outside firms are active in trying to develop this new product. The implication is that not only after investment the project ends with probability λdt, but also before the investment the option to invest will vanish with probability λdt. This holds for both the leader and the follower.

The resulting optimal investment decision of the follower is presented in the following proposition.

Proposition 6. *Given the capacity level K_L of the leader, the optimal investment threshold of the follower is given by*

$$X_F^*(K_L) = \frac{\beta_\lambda + 1}{\beta_\lambda - 1} \frac{\delta\left(r + \lambda - \mu\right)}{1 - \eta K_L}, \tag{39}$$

whereas the capacity level equals

$$K_F^*(K_L) = \frac{1 - \eta K_L}{\eta\left(\beta_\lambda + 1\right)}. \tag{40}$$

Since also in the previous section there was a finite option life for the follower, the follower's investment decision is exactly the same as in Proposition 3. Of course, K_L can be different, which we will find out below, so that the follower could still invest at a different time in a different capacity size.

In case of an entry deterrence strategy the leader invests as in the following proposition.

Proposition 7. *The leader will consider the entry deterrence strategy whenever X is in the interval $\left(X_1^{\text{det}}, X_2^{\text{det}}\right)$, where X_1^{det} is implicitly given by (31) and X_2^{det} is given by (32). The leader's entry* *deterrence investment threshold equals*

$$X_L^{\det}\left(K_L^{\det}\right)=\frac{\beta_\lambda}{\beta_\lambda-1}\frac{\delta\left(r+\lambda-\mu\right)}{1-\eta K_L^{\det}}, \tag{41}$$

and the investment size $K_L^{\det}\left(X_L^{\det}\right)$ is implicitly determined by

$$\frac{1-2\eta K_L^{\det}}{r+\lambda-\mu}X_L^{\det}-\delta+\left(\frac{X_L^{\det}}{X_F^*\left(K_L^{\det}\right)}\right)^{\beta_\lambda}\frac{K_L^{\det}\eta K_F^* X_F^*}{r+\lambda-\mu}\frac{1-\left(\beta_\lambda+1\right)\eta K_L^{\det}}{K_L^{\det}\left(1-\eta K_L^{\det}\right)}=0. \tag{42}$$

Expression (41) is an important one, because it helps to understand the various effects of the creative destruction parameter λ. If we take into account that λ also influences $K_L^{\det}$, as becomes apparent from expression (42), three effects can be distinguished, namely an *NPV effect* via λ itself, a *quantity effect via $K_L^{\det}$*, and a *strategic* effect via β_λ. The *NPV effect* takes into account that the project is expected to last shorter if λ increases, implying a smaller period during which revenues are earned. This makes the project less attractive, implying that the investment threshold $X_L^{\det}$ will increase.

The *quantity effect* results from the fact that we can obtain from (42) that $K_L^{\det}$ decreases in λ, having the obvious interpretation that the leader will invest less in the project when it has a shorter expected duration. A smaller investment means that the firm has a lower cash outflow, implying that revenues need not be that high to make an investment profitable. Hence, due to the quantity effect $X_L^{\det}$ will decrease.

The *strategic effect* is the effect coming from the fact that finite project duration results in a positive probability that the project has already stopped before the follower will enter. As also explained in the previous section, this mitigates the effect of competition on the leader's investment decision, and thus the negative effect of future follower entry on the expected net present value of the investment. For this reason investing becomes more attractive so that the strategic effect reduces the leader threshold $X_L^{\det}$.

In total, the leader threshold $X_L^{\det}$ is positively influenced by the *NPV effect,* and negatively by the *quantity* and the *strategic effect.* In principle the two expressions (41)-(42) can be used to solve for the threshold $X_L^{\det}$ and the investment size $K_L^{\det}$. The next proposition presents the outcome.

Proposition 8. *Under an entry deterrence strategy the leader's investment threshold is given by*

$$X_L^{\det}=\frac{\beta_\lambda+1}{\beta_\lambda-1}\delta\left(r+\lambda-\mu\right), \tag{43}$$

and the corresponding capacity size is equal to

$$K_L^{\det} = \frac{1}{\eta\left(\beta_\lambda + 1\right)}. \tag{44}$$

From the results of the last proposition and Proposition 4, while taking into account Corollary 3, we obtain that, when its investment option could expire in finite time, the leader invests earlier and therefore less. Note that for the follower this is a good situation, since it holds that for a given follower quantity the output price will be higher. Therefore, the follower will respond by investing earlier and more compared to the situation where the investment option of the leader has infinite life.

To analyze the effects of the creative destruction parameter λ, note that, since β_λ is increasing in λ, the effect of λ on $K_L^{\det}$ is negative. The effect of λ on $X_L^{\det}$ can be determined by the sign of the derivative of $X_L^{\det}$ as given in the following corollary.

Corollary 4. *The derivative of* (43) *with respect to* λ *is*

$$\frac{\partial X_L^{\det}}{\partial \lambda} = \frac{\delta \beta_\lambda \left(\sigma^2\left(\beta_\lambda^2 - 1\right) + 2\left(r + \lambda - \mu\right)\right)}{\left(\beta_\lambda - 1\right)\left(2\left(r + \lambda\right) + \sigma^2 \beta_\lambda^2\right)} > 0. \tag{45}$$

Corollary 4 implies that the *NPV effect* dominates the *quantity* and the *strategic effect.*

5 Duopoly with Symmetric Firms

We analyze a situation where the firms are completely symmetric, which implies that both firms are entitled to invest first. This means that it is not known beforehand which firm will be the leader or the follower. Hence, firm roles are endogenous in the duopoly investment game. To determine the equilibrium for such a scenario we follow the approach of, originally, Fudenberg and Tirole (1985) (see also Thijssen et al. (2012) and Riedel and Steg (2017)), which starts out with developing two different curves, as depicted in Figure 1. Note that we actually see four curves there, but this is because we depict two different situations: finite and infinite project life. The finite project life variant gets a subscript λ and the two graphs that relate to this case are the black ones. For each situation a leader and a follower curve is drawn. The leader curve connects points representing the leader value that results from investing immediately, taking into account that the follower invests at the corresponding follower threshold (see Proposition 6). Note that for small values of the geometric Brownian motion process, i.e. $X < X_{1\lambda}^{\det}$, the leader value equals zero, which is because the output price is too low for the leader to profitably invest in a capacity level larger than zero.

The follower curve connects follower values resulting from investing at the follower threshold. This explains why the follower curve is situated above the leader curve for small values of X. In between the values $X_{1\lambda}^{\mathrm{det}}$ and $\hat{X}_\lambda$ the leader applies the entry deterrence strategy, whereas for $X > \hat{X}_\lambda$ entry accommodation is applied (see also Huisman and Kort (2015)). Thus $\hat{X}_\lambda$ is defined by

$$\hat{X}_\lambda = \min\{X \in \left(X_{1\lambda}^{\mathrm{acc}}, X_{2\lambda}^{\mathrm{det}}\right) | V_{L\lambda}^{\mathrm{acc}}(X, K_{L\lambda}^{\mathrm{acc}}(X)) = V_{L\lambda}^{\mathrm{det}}(X, K_{L\lambda}^{\mathrm{det}}(X))\}.$$

Note that the follower curve jumps upwards for $X = \hat{X}_\lambda$. This is because, if the leader moves from an entry deterrence to an entry accommodation strategy, the leader's investment size jumps down, because one reason for overinvestment, namely that by investing more the follower invests later, disappears.

Having determined the graphs representing the leader and the follower value, we can determine the equilibrium. We do this for the situation where the market is small initially, i.e. $X_0 < X_P < X_{P\lambda}$. The firms will refrain from investing immediately, because as long as the follower curve is situated above the leader curve, a better strategy is to wait and invest at the follower threshold. On the other hand, if the firms wait with investing until $X > X_{P\lambda}$, a first mover advantage arises, because the leader value exceeds the follower value. In such a situation it would be better for one firm to invest at a slightly lower value of X to *preempt* its competitor so that it obtains the leader value. Therefore, in equilibrium the first investor invests at $X_{P\lambda}$, in capacity $K_{L\lambda}^{\mathrm{det}}(X_{P\lambda})$ (see Proposition 7) and the second investor waits with investment until X reaches the corresponding follower threshold $X_{F\lambda}^*(K_{L\lambda}^{\mathrm{det}}(X_{P\lambda}))$ and invests $K_{F\lambda}^*(K_{L\lambda}^{\mathrm{det}}(X_{P\lambda}))$. Because the preemption effect is so dominantly present, this equilibrium is called a preemption equilibrium and $X_{P\lambda}$ the preemption point determined by

$$V_{L\lambda}^{\mathrm{det}}(X_{P\lambda}, K_{L\lambda}^{\mathrm{det}}(X_{P\lambda})) = V_{F\lambda}(X_{P\lambda}, K_{L\lambda}^{\mathrm{det}}(X_{P\lambda}))\}. \tag{46}$$

All X-values without subscript λ in Figure 1 represent the case of $\lambda = 0$, which corresponds to the infinite project life variant. The preemption effect is less dominant if the project life is finite, which can be inferred from the fact that $X_{P\lambda} > X_P$. The finiteness of the project implies that the investment payoff is lower, and therefore firms are less inclined to be the first investor. So, like in the case of exogenous firm roles treated in Section 4, also here we get that the first investor will invest at a later point in time.

It is important to notice that both the leader and the follower curve, and thus also the corresponding preemption equilibrium, do not depend on whether the option to invest has a finite or infinite life for the first investor. The leader curve does not change because it is based on the leader investing immediately. The follower curve is based on the follower investing later or at

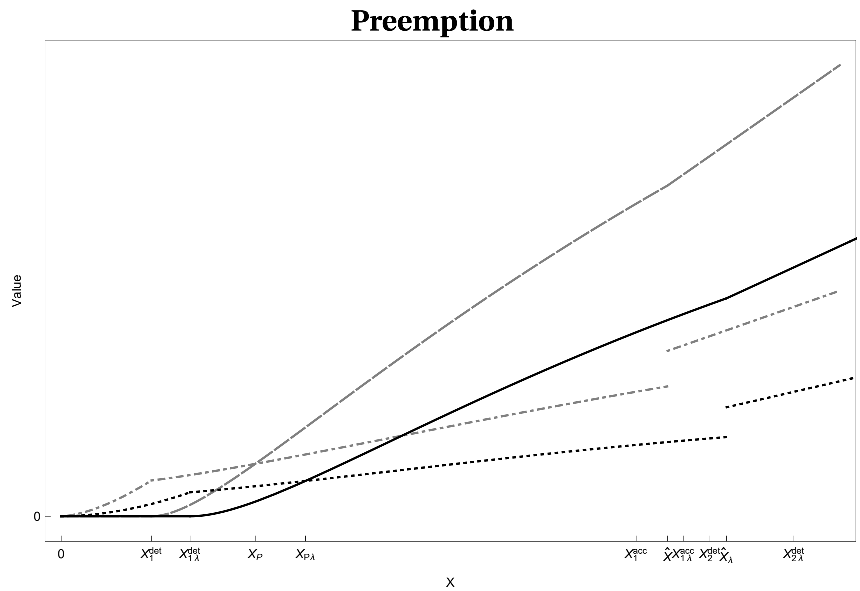

Figure 1: Optimal value functions for the leader and follower as a function of X. The parameter values are $r = 0.1; \mu = 0.06; \delta = 0.1; \eta = 0.05; \lambda = 0.02; \sigma = 0.1$. The gray lines represent the case of an infinite project length and in black both the project and the option are finite. The black solid line and gray dashed represent the value function of the leader and the black dotted and gray dot-dashed line the value function of the follower.

the same time as the leader. This implies that at the moment of the investment the option life of the follower is finite in both situations so there is no change either.

5.1 How important is it to realize project life is finite

Mainstream real options contributions consider the lifetime of an investment project to be infinite. However, in every day life we more and more realize that innovations are abound (McGrath (2019)) and product lifetime is mostly finite due to creative destruction. The analysis in this paper takes account of that and this section explores how important it is to explicitly model finite lifetime. To quantify, we derive the relative loss in value when the project life is finite, whereas firms ignore this feature in their investment decisions. To do so, we compare the value functions in two situations. First, we take the value functions of monopolist, leader and follower based on the strategies earlier derived in this paper. Second, we consider value functions resulting from standard real options strategies, i.e. firms invest assuming that project life is infinite, whereas in fact the project can end at each time with probability λdt. As a result, we define the relative loss of both leader and follower as follows:

$$\begin{aligned}\ell_i(\lambda) = &\left(\left(\frac{X}{X_{P\lambda}}\right)^{\beta_\lambda} V_{i\lambda}\left(X_{P\lambda}, X^*_{F\lambda}(K_{L\lambda}), K^*_{F\lambda}(K_{L\lambda}), K_{L\lambda}\right)\right.\\ &\left.- \left(\frac{X}{X_P}\right)^{\beta_\lambda} V_{i\lambda}\left(X_P, X^*_F(K_L), K^*_F(K_L), K_L\right)\right)\\ &/ \left(\frac{X}{X_{P\lambda}}\right)^{\beta_\lambda} V_{i\lambda}\left(X_{P\lambda}, X^*_{F\lambda}(K_{L\lambda}), K^*_{F\lambda}(K_{L\lambda}), K_{L\lambda}\right),\end{aligned} \tag{47}$$

where i is either L or F, the subscript λ denotes that the decision is optimal under a finite project life, the optimal decisions of the follower are given in Proposition 6, and $K_{L\lambda} = K^{\text{det}}_{L\lambda}(X_{P\lambda})$ and $K_L = K^{\text{det}}_L(X_P)$ are implicitly given by (42). For the monopolist the relative loss is analogously defined. Without subscript λ the strategy is determined as if the project life is infinite, i.e. $\lambda = 0$. Note that the initial value of $X \in (0, X_P)$, to which the values are discounted, is irrelevant as it drops out in the definition of the loss function. The loss function of the leader builds upon $V_{L\lambda}$ as given by (30) (for clarity we changed the notation, i.e. we explicitly added the follower's decisions as arguments and the λ dependence). For the follower, $V_{F\lambda}$ is given by (78) where the A_F is obtained by using the value matching and smooth pasting condition in combination with (51) at the decisions according to Proposition 6. For the monopolist the loss is analogously given by the difference of the value function (9) under the optimal decisions of Corollary 2 minus the strategies under an infinite option and project life, relative to the optimal value and all discounted to the same X.

The loss of not taking the finiteness of the project into account is shown in Figure 2. We conclude that, if the expected lifetime is shorter than 15 or 45 years (note that expected lifetime equals $1/\lambda$), the resulting suboptimal decisions lead to a loss that is more than 100% for both the follower and monopolist, and the leader respectively. The relative loss of the leader is bigger, because, when thinking that project life is infinite while it is in fact not, it is investing far too much at a too early stage. The latter is confirmed in Table 1, where we show the relevant solutions and value losses for three values of λ coinciding with an infinite expected lifetime, and an expected lifetime of 50 years and 10 years. Table 1 even shows that the "infinite leader strategy" is severely loss making when the expected lifetime in reality is 10 years.

The relative loss of the follower is not so high as that of the leader. The reason is that in infinite time the leader invests a lot, implying that, since capacities are strategic substitutes, the follower is already a cautious investor even when it thinks that time is infinite while it is not. However, even for the follower it holds that the infinite strategy is *loss-making* when the expected lifetime in reality is 10 years. The monopolist's relative loss is comparable to that of the follower.

That the leader's relative loss is so much higher is due to the preemption effect, resulting in

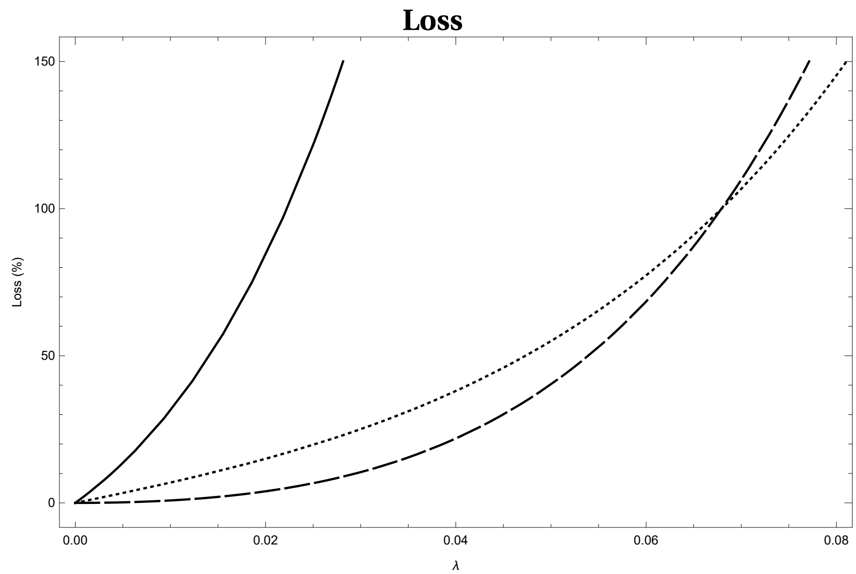

Figure 2: Loss due to ignoring finiteness of project for the leader, follower and monopolist as a function of λ. Loss of value functions for the monopolist, leader and follower as a function of λ. The parameter values are $r = 0.1; \mu = 0.06; \delta = 0.1; \eta = 0.05; \sigma = 0.1$. The dashed line represents the monopolist, the solid line represent the case of leader and the dotted line of the follower.

a strategy where the leader has to overinvest at an early point in time. Introducing this finite project life makes that the leader will invest at a considerable later time in a significantly smaller capacity size. Both of these effects result in a huge loss if the fact that project life is finite is not taken into account when the leader decides about its investment strategy.

We can conclude that it is of high importance to incorporate the finiteness of the lifetime of projects when determining the optimal entry and capacity strategies under uncertainty.

An increase in uncertainty results in the follower investing later. This is because more uncertainty increases the value of waiting with investment (see, e.g., Dixit and Pindyck (1994)). For the leader the same holds but on the other hand the investment project has become more attractive for the leader: the follower investing later means that the leader's monopoly period lasts longer. As a result, the preemption point is delayed but only a little. Figure 3 depicts the leader and follower curve for two different values of σ. The subscript σ indicates the higher $\sigma = 0.2$ compared to the assumption of $\sigma = 0.1$ that we used for the other numerical examples. Similarly, in Figure 4 we depict the losses when uncertainty is increased. The figure shows that, if firms invest as if the project life is infinite while it is not, the losses become smaller for both firms, as well as for the monopolist, when uncertainty goes up. Presumably this is because under increased uncertainty, firms not only invest later but also more (Dixit (1993)), and therefore investment sizes get closer to the level corresponding to the infinite project life case.

λ	$X_{P\lambda}$	$X^*_{F\lambda}(K_{L\lambda})$	$K_{L\lambda}$	$K^*_{F\lambda}(K_{L\lambda})$	$\ell_L(\lambda)$	$\ell_F(\lambda)$
0.1	0.0228	0.0350	3.48	4.26	3983.1	261.3
0.02	0.0132	0.0263	4.90	5.27	84.2	15.0
0	0.0105	0.0243	5.53	5.59	0	0
λ	$V_{L\lambda}(S_\lambda)$	$V_{L\lambda}(S)$	$V_{F\lambda}(S_\lambda)$	$V_{F\lambda}(S)$		
0.1	3559	-138192	3559	-5740		
0.02	540	85	540	459		
0	349	349	349	349		

λ	$X^*_{M\lambda}$	$K^*_{M\lambda}$	$\ell_M(\lambda)$	$V_{M\lambda}(S_\lambda)$	$V_{M\lambda}(S)$
0.1	0.0289	5.15	369.7	7469	-20144
0.02	0.0199	6.98	3.94	1206	1159
0	0.0176	7.73	0	805	805

Table 1: The upper subtable represents the duopoly values and the lower subtable the monopoly values for three different λs. The arguments of the value function S_λ represent the optimal strategies for a finite project with probability λ, while S represents the strategies that are optimal under an infinite project length and thus suboptimal when the finiteness of the project is ignored.

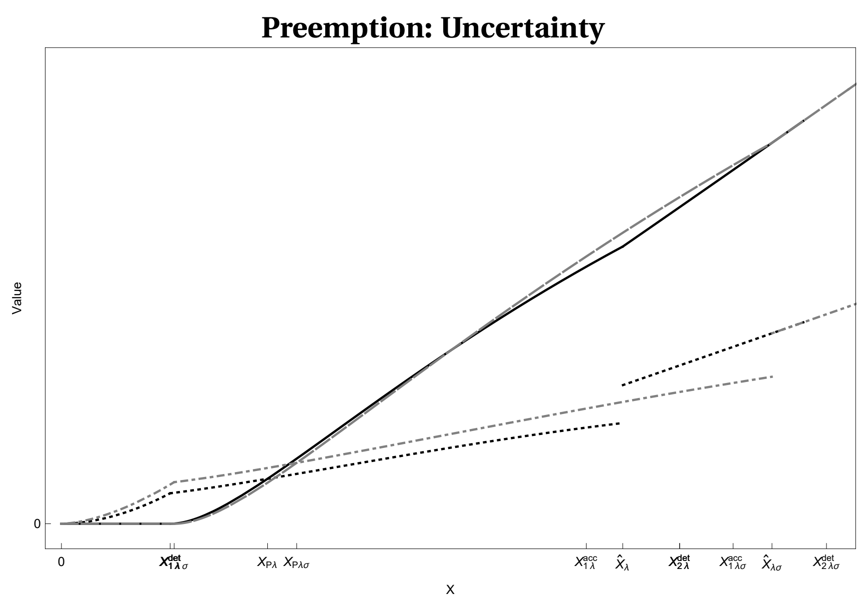

Figure 3: Optimal value functions for the leader and follower where the gray lines represent the case when $\sigma = 0.2$ and the black lines when $\sigma = 0.1$. The black solid line and gray dashed line represent the value function of the leader and the black dotted and gray dot-dashed line the value function of the follower.

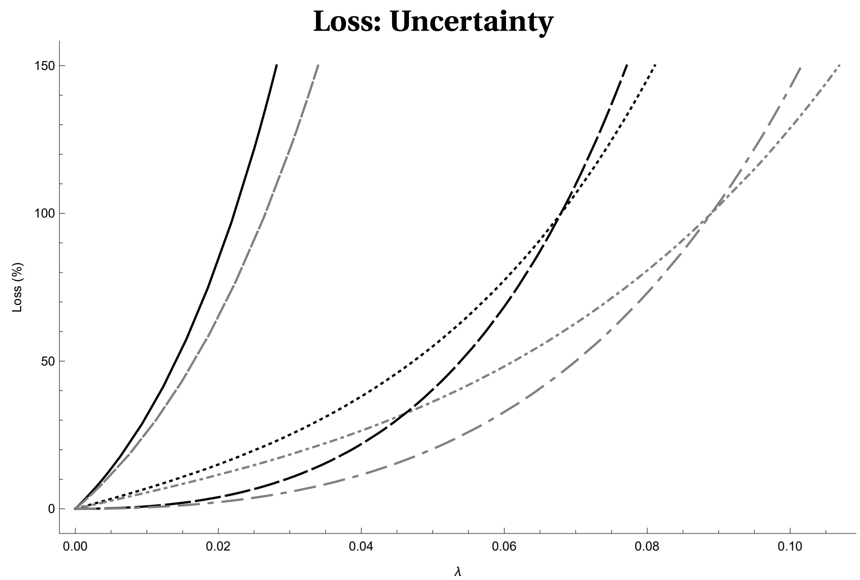

Figure 4: Loss due to ignoring finiteness of project for the leader, follower and monopolist for different levels of uncertainty. The gray lines represent the case when $\sigma = 0.2$ and the black lines when $\sigma = 0.1$. The black solid line and gray dashed line represent the loss of the leader, the black dotted and gray dot-dashed line the loss of the follower and the black dashed and gray dash-dash-dotted line the loss of the monopolist.

6 Conclusions

An often overlooked characteristic in economic research is the finiteness of investment projects. Due to technological progress drastic innovations appear that make current products obsolete. We show that having a finite life of the project delays the investment in a monopoly market while the investment size remains the same. If we take into account the possibility of future entry of a competitor, we see a similar picture.

Interesting is the case, where besides a finite project life, also the investment option will expire in finite time. We show that then there are multiple contradictory effects on the investment timing of the incumbent, such as an NPV effect, which delays the firm's investment, a quantity effect, and a strategic effect. The latter two accelerate the firm's investment, but still the total effect is such that also here investment is delayed. In general it holds that, compared to the scenario where the project life is finite and the option life is infinite, the firm will invest earlier and less.

In the duopoly game with endogenous firm roles there is an incentive to become the first investor, and a preemption equilibrium results. We show that this preemption effect is mitigated by the project life being finite. However, when firms mistakenly ignore the finiteness of the project when taking their investment decisions, the preemption effect considerably enlarges the already large value loss.

Future research could point to different aspects. First, the present paper considers that the lifetime of a product can end due to the occurrence of more innovative products on the market. If such an innovation is drastic, this event essentially stops all activities related to the current product not only of the focal firm, but also of its competitors. As an alternative it would be interesting to consider the finiteness of the economic lifetime of production factors. In such a case, the end of a project owned by one firm would not mean the end of the project of other firms too, instead these could still be continued. We intend to analyze such a scenario with independent events triggering the termination of projects in our future research. Second, a scenario could be considered where the firms themselves control the innovation activities with the implication that the innovation speed λdt, and thus the probability that the lifetime of the current product will end, will depend on the level of R&D activities. Third, effects of ambiguity could be studied, in the sense that firms are uncertain about the true values of the drift and the uncertainty parameters governing the demand system.

A Appendix

This appendix contains the proofs of the propositions.

A.1 Proof of Proposition 1

Maximizing value function (9) with respect to K gives the optimal capacity size K^* for every given level of X:

$$\frac{\left(f(K) + Kf'(K)\right)X}{r + \lambda - \mu} - \delta = 0. \tag{48}$$

Standard real options analysis, shows that the value of the option to invest, denoted by F is equal to (6). To determine the optimal trigger X^*, we employ the value matching and smooth pasting conditions:

$$F(X^*) = V(X^*), \tag{49}$$

$$F'(X)_{X=X^*} = V'(X)_{X=X^*}. \tag{50}$$

Substituting (9) into (49) and (50) leads to $V'(X)X = V(X)\beta$. Solving for X^* gives the first result. If we plug (10) into the first-order condition with respect to K as given by (48), then we get the implicit function (11).

A.2 Proof of Corollary 1

If the inverse demand is linear in the quantity as given by (12) then $f(K) = 1 - \eta K$. Substituting this into (11) and solving for K gives both results.

A.3 Proof of Proposition 2

Similar as in the proof of Proposition 1, the first-order condition with respect to K remains (48). The value matching and smooth pasting conditions now lead to $V'(X)X = V(X)\beta_\lambda$. Solving for X^* gives the results.

A.4 Proof of Corollary 2

If the inverse demand is linear in the quantity as given by (12) then $f(K) = 1 - \eta K$. Substituting this into the implicit function of Proposition 2 and solving for K gives both results.

A.5 Proof of Proposition 3

The value function of the follower at the moment of investment is denoted by V_F^* and depends on X, K_L and K_F, and is equal to

$$V_F(X, K_L, K_F) = \frac{XK_F\left(1 - \eta(K_L + K_F)\right)}{r + \lambda - \mu} - \delta K_F. \tag{51}$$

Maximizing with respect to K_F gives the optimal capacity size of the follower, given the level X and the capacity size of the leader K_L:

$$K_F^*(X, K_L) = \frac{1}{2\eta}\left(1 - \eta K_L - \frac{\delta\left(r + \lambda - \mu\right)}{X}\right). \tag{52}$$

Before the follower has invested, thus when $X < X_F^*(K_L)$, the firm holds an option to invest. The option value is

$$F_F(X) = A_F X^{\beta_\lambda}. \tag{53}$$

Solving the corresponding value matching and smooth pasting conditions gives

$$X_F^*(K_L, K_F) = \frac{\beta_\lambda}{\beta_\lambda - 1}\frac{\delta\left(r + \lambda - \mu\right)}{1 - \eta(K_L + K_F)}. \tag{54}$$

After solving the system of equations we obtain (27) and (28).

A.6 Proof of Proposition 4

The value function of the leader at the moment of investment for the deterrence strategy is given by (30)

$$V_L^{\det}(X,K_L) = \frac{K_L(1-\eta K_L)}{r+\lambda-\mu}X - \delta K_L - \left(\frac{X}{X_F^*(K_L)}\right)^{\beta_\lambda}\frac{K_L\eta K_F^*(K_L)}{r+\lambda-\mu}X_F^*(K_L).$$

Substituting (27) and (28) into this equation results in

$$V_L^{\det}(X,K_L) = \frac{K_L(1-\eta K_L)}{r+\lambda-\mu}X - \delta K_L - \left(\frac{X(\beta_\lambda-1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}\frac{K_L\delta}{\beta_\lambda-1}.$$

Maximizing with respect to K_L gives the following first-order condition:

$$\begin{aligned}\phi(X,K_L) = {} & \frac{(1-2\eta K_L)}{r+\lambda-\mu}X - \delta \\ & -\left(\frac{X(\beta_\lambda-1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}\frac{(1-(\beta_\lambda+1)\eta K_L)\delta}{(\beta_\lambda-1)(1-\eta K_L)} = 0. \end{aligned} \tag{55}$$

Solving (55) gives $K_L^{\det}(X)$ as given by (42). Setting $K_L=0$ in equation (55) gives equation $X_1^{\det}$. Define

$$\psi(X) = \frac{X}{r+\lambda-\mu} - \delta - \left(\frac{X(\beta_\lambda-1)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}\frac{\delta}{(\beta_\lambda-1)}, \tag{56}$$

then we have

$$\psi(0) = -\delta < 0, \tag{57}$$

$$\psi(X_F^*(0)) = \frac{\delta}{\beta_\lambda-1} > 0, \tag{58}$$

$$\frac{\partial\psi(X)}{\partial X} = \frac{1}{r+\lambda-\mu}\left(1-\frac{\beta_\lambda}{\beta_\lambda+1}\left(\frac{X(\beta_\lambda-1)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda-1}\right), \tag{59}$$

where (27) gives $X_F^*(0) = \frac{\beta_\lambda+1}{\beta_\lambda-1}\delta(r+\lambda-\mu)$ and $\beta_\lambda > 1$. For $X \in (0, X_F^*(0))$ it holds that

$$\frac{\partial\psi(X)}{\partial X} > 0. \tag{60}$$

Furthermore, the leader cannot use the deterrence strategy anymore if we have that $X_F^*\left(K_L^{\det}(X)\right) \leq X$. Let us define $X_2^{\det}$ as

$$X_F^*\left(K_L^{\det}(X_2^{\det})\right) = X_2^{\det}. \tag{61}$$

To determine $X_2^{\det}$ we substitute equation (27) for X into (55) which solves for K_L

$$K_L = \frac{1}{2\eta}. \tag{62}$$

Substituting this into (27) gives

$$X_2^{\det} = \frac{\beta_\lambda + 1}{\beta_\lambda - 1} 2\delta\left(r + \lambda - \mu\right). \tag{63}$$

Before the leader has invested, thus when $X < X_L^{\det}$, the firm holds an option to invest. The option value is

$$F_L^{\det}(X) = A_L^{\det} X^{\beta}, \tag{64}$$

when the option life of the investment is of infinite length. The value matching and smooth pasting conditions to determine $X_L^{\det}$ lead together to the condition $V'(X)X - V(X)\beta = 0$. Define

$$\begin{aligned}
\varphi(X, K_L) &= V'(X)X - V(X)\beta,\\
&= \left(\frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu}X - \beta_\lambda\left(\frac{X\left(\beta_\lambda-1\right)\left(1-\eta K_L\right)}{\left(\beta_\lambda+1\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda}\frac{K_L\delta}{\beta_\lambda-1}\right) -\\
&\beta\left(\frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu}X - \delta K_L - \left(\frac{X\left(\beta_\lambda-1\right)\left(1-\eta K_L\right)}{\left(\beta_\lambda+1\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda}\frac{K_L\delta}{\beta_\lambda-1}\right),\\
&= \frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu}X\left(1-\beta\right)\\
&-\left(\beta_\lambda-\beta\right)\left(\frac{X\left(\beta_\lambda-1\right)\left(1-\eta K_L\right)}{\left(\beta_\lambda+1\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda}\frac{K_L\delta}{\beta_\lambda-1} + \beta\delta K_L,\\
&= 0.
\end{aligned} \tag{65}$$

We now solve (55) and (65) simultaneously by the fact that $\phi(X, K_L) = 0$ and $\varphi(X, K_L) = 0$. Let $Z = \left(\frac{(\beta_\lambda-1)(1-\eta K_L)}{(\beta_\lambda+1)\delta(r+\lambda-\mu)}\right)^{\beta_\lambda}$ then

$$\begin{aligned}
\varphi(X, K_L) &= -\left(\beta_\lambda-\beta\right)Z\frac{K_L\delta}{\beta_\lambda-1}X^{\beta_\lambda} + \frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu}\left(1-\beta\right)X + \beta\delta K_L = 0,\\
\phi\left(X, K_L\right) &= -Z\frac{\left(1-\left(\beta_\lambda+1\right)\eta K_L\right)\delta}{\left(\beta_\lambda-1\right)\left(1-\eta K_L\right)}X^{\beta_\lambda} + \frac{\left(1-2\eta K_L\right)}{r+\lambda-\mu}X - \delta = 0.
\end{aligned} \tag{66}$$

These two equations are equal to each other when the capacity is equal to

$$K_L^{\text{det}} = \frac{1}{\eta\left(\beta+1\right)}. \tag{67}$$

Plugging this into $\phi(X, K_L^{\text{det}}) = 0$ and $\varphi(X, K_L^{\text{det}}) = 0$ leads to the implicit function for $X = X_L^{\text{det}}$ solving (34).

A.7 Proof of Corollary 3

Define (34) as $\Phi(X)$

$$\begin{aligned}\Phi(X) = X^{\beta_\lambda} \frac{\left(\beta_\lambda - \beta\right)\delta}{\beta\left(\beta_\lambda - 1\right)} \left(\frac{\beta\left(\beta_\lambda - 1\right)}{\left(1+\beta\right)\left(1+\beta_\lambda\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda} \\ + X\frac{\left(\beta-1\right)}{\left(r+\lambda-\mu\right)\left(\beta+1\right)} - \delta.\end{aligned} \tag{68}$$

We show that the implicit function that determines the leader's threshold is in between duopoly threshold under a finite option and the monopoly threshold under an infinite option. Plugging (43) into $\Phi(X)$ leads to

$$\Phi\left(\frac{\beta_\lambda+1}{\beta_\lambda-1}\delta\left(r+\lambda-\mu\right)\right) = \frac{\left(\beta_\lambda-\beta\right)\delta\left(\left(\beta+1\right)\left(\frac{\beta}{1+\beta}\right)^{\beta_\lambda} - 2\beta\right)}{\beta\left(1+\beta\right)\left(\beta_\lambda-1\right)} < 0, \tag{69}$$

since $\beta_\lambda > \beta > 1$. While plugging (16) into $\Phi(X)$ leads to

$$\Phi\left(\frac{\beta+1}{\beta-1}\delta\left(r+\lambda-\mu\right)\right) = \frac{\left(\beta_\lambda-\beta\right)\delta}{\beta\left(\beta_\lambda-1\right)}\left(\frac{\beta\left(\beta_\lambda-1\right)}{\left(\beta-1\right)\left(\beta_\lambda+1\right)}\right)^{\beta_\lambda} > 0. \tag{70}$$

Moreover, for $X > 0$,

$$\begin{aligned}\frac{\partial\Phi(X)}{\partial X} =& \beta_\lambda X^{\beta_\lambda-1}\frac{\left(\beta_\lambda-\beta\right)\delta}{\beta\left(\beta_\lambda-1\right)}\left(\frac{\beta\left(\beta_\lambda-1\right)}{\left(1+\beta\right)\left(1+\beta_\lambda\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda} \\ &+ \frac{\left(\beta-1\right)}{\left(r+\lambda-\mu\right)\left(\beta+1\right)} > 0.\end{aligned} \tag{71}$$

Hence, the optimal threshold of the leader under an infinite option, which is implied by $\Phi(X) = 0$, is in between these two strategies as displayed in (35).

A.8 Proof of Proposition 5

For the accommodation strategy, the value function of the leader is given by

$$V_L^{\mathrm{acc}}(X, K_L) = \frac{K_L\left(1-\eta\left(K_L + K_F^*(X, K_L)\right)\right)}{r+\lambda-\mu} X - \delta K_L. \tag{72}$$

Substituting (52) into (72) and maximizing with respect to K_L gives (38)

$$K_L^{\mathrm{acc}}(X) = \frac{1}{2\eta}\left(1 - \frac{\delta(r+\lambda-\mu)}{X}\right). \tag{73}$$

The leader will only use its accommodation strategy if the optimal quantity $Q_L^{\mathrm{acc}}(X)$ leads to immediate investment of the follower. So it should hold that $X_F^*(K_L^{\mathrm{acc}}(X)) \leq X$. We define X_1^{acc} as

$$X_F^*(K_L^{\mathrm{acc}}(X_1^{\mathrm{acc}})) = X_1^{\mathrm{acc}}. \tag{74}$$

Substitution of (27) and (73) into (74) and rearranging gives

$$X_1^{\mathrm{acc}} = \frac{(\beta_\lambda + 3)\,\delta}{\beta_\lambda - 1}\left(r+\lambda-\mu\right). \tag{75}$$

For the accommodation strategy, the value matching and smooth pasting conditions with an infinite option to invest leads to the condition $V'(X)X - V(X)\beta = 0$ where the value function is obtained by substituting (73) into (72)

$$V_L^{\mathrm{acc}}(X) = \frac{\left(X - \delta\left(r+\lambda-\mu\right)\right)^2}{8X\eta\left(r+\lambda-\mu\right)}. \tag{76}$$

From the value matching and smoothing pasting implied condition $V'(X)X - V(X)\beta = 0$ we obtain two roots of which $X = \delta(r+\lambda-\mu)$ is not a valid solution and thus we have that X_L^{acc}

$$X_L^{\mathrm{acc}} = \frac{1+\beta}{\beta-1}\delta\left(r+\lambda-\mu\right). \tag{77}$$

Thus the leader will consider the entry accommodation strategy whenever $X \geq \max\left(X_1^{\mathrm{acc}}, X_L^{\mathrm{acc}}\right)$

A.9 Proof of Proposition 6

This proof is similar to the proof of Proposition 3, though here the option life is finite. The value function of the follower at the moment of investment is denoted by V_F^* and depends on X, K_L and K_F, and is equal to (51). The optimal capacity size of the follower is unchanged and given by

(52). Before the follower has invested, thus when $X < X_F^*(K_L)$, the firm holds an option to invest. The option value is now

$$F_F(X) = A_F X^{\beta_\lambda}. \tag{78}$$

Solving the corresponding value matching and smooth pasting conditions gives

$$X_F^*(K_L, K_F) = \frac{\beta_\lambda}{\beta_\lambda - 1} \frac{\delta\left(r + \lambda - \mu\right)}{1 - \eta\left(K_L + K_F\right)}. \tag{79}$$

After solving the system of equations we obtain (39) and (40).

A.10 Proof of Proposition 7

This proof is similar to the proof of Proposition 4, though here the option life is finite. The derivations are identical up to (63). Before the leader has invested, thus when $X < X_L^{\text{det}}$, the firm holds an option to invest. The option value is

$$F_L^{\text{det}}(X) = A_L^{\text{det}} X^{\beta_\lambda}, \tag{80}$$

when the option life of the investment is of finite length. The value matching and smooth pasting conditions to determine X_L^{det} lead together to the condition $V'(X)X - V(X)\beta_\lambda = 0$. Define

$$\begin{aligned}
\varphi(X, K_L) &= V'(X)X - V(X)\beta_\lambda, \\
&= \left(\frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu} X - \beta_\lambda \left(\frac{X\left(\beta_\lambda - 1\right)\left(1-\eta K_L\right)}{\left(\beta_\lambda + 1\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda} \frac{K_L\delta}{\beta_\lambda - 1}\right) - \\
&\quad \beta_\lambda \left(\frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu} X - \delta K_L - \left(\frac{X\left(\beta_\lambda - 1\right)\left(1-\eta K_L\right)}{\left(\beta_\lambda + 1\right)\delta\left(r+\lambda-\mu\right)}\right)^{\beta_\lambda} \frac{K_L\delta}{\beta_\lambda - 1}\right), \\
&= \frac{K_L\left(1-\eta K_L\right)}{r+\lambda-\mu} X\left(1-\beta_\lambda\right) + \beta_\lambda \delta K_L, \\
&= 0,
\end{aligned} \tag{81}$$

which solves for

$$X_L^{\text{det}}\left(K_L^{\text{det}}\right) = \frac{\beta_\lambda}{\beta_\lambda - 1} \frac{\delta\left(r+\lambda-\mu\right)}{1 - \eta K_L^{\text{det}}}. \tag{82}$$

A.11 Proof of Proposition 8

Substituting (82) into (55) and solving for K_L^{det} gives (43). The corresponding threshold X_L^{det} can be calculated by substituting the optimal quantity into (82).

A.12 Proof of Corollary 4

Let

$$X = \frac{\beta_\lambda + a}{\beta_\lambda - 1}\delta\left(r + \lambda - \mu\right), \tag{83}$$

where $\beta_\lambda > 1$ is the positive root that solves (21) and is given by

$$\beta_\lambda = \frac{-(\mu - \frac{\sigma^2}{2}) + \sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2\sigma^2(r+\lambda)}}{\sigma^2}. \tag{84}$$

The derivative is

$$\frac{\partial X}{\partial \lambda} = -\frac{1+a}{(\beta_\lambda - 1)^2}\frac{\partial \beta_\lambda}{\partial \lambda}\delta(r + \lambda - \mu) + \frac{\beta_\lambda + a}{\beta_\lambda - 1}\delta, \tag{85}$$

where

$$\frac{\partial \beta_\lambda}{\partial \lambda} = \frac{1}{\sqrt{(\mu - \frac{\sigma^2}{2})^2 + 2\sigma^2\left(r+\lambda\right)}} > 0. \tag{86}$$

By rewriting (21) and (84), we can also express the derivative of β_λ with respect to λ as

$$\frac{\partial \beta_\lambda}{\partial \lambda} = \frac{\beta_\lambda}{r + \lambda + \frac{1}{2}\sigma^2\beta_\lambda^2}. \tag{87}$$

Plugging (87) into (85) leads to

$$\frac{\partial X}{\partial \lambda} = \frac{\delta}{\left(\beta_\lambda - 1\right)^2\left(2\left(r+\lambda\right) + \sigma^2\beta_\lambda^2\right)}\Upsilon, \tag{88}$$

where

$$\begin{aligned}\Upsilon = & -2a\left(r+\lambda\right) - 4\left(r + \lambda - \frac{1}{2}\mu\left(1+a\right)\right)\beta_\lambda + \\ & \left(2\left(r+\lambda\right) - a\sigma^2\right)\beta_\lambda^2 - \sigma^2\left(1-a\right)\beta_\lambda^3 + \sigma^2\beta_\lambda^4.\end{aligned} \tag{89}$$

We have to show that this derivative is positive. Since $\beta_\lambda > 1, \delta > 0$ and $r > \mu$ we thus concentrate on Υ. By applying twice the equality of (21) and rewriting, we obtain the following equalities that simplify the derivative to (90),

$$
\begin{aligned}
\Upsilon = & \overbrace{-2a\left(r+\lambda\right)}^{\text{use (21)}}-4\left(r+\lambda-\frac{1}{2}\mu\left(1+a\right)\right)\beta_\lambda+ \\
& \left(2\left(r+\lambda\right)-a\sigma^2\right)\beta_\lambda^2-\sigma^2\left(1-a\right)\beta_\lambda^3+\sigma^2\beta_\lambda^4, \\
& =\beta_\lambda\left(\sigma^2\beta_\lambda^2\left(\beta_\lambda-1\right)+a\sigma^2\left(\beta_\lambda-1\right)^2-2\left(\overbrace{\left(r+\lambda\right)}^{\text{use (21)}}-\mu\right)+2\left(r+\lambda\right)\left(\beta_\lambda-1\right)\right), \\
& =\beta_\lambda\left(\beta_\lambda-1\right)\left(\sigma^2\beta_\lambda^2+a\sigma^2\left(\beta_\lambda-1\right)-2\left(\mu+\frac{1}{2}\sigma^2\beta_\lambda\right)+2\left(r+\lambda\right)\right), \\
& =\beta_\lambda\left(\beta_\lambda-1\right)\left(\sigma^2\left(\beta_\lambda+a\right)\left(\beta_\lambda-1\right)+2\left(r+\lambda-\mu\right)\right).
\end{aligned}
$$

Hence,

$$
\frac{\partial X}{\partial \lambda} = \frac{\delta\beta_\lambda\left(\beta_\lambda-1\right)\left(\sigma^2\left(\beta_\lambda+a\right)\left(\beta_\lambda-1\right)+2\left(r+\lambda-\mu\right)\right)}{\left(\beta_\lambda-1\right)^2\left(2\left(r+\lambda\right)+\sigma^2\beta_\lambda^2\right)}. \tag{90}
$$

Since $r \geq \mu$ it follows that $r+\lambda-\mu \geq 0$ and because $\beta_\lambda > 1$ it follows that the derivative is positive for $a \geq -1$. For X_1^{acc}, $a=3$ and for X_L^{det}, $a=1$.

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