A Theory on R&D Investment Dynamics Under Optimal Effort Allocation*

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Abstract

We introduce a two-step model to study the investment dynamics of R&D investments. Entrepreneurs follow sequential decisions: first, they decide how to allocate effort towards short-term assets and a long-term R&D project; second, they decide the optimal timing to invest in the R&D project. Our model endogenously shows that the investment decision, in non-extreme events, is state-dependent so that it provides an additional explanation for the cyclical nature of R&D investment. This life-cycle hypothesis explains why firms tend to invest less in R&D as they mature. We then study which factors govern this hypothesis. We find that uncertainty amplifies the life-cycle hypothesis which corroborates the evidence that the life-span of firms operating in volatile industries is shorter than for firms in less volatile industries. We also retrieve that the effort allocation decision smooths out the positive relationship between the investment trigger and market uncertainty. We conclude that one-dimensional investment real options models exaggerate the influence of uncertainty in the investment trigger, by ignoring the role of the effort allocation decision. We then analyze individually how the quality of the innovation, technical uncertainty and the managing skills of entrepreneurs affect both the effort allocation and the investment decision, as well as its influence on the life-cycle hypothesis. These conclusions spawn important policy implications on how can firms try to counter the life-cycle trap.

Keywords: Real options; Effort Allocation; Investment Timing; Uncertainty; Business Life-Cycle; Innovation.
JEL codes: D81; D92; G31; O32.

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1 Introduction

The staggering Lev and Gu (2016)'s conclusion that performance in stock markets is increasingly explained by innovation-related factors has shed light on the importance of firm R&D investment. According to the authors, only 25 percent of the variation in market capitalization of public companies that are listed since 2000 can be explained by fixed assets on their balance sheet. For companies listing 50 years ago, the figure was between 70 and 80 percent. Intangible assets– encompassing branding, patents and information technology– make up much of this gap. Increasing competition at the worldwide level has pushed firms towards more dynamic innovation processes, as success has shifted from the traditional sources, such as economies of scale and other tangible-asset-based factors (Chandler 1990), to intangible ones, such as human capital and R&D efforts (Zingales 2000). Moreover, R&D expenditure as a percentage of GDP, a key indicator of government and private sector efforts to obtain a competitive advantage in science and technology, is continuously seeing new highs[1]. The economy has been re-shaped for the past decades, and so should the models be.

To analyze the dynamics of R&D investment, we introduce a two-step sequential dynamic model. Departing from the standard approach in which the managing task of running an R&D project is only associated with the definition of the firm's investment timing, the model herein developed assumes that entrepreneurs additionally have to *ex-ante* strategically decide how much effort to store to a potential long-term innovative project, along with effort towards short-term projects. Once this decision has been made, then the optimal investment policy is derived. In our model, we assume that the owner of the firm is also responsible for running the company. Under the perspective of an entrepreneur-central planner, we study the effort allocation and the investment decision and which variables govern these decisions.

The paper contributes to the literature by thoroughly modeling the R&D investment decision. We investigate how the effort allocation decision, the quality of the innovative project, market and technical uncertainty and the managing skills of entrepreneurs towards assets-in-place and innovative projects affect the investment timing. Furthermore, a key contribution of the paper is that we endogenously show that the R&D investment may become less likely as a firm matures. This life-cycle hypothesis allows us to offer an additional explanation for the cyclical nature of

[1] https://data.worldbank.org/indicator/GB.XPD.RSDV.GD.ZS

R&D based on an endogenous finding of our model.

Our approach differs from a large body of work that looks at this topic in two key points. First, we resort to real options methods to model the R&D investment process, rather than considering a static framework. Second, the model assumes that entrepreneurs decide both the optimal timing to implement a project, as standard in the literature, but also how to allocate their own effort to managing different projects: short-term assets and long-term innovative projects, the novelty of the model. As we shall see, it is due to the inclusion of this additional stage in the decision-making process of entrepreneurs that we retrieve the main results of our model.

The entrepreneurs' decision to invest in an R&D project is generally considered uncertain and irreversible. Additionally, this decision is a not now-or-never action, as they have the ability to wait, collect information and then make a wiser decision (McDonald and Siegel 1986), especially in R&D-heavy projects where delaying has an increased value (Aghion and Tirole 1994, Ferreira et al. 2012). When these three conditions are met, real options modeling is a superior methodology (Dixit and Pindyck 1994). The model, hence, recognizes not only the scale of investment and the allocation of effort as key decision steps, as shown in different approaches by the existing literature, but also adds the role of the decision's timing.

Modeling the allocation of effort between short and long-term projects has been the subject of recent literature. Edmans (2009) and Bhattacharya et al. (2017) provide theoretical models which translate the issue of this effort allocation trade-off into a static and deterministic framework. In these models, managers decide the scale of the investment and the allocation of effort towards different projects, conditional on the observed values of a set of exogenous variables, without any regard to the uncertainty of the decision. Manso (2011) and Ferreira et al. (2012) make a step further and consider the uncertainty in the process in a discrete approach, as the authors embed a bandit problem, a class of Bayesian problems, into the specific principal-agent framework. Our work differs from existing literature by acknowledging the continuous role that uncertainty has in the entrepreneurial decision-making process. To do so, the model treats the future outcomes of innovation endeavors as a stochastic variable. Our model provides, to the best of our knowledge, a novel theoretical representation of the effort allocation trade-off by resorting to real options methods.

We follow a multi-dimensional modeling approach that acknowledges the type of environment in which entrepreneurs operate and ultimately decide their allocation of effort between different projects (Holmstrom and Milgrom 1991). We model entrepreneurs' behavior when there are multiple tasks at their duty: running short-term and long-term projects. This dynamic works in a framework where the decision of allocation of effort between different tasks is naturally constrained by the mutually-exclusivity assumption of the model.

We analyze the optimal investment and the effort allocation decisions. We inspect the role of the quality of the innovation, measured by the incremental impact that the R&D project is expected to have on the firm's cash flows, and of the entrepreneurs' skill set, given by their contribution to the value of the project they direct, in both decisions. Then, we also investigate the relationship between uncertainty and the level of assets-in-place cash flows of the firm in these decisions. We confirm the general real options result that higher uncertainty leads firms to a later investment. However, we contribute to this literature by showing that this effect could be, perhaps, somewhat exaggerated. We find that the effort allocated to the R&D project also increases with uncertainty. Then, because we analytically show that the level of effort is negatively related with the investment timing, we find that the effort decision acts as a "cushion" that partially mitigates the increase in the investment trigger in the advent of an uncertainty spike.

Furthermore, we analytically derive the conditions in which the optimal investment policy is state dependent. In general terms, the life-cycle hypothesis is more likely to be observed when the quality of the innovation shock is high, when the ratio of ST skills to LT skills is high and when the technical uncertainty is low. We find that, under given conditions, the investment decision is affected by the current state of the industry so that a firm in the early stages of its life-cycle tends to invest sooner than if in a mature stage. This finding is also endogenous to our model and offers an additional explanation for the life-cycle pattern in R&D investment. This result is line with a vast literature of empirical studies documenting that R&D investment evolves negatively as firms develop into a mature status (e.g. Audretsch 1995, Agarwal 1998, Hill et al. 2014). Additionally, we retrieve that market uncertainty amplifies this life-cycle hypothesis. In industries where uncertainty is usually higher, the notion that the maturity status of a firm leads firms to defer investments in innovative projects is exacerbated. Thus, this endogenous result of our model potentially explains why firms operating in more volatile industries last less time than firms operating in less volatile industries.

The rest of the paper proceeds as follows. Section 2 presents the basic model derivation and the main results under a monopolist setting. Section 3 refers to the analysis of the investment and effort allocation decision. Section 4 sheds light on industry dynamics and the R&D preemptive game under a competitive setting. Section 5 includes the policy implications based on our findings. Section 6 presents the final remarks.

2 Basic Model

Setup. Consider a firm facing a chance to take an opportunity to invest in an R&D project. The firm is already active and produces a stream of cash flows through its own assets-in-place. The model is built from the perspective of an entrepreneur-central planner. Entrepreneurs are responsible for managing two types of ventures: existing short-term projects (ST) and long-term innovative projects (LT). The opportunity of investing in the R&D project is unique to the firm so that it faces no competition for the project *ex-ante*. Entrepreneurs while managing a portfolio of projects have to make two separate decisions. First, they decide on the effort allocated to run short-term projects, weight of w, and long-term projects, weight of $(1-w)$, where $0 \leqslant w \leqslant 1$. The effort allocated to short-term projects represents the regular tasks of running an already active firm. The effort allocated to long-term projects is interpreted as the effort applied in early stages of an R&D project. The particular tweak of the model's intuition is that entrepreneurs have to bear the opportunity cost of having to commit today to some effort level associated with the development of the R&D project, which ultimate outcome is uncertain. Second, they then run a classic option to invest based on the innovation breakthrough. Entrepreneurs are assumed to implement the project once a given threshold is crossed, maximizing their own value function. Entrepreneurs behave rationally and are value maximizers. We assume that when the investment happens the firm operates under monopoly. After the investment, entrepreneurs wait for the arrival of the innovation, which we consider to arrive randomly. Figure 1 illustrates the entrepreneurial path-dependent decision-making process.

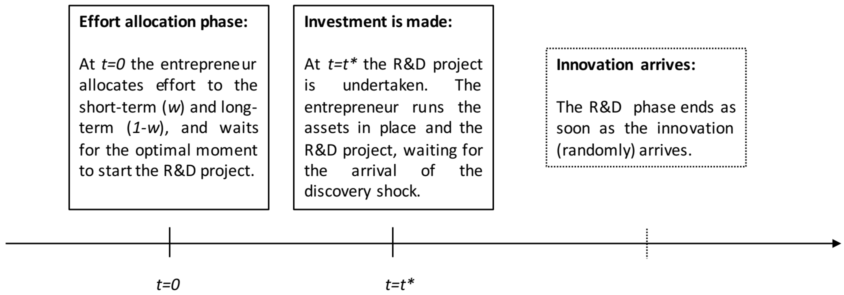


Figure 1: The path-dependent decision framework faced by entrepreneurs.

Entrepreneurs face a trade-off between investing now or delaying their decision in order to collect information and spur a wiser decision in an uncertain world. Importantly, once the project is implemented, the firm will bear a lump-sum cost (K) that refers to the total investment cost of the R&D project.

Let X represent the present value of the cash flows of an active firm, assumed to evolve stochastically as a Geometric Brownian Motion:

$$dX(t) = \alpha X(t)dt + \sigma X(t)dz(t), \tag{1}$$

where α ($\alpha = r - \delta$) is the instantaneous risk-neutral drift, σ is the instantaneous standard deviation, and dz is the increment of a standard Wiener process. We assume $X \equiv X(0) > 0$. Entrepreneurs are risk neutral, with the risk-adjusted required rate of return equal to the risk-free rate denoted by r. Additionally, δ (> 0) is the dividend-yield, corresponding the opportunity cost of deferring the implementation of the project instead of immediately decide to invest.

Let $V(X)$ represent the value of the firm. Since the entrepreneurs are the firm's single stakeholders, the following equation also represents their global welfare function:

$$V(X) = (1 + \theta_S w)X + (1 - w)\, E(X) \tag{2}$$

where w stands for the effort allocated by the entrepreneurs to the short-term, $(1 - w)$ for the effort allocated to long-term projects, X for the current cash flows generated by the firm's assets-in-place, θ_S for the impact of the entrepreneurs' management skills on the firm's assets-in-place cash flows, and $E(X)$ for the R&D investment option value.

Equation (2) represents the trade-off while deciding the effort allocation strategy. On the one hand, allocating effort towards short-term projects allows entrepreneurs to benefit from the value of the firm's assets-in-place. On the other hand, allocating effort towards managing the R&D option allows them to enjoy the upside of such project. Note that the model is designed so that even if entrepreneurs do not allocate any effort to managing the assets-in-place of the firm, there still is a baseline short-term firm value given by X. The level of short-term managerial skills (θ_S) contributes by adding value to this baseline threshold.

In order to understand how the effort allocation and the investment decisions are ultimately made, we shall now address the valuation of the R&D option, a key component in the entrepreneurs' decision-making process. Let us start by following a backwards procedure to derive the value of the entrepreneurs' welfare after the investment in R&D takes place (waiting for the random arrival of the technological shock). As being a contingent claim, and following the standard arguments[2], the correspondent post-investment second-order differential equation takes the following form:

$$\frac{1}{2}\sigma^2 X^2 \frac{\partial^2 H(X)}{\partial X^2} + \alpha X \frac{\partial H(X)}{\partial X} - rH(X) + \lambda[(1 + (1 - w)\theta_L \phi X) - H(X)] = 0 \qquad (3)$$

where one term is added to the traditional option valuation framework.

The last term on the left-hand side of equation (3) considers the different outcomes that may arise from the R&D stage. If entrepreneurs allocate effort to the discovery of any innovative technology and the outcome is a success, they will capture the incremental effect of the new technology on the existent cash flows by a factor of $\phi > 0$. Once again, the level of managerial skills – this time applied to long-term projects (θ_L) – adds value to a given baseline reference (ϕX). In general, the variables $\theta_{S,L}$ can be interpreted just like a Jensen's alpha. Talented entrepreneurs add value to the firm through the way they manage the firm's assets ($\theta_{S,L} > 1$), neutral-entrepreneurs are the ones whose influence on the project value is non-existent ($\theta_{S,L} = 1$), and less capable entrepreneurs are detrimental to the project value so that they actually destroy value from it ($\theta_{S,L} < 1$).

The probability of success during the research stages (technical uncertainty) is modeled as a Poisson arrival shock. From the time of this investment, the discovery process evolves randomly

[2]See Dixit and Pindyck (1994) for details.

according to a Poisson distribution with a constant hazard rate $\lambda(>0)$. In our model, λ is the mean arrival rate of an innovation shock, which, if successful, will add up to the cash flows of the company by an incremental factor of ϕ. This approach has its roots in the works of Loury (1979), Dasgupta and Stiglitz (1980), Lee and Wilde (1980), Reinganum (1983), Dixit (1988) and have been applied in multiple models since then (e.g. Weeds 2002).

The solution of the post-investment o.d.e takes the following form:

$$H(X) = \frac{X\left(1+\theta_L - w\theta_L\right)\lambda\phi}{r-\alpha+\lambda} \tag{4}$$

where we assume that once the investment is implemented there is no further flexibility. Therefore, only the solution for the non-homogeneous part of the equation is considered relevant.

Let us move back to the stage where the investment is in its "waiting region". The entrepreneurs run the assets-in-place and wait for the optimal time to invest in the R&D project. Our goal is to derive the ex-ante value of the option to invest $(E(X))$ of the entrepreneurs' welfare function (equation (2)). This option to undertake the investment in R&D is a call-type of option. Following standard arguments:

$$E(X) = AX^{\beta} \tag{5}$$

which, considering the effort allocation to be stored, comes:

$$(1-w)E(X) = AX^{\beta} \tag{6}$$

where A is a constant yet to be determined and β is the positive root of the fundamental quadratic:

$$Q(\beta) = \frac{1}{2}\sigma^2\beta\left(\beta-1\right)+\alpha\beta-r=0 \tag{7}$$

$$\beta = \frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{\alpha}{\sigma^2}-\frac{1}{2}\right)^2+\frac{2(r+\lambda)}{\sigma^2}}>1 \tag{8}$$

Furthermore, the option value for the entrepreneurs, E(X) is subject to the following boundary conditions:

$$\lim_{X\to X^*} E(X) = \frac{X^*\left(1+\theta_L - w\theta_L\right)\lambda\phi}{(1-w)\left(r-\alpha+\lambda\right)} - \frac{K}{(1-w)} \tag{9}$$

$$\lim_{X\to X^*} \frac{\partial E\left(X\right)}{\partial X} = \frac{\left(1+\theta_L - w\theta_L\right)\lambda\phi}{(1-w)\left(r-\alpha+\lambda\right)} \tag{10}$$

where X^* represents the entrepreneurs' trigger value for implementing the project.

The first boundary condition (equation (9)), generally called the "value-matching condition" establishes the payoff for the entrepreneurs when the project is implemented, so that for the level of cash flows at which is optimal to invest (X^*), the value of the option must equal the net present value that the entrepreneurs receive by undertaking the R&D project. Note that the first term is weighted by an augmented discount factor of $\left(\frac{\lambda}{r-\alpha+\lambda}\right)$, where the technical uncertainty associated with the outcome of the R&D stage (given by λ) adds up to the traditional market discount rate ($r - \alpha$). To some extent, this augmented discount factor represents a global probability adjusted for the usual time-value of money and also the risk intrinsic to an R&D project. The second term represents the implementation costs associated with the execution of the project (K). The second boundary condition (equation (10)) , known as "smooth-pasting condition" or "high-contact condition", ensures that $E\left(X\right)$ is continuously differentiable along X.

Solving both equations we derive the optimal trigger (X^*) and the constant A.

$$X^*\left(w\right) = \frac{\beta}{\beta-1}\frac{\left(r-\alpha+\lambda\right)K}{\left(1+\left(1-w\right)\theta_L\right)\lambda\phi} \tag{11}$$

$$A = \left(\frac{X^*(w)\left(1+(1-w)\theta_L\right)\lambda\phi}{(1-w)\left(r-\alpha+\lambda\right)} - \frac{K}{(1-w)}\right)\left(\frac{1}{X^*(w)}\right)^{\beta} \tag{12}$$

Therefore, we re-write the entrepreneurs' welfare function as:

$$V(X) = (1+\theta_S w)X + \left(\frac{X*\left(w\right)\left(1+\theta_L - w\theta_L\right)\lambda\phi}{r-\alpha+\lambda} - K\right)\left(\frac{X}{X^*(w)}\right)^{\beta} \tag{13}$$

To derive the final solutions we follow a sequential game:

- **1.** Entrepreneurs define their investment policy ($X^*(w)$) as a function of their effort allocation decision (w);

$$X^*(w) = \frac{\beta}{\beta - 1}\frac{(r - \alpha + \lambda) K}{(1 + (1 - w)\theta_L)\lambda\phi} \tag{14}$$

- **2.** Entrepreneurs derive the optimal effort allocation strategy (w^*) that maximizes their welfare $(V(X))$;

$$\max_w [V(X)] \equiv \max_w [(1 + \theta_S w)X + (1 - w) E(X)] \tag{15}$$

$$\text{s.t. } 0 \leq w \leq 1 \tag{16}$$

$$w^* = min\left(max\left(0; 1 + \frac{1}{\theta_L} - \frac{K\beta\left(\frac{\theta_S (r - \alpha + \lambda)}{\theta_L \lambda\phi}\right)^{\frac{\beta}{\beta-1}}}{X\theta_S(\beta - 1)} \right); 1 \right) \tag{17}$$

Note that the restriction imposed on the effort allocation, via w, means that entrepreneurs are not allowed to delegate management tasks to outside managers. For instance, a negative value of w would mean that entrepreneurs had allocated more than 100% of their effort to the R&D project only because they had the possibility of having outside managers directing the short-term projects. This would translate into a situation in which entrepreneurs had the possibility of "shorting" effort to the short-term project and "leverage" their position in the R&D project. Through the aforementioned constraint, this possibility is ignored by our model.

- **3.** Then, entrepreneurs internalize this effort strategy and compute their optimal investment policy $(X^*(w^*) \equiv X^*)$.

$$X^* = \begin{cases} \frac{\beta}{\beta - 1}\frac{(r - \alpha + \lambda) K}{(1 + \theta_L)\lambda\phi} & w = 0 \\ \bar{X}\left(\frac{r - \alpha + \lambda}{\lambda}\frac{\theta_S}{\theta_L \phi}\right)^{\frac{1}{1-\beta}} & 0 < w < 1 \\ \frac{\beta}{\beta - 1}\frac{(r - \alpha + \lambda) K}{\lambda\phi} & w = 1 \end{cases} \tag{18}$$

In extreme effort allocation strategies, in which firms are fully dedicated to run short-term projects or long-term projects, the investment decision does not depend on the level of the assets-in-place of the firm. However, for non-extreme situations, the investment decision becomes state-dependent. In this middle branch, for values of $\bar{X}$ lower than X^*, entrepreneurs find optimal to delay the investment decision. Nonetheless, this is only possible with $\left(\frac{r-\alpha+\lambda}{\lambda}\frac{\theta_S}{\theta_L\phi}\right)^{\frac{1}{1-\beta}} > 1$. In the subsequent sections, we will provide an analysis of the economic meaning of this condition which will allow us to distill the dynamics of R&D investment.

3 Investment Dynamics

In this section, we will provide a detailed analysis over the investment and the effort allocation decision. We will start by discussing the general intuition behind optimal trigger function (equation (18)) and develop the endogeneous life-cycle hypothesis. As we shall see, under certain conditions, our model predicts an explanation for the cyclical nature of R&D within firms. In the following subsections, we will then discuss in length what these conditions are and what is the economic reasoning behind those.

3.1 Life-cycle Hypothesis

The link between business life-cycle and R&D investing policies has been extensively covered by literature, and the relationship is usually deemed as negative. As firms mature, and the cash flows generated by their short-term assets increase, literature predicts a gradual decline in R&D intensity. For instance, during the growth stage, firms may overly invest to create a lasting cost or demand advantage over competitors (Anthony and Ramesh 1992). The idea derives from the seminal works of strategic literature (Porter 1980), where the underlying idea is that a firm maximizes revenue growth early in its life cycle, to create permanent cost or demand advantages over competitors, but in its mature stage, growth slows down and investments are less rewarding. Spence (1977) attempted to derived analytically these strategic findings, and modeled how firms are able to deter entry by creating capacity and incurring significant capital expenditures early in the life cycle, making the product market unattractive to potential entrants.

Throughout the years, additional alternative explanations arrived, always under the base conclusion that R&D investing evolves negatively as firms develop into a mature status. Miller and Friesen (1984) showed that firms during the growth stages focus on sales growth and R&D in order to gain competitive advantages. In the mature stage, as firms stabilize around a given sales growth figure, R&D gradually loses its relevancy. Additionally, Hill et al. (2014) argues that firms while in the growth stage all too often shift their attention to issues such as new product development, hence, relying more heavily on R&D. As mature firms bring in higher levels of brand awareness, R&D becomes less important. In line with these arguments, Audretsch (1995) shows that as industries move from an emerging status to a mature one, the ratio of new product innovation per R&D dollar expended tends to decline. Moreover, Agarwal (1998) documents that patenting activity increases during the initial stages of the business life-cycle, and subsequently declines as firms approach maturity. In short, there is rich literature advocating for a negative link between R&D and industry life-cycle[3]. In the present subsection, our model presents a theoretical explanation for the empirical evidence on the link between the firm's position in the life-cycle and its willingness to invest in R&D.

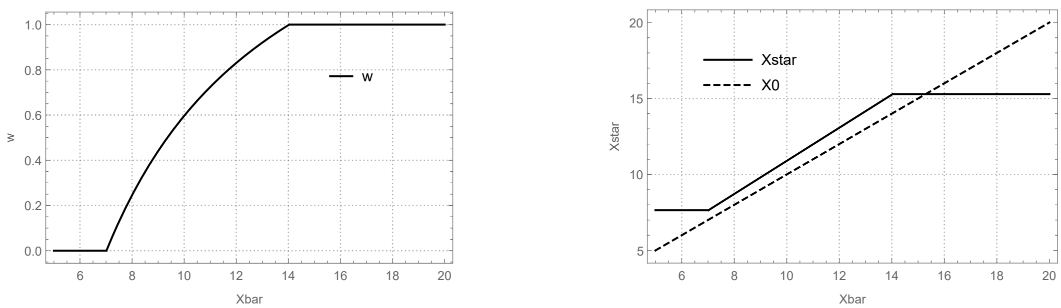
To do so, we proxy the relationship between the firm's stage in the life-cycle and its engagement in R&D activities by looking at the role of the level of the cash flows generated by the firm through its assets-in-place and we study its impact on the investing threshold. We build this proxy under the base assumption that as firms tend to maturity, the level of the assets-in-place-related cash flows tends to increase (Damodaran 2009, Jaafar and Halim 2016). Additionally, we consider that a firm's engagement in innovation should be interpreted as increasing whenever the investment trigger decreases, since lower thresholds translate, *ceteris paribus*, into a quicker investment decision. Thereby, we look at how different levels of X (a proxy of the current state of the firm in the life-cycle) relate to the investment trigger (a proxy for innovation engagement).

In mathematical terms, the relationship between the current state of the firm in the life-cycle and its engagement in innovative projects is given by the derivative of X^* with respect to $\bar{X}$. Doing so, we get:

$$\frac{\partial X^*}{\partial \bar{X}} = \begin{cases} 0 & w = 0 \\ \left(\dfrac{r-\alpha+\lambda}{\lambda}\dfrac{\theta_S}{\theta_L \phi}\right)^{\dfrac{1}{1-\beta}} & 0 < w < 1 \\ 0 & w = 1 \end{cases} \tag{19}$$

[3] Note that our model renders conclusions regarding both the business and the industry life-cycle since we have assumed that the investment in a successful R&D project generates a monopoly for the firm.

Naturally, the investment trigger is only state-dependent in the non-extreme cases of the effort allocation decision. Note that for $\frac{\partial X^*}{\partial \bar{X}} < 1$ the firm will always find optimal to implement the R&D project immediately since the cash flows of the firm $(\bar{X})$ will always be higher than the investment trigger (X^*). Nonetheless, for a value larger than 1, the firm will choose to delay the investment decisions. In the following figure, we depict this result. As we observe, under given conditions, when the entrepreneurs have the flexibility to adjust their effort position the trigger evolves positively with the level of assets-in-place cash flows of the firm (figure 2). In addition, Panel B also shows when the investment decision is taken. Whenever the dotted line, representing the level of short-term cash flows X^*, crosses the solid line $X*$, then it becomes optimal for the entrepreneur to invest in the project. Note that this dynamic will be present in the remaining figures in the paper. The same logic applies.



(a) Entrepreneurs' effort allocation to ST (w) as a function of the quality of the assets-in-place cash flows $(\bar{X})$

(b) Entrepreneurs' investment trigger (X^*) as a function of the assets-in-place cash flows $(\bar{X})$

Figure 2: Parameters are as follows: $K = 8, r = 0.12, \alpha = 0.02, \sigma = 0.3, \theta_S = \theta_L = 1, \lambda = 0.5$. In addition, $\bar{X} \in [5, 20]$.

Under given conditions, we retrieve that the investment trigger depends on the state variable. A derivative greater than 1 is in line with the theoretical predictions mentioned above, as higher levels of cash flows generated by a firm's assets-in-place lead to a higher investment trigger, *ceteris paribus*. In other words, as a firm/industry approaches the late-stages of the life-cycle, innovation is expected to lose relevancy, as the firm's criteria that trigger an R&D investment becomes increasingly daunting to attain. Importantly, this conclusion is endogenous to our model. Under these conditions:

Corollary 1. *When entrepreneurs have to choose an effort allocation strategy between short* *and long-term, and when they add value to the projects they are responsible for, under optimal conditions, situations arise where a positive relationship between the cash flows generated by a firm's assets-in-place and the optimal investment trigger solution is observed, so that firms tend to invest in R&D sooner (later) during the early (late) stages of the firm's life-cycle,ceteris paribus.*

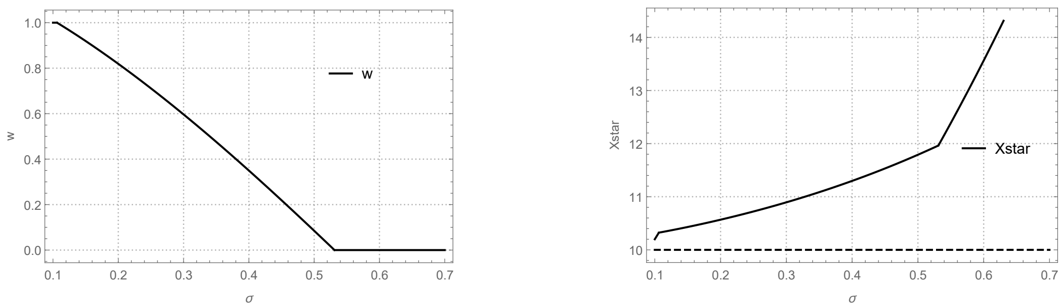
As an innovation-oriented firm produces innovation and converts R&D projects into assets-in-place, the firm gradually increases the fraction of its total value deriving from that assets-in-place. As this conversion takes place and the firm evolves through its life-cycle, X increases. This endogenous dynamic leads firms to become less willing to invest in future R&D projects, as the investment trigger also increases. The later an industry/a company is in the life-cycle, the less disruptive it should be expected to be. Only for industries/companies where the fraction of its total value is not yet significantly dependent on the assets-in-place, does our model predict firms to be especially keen on implementing R&D projects.

In the following subsections, we study the economic conditions under which the life-cycle hypothesis prevails. We will discuss how this dynamic depends on the level of market and technical uncertainty, the quality of innovation and the managerial skill set of entrepreneurs. In general terms, the life-cycle hypothesis is more likely to be observed when the quality of the innovation shock is high, when the ratio of ST skills to LT skills is high and when the technical uncertainty is low. Additionally, we will show next that market uncertainty may amplify the life-cycle hypothesis strength when existent.

3.2 Market Uncertainty

One of the key parameters governing the relationship between the current level of assets-in-place of the firm and the investment trigger of the R&D project is uncertainty. As we will show, changes in the level of uncertainty generate changes in terms of the entrepreneurs' effort allocation strategy and, ultimately, their investment decision.

Panel A of Figure 3 depicts the link between uncertainty and the effort allocated by entrepreneurs to short-term projects. According to the figure, the higher the uncertainty, measured by the volatility of the assets-in-place of the firm (σ), the more effort is allocated to the R&D project (lower w). This is the result of the fact that the higher the uncertainty, the greater the value of the R&D project option, a well-known result within the options field. Entrepreneurs respond to a higher option value by allocation more effort towards that project, so that they benefit from its greater value in their welfare function. Note that this negative relationship works until the boundaries of w are hit. For projects with great uncertainty, entrepreneurs will allocate 100% of their effort to this project and for safe-bet projects, they will allocate 100% to the short-term projects of the firm.



(a) Entrepreneurs' effort allocation to ST (w) as a function of uncertainty (σ)

(b) Entrepreneurs' investment trigger (X^*) as a function of uncertainty (σ)

Figure 3: Parameters are as follows: $\bar{X} = 10, K = 8, r = 0.12, \alpha = 0.02, \theta_S = \theta_L = 1, \phi = 1.3, \lambda = 0.5$. In addition, $\sigma \in [0.1, 0.7]$.

Panel B of Figure 3 analyzes how this dynamic translates into the investment decision. Remember, our sequential model stresses that entrepreneurs first decide on their effort allocation and only then they will decide how to invest in the R&D project. Accordingly, we find that higher uncertainty of this project leads them to, *ceteris paribus*, defer the investment. This occurs because, in line with real options standard literature, in the advent of higher uncertainty, the option to wait and delay the investment decision becomes more valuable. Therefore, higher uncertainty increases the value of the project, which in turns leads entrepreneurs to allocate more effort to that project but, simultaneously, to delay its implementation.

Even though the higher level of effort allocated to the R&D tends to, *ceteris paribus*, push down the trigger value[4], the impact of uncertainty in the option value is such that exerts a dominant effect over the impact of uncertainty in the effort allocation decision. That is why even when the effort allocated to the R&D project increases, the trigger is also increasing, rather than the opposite. In fact, note that in the range of high σ values in which the managers can

[4]The derivative of $X^*(w)$ with respect to w is positive, meaning that higher (lower) w increases (decreases) the trigger. Mathematically, we get $\frac{\partial X^*(w)}{\partial w} = \frac{K\beta\theta_L(r-\alpha+\lambda)}{(\beta-1)(1+\theta_L-w\theta_L)^2\lambda\phi} > 0$

no longer adjust their effort allocation decision, the slope of the line is even steeper. This occurs because now the effort allocation can no longer act as a cushion, so that the increases in X^* are even greater when uncertainty spikes. Uncertainty spikes lead to an increased investment trigger, as the effect via the option to wait exerts a dominant influence over the effect in the effort allocation decision. Therefore, we may extend our finding and state that perhaps one-dimensional investment real options models exaggerate the influence of uncertainty in the investment trigger, by ignoring the role of the effort allocation decision.

From the standpoint of the life-cycle hypothesis, the only impact that uncertainty has is in amplifying this dynamic when it is already present. The level of uncertainty does not define whether or not this hypothesis holds, but when it does, different levels of uncertainty can amplify this result[5]. Mathematically, higher σ leads to a lower β, given the well-known negative relationship between β and σ. Therefore, if $\left(\frac{r-\alpha+\lambda}{\lambda}\frac{\theta_S}{\theta_L\phi}\right) < 1$, for higher values of σ, we get a even higher $\left(\frac{r-\alpha+\lambda}{\lambda}\frac{\theta_S}{\theta_L\phi}\right)^{\frac{1}{1-\beta}}$. If so, the positive and higher than 1 relationship between X^* and $\bar{X}$ – our life-cycle condition – is amplified.

Economically, the idea that the firm position in the life-cycle negatively influences the willingness of firms to implement innovative projects is amplified in companies/industries in which the uncertainty is higher. We can extrapolate this idea and state that this is why sectors, such as utilities, in which historically volatility has been lower, companies can resist as innovators for a longer period of time. In sectors with higher volatility, such as the high-tech sector, uncertainty may be amplifying this life-cycle hypothesis which explains why firms in these industries tend to last less time. Therefore, we can state that:

Corollary 2. *Higher levels of market uncertainty amplify the deterring effect of the firm's assets-in-place cash flows on the investment trigger of R&D projects.*

In the next subsections, we will refer to the variables that are responsible for governing the existence or not of the life-cycle hypothesis.

3.3 Quality of the innovation shock

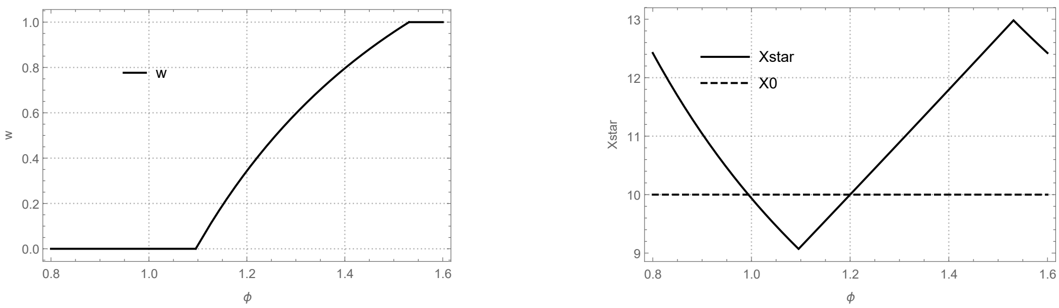
In this subsection, we study the role of the quality of the R&D project – measured by the incremental impact of this project in the firm's cash flows – in both the effort allocation and the

[5]The fact that β is always greater than 1 excludes the possibility of having uncertainty as a key variable deciding whether or not the life-cycle hypothesis holds.

investment decision. Different from the analysis above, the quality of the innovation is a key parameter in order to ensure the life-cycle hypothesis discussed above.

Panel A of Figure 4 shows that for sufficiently low values of the incremental impact of the R&D project on the firm's cash flows, entrepreneurs decide to allocate 100% of their effort towards the LT project. The better the project is, the less effort the manager allocates to it. This result may have two alternative explanations: (i) the better the project is, the less effort the entrepreneurs need to allocate to this project to attain a given welfare goal. This is a corollary of how their welfare function is designed. (ii) In addition, this effect can also be interpreted as a perverse reaction from the entrepreneurs in terms of their management role. We will explore in length this alternative.

As far as the investment is concerned, the relationship between the incremental impact of the innovation and the investment trigger is non-monotonic. For sufficiently low values of ϕ, so that entrepreneurs allocate 0% effort to ST projects and 100% effort to the LT project, as ϕ increases, the sooner managers want to implement it. In the region of ϕ value in which managers respond to an increase in ϕ through an increase in the effort allocated to ST projects (higher w), higher ϕs, the later managers want to invest. When the project is sufficiently good, so that managers no longer can increase the effort allocated to ST, then the quality of the project reverts to the initial situation in which better projects spur quicker investment decisions.



(a) Entrepreneurs' effort allocation to ST (w) as a function of the quality of the innovation (ϕ)

(b) Entrepreneurs' investment trigger (X^*) as a function of the quality of innovation (ϕ)

Figure 4: Parameters are as follows: $\bar{X} = 10, K = 8, r = 0.12, \alpha = 0.02, \sigma = 0.3, \theta_S = \theta_L = 1, \lambda = 0.5$. In addition, $\phi \in [0.8, 1.6]$.

Panel B of figure 4 displays the interaction between these two forces: the quality of the project, which leads, *ceteris paribus*, entrepreneurs to invest sooner; and the effort allocation decision, which spurs the opposite reaction. Only in extreme situations, in which managers no longer can adjust their effort allocation strategy, the quality of the project negatively influences the investment trigger. For medium-quality projects, the effect of the effort allocation decision exerts a dominant influence that delays the project implementation. Therefore, the process of effort allocation could be interpreted as having a perverse effect in the R&D investment decision whenever the quality of innovation (ϕ) changes. This perverse effect between w and ϕ could be interpreted as an inefficiency cost of having the same entrepreneur running both ST and LT projects.

As far as the life-cycle hypothesis, note that the likelihood of the derivative of X^* with respect to $\bar{X}$ being greater than 1 increases with ϕ, enforcing the life-cycle hypothesis described above. Mathematically, the life-cycle hypothesis is only held valid if the derivative ($\frac{dX^*}{d\bar{X}}$) is greater than 1, as also discussed above. For that to happen, necessarily,

$$\left(\frac{r-\alpha+\lambda}{\lambda}\frac{\theta_S}{\theta_L\phi}\right)<1 \tag{20}$$

From the following result, we find that the likelihood of this result increases with the quality of the innovation (ϕ), since:

$$\frac{d\left(\frac{r-\alpha+\lambda}{\lambda}\frac{\theta_S}{\theta_L\phi}\right)}{d\phi}=-\frac{\theta_S\left(r-\alpha+\lambda\right)}{\theta_L\lambda\phi^2}<0 \tag{21}$$

In these conditions, better projects imply higher investment triggers, *ceteris paribus.* Thus, we conclude that in non-extreme situations, in which firms are not fully focused on ST or LT projects, the life-cycle hypothesis holds. In these projects, the perverse effect between the effort allocation decision and the quality of the project justifies the existence of such a hypothesis.

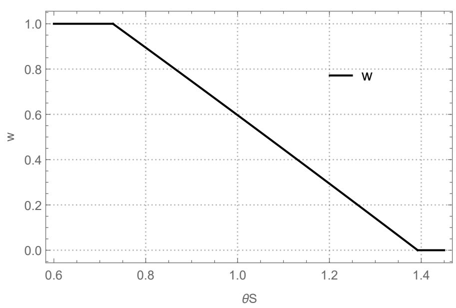
An important policy implication emerges. If firms willing to innovate have a manager at the helm of the firm that increases the effort allocated to the R&D project as the quality of such project increases, then the firm may build a defensive tactic against this life-cycle trap. Having managers of this kind would counter the perverse effect experienced in companies in which entrepreneurs run both ST and LT projects so that it could revert the relationship between the investment trigger and the stage that the firm is in the life-cycle. As we shall see next, having managers with a sufficiently high θ_S could fit this profile.

3.4 Skills

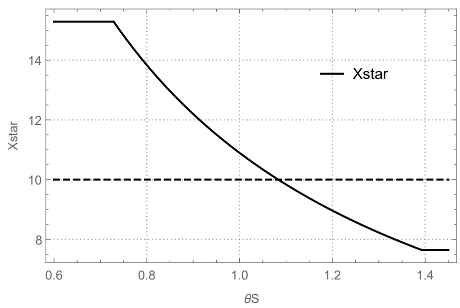
In this subsection, we study the role of the managing skills of entrepreneurs in the effort allocation and the investment decision. Then, we analyze the link between this variable and the life-cycle hypothesis. We start with the short-term skills.

Panel A of Figure 5 shows that for sufficiently low values of θ_S, entrepreneurs allocate 100% effort to ST. As their level of ST skills increases, the value they obtain via assets-in-place also increases, given the way the welfare function is designed. Thus, entrepreneurs engender a reallocation of resources in a way that allows them to allocate more effort towards the R&D project. In the most extreme case, if their ST skills are sufficiently high, entrepreneurs allocate 100% of their effort towards this innovative project. The quality of their skills towards ST projects gives entrepreneurs the flexibility to allocate effort to the LT project.

Simultaneously, as the level of ST skills increase, given the negative relationship between the effort allocated to a given project and the correspondent investment trigger, entrepreneurs will find optimal to invest increasingly sooner. Therefore, we retrieve that the quality of the managing skills towards the assets-in-place of the firm is positively correlated with the desire of managers to invest sooner in projects that will improve the value of the assets-in-place. These dynamics are illustrated in Panel B of Figure 5.



(a) Entrepreneurs' effort allocation to ST (w) as a function of the skills towards managing ST projects (θ_S)

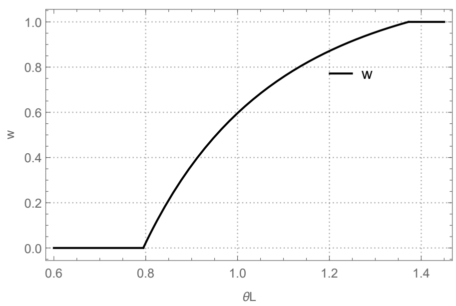


(b) Entrepreneurs' investment trigger (X^*) as a function of the skills towards managing ST projects (θ_S)

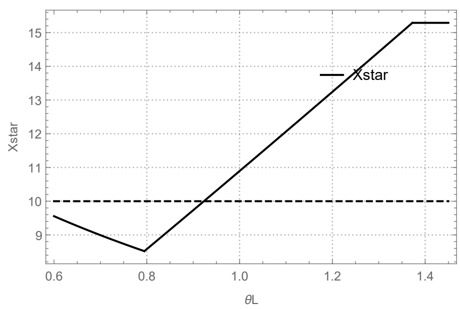
Figure 5: Parameters are as follows: $\bar{X} = 10, K = 8, r = 0.12, \alpha = 0.02, \sigma = 0.3, \theta_S = \theta_L = 1, \lambda = 0.5, \phi = 1.3$. In addition, $\theta_S \in [0.6, 1.45]$.

As far as the role of the LT skills of entrepreneurs, we retrieve similar conclusions to the impact of the quality of the innovation on the effort allocation and investment decisions. Just like with ϕ, increases in θ_L lead to an increase in the overall value of the investment opportunity. Entrepreneurs while managing the firm react by decreasing (increasing) the effort allocated to LT (ST) projects. This result is illustrated in Panel A of Figure 6. Once again, this result may have two alternative explanations: (i) the better the LT skills of the entrepreneurs are, the less effort the entrepreneurs need to allocate to this project to attain a given welfare goal. (ii) Or, this effect can also be interpreted as a perverse reaction from the entrepreneurs as far as their management role.

Similar to the previous case, Panel B of figure 4 displays the interaction between two forces: the LT skills of entrepreneurs, which lead, *ceteris paribus*, entrepreneurs to invest sooner; and the effort allocation decision, which spurs the opposite reaction. Only in extreme situations, in which managers no longer can adjust their effort allocation strategy, the LT skills of entrepreneurs negatively influence the investment trigger. For medium-quality skills, the effect of the effort allocation decision exerts a dominant influence that delays the project implementation. Thus, the process of effort allocation could be again interpreted as having a perverse effect in the R&D investment decision whenever the quality of the LT skills (θ_L) changes. This perverse effect between w and θ_L could be interpreted as another inefficiency cost of having the same entrepreneurs running all the firm's projects.



(a) Entrepreneurs' effort allocation to ST (w) as a function of the skills towards managing LT projects (θ_L)



(b) Entrepreneurs' investment trigger (X^*) as a function of the skills towards managing LT projects (θ_L)

Figure 6: Parameters are as follows: $\bar{X} = 10, K = 8, r = 0.12, \alpha = 0.02, \sigma = 0.3, \theta_S = 1, \lambda = 0.5, \phi = 1.3$. In addition, $\theta_L \in [0.6, 1.45]$.

As far as the life-cycle hypothesis, note that the likelihood of the derivative of X^* with respect to $\bar{X}$ being greater than 1 now decreases with the ratio of short-term skills relative to long-term skills ($\frac{\theta_S}{\theta_L}$), enforcing the life-cycle hypotehsis described above. Follow the standard approach of above, this conclusion is derived from the following result:

$$\frac{d\left(\dfrac{r-\alpha+\lambda}{\lambda}\dfrac{\theta_S}{\theta_L\phi}\right)}{d\frac{\theta_S}{\theta_L}} = \frac{(r-\alpha+\lambda)}{\lambda\phi} > 0 \tag{22}$$

In these conditions, entrepreneurs with a lower ratio of short-term skills relative to long-term skills imply higher investment triggers, *ceteris paribus.* In these non-extreme projects, the perverse effect between the effort allocation decision and the ratio of skills justifies the existence of such a hypothesis.

Another important policy implication emerges. Firms may build a defensive tactic against this life-cycle problem by acquiring managers with higher relative skills towards ST projects. This way, firms avoid the life-cycle trap and mitigate the perverse effect experienced in companies where both shor and long-term projects are managed by the same entrepreneurs so that they revert the relationship between the investment trigger and the stage that the firm is in the life-cycle.

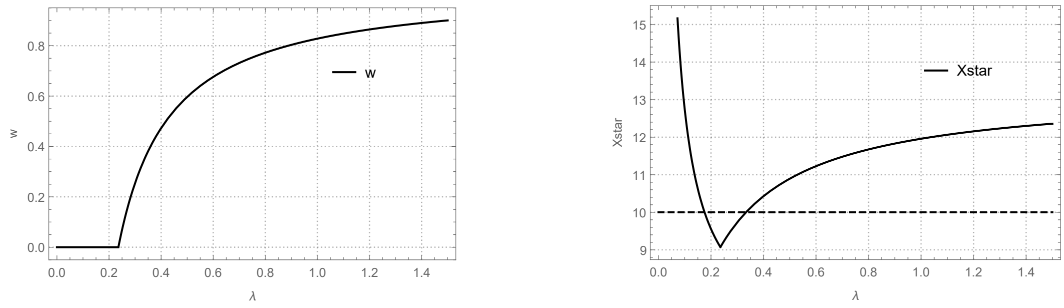
3.5 Technical Uncertainty

In the present subsection, we focus on the role of technical uncertainty – measured by the arrival rate of successful R&D projects (λ) – and we study how it affects the effort allocation and the investment timing decision. As usual, we then analyze under which conditions the life-cycle hypothesis prevails.

Panel A of Figure 7 displays the relationship between technical uncertainty and the effort allocation decision. We find that as the number of successful R&D projects arriving increases, i.e., the lower technical uncertainty given by a higher λ, the more effort is allocated by entrepreneurs towards ST projects. This result is similar to the one retrieved in the discussions about the role ϕ and θ_L. A lower level of technical uncertainty, *ceteris paribus*, increases the value of the project[6]. Therefore, entrepreneurs react by allocating less effort towards such a project. Then, as shown in Panel B, as the effort allocated towards the R&D project gradually decreases, the investment trigger increases. This effect occurs because, economically, knowing that after the investment takes place they will have to wait less time for a tech shock to arrive, it is optimal

[6]Because $\left(\frac{\lambda}{r-\alpha+\lambda}\right)$ increases with λ, a higher λ increases the present value of the option payoff. See equation 9.

for them to bear the investment cost K at a later stage.



(a) Entrepreneurs' effort allocation to ST (w) as a function of technical uncertainty (λ)

(b) Entrepreneurs' investment trigger (X^*) as a function of technical uncertainty (λ)

Figure 7: Parameters are as follows: $\bar{X} = 10, K = 8, r = 0.12, \alpha = 0.02, \sigma = 0.3, \theta_S = \theta_L = 1, \phi = 1.3$. In addition, $\lambda \in [0, 1.5]$.

As far as the life-cycle hypothesis goes, we retrieve that the state-dependency of the investment decision ultimately hinges upon a sufficiently high value of λ. Only for projects with low technical uncertainty, will the life-cycle hypothesis prevail. This is a consequence of the fact that entrepreneurs react to increasingly less uncertain LT projects by allocating less effort to such projects.

Note that these results have an important consequence. Let us assume that the firm has only one product so that the firm's assets-in-place cash flows are derived entirely from it. Therefore, the R&D project will have the function of improving the product, which will positively impact the value of that product (ϕ). We find that if a firm has a high level of technical uncertainty, where the arrival of this quality-enhancer innovation is less probable, the life-cycle hypothesis is more unlikely to prevail vis-à-vis a situation with a lower level of technical uncertainty, *ceteris paribus*. In other words, in product-industries in which innovation is not as frequent, the stage that the firm is in the life-cycle is less likely to be a drag in the investment timing decision. Therefore, in less stagnant industries, in which product innovation is recurrent, it is more probable that there is a natural innovation rotation force that leads firms to cease to be innovation-leading firms sooner via this life-cycle trap. At least, at this product level. Schilling and Hill (1998) have famously justified the product life-cycles shortening as a clear evidence of an accelerating trend in firms' new product development and innovation-seeking efforts. With our finding, we theorize this result.

4 Policy Implications

(...)

5 Industry Dynamics and R&D

TBD

6 Final Remarks

(...)

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