Optimal Investment in Human Capital under Migration Uncertainty

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Abstract

This paper develops a model of optimal education investment of an agent who has an option to emigrate. We distinguish between local and global human capital in a stochastic dynamic framework of optimal human capital investment. We analyze the time evolution of human capital in the source country and investigate the role of migration opportunities in the accumulation of different types of human capital. The analysis shows that the accumulation of human capital depends crucially on the level of uncertainty and the transferability of human capital across countries. Government subsidies are an important determinant of the composition of different types of human capital and can be crucial in alleviating the brain drain problem.

JEL Classification: J24, J61
Keywords: Human capital investment, Immigration, Real options, Skilled labor

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1 Introduction

How does the emigration of skilled workers affect source countries? Do source countries suffer from brain drain or can they benefit from the emigration of skilled workers? The four decades of research summarized by Docquier and Rapoport (2011) shows that countries can indeed experience beneficial brain drain (BBD) if, for example, there is uncertainty about the prospect of migration (see Mountford 1997, Stark et al. 1998 and Beine et al. 2001) or migration is on a temporary basis and migrants return to the source country (see Stark et al. 1997, Santos et al. 2003 and Dustmann et al. 2011). In these cases, the level of human capital in the source country is not necessarily depleted.

There is also evidence that prospects of migration increase the expected return to education. Individuals with migration prospects opt for more schooling in areas that are in demand overseas (International Office for Migration, 2003) and they put more effort in their studies (Kangasniemi et al. 2007). However, this evidence raises the question of how the composition of the human capital stock that is left in the source country as a result of migration changes over time. The prospect of migration can change the education choices of individuals who can invest their resources more in acquiring globally applicable skills that increase their mobility in international labor markets as opposed to locally applicable skills that increase their welfare mainly in the source country. If more individuals opt for education in more globally applicable skills, does the source country experience a surge in these type skills over time or does the country experience a "drain" of such skills as individuals emigrate? If the prospect of migration indeed affects the education choices, then even a country that experiences brain gain might end up with different sets of skills. The literature is relatively silent on the composition of the human capital stock that is left in the source country once individuals in a country emigrate.

In this paper, we develop a model of human capital investment under uncertainty that analyzes the education choices under uncertainty to shed some light on the interaction of migration options and human capital in the source country. In particular,
we investigate how human capital composition evolves over time in the source country when the skilled labor has the option to emigrate. We ask how government subsidies, destination countries’ immigration policies and the extent of applicability of country-specific human capital affect the brain drain phenomenon.

We distinguish between local and global human capital in our model. Local human refers to skills that generate value in the source country; examples of which include networking, having local language skills and knowing local technical standards. On the other hand, global human capital captures skills that are valued outside of the country too. Examples of global human capital include skills in the English language, computer programming, math and data science, professional certificates (e.g., CFA and PMP), and even a nursing degree. From the economic perspective, the major difference between the two is that the local human capital loses part of its value once the agent emigrates; whereas, global human capital can result in even a higher wage in the destination country.

In our model, the agent decides about optimal investment in the two types of human capital taking uncertainties into account. The uncertainty in the model comes from two major sources. The first source of uncertainty concerns the gap between the aggregate productivity between the destination country and the source country, which we assume is stochastic\footnote{We use the term aggregate productivity to keep the model concrete and economically meaningful. In a broad sense, the aggregate productivity can include soft factors such as social network, social capital, and life quality factors such as crime and air pollution. The migration literature shows the importance of such social factors. For example, \cite{Vergalli2008} draws attention to the role of an existing immigrant community in the destination country while \cite{DreherPoutvaara2005} link the study opportunities abroad to the subsequent migration flows. Thus, we invite the reader to hold a broad interpretation of the “aggregate productivity” term to account for these factors as well as other factors that are likely to attract the potential migrant to the destination country.}. The stochastic nature of the relative aggregate productivity between the two countries, and the possibility of migration in the future, generates an \textit{option value} for education and motivates the skilled labor to invest in sets of skills that may be more relevant in the country of immigration destination.

The destination country’s policy towards immigration constitutes the second source of uncertainty. We assume that the destination country screens applications for immi-
migration. This is in line with the recent tendency of destination countries to make immigration more selective (see Bhagwati and Hanson 2009) resulting in only a certain portion of the applicants admitted to the destination country.

The agent in our model holds an option to emigrate and determines the optimal time of emigration. The agent simultaneously optimizes her investment in human capital both in the source and the destination countries. Although human capital is perceived as an illiquid asset (Friedman 1962), we argue that not all human capital investments are illiquid to the same extent. A degree in the IT field can, for instance, substantially increase an Indian student’s migration prospect to a country like Germany that encourages immigration of skilled IT specialists. On the other hand, a degree in certain types of humanities may be more country-specific and need not increase the chances of emigration. In other words, accumulating skills that can be used globally is similar to acquiring a liquid asset. This is what we term global human capital: an individual can transfer her global human capital worldwide at little or no decrease in value.

The main results of the paper are the following. First, we show that targeted government subsidies of country-specific skills can be instrumental in reducing the brain drain effect as a result of migration. Government subsidies reduce the relative cost of obtaining education and discourage emigration. Importantly, however, subsidies can change the composition of skills in the source country: skilled labor invest considerably more in local human capital (i.e., country-specific skills). Second, we find that destination countries’ immigration policies have a nontrivial effect on the level and composition of human capital in the source country. Although lax immigration policies increase investment in global human capital and reduce investment in local human capital, it leads to an increase in the aggregate human capital only over the short- to medium-term. In the long-term, lax immigration policies of the destination countries lead to a decrease in the aggregate human capital in the source country.

2In 2000, Germany introduced special work visas to specialists in information, communications, and technology (Lowell 2002). Docquier and Rapoport (2011) report that the skilled migration to the OECD countries increased by 70% in the 1990s in contrast to a 30% increase for low-skilled migrants.
Third, our results highlight the importance of skilled labor’s ability to invest in their human capital in the destination country. In particular, source countries experience a more severe brain drain effect over the long-term when skilled labor have the opportunity to invest in their human capital further in the destination country. This has important implications for countries in which students prefer to pursue their education abroad. Finally, the model emphasizes the importance of economic integration between the source and the destination countries to the development of global human capital in the source country. Economic integration reduces the loss of local human capital due to emigration since a higher portion of local human capital is valued in the destination country. This encourages agents to accumulate more local human capital in the source country and staves off brain drain over time.

Our paper contributes to the literature by highlighting the effect of migration prospect on two distinct types of human capital and by analyzing the optimal human capital accumulation both in the source country and the destination country. We assume that, compared to the local human capital, global human capital can be more costly to accumulate; however, it carries an inherent option value: once the agent optimally decides to migrate, the average (and marginal) productivity of global human capital jumps according to the aggregate productivity of the destination country.

The rest of the paper is organized as follows. We discuss the connection to the literature in Section 2. Section 3 introduces the model. In Section 4, we discuss the implications of our model and carry out comparative statics. The limitations and possible extensions are discussed in Section 5. Section 6 summarizes and concludes the paper.

2 Relation to Literature

Our paper contributes to the literature on brain drain. Although the early brain drain literature recognizes the benefits associated with migration such as remittances and return migration (Bhagwati and Hamada 1974 and McCulloch and Yellen 1977),
studies such as Miyagiwa (1991) and Haque and Kim (1995) conclude that migration leads to the loss of the educated and the highly skilled people. This view has been increasingly challenged by several studies that argue that the possibility of emigration induces optimizing agents to invest more resources in human capital (Mountford 1997, Stark et al. 1998, Vidal 1998, Beine et al. 2001 and Katz and Rapoport 2005). Because only a fraction of those who invest in human capital can indeed emigrate, the average level of human capital and average productivity in the country of origin can increase even after emigration is netted out; a phenomenon labeled as ”brain gain.”

Similar to our paper, Poutvaara (2008) also distinguishes between the globally applicable skills from locally applicable skills. He studies the effects of migration on governments’ and individuals’ investment in internationally applicable and country-specific education. He demonstrates that the externalities associated with the migration possibility induces governments to underinvest in the internationally applicable skills. As opposed to Poutvaara (2008), we explicitly take into account the immigration policy of the destination country and study the evolution of human capital under both migration uncertainty and aggregate productivity uncertainty.

Our paper is also related to the real options literature that studies the impact of uncertainty on migration flows and human capital accumulation. Jacobs (2007) and Hwang et al. (2012) model human capital accumulation in a real options framework to study the effect of uncertainty on returns to human capital investment. The impact of uncertainty in wage differentials on migration flows has also been increasingly recognized in the literature. Burda (1995) observes that despite the high wage differential, migration from East Germany to West Germany after the fall of the Berlin Wall proceeded at a slow rate. He argues that the ability to delay emigration can explain this inertia. The positive value of delaying emigration is also documented in Locher (2002) and Anam et al. (2008). Furthermore, the stylized fact that migration flows tend to occur in waves (see Moretti 1999 and Moretto and Vergalli 2008 for further discussion.) suggests that the irreversibility associated with migration in the face of uncertainty plays an important role. More recently, Vergalli (2011) analyzes the role of migrant
communities and uncertainty in the migration (entry) and return (exit) decisions. We contribute to this literature by analyzing the impact of both the uncertainty about the aggregate productivity differential and the probability of migration on the accumulation of different sets of skills. Furthermore, our model captures the full dynamics of human capital investment by exploring the effect of accumulating human capital in the destination country as opposed to in the source country. This allows us to study the impact of student mobility on the brain drain.

3 The Model

Our model is based on the decision problem of an agent in the country of origin. The agent must determine both the timing of the emigration and her investment in human capital. For ease of exposition, we develop the model in the following subsections.

3.1 The Emigration Option

There are two countries, designated as the home (or source) and the destination (or host) countries. We assume that the decision to emigrate is completely irreversible. Return migration, therefore, is ruled out once the agent has emigrated from the home country to the destination country. The decision to emigrate from the source country to the destination country is motivated by an index of difference in aggregate productivity between the two countries. We let $x_h(t)$ and $x_d(t)$ denote the aggregate productivity in the source and the destination countries, respectively. For simplicity, we normalize the aggregate productivity in the source country and assume $x_h(t) = 1$. Define the ratio of productivity at time $t$ as:

$$ x(t) \equiv \frac{x_d(t)}{x_h(t)} $$

\textsuperscript{3}Section 4 discusses the modeling implications of relaxing this assumption.
The ratio is assumed to follow a geometric Brownian motion:

$$dx(t) = \mu x(t) dt + \sigma x(t) dB(t)$$  \hspace{1cm} (1)

where $dB(t)$ denotes the increments of a standard Brownian motion and $\mu$ and $\sigma$ are the constant drift and diffusion parameters. To keep the model simple, the “aggregate productivity” variable summarizes several factors that are relevant for migrations in one factor.

The agent’s time preference parameter, $r$, is specified exogenously with the standard assumption $r > \mu$.\footnote{This common assumption is necessary to prevent solutions with infinite present value.} Note that when $x(t) > 1$, the aggregate productivity in the destination country is higher than that in the source country.

Emigration to another country is costly. Besides the direct monetary costs of moving to another country, the individual incurs indirect costs in the form of losing established networks in the home country and transferability of existing benefits such as pension plans. The model summarizes these costs by $I$. Economically, $I$ can be considered as the opportunity cost of migration. Although the cost of migration can be incurred over time, we assume for parsimony that $I$ is incurred once when the agent decides to emigrate.

Given the above benefits and costs, the agent must choose the optimal time to emigrate to the destination country as well as the optimal rate of investment on local and global human capital. The interesting feature is the bi-directional feedback effect between the perceived migration probability and investment on human capital. Mathematically speaking, the model includes both the optimal stopping time problem, regarding migration decision, as well as the continuous control of human capital investment.

The agent emigrates when the aggregate productivity in the destination country is sufficiently higher than in the source country taking into account the opportunity cost of migration. Note that on the basis of the conventional NPV analysis, emigration
would take place when the expected discounted benefit from emigration exceeds the costs. However, when there is uncertainty regarding the future evolution of the gap of aggregate productivity, the agent might be better off by postponing the emigration decision and remaining in the source country if the aggregate productivity ratio is not expected to grow. The optimal emigration time is then characterized by the stopping time, $\tau$, defined as:

$$
\tau = \inf \{ t > 0 : x(t) \geq x^* \}
$$

where $x^*$ is the critical value of the aggregate productivity ratio that induces the agent to emigrate and must be endogenously determined.\footnote{See Dixit and Pindyck (1994), pp.103-104 for the conditions under which the optimal stopping time corresponds to a single trigger.}

Apart from the differences in aggregate productivity, the second source of uncertainty arises from the immigration policy of the destination country. The destination countries often have laws that regulate immigration. Depending on the restrictiveness of these regulations, the agent may not emigrate even if the aggregate productivity differences justify emigration. We denote by $\lambda$ the exogenous probability that the agent will be admitted to the destination country. The treatment of this case, however, is relegated to the appendix. In the sequel, we develop the model without this source of uncertainty.

In the next subsection, we describe the individual preferences and education choice given the above structure in the economy.

### 3.2 Human Capital Accumulation

Besides determining the optimal time to emigrate to the destination country, the agent must also determine her investment in human capital. The amount of accumulated human capital together with the aggregate productivity indices determine the welfare of the agent in both the source and the destination countries.
The agent accumulates two different types of human capital. The first is the *global* human capital. As briefly discussed in the Introduction, global human capital refers mainly to skills that can be applied in international labor markets. Obtaining an education in IT or developing language skills can be crucial in ensuring that the immigrant has higher aggregate productivity after migration. In other words, by accumulating global human capital, the agent acquires a relatively liquid asset: she can transfer her global human capital worldwide at little or no decrease in value. Let $g(t)$ denote the stock of global human capital at time $t$. The agent is assumed to invest in her global human capital at the rate $u(t)$. Furthermore, we assume that there is no depreciation in the knowledge stock. Thus, global human capital evolves as:

$$dg(t) = u(t)dt$$

where $u(t) \in [0, \bar{u}]$ and $g(0) = g_0$ is given. The constraint on the rate of human capital accumulation is motivated based on the natural, physical, and social limitations of humans. For example, one can study/practice only up to a certain number of hours per week. The learning resources are also limited; there is only a certain number of classes/books/courses to learn a topic.

As opposed to the global human capital, *local* human capital is specific to the source country. These skills are valued mainly in the source country and can only be partly transferred to the destination country. Studying the local law system can be an example of local human capital. We let $k_h(t)$ denote the level of local human capital the agent possesses at time $t$. The law of motion of $k_h(t)$ is given by:

$$dk_h(t) = q_h(t)dt$$

where $q_h(t) \in [0, \bar{q}]$ and $k_h(0)$ is given.

\textsuperscript{6}In fact, there may be entry barriers even for those with globally applicable skills. For instance, a medical doctor may be required to undergo a certain training period or take certain exams before she can practice. Our model can easily be extended to incorporate such settings.
In the source country, both types of skills are perfect substitutes. However, local human capital behaves like a "risky" investment because it will lose a (possibly large) fraction of its value in the case of migration. Thus, the problem of the agent resembles an optimal portfolio accumulation with one risk-less and one risky asset.\footnote{The difference with standard portfolio theory is that the riskiness of local human capital is subject to agent’s optimal decision. If the agent never emigrates, there is no risk for the local human capital.}

The agent can transfer only a fraction of her local human capital to the destination country. Let $\beta \in [0, 1]$ denote the portion of local human capital transferred to the destination country. The parameter $\beta$ can be perceived as measuring the extent of economic integration between the source and the destination countries. Migration of skilled labor need not flow from the developing to the developed countries (i.e., South-North immigration) but could also occur among the developed countries (i.e., North-North immigration).\footnote{For instance, in the aftermath of World War II, many young British scientists emigrated to the US in the pursuit of higher salary premium and better facilities for scientific research. See Bhagwati and Hanson (2009) for further discussion.} In particular, economic and political unions such as the European Union can enable individuals to transfer a larger part of their local human capital to the destination country. The transferability parameter can also relate to the cultural and historical proximity of the source and the destination countries. The more similar the countries are, the higher is the fraction of local human capital that can be transferred to the destination country.

An important contribution of this paper is to consider the full dynamics of human capital accumulation both before and after emigration. We assume that after emigration, the agent can no longer accumulate human capital related to the source country. Therefore, after the emigration at $\tau$, the level of local human capital is fixed at $k_h(\tau) = \bar{k}$. However, the agent continues to accumulate global human capital and local human capital pertaining to the destination country. Accumulation of further global human capital in the destination country can be seen as a proxy for student flows. Accumulation of local human capital related to the destination country, on the other hand, can be an important factor in facilitating the integration to the society in...
Let $k_d(t)$ denote the stock of local human capital in the destination country. We assume that:

$$dk_d(t) = q_d(t)dt$$

(4)

where $q_d(t) \in [0, \bar{q}]$ and $q_d(0)$ is given.

The accumulation of both global and local human capital comes at a cost. We assume the agent incurs quadratic costs both in the source and the destination countries:

$$c_h(t) = \frac{c_1 u(t)^2}{2} + \frac{c_2 q_h(t)^2}{2}$$

$$c_d(t) = \frac{c_3 u(t)^2}{2} + \frac{c_4 q_d(t)^2}{2}$$

(5)

where $c_i \in \mathbb{R}^{++}, i \in \{1, 2, 3, 4\}$ are constants. The parameters $c_i$ can be interpreted as a policy tool of the government. For example, a subsidy that supports the accumulation of local human capital in the source country can achieve this by lowering the cost of local human capital education relative to that of global human capital. Similarly, the government of the destination country can attract skilled migrants by lowering the cost of human capital accumulation.

The next subsection describes the individual’s preferences and payoffs in terms of her human capital and the aggregate productivity in the country of residence.

### 3.3 Preferences and Payoffs

The agent lives for a finite period of time $T$ and is assumed to be risk averse with a utility function $U(\cdot)$ in the source country and the destination country after emigration. Agents receive utility on their consumption. For simplicity we assume zero saving; thus, the agent’s consumption is equal to her productive output. Based on this formulation, the agent’s utility is a function of the total human capital that she has accumulated weighted by the aggregate productivity in each country. Thus, in the source country,

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9 An example could be a student studying in English in a non-English-speaking country but learning the local language at the same time.
the agent’s utility is determined by \( U(x_h[g + k_h]) \) whereas in the destination country, \( U(x_d[g + k_d + \beta \bar{K}] \) determines the utility. This formulation is motivated by the fact that a unit of human capital produces different outputs in different contexts. A skilled petroleum engineer in Venezuela has a different level of income/consumption in the 1980s versus the 2010s. The same petroleum engineer will earn a higher income if she migrates to Norway or Canada.

The net payoffs to the agent after accounting for the cost of accumulating human capital, \( \pi_h(s_h, t, m_h) \) and \( \pi_d(s_d, t, m_d) \), are then given by:

\[
\begin{align*}
\pi_h(s_h, t, m_h) &= U(g(t) + k_h(t)) - c_h(t) \\
\pi_d(s_d, t, m_d) &= U(x(g(t) + k_d(t) + \beta \bar{K})) - c_d(t)
\end{align*}
\]

(6)

Note that we have substituted \( x_h = 1 \) for the aggregate productivity in the source country, and \( x(t) \) for \( x_d(t) \) following our discussion in Section 3.1. We now characterize the problem and discuss the solution in the next section.

### 3.4 Agent’s Dynamic Optimization Problem

The agent simultaneously chooses her rate of investment in both types of human capital and the optimal time to emigrate to the destination country to maximize her lifetime expected utility. The optimal stopping time problem of migration is adapted to the filtration generated by the stochastic processes of relatively aggregate productivity, the stock of human capital, and the possibility of migration.

Let \( V_h(g, k_h, x, t) \) denote the value function before the emigration decision has been made. Then the problem can be posed as:

\[
V_h(s_h, t) = \max_{m_h, \tau} E_0 \left\{ \int_0^{\min(\tau, T)} \pi_h(s_h, t, m_h) e^{-rt} dt + e^{-r\tau} [V_d(s_d, t) - I]\right\}
\]

subject to equations (1), (2) and (3). The maximization is carried out with respect to
the controls \( m_h = \{u, q_h\} \) and the optimal time of emigration, \( \tau \). The first term in the expectation captures the utility accruing to the agent until her decision to emigrate at \( \tau \). After emigration, the agent continues to optimize her investment in human capital, which is captured by the value function \( V_d(s_d, t) \). Note that the cost of emigration, \( I \), is incurred at the emigration time, \( \tau \).

To solve the problem in equation (7), we proceed backward: assuming that the agent has already emigrated, we first analyze the agent’s optimization in the destination country. Then, we address the initial decision problem in equation (7).

After the agent emigrates, she optimizes her investment in \( g(t) \) and \( k_d(t) \) and solves:

\[
V_d(s_d, t) = \max_{m_d} \mathbb{E}_{\tau} \left\{ \int_{\tau}^{T} \pi_d(s_d, t, m_d) e^{-rt} dt \right\} 
\]

subject to equations (1), (2) and (4).

Note that in addition to receiving \( V_d(s_d, t) \) net of the emigration cost \( I \), the agent gives up the utility she would otherwise have derived in the home country had she decided not to emigrate. We denote the lost value of utility by \( V_s(s_d, t) \):

\[
V_s(s_d, t) = \max_{m_d} \mathbb{E}_{\tau} \left\{ \int_{\tau}^{T} \pi_h(s_d, t, m_d) e^{-rt} dt \right\} 
\]

In order to determine the optimal emigration time, the usual value-matching and smooth-pasting conditions, around the state variables, are invoked:

\[
\begin{align*}
V_d - I &= V_s \\
\frac{\partial V_d}{\partial x} &= \frac{\partial V_s}{\partial x}
\end{align*}
\]

The following proposition characterizes the optimal investment rates in the human capital both before and after emigration to the destination country.
Proposition 1. Before emigration, the agent’s investment policy is characterized by:

\[ u^*(t) = \frac{1}{c_1} \frac{\partial V}{\partial q} \]
\[ q^*_h(t) = \frac{1}{c_2} \frac{\partial V}{\partial k} \]

(11)

Optimal investment rates in global and local human capital after emigration are given by:

\[ u^*(t) = \frac{1}{c_3} \frac{\partial V}{\partial q} \]
\[ q^*_d(t) = \frac{1}{c_4} \frac{\partial V}{\partial k} \]

(12)

Proof. See Appendix A.

The investment rates before and after emigration are given by the ratio of the marginal product of human capital to its respective marginal cost. Note that government subsidies, that lower the costs of acquiring education, increase the agent’s investment in human capital. We can further characterize the solution presented in Proposition 1 by specifying the utility function. Suppose that the agent’s utility is of constant relative risk aversion (CRRA): \( U(H) = \frac{1}{1-\gamma} H^{1-\gamma} \) where \( H = g + k_h \)

\( (H = g + k_d + \beta \bar{k}) \) denotes the stock of global and local human capital weighted by the aggregate productivity in the source (destination) country. The value function, \( V(H) \), is then of the form \( V(H) = \frac{A_i}{1-\gamma} H^{1-\gamma} \) where \( A_i > 0, i \in h, d \) is a constant. The following corollary describes the agent’s investment behavior as a function of particular types of human capital she has accumulated.

Corollary 1. In both the source country and the destination country, the rate of investment in global human capital decreases in (i) the stock of global human capital accumulated and (ii) the stock of local human capital accumulated.

Similarly, the rate of investment in local human capital decreases in (i) the stock of global human capital accumulated and (ii) the stock of local human capital accumulated.


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Time Horizon $(T)$</td>
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<tr>
<td>Discount Rate $(r)$</td>
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<tr>
<td>Drift $(\mu)$</td>
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<tr>
<td>Volatility $(\sigma)$</td>
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<td>Initial Aggregate Productivity Difference $(x_0)$</td>
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<td>Max. Local Human Capital Investment Rate $(\bar{q})$</td>
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<tr>
<td>Max. Global Human Capital Investment Rate $(\bar{u})$</td>
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<tr>
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<td>Risk Aversion Parameter $(\gamma)$</td>
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<td>Migration Probability $(q)$</td>
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<tr>
<td>Transferability of Local Human Capital $(\beta)$</td>
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</tr>
</tbody>
</table>

Table 1: Baseline Parameter Values

in respective countries.

Proof. See Appendix A.

In the next section, we resort to numerical procedures to explore the optimal policies of the agent and how government policies affect the optimal investment behavior.

4 Numerical Analysis and Policy Implications

Our aim in this section is to qualitatively characterize the optimal path of levels of both the global and local human capital that remains in the source country when skilled labor have the opportunity to emigrate. We also analyze the impact of labor market factors as well as government policies on the agents’ endogenous decisions to opt for different types of education. The numerical analysis assumes a constant relative risk aversion (CRRA) utility. The benchmark case parameters are presented in Table 1.

4.1 Education Subsidies and Brain Drain

Governments often provide education subsidies to encourage investment in human capital. Subsidies in the form of flat income tax rates or tuition support and schol-
Figure 1: Local Human Capital Subsidy and Remaining Aggregate Global, Local and Total Human Capital in Source Country

Figure 1 studies the effect of a government subsidy that reduces the cost of accumulating local human capital relative to the cost of investing in global human capital. The figure compares the level of global (the left panel) and local (the middle panel) human capital accumulated under a government subsidy program to the benchmark case in which the government offers no such subsidy. As expected, a government subsidy leads to a decrease in global human capital accumulated while causing an increase in the accumulated local human capital over time. The effect of government subsidy becomes more pronounced over time: the wedge between accumulated human capital with subsidy and that without subsidy increases in both panels as the time passes. Furthermore, the government subsidy serves to reduce the gap between global human capital and local human capital that exists when there is no such subsidy. With the subsidy, local human capital remaining in the source country grows faster than the

Keane and Wolpin (1997) show, for example, that a $2,000 tuition subsidy increases college graduation rate by 8.4%.

As Poutvaara (2008) observes, in an open economy, the migration opportunity enables the destination country to partially capture the benefits of human capital investment in the source country. To encourage investment in local human capital and increase the aggregate level of human capital net of the migration, the source country can augment its provision of public subsidies to the accumulation of local human capital. In our model, the effect of government subsidies is captured by the cost parameters $c_1$ and $c_2$ in equation (5).
global human capital. This leads to a more balanced skill profile in the source country over time.

The right panel of Figure 1 shows that the total human capital in the source country is higher under a subsidy program. As in the left and the middle panels, the increase in total human capital in the source country becomes more pronounced over time. This result is reasonable: due to the migration option, the decrease in global human capital is much smaller in magnitude than the increase in local human capital, resulting in an overall increase in total human capital accumulated. Essentially, the government subsidy decreases the brain drain in the source country while changing the composition of skills that the skilled labor accumulate.

4.2 Uncertainty and Human Capital Accumulation

Recall that our model accommodates two sources of uncertainty. The first is the migration policy of the destination country. The destination country may, for instance, allocate quotas and place screening procedures before the migration. Therefore, the agent may not necessarily emigrate even if the aggregate productivity differences justify emigration (i.e., when $x_t > x^*$). We analyze the effect of an exogenous probability of migration, $\lambda_t$, on the formation of global and local human capital in the source country in Figure 2. The left panel of the figure shows that as the destination country rules regarding immigration become laxer, the amount of global human capital increases over time in the source country. Conversely, the level of local human capital in the source country is negatively associated with the probability of migration. Comparing the left and middle panels reveals that the wedge between global human capital and local human capital increases in the probability of migration. This result is important as it shows that destination countries’ policies towards migration can lead to a dearth of local human capital relative to global human capital in the source countries.

Surprisingly, the effect of the destination country’s immigration policy on total human capital is not straightforward. The right panel of Figure 2 shows that the relation between the probability of migration and aggregate human capital is not monotonic.
Figure 2: Probability of Migration and Human Capital Accumulation

In particular, total human capital is lower when the destination country is strict ($\lambda = 0$ in the figure) than when it admits immigrants ($\lambda = 0.5$ and $\lambda = 1$) for a certain period of time. After a certain period of time, however, the pattern reverses, and the total human capital under a stricter destination country policy exceeds that of a more lenient policy. This result is driven mainly by an "age effect." Recall from Table 1 our time horizon is 20 years. As time passes emigration becomes less likely since the probability that the aggregate productivity differential, $x_t$, exceeds the trigger level, $x^*$, decreases. This slows the investment in global human capital: the accumulated global human capital flattens as time passes as shown in the left panel. This is not the case when there is no possibility of migration. Thus, while over the short run, the prospect of migration can lead to a brain gain effect, this effect can be reversed in the long run.

Figure 3 explores the effect of the second source of uncertainty in the model on the level of human capital in the source country: volatility of aggregate productivity differential, $\sigma$, between the destination and the source countries. The left panel of the figure shows that when there is uncertainty, the level of global human capital increases significantly. On the other hand, there is a substantial decrease in the amount of local human capital the agent accumulates. This result is mainly driven by the migration option: as the aggregate productivity differential, $x_t$, becomes more volatile

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10 The age effect in this paper is similar to the effect of time to expiration on option prices in American call and put options. An option with a longer time to expiration is more valuable since the probability of the option ending in the money is higher.
the probability that the aggregate productivity differential will be sufficiently high to justify emigration (i.e., $x_t \geq x^*$) increases as well. This induces the agent to invest more in global human capital and substantially less in local human capital.

At the same time, the overall pattern depicted in the left and middle panels of Figure 3 can exacerbate the brain drain phenomenon in the source country. This is shown in the right panel of Figure 3. The aggregate human capital in the source country is highest when there is no volatility in the aggregate productivity differential. When there is uncertainty, the increase in the level of global human capital need not compensate the fall in the local human capital accumulated in the source country, resulting in brain drain. This suggests that government subsidies can be more crucial when the volatility in aggregate productivity differences is higher.

4.3 Transferability and Human Capital Accumulation

The migration literature shows that an individual is more likely to emigrate if the destination country is culturally close to the country of origin. A common language, for instance, can be an important factor (Chiswick and Miller 2003 and Pedersen et al. 2008). Similarly, an economic union or a more integrated labor market across countries such as the EU can facilitate emigration by increasing the transferability of local human capital. How does the prospect of emigrating to a destination country similar to the source country affect the brain drain phenomenon? We address this question by analyzing the effect of the parameter $\beta$. Recall that in our model, the
parameter $\beta$ proxies for the ease with which an agent can integrate into the destination country. It is also possible to think of $\beta$ as the extent to which the source and the destination countries are economically and institutionally integrated. For instance, if the two countries have similar bodies of business law, an agent in the source country can benefit from her knowledge of the local law system in the destination country.

The left and the middle panels of Figure 4 show that a stronger ability to transfer local human capital to the destination country results in a lower global human capital and a higher local human capital in the source country, respectively. The effect of $\beta$ is particularly pronounced for the accumulation of local human capital. A higher $\beta$ essentially makes local human capital more "liquid": the agent has the opportunity to use the source-country-specific skills in the destination country with little or no loss in the applicability of those skills. This increases the marginal value of the investment in local human capital.

The right panel shows that the source country enjoys a brain gain effect over time when the destination country is more similar to the source country. These results suggest that immigration among culturally similar or economically integrated areas could potentially benefit the source country.

### 4.4 Cost of Migration

Migration has two types of cost: an endogenous cost of losing local human capital and an exogenous cost of moving. The second component can be small if the country of destination is geographically and culturally close (e.g., moving from Germany to
Figure 5: Cost of Migration and Dynamics of Human Capital

Switzerland) or large if the physical and personal cost of the move is significant.

To analyze the impact of one-time migration cost, Figure 5 shows the impact of the cost of migration on the dynamics of human capital. We plot the dynamics of human capital for two levels of migration cost: a zero cost and a large (possibly prohibiting) level of cost. As expected, incentives to invest in global (local) human capital are lower (higher) when migration costs are high (the left and middle panels). Interestingly, aggregate human capital in the source country is greater when migration costs are high (the right panel). This is driven by two distinct channels. The first is the incentives effect: when migration costs are low, the agent has weaker incentives to invest in local human capital. Compared to a setting with substantial migration costs, weaker incentives create a greater discrepancy in the level of local human capital left in the source country over time (the middle panel). The second channel is directly linked to the migration option: when costs are low, a certain portion of those who have accumulated global human capital will eventually emigrate. This outflow of skilled labor explains the curvature of the global human capital in the left panel of Figure 5. Both the incentives and migration channels reinforce each other and result in a lower aggregate human capital level in the source country.

4.5 Student Mobility and Brain Drain

One crucial factor in explaining the brain drain phenomenon is the international flow of students. Countries such as the US and Canada attract foreign students from
developing countries. As Dreher and Poutvaara (2005) observe, studying abroad can provide a toehold for the students and ease the subsequent migration.

An important implication of the student flows for the source country is that agents can defer the accumulation of part of their human capital until after they have moved to the destination country. For instance, a student may choose to pursue studies in the IT field in the US and only invest resources to learn English in the source country before she moves to the US. In this case, the brain drain for the source country may be exacerbated compared to a case in which education is completed in the source country.

By analyzing the full dynamics of human capital investment both in the source and the destination countries, our model captures the effect of deferring human capital accumulation until after the migration decision. Figure 6 compares the setting in which the agent continues to accumulate human capital (both global and destination country-specific local) in the destination country to that in which human capital accumulation is assumed away in the destination country. The left panel of the figure shows that the level of global human capital with no further investment opportunity in the destination country is lower initially than the level of global human capital with the investment opportunity. However, the difference between the two settings is small. The pattern reverses itself as the model approaches the time horizon of the agent. This pattern is reasonable: when there is little time to take the emigration option, and the agent cannot invest in global human capital in the destination country, she accelerates her investment in global human capital in the source country. The middle
panel shows that the level of local human capital in the source country is substantially higher when agents do not have the opportunity to invest in human capital in the destination country. The ability to transfer at least part of the local human capital to the destination country induces skilled labor to accumulate more local human capital in the source country.

On the other hand, the results support the hypothesis of an exacerbated brain drain problem in the source country. This is shown in the right panel of the figure. When skilled labor has the opportunity to invest further in their human capital in the destination country the aggregate human capital decreases in the source country. The effect is particularly pronounced over time.

5 Caveats

In this section, we discuss the implications of relaxing some of the assumptions we made to keep the model tractable.

5.1 General Equilibrium Effects

Our model is based on the micro-level decision of individual agents (households). The feedback of migration decisions (and hence the human capital composition) on macro-level variables in the source (and potentially also the destination) country are not included in the model. As a result, variables such as aggregate productivity and wages are exogenous to the model.

These assumptions are valid as long as the share of outgoing and incoming human capital is small for source and destination countries. However, if the source or destination countries are significantly affected by the migration of skilled labor, they may change their policies. The source country may adopt support policies to retain its talent. The extreme case of such policies was observed in former communist countries, where many citizens were not allowed to leave the country. The more free-market version would be to provide loans to start-up companies, increase the wage of skilled
labor, incentives for part-time affiliations with research centers and universities, etc.

An appropriate framework for the general equilibrium is a mean-field game model (Huang 2013), in which the economy consists of a large number of individual agents, and the level of aggregate variables is determined by the aggregation over the continuum of agents.

5.2 Return Migration

Return migration is a common phenomenon among skilled workers. For a recent review of temporary and return migration see (Dustmann and Görlach 2016).

In the context of the model, if the agent can freely move back and forth between two countries, she gains a hedging option of returning to the source country in case the relative productivity in the destination country drops below one.

The problem, in this case, resembles a switching option model, where the agent chooses which regime of production (i.e., the source or the destination country) to be at each moment.

5.3 Endogenous Probability of Migration

We assume that the probability of migration is exogenous to agent’s human capital. One example of this is a US green card lottery, in which the human capital of the applicant plays no role. Under this scenario, the feedback is from the probability of migration to human capital investment and not the other way.

Alternatively, one can consider merit-based migration systems (e.g., Canada and Australia), in which a higher level of education and experience will significantly increase the likelihood of migration option. If this is also allowed in the model, the incentive for global human capital will increase. Moreover, a threshold behavior may emerge: if migration possibility is zero for global human capital below a certain range, agents with abilities below a certain level may push to the corner and only invest in the local human capital.
5.4 Remote Working

The globalized world economy, empowered with the advances in IT, has enabled high-skilled workers to even stay home and work for companies located in other countries. For example, it is becoming more common for programmers to work remotely for a company in another country.

In this case, the agent does not necessarily lose the local human capital but at the same time is able to enjoy selling her global human capital at a different rate than the domestic economy.

5.5 Obsolescence of Skills

Certain skills may turn obsolete over time as advances in technology and research become available. In certain areas, technology may even replace skilled workers. One case in point is the potential use of AI in the portfolio management services. Individuals would, therefore, need to either keep up with changes in their industries or even change fields. Our model does not incorporate such interactions between the broader disruptions in skill sets and the individual’s human capital accumulation choice. These interactions can be taken into account by introducing a depreciation factor and/or uncertainty into the human capital accumulation processes.

5.6 General Production Function

The baseline model considers local and global human capital as perfect substitutes (but with different levels of riskiness). An alternative modeling approach is to consider a CES production function using both types of human capital as inputs. Such a formulation allows for comparative statics on cases where local and global human capital are substitute or complement of each other.
6 Conclusion

This paper revisits the brain drain phenomenon and explores how the migration option affects agents’ education choices and the remaining human capital in the source country. Importantly, our model distinguishes between global human capital and local human capital. The model, therefore, investigates not only whether a source country experiences brain drain but also how the composition of human capital is affected by the prospect of migration and government policies. While the migration option favors the investment in global human capital, local human capital is important to provide subsistence in the source country.

The model shows that government subsidies of education, economic integration or cultural similarity with the destination countries can alleviate the brain drain phenomenon. However, the effect of these factors goes beyond whether the level of human capital in the source country increases: it changes the type of human capital the skilled labor invest in.

On the other hand, the possibility of continuing education in the destination country can be detrimental to the stock of human capital in the source country, especially in the long-run. This has important implications for countries where students move abroad to pursue their education. We also find that destination countries’ policies towards immigration are crucial to what extent source countries will experience a brain drain. While there seem to be gains to aggregate human capital in the source country in the short-run, the long-term effects can exacerbate the loss of skilled labor as they move abroad.
Appendix

Appendix A: Proofs

Proof of Proposition 1: The Bellman equation for value functions $V_i(s_i, t), i \in \{h, d\}$, is given by

$$rV_i(s_i, t) = \max_{m_i} \left\{ \pi_i(s_i, t, m_i) + \frac{1}{dt} \mathbb{E}(dV_i) \right\}$$  \hspace{1cm} (A-1)

Expanding the value functions by Itô's Lemma, and taking expectations, we obtain the following expressions for the value functions $dV_h(s_h, t)$ and $V_d(s_d, t)$:

$$\frac{dV_i}{dt} = \frac{\partial V_i}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_i}{\partial x^2} + \mu x \frac{\partial V_i}{\partial x} + u \frac{\partial V_i}{\partial g} + q_i \frac{\partial V_i}{\partial k_i}$$  \hspace{1cm} (A-2)

Substituting equation (A-2) into equation (A-1) yields:

$$0 = \max_{m_i} \left\{ \pi_i(s_i, t, m_i) + \frac{\partial V_i}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_i}{\partial x^2} + \mu x \frac{\partial V_i}{\partial x} + u \frac{\partial V_i}{\partial g} + q_i \frac{\partial V_i}{\partial k_i} - rV_i \right\}$$  \hspace{1cm} (A-3)

Optimizing with respect to the controls $m_i = \{u, q_i\}$ yields the expressions in the proposition. Using (12) and (11) in the Bellman equations yields a system of nonlinear, second-order PDEs. In particular, we have the following system before and after emigration, respectively:

$$\begin{aligned}
\frac{\partial V_h}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_h}{\partial x^2} + \mu x \frac{\partial V_h}{\partial x} + \frac{1}{2c_1} \left( \frac{\partial V_h}{\partial g} \right)^2 + \frac{1}{2c_2} \left( \frac{\partial V_h}{\partial k_h} \right)^2 - rV_h \\
+ \pi_h = 0
\end{aligned} \hspace{1cm} (A-4)$$

$$\begin{aligned}
\frac{\partial V_d}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_d}{\partial x^2} + \mu x \frac{\partial V_d}{\partial x} + \frac{1}{2c_3} \left( \frac{\partial V_d}{\partial g} \right)^2 + \frac{1}{2c_4} \left( \frac{\partial V_d}{\partial k_d} \right)^2 - rV_d \\
+ \pi_d = 0
\end{aligned}$$

Proof of Corollary 1: We only prove the corollary for the optimal investment rates in the source country. Similar arguments can be used for the optimal investment rates...
in the destination country.

By Proposition 1 we have

\[
\begin{align*}
    u^*(g, k_h) &= \frac{1}{c_1} \frac{\partial V_h}{\partial g}(g, k_h) \\
    q_h^*(g, k_h) &= \frac{1}{c_2} \frac{\partial V_h}{\partial k_h}(g, k_h)
\end{align*}
\]  
(A-5)

Based on a CRRA utility function, conjecture a value function of the form

\[
V_h(g, k_h) = A_h^{\gamma} (g + k_h)^{1-\gamma}
\]  
(A-6)

where $A_h > 0$ is a constant and $\gamma$ is the coefficient of risk aversion. We have

\[
\begin{align*}
\frac{\partial u^*}{\partial g} &= \frac{1}{c_1} \frac{\partial^2 V_h}{\partial g^2} = \frac{-\gamma}{c_1} A^{-\gamma}(g + k_h)^{-\gamma-1} < 0 \\
\frac{\partial u^*}{\partial k_h} &= \frac{1}{c_1} \frac{\partial^2 V_h}{\partial g \partial k_h} = \frac{-\gamma}{c_1} A^{-\gamma}(g + k_h)^{-\gamma-2} < 0 \\
\frac{\partial q_h^*}{\partial g} &= \frac{1}{c_2} \frac{\partial^2 V_h}{\partial k_h \partial g} = \frac{-\gamma}{c_2} A^{-\gamma}(g + k_h)^{-\gamma-2} < 0 \\
\frac{\partial q_h^*}{\partial k_h} &= \frac{1}{c_2} \frac{\partial^2 V_h}{\partial k_h^2} = \frac{-\gamma}{c_2} A^{-\gamma}(g + k_h)^{-\gamma-1} < 0
\end{align*}
\]  
(A-7)
Appendix B

This appendix incorporates the exogenous probability that the destination country admits an agent when the aggregate productivity differential justifies emigration. Let \( N(t) \) denote a Poisson process that tracks the visa policy of the destination country:

\[
dN = \begin{cases} 
1, & \text{with } \lambda dt \\
0, & \text{with } 1 - \lambda dt 
\end{cases} \tag{A-8}
\]

We say that an agent is accepted upon application to the destination country when the Poisson process has jumped. The parameter \( \lambda \) in equation (A-8) captures the intensity or the probability of acceptance. We assume that the processes \( N(t) \) and \( x(t) \) are independent.

Corresponding to the Poisson process in (A-8), we denote the arrival time of a visa by \( \tau_a \). Note that \( \tau_a \) is exponentially distributed with the parameter \( \lambda \). We assume that the agent is accepted to the destination country if the \( N(t) \) has jumped prior to the emigration decision of the agent. That is, the agent emigrates if \( \tau_a < \tau \). We can now reformulate the problem posed in equation (7):

\[
V_h(s_h, t) = \max_{m_h, \tau} \mathbb{E}_0 \left\{ \min(\tau, T) \int_0^{\min(\tau, T)} \pi_h(s_h, t, m_h)e^{-rt} dt \right. \\
+ I_{\{\tau_a < \tau < T\}}e^{-rt}[V_d(s_d, t) - I] \left\} \tag{A-9}
\]

Note that the agent’s problem conditional on emigration remains as specified in (9).

To solve (A-9) subject to equations (1), (2) and (3), we resort again to the Bellman equation in (A-1). By Itô’s lemma, the expression for \( dV_h \) is given by:

\[
\frac{dV_h}{dt} = \frac{\partial V_h}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_h}{\partial x^2} + \mu x \frac{\partial V_h}{\partial x} + u \frac{\partial V_h}{\partial g} + q_h \frac{\partial V_h}{\partial k_h} \\
+ dN[V_d - I - V_h] \tag{A-10}
\]
Substituting equation (A-10) into equation (A-1), we get:

\[
0 = \max_{m_h} \left\{ \pi_h(s_i, t, m_i) + \frac{\partial V_h}{\partial t} + \frac{1}{2} \sigma^2 x^2 \frac{\partial^2 V_h}{\partial x^2} + \mu x \frac{\partial V_h}{\partial x} + u \frac{\partial V_h}{\partial g} + q_h \frac{\partial V_h}{\partial k_h} - (r + q) V_h + \lambda (V_d - I) \right\} 
\]

(A-11)

It is important to note that the optimization in equation (A-11) yields the same investment policy as in Proposition 1. However, notice that the discount factor of the agent has now been augmented by the probability of acceptance to the destination country. The last term in (A-11) reflects the fact that the value after emigration can be captured only with intensity $\lambda$.

References


32


