The Impact of Product Market Characteristics on Firms' Strategies in Patent Litigation

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Abstract

We use a compound option model to investigate firms' decisions in patent litigation. The model incorporates settlement before and after filing of the lawsuit, the challenger's exit option and the incumbent's option to withdraw from litigation. We find that the challenger's profit relative to the incumbent's loss of profit (gain-to-loss ratio) is a key determinant of whether settlement occurs. The timing of settlement also depends on the relative costs of litigation and settlement. Overall, settlement is less likely for low gain-to-loss ratios, high probability of patent validity, and in more volatile product markets.

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1 Introduction

Patent infringement lawsuits are economically significant and have become more prevalent in recent years. The largest patent infringement verdict ever granted in the U.S. was over 2.5 billion dollars, when Gilead Sciences, Inc. was found to have wilfully infringed the patent rights of Merck in 2016. Not only are judgement amounts large, patent infringement cases also induce high ongoing litigation costs for the parties involved. The median cost of U.S. patent infringement cases in 2017 was $1.7 million[1]. Moreover, the total costs of litigation are much greater than the legal fees alone. Costs are large even for lawsuits that settle. Patent litigation lawsuits cost alleged infringers about $28.7 million in the mean and $2.9 million in the median (Bessen and Meurer, 2008). From 2000 to 2018, there were around 67 thousand patent infringement cases filed in the U.S. District Courts, and the number of companies that were plaintiffs and defendants reached 66 thousand and 100 thousand respectively[2]. The occurrence and the outcomes of patent infringement lawsuits thus reshape product markets and industry composition, and change the incentives for innovation.

The economics of litigation and settlements were investigated in the early literature (e.g., Landes, 1971; Ordover, Rubinstein et al., 1983; Bebchuk, 1984; Bebchuk, 1996). More recently, patent litigation has caught the attention of the economics literature, especially after the Leahy-Smith America Invents Act was enacted in 2012 (e.g., Bessen and Meurer, 2006; Choi and Spier, 2018; Lee, Oh, and Suh, 2018). Nevertheless, little is known regarding the impacts of product market characteristics on firms' strategies in patent infringement cases. Our goal is to fill that gap and provide baseline understanding of the following questions: how does the relationship between the infringed and the infringing products, in terms of market sizes and shares, affect firms' settlement decisions before litigation and their strategies during litigation? How do the two parties' relative cost savings from settlement affect case outcomes?

To answer these questions, we build a real options model of two all-equity firms with symmetric information. Both firms face demand uncertainty and have ongoing profits based on certain technologies, and one of them is endowed with patent rights. Settlement between the two firms

[1] Source:the American Intellectual Property Law Association's 2017 Economic Survey Report
[2] Source: Lex Machina

can occur before a patent infringement lawsuit is filed ("ex-ante settlement") as well as after filing the case ("ex-post settlement"). When settled, royalty payment is demanded by the patent holding firm, and paid by the alleged infringing firm proportional to its ongoing profit. New to the literature, we allow both the infringed firm (or the "incumbent") to withdraw from the litigation and the infringing firm (or the "challenger") to exit the market during litigation. Such modification is relevant both in practice and for the theory. Litigants compare the option values of different strategies when making decisions, taking consideration of any later options.

We obtain the following main results. First, the challenger's profit relative to the incumbent's loss of profit (the " gain-to-loss ratio") has a significant impact on the settlement decision. If the product market is such that the gain-to-loss ratio is below a critical level, for example when the two products are close substitutes and they have the same customers, then the two parties always renounce the possibility of settlement and wait for the court ruling. If the gain-to-loss ratio exceeds this level, for example when the two products are complements or their customers are located in different geographical areas, then settlement occurs when the demand level drops to a settlement threshold. Intuitively, when the gain-to-loss ratio is low, the challenger's cash flow capacity to pay for any royalty and litigation cost is restrictive. The incumbent thus optimally proceeds with the litigation in order to exhaust the challenger's resources and restore its monopolistic market power when the challenger is forced to exit.

Second, provided that the gain-to-loss ratio exceeds a critical level which makes the settlement possible, the relative cost of litigation and settlement is a key factor that determines the timing of settlement, that is whether settlement occurs before or after filing the case. When litigation is relatively more expensive, the incumbent's incentive to settle shifts from after filing to before filing. This result implies that policies which affect legal expenses and settlement cost impact how firms settle in patent infringement litigation.

Third, both the probability of patent validity and the product market demand volatility play an important role in the litigation outcomes. A high probability of patent validity increases the incumbent's winning probability at trial, and a high demand uncertainty increases the incumbent's value of not settling. Both forces reduce the incumbent's incentive to settle before or after litigation. The volatility result can be used to explain variations in the distribution of litigation outcomes cross industries or over time.

Our paper is one of the first studies to examine the effect of product market characteristics on firms' strategies in litigation and the litigation outcomes. We contribute to this literature by establishing the importance of the relationship between the plaintiff's and the defendant's product markets and re-examining the role of market volatility in the context of patent litigation. We believe that the gain-to-loss ratio is a first-order factor in determining firms' litigation strategies, similar to the previously argued judgment amount, litigation cost, and information asymmetry (e.g. Spier (2007); Hughes and Snyder (1995); Bebchuk (1984)).

Our paper adds to the recent discussion of how financing considerations affect litigation (e.g., Cohen et al. (2016); Choi and Spier (2018)). A number of papers have modeled both generic litigation and patent enforcement using litigation in real options models (e.g., Grundfest and Huang, 2005; Marco, 2005; Jeon, 2015). New to this literature, our model incorporates the possibility that the defendant may exit during litigation due to its inability to pay for the litigation cost. As Lee et al. (2018) find, defendants in patent litigation become much more financially constrained.

The paper proceeds as follows. In section 2, we provide an overview of the background and related literature on patent litigation. In section 3, we describe the model. In section 4, we develop and solve the real options model and obtain the values of the two firms under different strategies. Section 5 presents comparative statics of case outcomes and optimal strategies before and during the patent litigation. Section 6 concludes and discusses potential extensions of the model.

2 Background and the Related Literature

Empirical studies show that patent litigations are typically complex, time-consuming, expensive and are often difficult to win (e.g., Lanjouw and Schankerman, 1997; Bessen and Meurer, 2005). In the US, the average time from filing a case to judgment, based upon a jury trial, is 33 months. The total duration of the litigation process is expected to be 2 to 6 years. In addition, the litigation cost in a patent infringement case may be too high for firms to afford (e.g., Meurer, 1989; Elleman, 1996; Lanjouw and Schankerman, 2001; Chien, 2008). Lanjouw and Schankerman (2001) show that the cost of litigation diminishes the value of litigated intellectual property in the U.S.. Hu et al. (2017) find that small and medium enterprises face difficulties in trying to benefit from patent litigation. Moreover, on average, plaintiffs only have a 50% chance of winning a patent infringement case as the patent may be found to be invalid in court (e.g., Allison and Lemley, 1998; Moore, 2000). Due to the above characteristics of patent litigation, rather than litigating directly, the incumbent may prefer to settle or eventually abandon the lawsuit (Crampes and Langinier, 2002). A challenger faces a heavy burden of litigation cost can result in significant losses and even bankruptcy (Bessen and Meurer, 2008).

Settlement is the most common way for both parties to resolve the patent dispute and avoid the high litigation costs (e.g., Burton, 1980; Kesan and Ball, 2006). The rate of settlement in patent infringement cases is over 90 per cent and most settlement occurs soon after the suit is filed, or often before the pre-trial hearing is held (e.g., Cohen and Merrill, 2003; Cotropia, Kesan, and Schwartz, 2018). Besides the ex-post settlement, another kind of of settlement is pointed out by Choi and Gerlach (2017): ex-ante settlement that happens before the complaint is filed. Of the remaining 10% of patent infringement cases conditional on filing, two-thirds are abandoned or adjudicated on summary judgment of non-infringement before trial, as reported by Lex Machine.

A number of studies have focused on different aspects of firm's reactions in patent litigation, especially the timing of infringement by the challenger. Crampes and Langinier (2002) consider litigation and settlement in patent infringement cases in a static framework. They focus the situation when a firm is already a patent owner and investigate how intensively patent-protected markets should be watched and what the best reaction to infringement is. They show that these options also affect the challenger's entry, since licensing and threat of litigation can potentially deter the challenger's infringement. This finding is consistent with Aoki and Hu (1999), who investigated a cooperative approach to litigation and settlement, and found that a trial can be Pareto efficient and the threat of litigation can deter entry. In contrast, Meurer (1989) proposed that in a non-cooperative game approach, the litigation process can be seen as an opportunity to engage in a strategic behaviour when the validity of the patent can be challenged in court. In this case, a trial is just a failure to settle and not a deliberate choice.

Our paper is not the first to use a real options model to study litigation or patent value. Schwartz (2004) studies the value of patents and patent protected R&D projects. Marco (2005) investigates patent values when enforcement is costly and the winning probability is uncertain in patent litigation. Grundfest and Huang (2005) propose a two-stage real options model of with information revelation to explore the value of litigation. Jeon (2015) models the litigants' strategies in patent infringement cases as options and studies the circumstances in which settlement occurs.

We contribute to the literature by modeling the strategies of settlements and litigation with options as a whole. In particular, we incorporate the option value of the ex-post settlement into the value of litigation. We also consider the possibility of the challenger's exit during litigation. These two additional options make a significant impact on the outcome of patent litigation. We also build on the existing literature by investigating how the characteristics of firms and patent litigation affect the interaction between firms' decisions. We find an opposite result of firms' propensity to settle ex-ante regarding the possibility of patent validity and volatility from Jeon (2015). This is because the value of additional options we model increases the value of litigation, thus decreasing the likelihood of ex-ante settlement.

3 Model Setup

As in Jeon (2015), two risk-neutral all-equity firms operate in the same product market. An incumbent ("I") holds a patent, and a challenger ("C") has infringed the patent. Each firm generates a random net operating income of Πx, where Π is a positive constant, and x represents the demand shock which follows a geometric Brownian motion

$$dx_t = \mu x_t dt + \sigma x_t dW_t \tag{1}$$

$\mu < r, \sigma > 0, \mu < \frac{\sigma^2}{2}$, and W_t is the standard Brownian motion.

We assume the monopoly profit for the incumbent before infringement is $\Pi_1 x$. After infringement, this profit level will decrease to the duopoly profit $\Pi_2^I x$ for the incumbent whilst the challenger will receive a duopoly profit of $\Pi_2^C x$. If the challenger sells a highly differentiated product, it is possible to have $\Pi_1 < \Pi_2 = \Pi_2^I + \Pi_2^C$. On the other hand, with a challenger selling a close substitute in the same market, it is more likely that $\Pi_1 > \Pi_2 = \Pi_2^I + \Pi_2^C$ (Crampes and Langinier, 2002).

The game tree of our model is presented in Figure 1. At the initial stage, the incumbent can offer an ex-ante settlement, which the challenger may or may not accept, or sue the infringer. Then after litigation starts there are four possible strategies for the two parties: (1) the incumbent offers an ex-post settlement; (2) the challenger is unable to afford the cost and exits the market;

(3) the incumbent withdraws the lawsuit; (4) both wait for the court ruling.

[Insert Figure 1 here.]

Whilst the incumbent has the right to sue the challenger, it is not guaranteed that the judgment will be in favour of the incumbent. We assume that the patent is valid with probability p which is common knowledge. If the incumbent wins, the challenger is required to leave the market, while if the challenger wins, the two firms still share the market with duopoly profit $\Pi_2^I x$ and $\Pi_2^C x$ for the incumbent and the challenger respectively. We model the lengthy litigation process by assuming it takes an exponential time τ to reach the court's judgment. τ is assumed to be a Poisson process with rate parameter λ and which is independent of $(W_t)_{t\geq 0}$. The expected duration of the litigation process is λ^{-1}. Litigation incurs ongoing costs C_L^I and C_L^C for the incumbent and challenger respectively for the duration of litigation process. We use H_L^I and H_L^C to denote the expected discounted values of litigation costs for the incumbent and challenger respectively, where

$$
\begin{aligned}
H_L^I = E_t \int_0^\tau e^{-rt} C_L^I dt = \frac{C_L^I}{r+\lambda} \\
H_L^C = E_t \int_0^\tau e^{-rt} C_L^C dt = \frac{C_L^C}{r+\lambda}
\end{aligned}
\tag{2}
$$

The ongoing litigation cost is a heavy burden for both parties. The challenger faces the risk of bankruptcy and may exit the market if it can no longer pay the litigation cost. The litigation cost may also force the incumbent to withdraw the lawsuit.

Alternatively, the dispute can be resolved by settlement. In a settlement, the challenger agrees to pay a fraction of its profit as a royalty payment to the incumbent. In ex-ante settlement which occurs before filing the lawsuit, this fraction is denoted by θ_A, while in ex-post settlement this fraction is denoted by θ_P. Negotiating a settlement incurs costs. Suppose ex-ante settlement costs the incumbent a constant C_{SA}^I and the challenger C_{SA}^C. Ex-post settlement costs are C_{SP}^I and C_{SP}^C respectively. The settlement can only succeed when the two parties reach an agreement on the settlement timing and the royalty level. To determine the settlement threshold and royalty rate, we follow the method proposed by Lukas et al. (2012) and assume the incumbent offers the settlement contract and determines the level of royalty (θ_A or θ_P), whilst the challenger chooses the timing of settlement, i.e. when to accept the incumbent's settlement offer.

We let F denote the value functions of the incumbent and G denote the value functions of the challenger. For the party who makes the decision, a value-matching condition and an optimality condition are applied since this party will maximize his value function. For the reacting party, only the value-matching condition is applied to determine his value function.

4 Model Solutions

4.1 During Litigation

Using backward induction, we first investigate the stage when litigation has already started and litigants are waiting for the court's ruling.

We assume the probability of winning the trial p, is agreed by both parties. Before the final judgment at τ, the incumbent earns a flow profit of $\Pi_2^I x$. With probability p, he wins the trial and gets the monopoly profit $\Pi_1 x$ from then on. We use F_M to denote the incumbent's value function when he earns monopoly profit and $F_M = \frac{\Pi_1 x}{r-\mu}$. With probability $1-p$, the patent is found to be invalid and the incumbent loses. He then keeps sharing the profit with the challenger and gets $\Pi_2^I x$. The value function for the incumbent in this case is $F_D = \frac{\Pi_2^I x}{r-\mu}$. Thus, during litigation, the value function of the incumbent (F_{DL}) satisfies the following equation for any x:

$$\frac{1}{2}\frac{\partial^2 F_{DL}}{\partial x^2}x^2\sigma^2 + \mu x\frac{\partial F_{DL}}{\partial x} - C_L^I + \Pi_2^I x + \lambda(pF_M + (1-p)F_D - F_{DL}) = rF_{DL}. \tag{3}$$

For the challenger, he earns a flow duopoly profit $\Pi_2^C x$ before the trial. If he wins the trial with probability $1-p$, he continues its operation and shares the duopoly profit. His value function thus is $G_D = \frac{\Pi_2^C x}{r-\mu}$. If he loses with probability p, he has to leave the market and gets nothing. The challenger's value function (G_{DL}) satisfies the following equation at all times during litigation:

$$\frac{1}{2}\frac{\partial^2 G_{DL}}{\partial x^2}x^2\sigma^2 + \mu x\frac{\partial G_{DL}}{\partial x} - C_L^C + \Pi_2^C x + \lambda((1-p)G_D - G_{DL}) = rG_{DL}. \tag{4}$$

We can obtain the value functions during the litigation as follows

$$F_{DL}(x) = [\frac{\Pi_2^I}{r-\mu} + p\delta(\Pi_1 - \Pi_2^I)]x + B_I^{DL}x^{\beta_\lambda} + A_I^{DL}x^{\alpha_\lambda} - H_L^I, \tag{5}$$

$$G_{DL}(x) = (\frac{1}{r-\mu} - p\delta)\Pi_2^C x + B_C^{DL}x^{\beta_\lambda} + A_I^{DL}x^{\alpha_\lambda} - H_L^C, \tag{6}$$

where

$$\delta = \frac{\lambda}{(r-\mu)(r+\lambda-\mu)},$$

$$\alpha_\lambda = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\mu)}{\sigma^2}} > 1,$$

$$\beta_\lambda = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2(r+\mu)}{\sigma^2}} < 0,$$

$$H_L^I = \frac{C_L^I}{r+\lambda}, \quad H_L^C = \frac{C_L^C}{r+\lambda}.$$

There are two possibilities for the firm values of the incumbent $F_{DL}(x)$: the value of settling denoted by $F_{DLSP}(x)$ and the value of not settling denoted by $F_{DLNS}(x)$. The incumbent is the party who makes a decision about whether to offer settlement, so he compares the value of the two possibilities ($F_{DLSP}(x)$ and $F_{DLNS}(x)$) to determine whether to offer settlement or not. That is, the firm value of the incumbent during litigation is

$$F_{DL}(x) = max\{F_{DLSP}(x), F_{DLNS}(x)\} \tag{7}$$

Depending on the incumbent's strategy, the firm value of the challenger during litigation is

$$G_{DL}(x) = \begin{cases} G_{DLSP}(x), & \text{if} \quad F_{DL}(x) = F_{DLSP}(x) \\ G_{DLNS}(x). & \text{if} \quad F_{DL}(x) = F_{DLNS}(x) \end{cases} \tag{8}$$

Next, we use $F_{DLW}(x)$ to denote the incumbent's firm value who has to withdraw if the demand shock x falls below a threshold and $G_{DLW}(x)$ is the corresponding firm value of the challenger. Similarly, we use $G_{DLE}(x)$ to denote the firm value of the challenger who has to exit if the demand shock x decreases to some threshold and $F_{DLE}(x)$ is the firm value of the incumbent in these cases. If the demand shock reaches the threshold of the former case first, the firm value of not settling is the firm value when the incumbent withdraws, while if the demand shock first reaches the threshold of the latter case, the firm value of not settling is the firm value when the challenger exits. We follow Lambrecht (2001) in determining the order of withdrawal or exit and the calculation of these thresholds. Here, we use "the incumbent withdraws first" to represent the case if the incumbent first withdraws before the challenger exits. "The challenger exits first" stands for the case when the challenger exits before the incumbent withdraws the litigation.

The arbitrary constants B_I^{DL} and B_C^{DL} in Equation (5) can only be determined after considering the possible values for all different scenarios: (1) The incumbent withdraws before settlement or challenger exits. We use B_I^W and B_C^W to denote the arbitrary constants in such cases. (2) The challenger exits before settlement or incumbent withdraws. We use B_I^E and B_C^E as the notations. (3) Ex-post settlement occurs before either withdrawal by the incumbent or exit by the challenger, the arbitrary constants are denoted by B_I^{SP} and B_C^{SP}. When settlement does not occur during litigation. B_I^{NS} and B_C^{NS} denote such arbitrary constants.

According to Décamps et al. (2006), the decision maker will select the project which generates the highest net expected discounted profit that can be represented by the value of B_I.

Thus, the arbitrary constants are

$$B_I^{DL} = max\{B_I^{SP}(\theta_p^*), B_I^{NS}\}, \tag{9}$$

$$B_C^{DL} = \begin{cases} B_C^{SP}(\theta_p^*), & \text{if} \quad B_I^{NS} \leq B_I^{SP}, \\ B_C^{NS}, & \text{if} \quad B_I^{NS} > B_I^{SP}. \end{cases} \tag{10}$$

Note that the arbitrary constants under settlement $B_I^{SP}(\theta_p^*)$ and $B_C^{SP}(\theta_p^*)$ depend on θ_p^*, and θ_p^* is an optimal royalty level maximizing the incumbent's value during litigation including the option to settle.

In the following sections, we calculate these arbitrary constants and the corresponding thresholds of the demand shock to obtain the value functions of all outcomes during litigation for the two parties.

4.1.1 Withdrawal by the Incumbent

If the incumbent withdraws first, there are still two firms in the market, and both parties will share the market with profits at the same level as before. In this case, the firm value of incumbent and challenger are

$$
\begin{aligned}
F_W(x) = F_D(x) &= \frac{\Pi_2^I x}{r-\mu} \\
G_W(x) = G_D(x) &= \frac{\Pi_2^C x}{r-\mu}
\end{aligned} \tag{11}
$$

The value functions with an option to withdraw by the incumbent for two parties are

$$F_{DLW}(x) = [\frac{\Pi_2^I}{r-\mu} + p\delta(\Pi_1 - \Pi_2^I)]x + B_I^W x^{\beta_\lambda} - H_L^I \tag{12}$$

$$G_{DLW}(x) = (\frac{1}{r-\mu} - p\delta)\Pi_2^C x + B_C^W x^{\beta_\lambda} - H_L^C \tag{13}$$

Applying the value matching and smooth pasting conditions on the incumbent's value function and value matching condition on the challenger's value function, we derive the withdrawal trigger and arbitrary constants as follows:

$$x_w = \frac{\beta_\lambda H_L^I}{(\beta_\lambda - 1)p\delta(\Pi_1 - \Pi_2^I)} \tag{14}$$

$$B_I^W = \{H_L^I - p(\Pi_1 - \Pi_2^I)\delta x_w\}x_w^{-\beta_\lambda} \tag{15}$$

$$B_C^W = \{H_L^C + p\Pi_2^C\delta x_w\}x_w^{-\beta_\lambda} \tag{16}$$

4.1.2 Exit by the Challenger

If the challenger can no longer afford the litigation cost and goes bankrupt, the incumbent is in a monopolistic situation. The firm values of both parties that are given by

$$F_E(x) = F_M(x) = \frac{\Pi_1 x}{r-\mu} \tag{17}$$

$$G_E(x) = 0 \tag{18}$$

The value functions assuming that the trial proceeds to judgment while the challenger exits first for the two parties are

$$F_{DLE}(x) = [\frac{\Pi_2^I}{r-\mu} + p\delta(\Pi_1 - \Pi_2^I)]x + B_I^E x^{\beta_\lambda} - H_L^I \tag{19}$$

$$G_{DLE}(x) = (\frac{1}{r-\mu} - p\delta)\Pi_2^C x + B_C^E x^{\beta_\lambda} - H_L^C \tag{20}$$

At the exit trigger x_e, applying the value matching and the smooth pasting conditions on the challenger's value functions and value matching condition on incumbent's value functions, we have

$$x_e = \frac{\beta_\lambda H_L^C}{(\beta_\lambda - 1)(\frac{1}{r-\mu} - p\delta)\Pi_2^C} \tag{21}$$

$$B_I^E = \{H_L^I + (\frac{1}{r-\mu} - p\delta)(\Pi_1 - \Pi_2^I)x_e\}x_e^{-\beta_\lambda} \tag{22}$$

$$B_C^E = \{H_L^C - (\frac{1}{r-\mu} - p\delta)\Pi_2^C x_e\}x_e^{-\beta_\lambda} \tag{23}$$

The thresholds of withdrawal (x_e) and exit (x_w) are not the thresholds that determine the order of withdrawal or exit we discussed above. However, in most cases, the litigation process will stop when demand shock reaches $x_s = max\{x_e, x_w\}$, in the absence of the ex-post settlement. For ease of notation we use $x_e \gtrless x_w$ to distinguish the order of withdrawal or exit below.

Therefore,

$$F_{DLNS}(x) = [\frac{\Pi_2^I}{r-\mu} + p\delta(\Pi_1 - \Pi_2^I)]x + B_I^{NS} x^{\beta_\lambda} - H_L^I, \tag{24}$$

$$G_{DLNS}(x) = (\frac{1}{r-\mu} - p\delta)\Pi_2^C x + B_C^{NS} x^{\beta_\lambda} - H_L^C, \tag{25}$$

where

$$B_I^{NS} = \begin{cases} B_I^W, & \text{if} \quad x_e \le x_w \\ B_I^E. & \text{if} \quad x_e > x_w \end{cases}$$
$$B_C^{NS} = \begin{cases} B_C^W, & \text{if} \quad x_e \le x_w \\ B_C^E. & \text{if} \quad x_e > x_w \end{cases} \tag{26}$$

4.1.3 Ex-post Settlement

However, the incumbent still has an option instead of withdrawing the lawsuit. He can offer an ex-post settlement with required royalty θ_P. In this case, the incumbent can recoup some of his losses by collecting a fraction of the challenger's ongoing profit $\theta_P \Pi_2^C x$. Thus, the value functions of both parties after ex-post settlement can be obtained as follows,

$$
\begin{aligned}
F_{SP}(x) &= \frac{\Pi_2^I + \theta_P \Pi_2^C}{r-\mu} x - C_{SP}^I, \\
G_{SP}(x) &= \frac{(1-\theta_P)\Pi_2^C}{r-\mu} x - C_{SP}^C,
\end{aligned}
\tag{27}
$$

The value functions during litigation for the two parties assuming the incumbent may offer a ex-post settlement are

$$
F_{DLSP}(x) = [\frac{\Pi_2^I}{r-\mu} + p\delta(\Pi_1 - \Pi_2^I)]x + B_I^{SP} x^{\beta_\lambda} - H_L^I, \tag{28}
$$

$$
G_{DLSP}(x) = (\frac{1}{r-\mu} - p\delta)\Pi_2^C x + B_C^{SP} x^{\beta_\lambda} - H_L^C, \tag{29}
$$

To determine the optimal royalty level θ_P and the ex-post settlement threshold x_{sp}, we assume the challenger can choose the threshold of $x_{sp}(\theta_P)$ at which he accepts the settlement offered by the incumbent, given the royalty level offered by the incumbent (θ_P), while the incumbent determines this royalty θ_P by maximizing the value of option to settle taking account of the fact that the settlement threshold varies with the royalty level. The success of settlement depends on the consensus of both parties with the required royalty and settlement timing.

Applying the value matching and smooth pasting conditions on the challenger's value functions and value matching condition on the incumbent's value functions we derive the ex-post settlement trigger and coefficients for both parties for an arbitrary royalty level θ_p:[3]

$$x_{sp}(\theta_P) = \frac{\beta_\lambda (H_L^C - C_{SP}^C)}{(\beta_\lambda - 1)(\frac{\theta_P}{r-\mu} - p\delta)\Pi_2^C} \tag{30}$$

$$B_I^{SP}(\theta_P) = \{H_L^I - C_{SP}^I + (\frac{\theta_P \Pi_2^C}{r-\mu} - p\delta(\Pi_1 - \Pi_2^I))x_{sp}\}x_{sp}^{-\beta_\lambda} \tag{31}$$

$$B_C^{SP}(\theta_P) = \{H_L^C - C_{SP}^C + (p\delta - \frac{\theta_P}{r-\mu})\Pi_2^C x_{sp}\}x_{sp}^{-\beta_\lambda} \tag{32}$$

Note that the threshold and value functions of both parties vary with the settlement royalty rate θ_P. Maximizing B_I^{SP} with respect to θ_P, the incumbent can obtain a optimal θ_P represented by θ_P^* that maximizes the incumbent's value during litigation including the option to settle (i.e., $F_{DLSP}(x_{sp}, \theta_P^*)$).

$$\theta_P^* = \frac{p\delta(r-\mu)[(H_L^I - C_{SP}^I)\Pi_2^C(\beta_\lambda - 1) + (H_L^C - C_{SP}^C)(\Pi_2^C + (\beta_\lambda - 1)(\Pi_1 - \Pi_2^I))]}{\Pi_2^C[(H_L^I - C_{SP}^I)(\beta_\lambda - 1) + \beta_\lambda(H_L^C - C_{SP}^C)]} \tag{33}$$

In order to ensure $x_{sp} \geq 0, \theta_P^* \geq p\delta(r-\mu)$, thus we have $\Pi_1 > \Pi_2$. Although this royalty level θ_P^* can maximize the $F_{DLSP}(x_{sp})$, each parties will agree to the ex-post settlement only if their value including the option to settle is higher than the value to them of not settling during litigation.

The challenger will accept the offer only when his firm value of accepting the incumbent's settlement is higher than the value of not settling during litigation, that is

$$G_{DLSP}(x, \theta_P) \geq G_{DLNS}(x) \tag{34}$$

This implies, θ_P^* should be lower than a maximum θ_P denoted as θ_P^{Cmax} which satisfies equation (34), i.e.,

$$\theta_P^{Cmax} = \begin{cases} p\delta(r-\mu)\{1 + (h_L\gamma_c/\Phi)^{1-\frac{1}{\beta_\lambda}}[h_L(1-\beta_\lambda)/\Phi - \beta_\lambda]^{\frac{1}{\beta_\lambda}}\}, & \text{if} \quad B_C^{NS} = B_C^W \\ (1-p\delta(r-\mu))\gamma_c^{1-\frac{1}{\beta_\lambda}} + p\delta(r-\mu), & \text{if} \quad B_C^{NS} = B_C^E \end{cases} \tag{35}$$

and

$$\Phi = \frac{\Pi_2^C}{\Pi_1 - \Pi_2^I}, \quad h_L = \frac{H_L^C}{H_L^I}, \quad \gamma_c = \frac{H_L^C - C_{SP}^C}{H_L^C}. \tag{36}$$

[3]We only consider the case when $x_{sp} \leq x_l$.

Similarly, the incumbent will offer ex-post settlement only if his value including this ex-post settlement is higher than the value of not settling during litigation, that is

$$F_{DLSP}(x,\theta_P) \geq F_{DLNS}(x) \tag{37}$$

There is a range of θ_P denoted as θ_P^I satisfies the above equation (37) and we define the minimum royalty as θ_P^{Imin} and the maximum royalty as θ_P^{Imax}.

When the incumbent withdraws first (i.e $B_I^{NS} = B_I^W$), we simplify the equation (37) and obtain

$$[h_c(1-\beta_\lambda)(\frac{\theta_P^I}{r-\mu} - p\delta) - \beta_\lambda(\frac{\theta_P^I}{r-\mu} - p\delta/\Phi)](\frac{\theta_P^I}{r-\mu} - p\delta)^{\beta_\lambda - 1} - (\gamma_c h_L)^{\beta_\lambda - 1}(p\delta/\Phi)^{\beta_\lambda} \geq 0, \tag{38}$$

where $h_c = \frac{H_L^I - C_{SP}^I}{H_L^C - C_{SP}^C}$.

When the challenger exits first (i.e $B_I^{NS} = B_I^E$), we simplify the equation (37) and obtain

$$\Big[h_c(\frac{\theta_P^I}{r-\mu} - p\delta) + \frac{\beta_\lambda}{\beta_\lambda - 1}(\frac{\theta_P^I}{r-\mu} - \frac{p\delta}{\Phi})\Big](\frac{\theta_P^I}{r-\mu} - p\delta)^{\beta_\lambda - 1} - (\gamma_c)^{\beta_\lambda - 1}(\frac{1}{r-\mu} - p\delta)^{\beta_\lambda}(\frac{1}{h_L} + \frac{\beta_\lambda}{(\beta_\lambda - 1)\Phi}) \geq 0. \tag{39}$$

We define the maximum royalty in ex-post settlement as θ_P^{max}, which is determined by the challenger or the incumbent, that is

$$\theta_P^{max} = min\{\theta_P^{Cmax}, \theta_P^{Imax}\} \tag{40}$$

Therefore, the two parties can reach an agreement of ex-post settlement if $\theta_P^{Imin} \leq \theta_P^* \leq \theta_P^{max}$ and the royalty required by the incumbent is θ_P^* , but ex-post settlement may will not occur if $\theta_P^{max} < \theta_P^{Imin}$ or if there is no solution for equation (38) or (39), which means θ_P^{Imin} and θ_P^{Imax} do not exist.

4.2 Before Litigation

After the challenger infringes the incumbent' patent, he earns a duopoly profit $\Pi_2^C x$ while the incumbent's profit decreases to $\Pi_2^I x$. When the incumbent has identified the infringement, they can choose to litigate. We denote the demand level x when the incumbent chooses to start litigation by x_l and use $F_{BL}(x)$ and $G_{BL}(x)$ to denote the values for incumbent and challenger before the litigation. The value function before litigation for two parties have general solutions

$$\begin{aligned} F_{BL}(x) &= \frac{\Pi_2^I x}{r-\mu} + A_I^{BL} x^\alpha + B_I^{BL} x^\beta, \\ H_{BL}(x) &= \frac{\Pi_2^C x}{r-\mu} + A_C^{BL} x^\alpha + B_C^{BL} x^\beta, \end{aligned} \tag{41}$$

where

$$\begin{aligned} \alpha &= \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} > 1, \\ \beta &= \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{(\frac{1}{2} - \frac{\mu}{\sigma^2})^2 + \frac{2r}{\sigma^2}} < 0, \end{aligned}$$

When the demand shock decreases to zero, both parties earn zero profit, so we have the boundary conditions

$$F_{BL}(0) = G_{BL}(0)$$

Then, we have $B_I^{BL} = B_C^{BL} = 0$. Thus, the value functions for both parties before litigation are

$$\begin{aligned} F_{BL}(x) &= \frac{\Pi_2^I x}{r-\mu} + A_I^{BL} x^\alpha, \\ G_{BL}(x) &= \frac{\Pi_2^C x}{r-\mu} + A_C^{BL} x^\alpha. \end{aligned} \tag{42}$$

Similar to the during litigation case in section 4.1, the arbitrary constants A_I^{BL} and A_C^{BL} can only be determined after deriving the value functions for the two possible outcomes, i.e. litigation and ex-ante settlement. Here, we denote by A_I^L and A_C^L are arbitrary constants of the incumbent and the challenger when litigation is the outcome (i.e. when the incumbent chooses to litigate rather than settle ex-ante), and $F_{BLL}(x)$ and $G_{BLL}(x)$ are the corresponding value functions. A_I^{SA} and A_C^{SA} are the arbitrary constants of the incumbent and the challenger when the outcome is ex-ante settlement and $F_{BLSA}(x)$ and $G_{BLSA}(x)$ are the corresponding value functions.

The incumbent is the party who decides to litigate or offer an ex-ante settlement. Following the same method as discussed above, we obtain the arbitrary constants of the value functions before litigation

$$
A_I^{BL} = \max\{A_I^L, A_I^{SA}\}
$$

$$
A_C^{BL} = \begin{cases} A_C^L, & \text{if} \quad A_I^{BL} = A_I^L \\ A_C^{SA}. & \text{if} \quad A_I^{BL} = A_I^{SA} \end{cases} \tag{43}
$$

4.2.1 Litigation

During litigation, the value functions of two parties are

$$
\begin{aligned}
F_{DL}(x) &= [\frac{\Pi_2^I}{r-\mu} + p\delta(\Pi_1 - \Pi_2^I)]x + B_I^{DL} x^{\beta_\lambda} - H_L^I, \\
G_{DL}(x) &= (\frac{1}{r-\mu} - p\delta)\Pi_2^C x + B_C^{DL} x^{\beta_\lambda} - H_L^C,
\end{aligned} \tag{44}
$$

Before litigation, the value functions with the option to litigate of the two parties are

$$
\begin{aligned}
F_{BLL}(x) &= \frac{\Pi_2^I x}{r-\mu} + A_I^L x^\alpha, \\
G_{BLL}(x) &= \frac{\Pi_2^C x}{r-\mu} + A_C^L x^\alpha.
\end{aligned} \tag{45}
$$

Applying value matching and smooth-pasting conditions on the incumbent's value functions and value-matching on the challenger's value functions, we have that x_l satisfies the following equation,

$$
(\alpha - 1)x_l p(\Pi_1 - \Pi_2^I)\delta + (\alpha - \beta_\lambda)B_I^{DL} x_l^{\beta_\lambda} - \alpha H_L^I = 0, \tag{46}
$$

and the values of the arbitrary constants for the two parties

$$
\begin{aligned}
A_I^L &= \{p(\Pi_1 - \Pi_2^I)\delta x_l + B_I^{DL} x_l^{\beta_\lambda} - H_L^I\} x_l^{-\alpha}, \\
A_C^L &= \{-p\delta\Pi_2^C x_l + B_C^{DL} x_l^{\beta_\lambda} - H_L^C\} x_l^{-\alpha}.
\end{aligned} \tag{47}
$$

4.2.2 Ex-ante Settlement

Instead of litigation, the incumbent can also offer an ex-ante settlement to the challenger. In this case, the value functions of the two parties are

$$\begin{aligned} G_{SA}(x,\theta_A) &= \frac{(1-\theta_A)\Pi_2^C}{r-\mu}x - C_{SA}^C, \\ F_{SA}(x,\theta_A) &= \frac{\Pi_2^I + \theta_A\Pi_2^C}{r-\mu}x - C_{SA}^I. \end{aligned} \tag{48}$$

The value function with the option to settle before litigation of two parties are

$$\begin{aligned} F_{BLSA}(x) &= \frac{\Pi_2^I x}{r-\mu} + A_I^{SA}x^\alpha, \\ G_{BLSA}(x) &= \frac{\Pi_2^C x}{r-\mu} + A_C^{SA}x^\alpha. \end{aligned} \tag{49}$$

Based on the same method as ex-post settlement, we assume the challenger chooses the optimal threshold given the incumbent's royalty level offered θ_A. The challenger's value function then satisfies the value matching condition

$$G_{BLSA}(x) = G_{SA}(x,\theta_A).$$

which gives

$$A_C^{SA}(x_{sa},\theta_A) = -(\frac{\theta_A\Pi_2^C}{r-\mu}x_{sa} + C_{SA}^C)x_{sa}^{-\alpha} < 0. \tag{50}$$

This indicates the challenger's value incorporating the possibility of ex-ante settlement G_{BLSA} is smaller than the ongoing duopoly value $\frac{\Pi_2^c x}{r-\mu}$. Since A_C^{SA} is negative, the challenger's value function is concave and cannot be maximized by the smooth-pasting condition.

Maximizing this constant $A_C^{SA}(x_{sa},\theta_A)$ with respect to x_{sa} given the royalty level θ_A, we have

$$\frac{\partial}{\partial x_{sa}}A_C^{SA}(x_{sa}|\theta_A) = (\alpha-1)\frac{\theta_A\Pi_2^C}{r-\mu}x_{sa}^{-\alpha} + \alpha C_{SA}^C x_{sa}^{-\alpha-1} > 0$$

This implies that the gap between the duopoly value $\frac{\Pi_2^c x}{r-\mu}$ and value including the ex-ante settlement option G_{BLSA} narrows when x_{sa} increases. The challenger prefers to wait as long as possible before agreeing to settle ex-ante. however, ex-ante settlement must be agreed before litigation commenced. This imposes an upper bound on ex-ante settlement threshold $x_{sa} \leq x_l$. Together implies the challenger will wait until x_l before accepting ex-ante settlement .

In addition, the challenger will accept ex-ante settlement offer when his firm value assuming ex-ante settlement is higher than his value if he rejects and the incumbent proceeds to litigation, that is

$$G_{BLSA}(\theta_A, x) \geq G_{BLL}(x) \tag{51}$$

From which we can obtain a maximum royalty level that the challenger will be willing to accept

$$\theta_A^{Cmax} = -\frac{r-\mu}{x_{sa}\Pi_2^C}(A_C^L x_{sa}^\alpha + C_{SA}^C) \tag{52}$$

On the other hand, the incumbent will offer ex-ante settlement only when his value including the option to settle ex-ante is higher than his value incorporating the option to litigate, that is

$$F_{BLSA}(\theta_A, x) \geq F_{BLL}(x) \tag{53}$$

which gives the minimum royalty level that the incumbent requires

$$\theta_A^{Imin} = \frac{r-\mu}{x_{sa}\Pi_2^C}(A_I^L x_{sa}^\alpha + C_{SA}^I) \tag{54}$$

The incumbent's value function satisfies the value-matching condition $F_{BLSA}(x_{sa}) = F_{SA}(\theta_A|x_{sa})$, which gives

$$A_I^{SA}(\theta_A|x_{sa}) = (\frac{\theta_A\Pi_2^C}{r-\mu}x_{sa} - C_{SA}^C)x_{sa}^{-\alpha}. \tag{55}$$

The incumbent will choose the optimal θ_A^* by maximizing $A_I^{SA}(\theta_A|x_{sa})$ at $x_{sa} = x_l$. We have

$$\frac{\partial}{\partial\theta_A}A_I^{SA}(\theta_A|x_l) = \frac{\Pi_2^C}{r-\mu}x_l^{1-\alpha} > 0,$$

which indicates that incumbent's value with ex-ante settlement option increases with θ_A. The incumbent thus chooses the maximum royalty level which the challenger will accept (i.e $\theta_A^* = \theta_A^{Cmax}$).

In sum, there is the possibility of ex-ante settlement if $\theta_A^{Imin} \leq \theta_A^{Cmax}$, in which case royalty level chosen by the incumbent is $\theta_A^* = \theta_A^{Cmax}$, but the there is no settlement if $\theta_A^{Imin} > \theta_A^{Cmax}$.

5 Comparative Statics and their discussions

In this section, we investigate the possible outcomes arising from the model above and study the impact of different parameters on these outcomes.

Our base case parameter values are shown in Table 1. Here, we assume a moderate market scenario with the risk-free rate $r = 0.05$, the growth rate $\mu = 0.02$, and the volatility $\sigma = 0.3$. Consistent with the US patent legal system, we assume the average duration of litigation is 2.5 years and the probability of patent validity p is 0.5. We assume the costs of litigation and settlement are equal for both parties, but that competition reduces the overall market profit, so $\Pi_1(= 1.2) > \Pi_2(= 1)$ and the duopoly profits are asymmetric with $\Pi_2^C = 0.3$ and $\Pi_2^I = 0.7$. This represents an infringement where the challenger's product is a close substitute.

We measure the relative impact of the infringement on the challenger's and incumbent's profits via the gain-to-loss ratio defined as $\Phi = \frac{\Pi_2^C}{\Pi_1 - \Pi_2^I}$, which measures the challenger's value from sales of the substitute product as a proportion of the loss to the incumbent due to reduced sales. We assume competition reduces overall income, so $\Phi \leq 1$. Note in Jeon (2015), $\Pi_1 = \Pi_2$ which indicates no overall loss in market value from infringement. In contrast, we find that the gain-to-loss ratio is a key determinant of whether settlement occurs. We vary the gain-to-loss ratio by keeping fixed the duopoly profits of the two parties after infringement and varying the monopoly profit before infringement Π_1. Low gain-to-loss ratios thus correspond to situations where the overall profitability of the industry declined significantly after entry by a competitor, $\Pi_2 \ll \Pi_1$, due to stiff competition and/or close substitute products, whereas high gain-to-loss ratios suggest the competitor's entry into the market had relatively little impact on the incumbent's profits, perhaps because of greater differentiation between the competing products. Low gain-to-loss ratios limit the capacity for settlement, since the lower the gain-to-loss ratio, the less likely the challenger will be able to recompense the incumbent for his loss due to infringement by paying royalties.

The second key determinant of settlement feasibility is the potential for cost savings from settlement. Litigation incurs a continuing cost, which in present value terms is greater then the overall costs of settlement, both for each party and overall. We allow the cost saving from settlement ($H_L - C_S$) to differ between incumbent and challenger, and measure the relative cost savings as $rcs = \frac{H_L^C - C_S^C}{H_L^I - C_S^I}$. We find this is an important determinant of the timing of settlement, i.e. before or after commencement of litigation proceedings. Costs of settlement and litigation for both parties were assumed symmetric in Jeon (2015). We vary the relative cost savings by fixing the settlement costs of the two parties and the challenger's litigation cost H_L^C and changing the incumbent's litigation cost H_L^I. Thus low relative cost savings correspond to high litigation costs for the incumbent and therefore also high overall cost savings from settlement. Higher relative cost savings increase the challenger's incentive to settle but decrease the incumbent's incentive to settle. Due to these opposing effects, settlements are likely to occur when the relative cost savings are around 1.

[Insert Table 1 here.]

5.1 Royalties in Settlement

In this section, we investigate how the characteristics of patent litigation affect the capacity for settlement, i.e. whether the two parties will reach a settlement; the timing of settlement, i.e. whether such a settlement will occur before or after litigation has commenced (ex-ante or ex-post settlement); and the royalty payment if a settlement is feasible.

Recall the range of the royalty levels for ex-post settlement discussed in section 4.1.3. A minimum ex-post royalty level is required by the incumbent to compensate them for giving up the option of continuing with the litigation. There is also a maximum feasible ex-post royalty level which can in principle be determined by either the incumbent or the challenger because of the two possibilities as to why settlement may not occur. When the challenger exits first, the maximum royalty determined by the incumbent is always lower than the maximum royalty the challenger is willing to pay because in this case, the challenger is willing to pay more in order to avoid exit. On the other hand, when the incumbent withdraws first, the maximum royalty of the challenger will be lower than the maximum royalty determined by the incumbent as the challenger is in a better position to pay less.

Therefore,

$$\theta_P^{max} = \begin{cases} \theta_P^{Cmax}, & \text{if the incumbent withdraws first} \\ \theta_P^{Imax}, & \text{if the challenger exits first.} \end{cases} \tag{56}$$

Ex-post settlement is thus feasible if $\theta_P^{min} \leq \theta_P^{max}$; otherwise litigation will proceed until either the challenger exits, the incumbent withdraws from the litigation or judgment occurs.

Figure 2 shows how the minimum, maximum and optimal ex-post royalty levels change with the relative cost savings when the gain-to-loss ratio is 0.6. The red dotted line denotes the maximal royalty required by the incumbent and the red dashed line represents the maximal royalty determined by the challenger. The solid red line $= min(\theta_P^{Cmax}, \theta_P^{Imax})$ is the maximum royalty for ex-post settlement. The jump in the maximal royalty occurs because of the switching of the order of leaving the market. We use 'W' to denote the case when the incumbent withdraws first and 'Exit' to represent the case when the challenger exits first. The blue line is the minimum royalty determined by the incumbent and the green line is the optimal royalty that maximizes the incumbent's firm value.

The figure shows that ex-post settlement is likely to happen when the relative cost savings is around 1 as suggested before. In this region, the optimal royalty is in between the maximum and minimum royalty that are both determined by the incumbent. When the relative cost savings are too high, i.e. the challenger's cost savings are much greater than the incumbent's, the incumbent has lower cost savings from settlement and there is no royalty for which his value of settling is higher than the value of not settling (equation (39) has no solution). Therefore, there is no offer in this area. When the relative cost savings are low, although the incumbent can save a lot through settlement and is thus willing to make an offer, the challenger's cost savings from settlement are relatively low. The maximum royalty that the challenger is willing to pay is thus much lower than the minimum royalty required by the incumbent, so the offer is rejected by the challenger. Hence, although we have two regions where the agreement of ex-post settlement cannot be reached, the reasons are different.

[Insert Figure 2 here.]

Figure 3 shows how the ex-post royalties vary with the gain-to-loss ratio when the relative cost savings equal 1. We use a black dashed line to represent the change in the order of firms exiting the market (i.e either the incumbent withdraws first or the challenger exits first). When the gain-to-loss ratio is high, which indicates the challenger's increase in value is a relatively high proportion of the loss to the incumbent due to infringement, both the incumbent and the challenger have the incentive to settle because of the high litigation cost, and the challenger's profits are sufficiently high that a settlement can be reached which is beneficial to both parties. However, when the gain-to-loss ratio is low, i.e. the challenger's profit from infringement is a small proportion of the incumbent's loss from infringement, the incumbent's value of continuing with litigation to recover that loss is relatively high. Thus the capacity of the challenger to pay sufficiently high royalties to compensate the incumbent for giving up the option of continuing to judgment is limited: it is not worthwhile for the incumbent to make a settlement offer.

[Insert Figure 3 here.]

We also examine ex-ante royalties. We focus on the area of ex-post settlement in Figure 3, and find ex-ante settlement arises in the middle of this parameter region as shown in Figure 4. In this case, there is a maximum royalty that the challenger is willing to pay and a minimum royalty that the incumbent is willing to offer. We show that when the gain-to-loss ratio is high, which implies the challenger's profit from infringement is a relatively high proportion of the incumbent's loss from infringement, both parties are willing to settle ex-ante to avoid the high litigation cost and share the market profit. However, when the gain-to-loss ratio is close to 1, the challenger's profit is of a similar magnitude to the incumbent's loss. For these parameter values the incumbent will withdraw before the challenger would exit. In this case, ex-ante settlement will be rejected by the challenger, even though the incumbent is willing to offer. This case can occur when the challenger sells a highly differentiated product, making the overall profit after infringement Π_2 very close to the monopoly profit earned by the incumbent before infringement Π_1.

[Insert Figure 4 here.]

5.2 Outcomes: Settlement vs Continuing to litigate

5.2.1 Gain-to-loss Ratio and Relative Cost Savings

Figure 5 shows outcomes arising from different combinations of the gain-to-loss ratio Φ and relative cost savings rcs. Above the dashed line, the challenger exits first. Below the dashed line, the incumbent withdraws first. The green line is the incumbent's decision line to make an ex-post settlement: on the left-hand side of the green line, there is no offer; whereas on the right-hand side of the green line, the incumbent is willing to offer ex-post settlement. The red line is the challenger's decision line to accept the ex-post settlement: below the red line, ex-post settlement offered by the incumbent is rejected by the challenger; and above the red line, the ex-post offer made by the incumbent is accepted by the challenger. Finally, the blue line is the incumbent's decision line to make an ex-ante settlement: Inside the region that surrounded by the blue line, ex-ante settlement rather than litigation will occur. As the challenger will always accept if the incumbent offers, there is no decision line of ex-ante settlement for the challenger. This is because the incumbent will only offer the ex-ante settlement when his value of litigation is lower than his value of settling. For the challenger $A_C^L = A_C^{SA}$ for all cases, so the challenger is indifferent between accepting the offer or rejecting the offer.

[Insert Figure 5 here.]

To understand the impact of the gain-to-loss ratio, consider the impact of changing the monopoly profit on the gain-to-loss ratio and the incumbent's and challenger's incentives to settle vs continuing to litigate respectively. Increasing the monopoly profit increases the incumbent's loss from infringement, which decreases the gain-to-loss ratio. It also increases the incumbent's expected payoff from litigation and thus increases the incumbent's value of not settling. Since the incumbent will only choose to settle if the value of settling exceeds the value of continuing to litigate, this increases the minimum royalty required to induce the incumbent to make an ex-post settlement offer, but also decreases the incumbent's maximum feasible royalty offer (See Figure 3).

The incumbent's maximum feasible royalty level arises because increasing the royalty level decreases the threshold at which the challenger agrees to the settlement. A higher royalty level thus has two opposing effects: it increases the value of settlement to the incumbent once agreed, but delays the timing of the challenger's agreement to settle, which decreases the incumbent's pre-settlement value. The incumbent will only offer ex-post settlement if the value from settlement exceeds the value from continuing with the litigation. For too high royalty levels the timing of ex-post settlement, which is determined by the challenger, is sufficiently late (the threshold is sufficiently low) that settlement is no longer worthwhile for the incumbent. As the gain-to-loss ratio decreases, the range of feasible royalties shrinks, and for sufficiently low gain-to-loss ratios, ceases to exist. Then there is no ex-post royalty level which would give the incumbent a higher expected value than continuing to litigate, despite the higher costs involved.

Figure 5 also shows that the range of gain-to-loss ratios for which ex-post settlement is feasible depends on the cost savings. Recall low relative cost savings correspond to high incumbent litigation costs and hence high overall cost savings from settlement. In Figure 5, the greater the incumbent's cost savings from settlement (the lower the relative cost savings), the wider the range of gain-to-loss ratios for which the incumbent is willing to offer settlement. However, when the incumbent's litigation cost is high (relative cost savings are low), the challenger may reject the incumbent's offer. This is because the higher the incumbent's litigation cost, the greater the likelihood that they will be unable to continue to pay the costs and will thus withdraw the litigation. This increases the challenger's value of continuing, i.e. not settling, and hence decreases the likelihood of settlement. Nevertheless for sufficiently high gain-to-loss ratios, settlement, either ex-ante or ex-post, will occur. For ex-ante settlement, as we analyzed in section 5.1, both parties are willing to settle when the gain-to-loss ratio is high, but the two parties may not be able to reach an agreement when Φ is close to 1.

In summary, Figure 5 shows that the gain-to-loss ratio is an important determinant of the possible outcomes of patent infringement. When the gain-to-loss ratio is sufficiently low, there is no settlement, and when the gain-to-loss ratio is high, the dispute is resolved through settlement. Whether the settlement occurs ex-post or ex-ante depends on the relative cost savings.

The gain-to-loss ratio represents the change in the overall profit of the market before and after the entry by the challenger. If infringement by the challenger reduces the overall profit dramatically, which implies a low gain-to-loss ratio, the incumbent will continue to litigate rather than offering a settlement. On the other hand, if the reduction of overall profit is lower (i.e gain-to-loss ratio is high), the patent conflict can be settled. Product similarity is a key determinant of the decrease in overall market profit. This means if the challenger sells a product which is a close substitute, the conflict cannot be resolved through settlement.

The settlement outcome also depends on the relative cost savings. The relative cost savings affect the timing of the settlement if the two parties choose to settle. Higher relative cost savings increase the range of ex-ante settlement but decreases the range of ex-post settlement. High relative cost savings decreases the incumbent's incentive to settle ex-post as explained in section 5.1, thus it decreases the value of the ex-post settlement, lowering the litigation value and increasing the likelihood of settlement ex-ante.

5.2.2 Probability of Patent Validity

As well as the two key parameter combinations of the gain-to-loss ratio and relative cost savings, we also consider the impact of the probability of patent validity. This changes the outcomes of patent litigation significantly as shown in Figure 6. An increase in this probability, which means that the incumbent is more likely to win in the final judgment, has two effects. Firstly it decreases the challenger value during litigation, which means the "challenger exit" scenario (occurring before withdrawal from litigation by the incumbent) becomes relevant for a wider range of relative cost savings (for a given gain-to-loss ratio). This in turn implies that the incumbent's decision about whether to offer ex-post settlement becomes more important. Secondly, it increases the incumbent's expected value from the judgment. The resulting increase in the incumbent's value of not settling decreases the range of gain-to-loss ratios for which ex-post settlement is feasible. Overall, settlement, both ex-ante and ex-post, is feasible over a wider range of gain-to-loss ratios (e.g. closer substitute products) when the probability of patent validity is lower.

[Insert Figure 6 here.]

However, this result is in contrast with Jeon (2015) who concludes that the high probability of patent validity increases the incentive for both parties to settle ex-ante. He declares that the incumbent can use litigation as a threat to encourage the challenger to accept ex-ante settlement, causing the mutually beneficial settlement before litigation. There are two reasons causing the different result. First, in Jeon's model, the value of litigation does not include the value of the option to settle ex-post. This decreases the value of litigation. In his model, the ex-ante settlement value that the incumbent requires will be lower, so ex-ante settlement is more likely. In our model, however, we consider the option to settle ex-post when calculating the litigation value. This increases the litigation value and thus also increases the value required to settle ex-ante, making mutually beneficial ex-ante settlement more difficult to achieve. Second, Jeon did not include the exit option by the challenger during litigation. This option increases the challenger's incentive to settle ex-post, and thus increases the value of ex-post settlement. As we analyzed above, the litigation value in our model incorporates the ex-post settlement value, meaning the likelihood of ex-ante settlement is negatively affected by the likelihood of ex-post settlement, so ex-ante settlement will be more difficult to achieve if we include the possibility of the challenger's exit. This effect is even more significant when the probability of patent validity is high as shown in Figure 6, because the parameter range for the challenger's exit is larger in this case.

5.3 Product Market Volatility

Finally, we consider the effect of product market volatility on patent litigation outcomes. Figure 7 presents a comparison of the outcomes from high versus low uncertainty of demand shocks. When the volatility increases, the likelihood of settlement decreases. This is because the value of not settling during litigation increases with volatility, which is consistent with Dixit et al. (1994). Furthermore, the decrease in the likelihood of ex-post settlement is more significant than that of ex-ante settlement. A high volatility makes it less likely for the litigants to settle ex-post, but the likelihood of the ex-ante settlement changes only slightly.

[Insert Figure 7 here.]

6 Conclusion

In this paper, we develop a real options model that analyses firms' strategies and the outcomes in patent infringement by comparing the option value of each outcome. We consider the interaction between firms' decisions in resolving the litigation (i.e being forced to leave or agreeing to settle) and incorporate the option values of both ex-post settlement and outcomes when settlement is not offered into the value of litigation, enhancing the modeling of the incumbent's decision about whether to offer ex-ante settlement.

By focusing on the impact of parameters on the possible outcomes in patent infringement, we find the gain-to-loss ratio significantly influences firms' strategies. A minimum gain-to-loss ratio is required for firms to reach an agreement to settle. When the gain-to-loss ratio is below this level, which means that infringement significantly reduces the overall market profit, the incumbent will continue with the litigation knowing the challenger may exit the market before the incumbent would optimally choose to withdraw. Even when the incumbent would withdraw before the challenger would exit, the incumbent and the challenger will be unable to settle because the challenger's low profit from infringement means the challenger cannot afford the minimum royalty payment the incumbent would require. In general, product similarity will reduce the overall profit after infringement because of price competition. Therefore, infringement which reduces market profits significantly, for example with a close substitute product and intensive price competition, has a small possibility of settlement before the final judgment.

However, other parameters like the relative cost savings, the probability of patent validity and volatility also affect the likelihood of settlement. We find that both higher probability of patent validity and volatility decrease the likelihood of settlement both ex-ante and ex-post, while higher relative cost savings for the challenger decreases the likelihood of ex-post settlement but increases the likelihood of ex-ante settlement.

Whilst our models assumptions currently reflect the US legal system, where litigation costs are borne by each party, our model could be extended to compare outcomes with those under the English system, where the winning partys litigation costs are borne by the losing party in the trial. Different studies in the law and economics literature find contradictory results regarding the effect of cost allocation rules on litigants' strategies and litigation outcomes, but none of the existing models have been able to incorporate each party's (in)ability to pay costs awarded against them. Our model has the potential to provide such a new angle of public policy interest.

This model can also analyze the case when there is more than one challenger in the market. The incumbent will have an additional decision of which challenger is the best one to litigate against in order to maximize firm value, taking account of the fact that other parameters such as the probability of patent validity in follow-up cases may be affected.

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Figure 1: Strategies and payoffs of the incumbent ("I") and the challenger ("C")

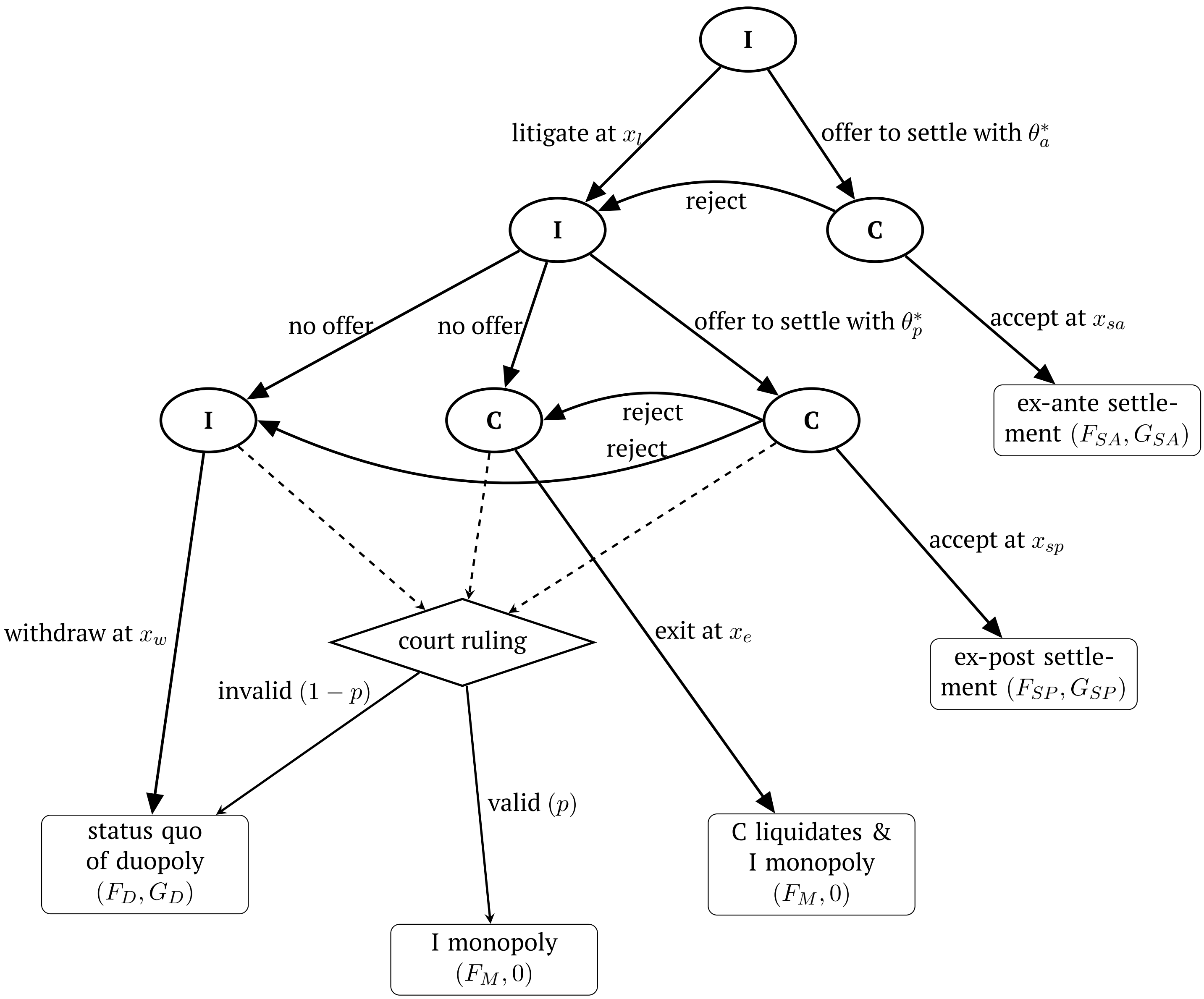


Figure 2: Ex-post royalties with relative cost savings

The blue line represents the minimal royalties required by the incumbent and green line is the optimal royalties determined by the incumbent. The red lines are the maximum feasible royalties for the incumbent or the challenger, where the dotted line is the one determined by the incumbent and the dashed line is the one determined by the challenger.

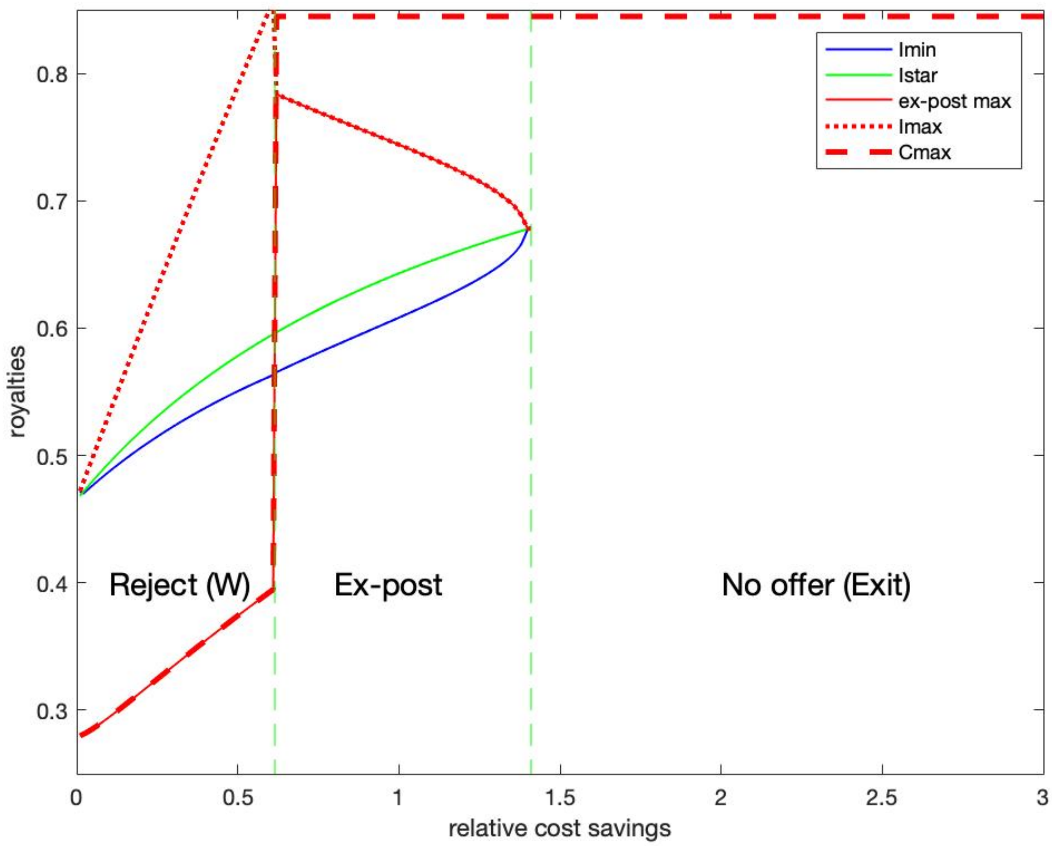


Figure 3: Ex-post royalties with gain-to-loss ratio

The blue line represents the minimal royalties required by the incumbent and green line is the optimal royalties determined by the incumbent. The red lines are the maximum feasible royalties for the incumbent or the challenger, where the dotted line is the one determined by the incumbent and the dashed line is the one determined by the challenger. The vertical black dashed line represents the boundary between regions where the challenger exits first and where the incumbent withdraws first.

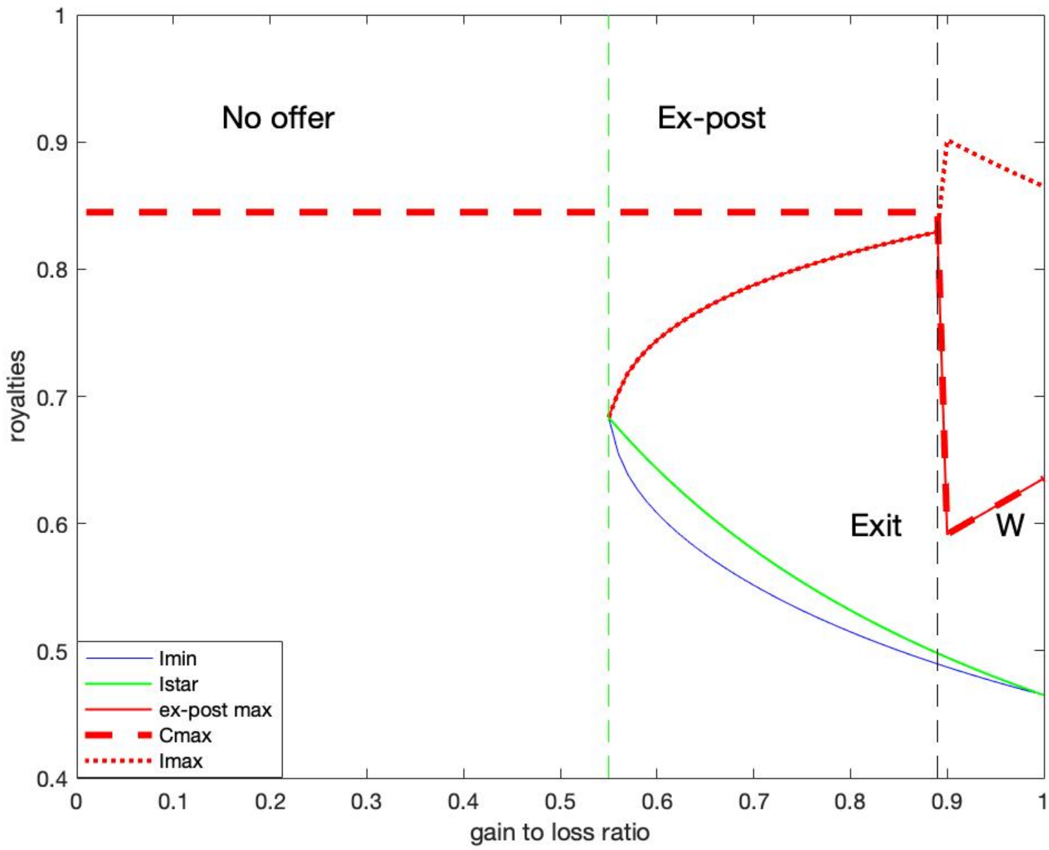


Figure 4: Ex-ante royalties with gain-to-loss ratio

The solid line is the challenger's maximum royalties and the dashed line is the incumbent's minimum royalties. The vertical black dashed line represents the boundary between regions where the challenger exits first and where the incumbent withdraws first.

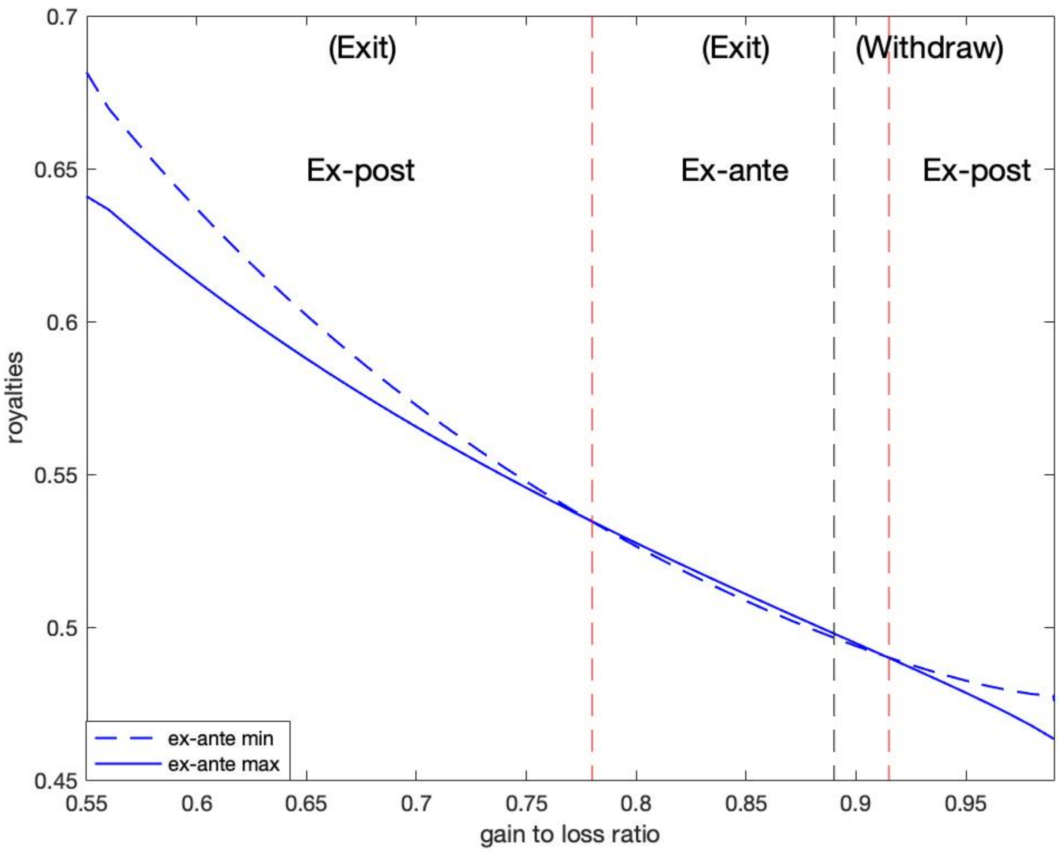


Figure 5: Possible outcomes in patent infringement

The green line is the threshold for the incumbent's decision whether or not to make an an ex-post settlement offer and the red line is the threshold for the challenger's decision whether or not to accept the ex-post settlement offer. The blue line is the boundary of the ex-ante settlement region. The black dashed line represents the boundary between regions where the challenger exits first and where the incumbent withdraws first.

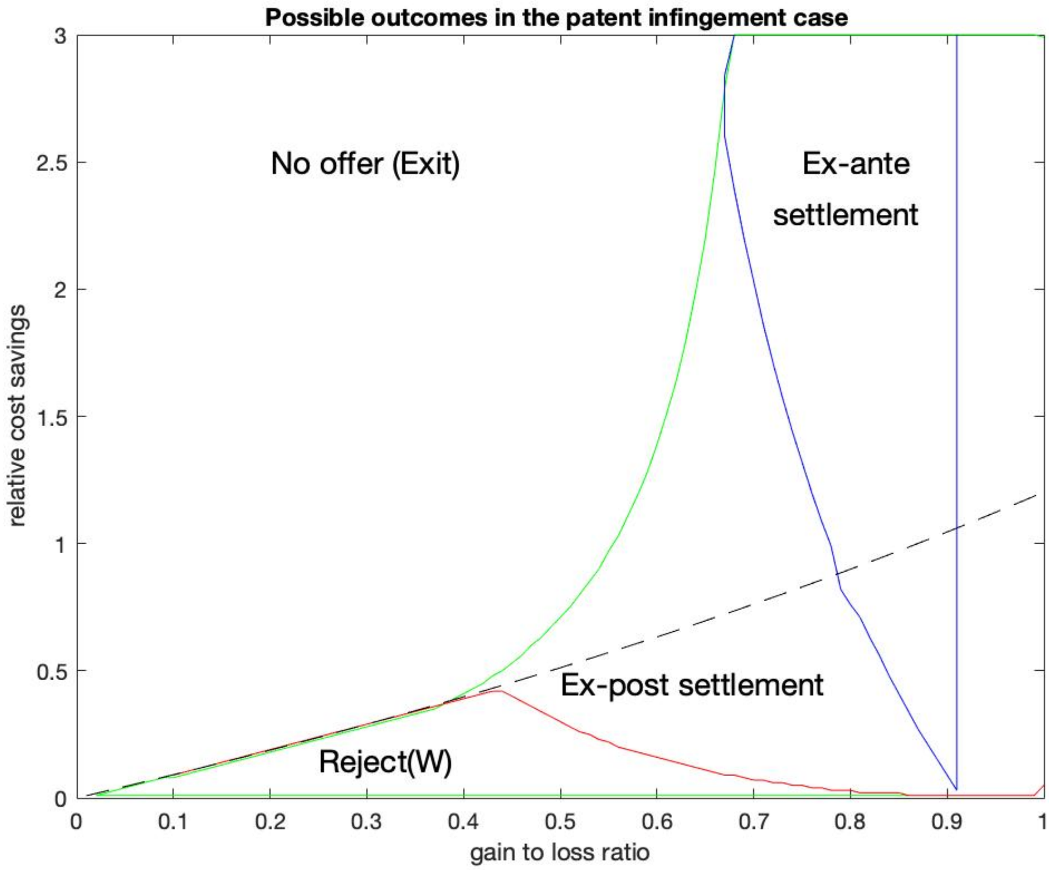


Figure 6: Possible outcomes with probability of patent validity

The green line is the threshold for the incumbent's decision whether or not to make an an ex-post settlement offer and the red line is the threshold for the challenger's decision whether or not to accept the ex-post settlement offer. The blue line is the boundary of the ex-ante settlement region. The black dashed line represents the boundary between regions where the challenger exits first and where the incumbent withdraws first.

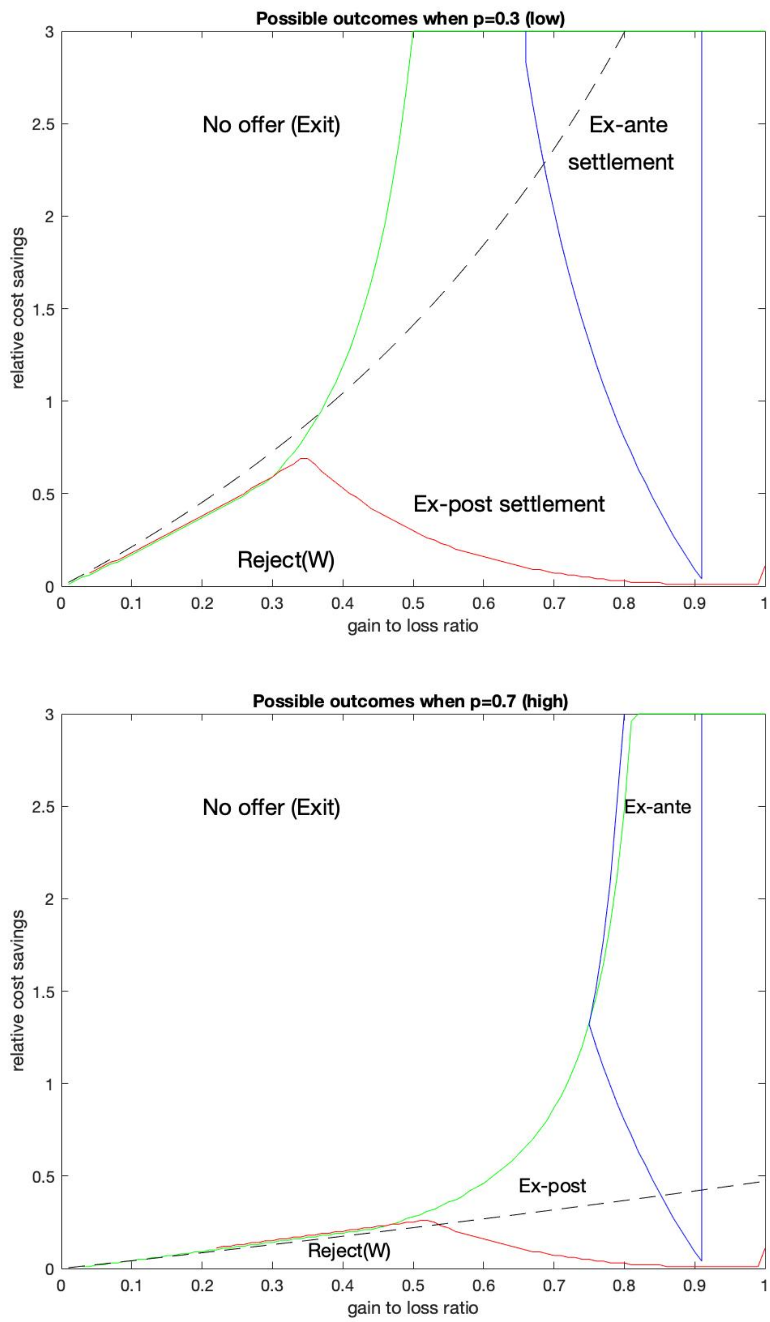
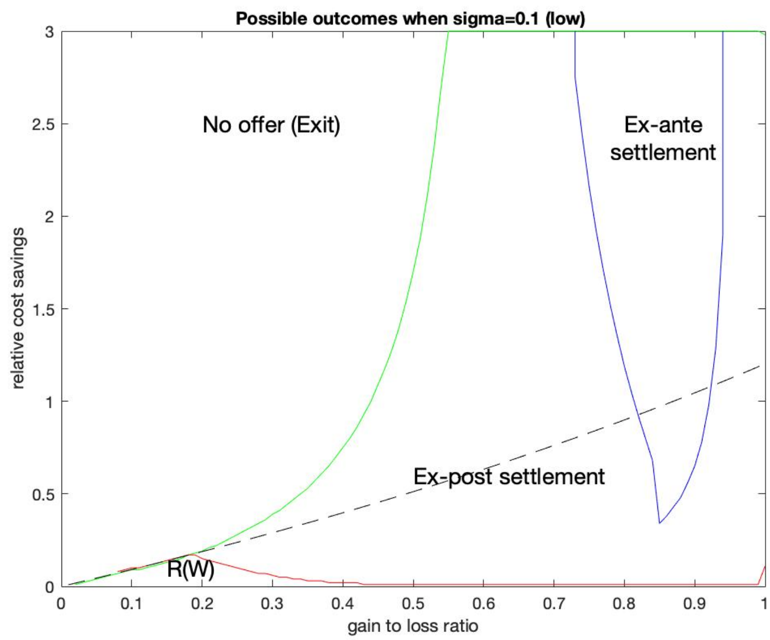


Figure 7: Possible outcomes with volatility

The green line is the threshold for the incumbent's decision whether or not to make an an ex-post settlement offer and the red line is the threshold for the challenger's decision whether or not to accept the ex-post settlement offer. The blue line is the boundary of the ex-ante settlement region. The black dashed line represents the boundary between regions where the challenger exits first and where the incumbent withdraws first.



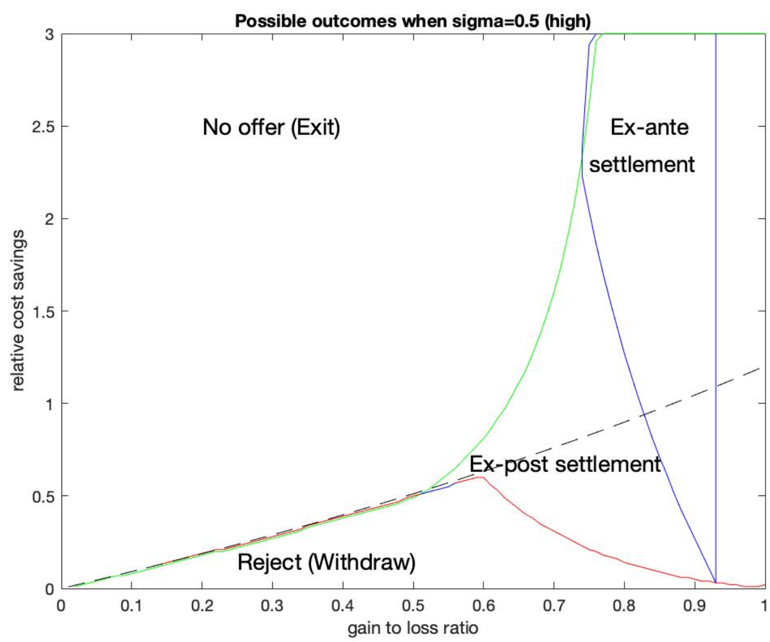


Table 1: Parameters in the benchmark case

We define the gain-to-loss ratio as $\Phi = \dfrac{\Pi_2^C}{\Pi_1 - \Pi_2^I}$ and the relative cost savings as $rcs = \dfrac{H_L^C - C_S^C}{H_L^I - C_S^I}$.

parameter	r	μ	λ	Π_2^I	Π_2^C	Π_1	Φ
value	0.05	0.02	$\frac{1}{2.5}$	0.7	0.3	1.2	0.6
parameter	p	σ	C_L^I	C_L^C	C_S^I	C_S^C	rcs
value	0.5	0.03	1	1	0.5	0.5	1