The Value of Turning-Point Detection for Optimal Investment

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Abstract

To capture the dynamic evolution of economic indicators and its impact on option pricing, we develop a regime-switching, real options framework for investment under uncertainty that facilitates time-varying transition probabilities. Considering a private firm with a perpetual option to invest, we use machine-learning techniques to forecast the evolution of transition probabilities and analyse how they affect the value of an investment opportunity. Results indicate that: (a) ignoring the dynamic evolution of transition probabilities can result in severe valuation errors; and (b) when the probability of a regime switch is low, the option value is greater in the good (bad) regime under time-varying than under fixed transition probabilities.

Keywords: investment analysis, time-varying transition probabilities, real options

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1 Introduction

Although firms recognise the cyclical nature of economic growth, predicting the dynamic evolution of economic indicators is notoriously hard. Theoretical models for investment under uncertainty have tried to analyse the evolution of business cycles and economic indicators via regime-switching models, however these are often based on restrictive assumption, such that transition probabilities are fixed (Driffill *et al.*, 2013; Goto *et al.*, 2017). Nevertheless, recent efforts have made significant breakthroughs in relaxing these assumptions. Bazzi *et al.* (2017) use data on industrial production growth to uncover three regimes for the mean and two regimes for the variance over the sample period considered. The corresponding transition probabilities are time varying. In particular, the high volatility regime appears to be much more persistent in the earlier part of the sample compared to the later part, while the converse holds for the low volatility regime.

Even though fixed transition probabilities may be a reasonable starting point, firms often have additional information about the likelihood of an economic turning point reflecting the emergence of a new market regime. For instance, leading indicators, such as building permits and interest rates (Stock & Watson, 1998) or the Purchasing Managers' Index (PMI) can is used to predict future economic conditions (Koenig, 2002). In fact, the PMI aims to capture executive's sentiment regarding future economic growth via a questionnaire about new orders, production and employment, and is an important indicator of the economy's health. Such indicators give firms reasons to adjust their expectations regarding future expansions and contractions. Nevertheless, traditional models often assume either a constant expected growth rate or, at best, fixed transition probabilities (FTP), thus disregarding important information that firms possess.

In this paper, we allow for time-varying transition probabilities by first describing how necessary parameters can be estimated from historical data, such as the likelihood of a regime shift, growth rate and price uncertainty. Subsequently these parameters are used to simulate future scenarios. Once we establish these scenarios, we can approximate the value of an investment opportunity by calculating the expected cash flow in each scenario. Hence, the contribution of the paper to the existing literature is threefold: **i.** we extend the current real options literature by incorporating TVTP; **ii.** we demonstrate the potential errors resulting from ignoring TVTP; and **iii.** we illustrate the model with a case study.

2 Related Work

Examples of early work that that develops a regime-switching model to describe policy uncertainty include Dixit & Pindyck (1994), who consider a tax credit on the investment cost that might be either present or absent depending on the current regime. In the same line of work, Hassett & Metcalf (1999) investigate a tax credit subject to regime switching, yet they allow the tax credit to be correlated with output price, and find that the policy uncertainty delays capital investments. Driffill *et al.* (2003) study how the evolution of business cycles may impact a monopolist's entry and exit decisions, and find that firms invest mainly during economic expansions and abandon in recessions. Guo *et al.* (2005) analyse the option to increase capacity incrementally within a regime-switching economy. They find that capacity expansions are subject to lumpy investment when a regime suddenly switches from recession to expansion, whereas firms will adjust capacity incrementally within a regime.

More recent examples of analytical models for investment within a regime switching environment include Driffill *et al.* (2013), who develop a model for analysing the state of the economy with discrete shifts between booms and busts. The value in each regime is found via a stochastic discount factor, which is greater during recessions than expansions, since investors marginal rate of substitution is greater during recessions. They find that Markov regime switching results in delayed investment. Grenadier *et al.* (2014) study how economic busts impact an agent's propensity to abandon real estate projects. Results indicate that after a bust, agents strategically decide to abandon unsuccessful projects in order to hide their abilities and thus blend in with the crowd. In the same line of work as Driffill *et al.* (2013), Goto *et al.* (2017) allow for competition in a regime-switching model, where two asymmetric firms compete to secure a period of monopoly profits. They show that even a disadvantaged firm can have an incentive to invest first, which occurs after a switch to the good regime where suddenly both firms find it lucrative to invest.

Although regime switching is a crucial feature affecting the viability of capital projects, the aforementioned papers ignore the time-varying nature of transition probabilities and the presence of unobservable regimes. Consequently, models that investigate the impact of the dynamic evolution of business cycles on a firm's propensity to invest remain somewhat underdeveloped. Nevertheless, in parallel, but separate from the the real options literature, statisticians and economists have developed analytical techniques to estimate and forecast

TVTP. In their pioneering model, Diebold *et al.* (1994) extend Hamilton (1989) by illustrating how transition probabilities can depend on other economic variables by employing an expectation-maximisation algorithm to estimate transition probabilities. Filardo (1994) uses maximum likelihood to investigate if the growth rate in US industrial production alternate between periods of expansion and contraction of varying duration. Their results indicate the existence of two states with different duration. Creal *et al.* (2013) introduce a framework to estimate time-varying parameters, which utilises the complete density of the dependent variable to calculate a score, which indicates how sensitive a likelihood function is to parameter changes. Bazzi *et al.* (2017) adopt the framework of Creal *et al.* (2013) to study TVTP and demonstrate its applicability to US industrial production.

Although these papers develop the necessary tools to estimate and forecast TVTP, they do not consider how TVTP may impact the value of an investment opportunity. However, the latter depends on the dynamic evolution of transition probabilities, which hinders analytical tractability, numerical methods must be adopted. Longstaff & Schwartz (2001) introduce a simulation method for valuing American options. Their main insight is to use ordinary least squares (OLS) to estimate the conditional expected payoff. The firm then decides whether to exercise the option by comparing the immediate payoff with the future expected value. Stentoft (2004) shows that the approximated option value converges to the true option value in a multi-period, multi-dimensional setting, yet convergence rates are uncertain in this general case. The flexibility of the approach lends itself well to a wide range of applications. For example, Boomsma *et al.* (2012) use this method to price real options under different support scheme, while Hsu & Schwartz (2008) investigate multistage R&D projects. Cortazar *et al.* (2008) extend Brennan & Schwartz's (1985) model by including three correlated stochastic processes which represent relevant commodity prices.

In this paper we develop a simulation method for investments under TVTP and investigate the tradeoff between waiting for more information against the possibility of a worsening business climate. First, we implement an analytical benchmark case and solve it both analytically and numerically. Next, we allow for TVTP to capture changing expectations through time. Consequently, we derive new insights regarding how expectations impact optimal investment decisions. Results indicate that, low regime uncertainty creates value in an expansionary period i.e. high growth. Although, small errors in the estimated parameters are amplified in the valuation calculation, the contribution from TVTP on option value is even greater in a period of economic expansion when a sudden regime switch is unlikely initially.

3 Assumptions and Notation

Uncertainty is modelled on a probability space $(\Omega, \mathcal{F}, P)$, where Ω is the set of all possible realisations of the economy, which is endowed with a filtration $\{\mathcal{F}_t; t \in [0, \infty)\}$ generated by relevant state variables, as in Longstaff & Schwartz (2001). We consider a firm with a perpetual option to invest in a project of infinite lifetime facing price uncertainty. The price process $\left\{P_t^{(\epsilon_t)}, t \geq 0\right\}$, where t denotes time and $\epsilon_t \in \{1, 2\}$ denotes the current regime, follows a Markov-modulated geometric Brownian motion (GBM) that is described in (1). We denote by μ_ϵ the annual growth rate, by σ_ϵ the annual volatility and by dZ_t the increment of the standard Brownian motion.

$$dP_t^{(\epsilon_t)} = \mu_{\epsilon_t} P_t^{(\epsilon_t)} dt + \sigma_{\epsilon_t} P_t^{(\epsilon_t)} dZ_t, \quad P_0 \equiv P > 0 \tag{1}$$

To facilitate a discrete approximation of $\left\{P_t^{(\epsilon_t)}, t \geq 0\right\}$, we let T be the duration of the time horizon and N the total number of time steps indexed by $n = 1, 2, \ldots, N,$, so that $\Delta t = T/N$. We set T sufficiently high so as to not impact the option value at $t = 0$, significantly. The analytical solution to (1) is indicated for the continuous-time case in (2) and the discrete approximation for a scenario is outlined in (3), where $\omega_{P,n} \sim \mathcal{N}(0, 1)$ (Miranda & Fackler, 2002).

$$P_{t+dt}^{(\epsilon)} = P_t^{(\epsilon)} \exp\left\{\left(\mu_\epsilon - 0.5\sigma_\epsilon^2\right) dt + \sigma_\epsilon dZ_t\right\} \tag{2}$$

$$P_{t+\Delta t}^{(\epsilon)} = P_t^{(\epsilon)} \exp\left\{\left(\mu_\epsilon - 0.5\sigma_\epsilon^2\right) \Delta t + \sigma_\epsilon \sqrt{\Delta t}\, \omega_{P,n}\right\} \tag{3}$$

In the FTP model, regime switching follows a time-homogenous Poisson process $\{J_t, t \geq 0\}$ that is independent of the process $\left\{P_t^{(\epsilon)}, t \geq 0\right\}$. Consequently, the time τ between two subsequent regime switches is exponentially distributed, i.e. $\tau \sim \exp\left(\lambda^{(\epsilon)}\right)$. The parameter $\lambda^{(\epsilon)}$ is the intensity of the Poisson process, and, thus, the probability of a regime switch within an infinitesimal time interval dt is $\lambda^{(\epsilon)} dt$. In contrast, under TVTP we assume that there is a state variable $\{W_t, t \geq 0\}$ determining the current transition probability. Normally, W_t is an observable time series with unknown data generating process and mapping to transition probabilities (Diebold *et al.*, 1994; Filardo, 1994). But, to facilitate numerical examples and avoid probabilities drifting towards an absorbing barrier (1 or 0), we assume that W_t is stationary and is generated by an autoregressive process, as shown in Bazzi *et al.* (2017).

The state variable W_t has an associated probability generating process $p_n^{(\epsilon,\hat{\epsilon})} = \Phi\left(\alpha^{(\epsilon)} W_t\right)$ (Ding, 2012), where $p_n^{(\epsilon,\hat{\epsilon})}$ is the probability of going from state ϵ to the other state $\hat{\epsilon}$ at time step n, $\Phi(\cdot)$ is the cumulative normal density function and $\alpha^{(\epsilon)}$ is a constant to be estimated. Although there are several candidate functions, mapping a time series process to probabilities, such as the logistic transformation (Bazzi *et al.*, 2017), any specification that maps W_t into a unit interval is a valid candidate (Filardo, 1994).

Furthermore, we assume that $\mu_1 > \mu_2$, which implies that the first regime can be interpreted as a period of expansion, while the second regime is characterized by lower growth or retraction. Additionally, economic retractions usually exhibit greater volatility, and, therefore, we assume that $\sigma_2 > \sigma_1$ (Driffill & Sola, 1998), and that $\lambda^{(2)} > \lambda^{(1)}$ since recessions often have a shorter duration than expansions (Harding & Pagan, 2002). In line with Goto *et al.* (2017), we assume that the discount rate, r, is constant across states. We denote by $V_c^{(\epsilon)}(P)$ and $G_c^{(\epsilon)}(P)$ the analytical expression for the value of the option to invest and the expected value of the active project, while $\widehat{V}_c^{(\epsilon)}(P)$ and $\widehat{G}_c^{(\epsilon)}(P)$ denote the corresponding value function obtained numerically. In each case, c denotes either FTP (f) or TVTP (v). Finally, we denote the project's output by D and the investment cost by K.

4 Numerical Solution Procedure

4.1 TVTP Estimation

In practice, the true probability of a regime switch cannot be observed and has to be estimated. This is commonly done as in Hamilton (2008), but, in our case, with a slight modification to capture TVTP as demonstrated in Bazzi *et al.* (2017).

4.2 Monte Carlo Simulation of Price Scenarios

Based on the estimated parameters, we can simulate M paths of the Markov-modulated GBM. This is illustrated in Figure 1 for $M = 7$. Notice that a regime switch may occur between any two points in time, and, also, that the first regime has greater growth rate (blue) compared to the second regime (red) on average, in line with the assumptions in Section 3.

We follow the algorithm outlined below to simulate price scenarios.

1. We adopt the approach of Hamilton (1989) and assign an initial state $\epsilon_{n,m}$ to each scenario $m = 1, 2, \ldots, M$, at $n = 1$ in accordance with the unconditional probability.

2. In the next time period $(n+1)$, we update the current state according to the transition probability independently for each scenario.

3. Calculate the value of the Markov-modulated GBM for all $m = 1, 2, \ldots, M$ using (3).

4. Repeat steps 2 and 3 until $n = N$.

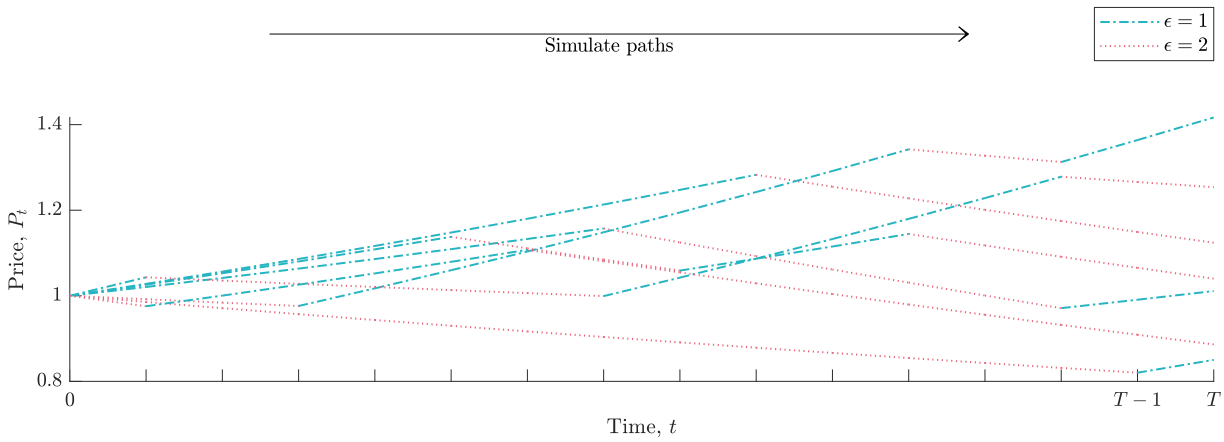

Figure 1: Illustrative example of simulation procedure

4.3 Option-Pricing Algorithm

Having simulated the paths of the price process, we now work backwards to evaluate the investment option as illustrated in Table 1. Each bracket contains the current state, the price observed in Figure 1 and the corresponding project value. For example, at time T for path 1 the firm is currently in the first regime and the output price is 1.42. Thus, the NPV at the expiration date is $G_f^{(1)}(1.42) - K = 6.78$, and in path 6 the firm chooses not to invest since the expected NPV is negative in regime 2. Prior to the final date, the firm compares the value of immediate exercise with the expected cash flows from continuing to wait, and will only exercise if immediate exercise is more valuable. Hence, the firm needs to identify the expected project value in the continuation region. In order to establish this expectation we use cross-sectional information in the simulated paths, which is done by regressing the subsequent realised cash flows from continuation on a set of basis functions with the current realisations as input for both states. Notice that when M is big some paths will have switched regime, while others have remained unchanged, and the future expectation reflects the regime uncertainty. The fitted value of this regression is an efficient unbiased estimate of the conditional expectation (Longstaff & Schwartz, 2001). For example, in the first path at $T-1$, it was not optimal to exercise the option. Thus the new project value is the discounted future value, i.e. $e^{-r\Delta t}6.78 = 6.74$, where $\Delta t = 2/30$ and $r = 0.1$.

Table 1: Illustrative table for the pricing algorithm: each bracket contains the state, price and project value. The brackets indicate the current state, the observed price and the corresponding project value.

Path	T-2	T-1	T
1	[1, 1.31, 6.69]	[1, 1.36, 6.74]	[1, 1.42, 6.78]
2	[2, 1.28, 3.69]	[2, 1.27, 3.55]	[2, 1.25, 3.42]
3	[2, 1.17, 2.58]	[2, 1.15, 2.30]	[2, 1.12, 2.03]
4	[2, 1.09, 1.68]	[2, 1.07, 1.41]	[2, 1.04, 1.13]
5	[1, 0.97, 1.95]	[1, 0.99, 1.96]	[1, 1.01, 1.98]
6	[2, 0.93, 0.00]	[2, 0.91, 0.00]	[2, 0.89, 0.00]
7	[2, 0.83, 0.07]	[1, 0.82, 0.07]	[1, 0.85, 0.07]

Hence, we extend Longstaff & Schwartz (2001) to value an American option when the price process follows a modulated GBM, and for each regime we approximate the option value by a linear combination of basis functions $L_j(\cdot)$ with coefficients $\beta_j^{(\epsilon)}$. Thus, $F^{(\epsilon)}(\cdot)$ is the approximated expected value in the continuation region as indicated in (4).

$$F^{(\epsilon)}\left(P_n^{(\epsilon)}\right) = \sum_{j=0}^{k} \beta_j^{(\epsilon)} L_j\left(P_n^{(\epsilon)}\right) \tag{4}$$

According to Longstaff & Schwartz (2001), there are several types of basis functions $L(\cdot)$ that can be used, e.g. Laguerre, Hermite, Legendre, Chebyshev, Gegenbauer and Jacobi polynomials. Longstaff & Schwartz (2001) find that results are robust to the different possible basis functions. Thus, we follow the same approach and use Laguerre polynomials with $k = 3$. The option-pricing algorithm can be summarised in the following steps.

1. Simulate $M \times N$ trajectories of the price process as described in Section 4.2, and calculate the expected value of immediate exercise by simulating the expected value for a *now-or-never* investment opportunity under TVTP.

2. We start at the final time step ($n = N$), and calculate the option value.

3. Step back to $n := n - 1$ and approximate the option value as in Longstaff & Schwartz (2001), via two regressions for each regime.

4. Calculate the conditional expectation for each scenario (m), and exercise the option only if immediate exercise yields a higher return.

5. Repeat step 3-4 until $n = 1$. At $n = 1$, calculate an average value for each regime and discount the two values one final time. The average values are then the option values for each state.

5 Simulation Example

For the numerical examples, we set the values for $\alpha^{(\epsilon)}$ such that the unconditional probability is the same as under FTP, i.e. $\lambda^{(\epsilon)}\Delta t = \Phi\left(\alpha^{(\epsilon)}\frac{\varphi_1}{1-\varphi_2}\right)$. Figure 2 illustrates the option value in regime one (left panel) and two (right panel) with FTP. Here, we adopt Goto *et al.* (2017) as a benchmark (green solid line) to the simulation approach (red dotted line). Notice that the confidence interval (CI) diminishes as M increases, yet at a lower rate when M is greater than 1500. Also, the mean is very close to the analytical solution for $M > 1500$.

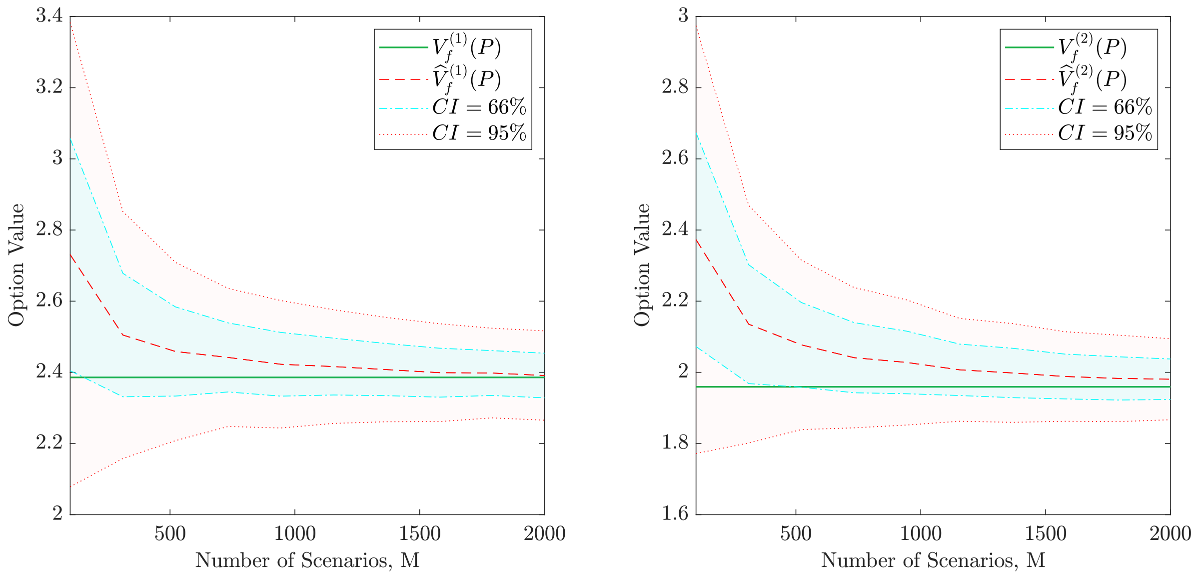

Figure 2: Impact of M on option value and estimation uncertainty for the first regime (left panel) and the second regime (right panel). The model is estimated one thousand times for each M in order to find the CIs, and $N = 2000$

To investigate the performance of the estimation procedure with TVTP, we consider two simulated paths which represent an observed process driving probabilities and a modulated GBM. The firm observes these time series and estimates the model parameters $(\hat{\theta})$. The resulting parameters are indicated in Table 2. Notice that both FTP and TVTP method are able to efficiently capture the first two moments in both regimes. Although, the true data generating process here is time-varying, the expected long-term transition intensities are $\lambda^{(1)} = 0.15$ and $\lambda^{(2)} = 0.3$ which the FTP model seems to capture well.

Table 2: True parameters and estimated parameters under FTP and TVTP, with $M = 1500$ and $N = 2000$

	μ_1	μ_2	σ_1	σ_2	$\lambda^{(1)}$	$\lambda^{(2)}$	$\alpha^{(1)}$	$\alpha^{(2)}$
True	3%	-2%	10%	20%	Na	Na	-0.097	-0.09
FTP	2.5%	-2.3%	10%	20%	0.16	0.26	Na	Na
TVTP	2.5%	-2.3%	10%	20%	Na	Na	-0.099	-0.091

6 Case Study

We investigate the implications of our proposed model through a case study. We assume that the PMI index is our leading indicator and that we are considering to invest in a firm with similar growth and risk characteristics as the S&P 500 index. In Figure 3 we plot the PMI (left panel) and S&P 500 (right panel) from 1948 until November 2018. The darker shades of grey are recessions according to NBER[1]. Notice that periods when the PMI index is less than 50, which means that the average sentiment among purchasing managers is negative, coincide well with recessions (left panel). Furthermore, the financial crisis and the dot-com bubble are also evident in the stock market and are classified as recessions according to NBER.

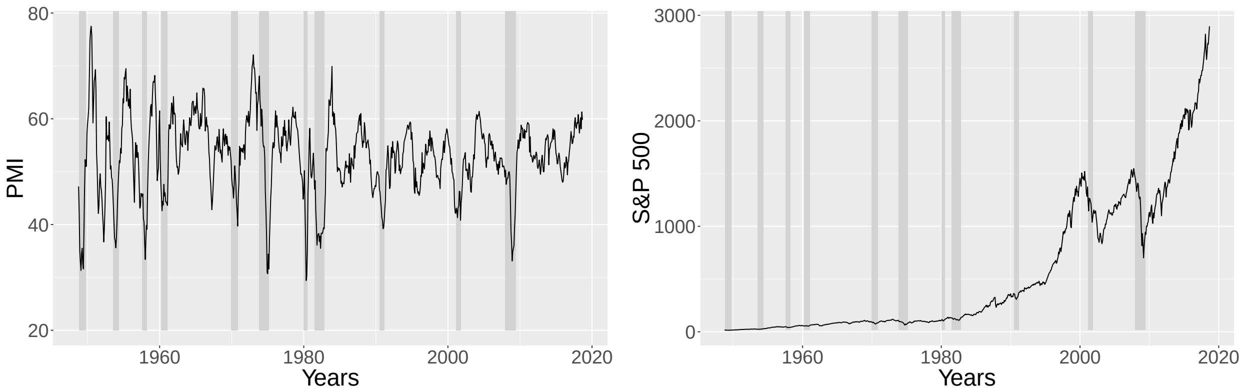

Figure 3: Historical values for PMI (left panel) and S&P 500 (right panel) with NBER business cycles as the shaded areas from 1948-2019.

The left panel in Figure 4 illustrates our model predictions for the current states, where the shaded areas are periods that the model forecasts a recession. Although our model predicts more regime transitions than NBER, it is able to capture the major recessions in the NBER dataset. In the right panel, each dot indicates the price for both scenarios for one model run. We run the model one thousand times in order to generate all the dots under different assumptions regarding TVTP and FTP. The average PMI over the period is 52, and at the time of doing this analysis, i.e. October 2018, the PMI index was 58. Hence, the index indicates an optimistic sentiment and we would assume that we are likely to stay in an expansion. More specifically, this implies that there is a higher likelihood for staying in regime 1 under TVTP. Indeed, that is what we observe in the right panel, where the value

[1] https://www.nber.org/cycles.html

is higher under TVTP (red dots) than under FTP (blue dots) for both regimes. In other words, by observing the PMI index at 58, the firm may find it unlikely that there will be a switch to the bad regime soon, and if they are currently in a recession, the firm assumes that the economy will enter an expansion shortly due to the positive sentiment.

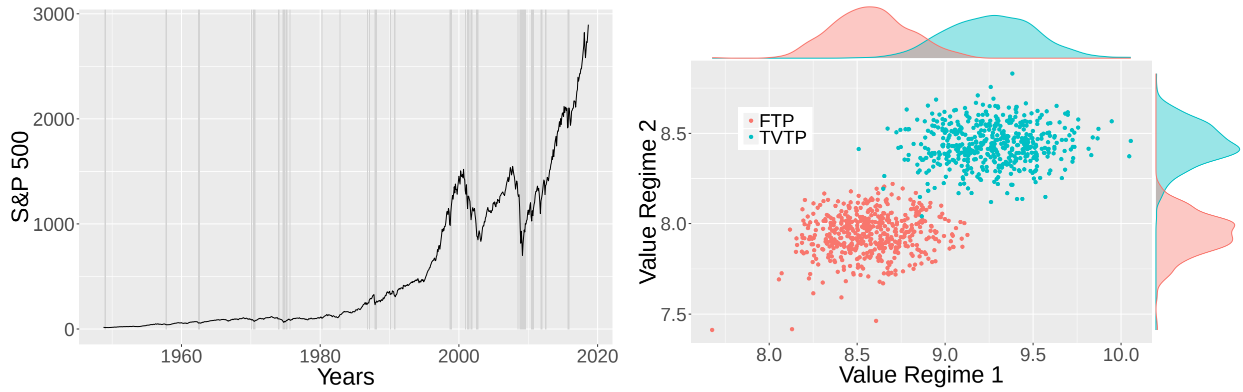

Figure 4: Predicted recessions indicated by shaded areas compared to the S&P 500 index (left panel) and option values under FTP and TVTP with estimated parameters based on the PMI and the S&P 500 index (right panel).

7 Conclusions

We develop a real options framework in order to address the problem of optimal investment under time-varying regime uncertainty and how future expectations impacts the option value. Although empirical evidence suggests that transition probabilities are time-varying (Filardo, 1994; Aloui & Jammazi, 2009), valuation frameworks that address TVTP are limited. Hence, our simulation approach extends the real options literature by developing a technique to price options under TVTP. More specifically, we capture the TVTP through an exogenous process which determines the transition probabilities, and thus impacts the option value. Results indicate that, when there is a high forecasted likelihood of an expansion, the project value increases substantially, and causes an increase in project value in both regimes. This is crucial for firms contemplating to invest in RE markets, where firms might have additional information indicating that markets are likely remain unchanged.

Apart from further empirically tests of the model, further analysis for different processes governing the transition probabilities and the price process are interesting extensions. For example, a mean-reverting process may be more suitable than a Markov-modulated GBM for firms connected to commodity markets, or transition probabilities could be cyclical (Bazzi *et al.*, 2017). Also, future work could consider other functional forms for calculating the transition probabilities.

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