Evaluating Flexibility for Oil Well Construction Services

Hernandes Coutinho Fagundes[1]

Rodrigo Tavares Nogueira[2]

Abstract

The antagonism between long term commitments and a highly uncertain scenario is notorious, however no few decisions are made in such conditions. Focusing on features connected to offshore subsea and well services, we search for value maximization adopting a flexible business model, where the contractor has the right to increase service level during the agreement period. The paper develops a practical and applied approach, modelling uncertainty as a mean reverting stochastic process and valuing decisions flexibility through real options theory and Cox-Ross-Rubinstein discrete method. The options consideration improved contract value and provided decision guidelines that can help managers to accomplish better dealings.

Keywords: real options; contract's flexibility; investment under uncertainty; switch options

I. Introduction

Oil exploration and production are developed through projects. Those complex investments begin with an opportunity evaluation, being subsequently organized in concept development, implementation, production and decommissioning. Researchers promoted studies about existing real options in opportunity evaluation (Dias 2001; Ma 2016) and production stages (Laine 1997; Jafarizadeh and Bratvold 2012), but little attention has been given to the options embedded in the project implementation phase, particularly those associated with oilfield

[1] Petróleo Brasileiro S.A.(Petrobras) / State University of Northern Rio de Janeiro, Campos dos Goytacazes, Brazil. E-mail: hernandesfagundes@gmail.com.

[2] Associate Professor, State University of Northern Rio de Janeiro, Campos dos Goytacazes, Brazil. E-mail: nogueirart@gmail.com

services.

Individually, the most important service provided during offshore installations are the maritime vessels, where activities are settled (rigs or boats), but each step is achieved with the support of others specialized services such as intervention workover and completion systems (IWOCS), well testing plants, slickline, wireline or coiled tubing equipment and subsea testing trees. Services are regularly provided as a full availability model, meaning that client pays a fixed fee (even when resources are in provider's storage) as a warranty for its readiness. In some cases, there is a perspective of extra payments during operations.

One important feature of these services is that they incorporate high specificity, restrict supply market, advanced technologies, noble materials or refined manufacturing processes. For the supplier, it means that a significant part of investment has no residual value and therefore it shall be depreciated during service commitment, leading to medium-term contracts, around four years long.

From a client point of view, the services characteristics ensures that they cannot be obtained in short term. Procurement process plus provider assemblies take regularly more than two years, and the results are affected by demand uncertainty during this time period. The decision between missed high value opportunities or capital investment in idle resources (as a high demand insurance) may both be undesirable.

For this reason, one of the hardest decisions during a contract planning refers to services quantification. The optimal specification depends on a demand prediction problem, considering expected variations, earnings and expenses. Recognizing the flexibility merit, we consider a business model that consents an additional initial payment to the supplier, in order to provide specific resources that usually contributes to a small fraction of total costs but are essential to reduce lead-time. In return, the client benefits from the ability to gather additional resources, in a high demand scenario, paying a fixed fee (entitled compensation charge) and the pre-agreed charges, until the end of the contract.

This paper addresses the additional value provided by this flexible arrangement. The challenges are related to cash flow and uncertainty modelling and also decisions criteria definition, allowing an undubious perspective of the best economic alternative, what can be rightly managed by real options theory.

The following sections are organized with chapter II providing a brief background of project valuation techniques and real options development. Section III models the demand behavior, as a mean reverting process. In addition, constitute the service contract's cash flow, valuing the project with traditional discounted cash flow techniques (DCF). Section IV discusses contract's valuation uncertainty, demonstrating lattices construction, with and without supply considerations. It also achieves an alternative (and more consistent) nonflexible valuation. Section V adds flexibility to the agreement, including the possibility to increase service level and applying real options techniques to determine it's worth. Section VI discusses the results and section VII summarizes conclusions.

II. Project Valuations Background

Valuation Techniques

According to de Souza and Lunkes (2016), investments valuation tools exist to advise managers in long term decisions, regarding costs relevance and behavior analysis. For this purpose, the most relevant prevailing technique was disseminated during the 70's and is known as discounted cash flow. Its main aspect is to consider capital adjustment in time and risk-discount relationship. The major DCF indexes are net present value (NPV) and internal rate of return (IRR).

Besides discounted cash flow acceptance, Dixit and Pindyck (1995) point out that in the end of the twentieth century, economic environment was substantially more unstable and unpredictable than early decades, demanding understanding of important alternatives, what is not granted by conventional analysis. Copeland and Antikarov (2001) and Brandão, Dyer, and Hahn (2005b) report that DCF technique is not able to reveal benefits that arise from flexibility, inherently assuming passive management in most situations. Also Trigeorgis (1996) argues that academics and managers are convinced of a failure to apply DCF methods to optimize resources allotment, mainly due to incapacity to handle manager's interactions.

The real options theory arose as an alternative to value flexibilities entrenched in projects, and its development has the same foundations of financial options, that were stablished by the seminal papers of Black and Scholes (1973) and Merton (1973). Still on the same decade, Myers (1977) conceived the term "real options" influenced by the original works of Black, Scholes and Merton. The real options theory relevance widely increased during the 90's, representing a robust opportunity to address issues related to DCF restraints.

The recent Real Options Theory dissemination was impelled by discrete approaches, benefited from the massive computational capacity increase. Wilmott (2009) comments that they provided higher transparency and reliability in comparison with traditional stochastic calculus, reaching an ampler and influential public. None of discrete methods are more significant than the approach by Cox, Ross, and Rubinstein (1979), constructing binomial lattices that converges into a lognormal diffusion process, known as geometric Brownian motion (GBM).

Mean Reverting Processes

When dealing with investments, the presumptions about uncertain behavior are very relevant. In some cases, it is interesting to identify a return tendency, pointing to an equilibrium state, in order that higher values are more likely to decrease and lower values are more likely to increase.

The stochastic motions that represent this feature are known as mean reverting. More formally, mean reverting processes are outlined as a Markovian process with changeable tendency, according to the relative position between the risky variable and the equilibrium level.

Mean reverting behavior might be intuitive recognized when related to market reactions, being qualified as the most appropriate process to describe commodities prices (Hahn and Dyer 2008; Samanez, Ferreira, and Nascimento 2014). The correlation is higher when feedbacks are faster, as exemplified by agricultural commodities.

It's possible to implement mean reversion with several models. Among them, the simpler and most popular is Ornstein-Uhlenbeck's. When the uncertain variable is not able to reach negative values (as prices), a modified version is recommended, known as geometrical Ornstein-Uhlenbeck process. Unlike the geometric Brownian motion, mean reverting processes' variance is limited by the long term reversion tendency (Samanez, Ferreira, and Nascimento 2014).

The work from Nelson and Plosser (1982) provided an important contribution about macroeconomic indicators and prediction capabilities, especially remarking random walk assumptions. Since it was published, discussions were raised about a lot of time series, questioning stationarity and the existence of some kind of reversion. According to Asteriou and Hall (2007), the distinction between a random movement and mean reverting behavior can be managed using econometric analysis, being unit root tests the most prominent class.

Real Options Categories

In real options theory, flexibilities are modelled as embedded options and classified according to its purpose. There are small variations among authors categories, but there is a common ground when referring to wait options, abandonment options, expansion/contraction options and switch options, as depicted in Dixit and Pindyck (1994), Trigeorgis (1996) or Copeland and Antikarov (2001). Switch options are frequently associated with process flexibilities, particularly input and output conversion, but represent a high versatile category, able to valuate most other real options, as far as the correct parameter interpretation is implemented.

III. Service Contracts Cash Flow

Data Regression

The proposed model considers demand (y_t) as the source of uncertainty for contract valuation. The variable is influenced by the requested services rate (as an exogenous risk), project conception's technical or preference modifications (as an endogenous risk) and time distribution pattern.

Figure 1 shows real data from a service applied to oil well construction. Time series includes 426 observations, weekly registered during almost eight years and two months.

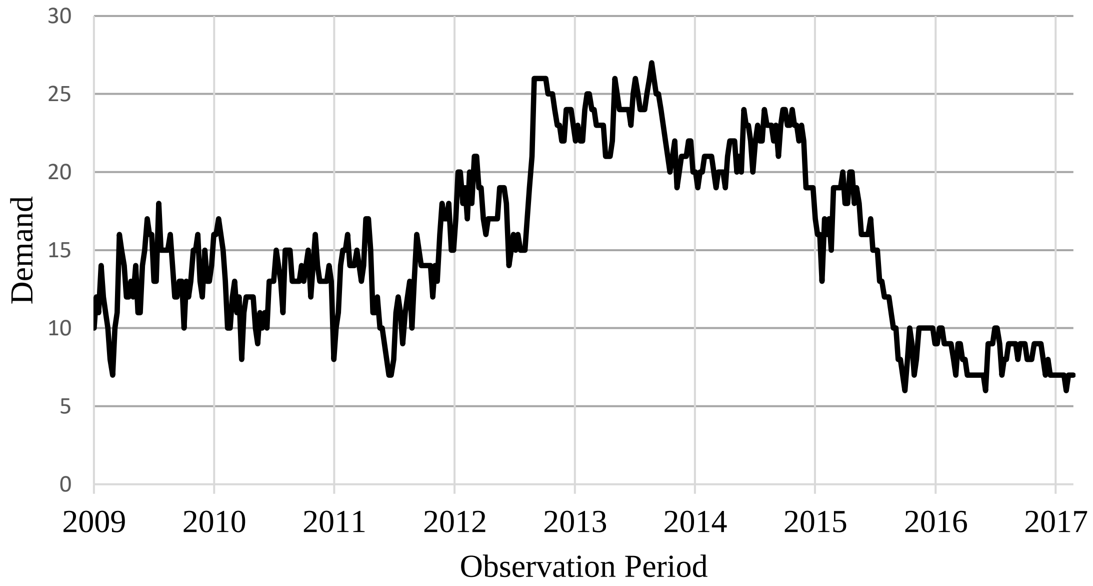

Figure 1. Time series representing demand of an oil well construction resource[3].

Stochastic processes are random sequences in time, describing uncertain variables behavior (Samanez, Ferreira, and Nascimento 2014) and its choice is one of the biggest sources

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3 Yearly marks are regularly plotted referring to the first measure date: 06/03/2009.

of discussion among current real options' works. Demand is a variable that can't assume negative values and the process representing it shall be able to capture its idiosyncrasy. This matter is frequently addressed using log transformed models, among which we select the geometric Brownian motion and the exponential Ornstein-Uhlenbeck mean reverting process, both accurate and simple enough to apply real options theory. Equation 1 represents these two stochastic processes, with a and b being constants and u_t the linear approximation error. The distinction between a random or mean reverting process shall be evaluated by the significance of the (b-1) term.

$$\Delta \mathrm{x_t} = a + (b - 1) * \mathrm{x_{t-1}} + u_t \quad (1)$$

$$x_t = \ln(y_t) \quad (2)$$

As discussed in Copeland and Antikarov (2001) and Samanez, Ferreira, and Nascimento (2014), equation 1 parameters are regressed using the ordinary least squares method (OLS) from log-returns data, what leads to results shown on Table 1. Since the regression is performed with one period lag, there is also one observation reduction relative to original data set.

Table 1 – Ordinary Least Squares regression parameters

Parameter	Value
a	0.1053
(b-1)	-0.0397
Observations	425

Stochastic Parameters

Residuals scattering allow us to identify a narrower dispersion related to variable higher values, typifying heteroskedasticity. A resource saturation elucidates this behavior, suppressing growth above a supply limit and accumulating neglected demands. As mentioned before, unit root tests usually evaluate the random walk hypothesis and once an altering variance was detected, we choose the Philips-Perron test that provides a heteroskedasticity robust verification (Hamilton

1994; Cavaliere 2005). The 5% significance critic value is defined as -2,87 and calculated test statistic is -28,71. Results refutes the unit root existence, stablishing mean reverting process as a suitable choice.

The exponential Ornstein-Uhlenbeck parameters are then calculated according to Copeland and Antikarov (2001) applying equations 3 to 6. In the mentioned equations, the parameter L represents how many observations are contained in one-year interval. Table 2 displays the results.

$$\sigma = \sigma_{\varepsilon} * \sqrt{L} * \sqrt{\frac{2*\ln(b)}{b^2-1}} \tag{3}$$

$$\bar{X} = \frac{a}{1-b} \tag{4}$$

$$\eta = -\ln(b) * L \tag{5}$$

$$y_{eq} = \exp\left(\bar{X} + \frac{\sigma^2}{2*\eta}\right) \tag{6}$$

Table 2 – Exponential Ornstein-Uhlenbeck process parameters

Parameter	Value
Volatility (σ)	82.288% p.a.
Reversion Speed (η)	2.107 p.a.
Equilibrium Level ($\bar{X}$)	2.652
Equilibrium Value (y_{eq})	16.65

DCF Valuation

Calculated parameters allow us to identify likely demand level and cash flow expected values. The monthly cash flow is proportional to demand but bounded by supply restraint, as described in equation 7. Service suppliers are regularly located in an intermediary process, serving several projects simultaneously, therefore we choose to link service benefits with implementation stage major resource, the maritime vessel. This approach is consistent with rig scheduling that presumes service availability for continuous operational capacity. In equation 7, $CF_{t,s}$ refers to cash flow in period t with the amount of available resources depicted by parameter s. CMU represents the monthly maritime vessel cost[4] and E operator symbolizes the expected value.

$$E(CF_{t,s}) = CMU * Min(E(y_t); s) \qquad (7)$$

The cash flow sum for every period until compromise's finalisation describes the current contract's value (CV_s^0), shown in equation 8. A risk-free rate (r) represents a limit situation, where revenues are certain (for practical purposes investors consider very low risks investments, such as treasury bonds). Risky projects, instead, are discounted using risk-adjusted rates (ψ). Companies usually define their convenient discount rate with the weighted average cost of capital technique or WACC (Bennouna, Meredith, and Marchant 2010).

The analysis presented in this paper considers a 48 months contract, which is a very common extent for such applications.

$$CV_s^0 = \sum_{t=1}^{48} \frac{E(CF_{t,s})}{(1+\psi)^t} \qquad (8)$$

The service monthly fee describes the expenses involved during the contract, for a specific number of resources (s), owing to their sureness, fees can be discounted using the risk-free rate. In this model they are assembled at time zero, representing an initial investment (I_s), as shown in equation 9. The net present value through DCF technique balances the revenues and expenses, as displayed in equation 10, parameter s representing resource quantity.

$$I_s = s * \frac{C_R}{r}(1 - (1 + r)^{-48}) \qquad (9)$$

$$NPV_s^D = CV_s^0 - I_s \qquad (10)$$

4 Considering a 3.2365 R$/US$ Brazilian real to American dollar ratio, according official closure exchange, provided by Brazilian Central Bank in 11/23/2017: http://www4.bcb.gov.br/pec/taxas/port/ptaxnpesq.asp?id=txcotacao.

The DCF method valuates the contract according to Figure 2, showing highest value when 16 initial resources (IR) are negotiated. In this case, contract's value reaches US$3,645 million during the overall period of four years. So far, the client has no capacity to change initial level along this interval.

Table 3 – Financial parameters value

Symbol	Value	Unit	Description
CMU	7,518,400	US$/month	Monthly maritime unit fee
C_R	328,930	US$/month	Monthly service fee
ψ	0.15	p.a.	Risk adjusted rate
r	0.06	p.a.	Risk free rate

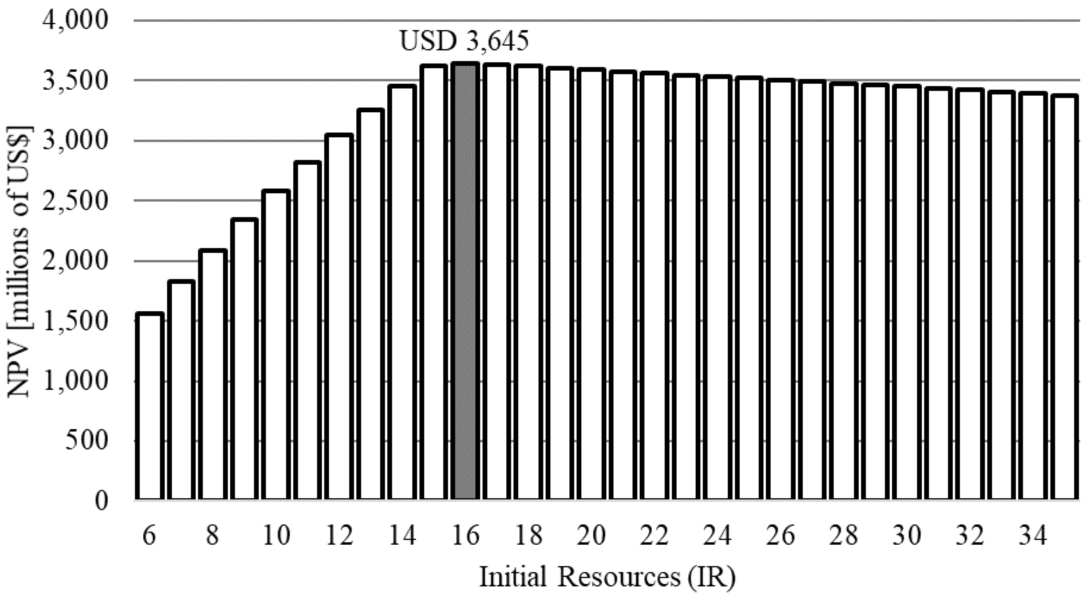

Figure 2. Contract's NPV for several supply levels according to DCF valuation.

IV. Event Tree Development

Volatility Definition

The DCF valuation ignores forecast's uncertainty, regarding demand and cash flow. Nevertheless, cash flows represented as dividends are capable of incorporate this feature in a discrete model. Under this aspect, the project pays dividend every month except in period zero, when no service has been provided.

According to Copeland and Antikarov (2001) and Brandão, Dyer, and Hahn (2005b) the dividend yield is based on contract's value, as a rate between period's expected cash flow and cum-dividend value (PV_c^t), shown in equations 11 and 12. For simplicity and without significant impact, we assume same risk-adjusted rate than DCF analysis. Notice that dividend yield is a period related calculus, fixed among same time states.

$$PV_t^c = \sum_{n=t}^{48} \frac{E(CF_n)}{(1+\psi)^{(n-t)}} \tag{11}$$

$$\delta_t = \frac{E(CF_t)}{PV_t^c} \tag{12}$$

Copeland and Antikarov (2001), supported by Samuelson (1965), demonstrated the arbitrary behavior of correctly anticipated cash flows. Thus, an uncertainty source incorporated in project's return rate will follow a random walk (GBM), even with a different cash flow pattern. This conclusion allows the development of event trees based on Cox-Ross-Rubinstein (CRR) method.

For the effort, we use Monte Carlo simulation to resolve volatility, based on works by Brandão, Dyer, and Hahn (2005a). Their methodology incorporates an adjustment from Copeland and Antikarov (2001) method, in order to avoid the parameter overestimation. Thereby, only the first cash flow (period 1) is simulated and posterior revenues are obtained by their expected values, considering the simulation result (equation 13). Through this procedure, we compute a 2.6867% volatility per month.

$$z = \ln\left(\frac{CF_1 + PV_1(E(CF_2), E(CF_3), \ldots, E(CF_n)|CF_1)}{PV_0^c}\right) \tag{13}$$

Main Tree

CRR methodology represents contract's uncertainty by means of discrete recombining lattices, with an upside (u) and a downside factor (d) designated by $u = e^{\sigma * \sqrt{\Delta t}}$ e $d = e^{-\sigma * \sqrt{\Delta t}}$ *r*espectively. Table 4 expresses the event tree parameters for the presented data.

Table 4 – Cox-Ross-Rubinstein event tree parameters

Symbol	Value	Description
σ	0.026867	Monthly volatility
u	1.027231	Monthly upside factor
d	0.973491	Monthly downside factor

Initially, the event tree does not consider supply consequences, thus there are no revenue limits, with equation 14 representing these boundless earnings.

$$R_t^* = CMU * y_t \tag{14}$$

Binomial tree has a chronological (forward) evaluation, with first node (time 0) calculated as the overall sum of cash flows expectation $\left(VP_{0,1}^c \ = PV_0^c\right)$. In the second step, we obtain ex-dividend contract's value by subtracting dividends amount from the cum-dividend appraisal $\left(VP_{t,n}^e \ = VP_{t,n}^c * (1 - \delta_t)\right)$. Following, every ex-dividend value evolves to next period upstate cum-dividend value $(VP_{t+1,n}^c = VP_{t,n}^e * u)$ and downstate cum-dividend value $(VP_{t+1,n+1}^c = VP_{t,n}^e * d)$. The resulting lattice is entitled main tree, as long it will ground every supply consideration.

Cognate Trees

Once supply capability limits the revenues absorption, the cash flow restriction (R_s^{max}) can lead to service quantification. This analysis starts in the last contract's period, moving backwards, and equations 15 and 16 express this phase. There, $V_{t,n,s}^c$ stands for cum-dividend value with s resources in period t and state n, same way, $V_{t,n,s}^e$ represents ex-dividend value as a function of period, state and resource quantity and $VP_{t,n}^{c}$ symbolizes the main tree cum-dividend value for period t and state n.

$$R_s^{max} = CMU * s \tag{15}$$

$$V_{t,n,s}^{c} = V_{t,n,s}^{e} + \min(\delta_t * VP_{t,n}^{c}; R_s^{max}) \tag{16}$$

In backward direction, both possible cum-dividend values (upside and downside) produce the prior ex-dividend value, ensuring a non-arbitrage condition. We do this through Market Asset Disclaimer technique (MAD) and replicating portfolio approach, using the main tree as an underlying asset, what is in accordance with Copeland and Antikarov (2001) discussions. The several constructed lattices incorporate the different service levels, being entitled cognate trees. Figure 3 illustrates cognate trees considering 6, 10, 15, 25 and 35 resources.

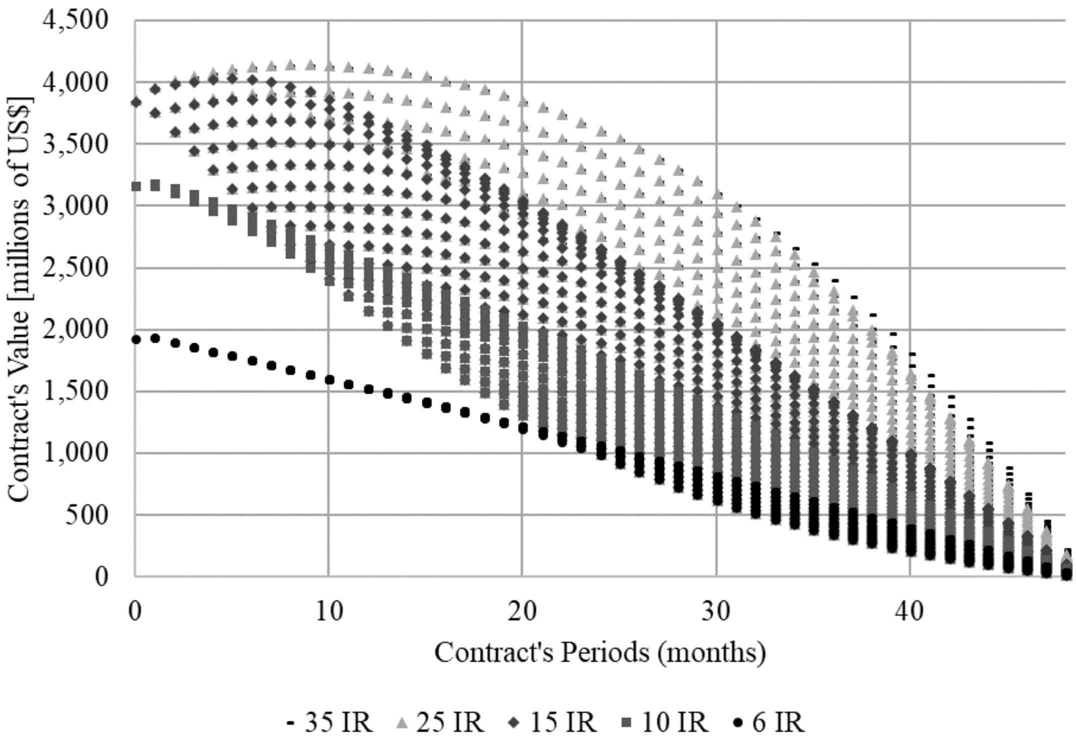

Figure 3. Cognate trees considering supply of 6, 10, 15, 25 and 35 resources.

Analysis permits contract's valuation without flexibility assumptions, just as was performed with DCF method, but with an additional advantage, the no-arbitrage condition. This assessment indicates the maximum NPV of US$3,641 million, obtained with 15 IR.

V. Flexible Contract's Valuation

The ability to increase service level is intuitive related to expansion options, nevertheless, models incorporating this category are often designed to provide a proportional change of project's value when options are exercised, what is not suitable in this analysis. Examples are found in Trigeorgis (1996), Copeland and Antikarov (2001) and Brandão, Dyer, and Hahn (2005b),

To capture non-proportional impacts and path dependence that arises from contracts' modifications, we appeal to switch options versatility. Using this perception, we are able to model service level changing as a commutation between different cognate trees, where the initial lattice represents the resource's quantity before decision and a second lattice the number after decision was materialized. In this case, the switch costs represent the option exercise price.

In this paper, we assume that switch options are able to change supply immediately, meaning that next period cash flows already reflect the adjustment. It's a common model simplification, as adopted in Hahn and Dyer (2008), which reduces uncertainty when compared to lagged consequences, hence options' value is conservative.

Last possible decision is made in last but one period (about last month's service level), which becomes the first backward evaluation. At this point, the first possibility is to remain at current resource level, equivalent to a "wait-to-see" decision. The associated value is calculated discounting next period possible estimates, as shown in equation 17. Respectively, $VO_{t,n,s}^{ew}$ and $VO_{t,n,s}^{c}$ represents flexible contract's ex-dividends and cum-dividends wait value (with s resources in period t and state n).

$$VO_{t,n,s}^{ew} = \frac{VO_{t+1,n,s}^{c}*q+VO_{t+1,n+1,s}^{c}*(1-q)}{1+r} \quad (17)$$

Option value's discount includes neutral risk probabilities, allowing risk-free rate employment (Trigeorgis 1996; Copeland and Antikarov 2001). The upside neutral risk probability (q) is obtained as displayed in equation 18 and downside neutral risk probability is complementary (1-q).

$$q = \frac{1+r-d}{u-d} \quad (18)$$

Besides keeping service level, flexibility grants the opportunity to increase it. As presented by Trigeorgis (1996), a switch option's exercise is comparable to an European call option to other cognate tree, as long as it represents a higher resource baseline. For this, we consider an exercise price that is proportional to the number of added resources (z), according equation 19. For each expanded service, the exercise price is constituted by a fixed cost of US$1.85 million, representing a compensation charge for mobilization expenditures and also pre-committed remaining rentals, hence early changes are more expensive than those performed on final periods.

$$EP_{z,t} = z*\left(1.85+\frac{C_R}{r}\left(1-(1+r)^{t-48}\right)\right) \quad (19)$$

These inputs enable best decision making, identifying the highest contract value among the "wait-and-see" choice and all available switch options, as demonstrated by equation 20. Complying previous observations, decision making occurs in an ex-dividend state, influencing next periods' cash flows.

$$VO_{t,n,s}^{e} = \max\{VO_{t,n,j}^{ew} - EP_{(j-s),t} \mid \forall\, j \geq s\} \quad (20)$$

Equation 20 is able to map out contract's flexible valuation for any period, state and resource level, establishing the highest amount but also the optimized decision that drives it. Proceeding backwards, the cum-dividend value is obtained including period's dividends, bounded by supply scheme (equation 21). Reaching the present time (period 0), we acquire contract's flexible valuation for each initial resource's strategy (and optimal decisions). The flexible contract's net present value for s initial resources (NPV_s^O) is calculated subtracting the IR's rental fee, in the shape of an initial investment (equation 22).

$$VO_{t,n,s}^{c} = VO_{t,n,s}^{e} + \min(\delta_t * VP_{t,n}^{c}; R_s^{max}) \quad (21)$$

$$NPV_s^O = VO_{0,1,s}^{e} - I_s \quad (22)$$

Figure 4 shows net present value for different IR levels, highlighting the most profitable choice. From that, we can observe that highest NPV arises from 12 IR and worth US$3,668 million, meaning flexibility added US$27 million to contract's value, when compared to an agreement without increase capacity.

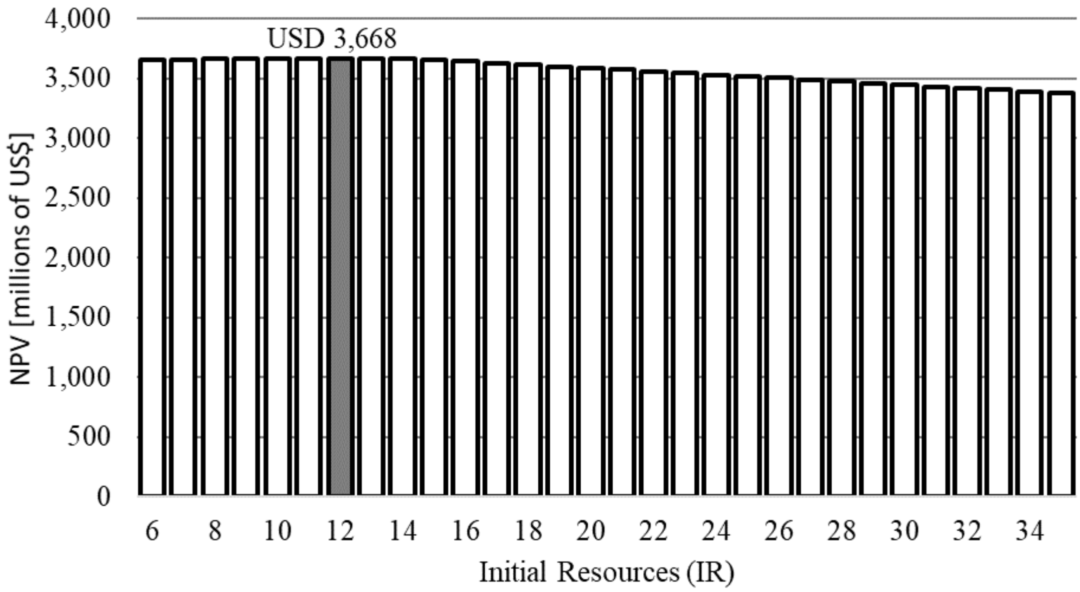

Figure 4. Flexible contract's NPV for several supply levels according to real options evaluation.

VI. Results Discussion

Optimal Path

The featured proceeding leads to objective discrete evaluation in accordance to Copeland and Antikarov (2001) observations, but with additional qualities. The most apparent difference is that while regular switch option examples evaluate two or three possibilities, this paper allows multiple correlated switch analysis. We also highlight the modelling of a variable exercise price (associated with remaining service fees) without additional intricacy or constraint.

Other important factor is that the model constructs a value mesh, represented by a three-dimensional matrix (period, scenario and resource quantity). When decisions are available, the one heading to best results can be registered as a fourth dimension, shown in Table 5 for 5th period's states. Through the main tree, dividend values are easily correlated to the demand level (routine measured), which allows the association between demand and the option trigger. This approach simplifies the manager's comprehension and real options acceptance.

Data on table 5 depicts ex-dividend value ($VO^e_{5,n,s}$) and consider resource variation between 6 and 12, along with optimal decisions that enable the presented valuation. As an example, this period's state 3 worth US$3,491 million if supplied by 6 resources ($VO^e_{5,3,6}$) considering that the manager will add 6 more services immediately. Alternatively, the same state worth US$3,578 million if supplied by 12 resources ($VO^e_{5,3,12}$) and in this case, the optimal decision is to keep the same service level.

Considering best decisions from the beginning, it's possible to draw an optimal path, to be followed as long as uncertainty reveals itself. Figure 5 illustrates the best decisions for the first nine months, where the number inside each node represent the optimal resource quantity.

In figure 5, we are able to verify that 12 is the best IR and the first switch opportunity occurs in the 4th month, when a high demand will suggest an increase to 13 resources. If demand growth persists, we may decide to increase supply to 14 in the 6th month.

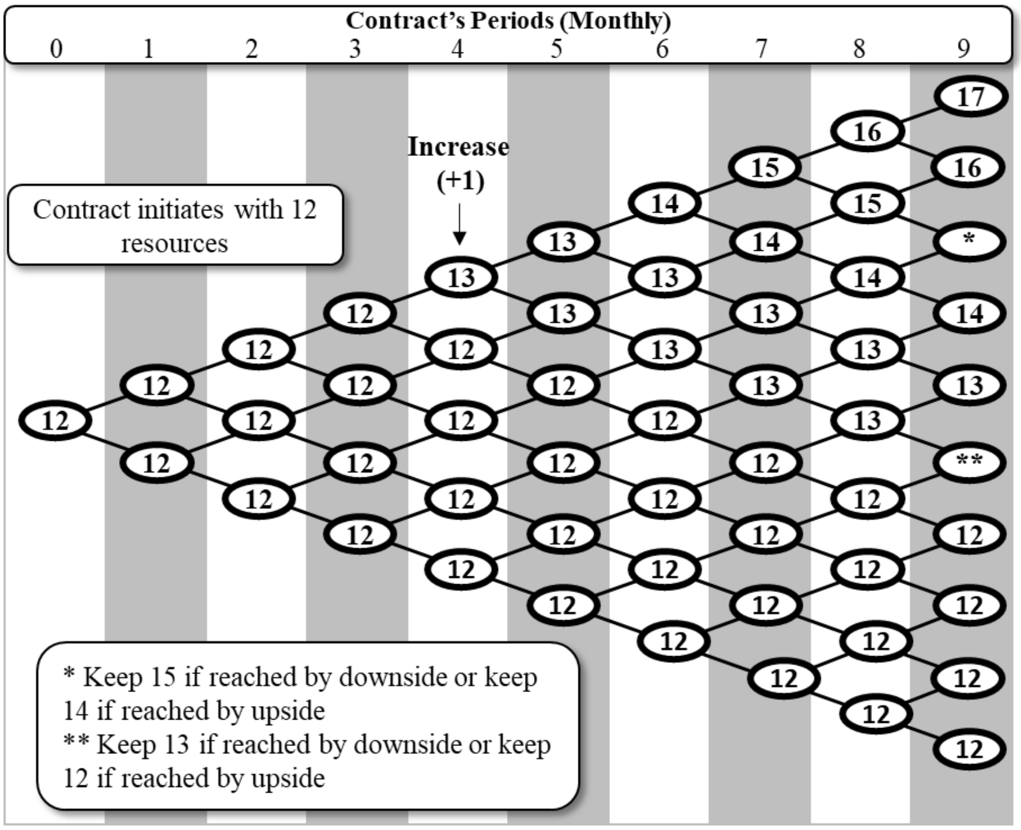

Figure 5. Optimal resource level in each possible state for the first nine contract's periods.

Table 5 – Ex-dividend value with options (millions of US$) and optimal target switch decision for different resource levels, at the beginning of 5th period.

	Contract's 5th Period Ex-dividend Value						
State/IRL*	6	7	8	9	10	11	12
1	3,877	3,892	3,906	3,921	3,936	3,950	3,965
2	3,679	3,693	3,708	3,722	3,737	3,752	3,766
3	3,491	3,505	3,520	3,534	3,549	3,564	3,578
4	3,312	3,326	3,341	3,356	3,370	3,385	3,400
5	3,143	3,158	3,172	3,187	3,202	3,216	3,229
6	2,983	2,998	3,013	3,027	3,042	3,056	3,067
	Associated Resource Level						
State/IRL*	6	7	8	9	10	11	12
1	13	13	13	13	13	13	13
2	13	13	13	13	13	13	13
3	12	12	12	12	12	12	12
4	12	12	12	12	12	12	12
5	11	11	11	11	11	11	12
6	10	10	10	10	10	11	12

*IRL = initial resource level, the number of resources at the beginning of 5th period.

In the 9th period there are particular situations where two optimal values coexist, depending of the previous state. For example, if state 3 ($VO^{e}_{9,3,s}$) is reached by a previous downside, we should keep 15 resources, but if reached by a previous upside it's optimal to keep 14, instead of expand to 15. The condition when best option is a function of previous state is entitled path dependence. Despite this aspect is often regarded as a serious aggravation of real options approach and even a restraint in binomial models (Triantis and Borison 2001), there is no additional complexity to handle it through the decision matrix. However, it's important to include Copeland and Tufano (2004) observation that challenges involving decisions fulfilment are bigger than those considering their identification, making management discretion an essential goal to achieve potential results.

Results Comparison

Comparing discounted cash flow's and real options' outputs (figures 2 and 4), the first method is clearly more sensible to IR variation. It's an expected effect, since flexibility enables to correct an underestimated decision, even with a compensation charge involved.

As already mentioned, the DCF nonflexible contracts evaluation contains a relevant difference in comparison with cognate trees, the discount factor. In fact, one of the most distinguished DCF critique is related to fixed discount rate assumptions, when they are indeed changeable (Trigeorgis 1996). The applied techniques (replicating portfolio and neutral probabilities) perform the required adjustments and provide more accurate assessments. Table 6 summarizes contract's valuation, achieved with the three discussed methodologies and their associated IR.

Table 6 – Contract's valuation without flexibility (DCF and CRR) and with options

Method	Value	IR
DCF (nonflexible)	US$3,645 million	16
Binomial tree (nonflexible)	US$3,641 million	15
Binomial tree (with options)	US$3,668 million	12

Figure 6 also assists procedures comparisson for different IR strategies. It represents the three discussed outcomes and one additional illustrative evaluation, using cognate trees but discounting values with WACC, instead of replicating portfolio technique. Our intention is to demonstate the similarity with the DCF results, when no-arbitrage condition is disregarded, an approach described as naïve according to Brandão, Dyer, and Hahn (2005b)

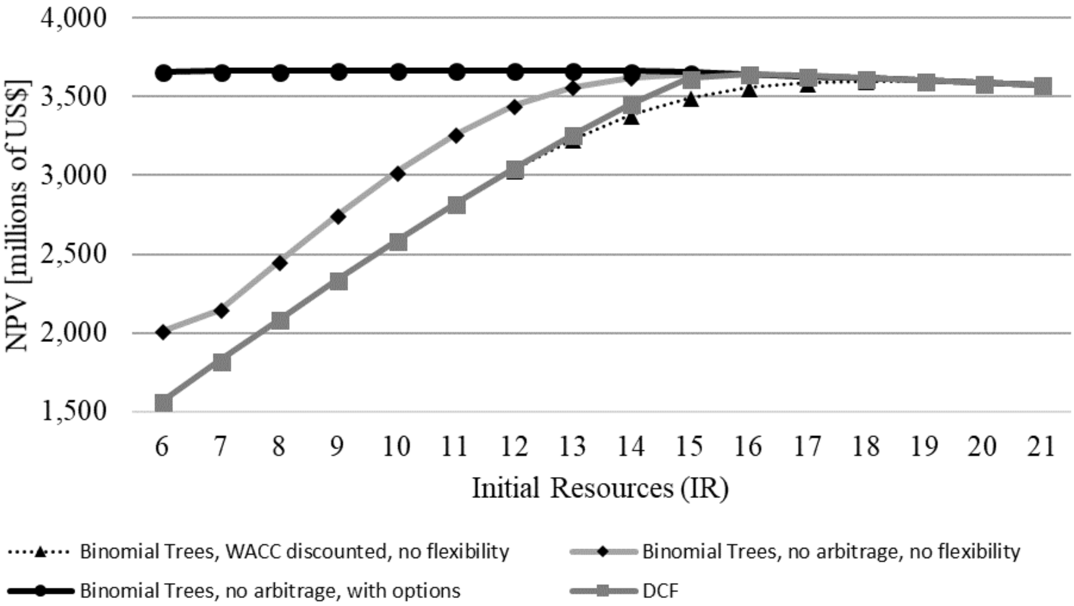

Figure 6. NPV obtained for a range of initial resources with different evaluation methodologies.

VII. Conclusions

In the effort to valuate a flexible agreement, where the client has the option to increase service level, we suggested lattices construction over three stages. The first step develops a main tree, incorporating contract's value volatility. This is accomplished with a Monte Carlo simulation in compliance with Brandão, Dyer, and Hahn (2005a), using the return rate as an unified source of uncertainty.

The second phase includes supply restrictions over the cash flows, creating cognate trees and providing a nonflexible contract's valuation. Third stage considers the flexibility to add services, equivalent to migrate for another (and better) cognate tree disbursing an exercise price. Even though the enlargement capability is intuitive related to expansion options, we use switch option versatility to address non-proportional value modifications and path dependence.

According to results, flexibility worth US$ 27 million in a four-year contract, what is significant, representing about 42% of 15 resources full costs (the IR recommended by a nonflexible evaluation). Also suggests a 20% reduction for initial service level, what is more consistent with management resolutions in an uncertain environment.

As a model choice, CRR discrete approach contributed with transparent option's triggers, what is critical to model acceptance by decision makers, as notified by Triantis and Borison (2001). Besides, the method provides an objective and unambiguous criterion for rental quantification, a massive concern for senior managers.

References

Asteriou, Dimitrios, and Stephen G Hall. 2007. *Applied Econometrics: a Modern Approach*. 1st ed. revisada ed. New York: Palgrave Macmillan.

Bennouna, Karim, Geoffrey G. Meredith, and Teresa Marchant. 2010. "Improved Capital Budgeting Decision Making: Evidence from Canada." *Management Decision* 48 (2):225-47.

Black, Fischer, and Myron Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of political economy* 81 (3):637-54.

Brandão, Luiz E, James S Dyer, and Warren J Hahn. 2005a. "Response to Comments on Brandão et al.(2005)." *Decision Analysis* 2 (2):103-9.
———. 2005b. "Using Binomial Decision Trees to Solve Real-Option Valuation Problems." *Decision Analysis* 2 (2):69-88.

Cavaliere, Giuseppe. 2005. "Unit Root Tests under Time-Varying Variances." *Econometric Reviews* 23 (3):259-92. doi: 10.1081/etc-200028215.

Copeland, Tom, and Vladimir Antikarov. 2001. *Real Options - A Practitioner's Guide*. New York: Texere LLC.

Copeland, Tom, and Peter Tufano. 2004. "A Real-World Way to Manage Real Options." *Harvard Business Review* 82 (3):90-9.

Cox, John C, Stephen A Ross, and Mark Rubinstein. 1979. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7 (3):229-63.

de Souza, Paula, and Rogério João Lunkes. 2016. "Capital Budgeting Practices by Large Brazilian Companies." *Contaduria y Administracion* 61 (3):514-34.

Dias, Marco Antônio Guimarães. 2001. "Selection of Alternatives of Investment in Information for Oil-Field Development Using Evolutionary Real Options Approach." In *5th Annual International Conference on Real Options*. Los Angeles: Citeseer.

Dixit, Avinash K, and Robert S Pindyck. 1994. *Investment Under Uncertainty*. 1 ed. ed. Princeton: Princeton University Press.
———. 1995. "The Options Approach to Capital Investment." *Harvard Business Review* 73 (3):105-15.

Hahn, Warren J., and James S. Dyer. 2008. "Discrete Time Modeling of Mean-Reverting Stochastic Processes for Real Option Valuation." *European Journal of Operational Research* 184 (2):534-48.

Hamilton, James Douglas. 1994. *Time Series Analysis*. Princeton, NJ: Princeton University Press.

Jafarizadeh, Babak, and Reidar Bratvold. 2012. "Two-Factor Oil-Price Model and Real Option Valuation: An Example of Oilfield Abandonment." *SPE Economics & Management* 4 (03):158-70.

Laine, JP. 1997. "Option Valuation of Field Development Projects." In *SPE Hydrocarbon Economics and Evaluation Symposium*, 11. Dallas: Society of Petroleum Engineers.

Ma, M. 2016. "Exploration Decision-Making on Energy Based on Improved Real Option Model and BP Neural Networks." *International Journal of Control and Automation* 9 (10):393-402.

Merton, Robert C. 1973. "Theory of Rational Option Pricing." *The Bell Journal of economics and management science*:141-83.

Myers, Stewart C. 1977. "Determinants of Corporate Borrowing." *Journal of Financial Economics* 5 (2):147-75.

Nelson, Charles R, and Charles R Plosser. 1982. "Trends and Random Walks in Macroeconmic Time Series: Some Evidence and Implications." *Journal of monetary economics* 10 (2):139-62.

Samanez, Carlos Patricio, Léo da Rocha Ferreira, and Carolina Caldas do Nascimento. 2014. "Evaluating the Economy Embedded in The Brazilian Ethanol-Gasoline Flex-Fuel Car: A Real Options Approach." *Applied Economics* 46 (14):1565-81.

Samuelson, Paul A. 1965. "Rational Theory of Warrant Pricing." *Industrial Management Review* 6 (2):p. 13-39.

Triantis, Alex, and Adam Borison. 2001. "Real Options: State of the Practice." *Journal of Applied Corporate Finance* 14 (2):8-24.

Trigeorgis, Lenos. 1996. *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. Cambridge (MA): MIT press..

Wilmott, P. 2009. *Frequently Asked Questions in Quantitative Finance* 2 ed. ed. Chichester: John Wiley & Sons.