When the sign (of an exposure) matters: real options, abatement and risk aversion

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Abstract

We show that risk aversion has opposite effects on investment thresholds depending on the sign of the underlying risk exposure. Investments which increase exposure to a risky revenue-type variable are delayed, but investments which decrease exposure to a risky cost, such as overseas production operations, energy costs for energy-intensive production processes or abatement investments in the face of potential future stochastic carbon prices, are brought forward, even if the incremental cash-flows are identical. Risk aversion also increases the attractiveness of splitting investments into multiple stages. Investment thresholds are affected by the incremental change in risk exposure, which is non-linear, so all a firm's real options should be valued as a whole. We illustrate the errors which can arise from ignoring this.

Keywords: Abatement options, risk aversion, staged investments

1 Extended Abstract

The impact of decision-makers' risk preferences on investment decisions is an area of increasing interest.[1] However, most extant studies consider only situations where the underlying risk to which investment gives exposure has a positive impact on firm profits or value (revenue-type variables). Risk aversion is just as relevant in circumstances where a firm's principal risk exposure relates to costs. Examples include airlines' exposure to oil or more generally energy prices for other energy-intensive manufacturers; exchange rate risk, particularly for firms with production overseas, or potential future scenarios where energy producers face a stochastic carbon price. In this paper we show that risk aversion has opposite effects on investment thresholds depending on the sign of the underlying exposure. Whereas investments which increase exposure to a risky ongoing stream of future cashflows are delayed by a decision-maker's risk aversion, investments which *reduce* exposure to a risky cost are instead *brought forward*, even if the incremental cash flows for the two investments are identical.

The reason is because of the difference in the magnitudes of risk exposure before and after investment: in the first example, risk exposure increases after investment whereas in the latter, exposure is reduced. Risk-averse decision-makers place a lower subjective value on investments with higher levels of risk exposure. This creates a disincentive for investments which increase risk exposure, raising their investment thresholds, and a corresponding incentive to invest at a lower threshold in risk-reducing projects. Examples include energy efficiency measures for energy intensive manufacturing processes, moving production to domestic facilities in the case of exchange rate exposure, and carbon capture and storage facilities in the presence of a stochastic

[1] See Hobson and Henderson (2007), Henderson (2007), Miao and Wang (2007), Whalley (2011), Chronopoulos, De Reyck and Siddiqui (2011, 2014), Chronopoulos and Lumbreras (2017).

carbon price.

We also investigate the incentives for staging investments which either increase exposure to revenue-type uncertainty or reduce exposure to cost-type uncertainty in the presence of risk aversion. Under risk neutrality, Kort, Murto and Pawlina (2010) have shown that the additional flexibility of the ability to split the timing of an investment, even into only two stages, is desirable: if the overall cost of staging is identical to the cost of investing in a single step, the additional flexibility of staging means it is strictly preferred. Kort, Murto and Pawlina (2010) show the flexibility cost premium firms are willing to pay for the flexibility of staging the investment can be substantial but that it decreases with uncertainty.[2] We show this premium also varies with risk aversion: since exposure to risk is mitigated in the intermediate state, risk aversion increases the attractiveness of staging investments still further.

Our work extends the analysis of investment under risk aversion. This is relevant to real options analysis because assets underlying investment projects are often non-traded. The valuation of claims on such 'non-traded' assets was developed in Henderson and Hobson (2002). Henderson (2007) finds this unhedgeable risk reduces real option value and hence reduces investment thresholds for a single lump-sum investment with revenue-type uncertainty. Miao and Wang (2007) also find risk aversion decreases investment thresholds for single investment options with lump-sum payoffs, but finds the opposite result with 'flow' payoffs, when investment produces a stream of risky future cash flows. Chronopoulos, De Reyck and Siddiqui (2011) confirm that risk aversion increases thresholds for investments which give rise to a stream of

[2]Guthrie (2012) and Chronopoulos, Hagspiel and Fleten (2017) extend this analysis to consider the additional effects of flexibility over capacity choice at each stage and find step-wise or modular investment is always preferred, but the magnitude of the preference decreases as volatility increases

cash flows with risky revenue-type uncertainty in the presence of operational flexibility and further investigate the impacts of abandonment, supsension and re-opening decisions.[3], All these papers either consider a single investment option in a risky asset or a cash flow stream where greater exposure is associated with increased profits, or a sequence of investment/disinvestment decisions where each decision is between a positive level of exposure or zero exposure. These assumptions allow for closed form solutions, but limit the investigation of more complex and realistic strategies which involve sequential investment, which we consider.

The downside of greater realism is the inability to obtain closed-form solutions; however we are able to obtain closed-form approximations of value functions and investment thresholds for small levels of risk aversion using asymptotic expansion, and confirm the intuition gained from these using numerical simulations. Using the asymptotic expansions we are able to show that for small levels of risk aversion, the threshold for investing in a project which eliminates exposure to an uncertain cost decreases with increasing risk aversion (i.e. such an investment is brought forward).

Specifically we show that the threshold for a cost abatement investment can, for small levels of the decision-maker's level of absolute risk aversion, γ be approximated by

$$P^* \approx P_0^* + \gamma P_1^* + \dots \tag{1}$$

where P_0^* is the investment threshold under risk-neutrality,

$$P_0^* = \left(\frac{\beta_1}{\beta_1 - 1}\right)\frac{L}{\delta} \tag{2}$$

$L = K - \frac{1}{r}(X_{After} - X_{Before})$ represents the overall cost of the investment, including up-front

[3]Chronopoulos, De Reyck and Siddiqui (2014) and Chronopoulos and Lumbertas (2017) consider the effects of risk aversion on competition and technology-adoption strategies respectively.

costs K and changes in ongoing fixed costs, $\delta = \frac{1}{r-\mu}(D_{After} - D_{Before})$ represents the change in exposure to the uncertain price P as a result of the investment, and β_1 is the positive root of the characteristic equation $\mathcal{Q}(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + \mu\beta - r = 0$, and the leading order correction due to risk aversion is given by

$$P_1^* = (P_0^*)^2 \phi\ \Omega(\beta_1) D_{Before} \tag{3}$$

where $\phi = \frac{1}{2(r-\mu)} r\sigma^2(1-\rho^2) > 0$ and

$$\Omega(\beta_1) = \frac{(\mathcal{Q}(2)(\beta_1\mathcal{Q}(\beta_1+1) - 2\mathcal{Q}(2\beta_1)) - (\beta_1-2)\mathcal{Q}(\beta_1+1)\mathcal{Q}(2\beta_1))}{(\beta_1-1)\mathcal{Q}(2)\mathcal{Q}(\beta_1+1)\mathcal{Q}2\beta_1)} > 0 \tag{4}$$

For cost abatement investments the exposure to the risky cost before investment, $D_{Before} < 0$, so $P_1^* < 0$ for these types of investment: increasing risk aversion (γ) decreases the overall investment threshold P^*.

One implication of this is that the greater the firm's prior exposure to the uncertain cost, the lower the threshold for investing in the abatement technology, all else equal.[4] This further implies that a risk averse decision-maker who can either make a lump-sum investment in abatement technology which will completely eliminate exposure to the uncertain future cost, or instead make the exposure-reducing investment in two stages, first reducing exposure to some

[4] To see this consider two firms 1 and 2 both exposed to the same cost but where firm 1 has a larger scale than firm 2, both able to abate their exposure with $X_{Before_1} - X_{After_1} = \alpha(X_{Before_2} - X_{After_2})$, $K_1 = \alpha K_2$, $D_{After_1} = D_{After_2} = 0$ and $D_{Before_1} = \alpha D_{Before_2}$ where $0 < \alpha < 1$. Since the investments have the same relative profitability, the leading order investment thresholds are identical, $P_0^{*(1)} = P_0^{*(2)}$. Both leading order adjustments to the threshold are negative, however $|P_1^{*(1)}| = \alpha|P_1^{*(2)}|$, so

$$P^{*(1)} \approx P_0^{*(1)} + \gamma P_1^{*(1)} = P_0^{*(1)} + \gamma\alpha P_1^{*(2)} < P_0^{*(2)} + \gamma P_1^{*(2)} \approx P^{*(2)}$$

remaining proportion α by paying the equivalent proportion of the costs, will always prefer the staged investment. This is in stark contrast to the risk-neutral setting, where a firm would be indifferent between multi-staged and single stage investment if each stage had the same and equal relative profitability (*i.e.* cost per unit of exposure reduction) as the lump-sum investment (see Kort, Murto and Pawlina (2010)). We confirm our analytic results with numerical compuations.

Our results arise because risk aversion in the presence of non-traded assets creates a nonlinear valuation problem: firm values V_i at each stage (i) satisfy a nonlinear differential equation:

$$\frac{\sigma^2 P^2}{2}\frac{\partial^2 V_i}{\partial P^2} + \mu P \frac{\partial V_i}{\partial P} - rV_i + \pi_i(P) - \frac{1}{2} r\gamma(1-\rho^2)\sigma^2 P^2 \left(\frac{\partial V_i}{\partial P}\right)^2 = 0 \tag{5}$$

together with appropriate boundary conditions, where $\pi_i(P)$ is the per-period cashflow from an infinitely-lived project, the underlying price P follows Geometric Brownian Motion $dP = \mu P dt + \sigma P dZ$, and we assume the decision-maker has constant absolute risk aversion γ and maximises her discounted utility of consumption.[5] The additional term in (5), $-\frac{1}{2} r\gamma(1-\rho^2)\sigma^2 P^2 \left(\frac{\partial V_H}{\partial P}\right)^2$, represents the cost of the unhedgeable risk; since it is negative, this always lowers firm values.

The nonlinearity of the problem means firm values must be calculated as a whole; it is no longer possible to value the incremental cash flows to each decision separately. In the scenario where a firm can aplit its investment into multiple stages, we find that, under risk aversion, the investment threshold for the first stage differs from the investment threshold for a project with the same incremental cashflows (and sign of change in risk exposure) at the last stage. However, the distinction between risky revenue-enhancing and risky cost-reducing investments shows this

[5]See Miao and Wang (2007).

effect at its most stark. Furthermore, ignoring these effects has consequences: waiting until the (higher) revenue-enhancing threshold to invest in the cost-reducing project would reduce firm value.

Our results suggest that, all else equal, firms with more risk-averse decision-makers should make investments which reduce their exposure to future uncertain costs (a form of operational hedging) at lower thresholds and hence earlier. This is in contrast to the effect of risk aversion on most other investments (which increase exposure to uncertain revenues), which are delayed and have higher investment thresholds. Furthermore, since they place a greater premium on the ability to stage investment, more risk averse decision-makers are also more likely to make partial cost-abatement decisions.

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