Understanding the Residual Value Risks of Highway Pavements in Public-Private Partnerships

A Real Option Approach
Abstract

To understand the residual value risks in public-private partnerships, we decide to conduct a real option analysis on the residual value risk. In this study, we identify two types of real options, the initial construction option and the maintenance option. The values of these options are estimated using least square Monte Carlo method to demonstrate the potential benefit of using the P3 model.

We find that under the P3 model, the private firms are more likely to explore the option values and achieve better asset performance, which will ultimately be translated into residual value and passed on to the public through proper contract design. Under the P3 model, private firms are more likely to explore these option values because their profit maximization function is integrated over the entire 30 years. Under the PSC model, private firms tend to be myopic and less likely to explore the real option values because their profit maximization function is constructed over a much shorter period of time which may or may not allow them to collect all the future benefits of the strategic investments in the initial construction or the maintenance program.
1 Introduction

This study disentangles the theoretical foundation of additional values created by P3 models and their impacts on the residual value risk by using real-option based approach. The approach is forward looking and aligns to the purpose of risk management. More importantly, the real-option approach takes into consideration the value of the lifecycle strategies (initial design and construction or maintenance program, or both) that can be considered.

\[ X(t) = \text{Performance measure,} \quad RL = \text{Residual service life} \]
\[ HBL = \text{Handback threshold level,} \quad RHL = \text{Rehabilitation threshold level} \]

Figure 1 illustrates two imaginary, somehow exaggerated extreme cases. The left panel represents the case of a superior long-life design with less need of maintenance and rehabilitation; whereas the right panel is the opposite. However, both scenarios have to satisfy the performance specifications and handback requirements. In the left panel, the initial construction uses a superior long-life design, which allows two maintenance programs, the larger less frequent maintenance program (represented by the dotted green curve), and the regular maintenance program (represented by the solid blue curve). In the right panel, the initial construction uses a baseline design, which allows two maintenance programs as well, the regular maintenance program (represented by the solid blue curve), and the small more frequent maintenance program (represented by the dotted brown curve). The impact of those lifecycle strategies on the residual value risk needs to be assessed. This is done by using the real-option valuation approach.

The baseline probability density function of the RVR is depicted in Figure 2(a). The shaded area left to zero is the probability that the residual value (excluding the embedded real option values) of a P3 asset is less than the residual value of a PSC. After including the added real option values (i.e., the built-in flexibilities) in P3 assets, it is expected that the RVR curve would move further to the right hand side, as sketched in Figure 2(b) because under the P3 contractual arrangement,
the private partner is free to choose the optimal initial construction design, the optimal timing and scale of maintenance program as discussed in Section 2.2.

![Figure 2: Residual Value Risk: (a) baseline, (b) with consideration of value of flexibility in P3s.](image)

2 Literature Review: the Traditional Valuation v.s. the Real Option Valuation

2.1 Traditional valuation approach

The residual value of a highway infrastructure is a complex function which depends on a variety of past and future variables. The variables in the past include the construction quality of the highway, the traffic characteristics and volume of the highway etc. The variables in the future include the timeliness of the maintenance program, the future traffic characteristics and demand etc. In a classical net present valuation (NPV) framework, all the variables are to be carefully measured, estimated, and translated into certain types of cash flows. These cash flows will be summed up or discounted to give the residual value in dollar terms. Many existing literature, such as Falls, Haas and Tighe (2004), has followed similar approach and documented several methods to measure the residual value of a highway infrastructure. These include book value (or historical costs), written down replacement cost (WDRC), replacement cost, and net salvage value. CIPFA’s 2010 report also defines a similar measurement using depreciated replacement cost (DRC)\(^1\), as summarized in the table below.

<table>
<thead>
<tr>
<th>Book value (or historical costs)</th>
<th>The value of an asset is defined as the historical costs (equals to the sum of construction cost, rehabilitation cost, and maintenance cost) subtract the depreciation.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Replacement cost</td>
<td>Replacement cost is the current market price required to return an asset to new condition.</td>
</tr>
</tbody>
</table>

\[ RC = AC \times TSM \]

AC is the average condition of the asset in a percentage of its initial condition.
TSM is the total square meters of the asset.

**Written down replacement cost (WDRC)**

Written down replacement cost is the current market price required to return an asset to new condition, adjusted for the deteriorated condition of the asset at the time of replacement.

\[
WDRC = (AC \times ARC) \times TSM
\]

AC is the average condition of the asset in a percentage of its initial condition.
ARC is the average replacement cost per square meter.
TSM is the total square meters of the asset.

**Net salvage value (NSV)**

NSV is defined as the difference between the cost to replace the asset and the cost to rehabilitate it.

\[
NSV = (ACC - AHC) \times TSM
\]

ACC is the average construction cost per square meter.
AHC is the average rehabilitation cost per square meter.
TSM is the total square meters of the asset.

**Depreciated replacement cost (DRC)**

DRC = GRC – ADI

GRC is the gross replacement cost, which is usually a function of asset dimensions, group/sub-group unit rate, relevant adjustment factors etc. ADI is the accumulated depreciation & impairment, where *depreciation* is the systematic consumption of economic benefits embodied in an asset over its service life arising from use, ageing, deterioration or obsolescence; And *impairment* is a reduction in Net Asset Value due to a sudden or unforeseen decrease in condition and/or performance of an asset compared to the previously assessed level which is not already recognised through depreciation.

### 2.2 Real option valuation approach

It is worth noting that all the traditional valuation methods described above are established in a deterministic framework. They are capable of dealing with the past variables, but always have limitations in handling the future variables. The challenge of modelling residual value is mainly caused by the uncertainty of these future variables, which could be partitioned into two systems, the engineering uncertainty such as the remaining life, and the financial uncertainty such as the interest rate. Due to the complexity of these two systems and the interactions between them, we propose focusing on the engineering uncertainty and leaving out the economic uncertainty till the next stage.

In the light of uncertainties involved in the residual value, we propose to use the real option based valuation methodology to model and value the flexibilities embedded in P3 projects. The real option valuation methodology is an ideal tool to model investment decisions under uncertainty (Dixit and Pindyck 1994). He and Pindyck (1992) develops a real option model specifically dealing with the firms’ investment decision with flexible production capacity. The real option model is widely applied in analyzing investment decisions in real estate industry (Grenadier 1995a and
Grenadier (2000) further applied a real option model to analyze firms’ investment decision with the flexibility of choosing the time to build. Grenadier (2002) derives a real option equilibrium investment decision in the framework of continuous-time Cournot-Nash.

A real option is generally an investment opportunity which allows the decision maker to pay a fixed strike price to make strategic investment in an early stage and receive unlimited potential upside gain in the future. This investment opportunity has to be a right not an obligation to the decision maker. In other words, the decision maker has the freedom to decide whether and when to exercise the real option, i.e. pay the strike price to make the strategic investment in the hope of receiving more benefit in the future. The future potential upside gain is mostly uncertain and depending on the probability distribution of the underlying asset quality and condition.

Figure 3 illustrates a stylized example of the decision process of exercising a real option. Suppose there is a real asset (the underlying asset) could be built or developed at any time \( t \) between day 0 and day \( T \), i.e., \( t \in [0, T] \). The construction cost is 25 million, which is usually defined as the strike/exercise price of this real option, \( K = 25 \) million. After the construction, the real asset is going to generate some future cash flows that are uncertain, depending on various economic or technological factors. The total benefit is usually modelled as the present value of all future cash flows, denoted as \( \bar{P}_A \). Since \( \bar{P}_A \) is a random variable, the decision maker never knows its exact value, but can form an expectation about it, denoted as \( E[\bar{P}_A] \).

The value of this real option investment opportunity, \( V \), depends on whether the real asset is developed optimally. If \( T = 1 \), this is a European style call option, the decision rule is quite straightforward. If \( E[\bar{P}_A] > K \), exercise the option; otherwise, walk away and let the option expires. The payoff of an European call is expressed as \( \max(0, E[\bar{P}_A] - K) \). However, if \( T \) is significantly large, e.g. \( T = 365 \) days or greater. This becomes an American style call option. The decision rule is a bit complex and shall be summarized in the table below.
### Boundary Conditions

<table>
<thead>
<tr>
<th>Boundary Conditions</th>
<th>Exercise or Not</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\hat{P}_A] &lt; K$</td>
<td>No</td>
<td>0</td>
</tr>
<tr>
<td>$E[\hat{P}_A] &gt; K$, and $\frac{\partial v}{\partial \hat{P}_A} &lt; 1$</td>
<td>No</td>
<td>0</td>
</tr>
<tr>
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<td>Yes</td>
<td>$E[\hat{P}_A] - K$</td>
</tr>
</tbody>
</table>

### 3 Model Assumptions and Methodology

#### 3.1 Residual value difference measured as the life cycle cost difference

As compared with PSC, P3 allows private firms more flexibility in construction and maintenance decisions as long as certain performance requirements are met. The objective of the real option analysis is to examine this flexibility and its impact on the residual value, whilst holding all other exogenous variables constant or relatively stable. Although the residual value is the asset value after the concession period, the P3 models are implemented mostly during the concession period, (i.e., after the initial construction but before year 30). Therefore, it is necessary to analyze the impact of P3 flexibility on residual value in conjunction with the asset value changes during the concession period. Detailed procedures are explained in the following steps, assuming a 30-year concession period.

1) Assume a road can be used for $T$ years providing it is always properly maintained regardless of the delivery models. i.e., a common life span is assumed here. The road will generate some utility, which may be estimated either as the discounted sum of toll or as the convenience / productivity that the society receives for the entire period of $T$ years. Denote this utility as $U$, which does not depend on the types of delivery model (PSC or P3).

2) Calculate the life cycle cost of this road. Denote it as $C$. Note this cost depends on the types of delivery model, either PSC or P3. Therefore, we shall have two different cost profile, $C_{PSC}$ and $C_{P3}$. This $C$ is non-linear in terms of the time because we have time varying maintenance cost (preventive and rehabilitation) profile.

3) The net value between year 0 and year 30 is calculated as $\int_0^{30} (U_t - C_{PSC}^t) dt$ and $\int_0^{30} (U_t - C_{P3}^t) dt$ for PSC assets and P3 assets, respectively.

4) After the 30 year concession period, the P3 asset is handed over to the public and will be maintained under PSC model again. The residual value is calculated as $\int_0^{T} (U_t - C_{PSC}^t) dt$ and $\int_0^{T} (U_t - C_{P3}^{PSC}^t) dt$ for PSC assets and P3 respectively. Depending on the assets’ condition under PSC or P3, they may require different maintenance work between year 30 and year $T$, in order to reach the common life span.
5) The life cycle value difference between PSC and P3 assets can be estimated as
\[ \int_0^{T} (C_{P3}^t - C_{PSC}^t) \, dt + \int_0^{T} (C_{P3PSC}^t - C_{PSC}^t) \, dt, \]
i.e., a life cycle cost difference based on a common life span.

6) Calculate the annualized life-cycle cost difference, multiply it by the estimated residual life for either P3 or PSC, we will get the residual value difference measured as the cost difference.

As indicated above, our analysis focuses on the effects of various cost decisions including the initial construction and maintenance, while assuming the same effects from various revenue management decisions and factors such as traffic characteristics and volume etc. The residual value difference between P3 and PSC will be reflected as the life cycle cost difference between these two systems.

### 3.2 The key RVR variables and the embedded real options

In part 1 of this report, “Understanding the Residual Value Risks in Public-Private Partnerships: An Empirical Study”, we have identified six major categories of factors that contribute to the RVR which include

1) The physical deterioration of the asset due to the design, construction, or excessive use;
2) The timeliness and effectiveness of the maintenance programs that the project company have used for the infrastructure over the contractual period;
3) The service demand in future;
4) The change in use or user’s expectation on the service the facility delivers;
5) Technological obsolescence;
6) The external damage on the components, structures and systems due to extreme events such as climate change, earthquake, and malicious damages.

Factor 1 and 2 are mainly decided by the private company and/or the government and therefore endogenous. Factor 3 to 6 are out of the control of either the private company or the public sector and therefore exogenous. The fundamental difference between a P3 project and a traditional concession project is the change of the decision maker who makes the strategic decisions, i.e., the real option investments on factor 1 and 2 in order to better cope with factor 3 to 6.

The real option investment is embedded in the interaction between factor 1 and 2, i.e., the initial construction quality and the maintenance program. Assuming same traffic characteristics and volume, high quality construction usually leads to savings in the maintenance. The decision maker could choose a low quality design which cost less in construction but more in the maintenance, or a high quality design which cost more in construction but less in the maintenance. In this case, the real option is defined as a call option to build a high quality asset which saves the maintenance cost. The extra cost in the initial construction is the fixed strike price of this call option, and the maintenance savings is the unlimited upside gain in the future.
The real option investment is also embedded within factor 2, the maintenance program. Under same traffic characteristics and volume, doing the maintenance in a timely and effective manner sometimes saves, sometimes costs more than a delayed maintenance after a severe damage. The private firm has to choose between small and more frequent maintenance work versus large but less frequent maintenance schedule. Assuming these two maintenance schedules are both technologically viable, the private firm wants to carefully examine and compare their uncertain economic benefits and costs. In this case, the real option is defined as a call option to do timely rehabilitation and maintenance activities in an optimal scale which may add additional feature or extend the asset life. The cost of timely rehabilitation and maintenance activities is the strike price of this call option, and the unlimited upside gain include the future maintenance savings or extended asset life.

As compared to the traditional project, a P3 project provides a better incentive structure for the private firms to make long-term strategic investment and explore the real option value, (i.e. be strategic). In a P3 project, the decision maker of the initial construction and maintenance program stays relatively stable. Usually, it is the same private company who builds and maintains the asset over the period of 20 to 30 years. In this system, the private company knows that if it makes strategic investment at present time, they will be able to claim unlimited upside gain in the future as the ownership does not change under normal circumstances. However, in a traditional project, the government makes the decisions about construction and maintenance, then contracts them out to private firms. Often, the initial construction company is different from the maintenance company. When the construction company spends extra money to build a high quality asset, there is a large possibility that it may not be able to claim the associated future benefits if the government awards the maintenance contract to another private company. Therefore, the private firms are more likely to focus on the short-term benefit of the initial construction and maintenance program (i.e. myopic), less likely to make long-term strategic investment and explore the real option value because they know that the long-term benefit may not belong to them.

The outperformance of P3 assets over PSC assets is mainly because of the cost (including maintenance and construction costs) optimization, not minimization during the concession period. Notice the cost optimization is different from cost minimization. The former allows the privates firms to achieve better pavement performance level at a fixed cost level by optimally allocating these costs during the entire concession period, whereas the latter only forces the private firms to achieve the smallest cost at a fixed performance level, which is not ideal from the point of view public authority.

3.3 Assumptions for the cost profile and the service life

In order to conduct a comprehensive cost-benefit analysis using the real option framework, we need to gain some understanding about the cost profile of P3 projects and PSC projects.

The data provided by PPP Canada about the rehabilitation profile for transportation projects procured under the P3 model shows that P3 projects typically inject approximately 5% of the construction cost value at each major rehabilitation interval, which occurs 3 - 4 times throughout
the duration of the concession period. The provided two P3 projects show an injection of 8 – 10% towards the end of the concession period. This significant investment towards the end of the contract term is likely to ensure that the asset will meet the handback requirements stipulated in project agreement.

The estimation of maintenance cost as a percentage of initial construction cost is mainly based on the data provided in the research report by ARA Inc. (2008). The ARA report shows that for PSC projects, the initial construction cost is roughly around 76% of the total costs which includes initial construction, rehabilitation, and regular maintenance costs during the entire asset life, which means the maintenance cost is around 32% of the initial construction costs. Hajek and Hein’s report also provides a typical maintenance example for pavement asset, from which we estimated the one time maintenance cost is 1% for SMF and 12% for LLF. Within 30 year concession period, this example performs two time SMF maintenance (2%) and two time LLF (18%). Thus, in the sensitivity analysis, we will vary the maintenance cost between 0.01% and 32% of the initial construction cost.

The repaired service life of rehabilitative maintenance work is estimated using the data from the MTO report by Lane and Kazmierowski (2005). Based on the service life of three consecutive AC overlays, we estimated the service life of rehabilitation as a percentage of the initial service life ranging from 56.14% to 63.16% with a mean of 59.06% and standard deviation of 5.53%. The change of rehabilitation service life as a percentage of the initial service life ranges from -5.26% to 5.26% with a mean of 1.75% and standard deviation of 2.48%.

The repaired service life of preventive maintenance work is estimated using the data from the MTO 2005 report. The service life of preventive maintenance work as a percentage of the initial service life ranges from 21.21% to 32.4%, with a mean of 26.90% and standard deviation of 9.84%. The change of preventive maintenance work as a percentage of the initial service life ranges from -8.04% to 0.60% with a mean of -2.48% and standard deviation of 16.16%.

The estimation of service life for different initial construction designs is based on the data provided in the MTO 2005 report. The expected service life is 28 years for DJPC (Doweled Jointed Plain Concrete) and 19 years for DFC (Dense Friction Course), 18 years for SMA (Stone Mastic Asphalt). Thus, it is reasonable to assume that the pavement assets could experience a mean service life difference of 30.41%, with a standard deviation of 18.59% due to different initial construction. The range of the service life difference is from 10.53% to 47.37% of the baseline construction service life.

3.4 The real option model for the residual value

3.4.1 Model 1: maintenance option

The maintenance option is defined as the choice between small more frequent versus large less frequent maintenance. During the entire concession period, the private firm has to decide (i) when to perform the maintenance; (ii) what type of maintenance to choose between a small more frequent maintenance (SMF) program and large less frequent maintenance (LLF) program. Denote
Let $H$ and $h$ be the repaired service life from LLF and SMF correspondingly. The LLF maintenance costs more and usually repairs longer service life, whereas the SMF maintenance costs less and usually repairs shorter service life. Therefore, we have $H > h$ and $H > h$. $H$ and $h$ are two random variables that follow the generalized Wiener’s process, defined as

$$dH = \mu(H)dt + \sigma(H)dz(H)$$

And,

$$dh = \mu(h)dt + \sigma(h)dz(h)$$

where $dz(H)$ and $dz(h)$ both has a standardized normal distribution $\phi(0,1)$. $\mu(H)$ and $\mu(h)$ are the long run mean of the repaired service life from LLF and SMF, respectively. $\sigma(H)$ and $\sigma(h)$ are the volatility of the repaired service life from LLF and SMF respectively. Notice, the initial value of $H$ is always greater than the initial value of $h$, i.e., $H_0 > h_0$. However, this inequality may not hold for $H_t$ and $h_t$ after the maintenance as $H_t$ and $h_t$ will evolve differently.

$H$ and $h$ also have different quality which causes the asset to deteriorate at different speed after the maintenance. Let $\beta$ and $\alpha$ be the factor elasticity for LLF and SMF maintenance cost, respectively, measuring the quality of repaired service life. We assume decreasing returns to scale, i.e., $\beta + \alpha < 1$, otherwise, the private firm would always increase the scale of SMF until the repaired service life exceeds that from LLF. We also assume $\beta > \alpha$ to represent LLF costs more than SMF to repair the same amount of service life. Therefore, the total benefit from maintenance at time $t$ is defined as $H^\beta_t$ and $h^\alpha_t$ for LLF and SMF respectively. The real option values of the LLF and SMF maintenance can be characterized as

$$V_{LLF} = \max_{M_t} \left[ \int_0^T \max(H^\beta_t - M_t, 0) \, dt \right]$$

$$V_{SMF} = \max_{N_t} \left[ \int_0^T \max(h^\alpha_t - N_t, 0) \, dt \right]$$

The private firm’s decision function may be summarized as

$$\max_{M_t, N_t} \left[ \int_0^T \max(H^\beta_t - M_t, 0) \, dt, \int_0^T \max(h^\alpha_t - N_t, 0) \, dt \right]$$

At any given time $t$ between 0 and $T$, if $H^\beta_t < M_t$, withhold the LLF maintenance. If $h^\alpha_t < N_t$, withhold the SMF maintenance. If $H^\beta_t > M_t$ and $h^\alpha_t > N_t$ both hold, then compare $H^\beta_t - M_t$ to $h^\alpha_t - N_t$. If $H^\beta_t - M_t > h^\alpha_t - N_t$, choose the LLF maintenance, if $H^\beta_t - M_t < h^\alpha_t - N_t$, choose SMF maintenance.

**3.4.2 Model 2: initial construction option**

The initial construction option is defined as the choice to increased upfront investment or additional features to prolong the life of asset. Suppose there are two types of initial construction design, the superior and the baseline. The private firm has an option to improve the quality of the design. Let $M_t$ and $N_t$ as the cost of LLF and SMF maintenance respectively at time $t$, measured as a percentage of total initial construction costs. Let $H$ and $h$ be the repaired service life from LLF and SMF correspondingly.

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asset or add additional features to prolong the life the asset by selecting the superior design. These improvements or additional features may be reflected in the pavement type or thickness, or any other advanced technologies that may be used in the improvements. The superior construction costs more than the baseline, and $C$ is the extra construction cost. Due to the superior construction, the asset will deteriorate at different speed and has different service life. We use $D$ to measure the extra service life of the asset due to superior construction. The initial value of $D$ is determined by the initial construction quality. Once the construction is done, $D$ is going to evolve due to many exogenous factors and become uncertain. Therefore, we assume $D$ follows a generalized Wiener process defined as

$$dD = \mu(D)dt + \sigma(D)dz(D)$$

and $dz(D)$ follows a standard normal distribution $\phi(0,1)$. $\mu(D)$ is the expected long run mean of the extra service life due to superior construction. $\sigma(D)$ is the volatility of the extra service due to superior construction. The private firm’s optimization construction decision can characterized as a real option valuation function

$$G_t = \int_0^T \max[V[D[I_0 - aexp(-bt^{-c})]] - C, 0]dt$$

$I_0$ is the initial PCI of the asset. $V(x)$ is the valuation function which is the sum of the discounted future cash flows. Parameters $a$, $b$, and $c$ are used to control the exponential declining process of the facility as discussed in Section 3.3, reflecting the common factors that deteriorates the asset such as the weather, the traffic characteristics and volume (i.e. all trucks, low volume, etc.).

At any given time $t$ between 0 and $T$, the private firm compares the upfront cost, $C$ with the uncertain future benefit, $V[D[I_0 - aexp(-bt^{-c})]]$. If the benefit is larger, the firm will choose the superior construction. If the benefit is smaller, the firm will choose the baseline construction.
3.5 The numerical demonstration of the real option value

*Figure 4: The evolvement of real option values for LLF maintenance and SMF maintenance*

*Figure 4* illustrates the real option value for LLF and SMF maintenance options. The yellow manifold represents the LLF maintenance. The green manifold represents the SMF maintenance. The two horizontal axes represent the repaired service life ($H$ and $h$) expressed as a percentage of the initial service life and the maintenance cost ($M$ and $N$) as a percentage of initial construction cost, respectively. The vertical axis is the maintenance real option value denominated as a percentage of initial service life.

In *Figure 4*, the repaired service life $H$ and $h$ change from 1% to 75% of the initial service life, the maintenance cost $M$ and $N$ change from 1% to 40% of the initial construction costs. When repaired service life is low and maintenance cost is high, neither LLF nor SMF maintenance should be performed (i.e., the options shall not be exercised). When the repaired service life increases, SMF option is exercised earlier than LLF option, i.e., SMF option required lower exercise price than the LLF option. However, once both options are exercised, the LLF option value increases faster than the SMF option value and will eventually dominates the SMF option value in a region where repaired service life is greater than 30% of the initial service life. Also, as maintenance cost increases, it would demand higher repaired service life to exercise these options and for the LLF option value to exceed that of the SMF option, which is consistent with the prevailing intuition in the industry practice.

Using the average LLF cost of 18% of the initial construction costs and average LLF repaired service life of 59% of the initial service life, the real option value from LLF maintenance is 56.72% of the initial construction cost during 30 year concession period. Using the average SMF cost of 2% of the initial construction costs and the average SMF repaired service life of 27% of the initial service life, the real option value from SMF maintenance is 27.25% of the initial construction cost during 30 year concession period.

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2 The range of maintenance costs, $M$ and $N$, and the range of repaired service life, $H$ and $h$ are based on our discussion in Section 2.1.2. Data and assumption, citing from *ARA Inc. (2008)* and Hajek and Hein's report.
service life, the real option value from SMF maintenance is 30.79% of the initial construction cost during 30 year concession period.

The practical implication of LLF and SMF options could be illustrated using a hypothetical numerical example of a 100 million dollar construction cost highway project. Assuming on average, the LLF maintenance costs 18 million dollars, repairs 59% of its initial service life, the LLF maintenance has a real option value of 56.72 million dollars. Similarly, assuming on average, the SMF maintenance costs 2 million, repairs 27% of its initial service life, the SMF maintenance has a real option value of 30.79 million.

Figure 5: The real option value for the strategic investment in initial construction.

**Figure 5** shows the real option value of the initial construction option. The two horizontal axes are the extra service life and the extra construction cost, respectively. The vertical axis is the initial construction real option value denominated as a percentage of baseline construction cost. The left graph represents the scenario of low extra service life volatility where $\sigma(D) = 0.1$, and the right graph represents the scenario of high extra service life volatility where $\sigma(D) = 0.2$.

**Figure 5** demonstrates the real option value for initial construction option which may or may not be exercised depending on the service life difference and construction cost difference between a superior design and a baseline design. When the manifold sits at the bottom surface, the construction option (option to choose the superior design) shall not be exercised because the extra service life does not justify the extra construction cost. Larger construction cost difference would require larger service life difference to justify the exercise of the real option. In order to observe enough comparative statics between the real option value and the differences in service life and

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3 The average cost and repaired service life are extracted from the preventative and rehabilitative maintenance schedule prescribed in Lane & Kazmierowski’s 2005 report.
construction cost, we vary the extra construction cost\(^4\) from 0\% to 100\% and vary the extra service life from 0\% of 90\% accordingly. Using an average extra service life of 66.67\%, and an average extra construction cost of 70\%\(^5\), the real option value of a superior design is 30.46\% or 1.73\% of the baseline construction cost when the extra service life volatility is either 20\% or 10\%, respectively.

Again, to illustrate the practical implication of the initial construction option, we use a hypothetical numerical example of a highway project which has 100 million dollar baseline construction cost and an expected service life of 18 years. If the superior design on average costs 70 million dollar more than the baseline design, the real option value of this superior design is 30.46 million if the extra service life volatility is 20\%, and 1.73 million if the extra service life volatility is 10\%.

In Figure 5, a simple comparison between the left \((\sigma(D) = 0.1)\) and right graphs \((\sigma(D) = 0.2)\) shows that the construction option value is higher when \(\sigma(D)\) is higher, i.e., the extra service life volatility due to superior construction has positive impact on the value of the initial construction option. Therefore, we decide to further investigate the relationship between the volatility of the expected benefits (e.g. the repaired service life from maintenance, or the extra service life from superior initial construction) and the value of the corresponding real options (e.g., the LLF and SMF maintenance options or the initial construction option).

\[\text{Cost difference} = \frac{\text{superior design cost}}{\text{baseline design cost}} - 1, \text{ and } \text{Extra service life} = \frac{\text{superior design service life}}{\text{baseline design service life}} - 1.\]

\(^4\) Extra construction cost = superior design cost − baseline design cost, and, Extra service life = superior design service life − baseline design service life.

\(^5\) According to MTO 2005 report, for the Dowelled JPC design (considered as a superior design), the typical initial construction cost is about $85/m\(^2\) with a typical service life of 30 years. For the Deep-Strength AC design (baseline design), the typical initial construction cost is about $50/m\(^2\) with a typical service life of 18 years. Therefore, the average extra cost is 70\% of the baseline cost, and the extra service life is 66.67\% of the baseline.
The three graphs in Figure 6 represent three levels of extra service life due to superior construction. The horizontal axis is the volatility of the extra service life. The vertical axis is the real option value for construction option of using superior design. Consistent with the theory, we find a positive relationship between option value and volatility of the underlying asset, i.e., the volatility of the extra service life. The option value increases as the extra service life increases as indicated in the left, middle, and right graphs. The option value also increases as the cost difference decreases as indicated by the lines with circle, star and triangle.

Figure 7 further verifies the positive relationships between the option value and the volatility, the option value and extra service life. It also verifies the negative relationship between the option value and the cost difference.

The observed positive relationship between the volatility of the expected benefits (e.g. the repaired service life from maintenance, or the extra service life from superior initial construction) and the value of the corresponding real options (e.g., the LLF and SMF maintenance options or the initial construction option) indicates that, for highway projects, when the repaired service life from maintenance is highly uncertain (i.e., has larger volatility), the LLF and SMF real option values are going to be larger. When the extra service life from superior construction is highly uncertain, the initial construction real option value is going to be larger. That is, the P3 model tends to create larger real option value for projects with larger uncertainties in maintenance program and construction quality.

Figure 7: Sensitivity analysis II: Construction option value at different extra cost levels

4 Conclusions and Recommendations
4.1 Conclusion of real option based analysis

The real option based analysis is used to compare the upfront extra costs with the potential future benefits (i.e. future cost savings). The flexibility of cost optimization is modelled as the two
different types of real options, the initial construction option and the maintenance options (SMF and LLF). With the average cost and benefit information extracted from the existing literatures, we estimated the average real option value for construction option and two maintenance options (LLF and SMF) using Lease Square Monte Carlo simulation. On average, the LLF and SMF maintenance option value could be 56.72% and 30.79% of the initial construction cost during 30 year concession period. And, as compared with a baseline design highway project which has an expected life of 18 years, the real option value of a superior design (costs 70% more and generates 66.67% extra service life on average) is 30.46% of the baseline construction cost if the extra service life volatility is 20% and 1.73% of the baseline construction cost if the extra service life volatility is 10%.

Under the P3 model, the private firms are more likely to explore these option values and achieve better asset performance, which are more likely to be translated into residual value and passed on to the public through proper contract design. Under the P3 model, private firms are more likely to explore these option values because their profit maximization function is integrated over the entire 30 years. Under the PSC model, private firms tend to be myopic and ignore the real option values because their profit maximization function is constructed over a much shorter period of time which may or may not allow them to collect all the future benefits of the strategic investments.

Under the same performance requirements, the P3 model tends to offer more construction and maintenance flexibilities than the PSC model. These flexibilities are likely to incentivize the private firms to act more strategically and less myopically, i.e., optimally allocating the costs to achieve better asset performance.

The implication of this report to the industry practitioners has twofold. The first is from the government’s perspective. When awarding the highway projects to the private firms, the government shall estimate the real option values contained in the construction stage and the maintenance stage, provide proper incentives either through a competitive bidding process or some sort of benefit sharing plan, so that the real option values could be optimally exploited and shared between the public and private parties. And in situations where the private firms are not sophisticated enough to realize and value the real options, the government shall provide sufficient guidance regarding the optimal exercise of real options and the estimation of real option values. When there is a large amount of technological uncertainty in the maintenance program or the construction quality, the government could design a risk sharing mechanism (such as a project buyback plan or an equity ownership within the P3 structure) to help the private firms manage the risks.

The second is from the private contractors’ perspective. When managing a P3 highway project, the private firms could use the real option exercise model developed in this report to optimally choose the maintenance program based on the information they have about the repaired service life and the associated maintenance cost. It could also be used to help the private firms decide whether and when to choose a superior initial construction design, especially when the benefit of the superior design is highly uncertain.
4.2 Limitations and Caveats

Based upon the findings of the real option bases analysis, we have the following recommendation for the management of the residual value risk in future P3 project management:

In practice, in order to determine the exact real option value for individual project, managers may need more precise information about the extra cost, the extra benefit and the corresponding volatility. The real option model may be used to solve for the optimal time to choose a more costly superior design versus a cheaper baseline design, providing they both satisfy the performance specification. It may also be used to compare the costs and benefits of a LLF maintenance versus a SMF maintenance, and to determine, in what conditions, it is optimal to choose one over the other.

The parameter values and ranges, such as the maintenance cost and corresponding repaired service life, the initial construction cost and the service life difference are mostly extracted from MTO 2005 report and ARA Inc. 2008 report. They are solely used for the purpose of the sensitivity analysis. Since these parameter values are not estimated from a large sample of cost profiles of P3 or PSC projects, they shall not be used to derive any statistical inferences.

In this study, when calculating the residual value, all the financial factors are compressed into one simplified pricing kernel, i.e., we assume away all the financial risks such as the term structure of interest rate, the possibility for the private firms to sell their equity ownership stake on the secondary market etc. Therefore, one way of furthering this study is to accommodate the financial risks into the residual value function and understand the dynamics between financial risks and engineering risks.
References