Optimal Timing and Entry Capacity of Investments with Uncertainty, Taxes and Subsidies*

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Abstract

We develop a real options model which examines the effect of subsidy and taxation policies on the timing and the size of the investment. We show that, when governments are not financial constrained regarding the tax-subsidy incentives, a higher depreciation rate, or a higher subsidy, or a lower tax rate, accelerates investment. However, we find an important difference between the use of a fixed and a variable subsidy, with the former inducing smaller size investments and the later encouraging larger size investments. We also find that the tax and depreciation rates do not affect the size investments. For a zero-cost (or neutral) investment incentive package, the effects of the variables described above on the timing and size of the investment may not be monotonic, and depend on the the model parameter which we adjust to absorb the effect of the variable change so as to keep the investment incentive package financially neutral. We also conclude that, for some economic contexts, a rise in the tax rate does not necessarily deters investment, and can actually accelerate investment with a larger scale.

Keywords: Taxation Policy; Investment Subsidy; Capacity Choice; Real Options; Uncertainty.

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1 Introduction

It has been extensively reported by the literature that the taxation policy has a significant effect on firms' investment behaviour. The use of subsidies to promote investments is also a well-established practice amongst government, and their effects on investment decisions has been extensively studied. Most of the available investment literature study the effect of a taxation policy, or a subsidy policy on the optimal time to invest. In this paper, however, we examine the simultaneous effect of taxes and subsidies on both the timing of the and size of the investment. We also consider assets depreciation and the use of a subsidy policy that involves a fixed and a variable subsidy.

Our paper is closely related to two main branches of literature. One, which studies the optimal timing and size (capacity choice) of the investment under uncertainty[1], neglecting taxes and subsidies, another, which investigates the effect of taxation policy and subsidies under uncertainty. For monopoly markets the available results show that higher uncertainty delays investments and but encourages investments with a larger entry-capacity (Bar-Ilan and Strange 1999, Dangl 1999, Décamps et al. 2006). Huisman and Kort (2015) extends this branch of literature to duopoly markets and conclude that the first entrant over-invests in order to deter the entrance of the second firm which invests later in a smaller capacity.

The effect of taxation policy on the timing of the investment, under uncertainty, has been extensively studied. Some of the available literature studies the combined effect of taxes and subsidies on firms' investment behaviour. Pennings (2000) show that, when the investment is irreversible, a rise in the tax credit, if financed with an increase in the corporate tax rate, accelerates investment. Agliardi (2001) extends previous investment analysis by considering the option to divest, and concludes that the investment threshold increases with the tax rate and decreases with the subsidy. Sarkar (2012) suggests that the combination of taxes and subsides may be a sensible strategy to promote investments when the governments' discount rates differ from those of the private firms.

The monotonic effect of a higher tax rate on investment timing was questioned by Alvarez and Koskela (2008), who shows that under progressive taxation, a higher tax rate do not necessarily deters investments if the tax exemption is lower than the sunk cost: (i) for lower volatilities,

[1] A recent review of this branch of literature is provided by Huberts et al. (2015).

investment timing is independent of the tax rate and the volatility, (ii) for medium volatilities, it is independent of both the tax rate and the volatility, and (iii) for higher volatilities, the investment threshold increases with the volatility and decreases with the tax rate, which is a "tax paradox". A similar tax paradox is found by Gries et al. (2012).

The effect of assets depreciation on investment timing is studied by Adkins and Paxson (2013), for a context of assets replacement investment, who conclude that the tax depreciation affects the replacement policy by decreasing the operating cost that triggers investment. On the other hand, Sureth (2002) shows that a higher depreciation rate may deter investments.

Neutral (or zero-cost) tax-subsidy systems (i.e., tax-subsidy incentives which are self-financed) are studied by Pennings (2000) and Wong (2012). Specifically, Pennings (2000) show that, when the investment is irreversible, a rise in the tax credit if financed with an increase in the corporate tax rate hastens investment. Wong (2012) finds the same result for the case of levered firms. The firm has an incentive to hasten its investment because of the agency conflicts arising from the commitment made by the government on the terms of the tax-subsidy program.

Our work also relates to that of Lukas and Thiergart (2016) who use a two-players real option game setting (a game between a firm and a government) to study the effect of uncertainty and government's investment incentive on optimal timing of the investment, and optimal financing scheme and scale of the investment, maximizing the government's revenue. Their results show that the investment subsidy reduces the underinvestment which arises from the nature of the investment game. Specifically, they find that a subsidized firm invests earlier in a lower amount of capital.

We develop a real options model that studies the effect of taxation policy instruments and subsidies on the timing ans size of the investment. We show that a lower tax rate, a higher assets depreciation rate, or a higher subsidy hasten investment. In addition, fixed subsidies induce smaller scale investments, and variable subsidies induce larger scale investments. Furthermore, the tax and depreciation rates do not affect the size of investment. However, when a neutral investment incentive package is implemented, so as the financial effects of a change in one of its components is offset by a changes in another component of the investment incentive package, we find very different results. Specifically, the effect on the timing and size of the investment of some of the models variables are not always monotonic and depend on the the component of the investment incentive package that is adjusted so as to keep it financially neutral. For example, a rise in the tax rate may have the paradoxical effect of promoting investment in a larger scale, and the government can, without increasing costs, induce earlier investments with a larger scale.

The paper unfolds as follows. Section 2 presents the models on investment timing and capacity choice with taxes and depreciations under three alternative demand functions. The impact of subsidies is analyzed in section 2. Neutral tax systems are studied in section 3. A robustness check for the case of a multiplicative demand function is presented in Section 4. 5 concludes.

2 The model

In this section, following Huisman and Kort (2015), we present a real option model that optimizes both the size and the timing of the investment considering the effect the taxation and depreciation policy, using a iso-elastic demand function. More specifically, let us assume that a monopolistic firm is considering investing in a new market whose output price obeys to the following (constant elasticity) demand function:

$$P(t) = X(t)Q(t)^{-\gamma} \tag{1}$$

where $Q(t)$ is the total market output, $\gamma \in (0, 1)$ is the price elasticity parameter, and $X(t)$ is an exogenous shock affecting the output price, which follows a geometric Brownian motion (gBm):

$$dX(t) = \alpha X(t)dt + \sigma X(t)dw(t) \tag{2}$$

where $X(t) > 0$, α (with $\alpha < r$) is the risk-neutral expected drift, r is the risk-free rate, σ the instantaneous volatility, $dw(t)$ is the increment of a Wiener process.

Let us also assume that the firm enters the market with an optimal capacity (Q) and the investment cost comprises two components: a fixed cost (k_0) and a cost (k_1) per output unit of Q. The total investment is, therefore, $k_0 + k_1 Q$. Following Huisman and Kort (2015), we assume that after investing the firm operates at full capacity.

The firm's objective function is given by:

$$V(X) = \max_{T\geqslant 0, Q\geqslant 0} E\left[\int_{t=T}^{\infty}\left(QX(t)Q(t)^{-\gamma}(1-\tau)+\lambda(k_0+k_1Q)e^{-\lambda(t-T)}\tau\right)e^{-rt}dt \right.$$
$$\left. -(k_0+k_1Q)e^{-rT}|X(0)=X\right] \qquad (3)$$

where τ is the corporate tax rate, T is the optimal time to invest, Q is the optimal entry capacity level, and $\lambda(k_0+k_1Q)\tau$ is the continuous depreciation tax shield value. For the sake of simplicity, we assume a declining balance depreciation at the rate of λ.

The solution for Equation (3) is attained in two steps (see Huisman and Kort (2015)). First, we select the optimal capacity ($Q^*(X)$) for a given $X(t)$, by:

$$\max_{Q\geqslant 0} E\left[\int_{t=0}^{\infty}\left(QX(t)Q(t)^{-\gamma}(1-\tau)+\lambda(k_0+k_1Q)e^{-\lambda t}\tau\right)e^{-rt}dt-(k_0+k_1Q)|X(0)=X\right] \qquad (4)$$

and obtain:

$$Q^*(X) = \left(\frac{(1-\gamma)(1-\tau)X}{(r-\alpha)k_1}\frac{r+\lambda}{r+\lambda(1-\tau)}\right)^{\frac{1}{\gamma}} \qquad (5)$$

The above finding is interesting because it shows that, if the firm targets the investment for a higher X (i.e., if it decides to invest later) that affects (increases) the optimal entry-capacity level. We also conclude that if the value of X at which the firm optimally enters the market changes as a consequence of a change in the corporate tax rate, or the depreciation rate (τ or λ, respectively), the effect on firm's behaviour of this change in X can be offset by adjusting the entry-capacity level. More specifically, we find that the size of the investment increases with X and decreases with both the corporate tax rate and the depreciation rate.

In the second step of the derivation of the solution for Equation (3), we obtain the optimal investment threshold X^*, by:

$$\max_{X^*}\left[\left(\frac{Q^*(X^*)X^*Q^*(X^*)^{-\gamma}(1-\tau)}{r-\alpha}-\frac{r+\lambda(1-\tau)}{r+\lambda}(k_0+k_1Q(X^*))\right)\left(\frac{X}{X^*}\right)^{\beta_1}\right] \qquad (6)$$

Proposition 1. *A firm holding a proprietary option to invest in a project whose cost comprises two components, a fixed cost (k_0) and a cost per output unit of the entry capacity level (k_1Q), in a market where a constant elasticity demand function ($P = XQ^{-\gamma}$) holds, invests should invest* *at the following investment threshold:*

$$X^* = \left(\frac{\beta_1(1-\gamma)k_0}{(\beta_1\gamma-1)k_1}\right)^{\gamma} \frac{r-\alpha}{(1-\gamma)(1-\tau)} \frac{r+\lambda(1-\tau)}{r+\lambda} k_1 \tag{7}$$

with capacity:

$$Q^* = \frac{\beta_1(1-\gamma)k_0}{(\beta_1\gamma-1)k_1} \tag{8}$$

where

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} \tag{9}$$

Two assumptions are imposed: (i) $\beta_1\gamma > 1$, otherwise firms would postpone investment forever (Dixit and Pindyck 1994), and (ii) $k_0 > 0$, otherwise firms would invest immediately (Huisman and Kort 2015).

To carry out a comparative statics of the tax policy instruments and uncertainty, we differentiate X^* and Q^* wit respect to the tax rate (τ), the depreciation rate (λ) and β_1 (note that $\frac{\partial \beta_1}{\partial \sigma} < 0$):

$$\frac{\partial X^*}{\partial \tau} = \left(\frac{\beta_1(1-\gamma)k_0}{(\beta_1\gamma-1)k_1}\right)^{\gamma} \frac{r-\alpha}{(1-\gamma)(1-\tau)^2} \frac{r}{r+\lambda} k_1 > 0 \tag{10}$$

$$\frac{\partial Q^*}{\partial \tau} = 0 \tag{11}$$

$$\frac{\partial X^*}{\partial \lambda} = -\left(\frac{\beta_1(1-\gamma)k_0}{(\beta_1\gamma-1)k_1}\right)^{\gamma} \frac{r-\alpha}{(1-\gamma)(1-\tau)} \frac{r\tau}{(r+\lambda)^2} k_1 < 0 \tag{12}$$

$$\frac{\partial Q^*}{\partial \lambda} = 0 \tag{13}$$

$$\frac{\partial X^*}{\partial \beta_1} = -\left(\frac{\beta_1(1-\gamma)k_0}{(\beta_1\gamma-1)k_1}\right)^{\gamma-1} \frac{\gamma}{(\beta_1\gamma-1)^2} \frac{r-\alpha}{1-\tau} \frac{r+\lambda(1-\tau)}{r+\lambda} k_0 < 0 \tag{14}$$

$$\frac{\partial Q^*}{\partial \beta_1} = -\frac{(1-\gamma)k_0}{(\beta_1\gamma-1)^2 k_1} < 0 \tag{15}$$

The effects of a tax policy comprising a tax rate and a depreciation rate are summarized in the following corollary:

Corollary 1. *For a firm with a proprietary option to invest in a project facing an iso-elastic demand function $P = XQ^{-\gamma}$, a higher tax rate and a lower depreciation rate deter investment* $\left(\frac{\partial X^*}{\partial \tau} > 0, \frac{\partial X^*}{\partial \lambda} < 0\right)$*, while they have no impact on the amount/capacity invested* $\left(\frac{\partial Q^*}{\partial \tau} = \frac{\partial Q^*}{\partial \lambda} = 0\right)$.

Changing the tax policy instruments can induce a different investment timing but the amount invested in unaffected.

The sign of the effect of uncertainty is the same as suggested by Huisman and Kort (2015):

Corollary 2. *For a firm with a proprietary option to invest in a project facing an iso-elastic demand function* $P = XQ^{-\gamma}$*, a higher uncertainty deters investment* $\left(\frac{\partial X^*}{\partial \sigma} > 0\right)$*, and induces investment in a larger capacity* $\left(\frac{\partial Q^*}{\partial \sigma} > 0\right)$.

However, the magnitude of the impact of a higher uncertainty on the investment delay depends on the current levels of the tax policy instruments.

Subsidies to investment

In this section we study the effects of an investment subsidy granted by the Government. When subsidies are included, the investment cost is reduced by the subsidy amount:

$$I_s = (k_0 - s_0) + (k_1 - s_1)Q \tag{16}$$

The solution is obtained substituting k_0 and k_1 for $k_0 - s_0$ and $k_1 - s_1$ in Proposition 1. Differentiating X^* and Q^* wit respect to the the subsidies (s_0 and s_1):

$$\frac{\partial X^*}{\partial s_0} = -\left(\frac{\beta_1(1-\gamma)(k_0 - s_0)}{(\beta_1\gamma - 1)(k_1 - s_1)}\right)^{\gamma-1} \frac{\beta_1\gamma}{\beta_1\gamma - 1} \frac{r-\alpha}{1-\tau} \frac{r+\lambda(1-\tau)}{r+\lambda} < 0 \tag{17}$$

$$\frac{\partial Q^*}{\partial s_0} = -\frac{\beta_1(1-\gamma)}{(\beta_1\gamma - 1)(k_1 - s_1)} < 0 \tag{18}$$

$$\frac{\partial X^*}{\partial s_1} = -\left(\frac{\beta_1(1-\gamma)(k_0 - s_0)}{(\beta_1\gamma - 1)(k_1 - s_1)}\right)^{\gamma} \frac{r-\alpha}{1-\tau} \frac{r+\lambda(1-\tau)}{r+\lambda} < 0 \tag{19}$$

$$\frac{\partial Q^*}{\partial s_1} = \frac{\beta_1(1-\gamma)(k_0 - s_0)}{(\beta_1\gamma - 1)(k_1 - s_1)^2} > 0 \tag{20}$$

The following corollaries on the effects of subsidies hold:

Corollary 3. *For a firm with a proprietary option to invest in a project facing an iso-elastic demand function* $P = XQ^{-\gamma}$*:*

- *A higher fixed subsidy* (s_0) *induces earlier investment in a smaller capacity* $\left(\frac{\partial X^*}{\partial s_0} < 0, \frac{\partial Q^*}{\partial s_0} < 0\right)$.
- *A higher variable subsidy* (s_1) *induces earlier investment in a larger capacity* $\left(\frac{\partial X^*}{\partial s_1} < 0, \frac{\partial Q^*}{\partial s_1} > 0\right)$.

Table 1 presents a summary of the comparative statics related with the tax-subsidy package and uncertainty. A lower tax rate, a higher depreciation rate or a higher subsidy hasten investment. Fixed subsidies induce smaller scale investments, and variable subsidies induce larger scale investments. The tax and depreciation rates do not affect the size of investment. A higher uncertainty deters investment and induces larger scale investments.

Table 1: Summary of comparative statics

	∂X^*	∂Q^*
$\partial \tau$	$+$	0
$\partial \lambda$	$-$	0
∂s_0	$-$	$-$
∂s_1	$-$	$+$
$\partial \sigma$	$+$	$+$

3 Neutral investment incentives

In line with Pennings (2000) we define a zero expected cost package in terms of the tax rate, depreciation rate and subsidy amount. The package (τ, λ, s_0, and s_1) is set in order to have a zero cost. Wong (2012) extends this analysis by considering both levered and unlevered firms. In any of these models, depreciations and investment scale were not considered.

The terms of a zero-cost package of tax and depreciation rates and subsidies ensure that the present value of taxes equals the cost of the subsidies:

$$
\begin{aligned}
s_0 + s_1 Q^* &= E\left[\int_{t=t^*}^{\infty} \left(Q^* P(X(t))\tau - \lambda(k_0 - s_0 + (k_1 - s_1)Q^*)e^{-\lambda(t-t^*)}\tau\right) e^{-r(t-t^*)} dt\right] \\
&= \left(\frac{Q^* X^* Q^{*-\gamma}}{r-\alpha} - \frac{\lambda}{r+\lambda}\left(k_0 - s_0 + (k_1 - s_1)Q^*\right)\right)\tau \qquad (21)
\end{aligned}
$$

where t^* is the first time X hits the threshold X^*. Please note that X^* and Q^* are affected by the package constituents. The solution is found solving simultaneously Equations (7), or (8), and (21).

The effects of the individual parameters must now consider the indirect effects. In particular, the neutral package is guaranteed by making the changes in one component be compensated by the changes in some another component of the package. The overall effects on the timing and scale are analyzed next. The comparison with the unrestricted package is also presented.

Figure 1(a) shows that, for an unrestricted package (UP), a decrease of the tax rate hasten investment. However, under a neutral package (NP) where the compensation component is the depreciations rate, an opposite effect occurs. For both the regimes, the scale reveals to be not affected (1(b)). Additionally, when the NP is ensured by the level of subsidies (fixed or variable), the tax rate effect reveals ambiguous, promoting or deterring investment depending on its level (see Figures 1(c) and 1(e)). The effects of taxation on the investment scale is also interesting. By jointly analyzing Figures 1(e) and 1(f) we see that an increase in τ negatively impacts the timing but does not change the scale for UP, while under NP there is a region where increases in tax rate hasten investment for an increasing capacity.

Figure 2 shows the impact of the depreciation rate on the investment timing and scale. Under UP, higher depreciations hasten investment but do not affect scale. However, under NP regime, all sorts of results can be found. Higher depreciation rates decreases or increases the trigger when neutralized by the tax rate or by the fixed subsidy, respectively (see Figure 2(c) and 2(d)), while shows an ambiguous effect when compensated by a variable subsidy (Figure 2(e)). These impacts on the timing happen along with an increase or decrease of the scale of the investment (Figure 2(d) and 2(f)).

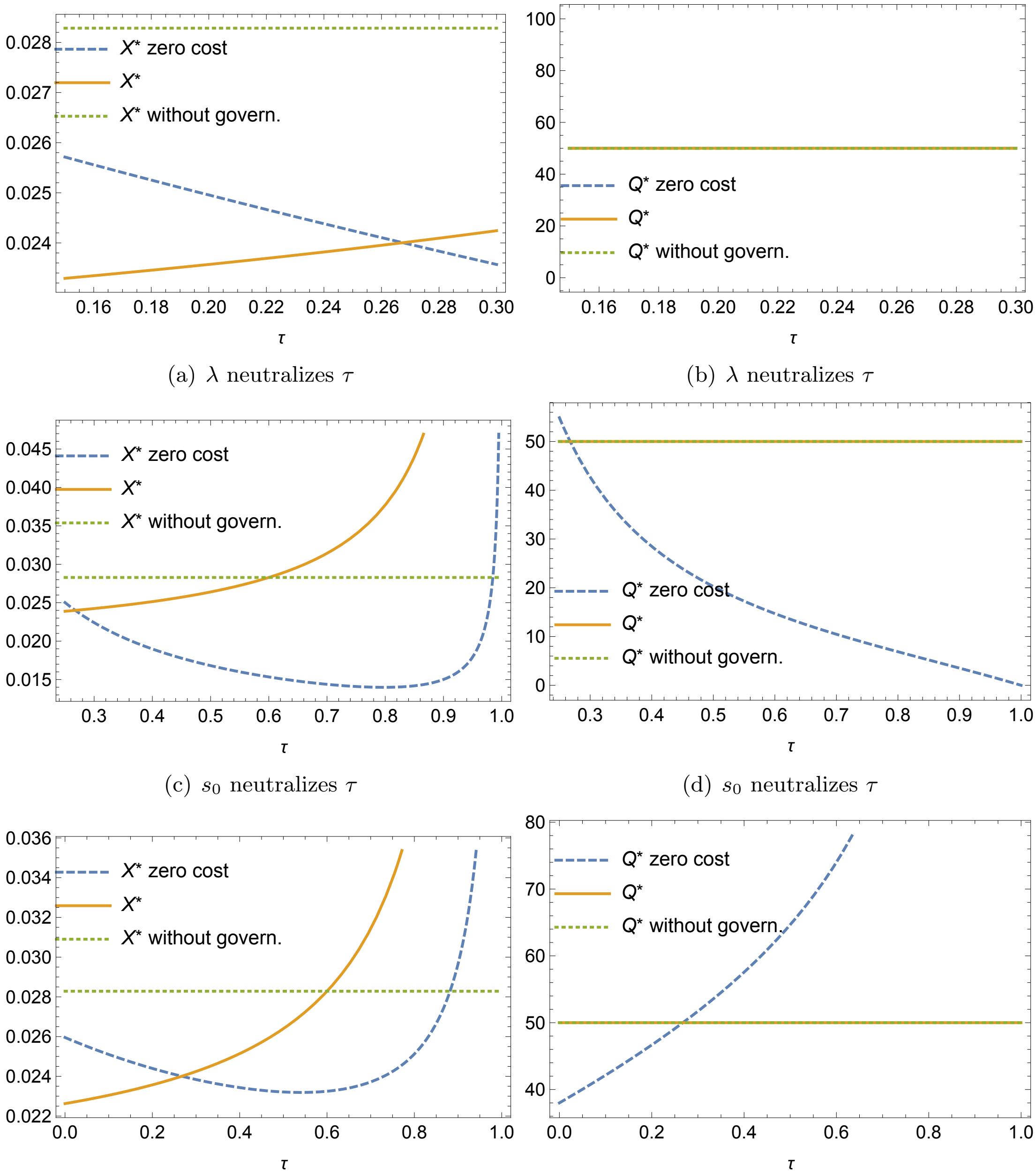
Let us now analyze the effect of the two types of subsidies. Starting with fixed subsidies (Figure 3), we see its general positive impact in speeding-up investment along with a reduction of scale. The only exception is when the fixed subsidy is compensated by the tax rate, where we can find ambiguous effects (Figure 3(a)). Regarding the variable subsidies (Figure 4), its impact is positive in hastening investment and promoting scale for the UP regime, while it reveals to deter investment under NP, independently from neutralizing variable.

4 The multiplicative demand function case

The robustness of our main results is checked replacing the iso-elastic demand function by a multiplicative demand function:

$$P(t) = X(t)(1 - \eta Q(t)) \tag{22}$$

where $\eta > 0$.



(a) λ neutralizes τ

(b) λ neutralizes τ

(c) s_0 neutralizes τ

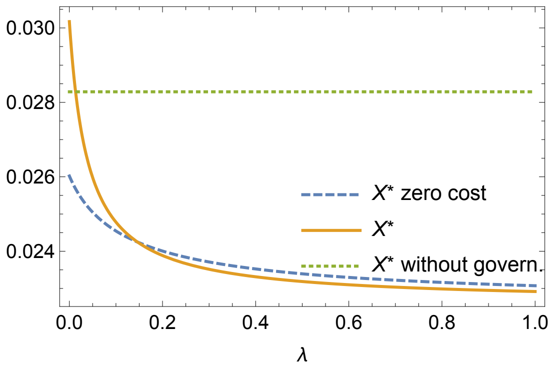
(d) s_0 neutralizes τ

(e) s_1 neutralizes τ

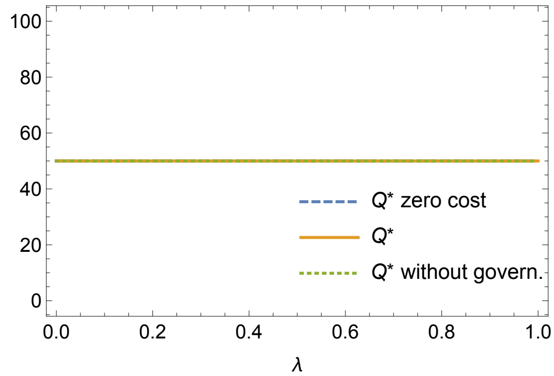
(f) s_1 neutralizes τ

$r = 0.04$, $\alpha = 0.02$, $\beta_1 = 2.5$, $\gamma = 0.5$, $\lambda = 0.2$, $k_0 = 1$, $k_1 = 0.1$, $s_0 = 0.2$, $s_1 = 0.02$, $\tau = 0.25$.

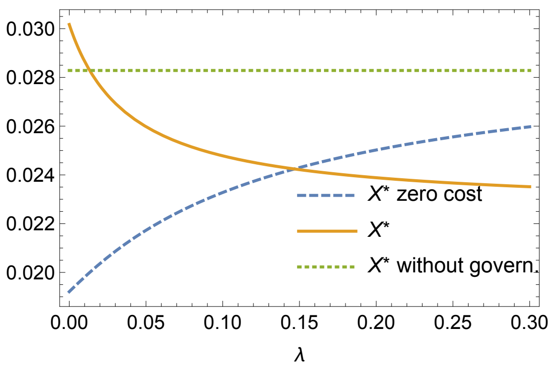
Figure 1: Effects of a neutral tax rate τ



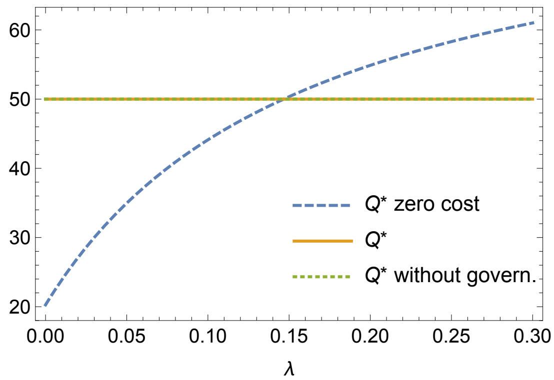
(a) τ neutralizes λ



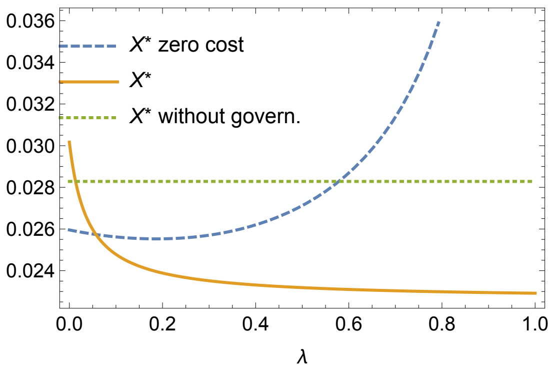
(b) τ neutralizes λ



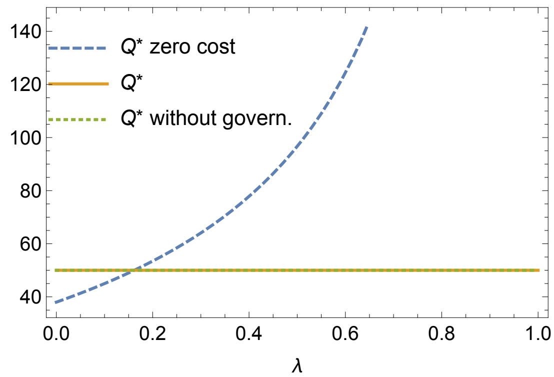
(c) s_0 neutralizes λ



(d) s_0 neutralizes λ



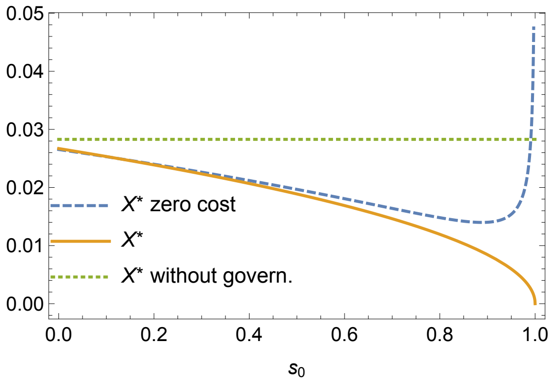
(e) s_1 neutralizes λ



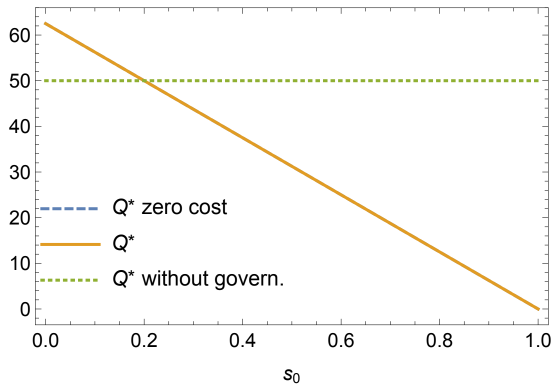
(f) s_1 neutralizes λ

$r = 0.04$, $\alpha = 0.02$, $\beta_1 = 2.5$, $\gamma = 0.5$, $\lambda = 0.2$, $k_0 = 1$, $k_1 = 0.1$, $s_0 = 0.2$, $s_1 = 0.02$, $\tau = 0.25$.

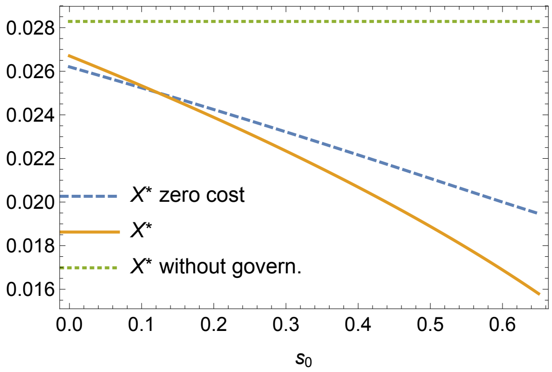
Figure 2: Effects of a neutral depreciation rate λ



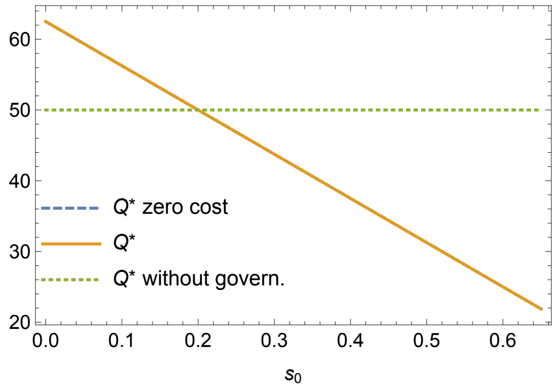
(a) τ neutralizes s_0



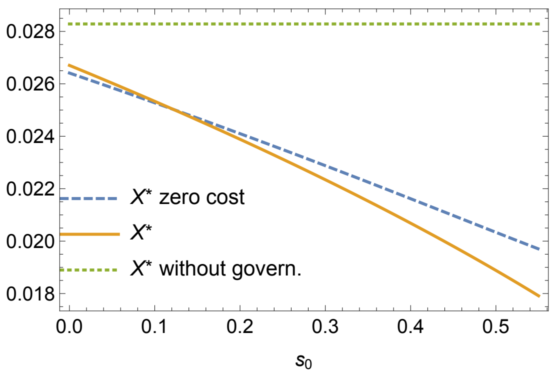
(b) τ neutralizes s_0



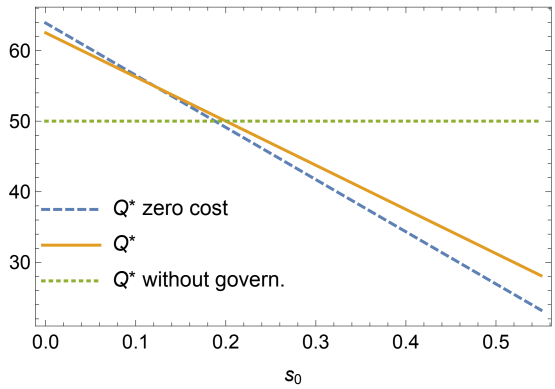
(c) λ neutralizes s_0



(d) λ neutralizes s_0



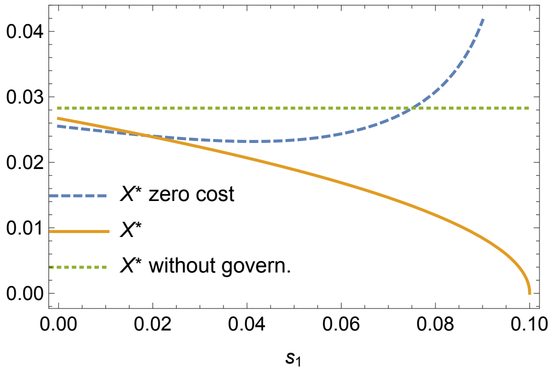
(e) s_1 neutralizes s_0



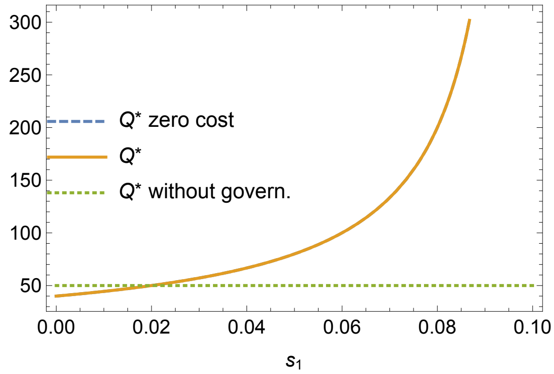
(f) s_1 neutralizes s_0

$r = 0.04$, $\alpha = 0.02$, $\beta_1 = 2.5$, $\gamma = 0.5$, $\lambda = 0.2$, $k_0 = 1$, $k_1 = 0.1$, $s_0 = 0.2$, $s_1 = 0.02$, $\tau = 0.25$.

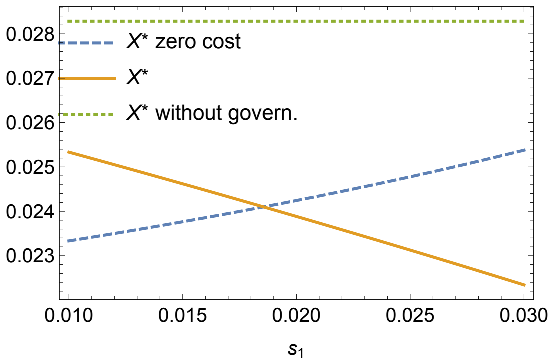
Figure 3: Effects of a neutral fixed subsidy s_0



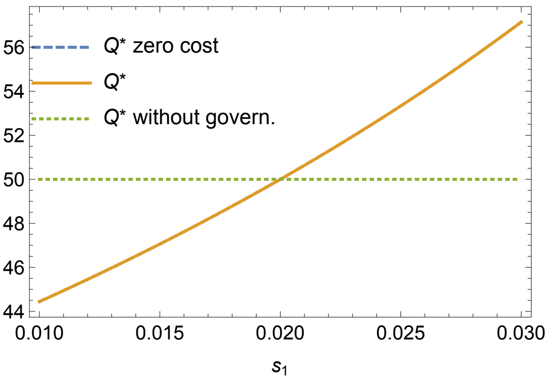
(a) τ neutralizes s_1



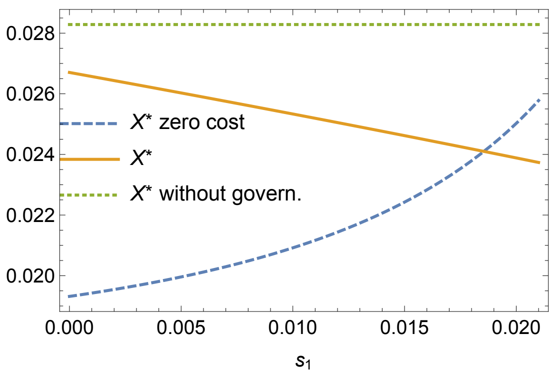
(b) τ neutralizes s_1



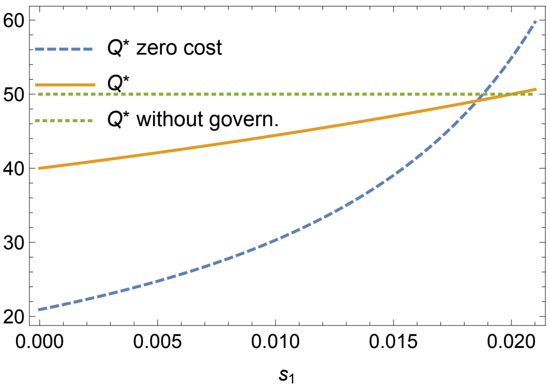
(c) λ neutralizes s_1



(d) λ neutralizes s_1



(e) s_0 neutralizes s_1



(f) s_0 neutralizes s_1

$r = 0.04$, $\alpha = 0.02$, $\beta_1 = 2.5$, $\gamma = 0.5$, $\lambda = 0.2$, $k_0 = 1$, $k_1 = 0.1$, $s_0 = 0.2$, $s_1 = 0.02$, $\tau = 0.25$.

Figure 4: Effects of a neutral variable subsidy s_1

Proposition 2. *A firm with a proprietary option to invest in a project facing an inverse multiplicative linear demand function $P = X(1 - \eta Q)$, that requires a net investment of $(k_1 - s_1)Q$, will optimally invest when X reaches the following threshold:*

$$X^* = \frac{\beta_1 + 1}{\beta_1 - 1} \frac{r - \alpha}{1 - \tau} \frac{r + \lambda(1 - \tau)}{r + \lambda} (k_1 - s_1) \tag{23}$$

and choose capacity

$$Q^* = \frac{1}{\eta(\beta_1 + 1)} \tag{24}$$

The effects of the fiscal package components are neutral in terms of the amount of investment ($\frac{\partial Q^*}{\partial \tau} = \frac{\partial Q^*}{\partial \lambda} = \frac{\partial Q^*}{\partial s_1} = 0$). The later effect differs from that observed in the case of a iso-elastic demand function, for which a higher subsidy reduces the size of investment. The difference is related with the presence of a fixed investment cost.

The effects on the investment timing are obtained differentiating X^* with respect to the fiscal policy package components :

$$\frac{\partial X^*}{\partial \tau} = \frac{\beta_1 + 1}{\beta_1 - 1} \frac{r - \alpha}{(1 - \tau)^2} \frac{r}{r + \lambda} (k_1 - s_1) > 0 \tag{25}$$

$$\frac{\partial X^*}{\partial \lambda} = -\frac{\beta_1 + 1}{\beta_1 - 1} \frac{r - \alpha}{1 - \tau} \frac{r\tau}{(r + \lambda)^2} (k_1 - s_1) < 0 \tag{26}$$

$$\frac{\partial X^*}{\partial s_1} = -\frac{X^*}{k_1 - s_1} < 0 \tag{27}$$

The sign of the effects are the same as in the iso-elastic case. The sign of effects of uncertainty on the scale and investment timing are also identical:

$$\frac{\partial X^*}{\partial \beta_1} = -\frac{2X^*}{(\beta_1 - 1)(\beta_1 + 1)} < 0 \tag{28}$$

$$\frac{\partial Q^*}{\partial \beta_1} = -\frac{1}{\eta(\beta_1 - 1)^2} < 0 \tag{29}$$

Excluding the effects of the variable subsidy to investment (s_1), the effects of the fiscal policy components, in an unrestricted package or in a neutral package are expected to remain the same.

5 Conclusion

We develop a real options model which examines the effect of subsidy and taxation policies on the timing and the size of the investment. We show that, when governments are not financial constrained regarding the tax-subsidy incentives, a higher depreciation rate, or a higher subsidy, or a lower tax rate, accelerates investment. However, we find an important difference between the use of a fixed and a variable subsidy, with the former inducing smaller size investments and the later encouraging larger size investments. We also find that the tax and depreciation rates do not affect the size investments. For a zero-cost (or neutral) investment incentive package, the effects of the variables described above on the timing and size of the investment may not be monotonic, and depend on the the model parameter which we adjust to absorb the effect of the variable change so as to keep the investment incentive package financially neutral. We also conclude that, for some economic contexts, a rise in the tax rate does not necessarily deters investment, and can actually accelerate investment with a larger scale.

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