Competition, innovation incentives and the *ex ante* conservation of capital (*Preliminary and incomplete*)

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Abstract

Dynamic innovation incentives incorporating the firm's ability to delay innovation are related to market characteristics. Because of the replacement effect these incentives are generallly larger with competition, but if there is even a slight limitation on appropriability (finite patent duration or small probability of imitation) a monopoly firm values a novel idea more highly if it is sufficiently pioneering, and this is more likely to occur in industries with high growth and volatility. If the size of innovation is endogenized, when growth and volatility are high competitive firms are more likely to opt for drastic innovations but innovate excessively from a social standpoint.

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1 Introduction

Because of the inherent uncertainty of invention it is unlikely that novel ideas arrive at the most opportune moment for commercial development. An important part of the value of innovation then consists of the option value associated with developing a novel idea at a moment at which economic conditions are most favorable. In a half-century of active research on the economics of innovation in the wake of Arrow [1]'s seminal contribution, a considerable literature has developed

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relating innovation incentives to the degree of competition firms face in the product market as well as uncertainty regarding the success of R&D investments.[1] In parallel the understanding of the option value associated with investments has also evolved, and it can continue to usefully inform innovation economics and policy by shedding light on the determinants of innovation when it is viewed as an investment decision. Notably in high growth or high volatility industries in which the value of expansion options is likely to be significant, the link between competition and innovation is less straightforward than the standard static analysis suggests, even when restricting attention to the polar cases of monopoly and competitive firms. Moreover, viewing innovation as an investment opportunity provides a better understanding of the type of innovation that firms ultimately choose to pursue, by highlighting the role played by industry characteristics in determining the magnitude of innovations.

This paper studies the innovation incentives of firms by representing innovation as an irreversible investment in which option value is likely to be important because the arrival of an innovation and its development cost are generally not coordinated events. Using a standard model of irreversible investment (Dixit and Pindyck [8]) to represent the dynamic innovation incentive of firms, one quickly sees that option value should not have any impact on the comparative innovation incentives of monopoly and competitive firms, because of the power of the replacement effect. The intuition behind this is straightforward: because of the replacement effect, the competitive payoff is greater than the monopoly payoff in every state, and it then follows naturally that the competitive option to develop the idea in any state is worth more than the corresponding monopoly option.

However, the importance of the replacement effect hinges upon a restrictive set of underlying assumptions, and much of the progress in the understanding of innovation incentives has resulted from relaxing these assumptions in various ways.[2] Viewing innovation as an investment

[1] Already a decade ago, Gilbert [13] refers to the wealth of economic models of innovation:

> "differences in market structure, the characteristics of innovations, and the dynamics of discovery lead to seemingly endless variations in the theoretical relationship between competition and expenditures on research and development or the outputs of research and development (R&D)."

[2] As Arrow observes ([1] p. 622):

> "The only ground for arguing that monopoly may create superior incentives to invent is that appropriability may be greater under monopoly than under competition. Whatever differences may exit in this direction must, of course, still be offset against the monopolist's disincentive created by his preinvention monopoly profits."

opportunity in a dynamic setting, it is straightforward to incorporate some specific limitations to the appropriability of invention into the analysis. For example, both the length of time for which a novel idea is protected by a patent and the scope of patents are limited. Moreover, information leakage is likely to occur over time, leading competitors to imitate or invent around a novel idea. Such limitations to appropriability as these are likely to favor monopoly innovation, which enjoys additional entry barriers in the product market. By integrating limits on appropriability such as finite patent duration or a random arrival of imitation into the standard model of irreversible innovative investment, the circumstances that favor monopoly innovation can be identified precisely.

If a novel idea is "pioneering" in that it emerges at a moment when the cost of development is currently high so that optimal time to develop an innovation is likely to be remote, then there is a greater dynamic innovation incentive under monopoly than under competition. This is in a sense a setback for any policy that aims to foster innovation by promoting those market structures that offer greater incentives for innovation through merger policy for instance, since in a stochastic dynamic setting the most favorable market structure changes over time with market conditions.[3] That imperfect appropriately has consequences for relative innovation incentives is not in itself particularly novel, but the real options approach allows more specific results to be derived that are relevant for policy. This is because this analysis points to those circumstances under which a monopoly market structure should not be viewed as an impediment to innovation, namely in industries in which volatility or growth are sufficiently high so that the value of an innovation resides mainly in the possibility of developing it in the future.[4] Conversely, it is when market conditions are more stable and development is likely to occur rapidly that the replacement effect

[3] This policy objective is described by Gilbert [13] as follows:

> "Studies also show that the social return to investment in R&D is higher than the private return[,] which suggests that policies that promote innovation can pay large dividends for society. One way to achieve these benefits is to promote industry structures that offer greater incentives for innovation, including policies toward mergers and laws that govern exclusionary conduct."

[4] Future earnings are notoriously important in the valuation of emerging internet businesses, as was the case for the emblematic internet firm Amazon in the late 1990s. Decades earlier, many important inventions of the twentieth century were made at Bell Labs, an organization that favored research on projects of unproven immediate commercial value (see, e.g., Gertner [12]). The main tehnical ideas behind mobile telephony for instance date back to 1947, but it wasn't until the FCC opened a block of frequencies a couple of decades later that cellular networks were actually developed. In the technology adoption literature, the importance of waiting and optimal timing is well-recognized (see Hoppe [16]), notably because of the expectation at any moment of further technical improvements.

will be relatively important and clearly favor competitive innovation.

Having at first focused attention on the timing of discrete investments (a notable exception is Bar-Ilan and Strange [2]) research on investment options has begun recently to account for both the timing and the magnitude of investment. In the framework for innovative investment developed in this paper, the size of investment is best understood as a decision regarding the magnitude of the innovation that the firm ultimately brings to market, such as a targeted level of unit cost reduction for a process innovation. Although some influential early work on innovation considers the choice of innovation size in a static setting (Dasgupta and Stiglitz [7]), we argue that it is in a dynamic context that this question takes on particular significance. Here as well, the real options approach allows the magnitude of innovation to be understood in relation to industry and market characteristics. For instance, it is when market growth and volatility are large that competitive firms are more likely to delay innovation and opt for so-called drastic innovations. Somewhat counterintuitively, competitive firms innovate "too much, too late". This occurs because of a mechanism described by Huisman and Kort [19] in the following way. As greater volatility raises the value of an investment option, it induces firms to invest later at a time when their marginal investment incentive is greater, either because marginal revenue has increased or because the incremental cost of investment is lower. In the innovation framework adopted here (which draws on a general specification proposed by Pennings [20]), this implies that competitive firms invest too much, too late as compared with the social innovation incentive.

Through its focus on the role of the uncertainty surrounding innovation in monopoly and competition rather than strategic oligopoly firms, the model of this paper is perhaps most closely related to some early "auction" models of innovation incentives, as surveyed by Reinganum [22], that study the scale of firm investments and its repercussion for the speed of innovation. As in these early models, innovation can occur more or less quickly as in the present paper, although this happens here as a result of the timing of innovative projects rather than of their intensity, and the focus remains on the polar cases of monopoly and competitive firms and the effect of limited appropriability, in contrast with the considerable literature in industrial organization that studies the relationship between innovation and competition through those models of strategic behavior in oligopoly that are admittedly most likely to directly inform such policy objectives as merger control. Among recent examples, Etro [9], Belleflamme and Vergari [3] and Chen and Schwartz [5] study intermediate or alternative forms of product market competition such as oligopoly, Stackelberg leadership, and coordination so as to identify specific circumstances that favor monopoly innovation. At least since D'Aspremont and Jacquemin [6] research on the choice of innovation size has similarly centered on strategic incentives in oligopolies so as to shed light on issues of policy relevance, such as R&D cooperation issue. Another influential line of research on the economics of innovation has explored strategic innovation incentives when firms race to obtain a patent or adopt a technology whose arrival is uncertain (Reinganum [21]) and more recent work in this area represents innovation as an investment option (Huisman [17], Weeds [25]) and integrates limited appropriability and strategic behavior in the presence of imitators into the analysis (Huisman and Kort [18], Femminis and Martini [11]). Finally, there are strong parallels between the study of innovation incentives for a firm and models of technology adoption, such as Farzin et al. [10]'s model of the optimal timing of technology adoption in a competitive setting that this paper complements by highlighting the effect of different market structures.

Section 2 develops a benchmark model of innovation incentives when a monopoly or competitive innovator face a stochastic development cost for a novel idea and decide upon the timing of investment, and verified that the replacement effect continues to hold in a dynamic setting. Section 3 allows for different forms of limited appropriability, such as limited patent duration or the arrival of imitation. In all cases, it emerges that even an arbitrarily small limitation on appropriability for competitive firms results in a greater dynamic innovation incentive for monopoly firms, when the current cost of development is sufficient high. Finally Section 4 allows for variable innovation size and endogenizes drastic innovation in the case of competitive firms. Section 5 concludes.

2 Dynamic innovation incentives

This section describes the value of innovation under monopolistic and competitive conditions in the product market, incorporating any option value associated with the right to develop a novel idea that may be present if a firm has leeway as to the timing of an innovation, so as to determine the dynamic incentives that firms may have to innovate in these different market structures.

2.1 Monopoly innovation option

The following paragraphs describe a canonical model of innovation for a monopoly firm along the lines of Dixit and Pindyck [8] that forms the basis for the analysis in the rest of the paper. The firm is taken to engage in a process innovation where uncertainty applies to the cost of innovation, and qualitatively similar results would obtain if product innovation or demand uncertainty were present instead.

Let $\underline{\pi}$ and $\overline{\pi}$, $\overline{\pi} > \underline{\pi} > 0$, denote flow profits before and after innovation. These flow profits may, for example, reflect a process innovation that reduces unit cost, *i.e.* $\underline{\pi} = \pi^{\mathrm{M}}(c_0)$ and $\overline{\pi} = \pi^{\mathrm{M}}(c_1)$, where π^{M} denotes the optimal monopoly profit function and c_0 and c_1, $c_1 < c_0$, denote pre and post-innovation unit cost. The discount rate $r > 0$ is assumed to be known and constant, so $\underline{\Pi} = \underline{\pi}/r$ and $\overline{\Pi} = \overline{\pi}/r$ represent the capitalized values of perpetual profit flows without and with innovation.

Innovation begins when the firm acquires the right to use a novel idea, but acquiring a novel idea does not directly lead to a higher profit flow. In order to be implemented, an idea must be developed by the firm in order to be viable, and this development involves a complementary expenditure in order to roll the novel idea out onto the shop floor or to bring it to market.

The fixed cost of developing an idea at a given time t, K_t, evolves according to a geometric Brownian motion with drift μ and volatility σ, $dK_t = \mu K_t dt + \sigma K_t dW_t$, with $\sigma > 0$. The option to delay development is therefore of potential value because of uncertainty regarding the future fixed cost of development (σ). The drift term may be interpreted either as a measure of resource scarcity that raises the cost of development over time (if $\mu > 0$) or technical progress and learning ($\mu < 0$), a form of market growth. In the latter case, the negative drift of the fixed cost is also a potential source of value for the option to delay. Once the fixed cost is paid, development is assumed to be instantaneous and riskless so that the firm immediately starts perceiving a corresponding flow profit in the product market.[5]

Under these conditions a monopoly firm holds a growth option with cost uncertainty. Its decision consists in choosing at which time to develop its idea, and its optimal policy is a trigger policy involving a stopping threshold of the development cost, denoted K^{M}, that defines the stochastic time at which it undertakes development, $\tau^{\mathrm{M}} = \inf\left\{t \geq 0, K_t \leq K^{\mathrm{M}}\right\}$.

For a given current value of the fixed cost $K_t = k$ standard arguments establish that under the above assumptions the dynamic innovation incentive of a monopoly firm is

$$V^{\mathrm{M}}(k) = \sup_{\tau \geq t} \mathbb{E}_t\left[\left(\overline{\Pi} - \underline{\Pi} - K_\tau\right) e^{-r\tau}\right] \tag{1}$$

$$= \begin{cases} \overline{\Pi} - \underline{\Pi} - k, \; k \leq K^{\mathrm{M}} \\ \frac{\left[K^{\mathrm{M}}\right]^{\gamma+1}}{\gamma} k^{-\gamma}, \; k > K^{\mathrm{M}} \end{cases} \tag{2}$$

[5]The main conclusions of the paper also hold if the success of innovation is stochastic. For instance if the arrival of innovation success follows a Poisson process with arrival rate $h > 0$ then the flow payoffs $\overline{\Pi}$ and Π^{C} are multiplied by $h/(r+h)$ and $(r+h)$ must be substituted for r in the discounting parameter γ, but the qualitative comparison of dynamic innovation incentives is unaffected.

where $K^{\mathrm{M}} = (\gamma / (\gamma+1)) \left(\overline{\Pi} - \underline{\Pi}\right)$ and

$$\gamma(r,\mu,\sigma) = \frac{\mu}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma^2}} \tag{3}$$

is a function of parameters that reflects expected discounting, written γ for short.[6] If $k > K^{\mathrm{M}}$ then the last line in (2), $V^{\mathrm{M}}(k) = \left(\overline{\Pi} - \underline{\Pi} - K^{\mathrm{M}}\right)\left(K^{\mathrm{M}}/k\right)^{\gamma}$, incorporates the value of the option to delay, and as $\partial\gamma/\partial\sigma < 0$ a straightforward envelope argument shows that $\partial V^{\mathrm{M}}/\partial\sigma > 0$ in this case.

2.2 Competitive innovation option and replacement effect

The opposite case of the dynamic innovation incentive of a monopoly firm (2) is the value of innovation to a firm facing a competitive fringe in the product market and thus earning no economic profit before innovation. In this case, the dynamic innovation incentive is also interpreted as the value that an inventor can extract upstream in a market for ideas. Let Π^{C} denote the present value of the competitive firm's post-innovation flow profit. Note that necessarily $\Pi^{\mathrm{C}} \leq \overline{\Pi}$ (a competitive firm cannot earn greater profit than a monopoly) and if $\Pi^{\mathrm{C}} = \overline{\Pi}$ the innovation is said to be *drastic*. Following similar arguments to the monopoly option case above, the dynamic innovation incentive of a firm facing product market competition is

$$V^{\mathrm{C}}(k) = \begin{cases} \Pi^{\mathrm{C}} - k,\ k \leq K^{\mathrm{C}} \\ \frac{\left[K^{\mathrm{C}}\right]^{\gamma+1}}{\gamma} k^{-\gamma},\ k > K^{\mathrm{C}} \end{cases} \tag{4}$$

where $K^{\mathrm{C}} = (\gamma/(\gamma+1))\,\Pi^{\mathrm{C}}$.

In his seminal paper Arrow [1] demonstrates that in a standard setting $\Pi^{\mathrm{C}} > \overline{\Pi} - \underline{\Pi}$ (the *replacement effect*). From his result it directly follows that $K^{\mathrm{C}} > K^{\mathrm{M}}$ and $V^{\mathrm{C}}(k) > V^{\mathrm{M}}(k)$. Product market competition is therefore associated with earlier innovation and a greater dynamic innovation incentive than monopoly, so that the replacement effect holds just as well as in a stochastic continuous time setting as in the static model. The intuition for this is very straightforward: if the competitive payoff is greater than the monopoly payoff in every single state of the development cost, then it naturally follows that the competitive option to develop the idea in any state is worth more than the corresponding monopoly option.

[6] The geometric Brownian motion has the property that for $k \geq K^{\mathrm{M}}$ and first hitting time τ^{M} defined in the text, $\mathbb{E}_k\left[e^{-r\tau^{\mathrm{M}}}\right] = \left(\frac{K^{\mathrm{M}}}{k}\right)^{\gamma}$. Here γ is the absolute value of the negative root of the fundamental quadratic (see Section A.1).

2.3 Social incentive

In addition to identifying the replacement effect Arrow compares the monopoly and competitive innovation incentives to the social incentive to innovate. In order to perform a benchmark comparison in the continuous time setting considered here, assume that the social discount rate is the same as that of firms and that consumers are risk-neutral.

Consider a process innovation that reduces unit cost from c_0 to c_1 and let $D(P)$ denote the demand function, assumed to be strictly decreasing on its domain. The capitalized surplus flows without and with innovation resulting from marginal cost pricing in the product market are $S_0^{\mathrm{C}} = (1/r)\int_{c_0}^{\infty} D(s)ds$ and $S_1^{\mathrm{C}} = (1/r)\int_{c_1}^{\infty} D(s)ds$. Similarly to (2) and (4), under these assumptions the social gain from innovation is defined by parts as

$$W^*(k) = \begin{cases} S_1^{\mathrm{C}} - S_0^{\mathrm{C}} - k,\ k \leq K^* \\ \frac{[K^*]^{\gamma+1}}{\gamma} k^{-\gamma},\ k > K^* \end{cases} \tag{5}$$

where $K^* = (\gamma/(\gamma+1))\left(S_1^{\mathrm{C}} - S_0^{\mathrm{C}}\right)$ is the socially optimal investment threshold. Since $S_1^{\mathrm{C}} - S_0^{\mathrm{C}} \geq \Pi^{\mathrm{C}}$, it follows directly that $K^* \geq K^{\mathrm{C}}$ so that innovation occurs later than is socially optimal under both competition and monopoly as may be expected. Similarly it is straightforward to verify that $W^*(k) \geq V^{\mathrm{C}}(k)$, so that both the monopoly and the competitive dynamic innovation incentives fall short of the social optimum.

As stated in the introduction a direct consequence of the replacement effect is that if an explanation of monopoly innovation is to be found, it must lie in some other industry characteristic that allows a monopoly firm to have a greater innovation incentive.[7] Certain limitations of appropriability, namely over time, arise naturally in a dynamic setting and are taken up in the next section.

3 Imperfect appropriability

As Arrow observes an inventor's incentive unravels entirely if there is perfect competition both in the product market and in the market for ideas. However conditions needn't be so extreme that permanent monopoly or perfect competition are the only possible upstream market structures. In a dynamic setting, an inventor typically possesses only a partial degree of (or temporary) monopoly power that it can exercise in the technology market. For example, the protection that

[7] For example if one of the firms is an incumbent in a duopoly, the efficiency effect (Gilbert and Newberry [14]) results in a greater innovation incentive for an incumbent monopolist.

an inventor derives from patenting may be of finite duration, or alternative inventions or imitations may arrive in the market. These mechanisms are examined in turn in this section: Section 3.1.1 describes a simple limiting case that emphasizes the existence of a range of development costs over which a monopolist values an innovation more highly than a competitive firm, and Sections 3.1.2 and 3.2 examine the robustness of this idea under intermediate and more realistic forms of imperfect appropriability.

3.1 Finitely-lived inventor monopoly

This section describes the effect of finitely-lived patents on dynamic innovation incentives, beginning with the extreme but illustrative case in which the competitive firm does not hold an option to delay before turning to the more realistic case of positive but finite patent length.

3.1.1 Spot monopoly

Consider first an inventor who only has instantaneous monopoly power, either because a substitute invention can emerge at the next instant so that firms must race to innovate or because the inventor cannot offer exclusivity to an innovating firm by committing not to subsequently resell the novel idea to a rival.

In this extreme case the inventor still has instantaneous pricing power in the market for ideas and can extract value for the novel idea but a competitive firm acquiring the novel idea must develop it immediately in order to realize any value in the product market. Its innovation incentive is therefore

$$V_m^{\mathrm{C}}(k) = \max\left\{\Pi^{\mathrm{C}} - k, 0\right\} \tag{6}$$

where the m subscript refers to the competitive firm's Marshallian or net present value investment threshold in this case, $K_m^{\mathrm{C}} = \Pi^{\mathrm{C}}$.

Comparing (2) and (6) yields the main intuition developed in the rest of the section: when the current cost of development k is sufficiently high, the value of the innovation option held by a monopoly firm outweighs the replacement effect so that the former has a greater dynamic innovation incentive than a competitive firm. Specifically, there exists a critical development cost threshold $\widehat{K}$ that determines which of the two market structures values innovation more highly.

In the "Schumpeterian" range over which the cost of development at the moment that a novel idea arrives is sufficiently high, innovation is valued more highly by a monopoly firm.[8]

This situation is represented in Figure 1: if a competitive innovator does not have the ability to time its investment optimally, the value of innovation to a monopolist in the product market, who has the possibility to postpone investment, is greater for sufficiently high levels of the development cost. The critical threshold $\widehat{K}$ is determined by the intersection of $V^{\mathrm{M}}(k)$ and $V_m^{\mathrm{C}}(k)$ (dark grey). Since an increase in input price volatility (σ) raises the option value part of $V^{\mathrm{M}}(k)$ while leaving $V_m^{\mathrm{C}}(k)$ unaffected, $\partial \widehat{K} / \partial \sigma < 0$. Greater cost uncertainty in such an industry increases the likelihood that it is monopoly that values innovation more highly.

Proposition 1 *If the inventor has an instantaneous monopoly in the market for ideas, there exists an development cost threshold* $\widehat{K}$ *such that for* $k \in \left(\widehat{K}, \infty\right)$ *a monopoly has a greater incentive to innovate than a competitive firm. Moreover* $\widehat{K}$ *decreases with input price volatility.*

With respect to policy guidelines, competitive innovation is preferable to monopoly if $k < \widehat{K}$, whereas if $k > \widehat{K}$ the greater innovation incentive lies with monopoly. However because the development cost fluctuates stochastically, up until investment has occurred there is a positive probability over any interval of time of moving to a region in which the alternative market structure has a greater innovation incentive, thus confounding any more general policy prescription.

[8] Schumpeter's discussion of investment in new equipment by a monopoly firm emphasizes both the firm's motive to delay investment in the face of uncertainty and the possibility of misinterpreting the firm's decision to wait as being driven by the preservation of existing profits rather than the optimal timing of adoption of new equipment or techniques ([23], p. 98):

> "There is however another element which profoundly affects behavior in this matter and which is being invariably overlooked. This is what might be called the *ex ante* conservation of capital in expectation of further improvement. [...] The real question then is at which link the concern should take action. The answer must be in the nature of a compromise between considerations that rest largely on guesses. But it will as a rule involve some waiting in order to see how the chain behaves. And to the outsider this may well look like trying to stifle improvement in order to to conserve *existing* capital values."

Discussions of the innovation advantages of large firms often cite their ability to diversify risk by holding a portfolio of R&D investments but stop short of identifying any option component that may be an integral part of the value of such a portfolio.

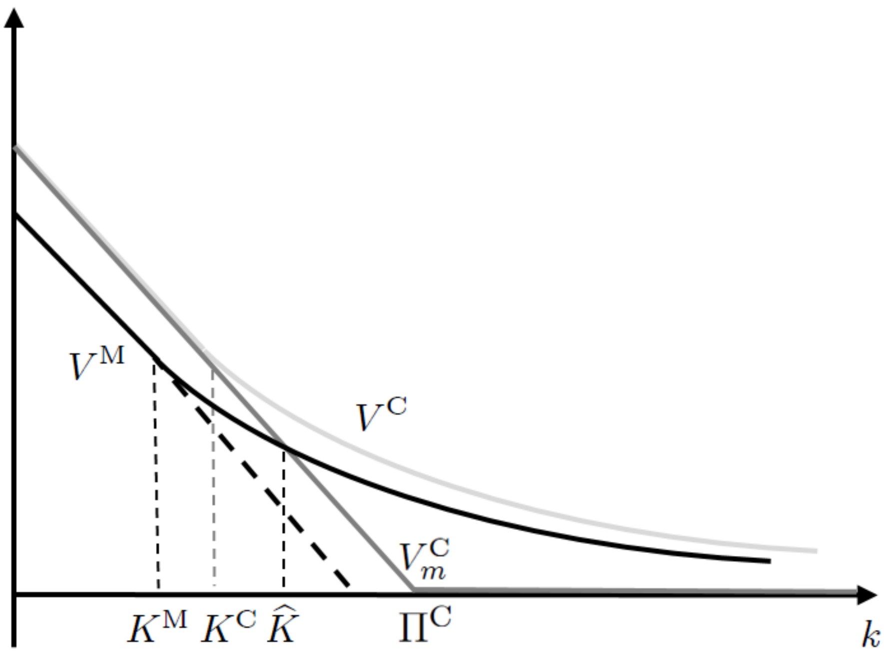

Figure 1: Because of the replacement effect the dynamic innovation incentive ($V^{\mathrm{C}}(k)$, light grey) is globally larger under competition than monopoly ($V^{\mathrm{C}}(k) > V^{\mathrm{M}}(k)$ for all k). However if the inventor has only instantaneous monopoly power, the competitive incentive to innovate ($V_m^{\mathrm{C}}(k)$, dark grey) is relatively smaller over the "Schumpeterian" range ($V_m^{\mathrm{C}}(k) < V^{\mathrm{M}}(k)$ for $k \in \left(\widehat{K}, \infty\right)$). Notably, for intermediate values of the development cost $k \in \left(\widehat{K}, \Pi^{\mathrm{C}}\right)$ the monopoly firm delays developing an innovation that a competitive firm would introduce immediately, but values the innovation more highly.

3.1.2 Patenting and the downstream innovation option

If there is a threat of competition from other inventors at the moment of discovery, an inventor is compelled to immediately patent a novel idea. The inventor then retains monopoly power in the market for ideas for a duration, taken to be of $T > 0$ years, until the patent expires. From the standpoint of the competitive firm that might acquire the novel idea, this means that innovation can be profitably developed only over a limited time range $t \in [0, T)$ since from the moment of patent expiry onward imitative competitor investment occurs at any moment at which the net present value of innovation is positive.

With such a finitely-lived monopoly any royalty that the inventor can extract has the form of an American option of finite maturity. Let $V_T^{\mathrm{C}}(k,t)$ denote its value. The valuation of such an option is a time inhomogeneous problem for which no general analytic formula is known to exist (the conditions that characterize V_T^{C} are given in Section A.2). Nevertheless some general properties of V_T^{C} suffice to establish a similar result regarding dynamic innovation incentives as in the spot monopoly case.

First extending the length of patent protection indefinitely yields the generic competitive dynamic incentive Section 2.2, *i.e.* $\lim_{T\to\infty} V_T^{\mathrm{C}}(k,t) = V^{\mathrm{C}}(k)$. Second, reducing the patent length to zero eliminates any value of waiting so that a competitive firms only perceives the net present value of developing the innovation, *i.e.* $V_0^{\mathrm{C}}(k,0) = V_m^{\mathrm{C}}(k)$ as in Section 3.1. Finally a revealed preference argument establishes that the incentive to innovate is monotonically increasing in the patent length T, *i.e.* $T' \geq T \Rightarrow V_{T'}^{\mathrm{C}}(k,t) \geq V_T^{\mathrm{C}}(k,t)$ with strict inequalities if $t < T$: a firm facing a longer period of patent protection $T' > T$ could simply set the same exercise threshold as a firm facing a shorter period, $K_{T'}(t) = K_T(t)$ for $t \in [0, T)$, in which case it holds the same value over $(0, T)$ with a positive residual value if time $t = T$ is reached and the option has not yet been exercised. Therefore, for any (t, T), $V_m^{\mathrm{C}}(k) \leq V_T^{\mathrm{C}}(k,t) \leq V^{\mathrm{C}}(k)$ (so $V_T^{\mathrm{C}}(k,t)$lies between the two grey curves in Figure 1).

In order to make a meaningful comparison of dynamic innovation incentives, suppose that patent length is sufficient for the static innovation incentive to be greater for a competitive firm ($T \geq (1/r)\ln\left(\Pi^{\mathrm{C}}/\left(\overline{\Pi} - \underline{\Pi}\right)\right)$). Then the comparison of competition and monopoly incentives is similar in nature to Section 3.1. For low enough k (e.g. if $k \leq K^{\mathrm{M}}$ so instantaneous investment at $t = 0$ is optimal for all T) the dynamic innovation incentive is lower at $t = 0$ for monopoly, $V^{\mathrm{M}}(k) < V_T^{\mathrm{C}}(k,0)$, and conversely $V^{\mathrm{M}}(k) > V_T^{\mathrm{C}}(k,0)$ for k large enough. To establish this latter part, let $p_{T,k} := \Pr\left\{\min_{s\in(0,T)} K_s \leq \left(1 - e^{-rT}\right)\Pi^{\mathrm{C}} \middle| K_0 = k\right\}$

denote the probability[9] that at some time before the patent expires the process K_t is in the region $\left(0, \left(1-e^{-rT}\right)\Pi^{\mathrm{C}}\right)$ over which the firm invests immediately. The value of the competitive option is then bounded above by the greatest value that it can expect to attain before time T is reached, so $V_T^{\mathrm{C}}(k,0) \leq p_{T,k}\left(1-e^{-rT}\right)\Pi^{\mathrm{C}}$ where $p_{T,k}$ is decreasing in k with $\lim_{k\to\infty} p_{T,k} = 0$. Therefore, there exists a threshold $\widehat{K}$ such that $V^{\mathrm{M}}(k) > V_T^{\mathrm{C}}(k,t)$ for all $k \geq \widehat{K}$ establishing an analogous result to Proposition 1.

3.2 Competition and imitation

As an alternative to patent protection, pioneering firms are thought to often rely on secrecy and lead times (Hall et al. [15]). Once an idea is developed, the idea itself or at the least its existence is revealed to potential competitors either directly or through a process of reverse engineering which may take more or less time. The onset of imitation by rival firms is then likely to be random from the point of view of the innovator. Moreover, concurrent research by other inventors may lead to the emergence of competing products, constituting another limitation to appropriation that is random from the point of view of the innovator.

For simplicity suppose that competing invention and information leakage regarding the firm's process innovation are represented by a single Poisson process with arrival rate $\lambda > 0$. Once information leaks to a competitor, imitation occurs and the innovator's profit flow is driven back to zero immediately. In the presence of such stochastic imitation, the dynamic innovation incentive of a competitive firm is

$$V_\lambda^{\mathrm{C}}(k) = \sup_{\tau\geq t}\mathbb{E}_t\left[\left(\int_0^\infty \left(1-e^{-rs}\right)\Pi^{\mathrm{C}}\lambda e^{-\lambda s}ds - K_\tau\right)e^{-r\tau}\right] \tag{7}$$

$$= \begin{cases} \frac{r}{r+\lambda}\Pi^{\mathrm{C}} - k, \; k \leq K_\lambda^{\mathrm{C}} \\ \frac{\left[K_\lambda^{\mathrm{C}}\right]^{\gamma'+1}}{\gamma'}k^{-\gamma'}, \; k > K_\lambda^{\mathrm{C}} \end{cases} \tag{8}$$

where

$$\gamma' = \frac{\mu}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} \tag{9}$$

and $K_\lambda^{\mathrm{C}} = (\gamma'/(\gamma'+1))(r/(r+\lambda))\Pi^{\mathrm{C}}$ (see Section A.3).

[9] Adapting the first hitting time distribution for arithmetic Brownian motion gives

$$p_{T,k} = \Pr\left\{\min_{s\in(0,T)} K_s \leq \Pi^{\mathrm{C}} \middle| K_0 = k\right\} = \int_0^T \frac{k-\Pi^{\mathrm{C}}}{\sqrt{2\pi\sigma^2 s^3}}\exp\left[-\frac{\left(k-\Pi^{\mathrm{C}}+\left(\mu-\frac{\sigma^2}{2}\right)s\right)^2}{2\sigma^2 s}\right]ds.$$

Note that the discounting parameter satisfies $\gamma' > \gamma$, with the random arrival of imitation playing in part a somewhat similar role to a jump in the stochastic process for K_t, and $\partial K_\lambda^{\mathrm{C}}/\partial\lambda < 0$. As a result, $K_\lambda^{\mathrm{C}} < K^{\mathrm{C}}$, from which it follows directly that $V_\lambda^{\mathrm{C}}(k) < V^{\mathrm{C}}(k)$. With uncertainty regarding the duration over which the firm perceives a profit flow from innovation, the firm has an option with different characteristics than when there is no imitation, and there exist two arrival rates thresholds, λ^{K} and λ^{Π}, $\lambda^{\mathrm{K}} > \lambda^{\Pi}$ (See section A.4) which are described in Proposition 2 below, which are critical to determining the comparative timing and value of innovation. For $\lambda = 0$, $K_\lambda^{\mathrm{C}} > K^{\mathrm{M}}$ because of the replacement effect and for $\lambda > 0$ it can then be established that:

Proposition 2 *With random imitation at arrival rate λ, a competitive firm develops a novel idea (earlier,later) than a monopoly if and only if $\lambda\,(<,>)\,\lambda^{\mathrm{K}}$. If $\lambda \geq \lambda^{\Pi}$ the dynamic innovation incentive is greater under monopoly for all k, whereas if $\lambda < \lambda^{\Pi}$ there exists a development cost $\widehat{K} \geq 0$ such that the competitive innovation incentive is (greater,lower) if and only if $k\,(<,>)\,\widehat{K}$.*

For $\lambda \geq \lambda^{\Pi}$ the arrival rate of a competitive innovator is sufficiently high to offset the replacement effect entirely and the innovation incentive of a competitive firm is globally lower than that of a monopoly firm. Conversely for $\lambda = 0$ the replacement effect always dominates any option value comparison, as seen in Section 2.2, and the competitive incentive is globally higher. The more interesting case to consider is the intermediate case of a moderate arrival rate $\lambda \in \left(0, \lambda^{\Pi}\right)$, which is represented in Figure 2. In this range of arrival rates either the monopoly's relatively greater option value or the static replacement effect may dominate the comparison of dynamic innovation incentives. Over a subset of this range $(0, \lambda_0)$ the separating threshold $\widehat{K}$ has an analytic expression and over $\left(\lambda_0, \lambda^{\Pi}\right)$, $\widehat{K}$ cannot be solved for directly but it is straightforward to establish that $\partial\widehat{K}/\partial\sigma$ so greater input price risk raises the likelihood that the dynamic innovation incentive is higher for a monopoly.[10]

The resulting economic interpretation runs along the same lines as in Section 3.1.1: if ideas are (moderately) imperfectly excludable then there exists a Schumpeterian range of development costs over which, if an innovation is sufficiently "pioneering" insofar as its current development cost is relatively high so that the moment of optimal development appears remote, then relative option value is more important than the replacement effect and the novel idea is worth more to a monopoly than a competitive firm. Of course, as the development cost fluctuates over time, up until investment occurs there is always a positive probability of moving from one part of the

[10] In contrast with Figure 2, for $\lambda \in \left(0, \lambda^{\Pi}\right)$ at $\widehat{K}$ a monopoly firm delays investment whereas a competitive firm invests immediately. Therefore an increase in σ raises $V^{\mathrm{M}}(k)$ while leaving $V_\lambda^{\mathrm{C}}(k)$ unaffected, shifting the point of intersection ($\widehat{K}$) leftward.

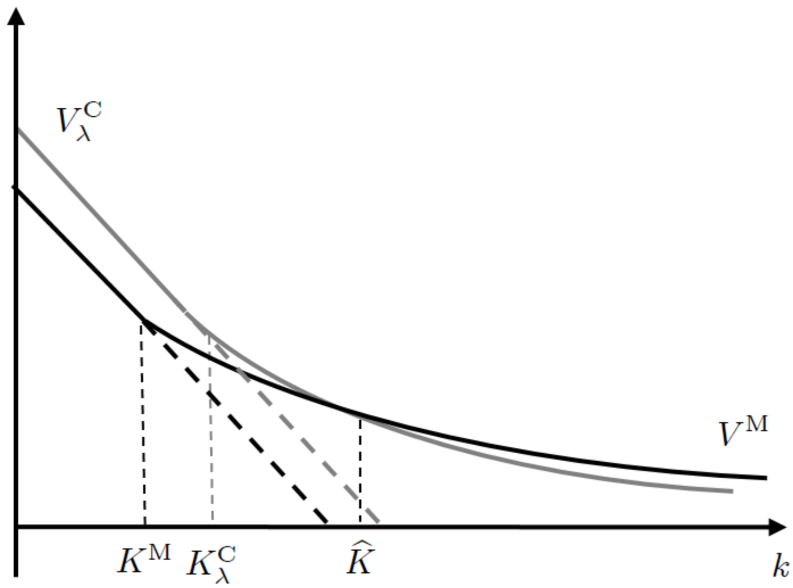

Figure 2: Competitive and monopoly innovation incentives V_λ^{C} and V^{M} with a low imitation arrival rate $\lambda \in (0, \lambda_0) \subset (0, \lambda^{\Pi})$. Over $\left(\widehat{K}, \infty\right)$ option value dominates the replacement effect and it is the monopoly firm that values a novel idea more highly.

range to the other, rendering any general prediction as to which market structure provides greater incentives for invention difficult. Moreover an arbitrarily small level of imperfect excludability is sufficient for such a situation to arise.

3.2.1 Social incentive and welfare

As in Section 2 the monopoly and competitive incentives still fall short of the social incentive, $\max\left\{V_\lambda^{\mathrm{C}}(k), V^{\mathrm{M}}(k)\right\} \leq W^*(k)$. Another welfare question involves the optimal level of leakage, which is determined by the enforcement of secrecy, a less-often studied but nevertheless relevant aspect of intellectual property.

The welfare associated with competitive innovation for a given level of the parameter λ is complex. The direct effect of competitive innovation on consumer surplus is neutral if the innovation is nondrastic but raises consumer surplus by $S_1^{\mathrm{M}} - S_0^{\mathrm{C}}$ if the innovation is drastic. Moreover, imitation when it occurs lowers the price in the product market to the new marginal cost c_1, resulting in a further increase in consumer surplus whose expected discounted value at the time that the firm innovates is $(\lambda/(r+\lambda))\left(S_1^{\mathrm{C}}-S_0^{\mathrm{C}}\right)$ if the innovation is nondrastic and $(\lambda/(r+\lambda))\left(S_1^{\mathrm{C}}-S_1^{\mathrm{M}}\right)$ otherwise. Thus,

$$W_\lambda^{\mathrm{C}}(k)=\begin{cases}\frac{r}{r+\lambda}\Pi^{\mathrm{C}}+\frac{\lambda}{r+\lambda}\left(S_1^{\mathrm{C}}-S_0^{\mathrm{C}}\right)-k,\ k\leq K_\lambda^{\mathrm{C}}\\ \left(\frac{K_\lambda^{\mathrm{C}}}{k}\right)^{\gamma'}\left(\frac{r}{r+\lambda}\Pi^{\mathrm{C}}+\frac{\lambda}{r+\lambda}\left(S_1^{\mathrm{C}}-S_0^{\mathrm{C}}\right)-K_\lambda^{\mathrm{C}}\right),\ k>K_\lambda^{\mathrm{C}}\end{cases}\tag{10}$$

if $P^{\mathrm{M}}(c_1)>c_0$,

$$=\begin{cases}\frac{r}{r+\lambda}\Pi^{\mathrm{C}}+\frac{r}{r+\lambda}S_1^{\mathrm{M}}+\frac{\lambda}{r+\lambda}S_1^{\mathrm{C}}-S_0^{\mathrm{C}}-k,\ k\leq K_\lambda^{\mathrm{C}}\\ \left(\frac{K_\lambda^{\mathrm{C}}}{k}\right)^{\gamma'}\left(\left(S_1^{\mathrm{M}}-S_0^{\mathrm{C}}\right)+\frac{r}{r+\lambda}\Pi^{\mathrm{C}}+\frac{\lambda}{r+\lambda}\left(S_1^{\mathrm{C}}-S_1^{\mathrm{M}}\right)-K_\lambda^{\mathrm{C}}\right),\ k>K_\lambda^{\mathrm{C}}\end{cases}$$

if $P^{\mathrm{M}}(c_1)\leq c_0$.

Assuming an interior solution, an optimal level of informal intellectual property enforcement solves $dW_\lambda^{\mathrm{C}}(k)/d\lambda=\partial\left(\left(\frac{K_\lambda^{\mathrm{C}}}{k}\right)^{\gamma'}\frac{r}{r+\lambda}\Pi^{\mathrm{C}}\right)/\partial\lambda+\partial\left[\left(\frac{K_\lambda^{\mathrm{C}}}{k}\right)^{\gamma'}\frac{\lambda}{r+\lambda}\left(S_1^{\mathrm{C}}-S_0^{\mathrm{C}}\right)\right]/\partial\lambda$ if the innovation is non-drastic and $dW_\lambda^{\mathrm{C}}(k)/d\lambda=\partial\left(\left(\frac{K_\lambda^{\mathrm{C}}}{k}\right)^{\gamma'}\frac{r}{r+\lambda}\Pi^{\mathrm{C}}\right)/\partial\lambda+\partial\left[\left(\frac{K_\lambda^{\mathrm{C}}}{k}\right)^{\gamma'}\left(\frac{r}{r+\lambda}S_1^{\mathrm{M}}+\frac{\lambda}{r+\lambda}S_1^{\mathrm{C}}-S_0^{\mathrm{C}}\right)\right]/\partial\lambda$ if the innovation is drastic. In either case, the sign of different effects is depends on the relative magnitude of option value, which is sensitive to the current level of the state variable k. When k is sufficiently large for instance, the effect on discounting dominates and $dW_\lambda^{\mathrm{C}}(k)/d\lambda<0$ so maximal enforcement is optimal (...)

4 Uncertainty and the size of innovation

Research on real options has begun to focus attention on both the timing and the magnitude of investment. This section complements the analysis of the dynamic innovation incentives of Section 2 by allowing firms to select the level of cost reduction achieved by a process innovation through a simultaneous choice of the timing and scale of investment. In order to outline the effect of innovation size more clearly, perfect appropriability is once again assumed in this section, in contrast with Section 3.

With variable investment size one expects the principle outlined earlier (Section 2.2) to continue to hold, namely that to the extent that a competitive firm has a comparatively greater incentive to innovate at any level of the development input's price and for any given investment size because of the replacement effect, the competitive option to innovate will be relatively more valuable than the monopoly option. Therefore the monopoly option with timing and size choice is presented below for the sake of completeness, but incorporating size does not contribute as much in terms of economic understanding as the subsequent competitive firm case. In the case of a competitive firm, endogenous innovation size is of greater interest because it can be linked to market conditions in a novel way. In the standard investment framework used here, it is when the volatility of the input price or learning in the input market are sufficiently large that a competitive firm is likely to opt for drastic innovation.

In order to study the choice of innovation size, assume that the unit cost of production is constant and equal to $c(x)$, where x denotes the firm's use of research input. Suppose that $c(0) > 0$ and that c is decreasing over the relevant range. The level of development effort of an innovating firm, $x \in \mathbb{R}_+$, is chosen at the moment of investment. The fixed cost of investment is then $K_t x$ where K_t is the unit cost of the development input, and as in the rest of the paper K_t evolves over time according to a geometric Brownian motion with drift μ and volatility σ.

4.1 Innovation size and dynamic innovation incentive: Monopoly case

With endogenous innovation size x, the post-innovation profit flow is a function $\overline{\Pi}(x)$ of the development effort, assumed to be twice differentiable, non-decreasing and concave. In order to solve the firm's investment problem with both timing and size choice, we proceed as in Huisman and Kort [19].[11]

If the firm invests when the input cost is K_t, at the moment of investment it chooses an innovation size x that solves the static problem

$$\max_{x>0} \overline{\Pi}(x) - K_t x. \tag{11}$$

Provided that $K_t < \lim_{x\to 0} \overline{\Pi}'(x)$ an interior optimum $x^{\mathrm{M}}(K_t)$ exists which is the solution of

$$\overline{\Pi}'(x) = K_t, \tag{12}$$

and otherwise $x^{\mathrm{M}}(K_t) = 0$. Provided that $\overline{\Pi}(x)$ is concave, as explained below, the optimal level of innovation size is nondecreasing in the investment threshold K_t.

At the moment investment occurs the value of innovation to a monopoly firm is a C^1 function of K_t, $\overline{\Pi}\left(x^{\mathrm{M}}(K_t)\right) - \overline{\Pi}(0) - K_t x^{\mathrm{M}}(K_t)$. The firm's investment problem is to determine a stopping

[11] An alternative way to understand the optimal investment policy that leads to the same result is to consider a firm making an ex-ante selection among an array of standard investment options of fixed size x, i.e. $\max_x V^{\mathrm{M}}(k;x)$ where $V^{\mathrm{M}}(k;x) = \begin{cases} \overline{\Pi}(x) - \underline{\Pi} - kx, & k \leq \frac{\gamma}{\gamma+1}\frac{\overline{\Pi}(x)-\underline{\Pi}}{x} \\ \frac{\gamma^{\gamma}\left(\overline{\Pi}(x)-\underline{\Pi}\right)^{\gamma+1}}{(\gamma+1)^{\gamma+1}x^{\gamma}k^{\gamma}}, & k > \frac{\gamma}{\gamma+1}\frac{\overline{\Pi}(x)-\underline{\Pi}}{x} \end{cases}$.

time τ to solve

$$\sup_{\tau \geq t} \mathbb{E}_t \left[\overline{\Pi} \left(x^{\mathrm{M}} \left(K_\tau \right) \right) - \overline{\Pi}(0) - K_\tau x^{\mathrm{M}} \left(K_\tau \right) \right]. \tag{13}$$

The problem (13) needn't have a straightforward solution in general. Before proceeding any further, it is necessary to impose some technical restrictions in order to ensure existence and uniqueness of an optimal investment threshold. Following Pennings [20] denote the investment elasticity by $\varepsilon_{\overline{\Pi}-\underline{\Pi}}(x) = x\overline{\Pi}'(x) / \left(\overline{\Pi}(x) - \overline{\Pi}(0)\right)$ and suppose that the following assumptions are satisfied:[12][13]

i) $\lim_{x\to 0} \overline{\Pi}'(x) < \infty$,

ii) $\lim_{x\to\infty} \varepsilon_{\overline{\Pi}-\underline{\Pi}}(x) = 0$,

iii) $d(\varepsilon_{\overline{\Pi}-\underline{\Pi}}(x))/dx < 0$.

Assumption *i)* is sufficient to ensure that $\lim_{x\to 0} \varepsilon_{\overline{\Pi}-\underline{\Pi}}(x) = 1$, which together with *ii)* and *iii)* suffices to guarantee a unique solution to the condition (16) below. A monopoly firm that follows a threshold investment policy obtains the value

$$V_x^{\mathrm{M}}(k) = \begin{cases} \overline{\Pi}\left(x^{\mathrm{M}}(k)\right) - \underline{\Pi} - kx^{\mathrm{M}}(k), \; k \leq K_x^{\mathrm{M}} \\ \frac{\left[K_x^{\mathrm{M}}\right]^{\gamma+1}}{\gamma} k^{-\gamma}, \; k > K_x^{\mathrm{M}} \end{cases} \tag{14}$$

where the optimal stopping threshold is defined implicitly by

$$K_x^{\mathrm{M}} = \frac{\gamma}{\gamma+1} \frac{\overline{\Pi}\left(x^{\mathrm{M}}\left(K_x^{\mathrm{M}}\right)\right) - \underline{\Pi}}{x^{\mathrm{M}}\left(K_x^{\mathrm{M}}\right)}. \tag{15}$$

The optimal investment threshold and innovation size for the monopoly firm jointly solve (12) and (15). Combining these yields a condition that implicitly defines the optimal innovation size,

$$\varepsilon_{\overline{\Pi}-\underline{\Pi}}(x^{\mathrm{M}}) = \frac{\gamma}{\gamma+1}. \tag{16}$$

The conditions imposed for existence and uniqueness ensure that $\varepsilon_{\left(\overline{\Pi}(x)-\underline{\Pi}\right)/x}$ is single-downcrossing at x^{M} and comparative statics with respect to γ and $\underline{\Pi}$ directly follow. For instance, a greater replacement effect is associated, somewhat counterintuitively from the perspective of the innovation literature, with more innovation (in magnitude) rather than less.

[12] Pennings gives $\overline{\Pi}(x) = x/(x+1)$ as an example and the functions $1 - e^{-x}$ and $x(a-x)$ also satisfy these assumptions (for the latter over the relevant range $x \in [0, a/2]$).

[13] If γ is known then an alternative is to substitute for *ii)* and *iii)* by *ii')* $\overline{\Pi}(x) - \overline{\Pi}(0)$ *is strictly* $1+(1/\gamma)-$*concave* (see Section A.5). A function $f : \mathbb{R} \to \mathbb{R}_+$ is $\rho-$concave if, for $\rho > 0$, $[f]^\rho$ is concave (see Caplin and Nalebuff [4]). The greater is ρ the more stringent is this restriction.

Proposition 3 *A monopoly firm earns the value $V_x^{\mathrm{M}}(k)$ by making an innovative investment of size x^{M} at the threshold K_x^{M}. An increase in uncertainty (lower γ) or a greater replacement effect (higher $\underline{\Pi}$) result in a greater innovation size (higher x^{M}) and more delay (lower K_x^{M}).*

4.2 Competitive investment option with size choice

A key difference between the competitive option with variable investment size and the monopoly option is that the type of pricing that occurs in the product market is related to the size of innovation. Provided that the pre-innovation unit cost $c(0)$ is sufficiently high there exists an innovation size, denoted by x^d, beyond which the innovation becomes drastic and the competitive firm prices as would a monopolist ($Q^{\mathrm{M}}(c(x^d)) = D\left(c(0)\right)$). The competitive firm's payoff from innovation therefore has the form

$$\Pi^{\mathrm{C}}(x) = \begin{cases} \left(c(0) - c(x)\right) Q^{\mathrm{C}}(c(0)),\ x < x^d \\ \overline{\Pi}(x)\,,\ x \geq x^d \end{cases} \tag{17}$$

where $Q^{\mathrm{C}}(c(0))$ denotes the competitive output level (which satisfies $Q^{\mathrm{C}} = D\left(c(0)\right)$). Note that $\Pi^{\mathrm{C}}(x)$ is continuous at x^d and furthermore that since $Q^{\mathrm{C}}(c(0)) = Q^{\mathrm{M}}(c(x^d))$, $\lim_{x^{d-}} \left(\Pi^{\mathrm{C}}(x)\right)' = -c'(x^d)Q^{\mathrm{C}}(c(0)) = -c'(x^d)Q^{\mathrm{M}}(c(x^d)) = \lim_{x^{d+}} \left(\Pi^{\mathrm{C}}(x)\right)'$ so $\Pi^{\mathrm{C}}(x)$ is C^1 over its range. A sufficient condition for the investment threshold to be well-defined is if each of the parts of Π^{C} satisfies the assumptions of the previous section.

Proceeding as in the monopoly case, a competitive firm that invests when the input cost is K_t chooses an innovation size x that solves the static problem

$$\max_{x>0} \Pi^{\mathrm{C}}(x) - K_t x. \tag{18}$$

Provided that $K_t < \lim_{x\to 0} -c'(x)\, Q^{\mathrm{C}}$ an interior optimum $x^{\mathrm{C}}(K_t)$ exists which solves

$$\left(\Pi^{\mathrm{C}}\right)'(x) = K_t, \tag{19}$$

and $x^{\mathrm{C}}(K_t) = 0$ otherwise. Letting $K^d = \left(c(0) - c(x^d)\right) Q^{\mathrm{C}}(c(0))$ denote the development cost that induces drastic innovation, with $K^d = 0$ if x^d is undefined, $x^{\mathrm{C}}(K_t)$ is defined implicitly by

$$\begin{cases} -c'(x)Q^{\mathrm{C}}(c(0)) = K_t, \text{ if } K_t > K^d \\ \overline{\Pi}'(x) = K_t, \text{ if } K_t \leq K^d \end{cases}. \tag{20}$$

Because the functions $-c$ and $\overline{\Pi}$ are twice-differentiable and concave $x^{\mathrm{C}}(K_t)$ is C^1 and decreasing over each of its parts. The remainder of the reasoning proceeds as in the monopoly case. The value of the firm at the moment of investment is $\Pi^{\mathrm{C}}\left(x^{\mathrm{C}}\left(K_t\right)\right) - K_t x^{\mathrm{C}}\left(K_t\right)$, and the firm determines a stopping time τ that solves

$$\sup_{\tau \geq t} \mathbb{E}_t\left[\Pi^{\mathrm{C}}\left(x^{\mathrm{C}}\left(K_\tau\right)\right) - K_\tau x^{\mathrm{C}}\left(K_\tau\right)\right]. \tag{21}$$

Following a threshold investment policy results in the value

$$V_x^{\mathrm{C}}(k) = \begin{cases} \Pi^{\mathrm{C}}\left(x^{\mathrm{C}}(k)\right) - k x^{\mathrm{C}}(k), \ k \leq K_x^{\mathrm{C}} \\ \frac{\left[K_x^{\mathrm{C}}\right]^{\gamma+1}}{\gamma} k^{-\gamma}, \ k > K_x^{\mathrm{C}} \end{cases} \tag{22}$$

and the optimal stopping threshold is defined implicitly by

$$K_x^{\mathrm{C}} = \frac{\gamma}{\gamma+1} \frac{\Pi^{\mathrm{C}}\left(x^{\mathrm{C}}\left(K_x^{\mathrm{C}}\right)\right)}{x^{\mathrm{C}}\left(K_x^{\mathrm{C}}\right)}. \tag{23}$$

The optimal investment threshold and innovation size jointly solve (20) and (23), and lead to a similar elasticity condition for optimality

$$\varepsilon_{\Pi^{\mathrm{C}}/x}\left(x^{\mathrm{C}}\right) = \frac{\gamma}{\gamma+1} \tag{24}$$

i.e.

$$-\frac{x^{\mathrm{C}} c'\left(x^{\mathrm{C}}\right)}{c(0) - c\left(x^{\mathrm{C}}\right)} = \frac{\gamma}{\gamma+1} \text{ if } x^{\mathrm{C}} \leq x^d, \ -\frac{x^{\mathrm{C}} c'\left(x^{\mathrm{C}}\right)}{\left(P\left(Q^{\mathrm{M}}(c(x^{\mathrm{C}}))\right) - c\left(x^{\mathrm{C}}\right)\right)} = \frac{\gamma}{\gamma+1} \text{ if } x^{\mathrm{C}} > x^d. \tag{25}$$

It follows directly that:

Proposition 4 *A competitive firm earns the value $V_x^{\mathrm{C}}(k)$ by making an innovative investment of size x^{C} at the threshold K_x^{C}. An increase in uncertainty (lower γ) results in a greater innovation size (higher x^{C}) and more delay (lower K_x^{C}).*

Because the real options framework relates investment to industry characteristics, endogenizing the size of innovation provides an understanding of the type of market conditions that are likely to foster drastic innovation. The greater the rate of learning in the input market (lower μ) or the level of uncertainty (higher σ), the more likely it is that a competitive firm opts for a drastic innovation.

Comparing dynamic innovation incentives for monopoly and competitive firms, the main intuition of Section 2 still holds with innovation size choice. The replacement effect implies that for a given x, $K_x^{\mathrm{C}}(x) \geq K_x^{\mathrm{M}}(x)$ whereas (20) does not lie below (12). Therefore, $x^{\mathrm{C}} < x^{\mathrm{M}}$ even if

$K_x^{\mathrm{C}} \geq K_x^{\mathrm{M}}$ and $V_x^{\mathrm{C}}(k) \geq V_x^{\mathrm{M}}(k)$. Similar conclusions hold regarding the social incentive to innovate, defined with respect to $S_x^{\mathrm{C}} = (1/r)\int_{c(x)}^{\infty} D(s)ds$. If the socially optimal innovation level also satisfies an elasticity condition $\varepsilon_{\left(S_x^{\mathrm{C}} - S_0^{\mathrm{C}}\right)}(x^*) = \gamma/(\gamma+1)$ then one concludes that competitive or monopoly firms innovate excessively ($x^{\mathrm{C}}, x^{\mathrm{M}} \geq x^*$), and in particular that in an intermediate range of the discounting term γ, competitive firms may opt for drastic innovation when it would be socially preferable not to do so.

4.3 Inventor size choice

-Suppose inventor makes costly choice of x and bargains with monopoly or competitive firm.

-no underinvestment and timing issues in competitive case

-possible investment in monopoly case, there is no cost of delay if k is high, if k is low cost of delaying agreement for monopolist is reduced by replacement effect ($\overline{\Pi} - \underline{\Pi}$) ; may have underinvestment issues

5 Conclusion

In a stochastic dynamic framework not just the replacement effect but also the current level of prices and the amount of uncertainty are key determinants of the relative competitive and monopoly innovation incentives. Developing these incentives in a standard model of investment establishes that even a slight imperfection in the degree of exclusivity afforded to a novel idea is sufficient for monopolies to value pioneering projects in which a novel idea arrives at a moment when the development cost is currently prohibitively high relatively more highly than will a firm facing product market competition, casting doubt on the value of a policy prescription to favor innovation by encouraging competition in industries where uncertainty and option value are likely to be important.

Expanding the analysis to allow firms to choose the size of innovation highlights the role of option value in explaining the emergence of drastic innovations, which are more likely to be chosen by a competitive innovator when uncertainty (volatility) is high. Integrating option value in this way enriches understanding of the dynamics of innovation, drawing a picture in which monopolistic firms are more closely associated with speculative, larger and long-term projects whereas the prevalence of drastic innovations in competitive markets may be closely related to the importance of uncertainty.

An extension that is not pursued here would be to incorporate uncertainty regarding invention as well, so that the moment at which invention arrives (and therefore the initial value of the monopoly or competitive innovation options) is unknown ex-ante.

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A Proofs and derivations

A.1 Benchmark option

The monopoly firm solves the optimal stopping problem

$$V^{\mathrm{M}}\left(K_t\right) = \sup_{\tau \geq t} \mathbb{E}_t \left[\left(\overline{\Pi} - \underline{\Pi} - K_\tau \right) e^{-r\tau} \right].$$

Letting $k = K_t$, $V^{\mathrm{M}}(k)$ in (1) is the expected value of a firm that invests when the current state of the process is k. Following standard arguments (see Dixit and Pindyck [8]) this function satisfies

$$rV^{\mathrm{M}}(k)\, dt = \mathbb{E}_t dV^{\mathrm{M}}\left(K_t\right). \tag{26}$$

Expanding the right-hand side using Itô's lemma yields the ordinary differential equation

$$rV^{\mathrm{M}} = \mu k \frac{dV^{\mathrm{M}}}{dk} + \frac{1}{2}\sigma^2 k^2 \frac{d^2 V^{\mathrm{M}}}{dk^2} \tag{27}$$

that V^{M} solves along with the boundary and smooth pasting conditions

$$\begin{aligned} V^{\mathrm{M}}\left(K^{\mathrm{M}}\right) &= \overline{\Pi} - \underline{\Pi} - K^{\mathrm{M}} \\ \lim_{k \to \infty} V^{\mathrm{M}}(k) &= 0 \\ \frac{dV^{\mathrm{M}}}{dk}\left(K^{\mathrm{M}}\right) &= -1. \end{aligned}$$

A solution to (27) has the form $V^{\mathrm{M}}(k) = A_1 k^{\beta_1} + A_2 k^{\beta_2}$. The associated fundamental quadratic is $0.5\sigma^2\beta(\beta - 1) + \beta\mu - r = 0$, which has two roots of which only

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}$$

is negative and satisfies the second boundary condition. It follows from the other conditions that

$$K^{\mathrm{M}} = \frac{-\beta}{1-\beta}\left(\overline{\Pi} - \underline{\Pi}\right)$$

and

$$A^{\mathrm{M}} = -\frac{1}{\beta}\left[K^{\mathrm{M}}\right]^{1-\beta}.$$

Setting $\gamma \equiv -\beta$ (so $\gamma > 0$) yields the expressions in the text.

A.2 Innovation option with finite patent length

Let $V_T^{\mathrm{C}}(k,t)$ denote the value of the novel idea to a competitive firm at time t with current development cost k and given the patent duration T. The optimal investment threshold, $K_T(t)$, is then time-dependent and the dynamic innovation incentive satisfies

$$rV_T^{\mathrm{C}} = \frac{\partial V_T^{\mathrm{C}}}{\partial t} + \mu K \frac{\partial V_T^{\mathrm{C}}}{\partial K} + \frac{1}{2}\sigma^2 K^2 \frac{\partial^2 V_T^{\mathrm{C}}}{\partial K^2} \tag{28}$$

over $t \in (0,T)$. The continuation region is $k > K_T(t)$ and $K_T(t)$ and $V_T^{\mathrm{C}}(k,t)$ satisfy the boundary and smooth-pasting conditions

$$\begin{aligned} &\lim_{k\to\infty} V_T^{\mathrm{C}}(k,t) = 0 \\ &V_T^{\mathrm{C}}(K_T(t),t) = \Pi^{\mathrm{C}}\left(1 - e^{-r(T-t)}\right) - K_T(t) \\ &\tfrac{\partial V_T^{\mathrm{C}}}{\partial K}(K_T(t),t) = -1 \\ &V_T^{\mathrm{C}}(K_T(T),T) = 0 \end{aligned} \tag{29}$$

along with $\lim_{t\to T} K_T(t) = \Pi^{\mathrm{C}}$.

A.3 Option with random imitation arrival

A probability λ per unit of time that a rival firm successfully imitates the firm's innovation (in which case all product market rent is competed away) corresponds to adding a jump to the stochastic process (Dixit and Pindyck [8]). The differential equation (27) then has the form

$$rV_\lambda^{\mathrm{C}} = \mu k \frac{dV_\lambda^{\mathrm{C}}}{dk} + \frac{1}{2}\sigma^2 k^2 \frac{d^2 V_\lambda^{\mathrm{C}}}{dk^2} - \lambda V_\lambda^{\mathrm{C}} \tag{30}$$

so the fundamental quadratic becomes $0.5\sigma^2\beta(\beta-1) + \beta\mu - (r+\lambda) = 0$. Proceeding as above yields the expression for γ' in the text.

To establish that $\partial K_\lambda^{\mathrm{C}}/\partial\lambda < 0$ and $K_\lambda^{\mathrm{C}} < K^{\mathrm{C}}$ as claimed in the text, it is necessary to verify that

$$f(\lambda) := \frac{\gamma'}{\gamma'+1}\frac{r}{r+\lambda} < \frac{\gamma}{\gamma+1} \tag{31}$$

for $\lambda > 0$. Note that $f(0) = 1$ and

$$f'(\lambda) = \frac{r\left(\frac{\partial\gamma'}{\partial\lambda}(r+\lambda) - \gamma'(\gamma'+1)\right)}{(\gamma'+1)^2(r+\lambda)^2} \tag{32}$$

$$= \frac{r\left(\frac{r+\lambda}{\sigma^2}\left(\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}\right)^{-0.5} - \gamma'(\gamma'+1)\right)}{(\gamma'+1)^2(r+\lambda)^2} \tag{33}$$

$$< \frac{r\left(\sqrt{\frac{r+\lambda}{2\sigma^2}} - \gamma'(\gamma'+1)\right)}{(\gamma'+1)^2(r+\lambda)^2} < 0. \tag{34}$$

Alternatively it can be assumed that competing innventions arrive at a rate $\lambda_0 > 0$ (an form of *ex-ante* threat to appropriability) and that information leaks leading to imitation at a rate λ_1 once innovation has occured. In that case, the innovation incentives becomes

$$V^{\mathrm{C}}_{\lambda_0}(k) = \begin{cases} \frac{r}{r+\lambda_1}\Pi^{\mathrm{C}} - k, \; k \le K^{\mathrm{C}}_{\lambda_0} \\ \frac{\left[K^{\mathrm{C}}_{\lambda}\right]^{\gamma'_0+1}}{\gamma'_0} k^{-\gamma'_0}, \; k > K^{\mathrm{C}}_{\lambda_0} \end{cases} \tag{35}$$

where

$$\gamma'_0 = \frac{\mu}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\mu}{\sigma^2}-\frac{1}{2}\right)^2 + \frac{2(r+\lambda_0)}{\sigma^2}} \text{ and } K^{\mathrm{C}}_{\lambda_0} = \frac{\gamma'_0}{\gamma'_0+1}\frac{r}{r+\lambda_1}\Pi^{\mathrm{C}} \tag{36}$$

and retains the asymptotic properties of V^{C}_{λ}.

A.4 Comparison of dynamic innovation incentives with random lead time

The proposition is proved by studying the piecewise-defined functions (2) and (8) for different values of the parameter λ.

Definition of λ^{K}

λ^{K} is defined implicitly by the condition $K^{\mathrm{C}}_{\lambda} = K^{\mathrm{M}}$. This condition can be expressed as

$$f_K(\lambda) := \frac{\gamma'}{\gamma'+1} = \frac{\gamma}{\gamma+1}\frac{\overline{\Pi} - \underline{\Pi}}{\Pi^{\mathrm{C}}}\left(1+\frac{\lambda}{r}\right) =: g_K(\lambda). \tag{37}$$

Here $f_K(0) = \frac{\gamma}{\gamma+1} > g_K(0)$. Both f_K and g_K are increasing, but f_K is concave:

$$f''_K(\lambda) = -\frac{1}{(\gamma'+1)^2}\left(\frac{\partial\gamma'}{\partial\lambda}\right)^2 + \frac{1}{\gamma'+1}\frac{\partial^2\gamma'}{\partial\lambda^2} < 0, \tag{38}$$

whereas g_K is linear. Therefore there exists a unique intersection in $(0, \infty)$, λ^{K}, establishing the first part of the proposition.

Definition of λ^{Π}

Suppose that $k \leq \min\left\{K_\lambda^{\mathrm{C}}, K^{\mathrm{M}}\right\}$. Then $V_\lambda^{\mathrm{C}}(k)\,(<,>)\,V^{\mathrm{M}}(k)$ if and only if

$$\frac{r}{r+\lambda}\Pi^{\mathrm{C}}\,(<,>)\,\overline{\Pi} - \underline{\Pi}. \tag{39}$$

Let $\lambda^{\Pi} = r\left(\left(\Pi^{\mathrm{C}}/\left(\overline{\Pi} - \underline{\Pi}\right)\right) - 1\right) > 0$ denote the arrival rate that equates the competitive and monopoly payoffs from instantaneous investment. Since $f_K\left(\lambda^{\Pi}\right) > g_K\left(\lambda^{\Pi}\right)$, it follows that $\lambda^{\Pi} < \lambda^{\mathrm{K}}$.

Comparison of innovation incentives for $\lambda > \lambda^{\Pi}$

Suppose that $\lambda > \lambda^{\Pi}$, so $\overline{\Pi} - \underline{\Pi} > (r/(r+\lambda))\,\Pi^{\mathrm{C}}$. There are two subcases to consider.

First, if $\lambda \in \left(\lambda^{\Pi}, \lambda^{\mathrm{K}}\right)$, then $K_\lambda^{\mathrm{C}} > K^{\mathrm{M}}$. For $k \in \left[0, K^{\mathrm{M}}\right]$, both firms invest immediately and $V_\lambda^{\mathrm{C}}(k) < V^{\mathrm{M}}(k)$ (by definition of λ^{Π}). For $k \in \left(K^{\mathrm{M}}, K_\lambda^{\mathrm{C}}\right)$, the monopoly firm delays investment whereas the competitive firm does not. Over this range $d\left(V^{\mathrm{M}} - V_\lambda^{\mathrm{C}}\right)/dk = -\left(K^{\mathrm{M}}/k\right)^{\gamma'+1} + 1 > 0$. The difference $V^{\mathrm{M}} - V_\lambda^{\mathrm{C}}$ is thus bounded above by $V^{\mathrm{M}}\left(K_\lambda^{\mathrm{C}}\right) - V_\lambda^{\mathrm{C}}\left(K_\lambda^{\mathrm{C}}\right) = \overline{\Pi} - \underline{\Pi} - (r/(r+\lambda))\,\Pi^{\mathrm{C}} > 0$. Finally for $k > K_\lambda^{\mathrm{C}}$, both firms delay investment and the ratio $V^{\mathrm{M}}/V_\lambda^{\mathrm{C}} = (\gamma'/\gamma)\left(K^{\mathrm{M}}\right)^{\gamma+1}\left(K_\lambda^{\mathrm{C}}\right)^{-\gamma'-1}k^{\gamma'-\gamma}$ increases in k. Therefore, for $\lambda \in \left(\lambda^{\Pi}, \lambda^{\mathrm{K}}\right)$, $V_\lambda^{\mathrm{C}}(k) < V^{\mathrm{M}}(k)$ for all k.

Second, if $\lambda \in \left[\lambda^{\mathrm{K}}, \infty\right)$, then $K_\lambda^{\mathrm{C}} \leq K^{\mathrm{M}}$. For $k \in \left[0, K^{\mathrm{C}}\right]$, both firms invest immediately and $V_\lambda^{\mathrm{C}}(k) < V^{\mathrm{M}}(k)$. If $\lambda > \lambda^{\mathrm{K}}$, then the interval $\left(K_\lambda^{\mathrm{C}}, K^{\mathrm{M}}\right)$ is non-empty and for $k \in \left(K_\lambda^{\mathrm{C}}, K^{\mathrm{M}}\right)$ the monopoly firm invests immediately whereas the competitive firm does not, and $d\left(V^{\mathrm{M}} - V_\lambda^{\mathrm{C}}\right)/dk = -\left(K^{\mathrm{M}}/k\right)^{\gamma'+1} + 1 < 0$. Over this range $V^{\mathrm{M}} - V_\lambda^{\mathrm{C}}$ is therefore bounded below by $V^{\mathrm{M}}\left(K^{\mathrm{M}}\right) - V_\lambda^{\mathrm{C}}\left(K^{\mathrm{M}}\right)$, and $V^{\mathrm{M}}\left(K^{\mathrm{M}}\right)/V_\lambda^{\mathrm{C}}\left(K^{\mathrm{M}}\right) = (\gamma'/\gamma)\left(K^{\mathrm{M}}/K_\lambda^{\mathrm{C}}\right)^{\gamma'+1} > 1$. Finally, for $k > K^{\mathrm{M}}$ both firms delay investment and $V^{\mathrm{M}}/V_\lambda^{\mathrm{C}}$ increases in k as in the previous subcase. Therefore, for $\lambda \in \left[\lambda^{\mathrm{K}}, \infty\right)$, $V_\lambda^{\mathrm{C}}(k) < V^{\mathrm{M}}(k)$ for all k.

To sum up, for $\lambda > \lambda^{\Pi}$, $V_\lambda^{\mathrm{C}}(k) < V^{\mathrm{M}}(k)$ for all k, and therefore in this range $\widehat{K} = 0$.

Comparison of innovation incentives for $\lambda \leq \lambda^{\Pi}$

If $\lambda < \lambda^{\Pi}$, then $K_\lambda^{\mathrm{C}} > K^{\mathrm{M}}$. For $k \in \left[0, K^{\mathrm{M}}\right]$ both firms invest immediately, and $V_\lambda^{\mathrm{C}}(k) > V^{\mathrm{M}}(k)$ over this range. Any intersection must therefore occur either over $\left(K^{\mathrm{M}}, K_\lambda^{\mathrm{C}}\right)$ or over $\left[K_\lambda^{\mathrm{C}}, \infty\right)$. To determine which of these occurs it is necessary to study the value of $V_\lambda^{\mathrm{C}}/V^{\mathrm{M}}$ at K_λ^{C}, where $V_\lambda^{\mathrm{C}}\left(K_\lambda^{\mathrm{C}}\right)/V^{\mathrm{M}}\left(K_\lambda^{\mathrm{C}}\right) = (\gamma/\gamma')\left(K_\lambda^{\mathrm{C}}/K^{\mathrm{M}}\right)^{\gamma+1}$. Note first that $\lim_{\lambda\to 0}\left(V_\lambda^{\mathrm{C}}\left(K_\lambda^{\mathrm{C}}\right)/V^{\mathrm{M}}\left(K_\lambda^{\mathrm{C}}\right)\right) =$

$\left(\Pi^{\mathrm{C}}/\left(\overline{\Pi}-\underline{\Pi}\right)\right)^{\gamma+1}>1$. Next,

$$\frac{\partial K_{\lambda}^{\mathrm{C}}}{\partial\lambda}=K_{\lambda}^{\mathrm{C}}\left(\frac{1}{\gamma'\left(\gamma'+1\right)}\frac{\partial\gamma'}{\partial\lambda}-\frac{1}{r+\lambda}\right), \tag{40}$$

so that after evaluating and simplifying

$$\frac{\partial\left(V_{\lambda}^{\mathrm{C}}\left(K_{\lambda}^{\mathrm{C}}\right)/V^{\mathrm{M}}\left(K_{\lambda}^{\mathrm{C}}\right)\right)}{\partial\lambda}=\frac{\gamma}{\gamma'}\left(\frac{K_{\lambda}^{\mathrm{C}}}{K^{\mathrm{M}}}\right)^{\gamma+1}\left(\frac{\gamma-\gamma'}{\gamma'\left(\gamma'+1\right)}\frac{\partial\gamma'}{\partial\lambda}-\frac{\gamma+1}{r+\lambda}\right)<0. \tag{41}$$

Finally $\lim_{\lambda\to\lambda^{\Pi}}\left(V_{\lambda}^{\mathrm{C}}\left(K_{\lambda}^{\mathrm{C}}\right)/V^{\mathrm{M}}\left(K_{\lambda}^{\mathrm{C}}\right)\right)=\left((\gamma+1)^{\gamma+1}/\gamma^{\gamma}\right)\left((\gamma')^{\gamma}/(\gamma'+1)^{\gamma+1}\right)$ and it is straightforward to establish that this latter term is smaller than 1 for any $\gamma'>\gamma$. Therefore, there exists a unique $\lambda_0\in\left(0,\lambda^{\Pi}\right)$ such that $V_{\lambda_0}^{\mathrm{C}}\left(K_{\lambda_0}^{\mathrm{C}}\right)/V^{\mathrm{M}}\left(K_{\lambda_0}^{\mathrm{C}}\right)=1$. There are thus two cases to consider.

First, if $\lambda>\lambda_0$ so $V_{\lambda}^{\mathrm{C}}\left(K_{\lambda}^{\mathrm{C}}\right)<V^{\mathrm{M}}\left(K_{\lambda}^{\mathrm{C}}\right)$, then $V_{\lambda}^{\mathrm{C}}(k)$ intersects $V^{\mathrm{M}}(k)$ in $\left(K^{\mathrm{M}},K_{\lambda}^{\mathrm{C}}\right)$, i.e. in a region in which a monopoly firm delays investment whereas a competitive firm invests immediately. In this case $\widehat{K}$ is defined as the root in $k\in\left(K^{\mathrm{M}},K_{\lambda}^{\mathrm{C}}\right)$ of

$$\frac{r}{r+\lambda}\Pi^{\mathrm{C}}-k=\frac{\left[K^{\mathrm{M}}\right]^{\gamma+1}}{\gamma}k^{-\gamma}. \tag{42}$$

Second, if $\lambda\leq\lambda_0$ so $V_{\lambda}^{\mathrm{C}}\left(K_{\lambda}^{\mathrm{C}}\right)\geq V^{\mathrm{M}}\left(K_{\lambda}^{\mathrm{C}}\right)$, then $V_{\lambda}^{\mathrm{C}}(k)$ intersects $V^{\mathrm{M}}(k)$ in $\left[K_{\lambda}^{\mathrm{C}},\infty\right)$ and both firms delay investment. In this case solving $V_{\lambda}^{\mathrm{C}}(k)=V^{\mathrm{M}}(k)$ directly yields

$$\widehat{K}=\left(\frac{\gamma}{\gamma'}\frac{\left[K_{\lambda}^{\mathrm{C}}\right]^{\gamma'+1}}{\left[K^{\mathrm{M}}\right]^{\gamma+1}}\right)^{\frac{1}{\gamma'-\gamma}}. \tag{43}$$

For $\lambda=\lambda^{\Pi}$, by continuity (42) determines the unique intersection to be at $\widehat{K}=K^{\mathrm{M}}$.

A.5 Innovation option with timing and size choice

A.5.1 Monopoly case

Pennings [20] provides sufficient conditions for an investment option with size and timing choice with a general payoff specification to be well-defined. The argument here runs along similar lines to his except that Assumption *ii.b*) requires the investment elasticity to cross $\gamma/(1+\gamma)$ once from above rather than to be globally decreasing.

In order to find V_x^{M} and the optimal investment threshold and size, applying Itô's lemma to $rV_x^{\mathrm{M}}(k)\,dt=\mathbb{E}_t dV_x^{\mathrm{M}}(K_t)$ yields

$$rV_x^{\mathrm{M}}=\mu k\frac{dV_x^{\mathrm{M}}}{dk}+\frac{1}{2}\sigma^2k^2\frac{d^2V_x^{\mathrm{M}}}{dk^2}. \tag{44}$$

The boundary and smooth-pasting conditions are

$$\begin{aligned} V_x^{\mathrm{M}}\left(K_x^{\mathrm{M}}\right) &= \eta\left(1-\eta\right)^{\frac{1-\eta}{\eta}}\left(\left[c\left(x^{\mathrm{M}}\left(K_x^{\mathrm{M}}\right)\right)\right]^{-\frac{1-\eta}{\eta}}-\left[c\left(0\right)\right]^{-\frac{1-\eta}{\eta}}\right)-K_x^{\mathrm{M}}x^{\mathrm{M}}\left(K_x^{\mathrm{M}}\right) \\ \lim_{k\to\infty} V_x^{\mathrm{M}}\left(k\right) &= 0 \\ \frac{dV_x^{\mathrm{M}}}{dk}\left(K_x^{\mathrm{M}}\right) &= -x^{\mathrm{M}}\left(K_x^{\mathrm{M}}\right) \end{aligned} \tag{45}$$

where the last (smooth pasting) condition results from the optimality of x^{M}, so that by the envelope theorem $\left(dV_x^{\mathrm{M}}/dk\right)\left(K_x^{\mathrm{M}}\right)=\left(\partial V_x^{\mathrm{M}}/\partial k\right)\left(K_x^{\mathrm{M}}\right)$. Solving these conditions for the posited form $V_x^{\mathrm{M}}\left(k\right)=Ak^{\beta}$ yields the value function and the candidate stopping threshold K_x^{M} given in the text. Uniqueness of K_x^{M} (and x^{M}) results from Assumption 1 in the text as follows.

Let

$$f\left(x\right):=\overline{\Pi}'(x) \tag{46}$$

and

$$g(x):=\frac{\gamma}{\gamma+1}\frac{\overline{\Pi}\left(x\right)-\underline{\Pi}}{x}. \tag{47}$$

The profit-maximizing innovation size if the firm invests at an optimal threshold, x^{M}, solves

$$f(x)=g(x). \tag{48}$$

To establish the existence of a solution to (48) proceed as follows. Because f and g are continuous a set of sufficient conditions is $\lim_{x\to 0}(f/g)>1$ and $\lim_{x\to\infty}(f/g)<1$.

For the limit at zero, $\lim_{x\to 0}(f/g)=(1+1/\gamma)\lim_{x\to 0}\left(x\overline{\Pi}'\left(x\right)/\overline{\Pi}\left(x\right)\right)$. Because $\overline{\Pi}$ is strictly concave, $\overline{\Pi}\left(x\right)<\overline{\Pi}\left(0\right)+x\overline{\Pi}'\left(0\right)=x\overline{\Pi}'\left(0\right)<\infty$. Therefore $\lim_{x\to 0}(f/g)<(1+1/\gamma)\lim_{x\to 0}\left(x\overline{\Pi}'\left(x\right)/x\overline{\Pi}'\left(0\right)\right)=(1+1/\gamma)\left(\lim_{x\to 0}\overline{\Pi}'\left(x\right)\right)/\overline{\Pi}'\left(0\right)=1+1/\gamma$.

The limit at infinity is obtained using l'Hôpital's rule

$$\lim_{x\to\infty}\frac{f}{g}=\lim_{x\to\infty}\frac{\gamma+1}{\gamma}\frac{x}{\frac{\overline{\Pi}(x)-\underline{\Pi}}{\overline{\Pi}'(x)}}=\frac{\gamma+1}{\gamma}\lim_{x\to\infty}\frac{1}{1-\frac{\left(\overline{\Pi}(x)-\underline{\Pi}\right)\overline{\Pi}''(x)}{\left[\overline{\Pi}'(x)\right]^2}}<1 \tag{49}$$

where the final inequality follows from strict $1+(1/\gamma)-$concavity of $\overline{\Pi}\left(x\right)-\underline{\Pi}$, which implies $1-\left(\left(\overline{\Pi}-\underline{\Pi}\right)\overline{\Pi}''/\left[\overline{\Pi}'\right]^2\right)>1+(1/\gamma)$. Therefore a solution to (48) exists.

The solution x^{M} is unique if $f-g$ is strictly single downcrossing, that is if $f(x^{\mathrm{M}})=g(x^{\mathrm{M}})\Rightarrow f'(x^{\mathrm{M}})<g'(x^{\mathrm{M}})$. Taking the condition $f'(x)<g'(x)$,

$$\overline{\Pi}''(x)<\frac{\gamma}{\gamma+1}\frac{x\overline{\Pi}'(x)-\left(\overline{\Pi}\left(x\right)-\underline{\Pi}\right)}{x^2}, \tag{50}$$

and evaluating at $x^{\text{M}} = (\gamma/(\gamma+1)) \left(\left(\overline{\Pi}\left(x^{\text{M}}\right) - \underline{\Pi} \right) / \overline{\Pi}'(x^{\text{M}}) \right)$ yields after simplification

$$\overline{\Pi}''(x^{\text{M}}) < -\frac{1}{\gamma} \frac{\left[\overline{\Pi}'(x^{\text{M}})\right]^2}{\overline{\Pi}\left(x^{\text{M}}\right) - \underline{\Pi}}, \tag{51}$$

which also follows from strict $1 + (1/\gamma)-$concavity of $\overline{\Pi}(x) - \underline{\Pi}$.

A.5.2 Competitive case

The competitive case proceeds like the monopoly case with the key difference that the benefit from innovation $\Pi^{\text{C}}(x)$ is defined by parts so existence and uniqueness of an optimal investment threshold x^{C} must be verified. Since each part is $1 + (1/\gamma)-$concave, independently applying the arguments of Section A.5.1 establishes that there exist a unique x_a and x_b such that

$$-\frac{x_a c'(x_a)}{c(0) - c(x_a)} = \frac{\gamma}{\gamma+1} \tag{52}$$

and

$$\frac{x_b \overline{\Pi}'(x_b)}{\overline{\Pi}(x_b)} = \frac{\gamma}{\gamma+1}. \tag{53}$$

At the same time, by definition at x^d,

$$\frac{-x^d c'\left(x^d\right)}{\left(c(0) - c\left(x^d\right)\right) Q^{\text{C}}(c(0))} = \frac{x^d \overline{\Pi}'\left(x^d\right)}{\overline{\Pi}'\left(x^d\right)}.$$

Since the left-hand sides of (52) and (53) are both single-crossing, either $x^{\text{C}} = x_a$ (if $-x^d c'\left(x^d\right) / \left(\left(c(0) - c\left(x^d\right)\right) Q^{\text{C}}\right.$ $\gamma/(\gamma+1)$) or $x^{\text{C}} = x_b$ (if $-x^d \overline{\Pi}'\left(x^d\right) / \overline{\Pi}'\left(x^d\right) > \gamma/(\gamma+1)$), and $x^{\text{C}} = x^d$ otherwise.