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The Impact of Technological Uncertainty on Project Scale

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Abstract

The interaction between uncertainty and managerial discretion is a crucial relationship in a firm's investment decision. Particularly, as a disruptive technology can precipitate the failure of a leading firm, a project under technological uncertainty can largely benefits from an investment strategy where the potential effects of a disruptive technology can be weighed in an incumbent technology project's valuation. Hence, in this thesis, a price-taking firm that has managerial discretion over both investment timing and the size of a project under price and technological uncertainty is considered. By constructing an analytical framework, it is shown that in comparison to solely price uncertainty, a project under low price and technological uncertainty will have both a lower optimal investment threshold and corresponding optimal capacity, whereas, under conditions of high price and technological uncertainty, a project will have a higher optimal investment threshold and corresponding optimal capacity. Additionally, directly revoking standard real options intuition, it is established through numerical results that the firm's optimal investment policy will be monotonically decreasing as a function of technological uncertainty.

Keywords: real options, capacity sizing, investment analysis, regime-switching

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Dedication

I would like to take this opportunity to formerly recognize the most kind and caring person I have ever had both the pleasure and honor of meeting in my entire life. This has been a person who has given up over two decades of her life to raise me, and also, has never in the entire history of our relation, ever asked me for anything in return. Hence, it is with great honor that I am able to dedicate my first, major milestone to my mother Toorandokht Binesh Oshikoji. I am very much appreciative of everything that you have done for me up until this point, and I carry the guidance, values, and morals that you instilled in me from a young age in everything I do. By adapting the sonnet *Amoretti LXXV: One Day I Wrote Her Name upon the Strand* by Edmund Spenser, I would like to exemplify to what extent my gratitude goes:

"My verse your vertues rare shall eternize,

And in the heavens write your glorious name:

Where whenas death shall all the world subdue,

Your name shall live, and later life renew."

Thank you so much.

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1. Introduction

Innovative, dynamic strategies for investments made under the blanket of technological uncertainty have become increasingly important in recent years with the rise of disruptive technology. As such technologies are conventionally preliminarily purchased by the lowest segment of the market as unproved, unpolished products, oftentimes their sale is associated with a lower price level and, consequently, a lower, expected revenue stream. In response, incumbent technology firms in the industry are often complacent to their inferior competitor's market position. However, in successful cases, where successive refinements have improved a technology to the extent that it becomes possible to take a significant portion of market share, a disruptive technology can reshape and revolutionize an entire industry. Recent examples of such supplantations can be referenced through a widespread number of cases. For example, classified ads have been replaced by Craigslist; long distance phone calls are now made with Skype; record stores are going out of business due to iTunes; research libraries are now at the consumer's fingertips with Google; Uber is redefining the entire taxi industry's business model; and even the most serious of news stations use Twitter (The Economist, 2015).

Hence, faced with the widespread effects of disruptive technologies, a growing number of incumbent firms must weigh the difficult choice between holding onto an existing market by following a repetitive business strategy and risking market share, or by aiming to capture new markets through embracing disruptive technologies and their risky adoption. Coined the innovator's dilemma by Clayton Christensen, the creator of the theory of disruptive technology (Christensen, 2000), this thesis aims to confront the investment problem faced by the established firm in order to properly value projects in rapidly changing industries by examining both investment timing and managerial discretion over project scale. For capitally intensive projects, discretion over project capacity is particularly crucial, since the installation of a large project increases a firm's exposure to downside risk in the case of a potential downturn in market settings, whereas the installation of a small project limits a firm's upside potential if market conditions were suddenly to become favourable (Chronopoulos et al., 2015). As such, a comprehensive business strategy aimed to counteract both a potential downturn in market settings and a limitation in upside potential of an investment project is an important issue for the modern firm.

Moreover, historical, empirical evidence further highlights the importance of managerial discretion in capacity choice. For example, in regards to the case of Kodak, the rise and fall of this monolithic corporation showcases the vulnerability of even the largest of firms to negligent capacity investment behaviour. Starting from its monopoly-like characteristics in the 1970's during which the firm had achieved an approximate 90% market share in film and an 85% market share in camera sales in the U.S. (Lucas and Goh, 2009), Kodak experienced large commercial success in both film and camera sales well into the 1980's. However, in the proceeding years, with the arrival of Sony's first electronic camera, Kodak, rather than prepare for the replacement of film through digital photography, chose to continually invest in film. This investment strategy continued despite, in 1986, Kodak's research labs developed the first mega-pixel camera, one of the milestones that Kodak's head of marketing intelligence had forecasted as a tipping point in terms of the viability of standalone digital photography (Mui, 2012). As a result, in the proceedings years, the company went from enjoying monopoly-like characteristics to a reduction in labour force by roughly 80% through retirements, lay-offs, and, finally, filed for Chapter 11 bankruptcy protection in January of 2012. Exuberantly denoted by Clayton Christensen, for Kodak, the rise of digital photography was comparable to being hit with a tsunami; the very technology that Kodak had helped to develop had led to its demise (The Economist, 2012).

Secondly, in the renewable energy industry, technological uncertainty plays a significant role in wind energy capacity installations as well as its respective valuation. Take into consideration in 2014, global wind energy capacity installations reached their highest point in newly installed wind energy capacity recorded to date at approximately 49 GW of additional global capacity (United Nations Environment Programme, 2015, Huang and McElroy, 2015). Based on this development, investment trends into wind energy have also experienced record-setting growth (United Nations Environment Programme, 2015). Therefore, as capacity installations and investment trends are augmenting, the development of wind turbine technology is of particular importance to consider within their respective power plant valuations. According to the McKinsey Global Institute, offshore wind turbine technology, as it is considerably less developed than onshore wind turbine technology, shows greater long term deployment potential despite significantly higher capital expenditure requirements. Similarly, as the offshore wind turbine technology matures, its costs are hypothesized to drop by more than 50% in capital expenditure requirements and operating expenses (Manyika et al., 2013). As this implicitly affects the onshore wind turbine market due to technological uncertainty, an onshore wind farm runs a correlated risk that its level of installations will become economically obsolete due to technological change before capital costs can be fully recovered and the investment provides positive cumulative cash flows (Venetsanos et al., 2002). Hence, given these potential long-term, production-shifting market characteristics and the intrinsic risk in a wind turbine investment, a firm currently considering investment into wind turbine technology must have the capability to incorporate a substantial change in onshore wind turbine market conditions as offshore wind turbine technology matures. Consequently, when considering optimal investment and capacity sizing from a firm's perspective into wind turbine technology, a deterministic valuation at face value provides a significantly inaccurate investment valuation as well as its resulting decision support information.

Hence, in the aforementioned cases, technological uncertainty plays a key role in the incumbent technology's valuation and development strategy. Furthermore, the underlying effects of disruptive technology highlight the need for responsive and efficient decision support information in industries as far-reaching as photography to renewable energy that are forced to deal with the implications of technological change. With relevance to even the most monolithic of firms, this thesis will examine a firm's choice in project scale under technological uncertainty in order to provide a model that can nondeterministically value the impact technological uncertainty has on project scale. Additionally, price uncertainty will also be regarded as it plays a direct role in the timing of the capacity investment decision. In order to construct an appropriate valuation, a real options, regime-switching model is proposed to effectively incorporate price and technological uncertainty in an irreversible investment decision. Under these circumstances, the question of how an investment decision in capacity sizing is affected by price and technological uncertainty is examined.

Presented by Dixit and Pindyck (1994), the real options theory provides a framework for valuing real assets in uncertain futures. Furthermore, there are two important analytical dimensions the real options model showcases about an investment problem. First off, a dynamic representation of the timing of the investment decision is used, whereas, in the traditional sense, a static timeframe was considered and weighed when making a final investment decision. Secondly, underlying factors are represented as stochastic processes. As such, stochastic processes can produce a more accurate representation of movements that fluctuate randomly and unpredictably. Accordingly, the resulting investment strategy becomes more restrictive as the strategy takes into further consideration both the qualitative and quantitative implications of the value of waiting for more information about uncertain future trends (Botterud and Korpås, 2007).

Moreover, regime-switching models often portray the tendency of financial markets to exhibit volatile behaviour with the phenomenon that the new behaviour often persists for several periods after a change has occurred. While the characteristics captured by regime-switching models are oftentimes identified by econometric procedures, they can also correspond with different periods in regulation, policy, and technological change (Ang and Timmermann, 2012). As such, regime-switching models can effectively capture the underlying effects a disruptive technology can have on incumbent technology market conditions.

Thus, the contributions of this paper are three-fold. First off, in order to derive the optimal investment threshold and the corresponding optimal capacity, an analytical framework combining both regime-switching and real options is proposed for investment opportunities under price and technological uncertainty. Second, in order to more closely scrutinize immediate investment policy, price and technological uncertainty are examined to see their interaction with optimal capacity sizing. Third, managerial insight is provided for capacity investment decisions through analytical and numerical results concerning both the qualitative and quantitative implications of the interactions between irreversible investment, disruptive technology, and managerial discretion over project scale.

In addition, the delimitations of the model concentrate solely on basic American call option characteristics. By doing so, the model forgoes the option to abandon the incumbent technology project post-investment if a regime-switch has occurred in the incumbent technology market conditions. As the abandonment option gives the firm the opportunity to sell a project's cash flows over the remainder of the project's lifetime, the investment decision's salvation value and, analogously, its American put option characteristics are ignored. Considering the project's liquidation value could further affect the project's optimal investment threshold and corresponding optimal capacity, it is important to note that the model serves solely as an approximation tool rather than one with complete precision.

In Section 2, literature regarding the analytical framework will be further evaluated. In Section 3, evidence supporting regime-switching will be presented through the lens of Clayton Christensen's theory of disruptive technology. Then, in Section 4, the mathematical tools implemented in the model will be examined. Subsequently, the regime-switching, real options model will be built in Section 5. First, investment excluding a regime-switch is analyzed in Section 5.2.1 where an analytical expression for optimal timing and capacity is derived. In Section 5.2.2, the penultimate investment decision under both price and technological uncertainty is examined, and a nonlinear solution requiring the numerical methods executed in Section 6 are implemented in order to gain managerial insight from the model. Within the same section, numerical results for the effects of regime-specific price uncertainty as well as technological uncertainty are regarded in order to illustrate their interaction with the optimal investment policy. Lastly, in Section 7, concluding remarks, limitations of the model, and suggestions for future research are offered.

2. Literature Review

In this section, literature related to the theoretical background of this paper is presented. It is systematically reviewed in two steps. First, real options theory will be broadly examined to present its historical background and its evolution as a framework. Particular nuance applications will also be observed to arrive at the conclusion of dynamic programming as the application of choice in the real options framework. Secondly, as this thesis concentrates on optimal timing and capacity sizing under price and technological uncertainty, the recent literature surrounding these concepts will additionally be examined. By doing so, this section aims to highlight the existing gap in academic literature to support modelling a lumpy investment under price and technological uncertainty with a regime-switching, real options model.

Real options theory complements the traditional discounted cash flow method, which originates from the classical work of Fisher (1930). In his valuation method, decision-making criteria for an investment decision is constructed by discounting the cash flows of a project in order to find its net present value, which is then subsequently used to evaluate the project's potential for investment. If the net present value of the project is positive, the investment is considered attractive; and in the case that the net present value is negative, the project is assumed to be unprofitable and abandoned. Conversely, using contingent claims analysis, Majd and Pindyck (1987) show how the traditional discounted cash flow method understates the value of an investment project by ignoring the inherent flexibility in the time to build and, as an outcome, showcase how adhering to the simple net present value rule can result in gross investment error. Furthermore, the real options framework considered by Majd and Pindyck (1987) was further implemented by McDonald and Siegel (1986) to address the standard problem of optimal investment timing in a project of given capacity size with the perpetual option to invest. Further discrediting the net present value rule, their findings quantify that for reasonable parameter values, sub-optimal investment timing can affect a project's value with a traditional net present value of zero by as much as 10-20%. Hence, through the acceptance of this criticism, the criteria governing a net present value calculation can be deemed insufficient, and highlights the necessity for an alternative investment valuation method.

However, although the contingent claims approach aims to fill this gap in literature, its limitations in assumptions restrict its application to span all investment opportunities. The standard real options textbook by Dixit and Pindyck (1994) seeks to overcome this shortcoming by extending the work of Mcdonald and Siegel (see Chapter 5 of Dixit and Pindyck (1994)) by considering both the contingent claims approach and, a more broader method, dynamic programming, to the firm's investment decision. By examining the relationship between these two approaches at the firm level, the authors highlight their specific merits for use in the context of irreversible investment and stochastic revenue streams. First off, contingent claims analysis works to construct a riskless portfolio through an appropriate long and short position. This portfolio, consisting of both the risky project and investment assets, tracks the project's uncertainty (Insley and Wirjanto, 2010). In equilibrium with no arbitrage opportunities, the portfolio must then earn the risk free rate of interest, which allows the value of the risky project to be determined. However, the limitations of contingent claims analysis dictate that any stochastic change in the project's value must be spanned by existing assets in the economy and that capital markets are sufficiently complete so that a dynamic portfolio of assets can perfectly correlate with the value of the project. In comparison, dynamic programming provides an application that is considerably more flexible in market parameters and does not require diversification of risk. Notwithstanding the relaxation of market assumptions, the exogenous discount rate implemented in dynamic programming highlights its subjective shortcomings. Regardless, in order to model an incumbent technology market dealing with innovation rates such as that of a disruptive technology, dynamic programming provides the required flexibility to model a production capacity investment decision under price and technological uncertainty.

As the breadth of disruptive technology effects are widespread, it is additionally important to note the numerous industries real options have analyzed and its extensions in application. Antecedents to the Mcdonald-Siegel investment model include seminal works by Myers (1977) who studied real option effects on corporate borrowing behaviour and Tourinho (1979) who pioneered real options application to an exhaustible, natural resource reserve. Furthermore, the field of real options spans the categories of real estate development (Titman, 1985, Capozza and Sick, 1994, Quigg, 1993), corporate strategy (Kester, 1984, Kulatilaka and Marks, 1988), research and development (Morris et al., 1991), and enterprise valuations (Chung and Charoenwong, 1991, Kellogg and Charnes, 2000), amongst others. Lastly, real options, petroleum literature is particularly well developed (Ekern, 1988, Cortazar and Schwartz, 1998, Kemna, 1993) given its exceptional fit for oil price uncertainty and the high stakes nature of petroleum projects. Furthermore, strategic real options literature provides useful extensions where managerial insight is added in addition to solving an investment timing problem. Combining the competitive real options model and a Markov-switching regime, Goto et al. (2012) study the investment problem of two asymmetric firms in the context of boom and recessive market conditions and find the investment threshold differences of a firm as a leader and as a follower are regime-dependent. Moreover, Chronopoulos and Siddiqui (2015) study the conventional investment problem where a firm considers the optimal time to undertake an investment project under both price and technological uncertainty. Implementing three different investment strategies: compulsive, laggard, and leapfrog; they find that under a compulsive strategy, technological uncertainty has a non-monotonic impact on the optimal investment decision. Hence, extensions and applications of real options literature are far-reaching while continually providing additional managerial insight to basic real options applications.

In the area of investment under technological uncertainty, optimal timing problems show various results with adoption rates of technologies. Early works include Balcer and Lippman (1984) who analyze the optimal timing of technology adoption using switching options. They find that the firm will adopt the current best technology practice after a certain threshold, and, in the case that technological uncertainty is increasing, new technology adoption will be delayed. Conversely, they also find that it may be profitable to purchase an incumbent technology that was considered unprofitable at its conception if after a certain period of time, no technological advances are made. Adopting the dynamic programming approach from Dixit and Pindyck’s (1994) real options framework, Farzin et al. (1998) extend the work of Balcer and Lippman (1984) by analyzing the optimal timing of technology adoption by a competitive firm when investment in new, improved technology is an irreversible investment decision and technological progress evolves according to a Poisson process. Including the correction by Doraszelski (2001), they find that a firm will defer the adoption of a new technology when it takes the value of waiting into consideration. Introducing both game-theoretic considerations and uncertainty to the real options framework, Huisman and Kort (2004) study a duopoly model where two firms have the option to invest in an incumbent technology under the uncertainty that a superior technology with an unknown arrival rate will become available as an investment option. Modelling the arrival rate according to a Poisson process and assuming that switching is not an option after investment in the incumbent technology has taken place, they find that investment is further delayed based on technological uncertainty, and the firm who invest second receives the highest payoff. Price uncertainty modeled by geometric Brownian motion also plays a substantial role as it induces a higher probability that the new technology will be adopted instead of the current technology.

Interestingly, as the predominant source of real options literature deals solely with investment timing while considering capacity sizing fixed, the strategic consequences of such a choice undermine the effects of managerial discretion over capacity size, while predominantly establishing the standard result that uncertainty directly correlates with the value of waiting. Supporting this switch, in his review of Dixit and Pindyck's textbook (1994), Hubbard (1994) states,

"(…) *the new view models… do not offer specific predictions about the level of investment. To go this extra step requires the specification of structural links between the marginal profitability of capital and the desired capital stock*" (page 1828).

As such, henceforth, the real options literature that deals with both optimal capacity sizing and timing will be reviewed. According to the survey by Huberts et al. (2015), three distinct areas of this type of literature prevail: continuous time models where investments have a lumpy structure, discrete time models, and incremental investment models. As the latter two models go beyond the scope of this thesis's application, lumpy investment models will be further regarded.

In the area of lumpy investment strategies in continuous time models, the firm generally invests at a later point in time and at a larger corresponding capacity size contradicting how uncertainty conventionally affects the firm's growth. Early examples include the work by Manne (1961), who was the first to determine that the firm invests in a larger capacity level when uncertainty increases by observing a stochastic capacity expansion problem. Continuing this work, Dangl (1999) sets up a model with both a concave investment cost function and a deterministic production cost function with price determined by both production quantity and a demand shift parameter assumed to undergo multiplicative geometric Brownian shocks. Under these conditions, he finds that increasing levels of demand uncertainty correlate with a delayed optimal investment strategy and increased project capacity. In the same year, Bar-Ilan and Strange (1999) examine capital stock as a capacity sizing and timing problem while assuming both clearance over production flexibility and a deterministic, marginal production cost. In comparison to Dangl (1999), their output price follows solely a geometric Brownian motion. Furthermore, adopting a similar method as that established by Dangl (1999), Bøckman et al. (2008) analyze hydropower projects. Although consistent with the exponential form of the concave investment cost function as that of Dangl (1999), they choose to model a convex cost function to match the limitations renewable energy projects conventionally exhibit; as the chosen capacity approaches a finite maximum capacity, each new unit of capacity displays diseconomies of scale. The contribution margin, which is indicated as the difference between electricity price and the marginal production cost, is also modelled by geometric Brownian motion. Similar to Bar-Ilan and Strange (1999), Kort et al. (2011) model both flexible and inflexible production in a firm's capacity investment decision. In order to do so, they assume clearance in the inflexible firm model, while varying utilization rates of installed capacity in the flexible firm model, and find that the flexible firm has a greater corresponding optimal capacity than that of the inflexible firm.

Returning to the area of technological uncertainty and simultaneously regarding capacity sizing, Della Seta et al. (2012) study investment in learning-curve technologies under price uncertainty and find that the characterization of the learning-curve leads to two opposite investment strategies. Revoking standard real options intuition, they find that in the case that the learning process is slower, the firm has a higher optimal investment threshold and a larger optimal capacity, whereas, if the learning-curve is steep, the firm invests earlier and at a limited capacity. In a similar vein, Hagspiel et al. (2013) study a price-setting firm facing a declining profit stream for its incumbent technology while weighing investment into an existing, disruptive technology. The firm has three available options to implement in its investment strategy: abandonment, call, and suspension. As in Dangl (1999), price is governed by an inverse demand function influenced by geometric Brownian shocks, and, in order to distinguish between booming- and recessive-like market conditions, regime-switching is implemented in the growth parameter settings of the geometric Brownian motion. Lastly, contrary to standard real options intuition and given a firm's optimal capacity choice, their findings conclude the investment threshold is monotonic as a function of uncertainty.

Hence, these academic papers highlight the various forms models have taken in order to examine the effects of various managerial discretions and flexibilities on optimal capacity sizing and timing. Particularly, the effect of technological uncertainty coupled with capacity sizing exhibits a field where common results both violate standard real options intuition and simultaneously do not provide ubiquitous results. In order to go further, Huberts et al. (2015) recommends that,

"*To add even more realism, future contributions could consider issues like… technological progress [and] innovation (...). As usual, researchers will face the trade-off between analyzing simple models that allow for full analytical solutions and designing more complex models that could only be solved using numerical methods.*"

As such, this thesis will contribute to the existing literature by adapting the real options approach to quantitatively analyse an incumbent technology, capacity investment under both price and technological uncertainty. In order to model technological uncertainty, as in Huisman and Kort (2004) and Farzin et al. (1998), the model uses a Poisson process to predict a regime-switch in incumbent technology market conditions. However, similar to Goto et al. (2012) and in comparison to Hagspiel et al. (2013), regime-specific market conditions denote both distinct boom- and recessive-like growth rates and volatilities to better model the effects of a disruptive technology. In order to model price uncertainty, the model uses geometric Brownian motion as is commonly implemented in the aforementioned capacity sizing literature. In order to have a conservative cost structure, a deterministic production cost function is assumed as in Dangl (1999), and drawing from Bøckman et al. (2008), a convex investment cost function is assumed so as to show the model's particular fit for the renewable energy industry as well as for projects exhibiting diseconomies of scale. Lastly, in order to more coherently study the effects of price and technological uncertainty on the investment decision, clearance, as in Bar-Ilan and Strange (1999) and Kort et al. (2011), is assumed.

Referencing Hubert's statement, although a full analytical solution is not provided due to the complexity of the model, numerical results show that, under technological uncertainty, if price uncertainty is low, firms invest earlier and in limited capacity, whereas, if price uncertainty is high, firms invest later and in extensive capacity. Additionally, directly revoking standard real options intuition, the numerical results establish that the firm's optimal investment policy will be monotonically decreasing as a function of technological uncertainty. In contrast to Chronopoulos and Siddiqui (2015), this seemingly counter-intuitive result occurs as a consequence of the assumption that investment will occur both irrevocably and irrespective of the regime the firm is operating within. Intuitively, the additional dynamics provided by a compulsive investment strategy would then be expected to shift this result towards a non-monotonic impact on the optimal investment decision.

3. Disruptive Technology

Disruptive technology represents a paradigm shift, and, once experienced, has the potential to create permanent change that can transform an entire industry. By experiencing such a shift, adopted technologies become embodied in both physical and human capital, and oftentimes allows for efficient economic value creation. Simultaneously, technology often disrupts, supplanting the status quo and rendering stagnant skill sets and organizational approaches irrelevant (Manyika et al., 2013). In order to effectively respond to these changes, grounded business action becomes paramount to a firm dealing with disruptive technological uncertainty. Take into consideration, IBM dealt with this dilemma by launching a new business unit to manufacture PCs, while continuing its core business development, mainframe computers. In a similar vein, Netflix took a more radical move, switching away from its previous business model, sending out rental DVDs by post, to streaming on-demand media to its customers (The Economist, 2015). Hence, grounded business action remains paramount in order to effectively respond to the ramifications disruptive technology has on both the firm and the market. Keeping this in consideration, it becomes important to incorporate the disruptive potential technologies display into the investment process. As such, this section concentrates on benchmarking and applying the effects of disruptive technology. In order to exemplify this, Christensen's theory of disruptive technology will first be defined and expounded upon so that a conceptual basis for disruptive technologies can be established. Second, in order to assume the relevance of a disruptive technology to an incumbent technology valuation, the laws of disruptive technology will be further delineated and analyzed. By doing so, this section aims to establish the relevance of disruptive technology to an incumbent technology investment decision and legitimize the proposition of regime-switching to aid in finding a solution to the investment dilemma.

3.1 The Theory of Disruptive Technology

In order to properly describe the effects of disruptive technology, it is helpful to first establish a basis on which to view technological change. Christensen's theory of disruptive technology is a heavily cited proposition rigorously developed in his textbook, *The Innovator's Dilemma* (Christensen, 2000), that aims to explain the phenomenon by which an innovation transforms an existing market or sector. Based on three crucial findings, the theory's ramifications can aid in characterizing a disruptive technology and its trends, which further provide a conceptual basis for a paradigm shift in incumbent technology valuations.

Initially, the first finding stipulates that the distinction between a sustaining technology and a disruptive one is an important strategic divergence. To clarify, whenever an innovation acts to upgrade a particular technology's performance in the market place, it can be considered sustainable, whereas the emergence of a disruptive technology constitutes an innovation that generally underperforms relative to the established product lines in a specific industry's market. However, over time, the disruptive technology can display characteristics of being cheaper, simpler, more compact, and, frequently, more pragmatic in comparison to the incumbent technology in the industry. Examples of this can be referenced through the hypothesized development trajectory of offshore wind turbine technology (Manyika et al., 2013); in order to operate within extreme weather conditions, innovative, costly materials such as carbon fibre are being introduced into offshore wind turbine blade technology to provide an elevated strength-to-weight ratio in blade characteristics (International Renewable Energy Agency, 2012, Douglas-Westwood, 2010). As this optimization, amongst others, acts to increase load capabilities and is predicted to drop in expenditure requirements over time, an offshore wind farm, in comparison to an onshore wind farm, can be expected to become simultaneously both more lucrative and efficient over time. Moreover, the success of a firm is contingent on the strategic classification of a disruptive technology versus a sustainable technology; a disruptive technology holds the potential of the failure of a leading firm, whereas a sustainable technology rarely precipitates such a consequence (Christensen, 2000). Therefore, it becomes crucial to have an innate understanding of both a sustainable and a disruptive technological change in a market to respond with grounded business action.

Secondly, the rate at which an incumbent technology evolves can surpass market needs and unknowingly invokes a vulnerability of market share as illustrated in Figure 1. Indicated by the upper-most trend line, conventionally, an incumbent firm overshoots customer needs by developing a technology to an extent where the customer no longer desires improvements and, ultimately, no longer display a willingness to pay for it. Moreover, a portion of the market becomes vulnerable as the least profitable customer segment in the market no longer displays a willingness to support the price demanded by the sustainable innovations. Furthermore, indicated by the lower trend line, a disruptive technology is initially embraced by the least profitable customer segment in the market and can ameliorate its own respective performance via sustainable innovations over a period of time. It follows then that they can compete to saturate fringe customer demands in the market and eventually take market share over time or create a new market as the disruptive technology becomes competitive in other key performance indicators. Hence, the trajectory of market needs compared to technological improvement plays a critical role in determining the vulnerability of a firm as well as the resulting incumbent technology market conditions.

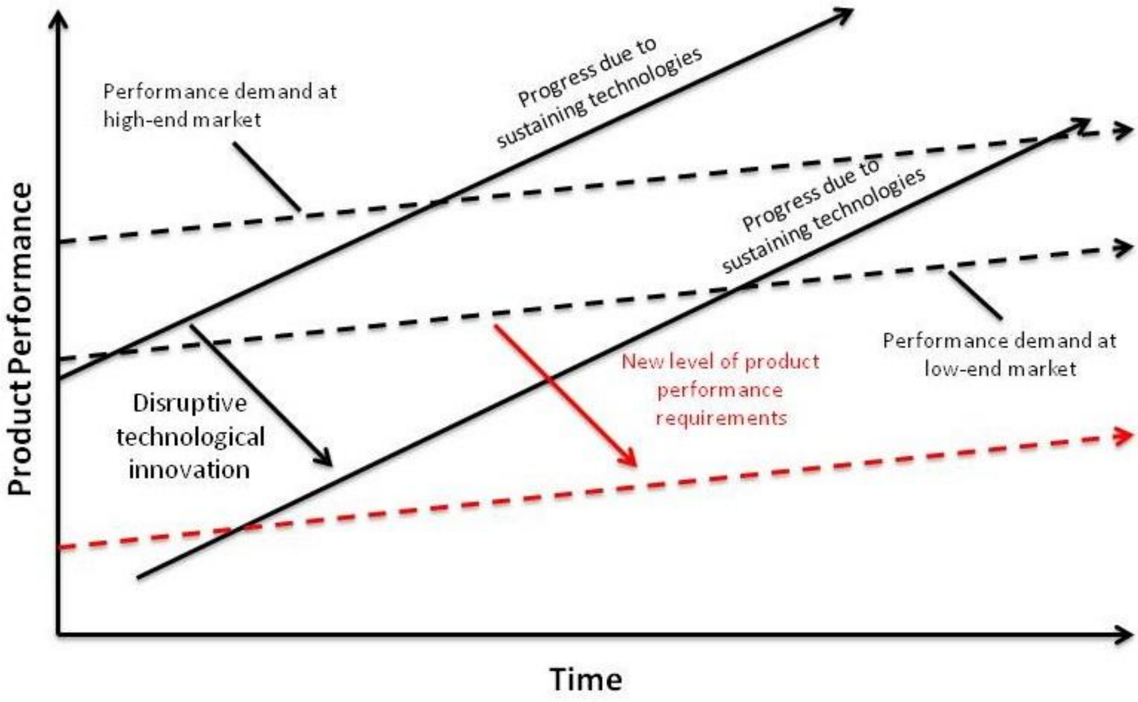

Figure 1: The Impact of Sustaining and Disruptive Technological Change (Christensen, 2000)

Third, the highest performing companies have well developed systems for maintaining the status quo by eliminating initiatives that do not directly coincide with customer demand. Similarly, the investment process a firm practices also ignores innovations that could potentially disrupt the market in which the firm operates within. As a result, adequate consideration to disruptive technologies does not occur until prospective technologies have decreased the long term profitability of the market for the incumbent firm. Effectively, the firm that continues to invest in an incumbent technology without properly weighing the effects a disruptive technology could have on the market unknowingly leaves portions of its market share vulnerable to the companies implementing the disruptive technology. Consequently, the very decision-making and resource allocation processes practiced by management, key to the success of well-established companies, are the very processes that act as the root cause of their demise in the face of technological uncertainty.

Hence, these findings illustrate the unabating effects a disruptive technology has on both a firm and an incumbent technology market. By not preparing for the potential change in market conditions and lacking effective business action, a firm could lose its position in the market as a result of not having the ability to properly weigh the aforementioned elements and, consequently, risk failure. Hence, as good management practice drives the failure of successful firms faced with disruptive technological change, then the conventional responses to companies' problems-planning better, working harder, becoming more customer-driven, and taking a longer-term perspective- all exacerbate the problem. As such, the solution to disruptive technologies lies within the laws of organizational nature which act to powerfully define what a firm can and cannot do (Christensen, 2000).

3.2 Laws of Disruptive Technology

It is proposed by Christensen that there are five organizational laws of disruptive technology that if properly harnessed lead to the success of a firm. In particular, the first and third law provide useful properties that a firm can effectively leverage in order to decide whether to invest in an incumbent technology given technological uncertainty or to divest into a disruptive technology. First off, the primary law indicates that a firm depends on its customers and investors for resources, whereas the third law stipulates that markets that do not exist cannot be analyzed. By critically examining these two laws, it provides not only grounded business action for a firm operating under technological uncertainty, but also gives a basis for implicitly defining an investment model using regime-switching.

First, the theory of resource dependence governs a firm's resource allocation. This principle dictates that the firm does not control its own flow of resources, but, rather, investors and customers are the forces within an organization that govern resource allocation, and firms that choose to digress from satiating these needs ultimately fail. Conversely, those that best satiate these needs are successful. As investment patterns are designed to dismiss a disruptive technology at its outset, the only instance in which mainstream firms have successfully established a timely position in a disruptive technology were those in which the firm's managers set up an autonomous organization charged with building an independent business around the disruptive technology (Christensen, 2000).

Therefore, the companies that can best succeed in these small, emerging markets are those that align their firms with forces of resource dependence by creating an independent organization.

Secondly, the third law of disruptive technology stems from the innovator's dilemma. As the strategies to manage a sustainable technology are generally predictable, the strategies are similarly competitively unimportant, whereas the leadership involved in fostering a disruptive innovation displays large, advantageous aspects. However, companies whose investment processes demand quantification of market sizes and financial returns before they can enter a market become paralyzed or make serious mistakes when faced with disruptive technologies (Christensen, 2000). As there are large first mover advantages in disruptive situations, leadership must take action before careful plans can be made. However, as this presents the innovator's dilemma, it becomes necessary to recognize the unpredictability of a new market. In order to overcome this aspect of innovation, Christensen suggests *discovery-driven planning*. Due to the fact that very little is known about disruptive markets, effective grounded business action is only applicable once a firm learns how best to implement a disruptive technology. Hence, in planning to learn, the mindset needed for the exploitation of a disruptive technology can be deduced after obtaining the necessary decision support information to resolve underlying technological uncertainty.

In tying these two laws of disruptive technology together as well as the theory of disruptive technology, regime-switching is implemented into the model due to its ability to incorporate the hypothesized effects from a disruptive technology into incumbent technology market conditions, as well as by providing a basis for responsive business action. By incorporating regime-switching into the real options model, a change in incumbent technology market conditions can implicitly reflect the hypothesized effects from the successful penetration of a disruptive technology in the market. Additionally, grounded business action can effectively be recommended based on the first and third organizational laws of disruptive technology. In response to these laws, the firm no longer must base its investment decision on a disruptive technology market but rather, implicitly on an incumbent technology market, which gives the possibility to use market information from an observable market instead of attempting to quantify the market size and financial return of a disruptive market. Additionally, in the case that the option to invest is out of the money, the strategic recommendations of either *discovery-driven planning* or, in the case of a successful venture, the creation of a separate enterprise can be given. In conclusion, both the effects of disruptive technology on incumbent technology market conditions and managerial insight can be provided through the use of regime-switching in the proposed real options model.

4. Mathematical Background

4.1 Itô's Lemma

Suppose the state variable $x(t)$ follows a simple Brownian motion as indicated in Equation (1). $a(x,t)$ and $b(x,t)$ are known, non-random functions, and dZ is the standard increment of a Wiener process.

$$dx = a(x,t)dt + b(x,t)dZ \tag{1}$$

Also, consider a function $F\big(x(t)\big)$ that is twice differentiable on $x(t)$ and to the first-order on the time variable t. Through conventional calculus, the total differential of the function $F\big(x(t)\big)$ can be expressed as Equation (2).

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial x}dx \tag{2}$$

Introducing the higher-order terms of dx by Taylor expansion, the differential dF expands to Equation (3).

$$dF = \frac{\partial F}{\partial t}dt + \frac{1}{1!}\frac{\partial F}{\partial x}dx + \frac{1}{2!}\frac{\partial^2 F}{\partial x^2}dx^2 + \frac{1}{3!}\frac{\partial^3 F}{\partial x^3}dx^3 + \cdots \tag{3}$$

In order to simplify Equation (3), the squared differential of the state variable dx^2 is first examined. Because the expected squared value of the Wiener increment is equal to the time derivative, $\mathbb{E}[dZ^2] = dt$, taking the expansion of dx^2 simplifies substantially as indicated in (4). Empirically, it is observed that as dt becomes infinitesimally small, the first and second term of the third line of (4) approach zero at a more rapid rate relative to dt. Hence, the differentials of time with a power greater than one can be ignored.

$$\begin{aligned} dx^2 &= (a(x,t)dt + b(x,t)dZ)(a(x,t)dt + b(x,t)dZ) \\ &= a^2(x,t)dt^2 + 2a(x,t)b(x,t)dtdZ + b^2(x,t)dZ^2 \\ &= a(x,t)^2dt^2 + 2a(x,t)b(x,t)dt^{\frac{3}{2}} + b^2(x,t)dt \\ &= b^2(x,t)dt \end{aligned} \tag{4}$$

Applying this same logic to any expansion of dx greater than the squared differential of dx in Equation (3) will generate an expression with each time differentials' exponent greater than one and, hence, also cancel. Therefore, Equation (3) simplifies to Equation (5).

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial x}(a(x,t)dt + b(x,t)dZ) + \frac{1}{2}\frac{\partial^2 F}{\partial x^2}b^2(x,t)dt \tag{5}$$

Collecting like terms, Itô's Lemma gives the total differential of the function $F\big(x(t)\big)$ generally as in Equation (6).

$$dF = \left(\frac{\partial F}{\partial t} + a(x,t)\frac{\partial F}{\partial x} + \frac{1}{2}b^2(x,t)\frac{\partial^2 F}{\partial x^2}\right)dt + b(x,t)\frac{\partial F}{\partial x}dZ \tag{6}$$

4.2 Markov-Modulated Geometric Brownian Motion

A stochastic variable is modelled by g*eometric Brownian motion with drift* if it is a specialized case of a continuous time stochastic process, x_t, which, indicated in Equation (7), can be found by adapting Equation (1) with $a(x,t) = \mu x_t$ and $b(x,t) = \sigma x_t$. This Itô process has four distinct components where dt is an infinitesimally small increment of time, dZ is an increment of the standard Brownian motion, and μx_t and σx_t are the expected instantaneous drift rate and the instantaneous variance rate respectively (Dixit and Pindyck, 1994).

$$dx_t = \mu x_t dt + \sigma x_t dZ \tag{7}$$

As changes in the process x_t over any finite interval of time are normally distributed, it becomes necessary to transform the underlying function so that it can be used to suitably model price. To do so, the relationship between the state variable x_t and its logarithm is examined, $F(x_t) = \log(x_t)$. Using Itô's Lemma, its rate of change, dF, can be expanded resulting in Equation (8).

$$dF = \frac{1}{x_t}dx_t - \frac{1}{2{x_t}^2}{dx_t}^2 \tag{8}$$

By inserting Equation (7) into Equation (8), the process followed by F becomes described as Equation (9). Hence, a change in a finite time interval t in F is normally distributed with a mean and variance, $dF \sim N\left(\left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right)$.

$$dF = \left(\mu - \frac{1}{2}\sigma^2\right)dt + \sigma dZ \tag{9}$$

This result can be used to find both the expected value and variance of x_t with its current, observable state, x_0, as indicated in (10).

$$(1)\ \mathbb{E}[x_t] = x_0 e^{\mu t} \text{ and } (2)\ V[x_t] = x_0^2 e^{2\mu t}\left(e^{\sigma^2 t} - 1\right) \tag{10}$$

Similarly, as this allows both the mean and variance of x_t to be found, it enables the expected present discounted value of x_t to be calculated over a period of time by using the result from (10) in Equation (11). In the case of perpetuity and an exogenous discount rate ρ, where the discount rate ρ exceeds the growth rate μ, the expectation provides a useful outcome for the valuation of an investment project integrated under a perpetual time frame.

$$\mathbb{E}\left[\int_0^\infty x_t e^{-\rho t} dt\right] = \frac{x_0}{\rho - \mu} \tag{11}$$

Secondly, as both the instantaneous drift and variance rate of geometric Brownian motion fail to capture the effect disruptive technologies are hypothesized to have on incumbent technology market conditions, the regime-switching model is introduced into geometric Brownian motion parameters to capture these effects. In itself, regime-switching often portrays the tendency financial markets have to exhibit volatile behaviour with the phenomenon that the new behaviour often persists for several periods after such a change has occurred. However, a key difference within this type of modelling occurs when looking at a regime-switch that can be classified as either irreversible or highly unlikely to reoccur. These changes, referred to as a change point process, were considered by Chib (1998) and further expounded upon through the examination of stock return dynamics by Pástor and Stambaugh (2001) and Pettenuzzo et al. (2014), amongst others. Within these processes, the characteristics captured by these specific regime-switching models aim to correspond with different periods in regulation, policy, and technological change (Ang and Timmermann, 2012). Logically, it follows that to document the effect a disruptive technological change has on incumbent technology market conditions, regime-switching should be implemented in its stochastic process.

In regime-switching models, there is an unobservable random state variable $\epsilon(t) = \{1, \dots, n\}$ that follows a Markov chain in the price process's time series, that indicates which regime, n, is realized in the economy. In a change point process, the regimes are no longer revisited after a state change has occurred, and can theoretically be considered as sustainable increments in disruptive technological change; with each subsequent regime visited, the disruptive technology has implicitly made an incremental, but significant change that is reflected in the incumbent technology's market conditions.

Mathematically, this state change can be modelled by a modified transition probability matrix $\boldsymbol{P}$ where the probability of returning to a previous regime is zero. More specifically, the regime-switch in a change point process is governed by a nxn transition probability matrix $\boldsymbol{P}$ with the probabilities $p_{i,j}$of switching from a regime i at time t to a regime j at time $t + dt$, as represented by the matrix in (12).

$$\boldsymbol{P} = \begin{bmatrix} p_{1,1} & \cdots & p_{1,n} \\ 0 & \ddots & \vdots \\ 0 & 0 & p_{n,n} \end{bmatrix} \tag{12}$$

Additionally, the sum of the probabilities of switching to a particular regime or staying within the realized regime sum to one for each respective regime as indicated in (13).

$$\sum_{j=1}^{n} p_{i,j} = 1, i \,\epsilon\, \{1, \dots, n\} \tag{13}$$

Moreover, each regime is assumed to be an independent price process that is governed by the strong Markov property; the current regime j is dependent upon only the most recent realized regime i, which corresponds to the transition probability matrix by the probability $p_{i,j}$ as indicated in (14).

$$P\left\{ x_{t,n} = j \backslash \epsilon = i \right\} = p_{i,j} \tag{14}$$

Practically, this implies that the point in time in which the process is applied is dependent upon only current available information. Then, it follows that when applied to a current state, the transition probability matrix is flexible in the sense that it can be applied at each step of the Markov chain regardless of the state of the disruptive technology or the incumbent technology market conditions. Hence, given these characteristics and the hypothesized switching probabilities, a regime-switching valuation can effectively capture technological uncertainty.

In addition, the change point process requires the probabilities $p_{i,j}$, which can be modelled using a Poisson jump process as denoted by dq in (15). This diffusion process aims to model an economic variable as a process that makes infrequent but discrete jumps. Consequently, the referenced jumps can be thought of as a substantial disruptive technological breakthrough that causes the market conditions for the incumbent technology to shift. Statistically, the Poisson jump process is subject to jumps of fixed or random size, for which the arrival times follow a Poisson distribution (Dixit and Pindyck, 1994). The jumps, u, represent events that can cause a structural break in the stochastic process being modelled, and which can in itself also be a random variable. The rate of occurrence or intensity of the Poisson process is reflected by the proportionality constant λ, and during a time interval of infinitesimal length dt, the probability that a jump will occur is given by λdt.

$$dq = \begin{cases} 0 \text{ with probability } 1 - \lambda dt \\ u \text{ with probability } \lambda dt \end{cases} \tag{15}$$

Finally, marrying these concepts together: geometric Brownian motion, regime-switching, and a Poisson jump process; the state variable x_t following Markov-modulated geometric Brownian motion is described in Equation (16).

$$dx_t = \mu_{\epsilon(t)} x_t dt + \sigma_{\epsilon(t)} x_t dZ_t \tag{16}$$

With the assumption that $\epsilon(t) \in \{1,2\}$, a Poisson jump process is modelled in the transition probability matrix (17).

$$\boldsymbol{P} = \begin{bmatrix} 1 - \lambda dt & \lambda dt \\ 0 & 1 \end{bmatrix} \tag{17}$$

Consequently, the growth rate and volatility are subject to the realized regime in the economy as indicated in (18).

$$(\mu_{\epsilon(t)}, \sigma_{\epsilon(t)}) = \begin{cases} (\mu_1, \sigma_1) \text{ if } \epsilon = 1 \\ (\mu_2, \sigma_2) \text{ if } \epsilon = 2 \end{cases} \tag{18}$$

By modelling the price process as such, Markov-modulated geometric Brownian motion can implicitly incorporate relevant information about a disruptive technology in incumbent technology market conditions to provide a more informed, capacity investment decision.

4.3 Dynamic Programming

Dynamic programming is a general tool used for dynamic optimization problems under uncertainty. It decomposes a sequence of decisions into two components: the immediate decision, and a valuation function that encapsulates the consequences of all subsequent decisions (Dixit and Pindyck, 1994). This decomposition can be formally described by Bellman's Principle of Optimality,

"An optimal policy has the property that, whatever the initial action, the remaining choices constitute an optimal policy with respect to the sub-problem starting at the state that results from the initial actions" (Bellman, 1954).

In order to clarify these assertions, the components of a dynamic optimization problem will be further analyzed in this section.

As indicated in Equation (19), during each period t, a maximization choice is represented by the control variable(s) u, which denotes the specific choices to be made by the firm. The firm's current status as it affects its operations and expansion opportunities is delineated by a state variable x. Both of these variables at time t affect the firm's immediate profit flow component, which can be denoted as $\pi_t(x_t, u_t)$. As the valuation function is evaluated from the perspective in period t, the expectation of the continuation value is taken, $\mathbb{E}_t[F(x_{t+1})]$, and further discounted to adjust to time t by the discount factor $\frac{1}{1+\rho}$.

$$F(x) = \max_{u_t} \left(\pi_t(x_t, u_t) + \frac{1}{1+\rho} \mathbb{E}[F(x_{t+1})] \right) \tag{19}$$

If there is no fixed finite time horizon for the decision problem, the dynamic optimization problem becomes simplified in the sense that the calendar date t ceases to have a direct impact on the valuation. In this setting, the objective function gets a recursive structure that facilitates theoretical analysis as well as numerical computation (Dixit and Pindyck, 1994) as illustrated in Equation (20). In this situation, x' denotes the evaluation of the state in the next period in relation to the current state x.

$$F(x) = \max_u \left(\pi(x, u) + \frac{1}{1 + \rho} \mathbb{E}[F(x') \backslash x, u] \right) \tag{20}$$

For a dynamic optimization problem in continuous time, the Bellman Equation (20) is reworked to consider a time period of infinitesimal length Δt in Equation (21).

$$F(x, t) = \max_u \left(\pi(x, u, t)\Delta t + \frac{1}{1 + \rho\Delta t} \mathbb{E}[F(x', t + \Delta t) \backslash x, u] \right) \tag{21}$$

By multiplying by a factor of $(1 + \rho\Delta t)$, dividing by Δt, and taking the limit as Δt goes to zero, Equation (21) becomes adapted for continuous time as indicated in Equation (22). In real options terminology, this equation can be interpreted as the entitlement to the flow of profits from an asset. In regards to the term, $\rho F(x, t)$, the understanding behind this component is the required rate of return a decision maker would demand from holding this asset. The immediate profit flow component signifies the cash flow received upon investment, which can be further considered the immediate payout or dividend of the asset. Secondly, the continuation component can be interpreted as the expected rate of capital gain on the asset.

$$\rho F(x, t) = \max_u \left(\pi(x, u, t) + \frac{1}{dt} \mathbb{E}[dF] \right) \tag{22}$$

To exemplify particular nuances of the solution of Equation (22), the optimization problem is simplified so that the option can be modelled excluding its immediate payout as in Equation (23). Additionally, for this purpose, it is assumed that the state variable x follows a geometric Brownian motion.

$$\rho F(x) dt = \mathbb{E}[dF] \tag{23}$$

Using Itô's Lemma, the right hand side of Equation (23) can be expanded with respect to the underlying stochastic component of the asset, dF, and, after simplification and rearrangement, results in the differential equation (24).

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F}{\partial x^2} + \mu x \frac{\partial \mathrm{F}}{\partial \mathrm{x}} - \rho F(x) = 0 \tag{24}$$

Additionally, the general solution of Equation (24) must adhere to three boundary conditions as indicated in (25) (Dixit and Pindyck, 1994). The first condition stems from the absorbing barrier of the stochastic process followed by the state variable x. Intuitively, this indicates that if the price process reaches zero, the option to invest will be of no value. Secondly, the second branch of (25) is known as the value-matching condition, and indicates the net value of the asset by subtracting a project's expected, discounted costs, I, from its expected, discounted revenues at the optimal investment threshold, $V(x *)$. Lastly, the third branch of (25) is the smooth-pasting condition, which guarantees that the derivatives of the functions, $F(x *)$ and $V(x *)$, meet tangentially at a certain threshold point.

$$\begin{cases} F(0) = 0 \\ F(x *) = V(x *) - I \\ F'(x *) = V'(x *) \end{cases} \tag{25}$$

Furthermore, in order to satisfy the first branch of (25), it is assumed that the general solution takes the functional format $F(x) = Ax^{\beta}$, which is then substituted into Equation (24). By doing so, Equation (24) reduces to the fundamental quadratic outlined in Equation (26).

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + \mu\beta - \rho = 0 \tag{26}$$

In order to find a solution to Equation (26), the quadratic formula is implemented to outline both the positive and negative roots of the solution, β_1 and β_2, indicated in the first and second branch of (27) respectively.

$$\begin{cases} \beta_1 = -\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) + \frac{\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2\rho}}{\sigma^2}, & \beta_1 > 1 \\ \beta_2 = -\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right) - \frac{\sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2\rho}}{\sigma^2}, & \beta_2 < 0 \end{cases} \tag{27}$$

It then follows that as the second-order, Cauchy-Euler differential equation (24) is linear in its dependent variable $F(x)$ and its derivatives, it has a general solutions that can be expressed as a linear combination of any two independent solutions as in Equation (28)

(Dixit and Pindyck, 1994). The endogenous constants, A_1 and A_2, remain undetermined, whereas β_1 and β_2 represent the aforementioned positive and negative roots of the proposed form of the solution. Notice as $\beta_2 < 0$ and the absorbing barrier $F(0) = 0$, the second term in Equation (28) goes to infinity as $x \to 0$. Hence, the second endogenous constant is set equal to zero, $A_2 = 0$, to mitigate this effect.

$$\begin{aligned} F(x) &= A_1 x^{\beta_1} + A_2 x^{\beta_2} \\ &= A_1 x^{\beta_1} \end{aligned} \tag{28}$$

Consequently, from these three boundary conditions and the proposed form of the solution, one can find the optimal investment policy by deriving both the optimal investment threshold and the value of the option to invest.

5. Analytical Formulations

5.1 Assumptions and Notations

Consider a situation in which a price-taking firm faces an investment decision in production capacity. Prior to investment, the firm is assumed to be generating no cash flow. It can be interpreted that the firm is considering investment in incumbent technology capacity while simultaneously weighing the possibility that an existing disruptive technology will potentially shift incumbent technology market conditions. As such, the dynamics of demand shock are governed by a Markov regime-switching model. In this model, the incumbent technology market has an exogenous output price denoted by the variable P_t where time, $t \geq 0$, is considered to be continuous. Specifically, the exogenous output price follows a Markov-modulated geometric Brownian motion as described in (29).

$$dP_t = \mu_{\epsilon(t)} P_t dt + \sigma_{\epsilon(t)} P_t dZ_t, \;\; P_0 \equiv P > 0 \tag{29}$$

In this stochastic differential equation, the incumbent technology's growth rate is denoted by $\mu_{\epsilon(t)}$, its volatility is represented by $\sigma_{\epsilon(t)}$, and dZ_t is the increment of the standard Brownian motion. Also, the firm implements a subjective discount rate, ρ, which is considered constant, and, intuitively, it follows that $\rho > \mu_{\epsilon(t)}$. The demand shift parameter, $\epsilon(t) \in \{1,2\}$, governs the switch between two regimes with both known growth rates and volatilities. As it is assumed that there are only two regimes in the economy, the state-dependent growth rates and volatilities take the form:

$$(\mu_{\epsilon(t)}, \sigma_{\epsilon(t)}) = \begin{cases} (\mu_1, \sigma_1) \text{ if } \epsilon = 1, \\ (\mu_2, \sigma_2) \text{ if } \epsilon = 2. \end{cases}$$

Within these two states, a specific incumbent technology market is represented. In the first regime, a booming incumbent technology market is assumed where the disruptive technology has not yet satiated or taken market demand. In the second regime, the resulting incumbent technology market models the demand shock a successful disruptive technology paradigm shift incurs. Consequently, the exogenous output price parameters in the incumbent technology market has both boom- and recession-like characteristics, which are reflected in regime one and regime two, respectively. Hence, as uncertainty is negatively related to economic conditions (Goto et al., 2012, Bloom, 2009), a larger growth rate is assumed in the first regime, $\mu_1 > \mu_2$, and a larger volatility is assumed in the second regime, $\sigma_1 < \sigma_2$.

In addition, the process $\epsilon(t)$ is assumed to follow a Poisson law such that $\epsilon(t)$ is a two-state Markov chain with the transition from regime one to regime two characterized by a jump with intensity λ. As such, the process $\epsilon(t)$ has the transition matrix between time t and $t + dt$:

$$\begin{bmatrix} 1 - \lambda dt & \lambda dt \\ 0 & 1 \end{bmatrix}.$$

This indicates that during an infinitesimal time interval $dt,$ there is a probability λdt that the booming incumbent technology market, $(\mu_1, \sigma_1),$ will shift to a recessive incumbent technology market, (μ_2, σ_2). Conversely, during the infinitesimal time interval dt, there is a probability $1 - \lambda dt$ that a regime-switch will not occur, and the incumbent technology market will continue in regime one.

Additionally, project scale is denoted by the state variable $K_{\epsilon(t)}$ when the firm has discretion over investment timing. However, when a firm exercises investment in a now-or-never investment opportunity, the capacity state variable is denoted by $\overline{K}_{\epsilon(t)}$. What is more, optimality is assumed by lower-case notation: $\tau_{\epsilon(t)}$ is the time at which the firm exercises the option to invest, $p_{\tau_{\epsilon(t)}}$ denotes the optimal investment threshold, and $k_{\epsilon(t)}$ $(\overline{k}_{\epsilon(t)})$ is the corresponding optimal capacity. In terms of costs, the firm must consider both an operating and an investment cost in order to effectively evaluate the investment opportunity. Over the production facility's lifetime, an operating cost component is assumed of the incumbent technology that is denoted by the deterministic variable, c. Likewise, the fixed and irrecoverable investment cost I is considered linked to capacity as displayed in (30).

$$I\left(K_{\epsilon(t)}\right) = aK_{\epsilon(t)} + bK_{\epsilon(t)}{}^{\gamma} \text{ and } \gamma > 1 \tag{30}$$

In Equation (30), a and b are regarded as constants whereas the parameter γ implies that the project investment costs exhibit diseconomies of scale as capacity sizing increases. The fundamental basis of this assumption is commonly seen in both the renewable energy industry as well as in a monopsonistic buyer environment in which a firm contemplates investment facing increasing prices due to increasing demand (Chronopoulos et al., 2015, Bøckman et al., 2008).

Moreover, immediate investment into additional capacity is considered the opportunity cost of foregoing investment into a risk-free asset with a constant rate of return, $r > 0$. Therefore, the net value of the investment project, $\varphi_{\epsilon(t)}$, takes this into consideration by aggregating the net present value of immediate investment, $G_{\epsilon(t)}$, the deterministic operating cost component, $cK_{\epsilon(t)}$, as well as the opportunity cost of investment, $rI(K_{\epsilon(t)})$. In the case that the firm chooses to delay investment into additional capacity, the firm then holds the perpetual option, $F_{\epsilon(t)}$, to invest in an incumbent technology project. Moreover, it is assumed that once exercise of the option has occurred, production starts instantaneously at full capacity with no operational flexibility to respond to exogenous demand factors. This assumption linked to production flexibility is referenced as the clearance assumption, and empirical evidence supporting this claim can be found in numerous pieces of literature (Chod and Rudi, 2005, Chronopoulos et al., 2015). For example, large integrated steel facilities exemplify this condition due to cost barriers to exit the steel industry. Pressure to cover fixed costs, the integrated steel industry's continuous production technology, and the high cost of shutting down furnaces reinforce the producer's resolve to continue production as normal. Hence, these obstacles induce integrated steelmakers to continue stable production in the face of diminishing returns (Madar, 2009).

5.2 The Model

In this section, an analytical framework for an incumbent technology, production capacity investment decision will be developed. In order to do so, the investment decision will be modelled through backward induction as an optimal stopping problem where the solution will be proposed through the usage of the Bellman equation, as well as the value-matching and smooth-pasting conditions. Furthermore, analytical expressions governing the value of the option to invest, the optimal investment threshold and the corresponding optimal capacity under price and technological uncertainty will be derived. In order to account for technological uncertainty, regime-switching will be incorporated into the model.

5.2.1 Regime 2

First off, an expression governing the firm's optimization objective is derived as indicated in (31). The inner maximization of (31) represents the net payout from immediate investment in the project. As the output price, P_t, is fixed and known at the time of investment, the analytical expression is maximized with respect to the capacity of the project, $\overline{K}_2$. Furthermore, the left-hand side of the maximization represents the option to invest. According to the Bellman principle, an investment opportunity over an infinitesimal time interval, $dt,$ is the equivalent of the expected rate of capital appreciation of an asset (Dixit and Pindyck, 1994), which is further interpreted according to the price process, $P + dP$. By combining these two respective valuations together, the firm can then derive the optimal stopping policy analogous to the same mechanism in which a financial call option is exercised.

$$F_2(P) = \max\left\{\mathbb{E}_P[F_2(P+dP)e^{-\rho dt}], \max_{\overline{K}_2} \varphi_2(P, \overline{K}_2)\right\} \quad (31)$$

In order to provide an overarching view of the optimal stopping policy, the dynamics of the option to invest are outlined in Figure 2. At time, τ_2, the firm exercises the option to invest at the corresponding output price, P_{τ_2}, and receives the expected value of a project with perpetual lifetime. Simultaneously, at the time of investment, managerial discretion over project scale is exercised, choosing capacity at K_2. Consequently, the resulting cash flows are determined by the project scale, which are, as will be determined, a function of the current output price.

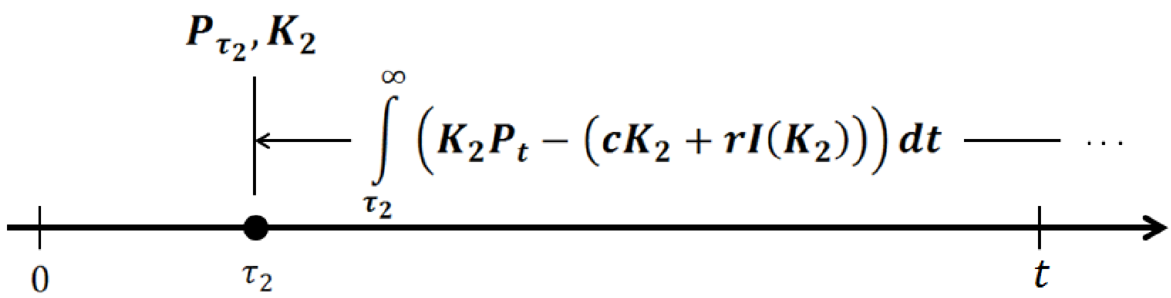

Figure 2: The Optimal Stopping Policy

Now-or- Never Investment

The immediate investment decision in regime two is characterized by the inner maximization of Equation (31). In this scenario, the firm is assumed to ignore the possibility to delay investment into the project, and takes the view that the investment is a now-or-never proposition; if the firm does not undertake the investment immediately, the investment proposition will cease to exist. The net value of the investment project, $\varphi_2,$ is derived by combining the project's stochastic revenue stream, and both the deterministic operating cost of the project as well as the opportunity cost of investment. The expectation operator, $\mathbb{E}_P$, conditional on the initial output price, P, of the price process allows an estimation to be made of the project's net present value based on its Markov properties and independent increments. Secondly, in order to discount the cash flows to the current timeframe, a subjective discount rate chosen by the firm is implemented represented on the left-hand side of (32). Lastly, because the investment decision is taken during the time frame in which a regime-switch has already occurred, the expected value of the exogenous output price process can be solved by referencing the geometric Brownian motion property exemplified in Equation (11).

$$\varphi_2(P) = \mathbb{E}_P\left[\int_0^\infty e^{-\rho t}\left(P_t\bar{K}_2 - \left(c\bar{K}_2 + rI(\bar{K}_2)\right)\right)dt\right] \tag{32}$$

As such, both the stochastic revenue stream and the aggregate costs of the project are effectively discounted at the firm's subjective discount rate over the lifetime of the project, and the net present value for a now-or-never investment becomes a function of project scale and the exogenous output price as derived in (33).

$$\varphi_2(P, \bar{K}_2) = \frac{P\bar{K}_2}{\rho - \mu_2} - \frac{c\bar{K}_2 + rI(\bar{K}_2)}{\rho} \tag{33}$$

Subsequently, as the analytical expression in (33) is exclusively reliant on managerial discretion over capacity, the firm must then determine the corresponding optimal capacity, $\bar{k}_2$, to maximize the value of the now-or-never investment decision. Hence, in order to find the optimal investment size, the partial derivative of the net value function with respect to production capacity is taken as represented below in (34). As this occurs when the marginal value of an extra unit of production capacity equals its marginal costs, the partial derivative is set equal to zero to uphold this condition (Bøckman et al., 2008).

$$\frac{\partial\left(\varphi_2(P, \bar{K}_2)\right)}{\partial \bar{K}_2} \rightarrow \frac{P}{\rho - \mu_2} - \frac{c + r\left(a + \gamma b \bar{k}_2^{\gamma - 1}\right)}{\rho} = 0 \tag{34}$$

Through algebraic rearrangements, the corresponding optimal capacity is then isolated, and the resulting analytical expression becomes a function of the current output price as summarized in (35). As the residual terms are assumed to be constant and known, the derivation provides a useful, observable function that serves as the optimal investment rule, and can be implemented by the decision maker based solely on current observable information.

Proposition 5.1 *A firm's now-or-never investment decision under price uncertainty has an optimal capacity defined as:*

$$\max_{\bar{K}_2} \varphi_2(P, \bar{K}_2) \rightarrow \bar{k}_2(P) = \left[\frac{1}{rb\gamma}\left(\frac{\rho P}{\rho - \mu_2} - c - ra\right)\right]^{\frac{1}{\gamma - 1}} \tag{35}$$

The Value of Waiting

The option to invest in regime two is represented by the left-most term in the outer maximization of Equation (31). In this scenario, the firm is assumed to have the option to defer investment in an incumbent technology project for the possibility of new information to arrive that might affect the desirability or timing of the expenditure. The firm's optimization objective is then further partitioned according to the optimal investment threshold, p_{τ_2}, as indicated in Equation (36). On the first branch on the right-hand side of Equation (36), the option to invest is modelled by selecting a time interval, $[t, t + dt]$, on which the option continues to be held and decomposing the investment opportunity into two components: its immediate payout, and its continuation value (Dixit and Pindyck, 1994). Considering the investment opportunity generates no cash flow until exercise, the option value comprises solely of its continuation value. In accordance with the Bellman principle, the value of the option then captures the discounted expected value of the capital appreciation of the incumbent technology project. Hence, the expected continuation value of holding the option beyond the infinitesimal time interval dt is discounted with the right-most term of the first branch, $e^{-\rho dt}$. Furthermore, as the option is a function of the current exogenous output price, the fluctuations in output price, dP, may accurately denote the stochastic nature of the capital appreciation of the incumbent technology project. On the second branch on the right-hand side of Equation (36), the current output price surpasses the optimal investment threshold, and indicates that immediate investment is optimal as the value of the project is greater than the value of holding the option to invest. In accordance with the firm's optimization objective, the corresponding optimal capacity as derived in Equation (35) is then chosen to maximize the expected net present value of the project.

$$F_2(P) = \begin{cases} \mathbb{E}_P[F_2(P + dP)e^{-\rho dt}] \;, & P < p_{\tau_2} \\ \dfrac{Pk_2}{\rho - \mu_2} - \dfrac{ck_2 + rI(k_2)}{\rho}, & P \geq p_{\tau_2} \end{cases} \tag{36}$$

In order to obtain a general solution to the first branch on the right-hand side of Equation (36), the expression, $\mathbb{E}_P[F_2(P + dP)e^{-\rho dt}]$, can then be expanded and simplified using Itô's Lemma and a Taylor series expansion at $-\rho dt = 0$ (see Appendix: $F_2(P)'s$ Differential Equation). Consequently, the resulting Bellman equation governing the solution takes the form of a second-order, homogenous, Cauchy-Euler differential equation as indicated below in (37).

$$\frac{1}{2}\sigma_2^2 P^2 \frac{d^2F_2}{dP^2} + \mu_2 P \frac{dF_2}{dP} - \rho F_2 = 0 \tag{37}$$

By noting that Equation (37) is linear in its dependent variable, F_2, and its derivatives, its general solution can then be expressed as a linear combination of any two independent solutions (Dixit and Pindyck, 1994) as in Equation (38). However, as the price process approaches zero and due to the negative root, β_2, the solution goes to infinity, $P^{\beta_2} \to \infty$. Consequently, the corresponding independent solution's endogenous constant must compensate for this condition, $B = 0$. Equation (38) then simplifies to solely to an independent solution with the positive root, β_1, and the corresponding endogenous constant A.

$$\begin{aligned} F_2(P) &= AP^{\beta_1} + BP^{\beta_2}, \text{ where } \beta_1 > 1 \text{ and } \beta_2 < 0 \\ &= AP^{\beta_1} \end{aligned} \tag{38}$$

As such, through the substitution of the general solution derived in Equation (38) for the first branch on the right-hand side of Equation (36), the firm's maximized net present value of its investment strategy can then be expressed as indicated in Equation (39).

$$F_2(P) = \begin{cases} AP^{\beta_1} \;, & P < p_{\tau_2} \\ \dfrac{Pk_2}{\rho - \mu_2} - \dfrac{ck_2 + rI(k_2)}{\rho}, & P \geq p_{\tau_2} \end{cases} \tag{39}$$

In order to determine the endogenous constant, A, and the optimal investment threshold, p_{τ_2}, the firm's maximization objective can then be used to derive the value-matching and smooth-pasting conditions implemented in Equation (40) respectively.

Conceptually, the first branch of Equation (40) illustrates that the value of the option to invest must match the net value obtained by its exercise, and the second branch of Equation (40) indicates that the project's present value must meet tangentially at the optimal investment threshold (Dixit and Pindyck, 1994).

$$\begin{cases} A{p_{\tau_2}}^{\beta_1} = \dfrac{p_{\tau_2}k_2}{\rho - \mu_2} - \dfrac{ck_2 + rI(k_2)}{\rho} \\ \beta_1 A{p_{\tau_2}}^{\beta_1 - 1} = \dfrac{k_2}{\rho - \mu_2} \end{cases} \tag{40}$$

Through algebraic rearrangements of the value-matching condition in Equation (40), the endogenous constant, A, can then easily be derived as indicated in Equation (41).

$$A = \frac{1}{p_{\tau_2}^{\beta_1}}\left[\frac{p_{\tau_2}k_2}{\rho - \mu_2} - \frac{ck_2 + rI(k_2)}{\rho}\right] \tag{41}$$

In order to derive the optimal investment threshold, the endogenous constant A is then substituted into the smooth-pasting condition of Equation (40) to garner the expression indicated in (42).

$$\beta_1\left(\frac{1}{p_{\tau_2}^{\beta_1}}\left[\frac{p_{\tau_2}k_2}{\rho - \mu_2} - \frac{ck_2 + rI(k_2)}{\rho}\right]\right){p_{\tau_2}}^{\beta_1 - 1} = \frac{k_2}{\rho - \mu_2} \tag{42}$$

Through numerous simplifications (see Appendix: Deriving the Optimal Investment Threshold), the optimal investment threshold can then be expressed as a function of project scale as indicated below.

$$p_{\tau_2}(k_2) = \frac{\beta_1(\rho - \mu_2)}{(\beta_1 - 1)}\frac{ck_2 + rI(k_2)}{\rho k_2} \tag{43}$$

In order to complete the solution, the corresponding optimal capacity at any point in the price process is then found by leveraging the now-or-never investment condition previously derived in Equation (35). By inserting Equation (43) into Equation (35) as displayed in Equation (44), the corresponding optimal capacity to any current price level can be found.

$$k_2 = \left[\frac{1}{rb\gamma}\left(\frac{\rho}{\rho - \mu_2}\left(\frac{\beta_1(\rho - \mu_2)}{(\beta_1 - 1)}\frac{ck_2 + rI(k_2)}{\rho k_2}\right) - c - ra\right)\right]^{\frac{1}{\gamma - 1}} \tag{44}$$

Through simplifications of Equation (44) (see Appendix: Deriving the Corresponding Optimal Capacity), an analytical expression governing the corresponding optimal capacity is found in (45).

$$k_2 = \left[\frac{c+ra}{rb}\,\frac{1}{\gamma(\beta_1-1)-\beta_1}\right]^{\frac{1}{\gamma-1}}, \qquad \gamma(\beta_1-1)-\beta_1>0 \tag{45}$$

Optimal Stopping

In order to gain a deeper understanding of the dynamics of the option to invest, the problem is also formulated as an optimal stopping problem. Referencing the integration displayed in Figure 2, the expectation operator is taken over the set of stopping times S generated by the Markov-modulated geometric Brownian motion augmented by the P-null sets as indicated in Equation (46).

$$F_2(P) = \sup_{\tau_2 \in S} \mathbb{E}_P\left[\int_{\tau_2}^{\infty} \Big(K_2P_t - \big(cK_2 + rI(K_2)\big)\Big)\,dt\right] \tag{46}$$

In order to make the appropriate integration transformation, the integral's bounds are then redefined according to the law of iterated expectations and the strong Markov property of geometric Brownian motion (Chronopoulos et al., 2015, Dias, 2004, Dixit and Pindyck, 1994), which states the dependence of the exogenous output price's movements rely solely on output price information available at the time of option exercise, P_{τ_2}. By doing so, the discount factor can then be factored out of the integration while simultaneously accounting for the stochastic nature of exercise. In doing so, the time at which the decision to exercise ceases to affect the integral's bounds as they are accounted for in the left-most argument in Equation (47). Hence, by factoring the stochastic discount factor, the calendar date τ_2 no longer bounds the integral, and the integration can be redefined over the perpetual time frame $[0,\infty)$.

$$F_2(P) = \sup_{\tau_2 \in S} \mathbb{E}_P[e^{-\rho\tau_2}]\,\mathbb{E}_{P_{\tau_2}}\left[\int_{0}^{\infty} \Big(K_2P_t - \big(cK_2 + rI(K_2)\big)\Big)\,dt\right] \tag{47}$$

To evaluate the expected value of the stochastic discount factor, the condition $P \leq P_{\tau_2}$ is assumed as well as observations are made of its respective boundary conditions (for further clarifications, see Appendix: The Expected Value of the Stochastic Discount

Factor). After application of the necessary conditions and assumptions, the stochastic discount factor equates to the leftmost term in (48), and the resulting integration becomes an unconstrained maximisation problem.

$$F_2(P) = \max_{P_{\tau_2} \geq P} \left(\frac{P}{P_{\tau_2}}\right)^{\beta_1} \left[\frac{P_{\tau_2} K_2}{\rho - \mu_2} - \frac{cK_2 + rI(K_2)}{\rho}\right] \tag{48}$$

Using this result, the endogenous constant, A, can then be determined through superimposing both the form of the general solution in (38) and the maximized net value of the option to invest as indicated in (49).

$$\begin{gathered} AP^{\beta_1} = \left(\frac{P}{p_{\tau_2}}\right)^{\beta_1} \left[\frac{p_{\tau_2} k_2}{\rho - \mu_2} - \frac{ck_2 + rI(k_2)}{\rho}\right] \\ \rightarrow A = \frac{1}{p_{\tau_2}^{\beta_1}} \left[\frac{p_{\tau_2} k_2}{\rho - \mu_2} - \frac{ck_2 + rI(k_2)}{\rho}\right] \end{gathered} \tag{49}$$

Secondly, in order to find an expression governing the optimal investment threshold, the maximization of the net value of the option $F_2(P)$ is taken with respect to the exogenous output price at the time of investment, P_{τ_2}, while noting that the condition for optimal capacity choice is a function of the exogenous output price at the time of investment. Hence, in order to find the first-order necessary condition, the product differentiation rule and the chain rule are applied to Equation (48), and the resulting first order necessary condition is displayed below in Equation (50).

$$\begin{aligned} \max_{p_{\tau_2},\, k_2(p_{\tau_2})} F_2(P) \rightarrow \beta_1 \left(-\frac{1}{p_{\tau_2}}\right) \left(\frac{P}{p_{\tau_2}}\right)^{\beta_1} \left[\frac{p_{\tau_2} k_2}{\rho - \mu_2} - \frac{ck_2 + rI(k_2)}{\rho}\right] \\ + \left(\frac{P}{p_{\tau_2}}\right)^{\beta_1} \left[\frac{k_2}{\rho - \mu_2} + k_2' \left(\frac{p_{\tau_2}}{\rho - \mu_2} - \frac{c + ra + b\gamma {k_2}^{\gamma - 1}}{\rho}\right)\right] = 0 \end{aligned} \tag{50}$$

Noting that the right-most term of Equation (50) contains the previously derived condition for optimal capacity choice as in Equation (34), the right-most term cancels to simplify the first-order necessary condition to:

$$\beta_1 \left(-\frac{1}{p_{\tau_2}}\right) \left(\frac{P}{p_{\tau_2}}\right)^{\beta_1} \left[\frac{p_{\tau_2} k_2}{\rho - \mu_2} - \frac{ck_2 + rI(k_2)}{\rho}\right] + \left(\frac{P}{p_{\tau_2}}\right)^{\beta_1} \left[\frac{k_2}{\rho - \mu_2}\right] = 0. \tag{51}$$

Through further algebraic simplifications, an analytical expression for the optimal investment threshold can then be derived akin to Equation (43), and the process to obtain the corresponding optimal capacity is replicated from Equation (44).

5.2.2 Regime 1

For the dynamics of investment in regime one, the real options approach must take into consideration not only the stochastic nature of the exogenous output price, but additionally, the likelihood that incumbent technology market conditions will be affected by the development of a disruptive technology. As such, the firm's optimization objective, indicated in Equation (52), must weigh the effect both price and technological uncertainty have on the value of the expected net present value of the maximization. On the first and second branch on the right-hand side of Equation (52), the firm's option to invest and the net present value of a now-or-never investment are modelled respectively.

$$F_2(P) = \max \begin{Bmatrix} \lambda dt \mathbb{E}_P[F_2(P + dP)e^{-\rho dt}] + (1 - \lambda dt)\mathbb{E}_P[F_1(P + dP)e^{-\rho dt}], \\ \max_{\bar{K}_2} \varphi_1(P, \bar{K}_1) \end{Bmatrix} \tag{52}$$

In order to provide a comprehensive view of the investment problem in regime one, the investment under price and technological uncertainty is illustrated as in Figure 3. As before, at time, τ_1, a firm exercises the option to invest at the corresponding current output price, P_{τ_1}, and receives the expected value of a project with perpetual lifetime while simultaneously choosing capacity at K_1. However, the resulting cash flows must take into consideration the likelihood of a future regime-switch, λ, indicated by the broken, one-way arrow.

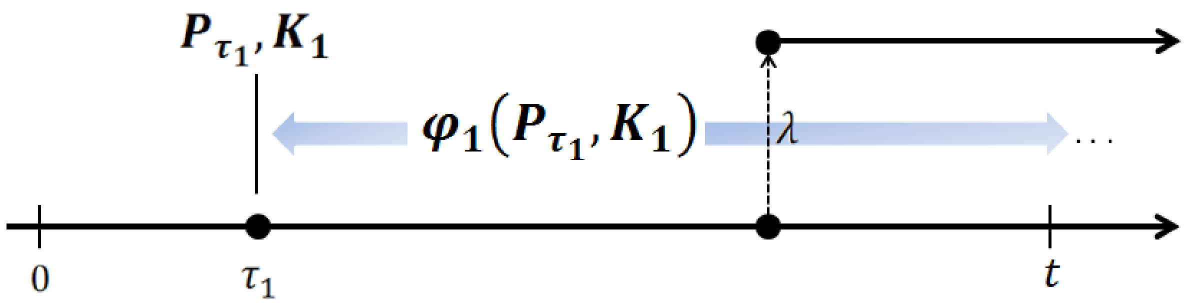

Figure 3: Investment under Price and Technological Uncertainty

Now-or-Never Investment

The immediate investment decision in regime one is again characterized by the maximization of the production capacity in an incumbent technology project. However, in this scenario, in addition to ignoring the possibility to delay investment, the firm must also weigh the possibility that the incumbent technology market conditions will be affected by the development of a disruptive technology. As such, the net value of the investment project, φ_1, is derived by incorporating a simultaneous system of ordinary differential equations governing the effects of a regime-switch. In order to observe these dynamics, irrespective of which regime the firm is operating within, the net value of immediate investment is defined as in Equation (53). In this decomposition, at time t, the net value of immediate investment in the incumbent technology project can be expressed as the sum of the operating profits over the infinitesimal time interval $[t, t + dt]$ and the continuation value of the project beyond the point $[t + dt]$. Consequently, the operating profits received upon immediate investment can be expressed as the revenue stream, $K_{\epsilon(t)}Pdt$, and the continuation value of the net present value of immediate investment can be expressed as the discounted, expected value of immediate investment, $\mathbb{E}_P\left[G_{\epsilon(t)}(P + dP)e^{-\rho dt}\right]$.

$$G_{\epsilon(t)}(P) = K_{\epsilon(t)}Pdt + \mathbb{E}_P\left[G_{\epsilon(t)}(P + dP)e^{-\rho dt}\right] \tag{53}$$

In order to derive the net present value of immediate investment in regime one, notice that within an infinitesimal time interval dt, there will be a regime-switch with probability λdt, or a continuation of operations in regime one with a probability of $1 - \lambda dt$. Hence, it follows that the expectation of the continuation value must be decomposed into two components to effectively accommodate for these two possible future outcomes indicated in Equation (54). Accordingly, the argument, $\lambda dt \mathbb{E}_P[G_2(P + dP)e^{-\rho dt}]$, represents the expectation of the continuation value of the project's revenue streams in regime two, whereas the right-most argument indicates the continuation value of the incumbent technology project in regime one.

$$G_1(P) = K_1Pdt + \lambda dt \mathbb{E}_P[G_2(P + dP)e^{-\rho dt}] + (1 - \lambda dt)\mathbb{E}_P[G_1(P + dP)e^{-\rho dt}] \tag{54}$$

Secondly, in order to find the net present value of immediate investment in regime two, Equation (53) is referenced, and, as a result, by adapting its logic and noting that the optimal capacity in regime two has already been derived, an expression governing the form of the solution in regime two can be derived as in Equation (55).

$$G_2(P) = k_2 P dt + \mathbb{E}_P[G_2(P + dP)e^{-\rho dt}] \tag{55}$$

Expanding Equation (54) and Equation (55) using Itô's Lemma (see Appendix: Deriving the Simultaneous System of Ordinary Differential Equations) yields the simultaneous system of ordinary differential equations indicated in (56).

$$\begin{cases} \frac{\sigma_1^2}{2} P^2 \frac{d^2 G_1}{dP^2} + \mu_1 P \frac{dG_1}{dP} - \rho G_1 + \lambda(G_2 - G_1) + K_1 P = 0 \\ \frac{\sigma_2^2}{2} P^2 \frac{d^2 G_2}{dP^2} + \mu_2 P \frac{dG_2}{dP} - \rho G_2 + k_2 P = 0 \end{cases} \tag{56}$$

Borrowing from the conjecture proposed by Goto et al. (2012), the functions that satisfy the simultaneous system of ordinary differential equations in (56) take a linear format as indicated in (57). Intuitively, as can be observed from Equation (33), the residual term in the revenue stream, $\pi_{\epsilon(t)}$, aims to define the factor by which the expected value of the revenue stream is discounted.

$$\begin{cases} G_1(P) = \pi_1 K_1 P \\ G_2(P) = \pi_2 k_2 P \end{cases} \tag{57}$$

By taking the first- and second-order derivatives of Equation (57) with respect to production capacity in each regime (see Appendix: Deriving the Discount Factor Function), the discount factor function in regime one, π_1, is derived. Indicated in Equation (58), π_1 can be interpreted as the effects of the development of a disruptive technology on the net present value of immediate investment in an incumbent technology. Interpretation by this format yields a function that effectively incorporates both the likelihood of a regime-switch, λ, and the respective growth conditions in each market, μ_1 and μ_2.

$$\pi_1 = \frac{\lambda k_2 + K_1(\rho - \mu_2)}{K_1(\rho + \lambda - \mu_1)(\rho - \mu_2)} \tag{58}$$

As such, a function governing the net value of the investment project, φ_1, can be proposed as indicated in (59). The first argument represents the project's discounted stochastic revenue stream, whereas the second argument again represents both the discounted deterministic operating cost of the project as well as the opportunity cost of the investment.

$$\varphi_1(P;\bar{K}_1) = \pi_1\bar{K}_1P - \frac{c\bar{K}_1 + rI(\bar{K}_1)}{\rho}$$
$$= \frac{\left(\lambda k_2 + \bar{K}_1(\rho-\mu_2)\right)P}{(\rho+\lambda-\mu_1)(\rho-\mu_2)} - \frac{c\bar{K}_1 + rI(\bar{K}_1)}{\rho} \tag{59}$$

As in regime two, the maximization of the value of immediate investment in regime one occurs when the marginal benefit of an extra unit of production capacity equals its marginal cost (Bøckman et al., 2008). As this is reliant on managerial discretion over capacity, the firm must then choose the optimal capacity, $\bar{k}_1$, so as to uphold this condition. Consequently, the partial derivative of the net value function with respect to production capacity is taken and set equal to zero as represented in Equation (60).

$$\frac{\partial\left(\varphi_1(P;\bar{K}_1)\right)}{\partial\bar{K}_1} \rightarrow \frac{P}{(\rho+\lambda-\mu_1)} - \frac{c + r\left(a + \gamma b\bar{k}_1^{\ \gamma-1}\right)}{\rho} = 0 \tag{60}$$

Through algebraic rearrangements of Equation (60), the optimal capacity is isolated, and the resulting analytical expression becomes a function of the current output price as summarized in (61). As the residual terms are assumed to be constant and known, the derivation provides a function that serves not only as the optimal investment rule under now-or-never investment conditions, but also, can further be used to investigate the impact technological uncertainty has on the now-or-never, optimal capacity.

Proposition 5.2 *A firm's now-or-never investment decision under price and technological uncertainty has an optimal capacity defined as:*

$$\max_{\bar{K}_1}\varphi_1(P,\bar{K}_1) \rightarrow \bar{k}_1(P) = \left[\frac{1}{rb\gamma}\left(\frac{\rho P}{(\rho+\lambda-\mu_1)} - c - ra\right)\right]^{\frac{1}{\gamma-1}} \tag{61}$$

By differentiating the expression of the optimal capacity in Equation (61) with respect to the transition probability, λ, the resulting derivative in Equation (62) exemplifies the relationship between optimal capacity and the transition probability of a regime-switch.

$$\frac{\partial\bar{k}_1}{\partial\lambda} = \left(\frac{\bar{k}_1}{\gamma-1}\right)\left(\frac{\rho P}{(\rho+\lambda-\mu_1)} - c - ra\right)^{-1}\left(\frac{-\rho P}{(\rho+\lambda-\mu_1)^2}\right) \tag{62}$$

More specifically, noting that the first argument is positive and the right-most argument in Equation (62) is negative, the impact technological uncertainty has on optimal capacity is defined by the residual term, $\frac{\rho P}{(\rho+\lambda-\mu_1)} - c - ra$. In order to more closely scrutinize this relationship, Equation (63) constructs this relationship by separating the terms to model both the marginal deterministic cost component as well as the marginal, discounted revenue stream.

$$\frac{P}{(\rho+\lambda-\mu_1)} = \frac{c+ra}{\rho} \tag{63}$$

As the marginal, discounted revenue stream must be greater than the marginal, deterministic cost component of the project for a firm to legitimize now-or-never investment, $\frac{P}{(\rho+\lambda-\mu_1)} > \frac{c-ra}{\rho}$, Proposition 5.3 indicates that the rate at which the now-or-never optimal capacity choice changes will be decreasing with increasing levels of technological uncertainty.

Proposition 5.3 $\frac{\partial \bar{k}_1}{\partial \lambda} < 0$.

The Value of Waiting

The option to invest in regime one is represented by the first branch on the right hand-side of the outer maximization in Equation (52). In this scenario, the firm is assumed to have the option to defer investment into an incumbent technology project for the possibility of new information to arrive in regards to both price and technological uncertainty. The firm's optimization objective is then further partitioned according to the optimal investment threshold, p_{τ_1}, as indicated in Equation (64). The value of the option, $F_1(P)$, reflects these implications and can be modelled by selecting a time interval, $[t, t+dt]$, on which the option continues to be held and decomposing it according to the project's discounted, expected capital appreciation. For the dynamics of the value of the option to invest, notice that within an infinitesimal time interval dt, there will be a regime-switch with probability λdt, or a continuation of the current regime with probability $1 - \lambda dt$. In the former case, the firm will hold the option to invest in regime one, $F_1(P)$, and, in the latter case, the firm will receive the option to invest in regime two, $F_2(P)$. On the second branch on the right-hand side of Equation (64), the current output price surpasses the optimal investment threshold, and indicates that immediate investment becomes optimal at the derived optimal capacity as in Proposition 5.2.

$$F_1(P) = \begin{cases} \lambda dt \mathbb{E}_P[F_2(P+dP)e^{-\rho dt}] + (1-\lambda dt)\mathbb{E}_P[F_1(P+dP)e^{-\rho dt}], & P < p_{\tau_1} \\ \dfrac{\left(\lambda k_2 + k_1(\rho - \mu_2)\right)P}{(\rho + \lambda - \mu_1)(\rho - \mu_2)} - \dfrac{ck_1 + rI(k_1)}{\rho} \qquad\qquad , & P \geq p_{\tau_1} \end{cases} \tag{64}$$

Noting that $F_2(P)$ has already been determined in the previous section, the differential equation in (64) is expanded using both Itô's Lemma and a Taylor series expansion at $-\rho dt = 0$ (see Appendix: $F_1(P)'s$ Differential Equation), which results in (65). Consequently, the resulting Bellman equation governing the solution takes the form of a second-order, non-homogenous, Cauchy-Euler differential equation.

$$\frac{\sigma_1^2}{2} P^2 \frac{d^2 F_1}{dP^2} + \mu_1 P \frac{dF_1}{dP} - (\rho + \lambda)F_1 + \lambda F_2 = 0 \tag{65}$$

By noting that the differential equation in (65) must be solved for both its homogenous and non-homogenous components, a general and a particular solution is conjectured in Equation (66). From the fundamental quadratic equation, it is known that the linear combination of independent solutions has both a positive and negative root, indicated by $\delta_1 > 1$ and $\delta_2 < 0$, respectively. Hence, as aforementioned, the solution becomes undefined as the exogenous output price approaches zero with regards to the independent solution with the negative root. As a result, the endogenous component $\hat{B}$ is set equal to zero to circumvent this limitation, and, consequently, the term, $\hat{B}P^{\delta_2}$, drops out of the form of the solution. Hence, the remaining linear, independent solution with a positive root, BP^{δ_1}, where B is an endogenous coefficient, constitutes the homogenous component of the solution in (66). Additionally, the non-homogenous component of the solution stems from the transition probability of a regime-switch as indicated by the term, λF_2. As the value of the option to invest in regime two takes the analogous form of a call option, max [$AP^{\beta_1}, 0$], it is proposed using the method of undetermined coefficients that the non-homogenous component of the solution takes the form $\hat{A}P^{\beta_1}$.

$$\begin{aligned} F_1(P) &= \hat{A}P^{\beta_1} + BP^{\delta_1} + \hat{B}P^{\delta_2} \\ &= \hat{A}P^{\beta_1} + BP^{\delta_1} \end{aligned} \tag{66}$$

Through the conjectured solution in (66), Equation (65) is manipulated to find an analytical expression for the endogenous constant $\hat{A}$ as indicated in Equation (67) (see Appendix: Deriving the Endogenous Constant).

$$\hat{A} = \frac{-\lambda(A)}{\frac{\sigma_1^2}{2}\big(\beta_1(\beta_1 - 1)\big) + \mu_1(\beta_1) - (\rho + \lambda)} \tag{67}$$

As such, through the substitution of the solution derived in Equation (66) for the first branch on the right-hand side of Equation (64), the firm's maximized net present value of its investment strategy can then be expressed as indicated in Equation (68).

$$F_1(P) = \begin{cases} \hat{A}P^{\beta_1} + BP^{\delta_1} & , \quad P < p_{\tau_1} \\ \dfrac{\big(\lambda k_2 + k_1(\rho - \mu_2)\big)P}{(\rho + \lambda - \mu_1)(\rho - \mu_2)} - \dfrac{ck_1 + rI(k_1)}{\rho}, & \quad P \geq p_{\tau_1} \end{cases} \tag{68}$$

Due to the mathematical limitations on nonlinear equations, the value-matching and smooth-pasting conditions cannot be used to solve for an analytical solution. Instead, the endogenous constant B, the optimal investment threshold p_{τ_1}, and its corresponding optimal capacity k_1, are determined through iterative, numeric methods using Matlab in the following section.

However, theoretically, an encompassing investment strategy can be advised to the firm facing an investment decision under price and technological uncertainty. First off, when the current output price for an incumbent technology is below the optimal investment threshold, the firm continues to hold the option as the value of the option surpasses the value of immediate exercise. Simultaneously, holding the option to invest in an incumbent technology is a precursor of *discovery-driven planning* as it implicitly warns the firm of the incumbent technology's potential demise. Hence, according to the first and third organizational laws of disruptive technology, a firm contemplating investment into an incumbent technology while holding the option to invest should weigh the strategic implications of *discovery-driven planning*.

Remark 5.1 *As long as the option to invest in an incumbent technology is held under technological uncertainty, a firm should implement discovery-driven planning in disruptive technology.*

6. Numerical Examples

For the numerical examples, the base case scenario assumes that the growth rates in regime one and two are $\mu_1 = .02$ and $\mu_2 = .01$ with their respective uncertainty in price indicated by $\sigma_1 = .2$ and $\sigma_2 = .3$. Additionally, the firm's subjective discount rate is $\rho = .1$, and the risk-free rate of return is $r = .05$. Moreover, the cost parameters are $a = 20, b = .2, c = 10$, and $\gamma = 3$. Lastly, technological uncertainty is set at $\lambda = .01$. Under these price- and cost-related parameters, Figure 4 illustrates both the scenario in which the firm values the option to invest under the given price uncertainty as well as under more volatile market conditions, $\sigma_2 = .35$. Under the base case scenario, the smooth-pasting condition is graphically represented by the tangential point between the graphs of the option value and the project value with the optimal investment threshold $p_{\tau_2} = 44.4758$ and corresponding optimal capacity $k_2 = 35.7852$. Under more volatile market conditions, the optimal investment threshold increases substantially, $p_{\tau_2} = 66.392$, in addition to its corresponding optimal capacity, $k_2 = 45.7416$.

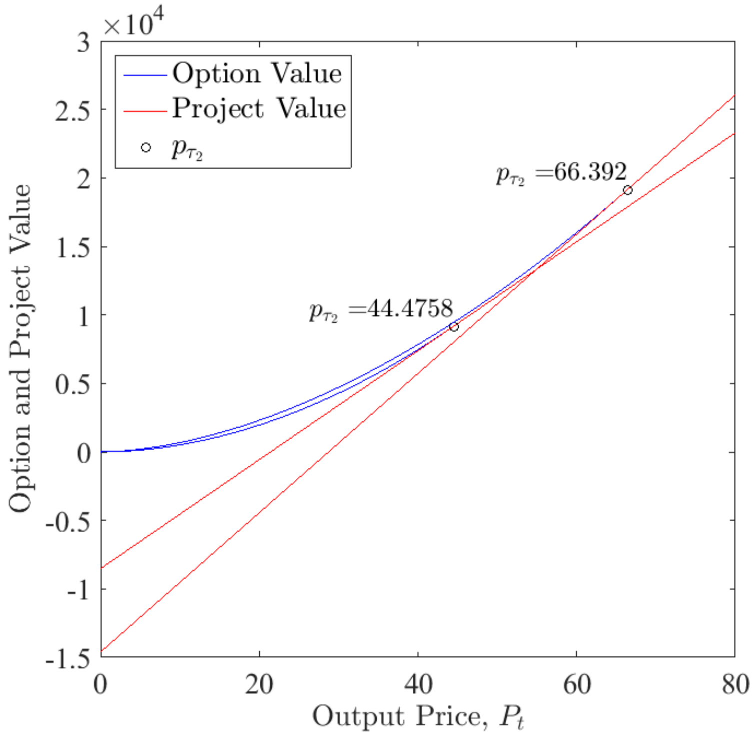

Figure 4: Option and project value in regime two: $\mu_2 = .01$ *and* $\sigma_2 = \{.3, .35\}$

Interestingly, by including technological uncertainty in regime one, Figure 5 indicates both a lower optimal investment threshold, $p_{\tau_1} = 19.5035$, and corresponding optimal capacity, $k_1 = 18.8596$, under base case scenario conditions. Further supporting this trend under more volatile market conditions, the firm's optimal policy depicts a lower optimal investment threshold $p_{\tau_1} = 27.0574$ with a corresponding optimal capacity $k_1 = 25.208$. Consequently, as the optimal investment threshold is significantly greater under solely price uncertainty, $p_{\tau_1} < p_{\tau_2}$, and the corresponding optimal capacity is significantly less under price and technological uncertainty, $k_1 < k_2$, these results revoke the standard real options intuition that in a more uncertain economic environment, uncertainty causes a firm to invest later and in larger capacity. Conversely, these results indicate that under price and technological uncertainty, the firm invests both earlier and in limited capacity.

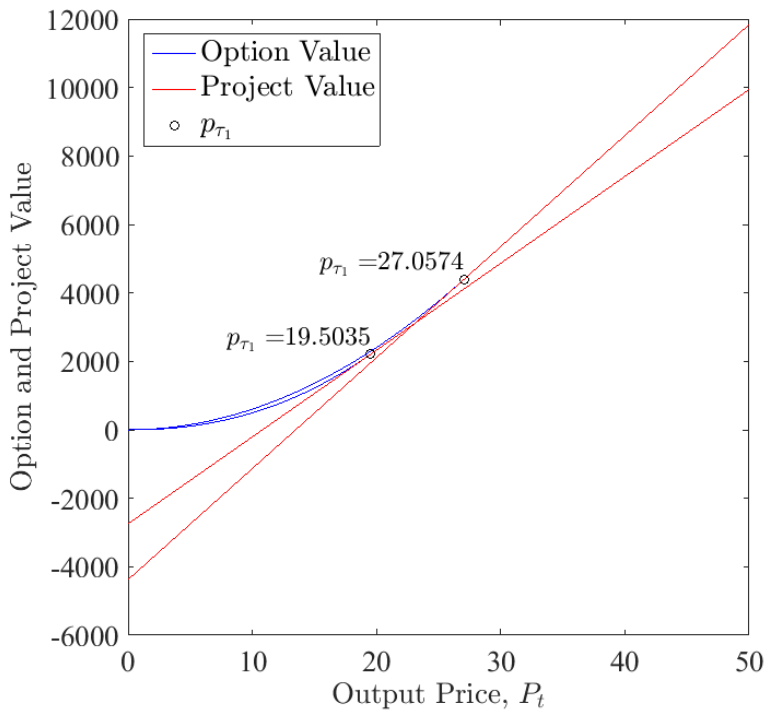

Figure 5: Option and project value in regime one: $\mu_1 = .02$ *and* $\sigma_1 = \{.2, .25\}$

Furthermore, in order to examine the robustness of these results, the impact of price uncertainty on both the optimal investment threshold and the corresponding optimal capacity are examined in each regime under base case scenario conditions. In regime two under volatilities of $\sigma_2 \in [.3, .4]$, the effect of price uncertainty on both the optimal investment threshold and the corresponding optimal capacity is illustrated in Figure 6. In line with standard real options intuition, the optimal investment threshold increases exponentially with higher levels of price uncertainty, $\frac{\partial p_{\tau_2}}{\partial \sigma_2} > 0$, and likewise, the corresponding optimal capacity also increases with higher levels of price uncertainty, $\frac{\partial k_2}{\partial \sigma_2} > 0$.

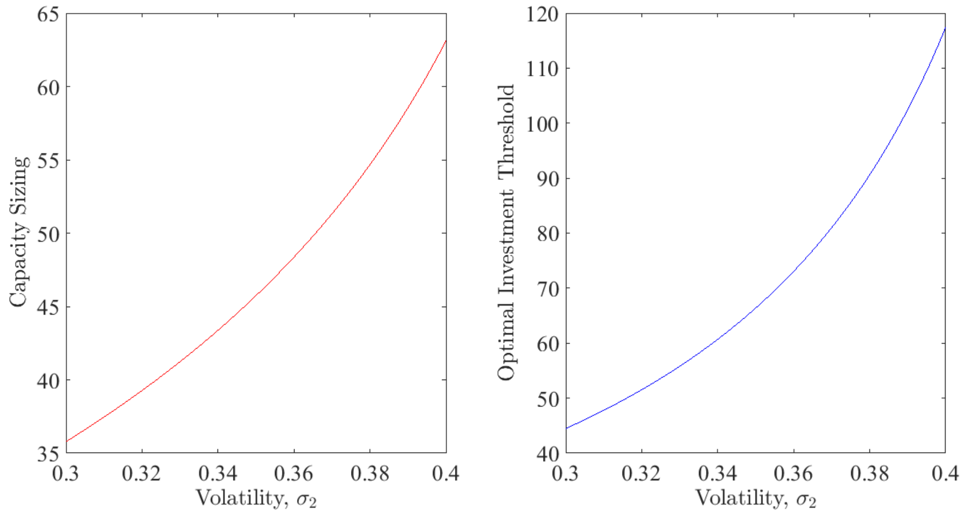

Figure 6: Capacity sizing and the optimal investment threshold in regime two with $\sigma_2 \in [.3, .4]$

Similarly, under identical values of price uncertainty in regime one, $\sigma_1 \in [.3, .4]$, the effect of price and technological uncertainty on the optimal investment threshold and the corresponding optimal capacity is illustrated in Figure 7. In this setting, the relationships between price uncertainty and the respective optimal policies indicate a positive trend as previously derived in regime two. However, in comparison to regime two, as can be discerned graphically at both the minimal and maximal values of the tested domain of σ_1, the optimal policies indicate that the start- and end-points are both lower and higher respectively. Additionally, further supporting a synergistic relationship among price and technological uncertainty, in the graphical representation of the optimal investment threshold and the corresponding optimal capacity, there is a sharper incline in which the exponential relationship between price uncertainty and the respective optimal policies are increasing.

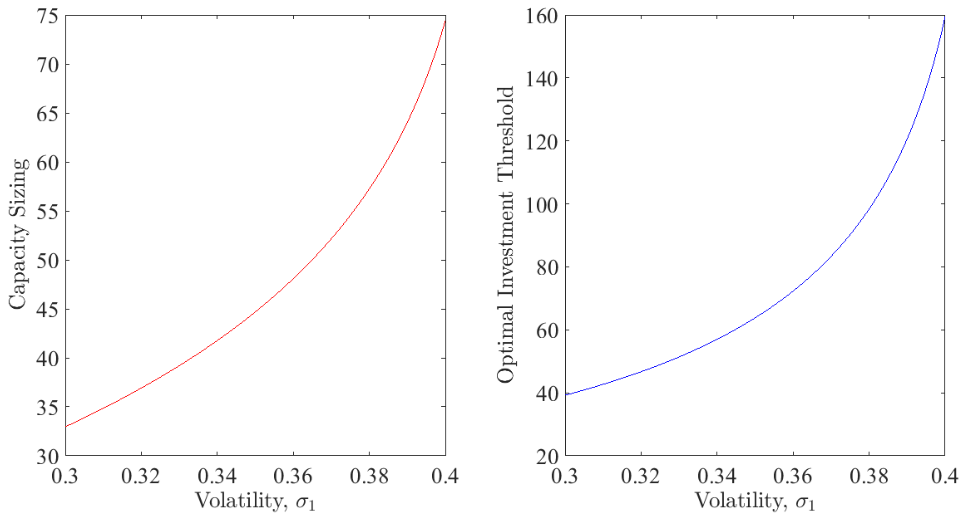

Figure 7: Capacity sizing and the optimal investment threshold in regime one with $\sigma_1 \in [.3, .4]$

Indeed, contradictory to standard real options intuition and in line with the previously derived numerical results, the graphical values displayed in Table 1 confirm the unique effect of multiple uncertainties in an irreversible, capacity investment decision. In the left-hand column, holding all other parameters fixed, the minimum and maximum values of the simulated price uncertainties from Figure 6 and Figure 7 are displayed, and following, the optimal investment thresholds and corresponding optimal capacities are noted in each subsequent column. As denoted in Corollary 6.1, although on average both the optimal investment threshold and capacity size are increasing, the effect of technological uncertainty interacts implicitly with varying levels of price uncertainty in a way such that the respective optimal policies increase with greater levels of price uncertainty and decrease with lower levels of price uncertainty relative to a context of solely price uncertainty.

Table 1: The effect of price uncertainty on the optimal investment policy

$\sigma_{\epsilon(t)}$	p_{τ_1}	p_{τ_2}	k_1	k_2
$.3$	*39.2255*	*44.4757*	*32.9565*	*35.7852*
$.4$	*159.4488*	*117.4418*	*74.4234*	*63.1112*

Corollary 6.1 *The impact of technological uncertainty on capacity sizing acts synergistically with price uncertainty to increase project scale at higher levels of price*

uncertainty and to decrease project scale at lower levels of price uncertainty. Moreover, optimal investment timing is affected in a similar format by which the delay in investment timing is subjugated by the level of price uncertainty.

In order to further elucidate the interaction between technological uncertainty and capacity sizing, the optimal policy of the firm in regime one is illustrated in Figure 8 under varying transition probabilities with real number solutions, $\lambda \in [0,.014]$. By noting that the optimal policy of the firm is monotonically decreasing as a function of technological uncertainty, the numerical results directly revoke the standard real options intuition that in a more uncertain economic environment, the firm has less incentive to invest and in a larger project. Conversely, indicated by Proposition 6.1, increasing levels of technological uncertainty decrease both the optimal capacity of the project and the optimal investment threshold. This is in contrast to Chronopoulos and Siddiqui (2015) who find that technological uncertainty has a non-monotonic impact on the optimal policy of a firm. The discrepancy in findings can be motivated by the investment strategy under which the optimal policy of the firm was evaluated under. Hence, modeling uncertainty under a lumpy investment strategy thereby suggests paradoxically that in a more uncertain economic environment, a firm both has both greater incentive to invest and in a smaller project.

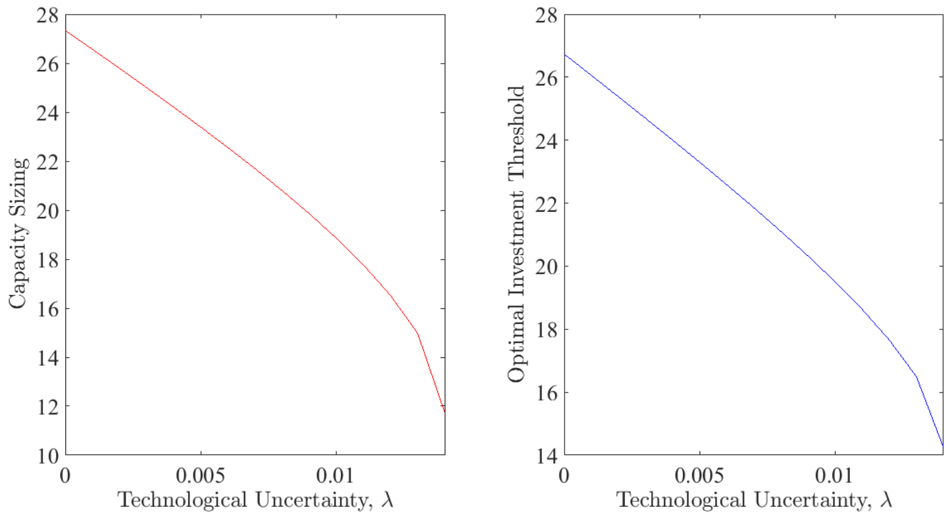

Figure 8: The impact of technological uncertainty on capacity sizing and the optimal investment threshold in regime one with $\lambda \in [0,.014]$

Proposition 6.1 $\frac{\partial k_1}{\partial \lambda} < 0$ and $\frac{\partial p_{\tau_1}}{\partial \lambda} < 0$.

7. Conclusion

This thesis set out to examine a firm's choice in project scale under both price and technological uncertainty, and, in order to address the potential disastrous effects of disruptive technology on a firm operating within an incumbent technology industry, established an analytical framework combining both regime-switching and real options. In the context of a continuous time model where investments have a lumpy structure, managerial insight was provided through the observation of the interactions between price and technological uncertainty as well as the optimal investment threshold and corresponding optimal capacity choice for a firm. Under these circumstances, the question of how an investment decision in capacity sizing is affected by price and technological uncertainty was scrutinized with the formulation of four propositions, one remark, a corollary, and numerical results to provide a comprehensive answer.

In further regards to the analytical results, conclusive derivations highlighting insight on the immediate investment decision were provided by both separately and simultaneously comparing technological and price uncertainty. Although results were confined by a nonlinear equation in regime one, the immediate investment decision in both scenarios was aided by deriving a guiding function where optimal capacity choice could be determined by the current output price of an incumbent technology. Furthermore, although expected, managerial insight was quantitatively reinforced by the conclusion that capital investment in an incumbent technology should decrease with increasing levels of technological uncertainty. Lastly, leveraging Christensen's theory of disruptive technology, further managerial insight was provided to the firm by advising *discovery-driven planning* when holding the option to invest under technological uncertainty.

In regards to the numerical results, the synergy between the two uncertainties was examined as well as their effect on the optimal investment threshold and the corresponding optimal capacity. Interestingly, although in standard capacity sizing literature it is generally concluded that uncertainty leads to delayed investment and larger optimal capacity sizing, this relationship was revoked when capacity sizing was examined under price and technological uncertainty. Indeed, under solely price uncertainty and managerial discretion over capacity sizing and timing, the conventional result was confirmed and graphically proven. However, by taking into consideration technological uncertainty and lower levels of price uncertainty, the optimal investment strategy, numerically derived, showed that a firm will generally invest earlier and limit capacity investment, whereas at higher levels of price uncertainty, a firm will invest even later and in larger capacity. Furthermore, revoking standard real options intuition and in contrast with Chronopoulos and Siddiqui (2015), it was shown that increasing levels of technological uncertainty directly correlate with a greater incentive to invest and in limited project scale. Hence, by revoking standard real options intuition and highlighted by the discrepancy in findings with Chronopoulos and Siddiqui (2015), this study emphasizes that the relationship between technological uncertainty and project scale requires further investigation.

As such, it is interesting to note the limitations of the model. Undeniably, although it is helpful to base managerial insight on an analytical framework with quantitative results, the complexity of the model poses severe restrictions on its overall applicability and utility. As the model can only support transition probabilities of $\lambda \in [0, .014]$ as well as a real solution set under specific parameter settings, it becomes exceedingly difficult to adapt the model for case-specific applications where a tailored-made solution can be provided for the firm. As this further limits the model's micro-level utility, managerial insight can only be broadly provided to the firm through implicit recommendations based on the aforementioned numerical results. To circumvent these shortcomings would require the respective uncertainties to be studied in a more deterministic sense or the firm would have to be willing to accept a solution with a complex numerical format. However, the consequences of such actions would severely constrict the managerial implications and insight that the model could realistically provide. Accordingly, as no simple solution exists, either a simpler model would need to be constructed to provide a full analytical solution or the trade-off between model complexity and design would have to be revisited with the firm to realistically decide an acceptable level of quantitative insight.

Nonetheless, in order to extend these results and to further examine the effect of technological uncertainty on project scale, there are many different avenues one could pursue based on this study. In the area of managerial flexibilities, one could additional see how allowing a firm the flexiblity to choose among differing investment strategies such as in Chronopoulos and Siddiqui (2015) could change the resulting optimal policy. Additionally, in order to examine the robustness of these results and further scrutinize flexbility, loosening of the clearnace assumption via discretion over the production decision as in Kort et al. (2011) could provide invalauble intuition to the firm's investment decision. Operational flexibility could also be examined as in Hagspiel et al. (2013) through the introduction of an abandonment option or, similarly, with the simultaneous relaxation of the clearance assumption, a suspension and resumption option could be introduced. Moreover, in a flexible context, game-theoretic considerations could also be considered as in Goto et al. (2012) to see how competitive analysis affects the firm's optimal policy. In terms of model parameters, although it would be expected to produce similar results in the majority of cases, modeling the output price akin to the inverse demand function used by Dangl (1999) would provide further interesting insight to capital intensive projects. Likewise, varying the type of stochastic process implemented in the model such as mean reversion or arithmetic Brownian motion could provide a means in which to further results as well as to provide the proper standpoint specific to the subjective, managerial view of output price. Lastly, in the area of uncertainty, as the model has a particular fit for the renewable energy industry given its convex investment cost function, environmental policy uncertainty would also be interesting to study, which could furthermore be extended by switching to a concave cost function to cover government policy uncertainty based on welfare analysis.

Appendix

$F_2(P)'s$ Differential Equation

The option to invest in regime two is as expressed in *(A- 1*).

$$F_2(P) = \mathbb{E}_P[F_2(P + dP)e^{-\rho dt}] \tag{A- 1}$$

In order to obtain an expression for $F_2(P)$, *(A- 1*) can be expanded using Itô's Lemma and a Taylor expansion at $-\rho dt = 0$. To do so, remark that a Taylor expansion at $-\rho dt = 0$ will obtain the expression in *(A- 2*).

$$e^{-\rho dt} = 1 + (-\rho dt) + \frac{(-\rho dt)^2}{2!} + \frac{(-\rho dt)^3}{3!} + \cdots \tag{A- 2}$$

Considering that higher order terms of dt reach the limit of zero at a more rapid pace than dt, the expected value of $e^{-\rho dt}$ can then be further simplified and factored out of the expression in *(A- 1*) as indicated in *(A- 3*). In order to denote the collection of higher order terms of dt, $o(dt)$ is included in the expression.

$$F_2(P) = (1 - \rho dt + o(dt))\mathbb{E}_P[F_2(P + dP)] \tag{A- 3}$$

Distributing the expectation in *(A- 3*), $F_2(P)$ can then be further partitioned as indicated in *(A- 4*).

$$F_2(P) = \mathbb{E}_P[F_2(P + dP)] - \rho dt\mathbb{E}_P[F_2(P + dP)] \tag{A- 4}$$

Using Itô's Lemma, *(A- 4*) expands and simplifies to *(A- 5*).

$$F_2(P) = F_2(P) + \frac{1}{2}\sigma_2^2 P^2 \frac{\partial^2 F_2}{\partial P^2} dt + \mu_2 P \frac{\partial F_2}{\partial P} dt - \rho dt F_2(P) + o(dt) \tag{A- 5}$$

Simplification of *(A- 5*), division by dt, and proceeding to the limit as $dt \to 0$, a second-order, homogenous, Cauchy-Euler differential equation is found as indicated in *(A- 6*).

$$0 = \frac{1}{2}\sigma_2^2 P^2 \frac{\partial^2 F_2}{\partial P^2} + \mu_2 P \frac{\partial F_2}{\partial P} - \rho F_2(P) \tag{A- 6}$$

Deriving the Optimal Investment Threshold

The smooth-pasting condition including the substitution of the endogenous constant A is indicated in *(A- 7)*.

$$\beta_1\left(\frac{1}{p_{\tau_2}^{\beta_1}}\left[\frac{p_{\tau_2}k_2}{\rho-\mu_2}-\frac{ck_2+rI(k_2)}{\rho}\right]\right)p_{\tau_2}{}^{\beta_1-1}=\frac{k_2}{\rho-\mu_2} \quad (A\text{- }7)$$

In order to isolate the optimal investment threshold, *(A- 7)* is rearranged through the distribution of β_1 and multiplication of p_{τ_2} as indicated in *(A- 8)*.

$$\frac{\beta_1 p_{\tau_2}k_2}{\rho-\mu_2}-\frac{\beta_1\big(ck_2+rI(k_2)\big)}{\rho}=\frac{p_{\tau_2}k_2}{\rho-\mu_2} \quad (A\text{- }8)$$

Grouping together like terms, *(A- 8)* further simplifies to *(A- 9)*.

$$(\beta_1-1)\frac{p_{\tau_2}k_2}{\rho-\mu_2}=\frac{\beta_1\big(ck_2+rI(k_2)\big)}{\rho} \quad (A\text{- }9)$$

An analytical expression for the optimal investment threshold is then found through algebra as indicated in *(A- 10)*.

$$p_{\tau_2}=\frac{\beta_1(\rho-\mu_2)}{(\beta_1-1)}\frac{ck_2+rI(k_2)}{\rho k_2} \quad (A\text{- }10)$$

Deriving the Corresponding Optimal Capacity

The now-or-never investment condition including the substitution of the optimal investment threshold p_{τ_2} is indicated in *(A- 11)*.

$$k_2=\left[\frac{1}{rb\gamma}\left(\frac{\rho}{\rho-\mu_2}\left(\frac{\beta_1(\rho-\mu_2)}{(\beta_1-1)}\frac{ck_2+rI(k_2)}{\rho k_2}\right)-c-ra\right)\right]^{\frac{1}{\gamma-1}} \quad (A\text{- }11)$$

Through the immediate simplification of *(A- 11)*, the resulting analytical expression is expressed as in *(A- 12)*.

$$k_2 = \left[\frac{1}{rb\gamma}\left(\left(\frac{\beta_1}{(\beta_1-1)}\left(c+r\left(a+bk_2^{\gamma-1}\right)\right)\right)-c-ra\right)\right]^{\frac{1}{\gamma-1}} \tag{A- 12}$$

In order to isolate the term, k_2, grouping together like terms, *(A- 12*) further simplifies to *(A- 13*).

$$rb\gamma\, k_2^{\gamma-1} - \frac{\beta_1}{(\beta_1-1)} rbk_2^{\gamma-1} = \left(\left(\frac{\beta_1}{(\beta_1-1)}(c+ra)\right)-c-ra\right) \tag{A- 13}$$

Through multiplication by the term, $\beta_1 - 1$, and the distribution of β_1, the expression becomes as indicated in *(A- 14*).

$$(\gamma(\beta_1-1)-\beta_1)rbk_2^{\gamma-1} = \beta_1 c + \beta_1 ra - \beta_1 c + c - \beta_1 ra + ra \tag{A- 14}$$

Through the cancellation of like-terms and algebraic manipulation, an analytical expression governing the corresponding optimal capacity is found as indicated in *(A- 15*).

$$k_2 = \left[\frac{c+ra}{rb}\frac{1}{\gamma(\beta_1-1)-\beta_1}\right]^{\frac{1}{\gamma-1}} \tag{A- 15}$$

The Expected Value of the Stochastic Discount Factor

The expected value of the stochastic discount factor is indicated in *(A- 16*).

$$f(P) = \mathbb{E}_P[e^{-\rho\tau_2}] \tag{A- 16}$$

Assuming that price, P, follows a geometric Brownian motion and τ_2 is the date at which the time process reaches the fixed output price, P_{τ_2}, dt can be chosen at an infinitesimally small level such that the probability that P reaches the fixed output price P_{τ_2} is an unlikely event (Dixit and Pindyck, 1994). Hence, it can be assumed that $P \leq P_{\tau_2}$. It then follows that the change in the price process, dP, can be modelled in a recursive-like fashion from a new level, $P + dP$, as indicated in *(A- 17*).

$$f(P) = e^{-\rho dt}\mathbb{E}_P[f(P+dP)] \tag{A- 17}$$

Through a Taylor expansion on $e^{-\rho dt}$ at $-\rho dt = 0$, using Itô's Lemma, and noting that higher order terms of dt reach the limit of zero at a more rapid pace than dt, the expression $f(P)$ can be re-written as in *(A- 18)*.

$$f(P) = [1 - \rho dt + o(dt)]\left[\frac{1}{2}{\sigma_2}^2 P^2 \frac{\partial^2 f}{\partial P^2} dt + \mu_2 P \frac{\partial f}{\partial P} dt + f(P) + o(dt)\right] \qquad (A\text{- }18)$$

Through algebraic simplifications, division by dt, and proceeding to the limit as $dt \to 0$, the equation then takes the form of a Cauchy-Euler, second-order, homogenous differential equation as in *(A- 19)*.

$$\frac{1}{2}{\sigma_2}^2 P^2 \frac{\partial^2 f}{\partial P^2} + \mu_2 P \frac{\partial f}{\partial P} - \rho f(P) = 0 \qquad (A\text{- }19)$$

As such, the general solution of $f(P)$ can be expressed as a linear combination of two independent solutions as indicated in *(A- 20)*, where β_1 is the positive root and β_2 is the negative root of the fundamental quadratic equation.

$$f(P) = A_1 P^{\beta_1} + A_2 P^{\beta_2} \qquad (A\text{- }20)$$

Furthermore, the endogenous constants A_1 and A_2 can be found by leveraging the model's boundary conditions. Logically, when the distance between P and P_{τ_2} is large, τ_2 is exceptionally large, which implies that $e^{-\rho\tau_2}$ approaches zero as indicated in *(A- 21)*.

$$f(0) = 0 \qquad (A\text{- }21)$$

Secondly, as P approaches P_{τ_2}, it is reasonable to assume that τ_2 is small, and consequently $e^{-\rho\tau_2}$ approaches one as indicated in *(A- 22)*.

$$f\left(P_{\tau_2}\right) = 1 \qquad (A\text{- }22)$$

Furthermore, as the term $A_2 P^{\beta_2}$ is undefined when $P = 0$, it further implies that $A_2 = 0$. Consolidating these findings implies that $A_1 P_{\tau_2}^{\beta_1} = 1$. Hence, *(A- 20)* can be re-written as *(A- 23)* and simplified to find the expected value of the stochastic discount factor.

$$f(P) = \left(\frac{1}{P_{\tau_2}^{\beta_1}}\right) P^{\beta_1} + (0)P^{\beta_2}$$
$$= \left(\frac{P}{P_{\tau_2}}\right)^{\beta_1} \quad (A\text{- }23)$$

Deriving the Simultaneous System of Ordinary Differential Equations

The differential equations of the net value of immediate investment in each respective regime are indicated in *(A- 24)*.

$$\begin{cases} G_1(P) = K_1 P dt + \ \lambda dt \mathbb{E}_P[G_2(P + dP)e^{-\rho dt}] + (1 - \lambda dt)\mathbb{E}_P[G_1(P + dP)e^{-\rho dt}] \\ G_2(P) = k_2 P dt + \mathbb{E}_P[G_2(P + dP)e^{-\rho dt}] \end{cases} \quad (A\text{- }24)$$

In order to expand the branches of *(A- 24)*, Itô’s Lemma and a Taylor expansion at $-\rho dt = 0$ are applied in a similar fashion as in *(A- 18)*, resulting in *(A- 25)*.

$$\begin{cases} G_1 = K_1 P dt + \ (1 - \rho dt)(\lambda dt)\left(G_2 + \frac{1}{2}\sigma_2^2 P \frac{\partial^2 G_2}{\partial P^2} dt + \mu_2 P \frac{\partial G_2}{\partial P} dt\right) \\ \qquad + (1 - \rho dt)(1 - \lambda dt)\left(G_1 + \frac{1}{2}\sigma_2^2 P^2 \frac{\partial^2 G_1}{\partial P^2} dt + \mu_2 P \frac{\partial G_1}{\partial P} dt\right) \\ G_2 = k_2 P dt + (1 - \rho dt)\left(G_2 + \frac{1}{2}\sigma_2^2 P \frac{\partial^2 G_2}{\partial P^2} dt + \mu_2 P \frac{\partial G_2}{\partial P} dt\right) \end{cases} \quad (A\text{- }25)$$

Working backwards, because $G_2(P + dP)$ is in the expectation operator of both branches of *(A- 24)*, its differential equation is first simplified as indicated in *(A- 26)*.

$$G_2(P) = k_2 P dt + (1)\left(G_2 + \frac{1}{2}\sigma_2^2 P \frac{\partial^2 G_2}{\partial P^2} dt + \mu_2 P \frac{\partial G_2}{\partial P} dt\right) - \rho G_2 dt + o(dt)$$
$$= k_2 P dt + \left(G_2 + \frac{1}{2}\sigma_2^2 P \frac{\partial^2 G_2}{\partial P^2} dt + \mu_2 P \frac{\partial G_2}{\partial P} dt - \rho G_2 dt\right) \quad (A\text{- }26)$$
$$\rightarrow 0 = k_2 P + \frac{1}{2}\sigma_2^2 P \frac{\partial^2 G_2}{\partial P^2} + \mu_2 P \frac{\partial G_2}{\partial P} - \rho G_2$$

Using the second branch of *(A- 26)*, the net value of immediate investment in regime one is then further simplified as indicated in *(A- 27)*.

$$G_1(P) = K_1 P dt + \ (\lambda dt)\left(G_2 + \frac{1}{2}\sigma_2^2 P \frac{\partial^2 G_2}{\partial P^2} dt + \mu_2 P \frac{\partial G_2}{\partial P} dt - \rho G_2 dt\right) \quad (A\text{- }27)$$

$$+(1-\rho dt-\lambda dt)\left(G_1+\frac{1}{2}\sigma_2^2P^2\frac{\partial^2G_1}{\partial P^2}dt+\mu_2P\frac{\partial G_1}{\partial P}dt\right)$$

$$=K_1Pdt+\ (\lambda dt)(G_2)+(1)\left(G_1+\frac{1}{2}\sigma_2^2P^2\frac{\partial^2G_1}{\partial P^2}dt+\mu_2P\frac{\partial G_1}{\partial P}dt\right)$$

$$-\rho dt(G_1)-\lambda dt(G_1)$$

$$\rightarrow 0=K_1P+\frac{1}{2}\sigma_2^2P^2\frac{\partial^2G_1}{\partial P^2}+\mu_2P\frac{\partial G_1}{\partial P}-\rho G_1+\lambda(G_2-G_1)$$

Deriving the Discount Factor Function

Regime One

Regime one is governed by the second-order, non-homogenous differential equation indicated in *(A- 28)*.

$$\frac{\sigma_1^2}{2}P^2\frac{d^2G_1}{dP^2}+\mu_1P\frac{dG_1}{dP}-\rho G_1+\lambda(G_2-G_1)+K_1P=0 \qquad (A\text{- }28)$$

By substitution from (57), *(A- 28)* becomes as indicated in *(A- 29)*.

$$\frac{\sigma_1^2}{2}P^2(0)+\mu_1P(\pi_1K_1)-\rho(\pi_1K_1P)+\lambda(\pi_2k_2P-\pi_1K_1P)+K_1P=0 \qquad (A\text{- }29)$$

Through simplification, *(A- 29)* reduces to *(A- 30)*.

$$\mu_1\pi_1K_1-\rho\pi_1K_1+\lambda(\pi_2k_2-\pi_1K_1)+K_1=0 \qquad (A\text{- }30)$$

By algebraic manipulation of *(A- 30)*, an analytical expression for π_1 is derived in *(A- 31)*.

$$\pi_1=\frac{\lambda\pi_2k_2+K_1}{(\rho+\lambda-\mu_1)K_1} \qquad (A\text{- }31)$$

Regime Two

Regime two is governed by the second-order, non-homogenous differential equation indicated in *(A- 32)*.

$$\frac{\sigma_2^2}{2}P^2\frac{d^2G_2}{dP^2}+\mu_2 P\frac{dG_2}{dP}-\rho G_2+k_2P=0 \qquad (A\text{- }32)$$

By substitution, from Equation (57), *(A- 32)* becomes as indicated in *(A- 33)*.

$$\frac{\sigma_2^2}{2}P^2(0)+\mu_2 P(\pi_2 k_2)-\rho(\pi_2 k_2 P)+k_2P=0 \qquad (A\text{- }33)$$

Through simplification, *(A- 33)* reduces to *(A- 34)*.

$$\mu_2\pi_2-\rho\pi_2+1=0 \qquad (A\text{- }34)$$

By algebraic manipulation of *(A- 34)*, an analytical expression for π_2 is derived in *(A- 35)*.

$$\pi_2=\frac{1}{\rho-\mu_2} \qquad (A\text{- }35)$$

By inserting *(A- 35)* into *(A- 31)* and through further algebraic manipulations, the discount factor function is derived in *(A- 36)*.

$$\pi_1=\frac{\lambda k_2+K_1(\rho-\mu_2)}{K_1(\rho+\lambda-\mu_1)(\rho-\mu_2)} \qquad (A\text{- }36)$$

$F_1(P)'s$ Differential Equation

The option to invest in regime one is as expressed in *(A- 37)*.

$$F_1(P)=\lambda dt\mathbb{E}_P[F_2(P+dP)e^{-\rho dt}]+(1-\lambda dt)\mathbb{E}_P[F_1(P+dP)e^{-\rho dt}] \qquad (A\text{- }37)$$

In order to expand *(A- 37)*, Itô's Lemma and a Taylor expansion at $-\rho dt=0$ are applied in a similar fashion as in *(A- 18)*, resulting in *(A- 38)*.

$$\begin{aligned}F_1(P)=&\big(1-\rho dt+o(dt)\big)(\lambda dt)\left(F_2+\frac{1}{2}\sigma_2^2P\frac{\partial^2F_2}{\partial P^2}dt+\mu_2P\frac{\partial F_2}{\partial P}dt+o(dt)\right)\\&+(1-\rho dt)(1-\lambda dt)\left(F_1+\frac{1}{2}\sigma_2^2P^2\frac{\partial^2F_1}{\partial P^2}dt+\mu_2P\frac{\partial F_1}{\partial P}dt\right)\end{aligned} \qquad (A\text{- }38)$$

Further simplifications of *(A- 38)* yield *(A- 39)*.

$$\begin{aligned} F_1(P) &= \lambda F_2(P)dt + \ (1-\lambda dt — \rho dt)\left(F_1 + \frac{1}{2}\sigma_2^2 P^2 \frac{\partial^2 F_1}{\partial P^2} dt + \mu_2 P \frac{\partial F_1}{\partial P} dt\right) \\ &= \lambda F_2(P)dt + F_1 + \frac{1}{2}\sigma_2^2 P^2 \frac{\partial^2 F_1}{\partial P^2} dt + \mu_2 P \frac{\partial F_1}{\partial P} dt - \lambda F_1 dt — \rho F_1 dt \end{aligned} \qquad (A\text{- }39)$$

By combining like-terms, division by dt, and proceeding to the limit as $dt \to 0$, a second-order, non-homogenous, Cauchy-Euler differential equation is found as indicated in *(A- 40)*.

$$\frac{\sigma_1^2}{2} P^2 \frac{d^2 F_1}{dP^2} + \mu_1 P \frac{dF_1}{dP} - (\rho + \lambda)F_1 + \lambda F_2 = 0 \qquad (A\text{- }40)$$

Deriving the Endogenous Constant $\hat{A}$

The form of the particular solution is indicated in *(A- 41)*.

$$F_1(P) = \hat{A}P^{\beta_1} \qquad (A\text{- }41)$$

Noting that $F_2(P) = AP^{\beta_1}$, insertion of *(A- 41)* into *(A- 40)* yields the expression *(A- 42)*.

$$\frac{\sigma_1^2}{2} P^2(\beta_1(\beta_1 - 1)\hat{A}P^{\beta_1 - 2}) + \mu_1 P(\beta_1 \hat{A}P^{\beta_1 - 1}) - (\rho + \lambda)(\hat{A}P^{\beta_1}) + \lambda(AP^{\beta_1}) = 0 \qquad (A\text{- }42)$$

By simplification and grouping together like-terms, *(A- 42)* becomes *(A- 43)*.

$$\hat{A}\left[\frac{\sigma_1^2}{2}\big(\beta_1(\beta_1 - 1)\big) + \mu_1(\beta_1) - (\rho + \lambda)\right] = -\lambda(A) \qquad (A\text{- }43)$$

Through algebraic manipulations of *(A- 43)*, an analytical expression for the endogenous constant $\hat{A}$ is then derived in *(A- 44)*.

$$\hat{A} = \frac{-\lambda(A)}{\frac{\sigma_1^2}{2}\big(\beta_1(\beta_1 - 1)\big) + \mu_1(\beta_1) - (\rho + \lambda)} \qquad (A\text{- }44)$$

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