Thermal Power Revamp Portfolio Valuation in Brazil: a Fuzzy-Real Options Approach

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Abstract

In Brazil, Energy Market players dedicate many efforts in valuation and optimal allocation of capital decision to implement projects, due the large candidate projects number in their investment portfolios. The strategic decisions of those players should choose the projects subset that will be implemented, because usually, they don´t have the financial resources to implement all then. Many are the risks presented, and greater are uncertainties, greater become the difficulties to value these investment decisions optimally. Complex problems and possible changes in economic and business scenarios can make it even harder. Classical investment portfolio valuation and optimization is based on the maximizing returns (NPV, IRR) and minimizing risks (NPV standard deviation, variance) concepts. But often, traditional methods of assessment may not be able to properly handle the projects managerial flexibilities (Real Options) and meet the great need for prediction for the management of risks and uncertainties due to possible intrinsic difficulties solution and mathematical modeling (multi-variable) problems. Thus, there is an ample room to alternative models´ development and implementation models, such as those based on Real Options Theory, including the use of Computational Intelligence methods. In this work is proposed a Fuzzy Real Options valuation to candidate projects, in a Thermal Power Generation market player, considering managerial flexibilities in uncertain environment. To project portfolio selection is provided the application of a Genetic Algorithm Optimization.

Keywords: Thermal Power Generation, Investment under Uncertainty, Real Options, Fuzzy Logic, Fuzzy-Real Options, Portfolio Optimization, Genetic Algorithms.

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Introduction

A complex problem to players in the Brazilian Energy and Electricity Generation Market is the valuation and optimal allocation of capital decision for investment projects implementation, due the large number of candidate projects in their portfolios and the involved uncertainties.

Decision makers must choose the subset of projects to be implemented, because generally large companies lack the financial resources (own or raised in the financial market) for the implementation of all candidate projects.

According to Aid (2012), the companies operating in this market have different investment opportunities. And the agents decision makers try to answer the following questions: *Should we invest in energy production or customers should be through positions in the wholesale markets?; Which type (s) of production active(s) should we invest?; How much of each active type the investor should own? We must invest now or wait?*

In fact, these issues could be summarized in one: *How should choose between different investment projects available?*

Those decisions are usually taken by investors to obtain the best performance / profitability possible, seeking to minimize the possible risks involved. And decisions should be based on business valuation and investment portfolio optimization approaches. Many are the risks presented to these players, and how much more risks and uncertainties, higher become the difficulties of these projects valuation and optimal investment decisions. Complex problems and possible changes in economic and business scenarios can make it even harder.

Several times, these managers and decision-makers do not use to their advantage the computational tools for the analysis of a large number of projects, based also on intuition, experience and simple methods and limited for making such important investment decisions. Of course, these tools can lead to wrong decisions. However, many of these cases of failure are not disclosed by companies (April, 2003).

In other cases, the classical methods are used on these investment projects valuation and optimization, which are based on the search of maximizing returns (NPV, IRR) along with minimizing the risk (NPV´s standard deviation, variance). But, these valuation traditional methods may not be able to properly handle the managerial flexibility (Real Options) characteristics of the projects and meet the great need for predictability for managing uncertainties, risks and uncertainties, because of the possible solution of intrinsic difficulties and mathematical modeling (multi-variable) of the problems and treatment of these real options. Thus, there´s still ample room for the development and implementation of alternative models, such as those based on Real Options Theory, including complemented by the use of Computational Intelligence Methods.

1. Thermal Power Generation in Brazil

According to Brazilian National Electric Energy Agency (ANEEL), the current electric energy generation and import installed capacity in Brazil is about 135,000 MW, of which about 86,000 MW in hydroelectric generation, and about 38,000 MW in conventional thermal power generation (gas, oil, biomass and coal) and also nuclear. According to the Annual Energy Balance (2013), the Energy Research Company (EPE), considering only the non-emergency capacity, the share of thermal generation (including nuclear), increased from 13% average in the period 1993-1999, to 15.3% in 2000 and 23.9% in 2012. The figure above shows the capacity expansion of hydro and thermal generation in MW in 1974-2012 period.

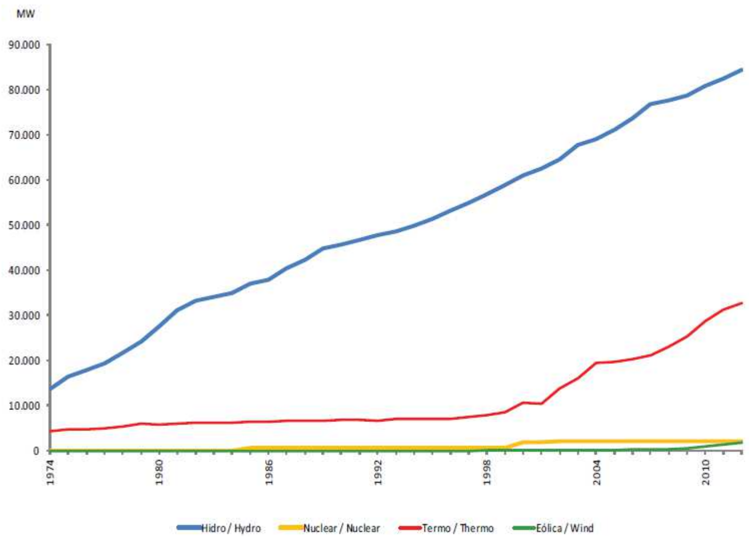

Figure 1 - Brazilian Power Generation Capacity Evolution

The boost in thermal generation verified from the year 2000 was, above all, the Government Thermoelectricity Priority Program (PPT), whose objective was to provide the construction of a large number of thermal power plants fueled by Natural Gas. Initially, the program was aimed at the implementation of 43 power plants with total capacity of 15,000 MW. Over the past few years, there followed adjustments in the program, with the inclusion of some projects and the exclusion of others.

The PPT was not fully implemented. But even the program has not produced the results initially desired, several other thermal plants started operating out of the program, and the increase represented in the national generating capacity was significant and contributed to steady the thermoelectric power as the secondary source of the Brazilian Electricity system.

1.1 Thermal Power Plants Operative Strategy and Trading Model

In Brazil, due to hydroelectric predominance, there is a low volatility in energy spot price in the short term and high volatility in the medium and long term, and the power plants in Brazilian System are in these context.

The Brazilian Market Energy sales contracts are financial instruments, it is known that a thermal power plant, that produces energy only during high spot price periods, can meet their energy supply contracts with a lower effective cost than the operation cost, because during the long periods of low prices, the plant can buy power for a much lower value in the spot market. In the medium-term volatility case, this flexible operation also allows the plant be disconnected in the months when the spot prices are low and operate on the basis in the months when they are high. In other words, the operational flexibility in thermal power plants is an attractive feature in the Brazilian System to raise the project profitability.

However, a flexible operation of thermal generators, associated with fuel market low diversification, could make the producers / fuel suppliers remuneration be too variable and, as the producer needs a stable cash flow to meet its financial commitments resulting from the substantial fixed exploration, production, processing and transportation investments and costs, they may impose on thermal generators a fuel take-or-pay (ToP) and/or ship-or-pay (SoP) contract. The first is a simply financial instrument to reduce the producer remuneration volatility, imposing the generator advance purchase of a certain minimum amount of fuel, monthly and annual, when it´s consumed or not, and the second aims at compensating the investment made in necessary infrastructure for the transport of fuel to the plant, similar to the costs associated with the use of transmission lines from the power grid.

The thermoelectric generating agents submit their production costs and availability to the National System Operator (Brazilian ISO - ONS), which then define its order, also state its operational inflexibility, that is, its restriction minimum generation, primarily due to the need to generating units conservation or resulting from minimum fuel purchase agreement (ToP). Through this inflexibility statement, thermal generators may impose their order to the Operator, even if their operating costs are high. However, according to the ISO current rules, only that generators capacity portion without inflexibility is considered in the energy pricing, ensuring that such restrictions are not onerous for the consumer, which results in an inefficient use of the thermal input, which generate a higher operating cost with the revenue they receive.

The net remuneration of an electrical energy generating company in Brazil depends essentially on the following factors (Street et al., 2006):

(i) Energy sale in the spot market, given by the product of short-term price (PLD) to the total energy produced, less operating costs (fuel costs and operation and maintenance (O & M) variables);

(ii) Because the risky sales in the spot market , due to the high volatility and prices asymmetry, bilateral contracts are used as a protection against the volatility and form the income second portion from a generator, which is the supply contracts sale, given by the product of contract price (P) by the contracted amount (Ec), reduced by the contracted amounts purchase costs in the spot market, which the price is a random variable, due to the uncertainty payment (default of the distribution company). This is the approach of so-called "contracts for quantity", where the risk supply is allocated to the generator;

(iii) The multilateral engagement in the auctions is signed contracts between the government auction each generator winner and all distributors. Although these contracts are backed by collateral clauses, there may be the perception of "credit" of some distributors by generators. This risk can be seen as a reduction in the price actually paid by them, which can be modeled by a random variable that encompasses the aggregate reductions of all distributors who are contracted with each generator.

It is observed that even in the event of default, the supply obligation remains the responsibility of the generator. Therefore, the expression of net income of thermoelectric generator for a period (monthly basis) you a hydrological series any can be expressed in a simplified way, by the following equation:

$$Rts = (Ec) + P\ (Gts - Ec)\ \pi ts - (GTS)\ ct - Cf$$

where:
Rts = Net operating income ($) (random variable);
Ec = Contracted energy amount (MWh);
P = Contract Price ($/MWh);
Gts = Thermo power plant order (MWh) (random variable);
πts = Spot price-PLD ($/MWh) (random variable);
ct = Cost operation variable (CVU), in period “t” ($/MWh). (known value);
Cft = Fixed cost in period “t” ($). (known value).

2. Real Options in Energy and Thermal Power Generation

Electricity Markets deregulation in several countries, incorporated a number of uncertainties and flexibilities to players, investors and risk managers in this economy segment. Thus, the traditional investment analysis tools can become limited in these uncertainties and flexibility treatment, making room for the Real Options Theory use in the investments analysis in the Brazilian Electricity Market context.

In recent decades, many studies in this area have been developed in the finance literature and investment analysis. The aim of this section is to address Real Options applications in Energy, specifically in Thermoelectric Generation, showing the theory use by companies, making a bibliographic review and contextualizing the objective of this work.

The increasing use by companies of Real Options Theory to capital investments valuation was approached by Triantis and Borison (2001), when an important survey of the Real Options practice, summarizing the experiences of 34 companies in this area was presented.

2.1 Real Options in Thermoelectric Valuation

A general approach to the types of real options existing in the economic valuation of thermoelectric was made by Griffes, Hsu and Kahn (1999), where they describe and show through simple examples, among others, the following options available*: Growth Option; Abandonment Option; Stand-by Option; Conversion Option; Repowering Option; Operational Flexibility Option.*

In Aid (2012), were also discussed the following alternatives, which can be identified as Real Options in investment opportunities valuation of a major energy company: Replacement of a turbine parts which would have a direct impact on production efficiency *(Exchange Option);* Plant closure before the end of its useful life *(Abandonment/Stop Option);* Lock, but without plant disassembly, so it can be used years later, with the improvement in market conditions *(Temporary Stop Option).*

In Brazil, some studies have also been developed in this area. In one, this identification was made by Angst (2007) for a Natural Gas Thermal Power Plant already installed on Southeast Subsystem and checked centrally by the ISO. In this work, the following options were discussed: *Input Exchange Option; Reduction Energy Supply to Electric System Option; Expansion Option.*

Using the Great Order Central Model Generation, Matsumura (2004) developed a model for investment decision on a new thermal power plant in the Brazilian Hydrothermal System, determining by Real Options the great moment of investment and considering that there is uncertainty in the investment return, this uncertainty related to future water flows and hydropower dispatch.

2.2 Operational Flexibility Option

The Real Option that has been most analyzed and priced in the thermal power valuation is the Operational Flexibility Option. Usually, this valuation is done considering that when the electricity spot price is above the variable cost is profitable to operate the plant. However, when the variable cost is higher, there is typically reduce the flexibility of generation level or stop operation, avoiding losses.

This analysis is very important, especially in the Brazilian System, where the order of these plants is done centrally by the Brazilian ISO, from the statement by the plant owner's agent, the variable cost/MWh generated.

Several studies used this idea in thermal power plants valuation. Deng, Johnson and Sogomonian (1998) considered that the fuel cost is the variable cost of thermal operation. In the proposed model for them to pay in time T refers to a unit of energy is described by the following equation:

CT (STE, STG, T) = Max (STE - H.STg, 0)

where:
CT (STE, STG, T) = Option Value to generate 1 MWh in the exercise time;
STE = Energy Spot Price ($/MWh);
STG = Fuel Spot price ($/MMBtu);
H = Heat Rate (MBtu/MWh).

The variable cost is defined as the product of Heat Rate (which measures the plant efficiency, indicating how many fuel units are necessary to produce 1 MWh) with the fuel spot price, being interpreted as the exercise price of a Call Option. The project value is determined by integrating the options value to generate every moment over the project useful life, according to the following equation:

$$V_{Ger} = \int_{t=0}^{VU} C_t (S^0{}_{E,} S^0{}_{G,} 0) dt$$

In this work, was considered that the spot prices of electricity and gas follow correlated stochastic processes of mean reversion.

Ethier (1999) made a similar assessment, complementing the previous model by introducing the possibility of jumps in the stochastic process of the electricity price. Winsen (1999) added to Deng, Johnson and Sogomonian formulation the possibility of protection through swap agreements, in which the fuel agrees to pay the price floating pool in exchange for fixed payments. In this paper a case study of a thermal power plant valuation in the Australian market, considering various procurement opportunities.

Johnson, Nagali and Romine (1999) separated the operational flexibility by duration: *Monthly Flexibility; Daily Flexibility; On-Peak /Off-Peak Flexibility.* In the words of Deng, Johnson & Sogomonian, they make a comparison of the value of operating a fuel between different flexibilities.

The operation has thermal restrictions that reduce the operating flexibility amount as was shown by Tseng and Graydon (1997). The constraints they considered were as follows: *Ramp Constraints* (there is a time required to restart the plant operation, as well as a cost associated with this operation, which depends on how long the plant was off); *Coupling constraints* (one unit of heat generation can´t switch between the connected mode and the off mode to an arbitrary frequency, i.e. once in either mode, is required staying for a minimum).

Investments analysis in thermal power plants in the Norwegian market under stochastic prices of Natural Gas and Electricity were made by Fleten and Nasakkala (2006). This paper also addresses the Investment Timing, Operational and Abandonment Options, whereas cash flow depends on the spread between the energy price generated and the gas price (Spark-Spread). In valuation, the authors considered three assumptions, below:

(i) The existence of forward contracts for gas and electricity, with the market complete without derivatives and arbitrage opportunities;
(ii) The spark spread following a Brownian Arithmetic Motion stochastic process;
(iii) The plant can be turned on and off instantly with a cost that can be amortized as a fixed cost.

It is noteworthy that, for the proposed model application, especially for the Brazilian case, should be considered the system and the market model specifities, and therefore, check the assumptions validity. For example, for the Brazilian System would not be appropriate to consider that the Spark-Spread follow a stochastic process, but rather, the direct application of the spot price of the ISO centralized optimal dispatch model (DOC) to calculate the plant profit.

2.3 Applications in Brazilian System

In Brazil, Castro (2000) studied the value of operational flexibility, performing various sensitivity analyzes. He incorporated in addition to Deng, Johnson and Sogomonian model, the Brazilian hydrothermal system characteristics and the bilateral energy contracts possibility, as shown by the following equation:

$$\pi_t = (P_c - P_{spot}) \times G_c + \max(P_{spot} - CO, 0) \times Gt$$

where:
π = Plant operating income for the period ($);
Pc = Plant contracted energy price ($/MWh);
Pspot = Energy spot price ($/MWh);
CO = Plant operating cost ($/MWh);
Gc = Contracted energy amount (MWh);
Gt = Produced energy amount (MWh);
$\max(P_{spot} - CO, 0)$ = Operational flexibility.

In addition to the operational flexibility, Gomes (2002), evaluated the best time to invest in a thermal and the wait option. In this work, several analyzes were made considering exogenous uncertainty in the supply expanding, through options models, also with the same exogenous uncertainty as a duopoly problem and beyond, uncertainty in demand was considered.

Moreira, Rocha and David (2002) addressed the optimal behavior investor would be a real asset pricing problem (Real Option), in which the investor should determine the optimal time of option exercise (in this case, investment in power plants) in order to maximize the investment value. In this case, the stochastic variable (option underlying asset) would be the plant profit, which varies for each nature state and each time in period. The investor should pay the investment (the strike price) to access the plant stochastic profit during the planning horizon. The law of this stochastic net revenue motion would be implicitly defined by the centralized optimal dispatch (DOC) model used by the ISO.

Marreco and Carpio (2006) considered that in each period, the ISO would exercise the option to choose between the thermal generation and hydropower to meet part of the demand. This operational flexibility would be equivalent to the option to choose the lowest cost fuel to meet the load. As model inputs, the authors considered the initial level of the hydroelectric reservoirs, the operational costs of thermal and affluent natural energy (ENA) watershed.

Damasceno do Nascimento (2008) developed the Real Option valuation of the conversion of a thermoelectric power plant connected to the Brazilian Interconnected Power System from Natural Gas to dual fuel (Natural Gas and Diesel). The Real Option Value was given by the difference between the dynamic cash flows of future plant remuneration, for the bi-fuel case *(PVflex)* and Natural Gas moved only *(PVng),* according to equation below:

$$V_{option} = PV_{flex} - PV_{NG}$$

As valuation method model, this study applied Monte Carlo simulations, considering the cash flow model and the associated uncertainties: the plant order level, the contracting level, contract price and the electricity spot price, costs unit variable (fuel costs) and the possible penalty paid by the plant if it is called to order by the ISO and not dispatch for lack of Natural Gas. In this case, the Option Value also could be defined as a “Flexibility Premium” (Kulatilaka and Trigeorgis, 1994).

3. Fuzzy-Real Options

3.1 Introduction

The origin of the Fuzzy Set Theory starts from the Zadeh's article (1965), which this kind of algebra was named and developed. It was developed to treat inaccurate elements in our decision-making processes, and these elements show how the theory factor that allows the treatment of practically all decisions in an uncertainty environment. According to Bellman and Zadeh (1970), informally, a Fuzzy Set is a class of objects where there is no fully defined barrier between the objects that belong to this class and those who do not belong to it.

The assessment by Real Options is usually carried out by the methods of Contingent Claims; Black & Scholes and Black, Scholes & Merton models (including dividends payments); Binomial Trees and Monte Carlo Simulation. All these methods use the probabilities theory for the uncertainties treatment, however, these uncertainties and inaccuracies in future scenarios estimates can be treated by other methods using other theories, such as the Fuzzy Sets Theory and Fuzzy Logic.

In classical Set Theory, simply an element belongs or not-belong to a given set. Or it can be said that the element is fully applicable to all or naughty. This characteristic function is called Bivalent Membership Function (Pacheco and Vellasco, 2007). This bivalent logic (true / false) is commonly used in financial applications (and presents the probabilities theory basic premise). However, the bivalent logic presents a problem because usually financial decisions need to be taken in environments and / or situations of uncertainty.

Uncertainties in the financial context and in investment ratings mean that shown virtually impossible to obtain correct and accurate estimates, for example, for future investment costs and cash flows. Fuzzy sets are sets in which elements have membership degrees, allowing, for example, the representation of *"an investment cost around 100 million dollars".* This means that the Fuzzy sets can be used to formalize the normally lack of accuracy existing in human decisions and as a form to represent vagueness, uncertainty, or imprecise knowledge, for example, to estimate future cash flows, where the human reason is particularly adaptable. The methodologies based on Fuzzy Sets can break the traditional line between qualitative and quantitative analysis, since the modeling reflects more the type of information being studied than the analysts and researchers preferences (Tarazo, 1997).

According to Ponsard (1988), specially in economic and financial areas, the use of Fuzzy Set Theory leads to results that could not be obtained by traditional methods. The Fuzzy Sets Theory and Fuzzy Logic have been adopted on financial options valuation models, such as the binomial pricing of an option with a “Fuzzy Pay-Off" (Muzzioli and Torricelli, 2000) and in the European Financial Options pricing by Black-Scholes Model (Yoshida, 2001).

Also, have been proposed Real Options valuation and pricing models, with Fuzzy Sets. Some pioneer studies have addressed these models, as the developed by Carlsson and Fuller (2003); Collan, Carlsson and Majlender (2003); and Majlender and Carlsson (2005).

Later, some studies showed "Fuzzy Real Options" applications to value investments in some areas, for example, Chen, Zhang, Lin and Yu (2007), which were carried out reviews of Real Options in Information Technology projects (IT), and Tolga and Kahraman (2008), which evaluated Research and Development (R&D) projects. Other specific Fuzzy models were also considered in the optionality value analysis to real investments in large-scale industrial area (Collan, 2004) and aviation (Datar and Matthews, 2007).

3.2 "Fuzzy Pay Off" for Real Options Assessment from Fuzzy Numbers

Some studies have addressed a practical method based on Probability Theory to calculate the Real Option value, as Mathews and Salmon (2007) and show that the method and the results would be equivalent to the Black-Scholes Model, under the mathematical point of view. The method is based on the probability distributions generation and Net Present Value (NPV) future project returns simulation. These future cash flows probability distributions are used to generate a Pay-Offs distribution, where negative results (subject to project closure) are truncated on the partition that results in the null Pay-Off, and where the Pay-Off distribution result average value is the Real Option value. This method shows that the Real Option value can be understood as the probability weighted average of Pay-Offs distribution.

Fuzzy numbers are used to represent the expected future distribution of possible costs and the investment project revenues, as well the profitability of these results through the NPV. The NPV is a Fuzzy Number, and represents this project resulting pay-offs distribution. As the method presented by Datar and Matthews (2007) considers that the pay-offs distribution positive results weighted average is the Real Option value, in the Fuzzy Numbers case, this weighted average is equal to the positive NPV average value results. This value is possibilistic and the mean value calculation is derived from the method disclosed by Carlson and Fuller (2001), defined below. The formula below shows the Real Option calculation from the Fuzzy NPV:

$$\mathrm{ROV} = \frac{\int_0^\infty A(x)\mathrm{d}x}{\int_{-\infty}^\infty A(x)\mathrm{d}x} \times E(A_+)$$

where:
ROV = Real Option Value;
A = Fuzzy NPV;
E(A+) = Fuzzy Value on the NPV positive side.

It is observed that when the Fuzzy Number that represents the NPV is totally positive, the Real Option value is the Fuzzy Number average value. And when the NPV is totally negative, the Real Option value is zero. The method components are merely the observation that the Real Option value is the weighted average of the project Pay-Off positive values (Fuzzy NPV) probability distribution; and for Fuzzy Numbers, the positive values probability-weighted average of the “Pay-Off” distribution is the weighted average of the fuzzy values of the positive Fuzzy NPV, when Fuzzy Numbers are used.

More recently, Collan, Fuller and Mezei (2009) used this method modified, with the NPV represented both as Triangular (3 points) and Trapezoidal (4 points) Fuzzy Numbers, confirming the applicability and simplicity of the method for calculating the Real Option compared to mathematically more complex methods. The Real Option probabilistic calculation rule is characterized by the present value of expected cash flows and the expected costs as Real Numbers ("Crisp"), which may not be a realistic consideration in many cases. Carlsson and Fuller (2003) address the probability theory use to these estimates can be defended for financial options valuation, because we can assume the existence of an efficient market, with many active and players, which justifies the validity the "law of large numbers," and consequently of Probability Theory.

In the Real Options case, it can be very different, for example, the option to postpone a large industrial investment will have different consequences of which would have the financial market, since the number of players on the type of industry may be much lower. The inaccuracies found in assessments and estimates of future cash flows are not stochastic in nature, and the use of probability theory provides a distorted level of precision and the notion that the consequences of postponement would be, somehow, repetitive. This case involves a genuine uncertainty as they just do not know the exact future cash flows levels. Thus, without the introduction of Fuzzy Real Options models, could not be possible to formulate this genuine uncertainty.

The proposed models, incorporating subjective valuations and statistical uncertainties, may offer to investors in real assets a much better understanding of the problem for making investment decisions.

3.3 Hybrid Approach to Real Option Valuation

In another study, Carlsson and Fuller (2000) present a more realistic way for the Real Option calculation rule (from the expected by Black & Scholes formula) by estimating the cash flows and Expected Costs present values by Trapezoidal Fuzzy Numbers:

$$\tilde{S}_0 = (s_1, s_2, \alpha, \beta), \quad \tilde{X} = (x_1, x_2, \alpha', \beta')$$

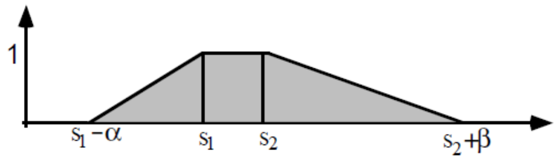

Figure 2 - Cash Flow as a Trapezoidal Fuzzy Number

Being proposed the following equation (heuristic) for the Fuzzy Real Option valuation:

$$\text{FROV} = \tilde{S}_0 e^{-\delta T} N(d_1) - \tilde{X} e^{-rT} N(d_2)$$

where:

$$d_1 = \frac{\ln\left(E(\tilde{S}_0)/E(\tilde{X})\right) + \left(r - \delta + \sigma^2/2\right)T}{\sigma\sqrt{T}},$$

$$d_2 = d_1 - \sigma\sqrt{T} = \frac{\ln\left(E(\tilde{S}_0)/E(\tilde{X})\right) + \left(r - \delta - \sigma^2/2\right)T}{\sigma\sqrt{T}}.$$

$E(\tilde{S}_0)$ = Possibilistic average value of the expected cash flows present value;

$E(\tilde{X})$ = Possibilistic average value of the expected costs present value;

$\sigma := \sigma(\tilde{S}_0)$ = Possibilistic standard deviation of the expected cash flows present value.

Using the algebraic equations for Trapezoidal Fuzzy Numbers operation：

$$\text{FROV} = (s_1, s_2, \alpha, \beta) e^{-\delta T} N(d_1) - (x_1, x_2, \alpha', \beta') e^{-rT} N(d_2)$$

$$\text{FROV} = (s_1 e^{-\delta T} N(d_1) - x_2 e^{-rT} N(d_2), s_2 e^{-\delta T} N(d_1) - x_1 e^{-rT} N(d_2),$$
$$\alpha e^{-\delta T} N(d_1) + \beta e^{-rT} N(d_2), \beta e^{-\delta T} N(d_1) + \alpha' e^{-rT} N(d_2)).$$

Thus, the Real Option Value also shows as a Fuzzy Number:

ROV = E (FROV)

3.4 Fuzzy Binomial Valuation Approach

According to Smit and Trigeorgis (2004), in Real Options valuation, the traditional NPV can be expanded as follows:

Expanded NPV = Static NPV + Real Option Value (from managerial flexibility)

where:
Static NPV = NPV obtained by the discount rate traditional method (also called passive NPV)
Expanded NPV = Expanded strategic NPV.

In Ho and Liao study (2011), is proposed a Binomial Fuzzy approach to the investment projects assessment with Real Options embedded. The project value is represented by its expanded NPV, but with the parameters for the calculation estimated by Fuzzy Numbers when it expanded NPV is estimated, thus being called Fuzzy Expanded NPV (FENPV). In the Call Option case, it is known that the asset base ascent and descent factors *"u"* and *"d"* are the most important to the option value, but it may not be so easy estimate these values in an accurate, given the uncertainty of the underlying volatility.

Often, the cash flow models applied to financial decision-making problems have some uncertainty degree. In the disabled historical data case, many decision makers tend to rely on the knowledge of experts on the financial information. The nature of this knowledge generally tends to be vague rather than random. Then, the study does not consider the probabilistic uncertainty, but the possibilistic uncertainty, using Fuzzy Numbers rather than statistical methods for estimating these parameters. Thus, the rise and fall factors are represented by Triangular Fuzzy Numbers $\tilde{u} = [u_1, u_2, u_3]$ and $\tilde{d} = [d_1, d_2, d_3]$, and rewriting the equations of the neutral probabilities risk as shown below:

$$\begin{cases} \tilde{P}_u \oplus \tilde{P}_d = \tilde{1} \\ \dfrac{\tilde{u} \otimes \tilde{P}_u}{1+\mathrm{r}} \oplus \dfrac{\tilde{d} \otimes \tilde{P}_d}{1+\mathrm{r}} = \tilde{1} \end{cases}$$

where:
$\tilde{P}_u = [P_{u1}, P_{u2}, P_{u3}]$ and $\tilde{P}_d = [P_{d1}, P_{d2}, P_{d3}]$

Like this:

$$\begin{cases} [P_{u1}, P_{u2}, P_{u3}] \oplus [P_{d1}, P_{d2}, P_{d3}] = [1, 1, 1] \\ \dfrac{[u_1, u_2, u_3] \otimes [P_{u1}, P_{u2}, P_{u3}]_u}{1+\mathrm{r}} \oplus \dfrac{[d_1, d_2, d_3] \otimes [P_{d1}, P_{d2}, P_{d3}]}{1+\mathrm{r}} = [1, 1, 1] \end{cases}$$

or:

$$\begin{cases} P_{ui} + P_{di} = 1 \\ \dfrac{u_i \times P_{ui}}{1+\mathrm{r}} + \dfrac{d_i \times P_{di}}{1+\mathrm{r}} = 1 \end{cases} \qquad \textit{for } i = 1, 2, 3$$

Can be solved by the following relationship:

$$P_{ui} = \frac{(1+\mathrm{r}) - d_i}{u_i - d_i}$$

$$P_{di} = \frac{u_i - (1+\mathrm{r})}{u_i - d_i}$$

As the free rate of return risk *"r"*, and the exercise price *"K"* are generally known and presented as "Crisp" Numbers, but the Option values "C_{1u}" and "C_{1d}" become Fuzzy Numbers as a result of the ascent and descent factors were *"fuzzyfied"*. Like this:

$\tilde{C}_{1u} = \max(\tilde{u}S_0 - K, 0)$ and $\tilde{C}_{1d} = \max(\tilde{d}S_0 - K, 0)$

May the ranking of 2 Triangular Fuzzy Numbers $\tilde{A} = [a_1, a_2, a_3]$ and $\tilde{B} = [b_1, b_2, b_3]$ be derived from the following equation:

$$Max(\tilde{A}, \tilde{B}) = [\max(a_1, b_1), \max(a_2, b_2), \max(a_3, b_3)]$$

Therefore, the Fuzzy Option pricing formula is:

$$\tilde{C}_0 = \frac{1}{1+r}[\tilde{P}_d \otimes \tilde{C}_{1d} \oplus \tilde{P}_u \otimes \tilde{C}_{1u}]$$

In a practical application, the present value of underlying asset would be determined by the investment project NPV, the exercise price is the additional investment to the option exercise. A managerial flexibility that allows future actions introduces an asymmetry in the distribution of project NPV odds. In the absence of managerial flexibility, the project NPV probabilities distribution should be considered symmetrical. However, there is a flexibility in the possibility of exercising options, the distribution is positively increased, being displaced to the right. The figure below illustrates the expected value distribution of the Fuzzy NPV (FENPV) shifted to the right:

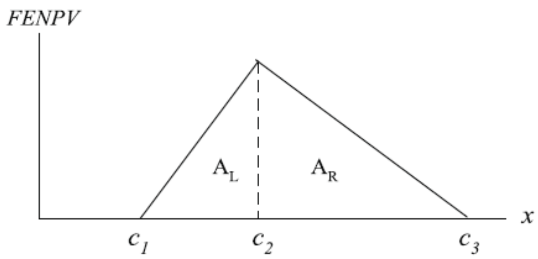

Figure 3 – Fuzzy NPV Distribution

It is observed that, similar results are obtained in the possibilistic distribution case, which is adopted in this study to characterize an investment project NPV. In short, the characteristics of the distribution shifted to the right also appear in FENPV of an investment project when the parameters (such as cash flows) are characterized as Fuzzy Numbers. Thus, a new method is proposed to compute the FENPV average value based on this shift to the right. This average value can be used to represent a "crisp" FENPV value. Anyway, different FENPVs can be compared according to their average values. Being $\tilde{C} = [c_1(\alpha), c_3(\alpha)]$ a Fuzzy Number and $\lambda \in [0,1]$. Then, the "$\tilde{C}$" average value is defined as:

$$E(\tilde{C}) = \int_0^1 [(1-\lambda)c_1(\alpha) + \lambda c_3(\alpha)]d\alpha$$

where:

$\lambda = \frac{AR}{AL + AR}$ = "Pessimistic-optimist weighted index"

Therefore, the Fuzzy NPV expected value can be calculated by:

$$E(FENPV) = \frac{(1-\lambda)c_1 + c_2 + \lambda c_3}{2}.$$

5 Thermal Power Investment Portfolio Valuation and Optimization

This section idea is to present the main objective of this work (in current stage), which is the proposal of a new methodology for economic valuation and optimization to a portfolio of investment revamp projects in thermal power plants installed in Brazil. As seen in previous sections, this assessment will be made combining the Real Options Theory to Fuzzy Numbers and Fuzzy Sets Theory, considering the various investment opportunities as a "Portfolio of Real Options", offered to the investor. The method to be used for the Real Option valuation will be reviewed by dynamic cash flows.

These cash flows are generated from Monte Carlo simulations, considering the plant remuneration and associated uncertainties (some uncertainties are represented as Fuzzy Numbers): the plant order level, contracting level, the contract price and electricity spot price (PLD), unit variable costs (fuel costs). The options value will take place from the difference between the plants cash flows with the flexibilities and the baseline without exercising these options, in the study horizon (120 months).

Each uncertainties have their adequate representation in the proposed model, being part of them represented as Fuzzy Numbers (thus, the model is identified as "Fuzzy Real Options"). In addition to the model and projects valuation, it is proposed to quantify the return and risk rates for the portfolio optimization of investment projects. For this optimization, is proposed a Genetic Algorithm application.

5.1 Investment Opportunities Portfolio / Real Options

The aim is to consider in the proposed model an investment projects portfolio in a real business environment, modeling uncertainties, inaccuracies, and the possibility of exercising real options in these projects.

The proposed model considers the following types of projects and their options:

. Extension of existing plants *(Expansion Option);*

. Simple thermal cycle plants conversion to combined cycle *(Repowering Option);*

. Natural Gas plants conversion to bi-fuel operation *(Switch Input Option).*

The portfolio example to be estudied, and the projects characteristics are presented below:

Project / Plant	Brazilian Submarket	Fuel	REVAMP Option	Original Power (MW)	REVAMP Power (MW)	Original HeatRate (MMBTU/MWh)	REVAMP HeatRate (MMBTU/MWh)	Original FixedCosts (MMBR$/Month)	REVAMP FixedCosts (MMBR$/Month)
P1	South	NG	Expansion	160	250	10,3	10,3	6,3	8,5
P2	South	NG	Combined Cycle Conversion	320	480	6,3	4,5	8	11
P3	Southeast	NG	Combined Cycle Conversion	250	375	6,872	4,8104	8,55	11,5
P4	Southeast	NG	Expansion	160	320	6,502	6,502	8	12,5
P5	Southeast	NG/DIESEL	Bi-fuel Conversion	360	360	7,769	9,1*	7,5	7,5
P6	Southeast	NG	Combined Cycle Conversion	160	200	4,499	3,1493	9	11,5
P7	Southeast	NG/DIESEL	Bi-fuel Conversion	90	90	8,689	10,5*	2,5	2,5
P8	Southeast	NG	Combined Cycle Conversion	150	225	9,417	6,5919	4,33	6
P9	Southeast	NG	Combined Cycle Conversion	340	510	5,87	4,109	7,5	9
P10	Northeast	NG/DIESEL	Bi-fuel Conversion	225	225	8,713	10*	5,95	5,95
P11	Northeast	NG	Combined Cycle Conversion	160	200	8,02	5,614	7,95	10
P12	Northeast	NG	Expansion	100	140	8,43	8,43	7,55	9
P13	North	NG	Expansion	150	180	9,228	9,228	11,86	11,86
P14	North	NG/DIESEL	Bi-fuel Conversion	150	150	9,228	11,2*	5,38	5,38
P15	North	NG	Expansion	200	260	6,85	6,85	8	9,2

***** - Diesel Heat Rate.

Table X – Project Portfolio Characteristics

5.2 Model Parameters

The base model reflects the most common features of a thermal power plant installed, operating and marketing in the Brazilian Electric System. Below, the work current state on the main model study, each of these parameters will be addressed:

5.2.1 Implementation / Start-up Times

The implementation period in investment projects is the time between the investment decision and the investment recovery start. This time has high influence on the NPV, so the estimate should be as accurate as possible.

However, the literature and the experience in the project management show that there is a high risk of real projects not to be implemented on the originally planned time, and how much more complex is the project, higher is the risk. Here, these investments are related to thermal power plants construction, expansions and revamps, and higher may be the uncertainties in these time estimates. Thus, these uncertain times can be represented by Triangular Fuzzy Numbers, incorporating this possibilistic uncertainty to the NPV model, based on the knowledge and experts information with a certain vagueness degree.

5.2.2 Investment Costs

As discussed in Section 4 ("Fuzzy Real Options"), the monetary values of investment costs to be considered for flexible NPVs and (consequently, for the options valuation) as well as the period of project implementation can also be represented as Fuzzy Sets, allowing for example represent *"an investment cost around 100 million dollars."* Thus, in the model, is used Triangular Fuzzy Numbers to represent these uncertain values. The figure below presents the projects implementation times and investment costs.

Project / Plant	Fuzzy Project Implementation Time (Months)			Fuzzy Project Investment Cost (MMBR$)		
	Optimistic	Most Likely	Pessimistic	Optimistic	Most Likely	Pessimistic
P1	21	24	27	105	140	205
P2	24	31	36	320	380	460
P3	20	24	28	200	225	275
P4	24	30	33	175	200	250
P5	12	18	22	25	35	55
P6	16	19	24	30	40	60
P7	10	15	20	7	10	15
P8	15	18	23	155	190	210
P9	24	35	41	360	410	480
P10	14	21	25	60	75	100
P11	14	17	22	64	80	110
P12	13	19	23	115	138	150
P13	13	18	24	13	17	21
P14	11	16	20	11	16	20
P15	15	19	25	100	120	140

Figure 6 – Projects Times and Costs

5.2.3 Thermoelectric Operating Costs (Unit Variable Costs)

The main components in the operating cost are the fuel prices (Natural Gas, Diesel, Fuel Oil and Ethanol) and their respective efficiencies in transforming thermal energy into electrical energy (Heat-Hate). This cost related to the plant operation are shown as a random variable and is called Unit Variable Cost (CVU), in ($/MWh).

To generate future fuel prices series in the Monte Carlo simulation, are considered the variations in CVUs would be the same in fuel prices, and they would follow a Geometric Brownian Motion (GBM) stochastic process. For the CVUs trend values (drift) and volatile estimative, a historical series with the monthly data sample will be used. As an example, below is a sample with 220 monthly data on the Natural Gas prices evolution, the same principle will be applied to the power plants to Diesel:

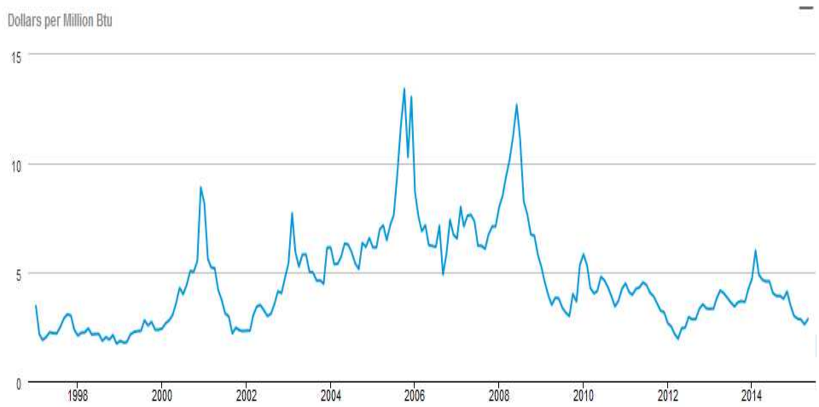

Figure 7 - Natural Gas Price Evolution

Figure 8 - Diesel Price Evolution

The Natural Gas and Diesel change rates in monthly prices (drift) can be calculated from the approach given by the formula below:

$$\alpha = \ln\left(P_t / P_{t-1} \right)$$

To calculate the volatility, the following equation is used, the formula would represent an unbiased estimator of the standard deviation for a sample:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} \left(u_i - \bar{u}\right)^2}{n-1}}$$

The table below shows these calculated parameters:

Parameter	Drift (α)	Volatility (σ)
Natural Gas	8,2% / year	35,2% / year
Diesel	7,8% / year	25,2% / year

Table X – MGB Stochastic Process Parameters

Therefore, changes in natural gas prices and the Diesel follow the process below:

dPgn = 0,082 . Pgn . dt + 0,352 . Pgn . dz

dPd = 0,078 . Pd . dt + 0,25 . Pd . dz

To stochastic processes simulations in fuel prices (Natural Gas and Diesel), applying "Ito Lemma", and an equation is derived:

$$P_1 = P_0 \times e^{\left(\mu - \sigma^2/2\right)\partial t + \sigma\varepsilon\sqrt{\partial t}}$$

Simply with the "μ" value as drift parameter, would be obtained the "Real GBM". However, to dynamic cash flows simulation discounted at Risk-free rate, it is necessary the "Risk neutral GBM" simulation. This risk neutral probability measure is an equivalent martingale measure, which allows discounting for risk-free rates the values obtained in stochastic process simulation stochastic. The equation below shows the "Risk Neutral Drift":

$$\mu = \alpha - \pi = r - \delta$$

where:
μ = risk neutral drift;
α = real drift;
π = risk premium;
r = risk-free rate;
δ = convenience yield

5.2.4 Energy Spot Price (PLD)

The Electricity Spot Price or Differences Settlement Price (PLD), given by Marginal Short Term Cost of the ISO Hydrothermal Dispatch Model, is certainly the main uncertainty component in the brazilian electricity market. As revenues from one generation plant depend on this variable, it must be properly represented in the investment projects valuation problem.

The future spot prices forecast is a very difficult task due to the hydrological characteristics of the river basin system. In addition, the calculation is done by a complex stochastic dynamic optimization problem. Thus, the future spot price will not be represented by a stochastic process, a representative sample containing 200 spot prices future series will be used, related to each different hydrological scenario. These future spot price series will be given by the SDDP (Stochastic Dual Dynamic Programing) computational system. These prices are calculated on a monthly basis and depend on several factors related to energy system operation, such as: past inflows, the basins current volume; the operation cost of the thermal; system costs and deficit limits exchanges between the submarkets; configuration provided for generating complex and demand projection.

For simplicity, is considered that plants would act as price takers in the subsystem Spot Price case, i.e. operating costs (CVUs) of these plants does not influence the subsystem Spot Price. This simplification seems adequate, given that the powers of these plants represent a very small percentage of each sub-market demand, and therefore, would have a very small influence on the PLD value pricing. As an example, the figure below shows this PLD development (outdated), to 1 of 200 hydrological series provided by SDDP system:

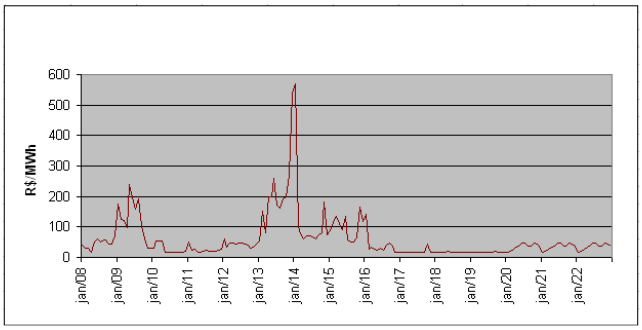

Figure 9 - Spot Price (PLD)

5.2.5 Contracting Level

In Brazil, thermal power plants can market their energy in 2 ways. First, they may sell part of its energy through agreements with distributors in the government auctions, and elsewhere in the Spot Market. Because of the high influence of hydrological regimes in the spot price of energy, the contracting level directly influences the plants cash flows, as well as demonstrates the investor risk aversion degree. The thermal contracting level is a very important factor in this plants cash flow, and therefore, the option value.

5.2.6 Energy Contract Price

The definition of the energy selling price for long-term contracts is an important factor for proper remuneration for a power plant. Moreover, this price should reflect the competition between generators for long-term contracts. Thus, a high price would encourage the loads to close contracts with other generators (thermoelectric or hydroelectric) that offered more favorable conditions. Low prices probably not lead to investments positive returns.

Despite thermal power plants can provide energy only through the spot market, this is very risky. The probability of no return on investment would be very high, although in these cases there are probabilities of very high profits. For setting this value, fixed values will be used, with reference to the offering of securities and sale of energy amounts in recent auctions. These auctions values are also known as ICB (Benefit-Cost Index).

5.3 Valuation Model

The present values of thermoelectric plants future rewards will be calculated according to the model presented in section 2.1. This model considers the operation of a plant if the revenues are higher than the operation cost, and to suspend the operation if the revenues are not sufficient to cover the operation cost. In the thermal plant case, this will only be dispatched by the ISO, if the operation cost (CVU) be less than electricity spot price (PLD). Some restrictions on the plant operation should be made. In this work is considered that there is no cost of entry into operation with any of the fuels. The shutdown time and restart with any of the fuels are considered snapshots, which can be neglected. The following equation represents this net payment:

Rt = Ec.P + (Gts - Ec) .PLDm - Gts.Cv - Cf

where:
Rt = Monthly net income ($);
C = Contracted energy, represented by the contracting level (MWh);
P = Energy contract price ($/MWh);
Gts = Energy amount generated by the plant in the period (MWh);
PLDm = Average energy spot price ($);
Cv = Operation Variable Unit Cost (CVU), ($ / MWh)
Cf = Monthly Fixed Cost ($)

Thus, the cash flow paths for the plant can be simulated in the period studied *(in the proposed case, t = 1 to t = 180).* The present value of this cash flow for is given by the following equation:

$$NPV_{static} = \sum_{t=1}^{180} \frac{RTg_{(t)}}{(1+r)^t}$$

5.4 Options Pricing

In the portfolio valuation, the various investment opportunities to be addressed will be represented in the model in order to allow the achievement of Fuzzy NPV values (returns) and standard deviations (risks) of these NPVs, to enable optimization through Genetic Algorithms. In the model proposed in this paper, in the case of an existing plant valuation, the returns will be calculated considering only the operational flexibility (generate or not generate) offered a proprietary generator agent of a thermal power plant in the Brazilian Electric System as shown in the previous section (Static NPV).

As already discussed, according Smit and Trigeorgis (2004), in the Real Options Valuation, the traditional NPV can be expanded:

Expanded NPV = Static NPV + Real Option Value (Managerial Flexibility)

Thus, additional Real Options values (expanded NPVs) represented by other investment opportunities *(Expansion Option, Repowering Option and Switch Input Option)* may be represented by the differences between the Present Values of dynamic cash flows, represented according the equation :

$$V_{option} = NPV_{expanded} - NPV_{static}$$

As early development, the following sections present the valuation models of flexible NPVs for each case:

5.4.1 Existing Plants Expansion *(Expansion Option)*

RTexp = Ec.P + (Gtexp - Ec) .PLDm - Gts.CV- Cf

where:
RTexp = monthly net revenues of the expanded plant ($);
Gtexp = amount of energy exported (generated) by the expanded plant (value higher than the original value).

5.4.2 Conversion of Simple Thermal Cycle Plants to Combined Cycle *(Repowering Option)*

RTrep = Ec.P + (Gts - Ec) .PLDm - Gts.CVrep - Cf

where:
RTrep = Monthly net revenue repowered plant (higher than the original value);
CVrep = CVU for the operation of repowered plant (less than the original value).

5.4.3 Conversion of Natural Gas plant to bi-fuel operation *(Switch Input Option)*

RTbi = Ec.P + Max [(Gts - Ec) .PLDm - Gts.CVg; (Gts - Ec) .PLDm - Gts.CValt] - Cf
where:
RTbi = Monthly net income of bi-fuel plant (equal or less than the original value);
CVg = CVU to operate the Natural Gas;
CValt = CVU for operation with alternative fuel (Diesel).

5.5 Valuation Process and Portfolio Optimization

The portfolio valuation, from the developed model is carry out Monte Carlo simulations combined with Fuzzy Sets Theory for the Real Options Valuation (incremental investment opportunities), obtaining as the returns measures and risks (Expect Fuzzy Real Option Value and Standard Deviation) results of each project.

In the first phase, the SDDP program is used to generate the sample of hydrological series (200 series), which presents electricity future spot prices samples. In addition to the hydrological series sample, is simulated the Unitary Variable Costs (CVU) sample value of fuel by MGB stochastic process. In the second stage , for each calculated value, the dynamic programming algorithm is applied backward in monthly net remuneration of the plant until the value at time t = 0 is found. In another step, from a Genetic Algorithm, these indicators of returns and risk will be used to optimal portfolio identification, considering profitability and / or minimizing portfolio risk approaches.

The figure below shows the values of the main parameters of the plant studied , and each of these parameters will be discussed below.

Parameter	Value
Spot Natural Gas Price (MGB P0)	8 US$/MMBTU
Spot Diesel Price (MGB P0)	9,5 US$/MMBTU
Energy Contract Level	80% Nominal Plants Power
Energy Contract Price	200 BR$
Exchange Rate	4 BR$/US$
Taxes	34%
Discount Rate	10% / year
Risk Free Discount Rate	5% / year
Study Horizon	120 months

Figure 9 – Portfolio Valuation Parameters (Base Case)

5.6 Valuation Results

Project	***FROV (MMBR$)***	***FRO Standard Deviation (MMBR$)***
P1	[154,90 ; 162,11 ; 169,65]	[55,08 ; 55,74 ; 56,58]
P2	[496,19 ; 531,11 ; 579,88]	[103,21 ; 104,08 ; 105,78]
P3	[303,16 ; 318,21 ; 334,46]	[87,12 ; 87,74 ; 88,60]
P4	[270,56 ; 283,70 ; 309,08]	[92,84 ; 93,57 ; 94,95]
P5	[53,27 ; 53,94 ; 54,42]	[121,73 ; 122,40 ; 122,86]
P6	[51,76 ; 54,18 ; 55,52]	[36,11 ; 36,96 ; 37,53]
P7	[13,55 ; 13,75 ; 14,03]	[30,67 ; 31,05 ; 31,37]
P8	[182,69 ; 194,88 ; 202,11]	[53,65 ; 54,65 ; 55,09]
P9	[451,32 ; 490,16 ; 562,69]	[107,44 ; 108,52 ; 111,09]
P10	[35,23 ; 35,54 ; 36,14]	[78,23 ; 78,99 ; 80,09]
P11	[113,28 ; 120,12 ; 124,06]	[38,60 ; 39,63 ; 40,16]
P12	[49,75 ; 52,80 ; 57,06]	[25,58 ; 26,12 ; 26,73]
P13	[20,16 ; 22,30 ; 25,51]	[19,45 ; 19,74 ; 20,30]
P14	[20,74 ; 20,90 ; 21,10]	[48,92 ; 49,21 ; 49,32]
P15	[128,85 ; 139,85 ; 147,03]	[33,19 ; 33,76 ; 34,23]

To ilustrate , the figures below show the valuation results for one project (P8) in the portfolio:

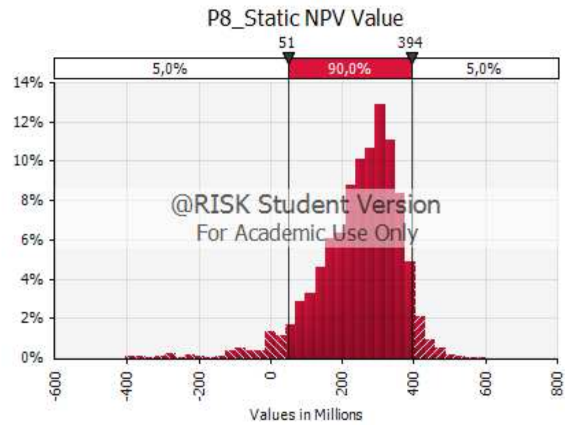

Figure 10 – Static NPV Distribution

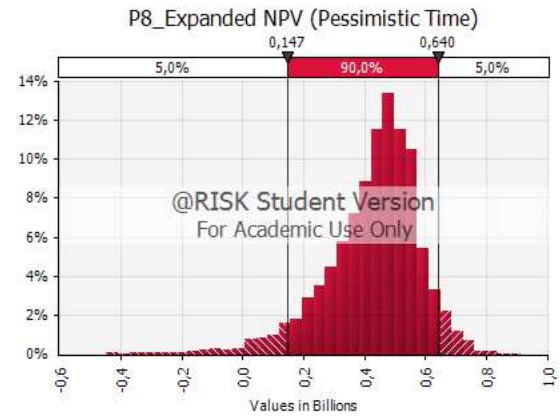

Figure 11 – Expanded NPV Distribution (Pessimistic Time)

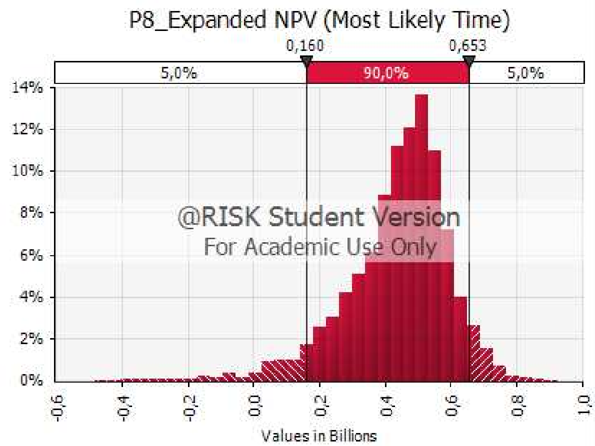

Figure 12 – Expanded NPV Distribution (Most Likely Time)

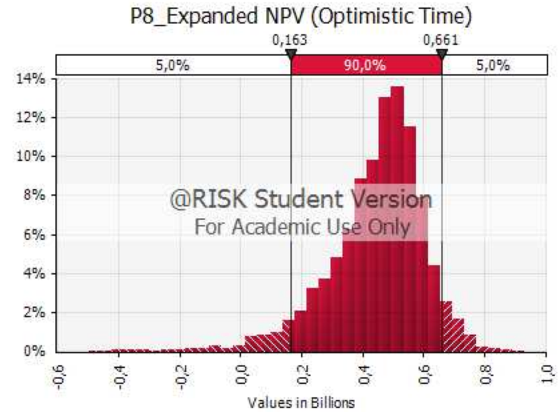

Figure 13 – Expanded NPV Distribution (Optimistic Time)

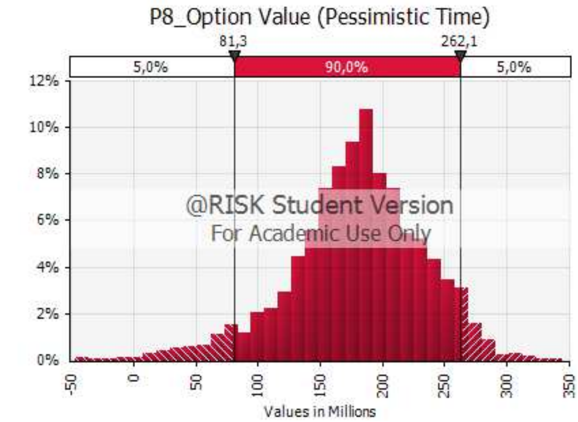

Figure 14 – Real Option Distribution (Pessimistic Time)

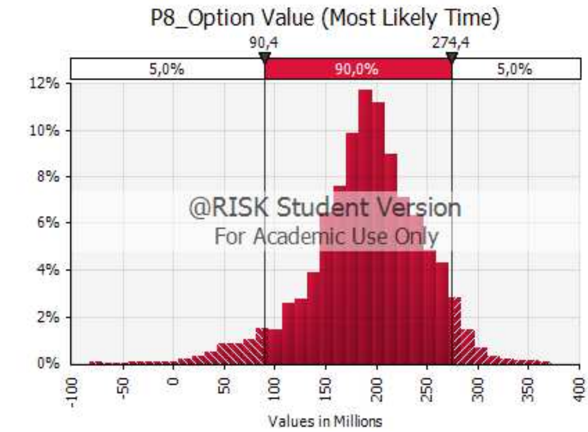

Figure 15 – Real Option Distribution (Most Likely Time)

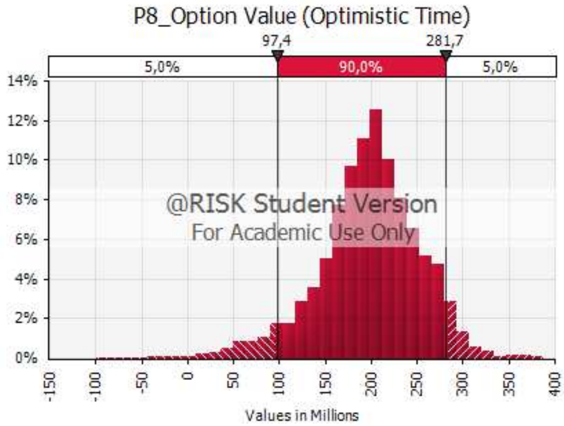

Figure 15 – Real Option Distribution (Optimistic Time)

5.7 Genetic Algorithm Optimization Results (to be completed)

5.8 Conclusions

The advanced decision methods based on Real Options and Fuzzy capital budgeting can explore the flexibility value inside and outside a project, and give further insight into the real investments uncertainties in thermal power generation. Some uncertainties are genuine and without introducing fuzzy real option model it would not be possible to formulate these uncertainties. The proposed model that incorporates subjective judgments and statistical uncertainties may give investors a better problem understanding when making these investment decisions.

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