Valuing Early Stage Investments with Market Related Timing Risk *

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Abstract

In this work, we build on a previous real options approach that utilizes managerial cash-flow estimates to value early stage project investments. The model is developed through the introduction of a market sector indicator, which is assumed to be correlated to a tradeable market index, and is used to drive the project's cash-flow estimates. Another indicator, assumed partially correlated to the market, is used to account for timing risk. This provides a mechanism for valuing real options of the cash-flow in a financially consistent manner under the risk-neutral minimum martingale measure. The method requires minimal subjective input of model parameters and is very easy to implement. Furthermore, we couple simulation with the results of our model to provide managers with a visualization of potential outcomes.

1 Introduction

Real option analysis (ROA) is recognized as a superior method to quantify the value of real-world investment opportunities where managerial flexibility can influence their worth, as compared to standard net present value (NPV) and discounted cash-flow (DCF) analysis. ROA stems from the work of Black and Scholes (1973) on financial option valuation. Myers (1977) recognized that both financial options and project decisions are exercised after uncertainties are resolved. Early techniques therefore applied the Black-Scholes equation directly to value put and call options on tangible assets (see, for example, Brennan and Schwartz (1985)). Since then, ROA has gained significant attention in academic and business publications, as well as textbooks (Copeland and Tufano (2004), Trigeorgis (1996)).

While a number of practical and theoretical approaches for real option valuation have been proposed in the literature, industry's adoption of real option valuation is limited, primarily due to the inherent complexity of the models (Block (2007)). A number of leading practical approaches, some of which have been embraced by industry, lack financial rigor, while many theoretical approaches are not practically implementable. Previously, we developed a real options analysis framework where we assumed that future cash-flow estimates are provided by the manager in the form of a probability density function (PDF) at each time period (Jaimungal and Lawryshyn (2015)). As was presented, the PDF can simply be triangular (representing likely, optimistic and pessimistic scenarios), normal, log-normal, or any other continuous density. Second, we assumed that there exists a *market sector indicator* that uniquely determines the cash-flow for each time period and that this indicator is a

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Markov process. The market sector indicator can be thought of as market size or other such value. Third, we assumed that there exists a tradable asset whose returns are correlated to the market sector indicator. While this assumption may seem somewhat restrictive, it is likely that in many market sectors it is possible to identify some form of market sector indicator for which historical data exists and whose correlation to a traded asset/index could readily be determined. One of the key ingredients of our original approach is that the process for the market sector indicator determines the managerial estimated cash-flows, thus ensuring that the cash-flows from one time period to the next are consistently correlated. A second key ingredient is that an appropriate risk-neutral measure is introduced through the minimal martingale measure (MMM) (Föllmer and Schweizer (1991)), thus ensuring consistency with financial theory in dealing with market and private risk, and eliminating the need for subjective estimates of the appropriate discount factor typically required in a discounted cash-flow (DCF) calculation. We then expanded our methodology to be able to account for managerial risk aversion. Furthermore, we developed an analytical approach for the case where the cash-flows are assumed to be normally distributed (Lawryshyn (2013)).

After using our method in a few practical settings where we valued early stage projects, we realized that a key ingredient was missing in much of the real options approaches, including ours – that of timing risk. In this work we develop a framework that provides a practical way to deal with timing risk. Furthermore, we allow the timing risk to be (partially) correlated to the market. We also include Monte Carlo simulation to provide a visualization of potential outcomes for the managers.

2 Methodology

A key assumption of many real options approaches (Copeland and Antikarov (2001), Datar and Mathews (2004) and Collan, Fullér, and Mezei (2009)), is that the risk profile of the project is reflected in the distribution of uncertainty provided by managerial cash-flow estimates. In Jaimungal and Lawryshyn (2010), we introduced a "Matching Method", where, as mentioned above, we assumed that there exists a *market sector indicator* Markov process that ultimately drives the managerial-supplied cash-flow estimates. The value of the method is the riskiness of the cash-flows are inherently accounted for by the managerial supplied distribution. A broader distribution necessarily implies a more risky cash-flow. As well, the method properly accounts for idiosyncratic and systematic risk (for full details of our methodology we refer the reader to Jaimungal and Lawryshyn (2015)). As we develop our method here, we set out two main objectives: 1) that the model be consistent with financial theory, and 2) that the methodology is easily adapted to managerial estimates.

A depiction of the project cash-flow scenario is provided in Figure 1. During product development, regular outlays of cash will be required for the project and these are depicted as K_0, occurring at times $\bar{T}_j$, $j = 1, 2, ..., \bar{n}$, where $\bar{n}$ is such that $\bar{T}_{\bar{n}}$ is less than the total available time for product development, t_{max}, as specified by managers[1]. At some point τ_m the project will be ready for the market, at which point a significant investment K will be required, after which cash-flows generated through revenue and operations are expected to be received. These cash-flows are uncertain and are estimated by managers. As discussed in Jaimungal and Lawryshyn (2015) the distribution of

[1]We note that in the current formulation we have assumed these to be equivalent, however in practice this does not have to be the case.

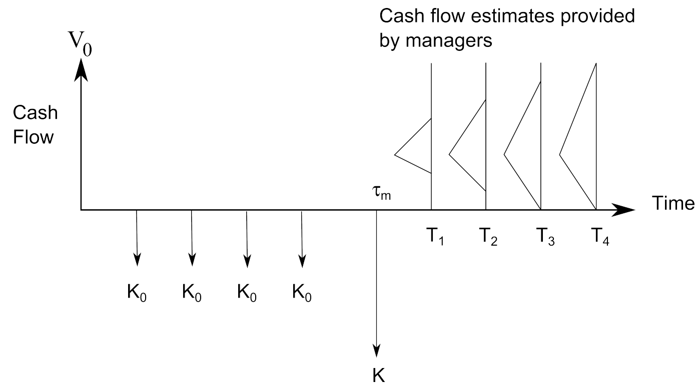

Figure 1: Cash-flow scenario. The timing, τ_m, is uncertain.

the cash-flow estimates can take any form, however, for practical reasons, in this formulation, we assume them to have a triangular density representing low, medium and high cash-flow estimates. The cash-flows are assumed to occur at times T_k, $k = 1, 2, ..., n$, where n is the number of cash-flows. In the present formulation, we assume these cash-flows are not dependent on τ_m, however, this assumption can be easily relaxed and in fact, using either fuzzy set theory or probabilistic methods, it is possible to distort these cash-flows appropriately.

To allow for proper valuation of both systematic and idiosyncratic risk, we assume there exists a traded index index that follows geometric Brownian motion (GBM),

$$\frac{dI_t}{I_t} = \mu dt + \sigma dB_t, \tag{1}$$

where B_t is a standard Brownian motion under the real-world measure $\mathbb{P}$. Following Jaimungal and Lawryshyn (2015) we assume there exists a market sector indicator that is partially correlated to the traded index and we assume it has a standard Brownian motion,

$$dS_t = \rho_S dB_t + \sqrt{1 - \rho_S^2} dW_t^S, \tag{2}$$

where W_t^S is a standard Brownian motion under the real-world measure $\mathbb{P}$ independent from B_t, and ρ_S is a constant ($-1 \leq \rho_S \leq 1$). Next, we introduce a collection of functions $\varphi_k(S_t)$ such that at each T_k, $S_k = \varphi_k(S_{T_k})$. Furthermore, at each cash-flow date T_k we match the the distribution S_k to the cash-flow distribution supplied by the manager $F^*(x)$. Thus, we require

$$\mathbb{P}(S_T < x) = F^*(x). \tag{3}$$

In our previous work (Jaimungal and Lawryshyn (2010)) it was shown that $\varphi_k(S_t)$ is determined as follows

$$\varphi_k(x) = F^{*-1}\left(\Phi\left(\frac{x}{\sqrt{T_k}}\right)\right), \tag{4}$$

where $\Phi(\bullet)$ is the standard normal distribution. We now have a very simple expression for φ which makes the valuation of risky cash-flows very simple.

As discussed in Jaimungal and Lawryshyn (2010), the real-world pricing measure should not be used, and instead, we propose the risk-neutral measure $\mathbb{Q}$, corresponding to a variance minimizing hedge. Under this risk-neutral measure, we have the following dynamics

$$\frac{dI_t}{I_t} = r\,dt + \sigma\,d\widehat{B}_t, \tag{5}$$

$$dS_t = \nu_S\,dt + \rho_S\,d\widehat{B}_t + \sqrt{1-\rho_S^2}\,d\widehat{W}_t^S, \tag{6}$$

where $\widehat{B}_t$ and $\widehat{W}_t^S$ are standard uncorrelated Brownian motions under the risk-neutral measure $\mathbb{Q}$ and the risk-neutral drift of the indicator is

$$\nu_S = -\rho_S\,\frac{\mu - r}{\sigma}. \tag{7}$$

We emphasize that the drift of the indicator is precisely the CAPM drift of an asset correlated to the market index and is a reflection of a deeper connection between the MMM and the CAPM as demonstrated in (Cerny 1999). Given this connection, our reliance on parameter estimation is similar to those invoked by standard DCF analysis when the weighted average cost of capital (WACC) is used to discount the cash-flows and the cost of equity is estimated using CAPM. In DCF analysis the CAPM drift is, however, estimated based on the company's beta, while in our approach, the CAPM drift derives from historical estimates of the sector indicator and traded index dynamics. Furthermore, the *riskiness* of the project is appropriately captured by the distribution of the cash-flows. Consequently, our approach is more robust and less subjective. Given the risk measure $\mathbb{Q}$, the values of the cash-flows can now be computed; i.e. at time τ_m the cash-flows can be valued as

$$U_{\tau_m}^{CF}(S_{\tau_m}) = \sum_{k=1}^{n} e^{-r(T_k-\tau_m)}\mathbb{E}^{\mathbb{Q}}\left[\varphi_k(S_{T_k})|S_{\tau_m}\right], \tag{8}$$

and the option value accounting for investment K at time $t < \tau_m$ as

$$U_t^{RO}(S) = e^{-r(\tau_m-t)}\mathbb{E}^{\mathbb{Q}}\left[(U_{\tau_m}^{CF}(S_{\tau_m}) - K)_+|S_t = S\right]. \tag{9}$$

Applying the discounted Feynman-Kac theorem, the value of the cash-flows and option $U = U_t(s)$ for $t \in [0, T_n]$ can be determined using the PDE

$$rU = \frac{\partial U}{\partial t} + \nu_S\frac{\partial U}{\partial s} + \frac{1}{2}\frac{\partial^2 U}{\partial s^2}, \tag{10}$$

where at each $t = T_k^+$ we have $U_{T_k^+}(S_{T_k^+}) = U_{T_k}(S_{T_k}) + \varphi_k(S_{T_k})$, at $t = \tau_m^+$ we have $U_{\tau_m^+}(S_{\tau_m^+}) = (U_{\tau_m}(S_{\tau_m}) - K)_+$, and at at $t = \bar{T}_j$ we have $U_{\bar{T}_j+}(S_{\bar{T}_j^+}) = \left(U_{\bar{T}_j}(S_{\bar{T}_j}) - K_0\right)_+$. The finite difference method was used to solve equation (10) where in the implementation we assumed $\frac{\partial U}{\partial s}$ is constant as $s \to \pm\infty$ and at each time step negative values were set to zero to account for an American style option where the project would be abandoned as soon as it no longer has value.

As mentioned, a key aspect of the formulation presented here is the timing risk associated with when the technological development of the project will be ready for revenue generation. As such, we ask the managers to provide a distribution estimate for τ_m. We introduce a second Brownian motion with drift μ_τ,

$$dG_t = \mu_\tau dt + \rho_\tau dB_t + \sqrt{1-\rho_\tau^2}dW_t^\tau, \tag{11}$$

where, similar to the cash-flow driver process, W_t^τ is another standard Brownian motion under the real-world measure $\mathbb{P}$ independent from B_t and W_t^S, and ρ_τ is a constant ($-1 \leq \rho_\tau \leq 1$). Under the risk-neutral measure the process in equation (11) becomes

$$d\widehat{G}_t = \nu_\tau dt + \rho_\tau d\widehat{B}_t + \sqrt{1-\rho_\tau^2}\, d\widehat{W}_t^\tau, \tag{12}$$

where $\widehat{W}_t^\tau$ is another standard uncorrelated Brownian motion under the risk-neutral measure $\mathbb{Q}$ and the risk-neutral drift is

$$\nu_\tau = \mu_\tau - \rho_\tau \frac{\mu - r}{\sigma}. \tag{13}$$

By choosing a boundary at at some value a we can calculate the distribution of the hitting time such that $\mathbb{P}(\tau_m \leq t)$, where $\tau_m = \min(t \geq 0, G_t = a)$, as

$$F_{\tau_m}(t) = e^{2\mu_\tau a}\Phi\left(\frac{-a - \mu_\tau t}{\sqrt{t}}\right) + 1 - \Phi\left(\frac{a - \mu_\tau t}{\sqrt{t}}\right). \tag{14}$$

We use equation (14) to match the managerial estimated distribution for τ_m to determine appropriate values for a and μ_τ and constrain $a > 0$.

Applying the Feynman-Kac theorem, the value of the project $V = V_t(s,g)$ can be determined using the PDE

$$rV = \frac{\partial V}{\partial t} + \nu_S \frac{\partial V}{\partial s} + \nu_\tau \frac{\partial V}{\partial g} + \frac{1}{2}\frac{\partial^2 V}{\partial s^2} + \frac{1}{2}\frac{\partial^2 V}{\partial g^2} + \frac{\partial^2 V}{\partial s \partial g}, \tag{15}$$

for $t \in [0, t_{max}]$ and $\tau_m \in [t_{min}, t_{max}]$, where t_{min} and t_{max} are the minimum and maximum times, as specified by the managers, for product development to be completed. The boundary conditions are imposed as follows:

- at $t = t_{max}$ product development has essentially run out of time and therefore we set $V_{t_{max}}(s,g) = 0, \forall s$ and $g < a$;
- at the hitting boundary where $g = a$ two conditions are possible:
  - for $t_{min} \leq t \leq t_{max}$, we set $\tau_m = t$ and we simply calculate the value of the option using equation (9) so that $V_{\tau_m}(s,a) = U_{\tau_m}^{RO}(s)$,
  - for $0 \leq t < t_{min}$ we have reached the boundary but the minimum time required for development has not yet been realized, therefore, we set $\tau_m = t_{min}$ and calculate $V_t(s,a) = U_t(s)$;
- for all $t < t_{max}$, as $g \to -\infty$ we assume $\frac{\partial V_t(s,g)}{\partial g}$ is constant;
- for all $t < t_{max}$, as $s \to \pm\infty$ we assume $\frac{\partial V_t(s,g)}{\partial s}$ is constant;
- for all $0 < t < t_{max}$ and $g < a$, we need to account for ongoing investments at each $t = \bar{T}_j$ so that $V_{\bar{T}_j^+}(s,g) = V_{\bar{T}_j}(s,g) - K_0$.

As above, an American option formulation was implemented to account for the fact that the project would be abandoned as soon as it has no value. Now that the exercise boundaries are known, Monte Carlow simulation can be used to present managers with possible outcome scenarios. The results will be presented in the following section.

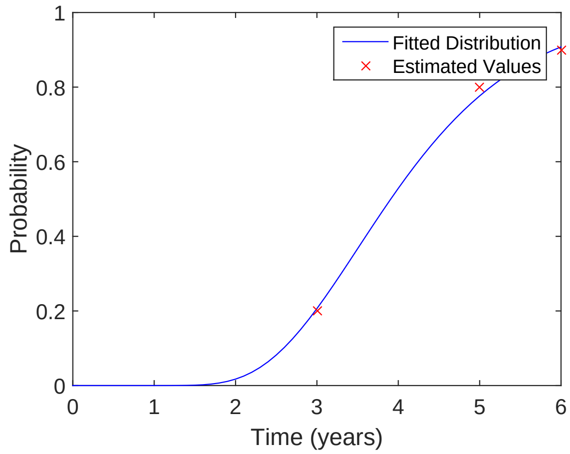

Figure 2: Fitted distribution for τ_m.

3 Results

Here we provide a practical implementation of the methodology. We assume that a company is interested in investing in an early stage R&D project. The managers estimate that the technology could be ready for market launch as early as 2 years from now, but if not launched within 6 years, it will be abandoned. Specifically, the managers estimate a 20% of market launch by year 3, 80% by year 5 and a 10% chance of no launch within the 6 years. The fitted distribution using equation (14) is plotted in Figure 2 and the fitted parameters were $a = 6.1717$ and $\mu_\tau = 1.499$.

The market parameters are assumed to be as follows:

- Risk-free rate: $r = 3\%$
- Expected market growth: $\mu = 10\%$
- Market volatility: $\sigma = 10\%$.

Managers have estimated the cash-flows to be as depicted in Table 1 and the correlation of the cash-flows to the traded index are estimated to be 0.5. The cost to enter the market for the technology K was estimated to be \$50 million and year per-market expenditures were estimated to be \$5 million. The value of the project was determined to be \$72.1 million for the case where the timing had a 0.5 correlation to the market index. The value surface for this case, as a function of S_t and G_t is plotted in Figure 3. The exercise boundary with a simulated path is shown in Figure 4. A histogram of simulated project outcome values is provided in Figure 5. The mean of the simulated results was \$72.9 million. As expected, the possible outcomes consist of either significant value, in the case where the project is successful, or losses of lower magnitude. The average value for the case of the project being a success was simulated to be \$ 120.6 million at a rate of 66%, whereas the average value, in the case of an unsuccessful outcome was simulated to be -\$19.9 million at a rate of 34%.

Table 1: Managerial Supplied Cash-Flows (Millions $).

	$\tau_m + 1$	$\tau_m + 2$	$\tau_m + 3$	$\tau_m + 4$	$\tau_m + 5$	$\tau_m + 6$
Low	10	20	20	10	10	5
Medium	30	40	50	60	60	50
High	50	70	100	120	130	120

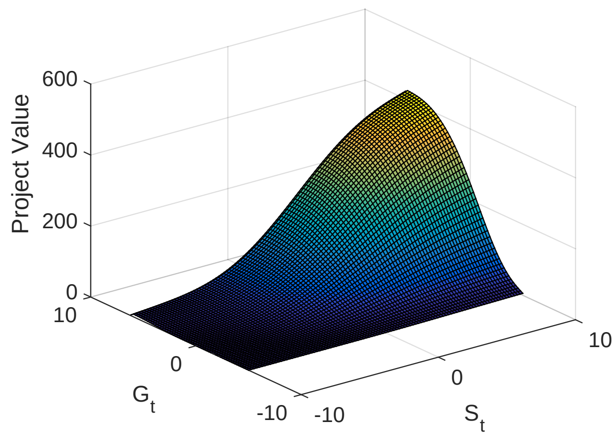

Figure 3: Project value as a function of S_t and G_t.

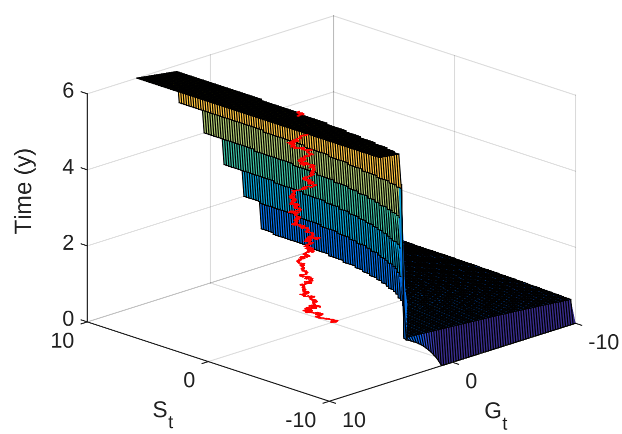

Figure 4: Exercise boundary with simulated path.

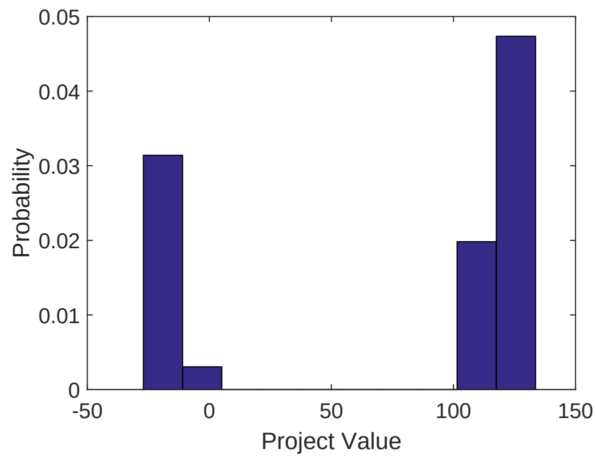

Figure 5: Simulated project value outcomes.

Figure 6 plots the project value as a function of both ρ_S and ρ_τ. As can be seen from the figure, the project value is more sensitive to ρ_S however, ρ_τ does impact the overall project value. In Figure 7 we plot the project value where the time to reach completion is progressively increased. As expected, the value of the project diminishes significantly. From a managerial perspective, this information is valuable, for it allows decision makers to determine various timing scenarios – and is especially valuable when comparing multiple project investment opportunities.

4 Conclusions

In this work we developed a method to account for timing risk of early stage project investments, in a real options context. The model is based on previous work where, through the introduction of a market sector indicator that is assumed to be correlated to a tradeable market index, is used to drive the project's cash-flow estimates. Another indicator, assumed partially correlated to the market, was used to account for timing risk. The methodology provides a mechanism for valuing real options of the cash-flow in a financially consistent manner under the risk-neutral minimum martingale measure. The method requires minimal subjective input of model parameters and is easy to implement. Furthermore, we coupled Monte Carlo simulation with the results of the model to provide managers with a visualization of potential outcomes.

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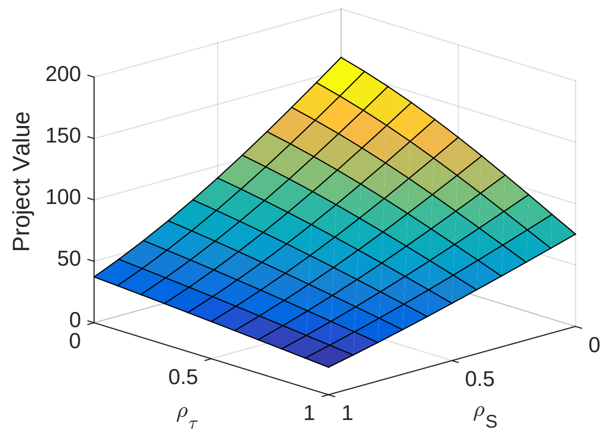

Figure 6: Project value as a function of ρ_S and ρ_τ.

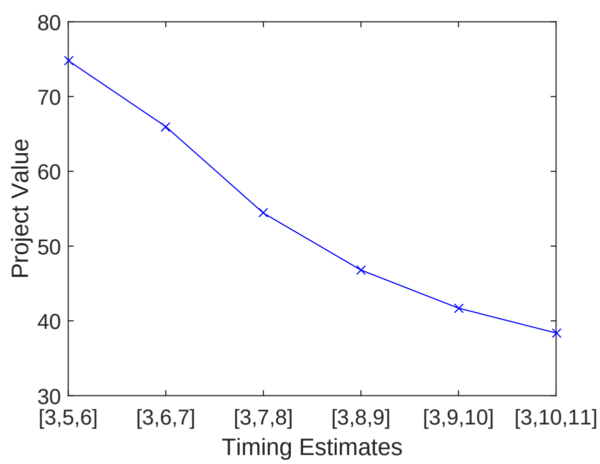

Figure 7: Project value as a function of ρ_S and ρ_τ.

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