The scaling up of capacity in duopoly under uncertainty [*in progress, comments welcome*]

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Abstract

The strategic benefit of commitment and the option value associated with flexibility are jointly examined in a model of capacity investment in a new market. When this is feasible, a first-mover optimally divides a discrete investment into two parts, a first installment that positions the firm strategically and a contingent remainder that accounts both for risk and simultaneous competition. A firm's superior ability to stage investment is thus seen to be a possible source of first-mover advantage in a two-period model. In a continuous time model, a leader firm is seen to adopt a capital accumulation strategy that durably maintains its rival on the brink of market entry, and to finally invest at an optimal threshold that consitutes a preemption trigger for its rival.

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1 Introduction

In industries in which imperfect competition reigns both uncertainty and the threat of competition play a significant role in investment decisions. Thus the strategic decisions of firms typically involve a classic trade-off between two opposing motives: on the one hand, uncertainty creates an option value which calls for waiting and flexibility, but on the other hand, the threat of competition gives rise to a strategic incentive to act quickly and to commit.[1] Although this trade-off is intuitive, specifying it further is a more daunting task that requires bringing several strands of research in economics and in finance together.

The main contribution of this paper is to set out a model of capacity investment in the presence of both uncertainty and competition in which a leader firm chooses to sink a part of its capacity investment early so as to gain a strategic advantage in the product market. Leadership is thus linked to a firm's relatively greater ability to fraction, or *stage*, an otherwise discrete investment. The temporal framework within which investment decisions occur is shown to affect the nature of the leader firm's strategy, insofar as in a continuous time setting the leader's optimal investment policy involves a form of brinkmanship that does not emerge in the two-period framework. Once a critical threshold is reached at which preemption would otherwise occur, the leader incrementally accumulates a sufficient level of capital so as to keep its rival just indifferent between entering and waiting, up until a specific optimal threshold is reached.

1.1 Literature review

The main ideas pertaining to the strategic commitment associated with capacity investment are laid out in Dixit [6]'s seminal article. Early on, several authors sought to incorporate both strategic commitment and flexibility into duopoly competition by introducing uncertainty into a two-period model of capacity investment. To mention just a few, Spencer and Brander [19] find that with a linear demand and cost specification the value of flexibility outweighs first-mover advantage when demand volatility is large enough, inducing a leader to forgo precommitment altogether. In an article whose primary focus is endogenous Stackelberg leadership, Maggi [13] identifies a sufficient

[1] See *e.g.* Chevalier-Roignant and Trigeorgis [4], p. 97:

> "In an uncertain competitive environment, any company considering a capital-intensive decision, like whether to invest in a new technological process or to develop a new product, faces a trade-off between investing early to build competitive advantage over rivals versus delaying investment to acquire more information and mitigate the potentially unfavorable consequences of market uncertainty."

condition for positive capacity precommitment to be optimal, which is discussed in further detail in Section 2 below. Under the assumption that the degree of uncertainty and hence the value of flexibility are relatively small, Maskin [14] finds that cost uncertainty in the product market competition stage reduces the degree of entry deterrence chosen by a leader firm.

Whereas the contributions listed above are cast in a two-period framework, the joint presence of competition and uncertainty naturally calls for a game-theoretic analysis of the continuous time real options that firms effectively hold. In what Azevedo and Paxson [1] describe as a standard real options game, two firms engage in a preemption race (Fudenberg and Tirole [9]) with stochastic payoffs (Thijssen et al. [22]) in which the rents accruing to the first-mover are dissipated by positional competition that leads *ex-ante* symmetric firms to enter too early. A noteworthy recent contribution in this area is due to Huisman and Kort [11], who allow firms to choose both the timing and scale of entry and are thus able to reexamine foundational concepts of the strategic investment literature such as entry deterrence through the lens of more recent models of irreversible investment.[2] In the standard real options game investments are discrete, and at the other end of the spectrum of possible investment strategies, entry competition between duopolists may also involve incremental investments, a study of which using closed-loop strategies is given by Chevalier-Roignant *et al.* [5].

There are several dimensions along which a standard real option game can, and has been, extended. The main innovation in the present paper is to allow for asymmetric firm strategies (rather than asymmetric fixed costs for instance, which constitute a more common extension), by taking the capital good to be divisible for one firm and indivisible for the other. Thus, in a hybrid form of competition drawing on both types of models described above, investment is assumed to be sequential in nature for one of the firms (as in Bertola and Cabalero [2], see also Stokey [20], Ch. 11), and discrete for the other. Aside from this asymmetry, the assumption that a single unit of capital must be accumulated is maintained although some authors have emphasized that industry development may follow a richer pattern than unit capital accumulation allows, such as the multiple investment rounds described by Boyer et al. [3], but this possibility is not pursued further here.

[2] "It might be interesting to see if Dixit's (1980) result that entry deterrence is ineffective if firms cannot commit to produce at full capacity ... is still true in a stochastic dynamic setting." [11], p. 395.

1.2 Results and outline

The remainder of this paper studies the flexibility and commitment motives in a model of investment with input price uncertainty. In Section 2, a leader firm that has the ability to segment its investment is seen to optimally commit part of its final capacity investment before the input price prevailing at the time of product market competition is known, so long as the probability of drastic bad news (of a shutout of the industry) is not too large. As a result, in this framework leadership may emerge not from a firm's advantage with respect to timing but from its greater ability to stage investment. This possibility is illustrated in the linear demand case in which the effects of competition and uncertainty can be disentangled.

As noted by some several authors, a more realistic approach to strategic investment involves incorporating richer dynamics into the analysis.[3] In Section 3 therefore, the investment policy of a leader firm is studied in a continuous time setting under the assumption it can engage in sequential investment whereas its rival's investment is discrete. It is shown that in equilibrium the leader firm begins investing as soon as the preemption threshold is reached so as to maintain its rival indifferent between entering or not, up until an optimal threshold is reached at which it completes its investment. Moreover, this optimal threshold is that level which, given the leader firm's accumulated capital stock at the moment of investment, equates its optimal standalone investment threshold and its rival's preemption threshold.

Section 4 concludes by revisiting a famous historical example of strategic commitment in light of the preceding analysis.

2 Commitment under uncertainty in a two-period framework with divisible and indivisible investments

Consider the standard (Stackelberg-Spence-Dixit) model of strategic capacity investment in which a leader firm shifts its *ex-post* product market reaction function through an irreversible *ex-ante* investment, resulting in a higher profit than when capacity investment decisions are simultaneous. An element of uncertainty can be introduced by supposing that the price of capital goods fluctuates

[3] From the onset, Dixit is quite clear as to the simplifications inherent in restricting the analysis to two periods whereas firm choices typically occur in a dynamic context: "It is as if the two players could see through the whole problem and implement the solution immediately." ([6] , p. 96) Smit and Trigeorgis [18] make a similar restriction and observe: "strategic commitment should not be seen as a one-time investment at the outset, but rather as a first necessary link in a chain of interrelated investment decisions."

over time, so that capacity pre-commitment requires purchasing some amount of capital before its price at the time of product market competition is known. A part of the opportunity cost of strategic commitment then reflects the leader firm's reduced ability to tailor its capacity to current economic conditions.

In such an environment, there are different ways to specify the choice set of the leader firm. For example Spencer and Brander [19] take *ex-ante* and *ex-post* investment to be mutually exclusive so a leader firm cannot rescale its capacity once a demand shock is observed. Then when the leader's *ex-ante* capacity investment is positive the product market equilibrium is invariably of the Stackelberg type, with the follower firm adapting its output to the leader's existing capacity. Evaluating the relative benefit of pre-commitment then involves comparing expected Stackelberg leader and Cournot equilibrium profits, and Brander and Spencer find that this comparison is ambiguous with a linear demand form, with strategic leadership preferred so long as uncertainty is not too large.

An alternative specification is to allow the leader firm to acquire positive amounts of capacity both *ex-ante* and *ex-post*, thus introducing the possibility of a partial or degree of commitment, as in Maggi [13]. This alternative specification can be motivated empirically, either if the leader firm's technology is such that no significant additional cost is incurred when new capital is added to existing capacity (so its capital cost function is linear rather than affine) or if the leader firm has access to such simple contractual devices as a fairly priced forward contract on the capital input.[4] In the above cases the leader firm can both pre-commit and subsequently increase capacity if it wishes to do so. This change in specification makes a crucial difference with regard to product market outcomes, as second period equilibrium then involves either Cournot and Stackelberg equilibria depending both on the degree of the leader's precommitment and on the *ex-post* price of capital (Figure 1 illustrates these outcomes for an arbitrary pre-commitment level K_0). As compared with the mutual exclusivity case, the leader firm's tactical product market flexibility lowers its opportunity cost of commitment.

To formalize these ideas, suppose that there are two firms in an industry. The demand function is denoted by $P(q)$ and assumed to be strictly decreasing and differentiable. Production follows a fixed proportions technology with constant returns to scale, and the price of labor is set to zero for simplicity. The *ex-ante* price of a unit of capital is denoted by r, with $0 < r < P(0)$ so as to restrict attention the more interesting case in which precommitment may be profitable. The price of a unit of capital at the product market competition stage is a random variable, denoted

[4] The possibility of making a nonrefundable deposit payment to a supplier on the capital input would play a similar role, up to an additional input price risk component.

by $\widetilde{r}'$. It is assumed that $\mathbb{E}\widetilde{r}' = r$ and that the discount rate between periods is zero, so that there is no inherent benefit either to delaying or to hastening investment. This price is assumed to be continuously distributed over a support $[\underline{r}, \overline{r}]$, with $0 \leq \underline{r} < \overline{r}$. Since technology and tastes are generally independent there is no reason to assume that *ex-post* production is viable, *i.e.* that $\overline{r} \leq P(0)$.

In addition suppose sufficient regularity conditions hold so that *i*) a unique Cournot equilibrium exists, *ii*) the Stackelberg leader profit is strictly quasiconcave, and *iii*) the interior equilibrium Stackelberg and Cournot outputs are distinct. This is true for instance if the industry revenue function is strictly concave and Cournot best responses are convex. Let the individual Cournot output be denoted by $q^C(r)$ and the Cournot profit by $\pi^C(r)$. For notational simplicity, let $\overline{q}^C := q^C(\underline{r})$ and $\underline{q}^C := q^C(\overline{r})$. Similarly, let $q^L(r)$ denote the equilibrium Stackelberg leader output, with lower bound $\underline{q}^L = \inf_{r \in [\underline{r}, \overline{r}]} q^L(r)$. Let $q_i^*(q_j; r)$, $i, j \in \{1, 2\}, i \neq j$, denote firm i's best response to a rival output q_j when the price of capital is r. Then the profit of a Stackelberg leader firm that has a capacity of K is $\pi^L(K; r) := K(P(K + q_i^*(K; r)) - r)$. Note finally that if $\overline{r} < P(0)$ then $\underline{q}^L > 0$.

Firm 1 is assumed to move first by acquiring K_0 units of capacity *ex ante* at a unit price r. When the value of $\widetilde{r}'$ is realized e*x post*, both firms may acquire capacity units and production and sales occur.

To determine firm 1's payoff and *ex-ante* decision to invest one reasons by backward induction. With cost uncertainty, the reaction functions in the product market stage vary according to the realization of the cost shock $\widetilde{r}'$. Visual inspection of the best responses (Figure 1) indicates that for a given capital investment K_0 the equilibrium in the product market stage is either of the sequential type $(q_1(K_0), q_2^*(q_1(K_0); r'))$ if the cost shock realization is relatively high or of the Cournot type $\left(q^C(r'), q^C(r')\right)$ if the cost shock realization is low enough that the leader firm prefers to expand its output beyond its initial capacity K_0. It is necessary to introduce the notation $q_1(K_0)$ here because if K_0 is sufficiently large, product market equilibrium may involve an output below K_0 for firm 1.[5] For simplicity in the discussion that follows the range of capacity commitment is taken to be $\left[0, q^C(0)\right]$, restricting attention to sequential equilibria of the form $(K_0, q_2^*(K_0; r'))$. Finally let $\widehat{r}(K_0)$ denote the critical level of the cost shock that delimits

[5] For a given capital price r let $\widehat{q}_1(r)$ denote the abscissa of the intersection of firm 1's zero capital cost reaction function $q_1^*(q_2; 0)$ and firm 2's full cost reaction function $q_2^*(q_1; r)$. In the Stackelberg-Spence-Dixit model, $\widehat{q}_1(r)$ is an upper bound of firm 1's equilibrium output. If firm 1's capacity satisfies $K_0 > q^C(0)$, then there exist cost levels $r_0 > 0$ so $\widehat{q}_1(r_0) \leq K_0$, in which case firm 1 does not produce up to its preinstalled capacity in product market equilibrium.

the ranges of Cournot and Stackelberg product market equilibrium, *i.e.* $\widehat{r}(K_0) = P(2K_0) + K_0 P'(2K_0)$.

Ex-ante, the payoff of firm 1 is

$$\begin{aligned}
\mathbb{E}\pi(K_0) &= \mathbb{E}_{\underline{r}\leq\widetilde{r}'\leq\widehat{r}}\left[P\left(2q^C\left(\widetilde{r}'\right)\right)q^C\left(\widetilde{r}'\right)-\widetilde{r}'\left(q^C\left(\widetilde{r}\right)-K_0\right)\right]+\mathbb{E}_{\widehat{r}<\widetilde{r}'\leq\overline{r}}\left[P\left(K_0+q_2^*\left(K_0;\widetilde{r}'\right)\right)K_0\right]-rK_0 \\
&= \mathbb{E}_{\underline{r}\leq\widetilde{r}'\leq\widehat{r}}\pi^C\left(\widetilde{r}'\right)+\mathbb{E}_{\widehat{r}<\widetilde{r}'\leq\overline{r}}\pi^L\left(K_0;\widetilde{r}'\right) \qquad (1)
\end{aligned}$$

where the second line follows from the assumptions of constant returns to scale in production and $\mathbb{E}\widetilde{r}' = r$. It is then possible to define the net benefit of strategic commitment to the capacity K_0, as compared with the value the leader firm would derive from retaining full flexibility and earning the expected Cournot payoff, as

$$\Delta(K_0) = \mathbb{E}_{\widehat{r}<\widetilde{r}'\leq\overline{r}}\left[\pi^L\left(K_0;\widetilde{r}'\right)-\pi^C\left(\widetilde{r}'\right)\right]. \qquad (2)$$

Note that strategic commitment is clearly advantageous ($\Delta(K_0) > 0$) if uncertainty is sufficiently small, since in this case the leader firm approaches the Stackelberg profit $\pi^L\left(q^L(r);r\right)$ which is greater in the limit than the expected Cournot profit of approximately $\pi^C(r)+\left(\sigma^2_{\widetilde{r}'}/2\right)\left(\pi^C\right)''(r)$. As cost uncertainty increases, the relative benefit of strategic commitment (2) is affected in several ways. One important channel is through the negative Cournot profit term. Since Cournot profit is convex in unit cost[6] whereas the direct effect of cost on the Stackelberg leader profit is linear, to the extent that the strategic effect of a cost change (on π^L through dq_2^*/dr) is not too large, greater uncertainty reduces the relative attractiveness of strategic commitment. Nevertheless the strategic benefit of commitment outweighs the value of additional flexibility for a broad range of cost uncertainty as described by the next proposition, originally due to Maggi [13]:

Proposition 1 *In the Stackelberg-Spence-Dixit model with capital cost uncertainty, if a shutout cannot occur in the industry (if $\overline{r} < P(0)$), strategic leadership is valuable and a positive commitment $K_0^* > \underline{q}^L$ is optimal.*

Proof If $\overline{r} < P(0)$ then $\underline{q}^L > 0$. Because interior Cournot and Stackelberg equilibria are distinct for a given capacity cost, $\widehat{r}\left(\underline{q}^L\right) < \overline{r}$. Letting f denote the density of $\widetilde{r}'$, the net benefit from setting $K_0 = \underline{q}^L$ is $\Delta\left(\underline{q}^L\right) = \int_{\widehat{r}\left(\underline{q}^L\right)}^{\overline{r}}\left[\pi^L\left(\underline{q}^L;s\right)-\pi^C(s)\right]f(s)ds$. For all $r > \widehat{r}\left(\underline{q}^L\right)$, $q^C(r) < \underline{q}^L$

[6] Let r' and r'' denote two capital cost levels, q' and q'' the associated Cournot equilibrium outputs, and take $r_\lambda = \lambda r' + (1-\lambda)r''$ with $\lambda \in (0,1)$. Letting q_λ denote the Cournot output at cost r_λ, by revealed preference $\lambda(P(2q')-r')q' > \lambda(P(2q_\lambda)-r')q_\lambda$ and $(1-\lambda)(P(2q')-r')q' > (1-\lambda)(P(2q_\lambda)-r')q_\lambda$ so that by summing, $\lambda\pi^C(r') + (1-\lambda)\pi^C(r'') > \pi^C(r_\lambda)$.

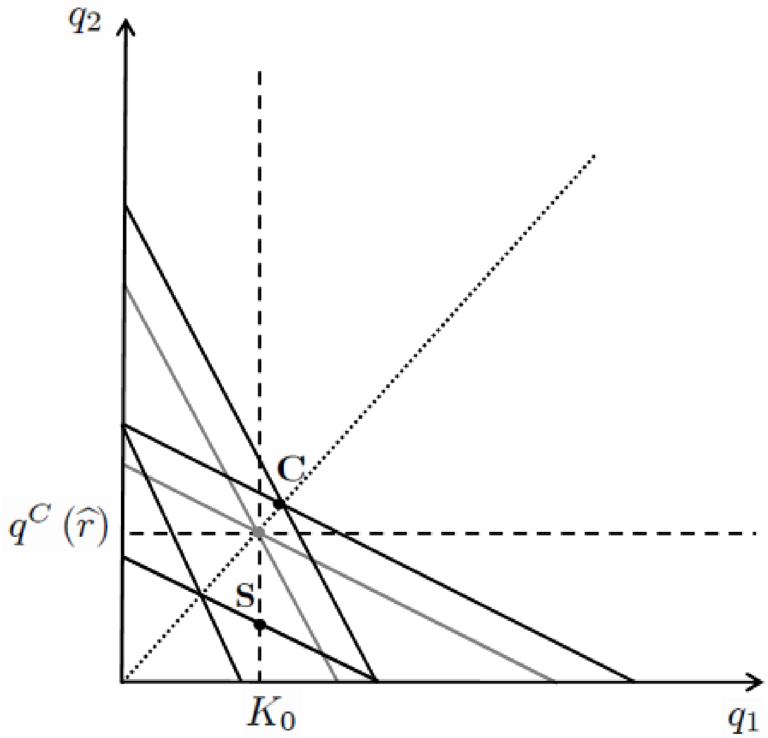

Figure 1: Cournot (C) and Stackelberg (S) product market equilibria in low ($\widetilde{r}' < r$) and high ($\widetilde{r}' > r$) cost states, for a given capital precommitment K_0 by firm 1. In grey the reaction curves defining the critical capital price threshold $\widehat{r}(K_0)$ that separates the two kinds of equilibria.

and $q^L(r) \geq \underline{q}^L$. Because π^L is strictly quasiconcave, it follows that for all $r > \widehat{r}\left(\underline{q}^L\right)$, $\pi^L\left(\underline{q}^L; r\right) > \pi^C(r)$. Since the integrand is positive over the non-empty interval $(\widehat{r}, \overline{r})$, $\Delta\left(\underline{q}^L\right) > 0$. Suppose that $K_0 < \underline{q}^L$. Then, $\widehat{r}(K_0) > \widehat{r}\left(\underline{q}^L\right)$ in which case both the integrand and range of integration in $\Delta(K_0) = \int_{\widehat{r}(K_0)}^{\overline{r}} \left[\pi^L(K_0; s) - \pi^C(s)\right] f(s)ds$ are smaller than in $\Delta\left(\underline{q}^L\right)$ so $\Delta(K_0) < \Delta\left(\underline{q}^L\right)$. $\square$

If $\overline{r} \geq P(0)$, there is a positive probability of "drastic" bad news, in which case the option value of flexibility can be sufficient to outweigh the benefit of drastic commitment (for an example, set $a = .5$ in Example 1 below). Otherwise, capacity commitment is optimal for the leader and moreover the optimal capacity is at least as large as the capacity the firm chooses in the worst cost realization.

2.1 Staging and endogenous Stackelberg leadership

The main result given above, Proposition 1, forms the basis of a further observation regarding the relationship between the ability to stage investment and first-mover advantage. In a framework combining elements of the analyses of both Spencer and Brander [19] and Maggi [13], suppose that firms strategies are asymmetric in that firm 1 can segment its investment whereas firm 2 cannot, but that firm roles are not set exogenously. That is, assume now that both firms have the ability to acquire a positive amount of capital K_0^i, $i = 1, 2$, in the first period. Leadership, should it now arise, is then endogenous.

Although a more general result is beyond the scope of this paper (note that Spencer and Brander [19] focus on a linear demand specification), a particularly simple case of endogenous leadership arises if uncertainty is such that the firm that can segment its investment (firm 1) finds commitment profitable whereas the firm that invests exclusively (firm 2) does not. In a unique subgame perfect Nash equilibrium, it is firm 1 that invests, earning an additional profit that corresponds to its first-mover advantage. In this case, first-mover advantage may be properly attributed to the firm's greater ability to segment investment.

The following example illustrates this possibility.

Example 1 *Let inverse demand in a duopoly be* $P(Q) = 1 - Q$. *The unit cost of capital is* $r_0 = .5$ ex-ante *and* $\widetilde{r}_1 = .5 + \widetilde{\varepsilon}$ *with* $\widetilde{\varepsilon} = [.5, .5 : .5 - a, .5 + a]$ ex-post. *Assume that* $.3 < a < .5$, *so as to rule out certain cumbersome corner solutions in the product market stage. Note that uncertainty is large enough so that both Cournot and Stackelberg equilibria may result in the product market stage when commitment is possible (as* $a > .1$*). Firm* 1 *has the ability to stage investment (that is, to make a positive investment both* ex-ante *and* ex-post*) whereas* ex-ante *and* ex-post *investment are mutually exclusive for firm* 2.

To identify the equilibrium of the two-period investment game, consider firm 2 *first and assume that firm* 1 *does not invest initially*, i.e. $K_0^1 = 0$. *In this case, for firm* 2 *the expected profit from an optimal positive pre-commitment (setting* $K_0^2 = .25$*) is* $1/32$ *whereas the expected Cournot profit is* $\left(1 + 4a^2\right)/36$, *so firm* 2 *prefers not to pre-commit (see Section A.1). Moreover, if pre-commitment is not profitable for firm* 2 *when* $K_0^1 = 0$, *it is not profitable if* $K_0^1 > 0$ *either. Hence, in an equilibrium of the investment game,* $K_0^{2*} = 0$.

Computing firm 1*'s payoff directly given* $K_0^2 = 0$, *it is possible to show that firm* 1 *optimally invests so as to acquire the high cost Stackelberg leader capacity* $K_0^{1*} = .25 - .5a$. *Thus, for the chosen parameter values, the unique equilibrium of the investment game is* $\left(K_0^{1*}, K_0^{2*}\right) = (.25 - .5a, 0)$. *Because of its ability to stage investment, firm* 1 *is thus an endogenous leader whereas firm* 2 *acts* *as a follower. Firm* 1 *earns an expected profit from committing to its optimal initial capacity of* $\left(17 - 4a + 68a^2\right)/576$ *so that the expected first-mover advantage is* $(.5 - a)^2/32 > 0$*. Note firm* 2 *would derive no such value from leadership, even if it were an exogenously designated leader.*

The analysis of the relative importance of the commitment and flexibility motives conducted in this section naturally rests on several restrictive assumptions that are standard for this framework, such as product homogeneity and the absence of any substitution between inputs. But in addition, there are some features of strategic investment do not emerge clearly in a two-period model, and these are developed in a continuous time model in the next section.

3 Stackelberg leadership in a dynamic setting

In a new market, entry decisions may occur in a continuous time framework and there may be several approaches to rendering the key ideas of the Stackelberg model regarding strategic commitment.[7] If two firms contemplate entry in a new market in the face of cost or demand uncertainty, the sequence of firm entry might first of all be determined exogenously. In this case, a diffusion equilibrium results (Reinganum [17]) in which each of the firms invests at an optimal monopoly or duopoly threshold, and the leader earns a positional rent corresponding to its phase of monopoly profit. An second approach would be to assume that one firm has sunk a portion of the fixed cost of investment *ex-ante*. In this other case, firms engage in a preemption game with asymmetric fixed costs, as described in chapter 8 of Huisman [10]. Equilibrium is then either of the preemptive or the sequential investment type depending on the degree of cost asymmetry, that is to say on the magnitude of the leader firm's investment. In this case, the leader firm also earns a positive rent, but this rent is related to the greater threat the leader exerts on its rival if it has the role of a follower, as it would cut its rival's monopoly phase short earlier due to its more aggressive duopoly investment threshold.

But there are still other ways in which the sequencing of investment and the role of sunk capacity expenditure may be cast in a dynamic framework. In the model that follows, firms are assumed to have endogenous roles as leader and follower and to have initially identical fixed costs. Naturally some asymmetry must be introduced in order for a first-mover advantage to arise (absent which entry competition would constitute a standard real option game), but this is done through contrasting assumptions with respect to each firm's choice set. One of the firms (firm

[7] Besides the motivations described in this paragraph, first-mover advantage may also arise for entirely different reasons that are beyond the scope of the present paper. For instance if there may ne firm asymmetries in the product market as a result of endogenous capacity asymmetry (see Huisman and Kort [11]).

1) is assumed to be more reactive at any moment by virtue of its ability to break a lump-sum investment into finer parts so as to gradually build up to the level of capital required to operate in the market, whereas its rival (firm 2) takes investment to be a binary decision at all points in time.

3.1 Assumptions and firm payoffs

Two firms compete to enter the product market. Entering requires that a firm acquire a unit of capital. Demand and variable cost are stationary and generate flow profits of π_M or π_D to active firms depending on whether the industry is currently a monopoly or a duopoly, with $\pi_M > \pi_D$. The common discount rate ρ is positive and constant, so the present values of monopoly and duopoly profit streams are given by $\Pi_M := \pi_M/\rho$ and $\Pi_D := \pi_D/\rho$. The unit price of capital follows a geometric Brownian motion $dX = \mu X dt + \sigma X dW$, with $\sigma \geq 0$, $\sigma + |\mu| > 0$. For the heuristic discussion of this section assume a sufficiently large initial value $X(0) > \Pi_M$ so that firms initially prefer to wait rather than invest immediately. Capital is firm-specific and has no resale value.

Firms are assumed to have qualitatively different strategies. Firm 1's strategy is a nonincreasing left-continuous function $K^1 : [0, X(0)] \to [0,1]$ with $\lim_0 K^1(X) = 1$, which describes its capital accumulation policy or expansion strategy in the absence of investment by its rival.[8] Let $X^1 := \inf\left\{X \geq 0, K^1(X) < 1\right\}$ denote firm 1's investment threshold in accordance with its capital accumulation strategy, and $\overline{K}^1(X) = \lim_{X \to X^1+} K^1(X)$ denote firm 1's existing capital stock at the moment that a threshold X is reached for the first time from above. Firm 2's strategy is an investment threshold $X^2 \in [0, X(0)]$.

If $X^i > X^j$, $i, j \in \{1,2\}$, $i \neq j$, then firm i is the leader and firm j is the follower. Firm i then invests when the input price reaches X^i and earns a monopoly profit flow from that moment up until the time at which the input price reaches X^j and firm j invests. In this case, as a follower firm j chooses a new investment threshold $X_F^j \in \left[0, X^i\right]$. If $X^i = X^j$, then the leader role is assumed here to be attributed to either firm with equal probability. Note in particular that firm 1's expansion strategy is assumed to apply provided that no investment has yet occurred, so that if firm 1 takes on the follower role at the threshold X^2, its accumulated capital stock is taken to be $\overline{K}^1\left(X^2\right)$ and it is free to subsequently revise its expansion strategy. Finally, as discussed in the previous section a number of different rationales may justify the asymmetry in firm strategies.

[8] Thus firm 1 is assumed to have the ability to solve a singular stochastic control problem as per Kobila [12] so as to implement the capital accumulation process $K_t^1 = \sup\left\{K^1(X_s), s \in [0,t)\right\}$.

For instance, this might be due to a difference between the technologies of the two firms, or firm 1 may be thought of as having a long-term relationship with an input supplier that allows it to transact arbitrarily small amounts of the capital input at the current spot price.

Before determining firm behavior in the initial investment stage note first that once one the threshold input price $X^1 \vee X^2$ is reached at which one of the firms invests, any remaining firm i holds a growth option on a perpetual stream of duopoly profits. Standard arguments establish that the follower firm's optimal policy is then to invest when an optimal duopoly investment threshold is attained (Dixit and Pindyck [8]).

Consider first the case of firm 1's follower option when its rival invests when the current level of the input price reaches $X = X^2$. If as a follower it holds a capital stock $\overline{K}^1(X) < 1$, it may delay investing and its remaining fixed cost has the form $X\left(1 - \overline{K}^1(X)\right)$. Its optimal policy is then to complete its investment at the input price threshold $X_F^1\left(K^1, X\right) = (\beta/(\beta+1))\left(\Pi_D/\left(1 - \overline{K}^1(X)\right)\right)$, where the function of parameters

$$\beta(\mu, \rho, \sigma) = \frac{\mu}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\mu}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} \tag{3}$$

is denoted β for short (see Section A.2 derivation) and satisfies $\beta > 0, \partial\beta/\partial\mu, \partial\beta/\partial\rho > 0, \partial\beta/\partial\sigma < 0$. Note that because K^1 is a nonincreasing function n of X, the threshold X_F^1 is nonincreasing in X. The value of the duopoly follower option for firm 1 is then

$$F^1\left(X; K^1\right) = \begin{cases} \Pi_D - X\left(1 - \overline{K}^1(X)\right), & X \leq X_F^1\left(K^1, X\right) \\ A\left[X\left(1 - \overline{K}^1(X)\right)\right]^{-\beta}, & X > X_F^1\left(K^1, X\right) \end{cases} \tag{4}$$

where $A \equiv \beta^\beta \Pi_D^{\beta+1}/(\beta+1)^{\beta+1}$.

Since it does not expand capacity incrementally by assumption, firm 2's duopoly follower option has an exercise threshold $X_F^2 = (\beta/(\beta+1))\,\Pi_D$ and value

$$F^2(X) = \begin{cases} \Pi_D - X, & X \leq X_F^2 \\ AX^{-\beta}, & X > X_F^2 \end{cases}. \tag{5}$$

The payoffs for each firm if it has the leader role run as follows. If firm 1 invests as a leader at a threshold $X = X^1$, firm 2 invests as a follower at the threshold $X_F^{2*} = \min\left\{X, X_F^2\right\}$. Moreover when the threshold X^1 is reached firm 1 has acquired a share $\overline{K}^1(X)$ of its capital input. The instantaneous value of leadership to firm 1 is thus

$$L^1\left(X; K^1\right) = \Pi_M - X\left(1 - \overline{K}^1(X)\right) - (\Pi_M - \Pi_D)\left(\frac{X_F^{2*}}{X}\right)^\beta. \tag{6}$$

If firm 2 invests as a leader at a threshold $X = X^2$, firm 1 invests as a follower at the threshold $X_F^{1*} = \min\left\{X, X_F^1\left(K^1, X\right)\right\}$. The instantaneous value of leadership for firm 2 is thus

$$L^2\left(X; K^1\right) = \Pi_M - X - \left(\Pi_M - \Pi_D\right)\left(\frac{X_F^{1*}}{X}\right)^\beta. \tag{7}$$

Thus, the discounted expected payoffs at the beginning of the game are:

Firm 1

$$V^1\left(K^1, X^2\right) = \begin{cases} L^1\left(X^1; K^1\right)\left(\frac{X^1}{X(0)}\right)^\beta + \int\limits_{X^1+}^{X(0)} Z\left(\frac{Z}{X(0)}\right)^\beta dK^1(Z), & X^1 > X^2 \\ \left[\frac{1}{2}L^1\left(X^1; K^1\right) + \frac{1}{2}F^1\left(X^1; K^1\right)\right]\left(\frac{X^1}{X(0)}\right)^\beta + \int\limits_{X^1+}^{X(0)} Z\left(\frac{Z}{X(0)}\right)^\beta dK^1(Z), & X^1 = X^2 \\ F^1\left(X^2; K^1\right)\left(\frac{X^2}{X(0)}\right)^\beta + \int\limits_{X^2+}^{X(0)} Z\left(\frac{Z}{X(0)}\right)^\beta dK^1(Z), & X^1 < X^2 \end{cases}. \tag{8}$$

Note that (8) incorporates the expected cost of firm 1's expansion strategy up until the first investment threshold is reached. At a given input price threshold X its incremental investment is $-XdK^1(X)$, and the cumulative cost is given by the right integral terms.

Firm 2

$$V^2\left(K^1, X^2\right) = \begin{cases} L^2\left(K^1, X^2\right)\left(\frac{X^1}{X(0)}\right)^\beta, & X^2 > X^1 \\ \left[\frac{1}{2}L^2\left(X^2; K^1\right) + \frac{1}{2}F^2\left(X^2\right)\right]\left(\frac{X^2}{X(0)}\right)^\beta, & X^2 = X^1 \\ F^2\left(X^1\right)\left(\frac{X^2}{X(0)}\right)^\beta, & X^2 < X^1 \end{cases}. \tag{9}$$

3.2 Equilibrium

An $\varepsilon-$equilibrium of the investment game is a pair of strategies $\left(K_\varepsilon^1, X_\varepsilon^2\right)$ such that $V^1\left(K_\varepsilon^1, X_\varepsilon^2\right) \geq V^1\left(K^1, X_\varepsilon^2\right) - \varepsilon$, all K^1, and $V^2\left(K_\varepsilon^1, X_\varepsilon^2\right) \geq V^2\left(K_\varepsilon^1, X^2\right) - \varepsilon$, all X^2. This subsection describes such an equilibrium so as to set out the key economic insights of the model in an intuitive way. Section B in the appendix provides some further specification of the model, in particular regarding simultaneous investment, so as to derive the unique Markov perfect equilibrium towards which the $\varepsilon-$equilibrium converges, but the discussion given here covers the main economic aspects.

To begin with, assume for the moment that firm 1 has a capital stock $\overline{K}^1$ that is given exogenously and that both firms can make only a single investment. This situation corresponds to a standard preemption game with asymmetric fixed costs, in which firm 1's relative cost advantage is $k \equiv 1 - \overline{K}^1$. In the following, understand the argument in $\overline{K}^1$ in the payoff functions to refer to the appropriate discrete investment strategy for firm 1, *i.e.* $K^1(X) = \mathbf{1}_{X \leq X^1} + \overline{K}^1 \mathbf{1}_{X > X^1}$. In this simplified game, the leader and follower payoffs $L^i\left(X, \overline{K}^1\right)$ and $F^i\left(X, \overline{K}^1\right)$ that arise for each firm i, are associated with the concept of a preemption threshold, denoted X_P^i. When a solution to this equation exists, $X_P^i(k)$ is defined as the upper root in X of the condition $L^i(X, 1-k) = F^i(X, 1-k)$, which gives the least upper bound of the input prices at which firm i strictly prefers the leadership. In the case when $k = 1$, firms are symmetric and the preemption threshold is denoted X_P. A standard result in the asymmetric preemption game of new market entry is that equilibria are either preemptive or sequential in nature (Huisman [10]), with initial investment occurring at $X_L^1(k) \vee X_P^2(k)$ where $X_L^1(k) = (\beta/(\beta+1))(\Pi_M/k)$ denotes firm 1's leader threshold and $X_P^2(k)$ is firm 2's preemption threshold as defined above. Moreover there exists a unique level of cost asymmetry, k^*, such that $X_L^1(k^*) = X_P^2(k^*) := X^*$, i.e. such that the equilibrium is preemptive but has the property that at firm 1's monopoly optimum threshold firm 2 is just indifferent between investing or not.

Returning to the setting with incremental investment by firm 1, the input price threshold at which firm 2 prefers to invest as a leader depends on firm 1's capital accumulation strategy, and plays a key role in the analysis. For a given input price $X > X_F^2$ and current capital stock of the first firm $\overline{K}^1$, firm 2 is indifferent between the leadership role and investing as a follower if $L^2\left(X, \overline{K}^1\right) = F^2(X)$, *i.e.* if X and $\overline{K}^1$ satisfy the condition

$$\Pi_M - X - (\Pi_M - \Pi_D)\left(\frac{X_F^2}{X\left(1 - \overline{K}^1\right)}\right)^{\beta} = AX^{-\beta}. \tag{10}$$

It is then straightforward to show that there exists an upper bound on $\overline{K}^1$, denoted by $\overline{K}$, below which (10) has a well-defined upper root in X. Letting X_P^2 denote this upper root, X_P^2 is a differentiable and strictly increasing function of k over $\left(1 - \overline{K}, 1\right)$, with $X_P^2(1) = X_P$ and $X_P^2(1 - \overline{K}) = (\beta/(\beta+1))\,\Pi_M$. A larger capital stock held by firm 1 thus lowers the threshold at which firm 2 finds it just profitable to invest as a leader. Conversely (10) yields a closed form expression for $K_P(X)$, the level of sunk investment by firm 1 that is required in order to keep firm 2 just indifferent between the leader and follower roles when the current input price is X. Note that $K_P(X)$ is a continuous and decreasing function of X over the relevant range $\left(X_L^1(1), X_P\right)$.[9]

[9] See Section A.3 for a derivation of $\overline{K} = 1 - \left((\beta+1)\left((\Pi_M/\Pi_D) - 1\right) / \left((\Pi_M/\Pi_D)^{\beta+1} - 1\right)\right)^{\frac{1}{\beta}}$ and characterization of the function $K_P(X)$.

Having defined $K_P(X)$, an $\varepsilon-$equilibrium of the investment game can now be described. Choose δ small enough and consider the following pair of strategies:

$$K_\varepsilon^1(X) = \begin{cases} 0, & X > X_P + \delta \\ \delta, & X_P < X \leq X_P + \delta \\ K_P(X) + \delta, & X^* < X \leq X_P \\ 1, & X \leq X^* \end{cases} \quad \text{and } X_\varepsilon^2 = X^* - \delta. \tag{11}$$

With the strategies (11), firm 1 starts to invest slightly before the standard preemption threshold for symmetric firms is reached. When the input price reaches a lower threshold for the first time, firm 1 invests so as to maintain its level of capital slightly (δ units) above that required to dissuade firm 2 from immediate entry ($K_P(X)$ units). Firm 1 continues to do so until the threshold X^* is reached, at which time it completes its investment. Then,

Proposition 1 *For δ low enough the pair of strategies $\left(K_\varepsilon^1, X_\varepsilon^2\right)$ is an $\varepsilon-$equilibrium of the investment game.*

An intuitive argument for the $\varepsilon-$equilibrium in Proposition 1 runs as follows (see section A.4 for a formal derivation). Absent any incremental investment, firms would play a standard preemption game whose symmetric equilibrium has both firms investing at the threshold X_P, and both would earn a follower payoff because of rent dissipation. However, around X_P the discounted payoff to leading is decreasing in the input price threshold and any firm would prefer to push back investment by lowering the equilibrium threshold if it could do so without incurring too much additional cost. Because it can invest incrementally, firm 1 has the ability to acquire an arbitrarily small capital stock slightly before the preemption threshold X_P is reached. If it does this, then from that moment on the investment game between the firms is intuitively similar to a dynamic agency problem. Firm 1 incrementally accumulates sufficient capital (setting $dK^1 = dK_P$) to dissuade firm 2 from investing, up until an optimal threshold is reached at which it completes its investment by acquiring the remaining $1-\delta-K_P(X^*)$ units of capital.[10] Because this investment policy maintains firm 2 indifferent between investing or not for a period of time, it may be thought of as a form of brinkmanship by the leader firm that does not arise in the two-period model.

To verify that investment at the threshold X^* is optimal for firm 1, let $\delta \to 0$ so firm 1's

[10] Put another way, for input prices between X^* and X_P levels of capital below $K_P(X)$ $(+\delta)$ constitute a forbidden region with firm 1 adjusting its level of capital to remain at the boundary $K_P(X)$ until X^* is reached.

optimization problem approaches

$$\max_{K^1(X)} L^1\left(K^1, X^1\right)\left(\frac{X^1}{X(0)}\right)^\beta + \int_{X^1+}^{X(0)} Z\left(\frac{Z}{X(0)}\right)^\beta dK^1(Z)$$

$$\text{s.t. } K^1(X) \geq K_P(X), \text{ all } X > X^1.$$

The solution to this problem is straightforward. Carrying additional capital is costly and firm 1 minimizes its capital holdings conditional upon dissuading preemptive investment by firm 2. Substituting the constraint into the objective and reorganizing terms, firm 1's optimization problem is effectively to choose an investment threshold $X^1 \in [X_L, X_P]$ so as to maximize the payoff

$$\widehat{V}^1\left(X^1\right) = \left[\Pi_M - X^1\left(1 - K_P\left(X^1\right)\right)\right]\left(\frac{X^1}{X(0)}\right)^\beta - \left(\Pi_M - \Pi_D\right)\left(\frac{X_F^2}{X(0)}\right)^\beta + \int_{X^1}^{X_P} \frac{Z^{\beta+1}}{[X(0)]^\beta} dK_P(Z). \tag{12}$$

This payoff is quasiconcave in X^1 and optimization yields the first-order condition

$$\beta\Pi_M - (\beta+1)\left(1 - K_P\left(X^1\right)\right)X^1 = 0 \tag{13}$$

after normalization by $\left[X^1\right]^{\beta-1} / [X(0)]^\beta$.

At an optimum therefore, firm 1 completes its investment when it has sunk the expenditure on fraction $K_P(X^*) = 1 - k^*$ of the capital good where k^* has the property described earlier, namely that $X_L^1(k^*) = X_P^2(k^*)$. The intuition behind the first-order condition (13) is that when firm 1 considers delaying market entry a little more, the incremental cost of its flow investment is negligible compared to the cost of holding the capital stock $K_P(X)$. The optimal investment decision thus depends on a standard trade-off in which the cost of holding a larger capital stock reduces the firm's expected marginal benefit from delaying entry. The solution to (13) is necessarily an optimal monopoly threshold for firm 1's given its accumulated capital stock, but must also constitute a preemption threshold for firm 2, and these requirements are only met at the threshold X^*.

Corollary 1 *In an equilibrium of the investment game firm* 1 *invests at an input price* X^* *such that its optimal threshold and its rival's preemption threshold are equal* ($X_L^1(k^*) = X_P^2(k^*)$)*.*

3.2.1 Discussion

Comparative statics The effects of changes in uncertainty or the intensity of competition on the first-order condition (13) are not easily disentangled since part of their effect is linked to the preemption equilibrium condition through the function K_P. The optimal cost asymmetry at the moment of investment, k^*, is scale-free, whereas the optimal investment threshold X^* is proportional to X_F^2. A comparative static result that allows some economic insight into the latter is to allow Π_M to vary. Doing this affects the preemption equilibrium and first-mover advantage but leaves the duopoly option value $F^2(X)$ which constitutes a form of reservation value for firms in the investment game unchanged.

From the first-order condition, it follows that in equilibrium $(1 - K_P(X^*))\, X^* = (\Pi_M/\Pi_D)\, X_F^2$. As at the moment of investment $\overline{K}^1 = K_P(X^*) = 1 - k^*$, substituting into (10) and rearranging yields

$$A\,[X^*]^{-\beta} + X^* = \Pi_M - (\Pi_M - \Pi_D)\left(\frac{\Pi_D}{\Pi_M}\right)^{\beta}. \tag{14}$$

The left-hand side of (14) is an increasing function of X^* (its derivative is $1 - \left(X_F^2/X^*\right)^{\beta+1} > 0$) whereas the right-hand side can be shown to be increasing in Π_M. Therefore, $dX^*/d\Pi_M > 0$, *i.e.* a greater first-mover advantage in the product market results in earlier market entry as may be expected.

The sign of $dk^*/d\Pi_M$ is more revealing. From (13),

$$\frac{dk^*}{d\Pi_M} = \frac{\beta}{\beta+1}\frac{1}{X^*}\left(1 - \varepsilon_{X^*/\Pi_M}\right) \tag{15}$$

so that this sign depends on the magnitude the elasticity of X^* with respect to Π_M. Direct calculation (see Section A.5) establishes that $dk^*/d\Pi_M > 0$ in this model (and hence $\varepsilon_{X^*/\Pi_M} < 1$).

Thus, a greater incentive for strategic investment leads to earlier investment and a relatively lower degree of precommitment at the moment of investment. This seemingly paradoxical result arises because the firm chooses not to wait as long to approach an optimal monopoly option value, but instead attributes relatively more weight the benefit derived from successful preemption and a longer monopoly phase.

Entry deterrence Lastly, strategic investment is often studied in the context of entry deterrence. To address this issue in a simple way, suppose that there exists an action that firms can take at the moment of entry that dissuades subsequent entry by their rival, at an additional cost f. A given firm i investing at a threshold $X = X^i$ finds its optimal to deter if the cost of deterrence is lower than the discounted profit loss due to the rival's entry, *i.e.*

$$f < (\Pi_M - \Pi_D)\left(\frac{X_F^{j*}}{X}\right)^{\beta}. \tag{16}$$

Suppose that f is low enough that (16) holds for both firms. Then, if firms cannot commit not to deter entry when they invest, deterrence necessarily occurs and firm 2's reservation value in the investment game is driven to zero, so that it finds it profitable to invest as soon as its Marshallian threshold $X_{\mathrm{NPV}} = \Pi_M - f$ is reached. But in that case, firm 1 can no longer use sequential investment to push back its rival's investment by accumulating capital incrementally. The possibility of deterrence thus underscores the fact that firm 1's brinkmanship strategy hinges on its ability to progressively scale its threat to enter as a follower at the threshold X_F^1, whereas when firms have the ability to shut out a rival, such a strategy cannot be employed.

4 Conclusion

This paper has sought to develop a framework that accounts for both strategic commitment, such as set out in Dixit [6]'s model of sequential capacity choice in duopoly, and option value, as in Dixit and Pindyck [8]'s framework for investment under uncertainty, so as to highlight linkages that exist between these approaches to investment. In a two-period framework the ability to segment, or stage, investment confers an additional value to strategic commitment in the face of uncertainty and under certain conditions this ability may be a source of endogenous first-mover advantage. In a dynamic framework, a leader firm is seen to follow a strategy of brinkmanship leading to investment at a threshold that constitutes an optimum for one firm and a preemption threshold for the other.

Some historic military actions have provided valuable illustrations of otherwise abstract game theoretic concepts. Notably, the notion of strategic commitment can be understood with reference to the Spanish conquistador Hernán Cortés' order to scuttle his fleet before marching on Mexico. As Dixit and Nalebuff [7] explain, this destruction served the double purpose of both compelling Cortés' men to fight and of signalling their determination to their enemy. As the loss of his ships involved a clear irreversibility and moreover sent an unambiguous public signal to the Aztecs whom the Spaniards had encountered, this episode provides an eloquent illustration of the strategic effect of the commitments similar to those that are studied by researchers in the context of investment decisions.

In light of the analysis of the present paper, the story of Cortés can perhaps be retold, with an eye towards the dimensions of both strategy and uncertainty. The historical record cited by Dixit and Nalebuff, Prescott [16], is instructive in this regard. The destruction of the fleet did not occur right upon the arrival of the Spaniards, but rather several months after Cortés and his men first landed further down the Mexican coast on the shores of the Yucatán. They had by then already begun to found of a colony and what would become the city of Veracruz. There had been several exchanges between the Spaniards and both local Totonac populations in Cempoala and Aztec emissaries. Moreover, some of Cortés' men had just conspired to commandeer one of the ships and escape back to Cuba. Seen in its broader context, Cortés' strategic commitment appears not as an isolated or impetuous action, but as the result of a progressive process of learning over the course of which the conquistador and his men acquired a growing familiarity with their new environment. Moreover, Cortés' actions over these first months can be thought of as involving not a single strategic move, but a series of them. Upon setting sail, an ongoing dispute with the governor of Cuba, Diego Velázquez, had already complicated the possibility of turning the expedition back. An alliance concluded with the Totonacs had led to their dismissal of Aztec emissaries and positioned the Spaniards within the patchwork of existing local disputes. Moreover, a colony had begun to be founded, so that one could make the case that it is only when the prevailing element of uncertainty had been sufficiently reduced through a chain of more or less incremental prior actions that the destruction of the fleet could actually take place.

A Appendix

A.1 Example 1

In addition to the notation in the text, let $\overline{q}^L$ denote the Stackelberg leader quantities in the low cost state, and $\overline{\pi}^C, \underline{\pi}^C$ the Cournot profits in low and high cost states. Since $a > .1$ by assumption, $\underline{q}^L = (.5 - a)/2 < (.5 + a)/3 = \overline{q}^C$, so equilibrium Cournot and Stackelberg outputs are ranked as $0 \le \underline{q}^C < \underline{q}^L < \overline{q}^C < \overline{q}^L$. Since investing beyond the largest Stackelberg capacity can never be optimal for the leader firm, the relevant range of initial capacities to consider is $K_0^i \in \left[0, \overline{q}^L\right]$.

Consider first firm 2 and suppose that $K_0^1 = 0$. Then, as in Spencer and Brander [19] firm 2 chooses K_0^2 so as to maximize the expected Stackelberg leader profit

$$\begin{aligned} \mathbb{E}\pi\left(K_0^2; \widetilde{r}\right) &= .5K_0^2\left(1 - K_0^2 - q_1^*\left(K_0^2; .5 + a\right)\right) + .5K_0^2\left(1 - K_0^2 - q_1^*\left(K_0^2; .5 - a\right)\right) - .5K_0^2 \\ &= K_0^2\left(.25 - .5K_0^2\right), \end{aligned} \tag{17}$$

which yields an optimal initial investment of $K_0^{2*} = .25$, and $\mathbb{E}\pi\left(K_0^{2*}; \widetilde{r}\right) = 1/32$. The expected Cournot profit (if $K_0^1 = K_0^2 = 0$) is

$$\mathbb{E}\pi\left(0; \widetilde{r}\right) = .5\left(.5 - a\right)^2/9 + .5\left(.5 + a\right)^2/9 = \left(1 + 4a^2\right)/36. \tag{18}$$

Then $\mathbb{E}\pi\left(K_0^{2*}; \widetilde{r}\right) < \mathbb{E}\pi\left(0; \widetilde{r}\right)$ as $a > .3 (> 1/\sqrt{32})$.

Now consider firm 1, and note that $K_0^2 = 0$ in equilibrium. To derive firm 1's *ex-ante* expected profit, note first that in the product market stage, for a given K_0^1 firms choose Cournot outputs in the low (high) cost state if and only if $K_0^1 \le \overline{q}^C$ ($K_0^1 \le \underline{q}^C$). Otherwise, firm 1's equilibrium output is $\min\left\{K_0, \widehat{q}_1(r)\right\}$ where $\widehat{q}_1(r)$ denotes firm 1's output at the intersection of its $r = 0$ capital cost reaction function with firm 2's reaction $q_2^*(q_1; r)$ in $\mathbb{R}_+^2$, when it exists. In the low cost state, $q_2^*(q_1; .5 - a) = (.5 + a - q_1)/2$ intersects $q_1^*(q_2; 0) = .5(1 - q_2)$ at $\widehat{q}_1 = .5 - (a/3)$. Note that $\widehat{q}_1 \ge \overline{q}^C$ since $a \le .5$ and that $\widehat{q}_1 \le \overline{q}^L$ for $a \ge .3$ so firm 1's output choice is indeed be constrained by its outer reaction function over the relevant range of values of K_0^1. In the high cost state the relevant reaction functions are $q_2^*(q_1; .5 + a) = (.5 - a - q_1)/2$ and $q_1^*(q_2; 0) = .5(1 - q_2)$, whose intersection occurs for $\widehat{q}_1 = .5 + (a/3)$. In this latter case, $\widehat{q}_1 \ge \overline{q}^L$ since $a \le .5$ so the constraint is not binding.

By Proposition 1, firm 1 optimally sets $K_0^1 \ge \underline{q}^L$. The *ex-ante* payoff of firm 1, $\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right)$, is thus defined piecewise over the relevant range as follows.

$\underline{q}^L \leq K_0^1 \leq \overline{q}^C$: firms choose Cournot outputs in the low cost state and leader and follower outputs K_0^1 and $q_2^*\left(K_0^1; .5+a\right)$ in the high cost state. Therefore

$$\begin{aligned}
\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right) &= .5\left(\overline{q}^C\left(1-2\overline{q}^C\right) - (.5-a)\left(\overline{q}^C - K_0^1\right)\right) + .5K_0^1\left(1 - K_0^1 - q_2^*\left(K_0^1; .5+a\right)\right) - .5K_0^1 && (19) \\
&= .5\overline{\pi}^C + .5K_0^1\left(.5 - a - K_0^1 - q_2^*\left(K_0^1; .5+a\right)\right). && (20)
\end{aligned}$$

Note that whereas trivially $q_2^*\left(\underline{q}^L; .5+a\right) > 0$ for all $a \leq .5$, the non-negativity constraint may bind firm 2's best response for some larger values of K_0^1. As $a \geq .25$, $q_2^*\left(\overline{q}^C; .5+a\right) = \left(.5 - a - \overline{q}^C\right)/2 = (1-4a)/6 \leq 0$ so $q_2^*\left(K_0^1; .5+a\right) = 0$ over an interval $\left[.5-a, \overline{q}^C\right] \subset \left(\underline{q}^L, \overline{q}^C\right]$. Therefore (19) can be written

$$\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right) = \begin{cases} .5\overline{\pi}^C + .25K_0^1\left(.5 - a - K_0^1\right), & \underline{q}^L \leq K_0^1 \leq .5 - a \\ .5\overline{\pi}^C + .5K_0\left(.5 - a - K_0\right), & .5 - a \leq K_0 \leq \overline{q}^C \end{cases}. \tag{21}$$

$\overline{q}^C < K_0^1 \leq \overline{q}^L$: firms choose leader and follower outputs $q_1 = \min\left\{K_0^1, \widehat{q}_1\right\}$ and $q_2^*\left(q_1; .5-a\right)$ in the low cost state and K_0^1 and $q_2^*\left(K_0^1; .5+a\right) = 0$ in the high cost state. Recall also from above that $\widehat{q}_1 \in \left[\overline{q}^C, \overline{q}^L\right]$. For $K_0^1 \leq \widehat{q}_1$, $q_1 = K_0^1$ and the leader firm's expected profit is

$$\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right) = .5K_0^1\left(1 - K_0^1 - q_2^*\left(K_0^1; .5-a\right)\right) + .5K_0^1\left(1 - K_0^1\right) - .5K_0^1 = .125K_0^1\left(3 - 2a - 6K_0^1\right). \tag{22}$$

For $K_0^1 > \widehat{q}_1$, the leader firm's expected profit is

$$\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right) = .5\widehat{q}_1\left(1 - \widehat{q}_1 - q_2^*\left(\widehat{q}_1; .5-a\right)\right) + .5K_0^1\left(1 - K_0^1\right) - .5K_0^1 = .125 - (a/12) - \left(a^2/9\right) - .5\left(K_0^1\right)^2. \tag{23}$$

So that over $\left(\overline{q}^C, \overline{q}^L\right]$,

$$\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right) = \begin{cases} 125K_0^1\left(3 - 2a - 6K_0^1\right), & \overline{q}^C \leq K_0^1 \leq \min\left\{\overline{q}^L, \widehat{q}_1\right\} \\ .125 - (a/12) - \left(a^2/9\right) - .5\left(K_0^1\right)^2, & K_0^1 \geq \min\left\{\overline{q}^L, \widehat{q}_1\right\} \end{cases}. \tag{24}$$

The leader's optimal capacity maximizes $\mathbb{E}\pi\left(K_0^1; \widetilde{r}\right)$. Over $\left[\underline{q}^L, \overline{q}^C\right]$, there is a local maximum $K_0^{1*} = .25 - .5a$ $(= \underline{q}^L)$. The resulting expected profit is $\mathbb{E}\pi\left(\underline{q}^L; \widetilde{r}\right) = .5\overline{\pi}^C + .25\,(.25 - .5a)^2$. The remaining piece of the expected profit, (24), is concave over $\left[\overline{q}^C, \min\left\{\overline{q}^L, \widehat{q}_1\right\}\right]$ and decreasing thereafter. Over the latter interval, $d\mathbb{E}\pi/dK_0^1 = .125\left(3 - 2a - 12K_0^1\right)$ so that $\left(d\mathbb{E}\pi/dK_0^1\right)\left(\overline{q}^C\right) = .125\,(1-6a) < 0$ and expected profit over $\left[\overline{q}^C, \overline{q}^L\right]$ is thus maximized at $\overline{q}^C$. Therefore, K_0^{1*} as defined above is a global maximum of expected profit.

Substituting values into (21) and rearranging gives

$$\mathbb{E}\pi\left(K_0^{1*}; \widetilde{r}\right) = .5\frac{(.5+a)^2}{9} + .5\frac{(.5-a)^2}{8} = \frac{17 - 4a + 68a^2}{576}. \tag{25}$$

The value of exercising strategic leadership to firm 1 is $\mathbb{E}\pi\left(K_0^{1*};\widetilde{r}\right) - \mathbb{E}\pi\left(0;\widetilde{r}\right)$, *i.e.* the difference between (25) and (18), is $(1-2a)^2/576$. Firm 1's first-mover advantage is taken to be the difference in profit between firm 1 and firm 2 at equilibrium. As firm 2 earns

$$\mathbb{E}\widehat{\pi}\left(K_0^{1*};\widetilde{r}\right) = .5\frac{(.5+a)^2}{9} + .5\frac{(.5-a)^2}{16}, \tag{26}$$

the first-mover advantage here is simply $(.5-a)^2/32$.

A.2 Duopoly option payoff

For a given current price $X_t = X$ of the capital good let $F(X)$ denote the expected value of an option on the duopoly profit stream whose net present value is Π_D. The function F is known to be twice differentiable and to satisfy

$$\rho F(X)\,dt = \mathbb{E}dF(X). \tag{27}$$

Standard reasoning (expanding the right-hand side of (27) around X_t using Itô's lemma and taking the expectation, see Dixit and Pindyck [8]) yields the differential equation that F solves

$$\rho F(X) = \mu X F'(X) + \frac{\sigma^2}{2} X^2 F''(X) \tag{28}$$

along with the different boundary and smooth pasting conditions $F(\infty) = 0$, $F(X_F) = \Pi_D - X_F$, and $F'(X_F) = -1$. The solution to (28) has the form $F(X) = AX^b$. The corresponding fundamental quadratic $\frac{\sigma^2}{2}b(b-1) + b\mu - \rho = 0$ has two roots of which only the negative lower root $b' \equiv -\beta := \left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right) - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2\rho}{\sigma^2}}$ satisfies the first boundary condition. Solving then yields the optimal investment threshold $X_F = \frac{\beta}{\beta+1}\Pi_D$ and option value

$$F(X_t) = \begin{cases} \Pi_D - X_t, & X_t \leq X_F \\ \frac{\beta^\beta}{(\beta+1)^{\beta+1}}\Pi_D^{\beta+1}X_t^{-\beta}, & X_t \geq X_F \end{cases}. \tag{29}$$

A.3 Firm 2 preemption threshold

As described in the text for a given accumulated capital $\overline{K}^1$ firm 2's preemption threshold X_P^2 when it exists is the upper root of the condition

$$\left(\Pi_M - X_P^2\right)\left(X_P^2\right)^\beta - \left(\Pi_M - \Pi_D\right)\left(\frac{\beta}{\beta+1}\frac{\Pi_D}{1-\overline{K}^1}\right)^\beta = \frac{\beta^\beta \Pi_D^{\beta+1}}{(\beta+1)^{\beta+1}}. \tag{30}$$

First write (30) more compactly by normalizing by $\Pi_D^{\beta+1}$ and setting $x \equiv X_P^2/\Pi_D$ and $m \equiv \Pi_M/\Pi_D$. Then the condition is equivalent to

$$f(x) := (m-x)\,x^{\beta} = \left(\frac{\beta}{\beta+1}\right)^{\beta}\left(\frac{1}{\beta+1}+\frac{m-1}{\left(1-\overline{K}^1\right)^{\beta}}\right) =: g\left(\overline{K}^1\right). \tag{31}$$

The left hand side is a concave function f with $f(0) = f(m) = 0$ that attains a maximum value $\beta^{\beta}m^{\beta+1}/(\beta+1)^{\beta+1}$ at $x_0 = \beta m/(\beta+1)$. The right-hand side is an increasing function g over $[0,1)$ with $g(0) = \beta^{\beta}((\beta+1)m-\beta)/(\beta+1)^{\beta+1}$ and $\lim_{\overline{K}^1\to 1} g\left(\overline{K}^1\right) = \infty$. After simplification $g(0) < f(x_0)$ if and only if

$$m^{\beta+1} - (\beta+1)\,m + \beta > 0, \tag{32}$$

which holds for $m > 1$. The upper root in x of (31) exists so long as $\overline{K}^1$ is not too large. Letting $\overline{K}$ denote the solution to $g\left(\overline{K}\right) = f(x_0)$, *i.e.*

$$\overline{K} = 1 - \left(\frac{(\beta+1)(m-1)}{m^{\beta+1}-1}\right)^{\frac{1}{\beta}}, \tag{33}$$

then x (and X_P^2) is well-defined for $K^1 \in \left[0, \overline{K}\right]$. Economically, $\overline{K}$ is the upper bound of the capital stocks over which equilibrium in a preemption game between the two firms is of the preemptive type, and for $\overline{K}^1 > \overline{K}$ it is of the sequential (Stackelberg) type. From 30 and when firm 2's preemption threshold X_P^2 is well-defined one may explicitly define $K_P : [\beta\Pi_M/(\beta+1), X_P] \to \left[0, \overline{K}\right]$ as

$$K_P(X) = 1 - \left(\frac{1}{m-1}\left(\left(m - \frac{X}{X_F^2}\right)\left(\frac{X}{X_F^2}\right)^{\beta} - \frac{1}{\beta+1}\right)\right)^{-\frac{1}{\beta}} \tag{34}$$

which is the minimum capital stock that firm 1 must hold when the price of the capital good is X in order to dissuade instantaneous leader investment by firm 2.

Note that as a result of this derivation:

Proposition 2 *There is an upper bound $\overline{K} < 1$ on the amount of capital that firm 1 precommits before investing.*

A.4 $\varepsilon-$equilibrium derivation

In equilibrium firm 2's payoff from the threshold X_{ε}^2 is $F^2(X^*)\,(X^*/X(0))^{\beta}$, but it cannot do better either by delaying or by preempting, as in the latter case for instance, choosing $X^{2\prime} = X^* + \delta'$ for some small δ'. Given firm 1's capital accumulation policy, $K_\varepsilon^1\left(X^{2\prime}\right) > K_P\left(X^{2\prime}\right)$ and therefore such a choice by firm 2 would yield a payoff $L^2(\overline{K}^1, X^{2\prime}) < F^2(X^{2\prime}) = F^2(X^*)$.

The are a greater number of deviations from the equilibrium strategy to consider for firm 1, of which many can be directly eliminated. For instance, incremental investment at any rate greater than $|dK_P(X)|$ holding the investment threshold constant results in a stochastically dominated payoff that only raises costs without altering the benefit derived from entry. For firm 1 the most profitable deviation from the equilibrium strategy K_ε^1 given that it carries an additional capital stock of δ units is to invest optimally at the threshold $X_\delta^1 = (\beta/(\beta+1))\left(\Pi_M/\left(1-K_P\left(X_\delta^1\right)-\delta\right)\right)$, *i.e.*

$$K_\varepsilon^{1\prime}(X) = \begin{cases} 0, & X > X_P + \delta \\ \delta, & X_P < X \leq X_P + \delta \\ K_P(X) + \delta, & X_\delta^1 < X \leq X_P \\ 1, & X \leq X_\delta^1 \end{cases}. \tag{35}$$

If firm 1 follows this strategy, it earns a payoff

$$\widehat{V}^1\left(X_\delta^1\right) = \left[\Pi_M - X_\delta^1\left(1 - K_P\left(X_\delta^1\right)\right)\right]\left(\frac{X_\delta^1}{X(0)}\right)^\beta - \left(\Pi_M - \Pi_D\right)\left(\frac{X_F^2}{X(0)}\right)^\beta + \int_{X_\delta^1}^{X_P} \frac{Z^{\beta+1}}{[X(0)]^\beta} dK_P(Z) \tag{36}$$

and the difference,

$$\widehat{V}^1\left(X_\delta^1\right) - \widehat{V}^1\left(X^*\right) = \left[\Pi_M - X_\delta^1\left(1 - K_P\left(X_\delta^1\right)\right)\right]\left(\frac{X_\delta^1}{X(0)}\right)^\beta - \left[\Pi_M - X^*\left(1 - K_P\left(X^*\right)\right)\right]\left(\frac{X^*}{X(0)}\right)^\beta \geq 0, \tag{37}$$

vanishes as $\delta \to 0$ by continuity of K_P and $\widehat{V}^1$.

A.5 k^* comparative static

Combining the first-order condition,

$$X^* = \frac{\beta}{\beta+1}\frac{\Pi_M}{k^*}. \tag{38}$$

and definition of k in (34)

$$k^* = \left\{\frac{\Pi_D}{\Pi_M - \Pi_D}\left[\left(\frac{\Pi_M}{\Pi_D} - \frac{X}{X_F^2}\right)\left(\frac{X}{X_F^2}\right)^\beta - \frac{1}{\beta+1}\right]\right\}^{-\frac{1}{\beta}} \tag{39}$$

yields after rearrangement

$$\left(\frac{\Pi_M}{\Pi_D}\right)^{\beta+1}[k^*]^{-(\beta+1)}+\left[-\left(\frac{\Pi_M}{\Pi_D}\right)^{\beta+1}+\frac{\Pi_M}{\Pi_D}-1\right][k^*]^{-\beta}+\frac{1}{\beta+1}=0. \tag{40}$$

Letting $m\equiv\Pi_M/\Pi_D$, k^* is implicitly defined by a condition $F(k,m,\beta)=0$. Then

$$\begin{aligned} F_k &= -(\beta+1)\,m^{\beta+1}[k^*]^{-(\beta+2)}-\beta\left(-m^{\beta+1}+m-1\right)[k^*]^{-(\beta+1)} && (41)\\ &= -\left((1+\beta k^*)\,m^{\beta+1}+\beta\,(m-1)\,k^*\right)[k^*]^{-(\beta+2)}<0 && (42)\end{aligned}$$

and

$$\begin{aligned} F_m &= (\beta+1)\,m^{\beta}[k^*]^{-(\beta+1)}+\left(-(\beta+1)\,m^{\beta}+1\right)[k^*]^{-\beta} && (43)\\ &= \left(1+(\beta+1)\,m^{\beta}\,(1-k^*)\right)[k^*]^{-(\beta+1)}>0. && (44)\end{aligned}$$

Since $dk^*/d\Pi_M$ has the sign of $-F_m/F_k$ it follows that $dk^*/d\Pi_M>0$.

B Markov perfect equilibrium of the investment game

Further specifications are necessary in order to show that the $\varepsilon-$equilibrium in Section 3 converges to an appropriate Markov perfect equilibrium as ε becomes arbitrarily small. In order to focus on the game-theoretic aspects, this is done in the case of a deterministic input price and further specifics of the stochastic case are left for future work.

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