Subsidised Capacity Investment under Uncertainty

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Abstract

After privatisation, policy measures are implemented to align the private firm's decisions to the welfare maximiser's decisions. This research work considers subsidy supports such as price support and reimbursed investment cost support. We analyses how the subsidy supports affects the investment decisions for profit and welfare maximisers under uncertainty, and how to align their decisions by optimal subsidy policies. An example is from the green energy investment, where a power firm wants to undertake an investment to build a power plant. The commodity price with no subsidies at time t in the market is denoted by $p(t)$ and

$$p(t) = X(t) - \eta K(t), \tag{1}$$

with $K(t)$ as the total market output, $\eta > 0$ is a constant, and $X(t)$ is assumed to follow a geometric Brownian motion:

$$dX(t) = \mu X(t)\, dt + \sigma X(t)\, d\omega(t), \tag{2}$$

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Support Scheme	$P_i(t)$	$I(K)$
Flexible Feed-in Price Support ($i = P$)	$(1 + S_P)\,p(t)$	δK
Fixed Feed-in Price Support ($i = F$)	$p(t) + S_F$	δK
Reimbursed Investment Costs Support ($i = G$)	$p(t)$	$(1 - S_G)\,\delta K$

Table 1: Notations for $P_i(t)$ and $I(K)$.

in which μ is the drift rate, $d\omega\,(t)$ is the increment of a Wiener process, and $\sigma > 0$ is a constant. The firm is risk neutral and discounts against rate r. We assume $r > 2\mu + \sigma^2$. Otherwise, the investment may never occur. Based on this market uncertainty, three kinds of subsidies are considered: feed-in premiums price support (flexible and fixed), reimbursed investment cost (or investment tax credit) and feed-in tariff support. Suppose the subsidized price is $P(t)$, then $P(t)$ is defined as in Table 1, where $I(K)$ is investment costs and S_i with $in \in \{P, F, G\}$ is the subsidy rate. The unit cost of capacity is $\delta > 0$.

The analytical results show that subsidy encourages earlier investment at the cost of investing less. For the flexible price support, fixed price support and reimbursed investment cost support, the optimal investment decisions for the profit maximiser are summarised as

Proposition 1. *Denote*

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}} > 2. \tag{3}$$

The value of the monopolist facing flexible feed-in premium subsidy support is equal to

$$V_P(X) = \begin{cases} A_P X^{\beta} & \text{if } X < X_P^*, \\ \frac{(1+S_P)XK}{r-\mu} - \frac{(1+S_P)\eta K^2}{r} - \delta K & \text{if } X \geq X_P^*, \end{cases} \tag{4}$$

in which

$$A_P = \frac{r\delta}{\beta(\beta-2)(r-\mu)\eta}\left(\frac{\beta\delta(r-\mu)}{(\beta-2)(1+S_P)}\right)^{1-\beta}. \tag{5}$$

The optimal investment threshold X_P^* and the corresponding investment capacity $K_P^*(X_P^*)$ under the flexible feed-in premium subsidy for $S_P \geq 0$ are given by

$$X_P^* = \frac{\beta\delta(r-\mu)}{(\beta-2)(1+S_P)}, \tag{6}$$

$$K_P^* \equiv K_P^*(X_P^*) = \frac{r\delta}{(\beta-2)(1+S_P)\eta}. \tag{7}$$

The value of the monopolist facing fixed feed-in premium subsidy support is equal to

$$V_F(X) = \begin{cases} A_F X^\beta & \text{if } X < X_F^*, \\ \frac{XK}{r-\mu} - \frac{\eta K^2}{r} + \frac{S_F K}{r} - \delta K & \text{if } X \geq X_F^*, \end{cases} \tag{8}$$

with

$$A_F = \frac{r\left(\delta - \frac{S_F}{r}\right)}{\beta(\beta-2)(r-\mu)\eta} \left(\frac{\beta(r-\mu)\left(\delta - \frac{S_F}{r}\right)}{\beta - 2} \right)^{1-\beta}. \tag{9}$$

The optimal investment threshold X_F^* and the optimal investment capacity $K_F^*(X_F^*)$ under the fixed feed-in premium subsidy when $\delta - S_F/r > 0$ are given as

$$X_F^* = \frac{\beta(r-\mu)}{\beta-2}\left(\delta - \frac{S_F}{r}\right), \tag{10}$$

$$K_F^* \equiv K_F^*(X_F^*) = \frac{r}{(\beta-2)\eta}\left(\delta - \frac{S_F}{r}\right). \tag{11}$$

The value of the monopolist facing reimbursed investment cost subsidy support is equal to

$$V_G(X) = \begin{cases} A_G X^\beta & \text{if } X < X_G^*, \\ \frac{XK}{r-\mu} - \frac{\eta K^2}{r} - \delta\left(1 - S_G\right) K & \text{if } X \geq X_G^*, \end{cases} \tag{12}$$

where

$$A_G = \frac{r\delta(1-S_G)}{\beta(\beta-2)(r-\mu)\eta} \left(\frac{\beta\delta(r-\mu)(1-S_G)}{\beta-2} \right)^{1-\beta}. \tag{13}$$

The optimal investment threshold X_G^* and the optimal investment capacity $K_G^*(X_G^*)$ under reimbursed investment cost subsidy when $0 \leq S_G \leq 1$ are

$$X_G^* = \frac{\beta\delta(r-\mu)}{\beta-2}\left(1 - S_G\right), \tag{14}$$

$$K_G^* \equiv K_G^*(X_G^*) = \frac{r\delta}{(\beta-2)\eta}\left(1 - S_G\right). \tag{15}$$

For $i \in \{P, F, G\}$, if $X_i^* \leq X(0)$, then the firm invests immediately at $X(0)$ with capacity levels $K_i^* = K_i^*(X(0))$.

There exists optimal subsidy policies to align the firm's investment decisions to decisions that maximise social welfare.

Proposition 2. *The government does not provide subsidy support to the firm until the threshold $\frac{\beta\delta(r-\mu)}{\beta-2}$ is reached and for $\beta > 4$, the optimal subsidy rates to align the profit* *maximiser's optimal capacity with the welfare maximiser's optimal capacity are*

$$S_P^* = \frac{2}{\beta - 4}, \tag{16}$$

$$S_F^* = \frac{2r\delta}{\beta - 2}, \tag{17}$$

$$S_G^* = \frac{2}{\beta - 2}. \tag{18}$$

Otherwise, there is no optimal subsidy rates to align the profit and welfare maximiser's investment decisions.

Welfare analysis is done to get how much the total surplus increases because optimal subsidy policy aligns two parties' decisions, and how much the total surplus loss because of the subsidy costs. Comparison is also carried out among three subsidy supports.