Greenfield Investment or Acquisition? The Decision under Hidden Competition*

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Abstract

In this paper we extend the literature on real options under hidden competition. In addition to the decision of investing or waiting, we consider the realistic alternative of acquiring the hidden rival, after his appearance in the market. The model that supports the decisions regarding the timing, the best alternative available, as well as the optimal scale for the project is derived. We also introduce and analyze the conditions under which an acquisition is preferable to the greenfield investment, which can be useful for supporting the decision making in real world.

Keywords: Real Options; Hidden Competition; Acquisitions; Greenfield Investment; **JEL codes:**

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1 Introduction

The effect of competition in the investment decisions of companies is well established in the real options literature. The so-called "real options game models"¹ build the dynamics of investment where firms integrate the behavior of the competitors in their our decisions, acting in a optimal manner. After the seminal work of Smets (1993), where the leader/follower optimal investment decision under uncertainty was developed, many contributions appeared since then, for instance, Grenadier (1996), Weeds (2002), Shackleton et al. (2004), Pawlina and Kort (2006), Bouis et al. (2009), and Pereira and Rodrigues (2014), among many others.

A common assumption of these papers is that they consider full information about the project value drivers for all the competitors, and so it is possible for them to endogenize all the information for finding the optimal competitive behavior.

Some notable exceptions to the full information setting are Lambrecht and Perraudin (2003), that considers incomplete information about the competitor's investment cost, Hsu and Lambrecht (2007), that builds on Lambrecht and Perraudin (2003) in the context of patent racing, and Nishihara and Fukushima (2008) where a start-up firm has incomplete information about the behavior of a large competitor.

A recent trend in the literature assumes a more extreme setting where firms act in a total hidden competition environment (Armada et al. (2011), Pereira and Armada (2013), Lavrutich et al. (2014), and Huberts et al. (2015)). Under this setting, the potential competitors remain hidden, not unveiling their intention to enter the market, until the moment they decide to do so. In this context, the threat comes not from the known competitors, but instead from "three guys in a garage" developing new, sometimes radical and unexpected, business ideas. Many examples can be given in different industries: "(...) before its legendary rise, Apple was just three guys in a garage in Los Altos, California"², "Three Guys in a Garage Are Turning Your Eyes Into Powerful Remote Controls"³, "WestconGroup: From Garage Guys To Global Distribution Powerhouse"⁴, or the meaningful Michael Bloomberg's confession: "My great fear is that there are three guys in a garage right now doing the same thing to us that we did to Reuters and Dow Jones" Ottoo (2000). Common to these examples is the fact that, in many situations, competition can

¹Refer to Azevedo and Paxson (2014) and Chevalier-Roignant et al. (2011) for an extensive review of the main developments in dynamic real options games over the last two decades.

²http://www.businessinsider.com/history-of-apple-in-photos-2015-8

³ https://pando.com/2012/10/05/three-guys-in-a-garage-are-turning-your-eyes-into-powerful-remote-controls/

⁴ http://www.crn.com/slide-shows/managed-services/300077946/westcongroup-from-garage-guys-to-global-distribution-powerhouse.htm

come apparently from nowhere, starred by someone not yet known. In this competitive context, a company willing to invest needs to account for the possibility of being preempted by a hidden rival, without the possibility to endogenize his behavior. In fact, the entrance of a hidden competitor is an exogenous event that has some probability of occurrence, and for which a company has no control measures.

Assuming the market accommodates a finite number of active firms, the entrance of a hidden rival leads to sudden decrease of the available places, which, in turn, may speed up a firm's investment. In this setting, the investment opportunity is approaching maturity as new competitors enter into the market and disappears when the last place gets occupied. A firm's optimal behavior should be balance between the benefits of waiting and the risks of being preempted by a hidden rival. In other words, in every moment in time, two alternative decisions are considered by a firm: either to invest or to postpone the entrance into the market. Naturally, the risk of a competitive damage only exists in the latter situation.

Our paper builds on this piece of literature, namely on Pereira and Armada (2013). However, we realize that a third alternative is available to a firm: the option to acquire the hidden rival after his appearance. This possibility introduces more flexibility in the investment process, and impacts the decision. Acquiring the rival allows not only to eliminate any further risk arising from the hidden competition, but also to benefit from the potential synergies. According to this idea, it could be beneficial for a firm to acquire the hidden rival after his appearance in the market, even when more places remain available for greenfield investment. In this paper, we show the conditions where an acquisition is better than greenfield investment.

Many examples can be found real world. In 2012, 18 months after its launch, Facebook purchased the photo-sharing network Instagram for \$1 billion. In 2014, Google bought London-based artificial intelligence company DeepMind for more than \$500 million. Finally, in 2002, eBay acquired online payments company PayPal in a deal valued at \$1.5 billion. In all these examples, the decision was to buy the competitor, instead of investing to compete with him.

Naturally, we also have many situations where companies decided not to acquire the new rival, but instead, opt for the greenfield investment. A famous example is the decision IBM toke to keep developing its own personal computer (IBM PC), instead of approaching the *young* Apple at the time Macintosh (with the revolutionary graphic interface) was launched.

The conditions that make one alternative (for instance, the greenfield investment) preferable than the other (the acquisition), under the context of hidden competition, is the main focus of this paper. The model herein proposed aims to support decisions under this setting.

This paper contributes to the literature in several ways. Firstly, we extend the liter-

ature on real options under hidden competition by considering the realistic alternative of acquiring the hidden rival, after his appearance in the market (in addition to the greenfield investment and to the waiting option). Secondly, we derive the model that supports the decisions regarding the timing, the best alternative available, as well as the optimal scale for the project (and the latter has not been addressed in the related literature). Finally, we introduce and analyze the conditions under which an acquisition is preferable to the greenfield investment, which can be useful for supporting the decision making in real world.

The paper unfolds as follows: Section 2 presents in detail the derivation of the model considering the different stages in the market, as well the corresponding optimal alternatives; Section 3 analyses the results based on a numerical example; and Section 4 concludes.

2 The Model

Consider a firm facing the chance to enter in a market where only N companies can be placed. For the sake of convenience, assume the market is a duopoly, and so N = 2. The firm acts in a hidden competition environment, where potential competitors remain unrevealed until one observes their entrance into the market. Given the places available, three stages can be considered: the first one when no company is in the market, and so the two places remain available (N = 2), then when a company takes a place in the market and only one more is available (N = 1), and finally, the last stage, when two companies are in and there is no available room for more players (N = 0).

In each of these stages the firm has different options to consider. When there is no company in the market and the two places are available, the firm has to decide either to launch the project or to wait, facing, in the latter case, the risk of an sudden entrance of a hidden rival. If an entrance occurs while waiting to invest, the firm maintains the same options as in previous stage (one place remains in the market), but a new one arises: the option to acquire the rival company. If the firm continues to wait and a second hidden rival enters the market, the opportunity for greenfield disappears but remains the chance to acquire the first hidden entrant. All the set of options are resumed in Figure 1.

Let $F_N(X)$ represent the value of the investment opportunity as a function of the available places in the market (N) and the present value of the expected cash flows (X). We assume that X follows a geometric brownian motion process:

$$dX = \alpha X dt + \sigma X dW \tag{1}$$

where X > 0, and $\alpha = r - \delta$ represents the expected drift (r is the risk free rate and δ is the dividend-yield), σ the instantaneous volatility, dW is the increment of a Wiener process. Risk-neutrality is assumed.



Figure 1: The options the firm has in each stage. N stands for the places available in the market.

Additionally, we assume that $N \in \{0, 1, 2\}$ follows a Poisson process:

$$dN = -dq \tag{2}$$

where

$$dq = \begin{cases} 1 \text{ with probability } \lambda_N dt \\ 0 \text{ with probability } 1 - \lambda_N dt \end{cases}$$
(3)

The increment dq corresponds to a decrease in the available places in the market due to an entrance of a hidden rival. The parameter λ_N is the mean arrival rate during the period of time dt. We assume that λ_N increases as N decrease, since it would be more likely the entrance of a hidden rival when the places are reduced, and so $\lambda_2 < \lambda_1$.

Regarding the places in the market, a first condition can be set:

$$F_0(X) = 0 \tag{4}$$

which states that if there is no available room for more companies in the market, the option to invest should be worthless. s Based on the standard arguments, $F_N(X)$ must satisfy the following ordinary differential equation (o.d.e.), in the continuation region:

$$\frac{1}{2}\sigma^2 V^2 F_N''(X) + \alpha F_N'(X) - rF_N(x) + \lambda_N [F_{N-1}(X) - F_N(X)] = 0$$
(5)

where the last term captures the expected loss due to the entrance of a rival.

The solution for $F_N(X)$ depends on $F_{N-1}(X)$ and we need to move backwards in order to find it. Remember the value in the very last stage is given by Equation (4), and so we start at N = 1.

2.1 The solutions for N = 1

Let us assume that one hidden rival is already placed in the market, capturing a market share s. One may say that, in a context of hidden competition, the market share for the hidden rival is, by definition, unknown *ex-ante*. However, we can argue that in such a context the best guess is to assume the market share for the hidden rival is equal to the one the firm would choose if entering first in the market. Naturally, if a firm captures the share s, the other firm will get the remaining (1 - s). As will see later on, the optimal market share, s, will be endogenously determined in our model.s

Since one more place is available the firm needs to choose among the following three alternatives: the greenfield investment, the acquisition of the rival now placed in the market or the maintenance of both options, by deferring the decision. Let as start with the alternative for greenfield investment.

2.1.1 The value of the option to invest

When only one more place is available in the market, Equation (5) can be re-arranged as follows:

$$\frac{1}{2}\sigma^2 x^2 F_1''(X) + \alpha x F_1'(X) - (r+\lambda_1)F_1(X) = -\lambda_1 F_0(X)$$
(6)

Accounting for the condition presented in Equation (4), the right-hand side equals zero and the general solution is well known:

$$F_1(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2} \tag{7}$$

where β_1 and β_2 are, respectively, the positive and the negative roots of the characteristic equation $0.5\sigma^2\beta(\beta-1) + \alpha\beta - (r+\lambda_1) = 0$, and A_1 and A_2 are arbitrary constants that need to be determined.

Given the fact that the project must be worthless if not producing any cash flows, i.e.:

$$F_N(0) = 0 \tag{8}$$

the constant A_2 must be set equal to zero. The other constant A_1 can be found along with the optimal investment trigger (X_1^*) , using the value-matching and smooth pasting conditions, respectively:

$$F_1(X_1^*) = X_1^*(1-s) - I_{(1-s)}$$
(9)

$$F_1'(X_1^*) = 1 - s \tag{10}$$

where $I_{(1-s)}$ stands for the investment needed for capturing the market share (1-s). Equation (9) sets the Net Present Value when the investment is optimal, and Equation (10) ensures the function is continuously differentiable along X. Solving the equations we get:

$$F_1(X) = \begin{cases} \left(X^*(1-s) - I_{(1-s)}\right) \left(\frac{X}{X_1^*}\right)^{\beta_1} & \text{if } X < X^* \\ X(1-s) - I_{(1-s)} & \text{if } X \ge X^* \end{cases}$$
(11)

where

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda_1)}{\sigma^2}} > 1$$
(12)

and the optimal trigger to invest is:

$$X_1^* = \frac{\beta_1}{\beta_1 - 1} \frac{1}{1 - s} I_{(1 - s)} \tag{13}$$

2.1.2 The value of the option to acquire the rival firm

Consider now the option to acquire the rival company that has entered the market. We follow recent literature by assuming that the takeover is the result of a noncooperative bargaining game (as in Lukas and Welling (2012)). In particular, while the firm offers the incumbent a premium $\Psi > 0$ in exchange for some synergies $\xi > 0$ (as a percentage of firm's value) the target firm has to time the asset sale. Moreover, we will assume that transaction cost arise for both the target firm and the acquirer, i.e. εC and $(1 - \varepsilon)C$, respectively. Because the rival firm was hidden before, we will assume that he optimally chooses the same market share when he had decided to enter the market, i.e. s, previously. Hence his timing decision to sell the target with respect to $\bar{X}_1(s)$ solves the following optimization problem:

$$f_1(x) = \max_{\tau} \left[\mathbf{E} \left[\left((\Psi - 1) \bar{X}_1 s - \varepsilon C \right) e^{-r\tau} \right] \right], \tag{14}$$

$$= \left((\Psi - 1)\bar{X}_1 s - \varepsilon C \right) \left(\frac{X}{\bar{X}_1} \right)^{\gamma_1}$$
(15)

Consequently, the acquisition occurs once X hits an optimal trigger value from below, i.e. $X = \overline{X}_1$ with:

$$\bar{X}_1 = \frac{\gamma_1}{\gamma_1 - 1} \frac{\varepsilon C}{((\Psi - 1)s)} \tag{16}$$

Obviously, the timing decision depends on the premium $\bar{X}_1(\Psi)$ offered by the acquiring firm. As the acquirer is interested in maximizing his option value, the optimal premium is the solution to the following optimization problem, i.e.:

$$\max_{\Psi} \left[\left((\xi - \Psi) \bar{X}_1 s - (1 - \varepsilon) C \right) \left(\frac{X}{\bar{X}_1} \right)^{\gamma_1} \right]$$
(17)

It follows that the optimal premium Ψ^* paid results to:

$$\Psi^* = 1 + \frac{\varepsilon(\xi - 1)(\gamma_1 - 1)}{\varepsilon + (\gamma_1 - 1)}$$
(18)

2.1.3 The decision between the two alternatives

If N = 2 changes to N = 1 this will trigger the choice between the two alternatives (the acquisition or the greenfield investment) based on the value maximization rule:

$$\Omega(X) = \max[F_1(X), f_1(X)] \tag{19}$$

This means that the firm will decide for the alternative of most value, immediately after the entrance of a hidden rival into the market. After identifying the best alternative, the firm starts monitoring the state variable, exercising the option at the corresponding optimal moment.

2.2 The solutions for N = 2

Let us now move backwards to the initial stage, where no one is in the market. Based on the generic differential equation presented in (5), and considering the rule stated in (19), the value function $F_2(x)$ must satisfy appropriate o.d.e.s presented below.

2.2.1 For $\Omega(X) = F_1(X)$

Under the circumstances where greenfield investment is optimal at N = 1, the value function $F_2(X)$ must satisfy the following o.d.e.:

$$\frac{1}{2}\sigma^2 X^2 F_2''(X) + \alpha X F_2'(X) - (r+\lambda_2) F_2(X) = \begin{cases} -\lambda_2 \left(X_1^*(1-s) - I_{(1-s)} \right) \left(\frac{X}{X_1^*} \right)^{\beta_1} &, X < X_1^* \\ -\lambda_2 \left(X(1-s) - I_{(1-s)} \right) &, X \ge X_1^* \end{cases}$$
(20)

Following the standard procedures, and for the current level of the state variable, X, the value of the option to invest at the stage N = 2 comes:

$$F_{2}(X) = \begin{cases} b_{1}X^{\eta_{1}} + c_{1}\left(X_{1}^{*}(1-s) - I_{(1-s)}\right)\left(\frac{X}{X_{1}^{*}}\right)^{\beta_{1}} &, X < X_{1}^{*} \\ b_{3}X^{\eta_{1}} + b_{4}X^{\eta_{2}} + c_{2}X(1-s) - c_{3}I_{(1-s)} &, x_{1}^{*} \le X < X_{2}^{*} \\ Xs - I_{(s)} &, X \ge x_{2}^{*} \end{cases}$$
(21)

where $c_1 = \frac{\lambda_2}{\lambda_2 - \lambda_1}$, $c_2 = \frac{\lambda_2}{r - \alpha + \lambda_2}$, and $c_3 = \frac{\lambda_2}{r + \lambda_2}$. $I_{(s)}$ stands for the investment that it is required to capture the market share s. Additionally, η_1 is similar to Equation (12) but

replacing λ_1 by λ_2 , and

$$\eta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r+\lambda_2)}{\sigma^2}} < 0 \tag{22}$$

The solutions for the unknowns b_1 , b_3 , b_4 , and X_2^* are obtained by solving the following set of equations:

$$\begin{split} b_1 X_1^{*\eta_1} + c_1 \left(X_1^*(1-s) - I_{(1-s)} \right) &= b_3 X_1^{*\eta_1} + b_4 X_1^{*\eta_2} + c_2 X_1^*(1-s) - c_3 I_{(1-s)} \\ \eta_1 b_1 X_1^{*\eta_1 - 1} + c_1(1-s) &= \eta_1 b_3 X_1^{*\eta_1 - 1} + \eta_2 b_4 X_1^{*\eta_2 - 1} + c_2(1-s) \\ b_3 X_2^{*\eta_1} + b_4 X_2^{*\eta_2} + c_2 X_2^*(1-s) - c_3 I_{(1-s)} &= X_2^* s - I_{(s)} \\ \eta_1 b_3 X_2^{*\eta_1 - 1} + \eta_2 b_4 X_2^{*\eta_2 - 1} + c_2(1-s) &= s \end{split}$$

which together ensure that $F_2(X)$ is continuous and differentiable along x.

The solutions presented above are appropriate for conditions that lead to $X_1^* < X_2^*$ (i.e., the triggers for investing is smaller when only one place is available in the market). However, if the greenfield is comparatively more attractive at N = 1 than at N = 2(namely, when a larger market share is captured by the firm in the later stage greenfield investment), an inverted order of the triggers, $X_2^* < X_1^*$, may occur. For obtaining the solutions for all range of s this inverted order must also be considered. In this case, the solutions for $F_2(X)$ and for X_2^* are as follows:

$$F_2(X) = \begin{cases} b_1 X^{\eta_1} + c_1 \left(X_1^* (1-s) - I_{(1-s)} \right) \left(\frac{X}{X_1^*} \right)^{\beta_1} &, X < X_2^* \\ X_s - I_{(s)} &, X \ge X_2^* \end{cases}$$
(23)

where the trigger X_2^* is the numerical solution of the equation:

$$(\eta_1 - 1)X_2^*s - \eta_1 I_{(s)} + (\beta_1 - \eta_1)c_1 \left(X_1^*(1 - s) - I_{(1 - s)}\right) \left(\frac{X_2^*}{X_1^*}\right)^{\beta_1} = 0$$
(24)

subject to $X_2^* < X_1^*$.

The inverted order of the triggers occurs whenever the market share s is between s_a and s_b , the two roots of the following quadratic equation:

$$(\eta_1 - 1)\frac{\beta_1}{\beta_1 - 1} - \eta_1 \frac{s}{1 - s} + \frac{\beta_1 - \eta_1}{\beta_1 - 1}c_1 \frac{1 - s}{s} = 0$$
(25)

2.2.2 For $\Omega(X) = f_1(X)$

For cases where the acquisition of the hidden rival is optimal, the solutions regarding $F_2(x)$ are as follows:

$$\frac{1}{2}\sigma^2 X^2 F_2''(X) + \alpha X F_2'(X) - (r+\lambda_2) F_2(X) = \begin{cases} -\lambda_2 \left(\xi - \Psi^*\right) s \bar{X}_1 - (1-\varepsilon) C\right) \left(\frac{X}{\bar{X}_1}\right)^{\gamma_1} &, X < \bar{X}_1 \\ -\lambda_2 \left((\xi - \Psi^*) s X - (1-\varepsilon) C\right) &, X \ge \bar{X}_1 \end{cases}$$
(26)

Accordingly, the value of the option to invest at the stage N = 2 is:

$$F_{2}(X) = \begin{cases} B_{1}X^{\eta_{1}} + \left((\xi - \Psi^{*})s\bar{X}_{1} - (1 - \varepsilon)C\right)\left(\frac{X}{\bar{X}_{1}}\right)^{\gamma_{1}} & , X < \bar{X}_{1} \\ B_{3}X^{\eta_{1}} + B_{4}X^{\eta_{2}} + c_{2}(\xi - \Psi^{*})s\bar{X}_{1} - c_{3}(1 - \varepsilon)C & , \bar{X}_{1} \le X < \bar{X}_{2} \\ Xs - I_{(s)} & , X \ge \bar{X}_{2} \end{cases}$$
(27)

where c_1 , c_2 , c_3 , and η_1 are as previously presented.

The solutions for the unknowns B_1 , B_3 , B_4 , and \bar{X}_2 , are obtained by solving the equations:

$$B_{1}\bar{X}_{1}^{\eta_{1}} + \left((\xi - \Psi^{*})s\bar{X}_{1} - (1 - \varepsilon)C \right) = B_{3}\bar{X}_{1}^{\eta_{1}} + B_{4}\bar{X}_{1}^{\eta_{2}} + c_{2}(\xi - \Psi^{*})s\bar{X}_{1} - c_{3}(1 - \varepsilon)C$$

$$\eta_{1}B_{1}\bar{X}_{1}^{\eta_{1}-1} + (\xi - \Psi^{*})s = \eta_{1}B_{3}\bar{X}_{1}^{\eta_{1}-1} + \eta_{2}B_{4}\bar{X}_{1}^{\eta_{2}-1} + c_{2}(\xi - \Psi^{*})s$$

$$B_{3}\bar{X}_{2}^{\eta_{1}} + B_{4}\bar{X}_{2}^{\eta_{2}} + c_{2}(\xi - \Psi^{*})s\bar{X}_{2} - c_{3}(1 - \varepsilon)C = X_{2}^{*}s - I_{(s)}$$

$$\eta_{1}B_{3}\bar{X}_{2}^{\eta_{2}-1} + \eta_{2}B_{4}\bar{X}_{2}^{\eta_{2}-1} + c_{2}(\xi - \Psi^{*})s = s$$

The solutions resulting from the equations above lead to $\bar{X}_1 < \bar{X}_2$ (i.e., the trigger for the M&A, in stage N = 1, is smaller than the triggers to launch the greenfield investment at N = 2). However, as previously, for some market shares an inverted order in the triggers occurs, which must be considered for computing the triggers for all set of s. Under these circumstances, the value function F(2) is

$$F_{2}(X) = \begin{cases} B_{1}X^{\eta_{1}} + \left((\xi - \Psi^{*})s\bar{X}_{1} - (1 - \varepsilon)C\right)\left(\frac{X}{\bar{X}_{1}}\right)^{\gamma_{1}} , X < \bar{X}_{2} \\ Xs - I_{(s)} , X \ge \bar{X}_{2} \end{cases}$$
(28)

and the trigger \bar{X}_2 corresponds to the numerical solution of the equation:

$$(\eta_1 - 1)\bar{X}_2 s - \eta_1 I_{(s)} + (\gamma_1 - \eta_1) \left((\xi - \Psi^*)\bar{X}_1 s - (1 - \varepsilon)C \right) \left(\frac{\bar{X}_2}{\bar{X}_1} \right)^{\gamma_1} = 0$$
(29)

subject to $\bar{X}_2 < \bar{X}_1$.

The inverted order occurs whenever

$$s < \frac{H}{H + \eta_1 z} \tag{30}$$

where

$$H = (\gamma_1 - \eta_1) \left((\xi - \Psi^*) \frac{\gamma_1}{\gamma_1 - 1} \frac{\varepsilon C}{\Psi^* - 1} - (1 - \varepsilon) C \right) + (\eta_1 - 1) \frac{\gamma}{\gamma - 1} \frac{\varepsilon C}{\Psi^* - 1}$$
(31)

2.3 The optimal market share

While we have derived the flexibility value for the overall firm strategy, the firm has not yet chosen the optimal scale of the project, i.e. how much it will invest in order to capture a certain market share. Naturally, the higher the scale (and so the market share) the higher the investment that needs to be spent. In particular, we seek for a functional relation between the market share, $s \in (0, 1)$, and the investment, I, such that I'(s) > 0, I''(s) > 0and that $I(s) \to +\infty$ as $s \to 1$, which means that capturing all the market is prohibitively expensive. The following expression is used:

$$I(s) = \frac{s}{1-s}z\tag{32}$$

where z is a scale parameter.

Based on this relation an optimal market share need to be chosen such that it maximizes the overall value of the investment opportunity for the firm. Thus, we have:

$$\max_{a^*}(F_2(X, s^*)) \tag{33}$$

which leads the corresponding required investment:

$$I(s^*) \equiv I_{(s^*)} \tag{34}$$

3 Numerical Example

For the numerical example we use the following parameter values: X = 0.5, $\mu = 0.1$, $\sigma = 0.25$, $\delta = 0.05$, $\lambda_1 = 0.5$, $\lambda_2 = 0.05$, C = 4, $\epsilon = 0.1$, and for the cost function we use I(s) = z * (s/1 - s) with z = 9. Let us first assume that a hidden competitor has already entered the market, i.e. N = 1 and that the synergies that the hidden competitor would have generated are low, i.e. $\xi_L = 1.03$. As Figure 2 (a) indicates the option value for performing the greenfield investment is thus always greater than due to an M&A, irrespectively which market share s is chosen previously. As uncertainty increases, both strategies become more valuable, i.e. their option values increase and we also observe that the difference between $f_1(X_0, s)$ and $F_1(X_0, s)$ is smaller for low market shares.

To what extend are the investment thresholds x_1^* and x_2^* affected by varying market size. The results indicate that x_1^* monotonically decreases as the market share s increases which is reflected by Equation (13) and the fact that $I_{1-s} = z(1-s)/s$. Consequently, the more market share the firm has missed to secure in the first stage the less market share it can attract while the competitor is in the market in stage two. Obviously, the low market share comes at a lower cost and as a result the optimal investment threshold decreases as market share increases. Moreover, the usual result stands out, i.e. the higher the cash flow uncertainty the higher the optimal investment threshold becomes. Finally, the optimal investment threshold decreases as the threat of a new entrant increases in N = 1, i.e. as λ_1 increases. This result is in line with previous results and indicates that the firm's propensity to invest in the greenfield investment in the second stage will increase as the probability of a new hidden competitor increases (see e.g. Pereira and Armada (2013)).

Now let us assume that the firm is faced with the decision to enter the market by means of a greenfield investment in N = 1. Ceteris paribus, the firm anticipates that a greenfield investment in the subsequent stage is more favorable and due to the fact that no hidden competitor has entered so far it will invest as soon as the cash flow x(t) hits the optimal investment threshold X_2^* from below. As Figure 3 (b) reveals this optimal threshold increases as the market share increases. This is due to the fact that a higher market share implies higher sunk cost which raise the opportunity cost of giving up the option to invest. Obviously, the subsequent option value due to greenfield investment when competition has materialized is also priced in which dampens the increase of X_2^* for situations characterized by high sunk costs. Moreover, we find that the firm's propensity to invest will increase should the probability of a new entrant in either N = 1 or N = 2increase, i.e. X_2^* will decrease as λ_1 and λ_2 increase.

While these results replicate previous ones, we, however, find that the choice of market share renders the firm's overall entry strategy significantly. As can be seen from Figure 2, for high market shares we have $X_2^* > X_1^*$ indicating that postponing investment N = 2 is very attractive for the firm. However, it will speed up investment should a new competitor enter the market. On the contrary, for low market shares situations might occur where the firm will prolong waiting in reaction of the entry of a new competitor, i.e. when $X_1^* > X_2^*$. Put different, the threat of competition will not lead to an increased propensity to speed up investment.

Finally, Figure 2 (c) depicts how the choice of market share affects the overall option value for the firm at N = 2, i.e. $F(X_0, s)$. While an increase in cash flow volatility increases the option value an increased probability that a new entrant enters either in N = 1 or N = 2 reduces the option value. More importantly, however, the figure also reveals that $F(X_0, s)$ exhibits an inverted U-shape with respect to the market share s. Consequently, an increased commitment to invest more in order to obtain a larger stake in the market leads to a larger flexibility value. However, at a certain market share the overall option value decreases indicating that it becomes less valuable to invest too much. Obviously, an optimal market share exists for which the option value becomes maximal.

Solving for the optimal market share leads to the following results (see Figure 3). There is an ambiguous relationship between the optimal market share s^* and uncertainty. In



Figure 2: Greenfield investment dominant market entry where (a) shows the option values $F_1(s)$ and $f_1(s)$ as a function of market share s for different uncertainty levels, (b) the critical investment thresholds x_1^* and x_2^* as a function of market share s for different uncertainty levels, and (c) the option value of the overall greenfield investment at N = 2, i.e. $F_2(s)$ as a function of market share s.

particular, the results indicate that for low levels of uncertainty the optimal market share will decrease as uncertainty increases. Contrary, for higher levels of cash flow uncertainty the firm will opt for a higher market share as uncertainty increases. If the threat of a new entrant becomes more severe, i.e. λ_1 increases the U-shape pattern remains, however, the overall market share increases indicating that the firm will thus prefer to choose a larger market share. Alike, an increase in λ_2 will lead to higher (lower) levels of optimal market shares when uncertainty is low (high). Regarding the impact of λ_1 on the propensity to invest at N = 1 and N = 2 we find mixed results. While an increase in λ_1 leads to an increased propensity to invest in the Greenfield investment once a hidden competitor has already entered we find that the opposite is the case once a competitor has not yet materialized at all (see Figure 3 (b) and (c)). Consequently, an increase of λ_1 indicating that the last seat is about to be taken earlier leads to an increase of X_2^* which is opposite to the results of previous literature (e.g. Armada et al. (2011), Pereira and Armada (2013)). Obvious, in N = 2, the firm is also subject to the threat that a new competitor enters the market. Here we find that an increase in λ_2 will lead to a (lower) higher optimal investment threshold X_2^* (X_1^*) when uncertainty is low (high). Figure 3 summarizes these findings.



Figure 3: Greenfield investment dominant market entry where (a) shows the optimal market share $s(\sigma)$ as a function of uncertainty, (b) shows the critical investment thresholds X_2^* as a function of uncertainty, and (c) shows the critical investment thresholds X_1^* as a function of uncertainty.

Finally, we find an ambiguous timing-scale effect. More precisely, when uncertainty is generally high we find that an increase of cash flow uncertainty leads to higher investment levels and larger optimal investment threshold indicating a positive timing-scale effect. For low levels of cash flow uncertainty, however, an increase of σ leads to an decrease in s^* while X^* increases (Figure 3).

As synergies increases from $\xi_L = 1.03$ to $\xi_H = 1.25$, the M&A strategy becomes dominant in N = 1, i.e. once a hidden competitor enters (see Figure 4). Hence, the firm favours to acquire the new entrant instead of committing to organic growth. As opposed to the previous Greenfield investment where missing to secure s in N = 2 leads to having (1 - s) in N = 1 the firm's option to acquire the rival secures any previous committed market share. Hence, acquiring the rival acts as some kind of natural hedge. A direct result is that the option value assigned to acquiring the rival in N = 1 is no longer inverted U-shaped and increases as market share increases. In addition, a higher market share lowers the optimal acquisition threshold \bar{X}_1 . Obviously, as cash flow uncertainty increases the option value f_1 and \bar{X}_1 increase, too replicating well known results. Apart from securing any previously fixed market share the additional advantage of committing to an M&A in N = 1 is that it rules out the threat that the last seat might be taken in the market by another hidden firm. Consequently, λ_1 does not impact the decision making.



Figure 4: M&A dominant market entry where (a) shows the option values $F_1(s)$ and $f_1(s)$ as a function of market share s for different uncertainty levels, (b) the critical investment thresholds x_1^* and x_2^* as a function of market share s for different uncertainty levels, and (c) the option value of the overall greenfield investment at N = 2, i.e. $F_2(s)$ as a function of market share s.

Turning to the overall option value when N = 2 we find that the option value is again inverted U-shaped as market share increases. This effect is based on the same forces as described previously, i.e. increasing cost to scale which drive the Greenfield investment decision. As expected an increase in *s* leads to a higher investment threshold \bar{X}_2 . Moreover, we see from Figure 4 that an increase in cash flow uncertainty increases F_2 and \bar{X}_2 , respectively.

Solving for the optimal market share leads to the following results (see Figure 4).

Firstly, as uncertainty increases the firm will opt for a higher market share in N = 2. In comparison to the alternative Greenfield-Greenfield entry strategy discussed before, we do not find a U-shaped relationship and the results further indicate that the Greenfield-M&A entry strategy leads to smaller market shares and lower investment levels, respectively. However, the possibility of securing the intended market share by means of a natural hedge, i.e. having the option to buy the first-moving firm later on serves as an incentive to invest earlier. As Figure 4 reveals the optimal Greenfield investment threshold \bar{X}_2 is smaller than X_2^* . Obviously, as uncertainty increases the propensity to delay investment increases, too.

In line with the literature we find that an increased threat of being preempted by a hidden competitor increases the propensity to speed-up investment, i.e. \bar{X}_2 decreases. This goes hand-in-hand with a lower optimal market share indicating lower levels of investment. Interestingly, the optimal acquisition threshold \bar{X}_1 exhibits a U-shape pattern with respect to uncertainty. Consequently, for low levels of cash flow uncertainty, an increase in σ will increase the propensity to acquire the rival sooner while for higher levels of uncertainty an increase in σ will have the opposite effect. Moreover, due to the fact that an increased threat of being preempted in N = 2, i.e. a larger λ_2 , leads to an adjustment of the optimal market share we will see that such a threat also translates into an increased propensity to delay the acquisition once the hidden competitor has materialized in N = 1 (see Figure 5).

A final question remains which is linked to the interaction of optimal market share and the impact of investment cost under uncertainty. In particular, we want to check whether less expensive market entries lead to higher market power. From Eq. (X) it becomes apparent that z measures how strongly market share affects cost. Exemplary, for low values of z an increase increase in market share leads to an overall smaller increase of investment cost than for high values of z. Our results, however, indicate that the optimal choice of market share is less sensitive to z. As Figure (6) indicates, the firm will in general attempt to capture a market share larger than fifty percent. This is independent from whether the firm prefers greenfield or M&A should a hidden competitor preemt the firm. Moreover, even for very low values of z a favor for a majority market share sustains.

From the aforementioned comparative-static analysis we can deduce the following hypothesis:

Hypothesis 1: For markets that are characterized by a high threat of new entrants in both stages, i.e. N = 1, 2, which do not generate valuable synergies the first-moving firm will favor a higher (lower) market share should the operate in highly volatile (less volatile) industries. In such a setting, firm's operating in less volatile industries have a tendency to postpone rather to speed up greenfield investment as a sequence of hidden competition.

Hypothesis 2: For markets that a less volatile with strong synergy potential among the firms exhibit a higher probability that an entrepreneurial firm invests soon followed by



Figure 5: M&A dominant market entry where (a) shows the optimal market share s as a function of uncertainty, (b) shows the critical investment thresholds X_2^* as a function of uncertainty, and (c) shows the critical investment thresholds X_1^* as a function of uncertainty.

a greater propensity of consolidation after the first-entrant has materialized.

Hypothesis 3: If there is a promising market for $M \mathscr{C}As$ due to strong synergies potentials, then these industries exhibit lower levels of investment at the time the first-moving firm materializes.

Hypothesis 4: In industries with less promising market for $M \mathcal{C}As$, first-movers tend to capture larger stakes of the market, independently from the level of the uncertainty.

4 Conclusions

In this paper we extended the literature on real options under hidden competition. In addition to the decision of investing or waiting, we consider the realistic alternative of acquiring the hidden rival, after his appearance in the market. The model that supports the decisions regarding the timing, the best alternative available, as well as the optimal scale for the project is derived. We also introduce and analyze the conditions under which an acquisition is preferable to the greenfield investment, which can be useful for supporting the decision making in real world.

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Figure 6: Optimal market share as a function of uncertainty and z.