Optimal timing of technology adoption by incumbents: war of attrition versus preemption*

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Abstract

This paper considers two incumbent firms with an option to adopt a horizontally and vertically differentiated technology. The firms engage in a Stackelberg competition where they decide upon both the investment moment and the investment size. I find that adoption kills the old technology only when innovation is radical. When the degree of innovation is small and when the products are not close substitutes a war of attrition arises. Otherwise the firms end up in a preemption equilibrium. When a second-mover advantage is present, firms either want to stay alone on the old market or want to set a larger capacity as Stackelberg follower. This paper also shows that market uncertainty increases the first-mover advantage while at the same time it makes it more attractive for the endogenous follower to forego adoption.

Keywords: War of attrition, Second-mover advantage, Innovation, Capacity choice, Investment under Uncertainty, Oligopoly, Real-Option Games

JEL classification:

1 Introduction

Timing of technology adoption has received much attention over the years. In particular, is has been shown that games face different outcomes under different set-ups. Since Jensen (1992) it has been established that, under uncertainty, a second-mover advantage might arise, reflected by the presence of a war of attrition

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instead of the more traditional preemption run. Hoppe (2002) serves a decent overview of the literature and her conclusion has remained relatively consistent ever since. Firms are inclined to wait for new information - as e.g. the adoption by a competitor disclosing the true market characteristics - if the uncertainty about the profitablity of the new technology imposes that the expected post-adoption profit is too small. However, this hinges on the asymmetry between the prior and posterior beliefs on the new technology's profitability. Nevertheless, in this paper I show that this is not a nessecity. In particular, I show that a second-mover advantage can arrise even when waiting does not lead to new information, nor to the emergence of new technologies. This is in line with Golder and Tellis (1993), who conclude that, by studying 500 brands in 50 product categories, followers are more often market leaders and tend to be more successful. I moreover show that, after the adoption of one firm, other firms might decide not to undertake the adoption, even though the new technology yields an innovation with respect to the old technology. The empirical work on the diffusion of chairlifts by Mulligan and Llinares (2003) shows that adoption of a technological innovation by a firm decreases the likelihood that a local competitor will also adopt it. Despite well observed in real life, incentives to not undertake (profitable) adoption has not been a common observation in the theoretical literature[1].

A good example of these markets is one that arose very recently. Under the discussion of sustainability, the submarket for organic food and durable products gained much popularity over the past decades. New technologies are developped to manufacture products that are of a better quality, but also more expensive. Since only part of the consumers is willing to pay a higher price the market for traditional products never disappeared. Therefore, not all firms decided to adopt the new technology, which has lead to a variety of products consumers face nowadays. Especially in this sector, for different reasons, firms were reluctant to embrace the new technologies. However, many firms found it better to adopt than to stay on a crowded submarket. For some of them, this was a matter of conscious, but for many other firms this decision was a strategic choice.

This paper studies two incumbents with homogenous goods facing the option to adopt a better technology. When adopting, a firm replaces its current product by a new product that is horizontally and vertically differentiated from the earlier product. Vertical differentiation captures the innovative improvement relative to the old technology, or differently, the quality improvement, while the horizontal differentiation reflects the cross sensitivity between the products' prices, i.e. the degree to which the products are substitutes. Both products are part of the same market that is subject to exogenous stochastic shocks affecting the products' prices, so that the timing of technology is partially dependent on the firm's expectation of the market's future. Firms do not only decide upon the optimal adoption moment, but also upon the capacity size they set under the new technology. I find that it depends on the degree of differentiation whether only one firm adopts or whether both firms leave the old submarket. In particular, when both the vertical differentiation,

[1] The static game with reversible adoption by Reinganum (1983) includes imperfect information about the other firm's adoption. Here, the possibility that firms do not adopt is not an endogenous result.

i.e. the innovative improvement, and the cross sensitivity are small, only one firm undertakes the adoption. In such a situation, it serves both firms' interest if one of them decides not to switch to the new product, since serving your own submarket yields a higher profit than to serve the same submarket with a competitor. However, if the new technology is much more of an improvement or if the products are closer substitutes, the second firm decides to also substitute products after the leader's investment. One could fairly say that innovation kills the old technology when innovation is radical, but it does not when innovation is incremental.

Firms then face a second-mover advantage as a result of this: they prefer to stay with the old technology instead of undertaking the adoption. This happens only when products are poor substitutes and the quality improvement is small, so that no firm has the incentive to move first. Nevertheless, when both firms stay, they are both worse off and therefore optimally investment is undertaken by one firm, but neither firm prefers to do so. However, if the market uncertainty is small, a second type of second-mover advantage comes into play. Then, if the new technology is more vertically differentiated or if the cross-sensitivity is larger both firms want to adopt, but neither of them wants to adopt first. Smaller uncertainty levels accelerate investment so that adoption is undertaken in the situation where the first mover sets a small capacity size. Being a late mover induces a larger capacity size, which makes that firms prefer to be a late mover.

In the context of asymmetric firms, that is, when firms have different capacities on the submarket corresponding to the old technology, this paper shows that there are situations where one firm has a second-mover advantage while the other one has a first-mover advantage. Moreover, it shows that in these settings not always the firm with the first-mover advantage undertakes adoption first. In fact, it can happen that the firm with the second-mover advantage is the only adopter in the game. This happens since, although the firm prefers its competitor to leave the old submarket in an early stage of the game, it is worse off when the competitor undertakes the adoption in a later stage of the game. As a result, the firm with the second-mover advantage undertakes the adoption.

Earlier models on timing of technology adoptions under uncertainty include the seminal work by Reinganum (1981), building a game-theoretic framework and showing the diffusion of adoption dates, by Jensen (1982), where the probabilities of the outcomes of the project are uncertain, and by Fudenberg and Tirole (1985), loosening the precommitment assumption resulting in the firms' preemptive behavior. Other papers on the timing of technology adoption include Götz (1999), studying the diffusion of adoption under monopolistic competition, Riordan (1992), including regulations, and Hendricks (1992) on reputations, looking at the innovative capabilities of rival firms. The effect of R&D on the adoption moments was studied after Dutta, Lach, and Rustichini (1995).

Among others, Gal-Or (1985) considered the differences between being leader of follower. She concluded that, without uncertainty, the preferred position in the order depends on the slope of the players' reaction curves. Since then, different models have been introduced studying the adoption behavior of firms resulting from uncertainty regarding the new technology. In Hoppe (2000) the new technology can be good with probability p and bad with probability $1 - p$. When p is believed to be relatively small, Hoppe finds that

Information spillover	Choi (1997), Hoppe (2000), Frisell (2003) Ferreira, Ferreira, Ferreira, and Pinto (2015) Liu (2005), Thijssen, Huisman, and Kort (2006) Jensen (2003)
Observation costs and/or imperfect information	Bagwell (1995), Vardy (2004), Yoon (2009)
Cost asymmetry	Amir and Stepanova (2006), Meza and Tombak (2009) Harsanyi and Selten (1988)
Technology choice + Internal organization	Kopel and Löffler (2008)
Uncertain time lags	Stenbacka and Tombak (1994), Götz (2000)
Very costly R&D	Hoppe and Lehmann-Grube (2001)
Exogenous	Tran, Sibley, and Wilkie (2012)

Table 1: Literature where being a late mover can be beneficial

firms gain from being a late mover, since after the first investment the true type is revealed. Information spillover is a common cause for the presence of such behavior. Table 1 shows a small summary of the papers in this field. Not only information spillovers, but also other factors could imply the advantage of a late mover. Amir and Stepanova (2006) consider asymmetric firms and differentiated products. They find that a second-mover advantage applies to the low cost firm when costs are sufficiently different or when costs are high for both firms. Also asymmetry in information quality (Yoon (2009)), expensive R&D (Hoppe and Lehmann-Grube (2001)) or uncertainty about implementation time (Stenbacka and Tombak (1994)) lead to a similar result. Bagwell (1995) and Vardy (2004) find that the first-mover advantage is completely lost if the first mover's choice is imperfectly observed or if there are observation costs. A final stand of papers studies the effect of a second-mover advantage by introducing a larger fixed costs when producing two products (Bárcena-Ruiz and Olaizola (2008)) or by assuming the second-mover advantage to be exogenously given (Tran, Sibley, and Wilkie (2012)). Although this paper also includes uncertainty, one should notice that this type of uncertainty is different from the literature. In this paper uncertainty affects both submarkets equivalently and does not lead to imperfect information or any asymmetries. Moreover, in this set-up, uncertainty is not resolved after the first mover's adoption so that this cannot incentify the rival firm to be a late mover, which is the case in the papers mentioned above. Overviews of earlier work can be found in Hoppe (2002), Karshenas and Stoneman (1995) and Reinganum (1989).

Perhaps the closest paper is the work by Steg and Thijssen (2015) where a switching option is offered in a duopoly game. Profits are assumed to be zero if both firms are active on the same market, so either of them has to switch to a different market. Each market is subject to different market shocks, continuously altering the relative profitability among markets. In case the other market is, relative to the current market, more profitable firms behave in a preemptive manner, while a war of attrition is present otherwise. Their results are, therefore, comparable, but there are some substantial differences. First, since duopolies are assumed to make no profits, the follower is automatically assumed not to switch. In this paper this is not the case and arrises as an endogenous result. Second, since the market dynamics are characterized by different stochastic processes for each market, one could alternate between a second-mover advantage and a first-mover advantage constantly. In this model, I speak of heterogeneous products on the same market so that being a first or late mover is not only a result of the state variable but also the technological improvement, the degree to which products are substitutes, the current capacities and the strategic advantages of being leader and follower play a significant role.

Although Dawid, Kopel, and Kort (2013) do not consider technology adoption, qualitatively their results are comparable. In their model there is no vertical differentiation and rather than adoption, firms consider to expand their production lines by offering a second product that forms a substitute. Here, large firms can prevent that small firms also undertake investment. Partially this can be explained by the presence of the cannibalization effect, where prices are reduced each time investment is undertaken. This comes in since both the first mover and the second mover remain present on both submarkets.

The stochastic nature of the demand process in this paper's model shows that the number of adopters and the willingness to invest is dependent on the level of uncertainty. More specifically, when considering large market uncertainty firms delay their investments. This results in a situation where first movers invest when the market is more profitable. Intuitively one would expect that in these situations both firms would undertake investment. However, since the old submarket is subject to the same market development, this type of profitability does not trigger the second firm to also undertake investment. Contrarily, for high levels of uncertainty the late mover more willingly does not undertake the adoption. This comes as a result of a large capacity set by the leader. Hence, only for small levels of uncertainty, when the leader sets a small capacity, firms are eager to be a second adopter.

After describing the model characteristics in Section 2, this paper will continue by studying a model where the investment order is fixed. Standard in these games, first the follower's reaction curves are determined, in Section 3, before optimizing the leader's decisions, which is done in Section 4. The second part of this paper deals with the adoption under endogenous firm roles, i.e. where the investment order is endogenously determined, described by Section 5. Finally, Section 6 analyses the effect of uncertainty and discusses the robustness of the model. Section 7 concludes this paper.

2 Model

This paper considers two (asymmetric) firms on an established, currently homogeneous, market. Both firms have the option to substitute their current product for a new product that is both vertically and horizontally differentiated from the existing product in continuous time $t \in \mathbb{R}_+$. In this paper's terminology, a firm is said to undertake investment the moment it decides to exercise the option and to adopt the new technology. The horizontal differentiation, i.e. the degree to which products are close subsitutes, is captured by the parameter ω and the vertical differentation, i.e. the degree of technological improvement, by parameter ν. After the adoption by one firm, which will then be called the leader (L), both firms serve different submarkets. However, after successive investment by the other firm, the follower (F), both firms are only active on the new submarket. I assume firms are perfectly informed about the other firm's movements and time information legs are asummed to be negligable (closed-loop). Before investment, the price on the old market is equal to

$$p^o(x(t), t) = x(t)(1 - \eta q_L^o - \eta q_F^o),$$

where η is the sensitivity parameter measuring the sensitivity of the leader's old capacity size q_L^o and the follower's old capacity size q_F^o on the price and where $x = \big(x(t)\big)_{t \geq 0}$ is an exogenous shock process assumed to follow a geometric Brownian motion, i.e.

$$\frac{\mathrm{d}x}{x} = \mu \mathrm{d}t + \sigma \mathrm{d}z.$$

Here μ and $\sigma > 0$ are the trend and volatility parameters and $z(t)$ is a Wiener process.[2] After the leader's investment, where it installs capacity q_L^n on the new market, prices equal

$$p^o(x(t), t) = x(t)(1 - \eta q_F^o - \omega q_L^n),$$
$$p^n(x(t), t) = x(t)(\nu - \eta q_L^n - \omega q_F^o).$$

It is assumed that $\nu \geq 1$ and $\omega \leq \eta$.[3] Note that products are more differentiated for larger ν and smaller ω. Moreover, if $\nu = 1$ and $\omega = \eta$ one can speak of homogenous products.

Investment in this model does not only consist of determining the optimal investment moment but also incorporates the optimal investment size. At investment a firm is confronted with adoption costs, which are assumed to be linear in the firm's new capacity and are captured by marginal investment cost parameter δ. Finally, after investment of both firms, the old market ceases to exist and the new market is defined by the price function

$$p^n(x(t), t) = x(t)(\nu - \eta q_L^n - \eta q_F^n).$$

The optimal investment moment is determined on the basis of real options theory (see, e.g. Dixit and Pindyck (1994)). As of now, the denotation of time t shall be omitted to simplify notation. There are two additional assumptions. First, the current value of the shock process $X = x(0)$ is sufficiently small, so that none of the firms is inclined to undertake immediate investment. This allows us to examine all incentives and expore the different investment strategies. Secondly, firms are commited to produce the amount their capacity dictates. This assumption is widely used in the literature on capacity constrained oligopolies (e.g.

[2]Throughout the paper I will refer to the current value of the process $x(t)$ as X.

[3]Although it has no consequences qualitatively, I generally assume ω to be nonnegative.

Deneckere *et al.* (1997), Chod and Rudi (2005), Anand and Girotra (2007), Goyal and Netessine (2007) and Huisman and Kort (2015)).

In line with standard game theory, this model is solved backwards. After obtaining the follower's reaction curves, the leader's optimal strategies are determined. This paper deals with two models. In the first model it is assumed that the investment order is exogenously determined. This is done in Section 3 and Section 4. Section 5 allows for an endogenously determined order of investment.

3 Follower's decision

The follower (F) is defined as the firm that undertakes the investment secondly. This means, at the time the follower is evaluating its investment option, the other firm, the leader (L), has already invested and has substituted the old product in favor of the new product. At investment, the follower also carries out the substitution and both are active on the new market, which leads to the following value function for the follower at its investment,

$$\begin{aligned} V_F(X, q_L^n) &= \max_{q_F^n \geq 0} \left\{ \mathbb{E}\left[\int_0^\infty q_F^n \; p^n(x(s), s) e^{-rs} \; \mathrm{d}s \mid \mathcal{F}(\tau_F) \right] - \delta q_F^n \right\} \\ &= \max_{q_F^n \geq 0} \left\{ \frac{X}{r-\mu} q_F^n (\nu - \eta(q_F^n + q_L^n)) - \delta q_F^n \right\}, \end{aligned}$$

where τ_F is the investment moment of the follower[4] and $\mathcal{F}$ is the filtration with observations of the shock process. The value function consists of two terms. The first term represents the expected discounted cash inflow stream from production whereas the second term reflects the investment costs. The firm chooses q_F^n in such a way that it optimizes its value,

$$q_F^{n*}(X, q_L^n) = \frac{1}{2\eta} \left[\nu - \eta q_L^n - \frac{\delta(r-\mu)}{X} \right].$$

Conforming the real options literature on investment decisions, a firm decides to undertake an investment instantly when the exogenous shock process hits the trigger X_F^*, at which point the value of waiting no longer constitutes a larger value than the value of investment. The value of waiting (see, e.g., Dixit and Pindyck (1994)) combines the option value and the value of current activity respectively,

$$V_F(X, q_L^n) = A_F X^\beta + \frac{X}{r-\mu} q_F^o (1 - \eta q_F^o - \omega q_L^n),$$

where the value of β is defined as

$$\beta = \frac{1}{2} - \frac{\mu}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}}.$$

[4]Notice that, although ex-ante the adoption moment of the follower is stochastic, it is not here since this is the determination of the follower's profits at the moment of its investment. For the same reason, $X = x(\tau_F)$ is deterministic since adoption takes place at $t = \tau_F$.

The value of X_F^* and A_F follow from the smooth pasting and value matching conditions (see Appendix A). However, these conditions do not always return solutions. This means that, under certain conditions, there is no solution to the follower's optimization problem. More specifically, one can show that the investment trigger X_F^* exists if and only if

$$4\eta q_F^o(1-\eta q_F^o-\omega q_L^n) < (\nu-\eta q_L^n)^2. \tag{1}$$

This implies that when the inequality does not hold, the second firm decides to never adopt the new technology. The net gain from undertaking investment is smaller than the loss one faces when loosing the old market. This occurs since, when undertaking investment, the follower shares the new product's market with the leader, whilst serving only its own submarket when not undertaking investment. For that reason the firm optimally abstains from investment. The first thing one could notice is the absence of the marginal investment cost δ in the equation. Generally, a larger cost would delay investment and impose a larger capacity. However, the binary decision whether or not to invest, is not affected by this parameter. After all, for large δ, investment is only delayed, as a result of a one-off investment cost, while not affecting the profitability on the new market relative to the old market. For that reason one does not take δ into account. A more thorough analysis of (1) will be provided, but let me first state the following proposition.

Proposition 1 *Define X as the current value of the stochastic demand process. Define q_F^o as the follower's capacity before investment and similarly define q_L^n as the leader's capacity on the new market. Then, for sufficiently large values of x the follower undertakes investment if and only if*

$$\nu > \eta q_L^n + \sqrt{4\eta q_F^o(1-\eta q_F^o-\omega q_L^n)}. \tag{2}$$

Assume (2) holds. Then there exists a pair (q_F^{opt}, X_F^*) such that the follower postpones investment for $X < X_F^*$ and invests q_F^{opt} when x reaches X_F^*. However, the follower undertakes immediate investment for $X \geq X_F^*$ and sets capacity equal to

$$q_F^{n*}(X, q_L^n) = \frac{1}{2\eta}\left[\nu - \eta q_L^n - \frac{\delta(r-\mu)}{X}\right]. \tag{3}$$

The follower's investment trigger is defined as

$$X_F^*(q_L^n) = \frac{\delta(r-\mu)}{\beta-1}\frac{\beta(\nu-\eta q_L^n)+\sqrt{(\nu-\eta q_L^n)^2+(\beta^2-1)4\eta q_F^o(1-\eta q_F^o-\omega q_L^n)}}{(\nu-\eta q_L^n)^2-4\eta q_F^o(1-\eta q_F^o-\omega q_L^n)}. \tag{4}$$

The corresponding capacity size then equals $q_F^{opt} = q_F^{n*}(X_F^*, q_L^n)$. As a result, the follower's value function is defined as

$$V_F(X, q_L^n) = \begin{cases} A_F X^\beta + \frac{X}{r-\mu} q_F^o(1-\eta q_F^o - \omega q_L^n) & \text{if } X < X_F^*, \\ \frac{X}{r-\mu} q_F^{n*}(X, q_L^n)(\nu - \eta(q_L^n + q_F^{n*}(X, q_L^n))) - \delta q_F^{n*}(X, q_L^n) & \text{if } X \geq X_F^*. \end{cases}$$

Assume (2) does not hold. Then the follower never undertakes investment, in which case $X_F^* \to \infty$. The follower obtains the value of current production under horizontal competition,

$$V_F(X, q_L^n) = \frac{X}{r-\mu} q_F^o(1-\eta q_F^o-\omega q_L^n).$$

The presence of the follower on the new market hinges on (1). Here, the set of potential actions for the follower seems to inherently depend on its current capacity and in the second place on the leader's newly chosen capacity size. For sufficiently large q_F^o this inequality is always true. However, this is not the case for all q_F^o and would then depend on the value of the leader's capacity q_L^n. Correspondingly, one can distinguish three regions with respect to q_F^o. Figure 1 illustrates these regions graphically.

Proposition 2 *Define q_F^o as the follower's capacity on the old market and define q_L^n as the leader's capacity on the new market. Let X_F^* be defined as in Proposition 1. Assume $q_L^n < \frac{1}{\eta}$. Then one can define*

$$\tilde{q}_F^1 = \frac{\eta - \omega\nu}{\eta^2}, \ \tilde{q}_F^2 = \frac{\eta - \omega\nu}{\eta^2 - \omega^2} \ \text{and} \ \tilde{q}_F^3 = \frac{1}{\eta}$$

such that

(i) for $q_F^o \in \mathcal{R}_1 \equiv [0, \tilde{q}_F^1)$, the follower's trigger is an increasing function with respect to q_L^n for $q_L^n < \tilde{q}_L^1$ and is defined as $X_F^* \to \infty$ for $q_L^n \geq \tilde{q}_L^1$,

(ii) for $q_F^o \in \mathcal{R}_2 \equiv [\tilde{q}_F^1, \tilde{q}_F^2)$, the follower's trigger is an increasing function with respect to q_L^n for $q_L^n < \tilde{q}_L^1$, is defined as $X_F^* \to \infty$ for $q_L^n \in [\tilde{q}_L^1, \tilde{q}_L^2]$ and is a decreasing function for $\tilde{q}_L^2 < q_L^n < \tilde{q}_L^3$. For $q_L^n > \tilde{q}_L^3$ the follower immediately invests.

(iii) for $q_F^o \in \mathcal{R}_3 \equiv [\tilde{q}_F^2, \tilde{q}_F^3]$, the follower's trigger is a nonmonotonic function with respect to q_L^n for $q_L^n \leq \tilde{q}_L^3$ that is first increasing and then decreasing. For $q_L^n > \tilde{q}_L^3$ the follower immediately invests.

The borders with respect to q_L^n are defined as

$$\tilde{q}_L^1 = \frac{1}{\eta}\left[\nu - 2\omega q_F^o - 2\sqrt{q_F^o(\eta - \omega\nu) - (q_F^o)^2(\eta^2 - \omega^2)}\right], \tag{5}$$

$$\tilde{q}_L^2 = \frac{1}{\eta}\left[\nu - 2\omega q_F^o + 2\sqrt{q_F^o(\eta - \omega\nu) - (q_F^o)^2(\eta^2 - \omega^2)}\right], \tag{6}$$

$$\tilde{q}_L^3 = \{q_L^n \ : \ (\beta^2 - 1)4\eta q_F^o(1 - \eta q_F^o - \omega q_L^n) + (\nu - \eta q_L^n)^2 = 0\}. \tag{7}$$

Region 1 In Region 1, that is, for $q_F^o < \tilde{q}_F^1$it is found that for small values of q_L^n the follower undertakes investment for sufficiently large values of x, while large values of q_L^n incentify the follower to never undertake investment. This signifies that the leader can block any investment on the new market by the competitor by setting a sufficiently large capacity size. This result expands the set of possible strategies believed for the leader. In a more traditional set-up where firms do not substitute but expand their production lines (see, e.g., ...), firms are believed to always undertake investment. Substituting products induces that it could be unbeneficial for firms to undertake the adoption, even though the second product is better. Intuitively, serving your own submarket is more profitable than sharing a submarket with a (large) competitor. For that reason, the follower abstains from participation in the new market when the leader sets a relatively large capacity.

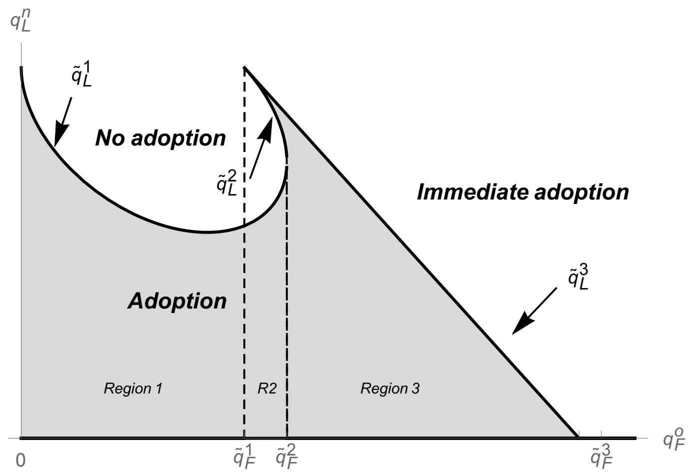


Figure 1: Regions with respect to the follower's adoption decision.

Region 2 Capacities affect both products through their corresponding price functions differently. There could emerge a situation where setting an (exceedingly) large capacity size as a leader may still contribute to a positive price on the new market, while only a very small price on the old market. A suchlike situation can only occur for sufficiently large values of q_F^o. This defines Region 2, with $\tilde{q}_F^1 < q_F^o < \tilde{q}_F^2$. Here, similar to the analysis for Region 1, small values of the leader's new capacity allow the follower to undertake investment and medium sized values of the leader's capacity would *segregate* the firms with respect to their submarkets, i.e. the follower will never enter the new market. However, for large values of the leader's new capacity, it becomes rewarding again to undertake investment for the follower and leave the old market. In the latter case, the leader's new capacity marginalizes the profitability on the old market, which triggers the follower to leave it[5]. The leader would find the employment of such a strategy satisfactory for in such a case the follower would trade his large capacity on the old market for a very small capacity on the new market. This is, being a small firm on the new market as a follower is preferred by the leader over being a large firm on the old market since, in that case, it holds that $\eta q_F^n < \omega q_L^o$.

Figure 2 features an example[6] of the follower's trigger for values of q_F^o that belong to the Region 2. For small values of q_L^n the follower's trigger is increasing. The gray area depicts the region where the follower is delayed and in the superjacent white region the follower undertakes investment immediately. For intermediate values of q_L^n the follower optimally never adopts the new technology, but it does in case of large values of q_L^n as shown in the figure. For Region 1, one obtains a similar picture, but then without the second gray area on the right.

[5]Technically, this is also possible for Region 1. This would, however, require values for the leader's capacity that would make the price on the new market negative.

[6]Examples in this paper use the following parametrization: $\mu = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.2,\ \omega = 0.13,\ \nu = 1.05$ and $\delta = 1000$.

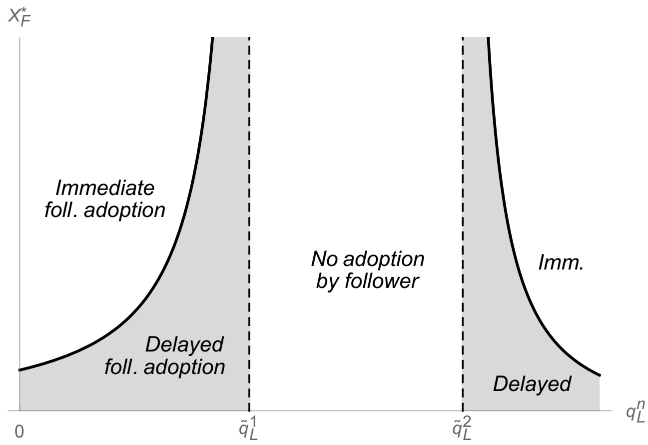


Figure 2: The follower's trigger X_F^* for different values of the leader's new capacity q_L^n differentiating the different strategic regions in Region 2, where $q_F^o = 2.7$.

Region 3 For large values of q_F^o the region where the follower undertakes no action ceases to exist. Instead, the follower always undertakes investment for sufficiently large values of the shock process, irrespective of the leader's capacity size. The reason for leaving the market, though, does depend on the value of the leader's capacity. For small values of q_L^n this is because the new market is more attractive. Nevertheless, for large values of q_L^n the follower is forced to leave the old market since the introduction of the new product cuts the profitability on the old market. Hence, one then ends up in Region 3, if $q_F^o \geq \tilde{q}_F^2$.

3.1 Discussion

Evidently, the existence of the regions relies on the parametrization. When considering the situation where both products are kept on the market, each produced by a different firm, one studies the presence of the *Segregation Region*. Let the Segregation Region be defined as the region with respect to q_L^n where the follower never undertakes investment. The vertical differentiation parameter ν has a negative effect on the magnitude of this strategy. In other words, the degree of existence of the segregation strategy, and hence the possibility to separate markets, is negatively influenced by the degree of innovation. From the follower's perspective, a more profitable new market makes it more interesting for the follower to adopt the new technology, which is captured by the negative relation with $\tilde{q}_F^1$ and $\tilde{q}_F^2$. This effect is also apparent from equation (2). This makes it harder for the leader to block the follower from investing, captured by a positive relation between ν and $\tilde{q}_L^1$.

- ≫ The values of $\tilde{q}_F^1$ and $\tilde{q}_F^2$ are only positive in case $\eta > \omega\nu$, i.e. in a scenario with a stronger degree of innovation, that is, a larger vertical differentiation parameter ν, the follower has more incentives to enter the new market which results in a fading likelihood for the segregation strategy to emerge. Nevertheless, the gap between the horizontal differentation parameter ω and the sensitivity parameter

η should be sufficiently large. In case, though, the horizontal differentiation parameter takes a small value, firms are more eager to hold on to their own submarkets.

≫ The negative relation between ν and $\tilde{q}_L^1$ follows directly from (5). Nevertheless, one can verify that the minimum of $\tilde{q}_L^1$ with respect to q_F^o takes place at the coordinates

$$(q_F^o, q_L^n) = \left(\frac{\eta - \omega\nu}{2\eta(\eta - \omega)} \ , \ \frac{\nu - 1}{\eta - \omega} \right).$$

Here, one can easily check that the minimum bears a positive relation with ν. Moreover, it can be shown that $\tilde{q}_L^1$ is a decreasing function of ν, as is $\tilde{q}_L^2$ a decreasing function of ν. Therefore, as could also be concluded from (2), for a given level of q_F^o, the Segregation Region is expanding with respect to the vertical differentiation ν. However, in order for $\eta > \omega\nu$, the latter parameter cannot be too large. This underlines that, for a given value of q_F^o, the extent to which segregation is a noticeably present strategy is smaller for a larger degree of innovation. Moreover, it is apparent that is negatively influenced by the relative mutual sensitivity of the products.

4 Leader's decision

This section serves to study the leader's decision whether or not to obviate the other firm's adoption. It is therefore assumed that the follower's initial capacity size falls in Region 1 or Region 2. This is still under the framework of exogenous firm roles; the case where the investment order is not exogenously chosen, but endogenously determined, is studied in the next section.

If the leader chooses q_L^n to be in the interval $[\tilde{q}_L^1, \tilde{q}_L^2]$ the follower chooses to never undertake investment. This defines the leader's *segregation strategy.* However, if the leader chooses q_L^n to between outside the interval $[\tilde{q}_L^1, \tilde{q}_L^2]$ the follower is prone to adopt. It then depends on the current value of x whether the follower undertakes an investment immediately or waits for the shock process to reach a sufficiently high level. If, for a given level of $X = x(0)$, the leader's capacity is chosen in such a way that $X_F^* > X$, the follower is temporarily deterred. All capacity levels corresponding to a delayed investment by the follower form the leader's *deterrence strategy* (see, e.g. Huisman and Kort (2015)). The complementary levels, the levels of q_L^n for which $X_F^* \leq X$, lead to an immediate investment by the follower. These choices constitute the *accommodation strategy.*

4.1 Investment under segregation

Here, the leader chooses to employ the strategy where the follower does not adopt. The leader's value at investment in such a setting comes down to

$$V_L^{seg}(X, q_L^n) = \mathbb{E}\left[\int_0^\infty q_L^n \ p^n(x(s), s)e^{-rs} \ \mathrm{d}s \mid \mathcal{F}(\tau_L)\right] - \delta q_L^n = \frac{X}{r - \mu} q_L^n(\nu - \eta q_L^n - \omega q_F^o) - \delta q_L^n.$$

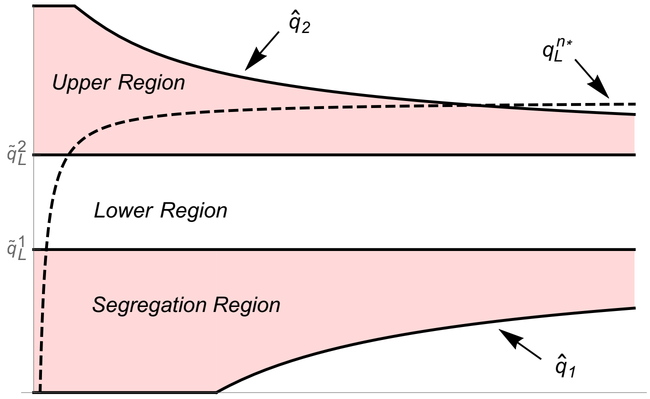


Figure 3: Example of optimal capacity q_L^n and the borders to q_L^n for different values of X.

The value function consists of two terms, reflecting, similar to the case of the follower, the expected discounted cash inflow stream from production and the investment costs.

At investment, the leader chooses its capacity such that it optimizes V_L with respect to q_L^n, which, under the segregation strategy, comes down to

$$q_L^{seg}(X) = \frac{1}{2\eta}\left[\nu - \omega q_F^o - \frac{\delta(r-\mu)}{X}\right].$$

Generally speaking, the optimal capacity size of the leader is defined as q_L^{n*}, which corresponds to q_L^{seg} in case optimal investment takes place while blocking the other firm's investment, corresponds to q_L^{det} in case optimal investment is chosen to be done under the deterrence strategy and similarly to q_L^{acc} under the accommodation strategy. Investment under the segregation strategy will only be considered by the leader if q_L^{seg} falls in the Segregation Region, i.e. $q_L^n > \tilde{q}_L^1$ for $\mathcal{R}_1$ and $q_L^n \in [\tilde{q}_L^1, \tilde{q}_L^2]$ for $\mathcal{R}_2$. To see when this happens one first needs to invert X_F^* as defined in Proposition 2 with respect to q_L^n, so that e.g. Figure 2 is transformed into Figure 3. Figure 3 shows the inverted curves for Region 2, mapping each value of the shock process x into a value $\hat{q}_1$ and $\hat{q}_2$ at which the follower is indifferent between waiting and investing. This is, if, for a given value of x, the optimal investment size falls to the left of $\hat{q}_1$ or $\hat{q}_2$, let me call these the *Upper Region* and *Lower Region* resp., the follower's investment is delayed. However, if the optimal investment size falls to the right, both firms simultaneously invest. Moreover, it shows $\tilde{q}_L^1$ and $\tilde{q}_L^2$ denoting the borders to the region where the follower's investment is blocked (see Proposition 2), the equivalent *Segregation Region*. Since q_L^{seg} is an increasing function of X one can obviously conclude that q_L^{seg} falls into the Segregation Region for a convex interval of X.

Note that this figure corresponds to the scenario where q_F^o lies in Region 2 ($\mathcal{R}_2$). Figures resulting from Region 1 ($\mathcal{R}_1$) and Region 3 ($\mathcal{R}_3$) are similar to Figure 3 but the former does not contain the Upper Region and latter only contains the Lower Region.

Lemma 1 *Consider the leader's optimization problem. If and only if*

$$\nu \geq \frac{8\eta}{5\omega + \sqrt{16\eta^2 - 7\omega^2}}, \tag{8}$$

then there exist no q_F^o such that there exist $X < \infty$ at which the leader's value function V_L^{det} is maximized in the Segregation Region.

This Lemma shows that for a large degree of vertical differentiation, the follower will always undertake the adoption. This is in line with the results from the follower's analysis. For smaller values of ν there exist q_F^o such that the leader prefers to block the follower's adoption, see Appendix C.

4.2 Investment under deterrence

Under the deterrence strategy the leader obtains the value given by the segregation strategy, corrected by a term reflecting the future investment by the follower at time $t = \tau_F$,

$$\begin{aligned} V_L^{det}(X, q_L^n) &= \mathbb{E}\left[\int_{s=0}^{\tau_F} q_L^n \; x(s)(1 - \eta q_L^n - \omega q_F^o) e^{-rs} \mathrm{d}s + \int_{s=\tau_F}^{\infty} q_L^n \; x(s)(1 - \eta q_L^n - \eta q_F^n) e^{-rs} \mathrm{d}s \;\Big|\; \mathcal{F}(\tau_L) \right] - \delta q_L^n \\ &= \frac{X}{r-\mu} q_L^n (\nu - \eta q_L^n - \omega q_F^o) + \frac{X_F^*}{r-\mu} q_L^n (\omega q_F^o - \eta q_F^n) \left(\frac{X}{X_F^*} \right)^{\beta} - \delta q_L^n. \end{aligned}$$

Investment under the deterrence strategy is only considered feasible when the optimal investment size q_L^{det}, the value of q_L^n that optimizes V_L^{det}, falls in the Upper or Lower Region.

Figure 3 shows an example of the optimal investment by the leader. Clearly, for small values of X the leader invests in the Lower Region, delaying the follower's investment. For increasing $X = x(0)$ the leader, consecutively, optimally, invests in the Segregation Region, the Upper Region and eventually it chooses accommodation as an optimal strategy[7].

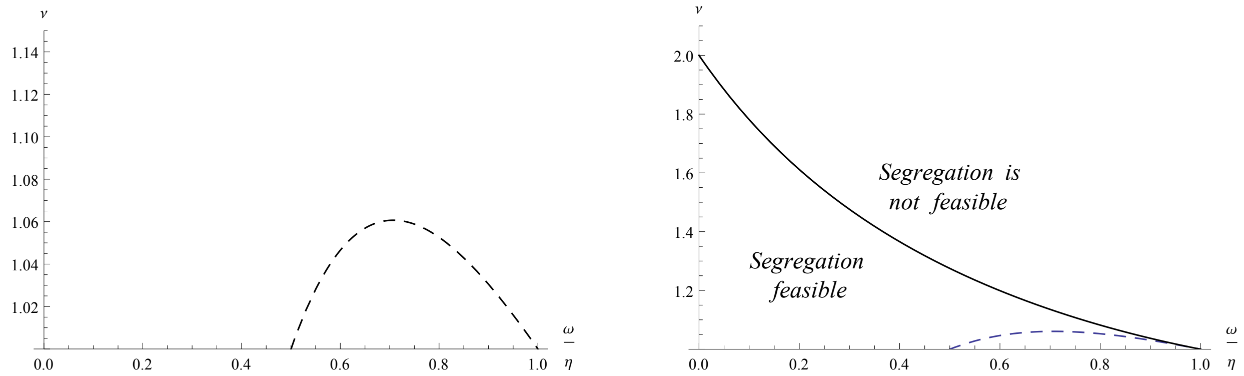
Lemma 2 *Consider the leader's optimization problem. If and only if*

$$\nu < \frac{3\omega\eta}{\eta^2 + 2\omega^2}, \tag{9}$$

then there exists a $\bar{q}_F^2$ such that for $q_F^o \in [\bar{q}_F^2, \tilde{q}_F^2]$ there exist $X < \infty$ at which the leader's value function V_L^{det} is maximized in the Upper Region.

The range of all values satisfying (9) are depicted in Figure 4a. If the leader decided to undertake investment in the Upper Region, then the price for the old product is minimized, in which way the follower is forced to undertake adoption. This strategy is not feasible for relatively large values of the vertical differentiation parameter ν. Intuitively, a large degree of innovation, raises the leader's incentives to not share the market with a competitor - which would lead to optimal investment in the Segregation Region. Moreover, is ν is large, adoption becomes sufficiently attractive for the follower. This implies that, under the

[7] In the proof of Lemma 2 I show that, if investment in the Upper Region is feasible, this order always holds.



(a) Region where there are candidate interior solutions in the Upper Region, see Lemma 2.

(b) Region where there are candidate interior solutions in the Segregation Region, see equation (23).

Figure 4: Parameter sets that lead to interior solutions the Upper Region and the Segregation Region.

deterrence strategy, investment would take place in the Lower Region. For smaller values of ν, investment only takes place in the Upper Region when the follower's old capacity is noticeably present on the new market, i.e. a relatively large value of ω. However, when both products are close substitutes, i.e. in the case where $\frac{\omega}{\eta}$ is close to 1, both firms have more incentives to trade their capacities for the new market is more profitable. This explains the convex shape of the region.

4.3 Investment under accommodation

Under the accommodation strategy the leader sets its capacity in such a way that $X_F^*(q_L^n) \leq X$. Both firms then decide to simultaneously undertake investment, while the leader sets its capacity first (a la Stackelberg). It obtains,

$$\begin{aligned} V_L^{acc}(X, q_L^n) &= \mathbb{E}\left[\int_{s=0}^{\infty} q_L^n \; x(s)(1 - \eta(q_L^n + q_F^{n*}(X, q_L^n))e^{-rs}\mathrm{d}s \;\Big|\; \mathcal{F}(\tau_L) \right] - \delta q_L^n \\ &= \frac{X}{r-\mu} \tfrac{1}{2} q_L^n(\nu - \eta q_L^n) - \tfrac{1}{2}\delta q_L^n. \end{aligned} \tag{10}$$

At the moment of investment, capacity is chosen to maximize (10),

$$q_L^{acc}(X) = \frac{1}{2\eta}\left(\nu - \frac{\delta(r-\mu)}{X}\right).$$

Notice that $q_L^{acc} > q_L^{seg}$ for all X. Entry accommodation can only be played when q_L^{acc} falls in the region where entry accommodation is feasible, i.e. below $\hat{q}_1$ and above $\hat{q}_2$. One can show that the capacity size q_L^{acc} is an increasing function of X, starting in the Lower Region where deterrence is optimal, i.e. the smallest value of X for which q_L^{acc} becomes positive lies in the region where the follower's investment is delayed. Then, either, the function intersects with $\hat{q}_1$ so that accommodation becomes feasible there, or, it intersects with $\tilde{q}_L^1$ so that it never reaches the accommodation region associated with the Lower Region. Hence, for the former scenario, accommodation becomes feasible at $X = X_1^{acc}$, where

$$X_1^{acc} = \{X \in \mathbb{R} \mid q_L^{acc}(X) = \hat{q}_1(X)\}.$$

In the latter scenario, the curve could intersect with $\hat{q}_2$, in which case accommodation becomes feasible in the Upper Region. One can show that this intersection point in both scenarios is characterized by the same equation as defined in the following Proposition.

Proposition 3 *Let $q_F^o \in \mathcal{R}_1 \cup \mathcal{R}_2$. Assume $\nu > \frac{2\eta}{\eta+\omega}$, then the accommodation strategy becomes feasible at X_1^{acc}, where*

$$X_1^{acc} = \delta(r-\mu)\frac{\frac{\beta+1}{\beta-1}\nu + 2\omega q_F^o + \sqrt{\nu^2\left[\left(\frac{\beta+1}{\beta-1}\right)^2 - \frac{1}{4}\frac{\beta+3}{\beta-1}\right] + 4(q_F^o)^2(\omega^2-\eta^2) + q_F^o\left(2\omega\nu + 4\eta\frac{\beta+3}{\beta-1}\right)}}{\frac{1}{2}\nu^2 - 4q_F^o(2\eta - \omega\nu - 2\eta^2 q_F^o)}. \tag{11}$$

Notice that $\frac{2\eta}{\eta+\omega}$ is strictly larger than 1. Intuitively, for the accommodation strategy to be feasible for all q_F^o, one needs the new product to be substantially more profitable in order to undertake investment simultaneously, i.e. one needs a sufficiently large ν. However, the requirement becomes less rigorous the closer ω en η are. When prices become more sensitive to other products the incentive to stay with the old product becomes lower. Also notice that $\frac{2\eta}{\eta+\omega} < \frac{\eta}{\omega}$, which means that there exist ν such that the accommodation strategy is always feasible and such that $\eta - \nu\omega > 0$, so that $\mathcal{R}_1$ and $\mathcal{R}_2$ are nonempty.

In the situation where $\nu \leq \frac{2\eta}{\eta+\omega}$, entry accommodation is not feasible for all values of $q_F^o \in \mathcal{R}_1 \cup \mathcal{R}_2$. Appendix C elaborates on these cases.

For convenience, let $\mathcal{S}(q_F^o)$ denote the Segregation Region with respect to the leader's new capacity for given q_F^o. I will omit the given argument unless necessary. Then the following Corollory shows that accommodation and segregation are two mutuallty exclusive strategies, for $\mathcal{R}_1$ and for cases in $\mathcal{R}_2$.

Corollary 1 *Assume there exists an $X \in \mathbb{R}$ such that $q_L^{n*} \in \mathcal{S}$.*

(i) Let $q_F^o \in \mathcal{R}_1$. Then $X_1^{acc} \to \infty$, i.e. the accommodation strategy is infeasible.

(ii) Let $q_F^o \in \mathcal{R}_2$ and assume $\nu \geq \frac{4\omega\eta}{\eta^2+3\omega^2}$. Then $X_1^{acc} \to \infty$, i.e. the accommodation strategy is infeasible.

4.4 Optimal investment under the presence of segregation

I will now study the optimal investment decision for the situation where investment under the segregation strategy is possible. This means that there exists at least one X such that $q_L^{n*} \in \mathcal{S}$. As shown before, for different values of X the leader prefers different strategies. The optimal moment, i.e. the optimal value of X at which the leader performs the adoption, then determines whether the follower also (eventually) undertakes investment.

4.4.1 Region 1

Notice that in $\mathcal{R}_1$, under the presence of segregation, accommodation is infeasible. As a result, either the follower never undertakes adoption or the leader delays the follower's investment: for small X optimal investment takes place under deterrence strategy which delays the follower's investment and for larger X the leader sets a sufficiently large capacity such that the follower will never substitute its current product. Define,

$$X_1^{seg} = \frac{\delta(r-\mu)}{\nu - \omega q_F^o - 2\eta \tilde{q}_L^1},$$

then $q_L^{n*} \in \mathcal{D}$ for $X < X_1^{seg}$ and $q_L^{n*} \in \mathcal{S}$ otherwise. Here, $\mathcal{D}$ is defined as the region with respect to q_L^n, for given q_F^o, where the leader delays the follower's investment and $\mathcal{S}$ is defined as the region with respect to q_L^n, for given q_F^o, where the leader's investment hinders the follower. Naturally, $\mathcal{A}$ is then defined as the region where the follower is accommodated[8]. The significance of the Segregation Region $\mathcal{S}$ depends on the parameterization. X_1^{seg} is only defined as long as $\nu - \omega q_F^o - 2\eta\tilde{q}_L^1 > 0$, i.e. if the opposite is true, then it holds that $q_L^{seg} < \tilde{q}_L^1$ for all X. This implies that in that case optimal investment, under the deterrence strategy, always takes place in $\mathcal{D}$[9]. Hence,

$$X_1^{seg} = \begin{cases} \frac{\delta(r-\mu)}{\nu-\omega q_F^o - 2\eta\tilde{q}_L^1} & \text{if } \nu - \omega q_F^o - 2\eta\tilde{q}_L^1 > 0, \\ \infty & \text{otherwise.} \end{cases}$$

Since the investment size q_L^{seg} is an increasing function of X it is concluded that the segregation strategy is only feasible for $X \geq X_1^{seg}$. As discussed in Section 4.1, deterrence is only feasible for $q_L^{n*} > \hat{q}_1$. The deterrence strategy is, thus, only present when $\hat{q}_1 < q_L^{det} < \tilde{q}_L^1$, where,

$$\hat{q}_1(X) = \{q_L^n \in [0, \tilde{q}_L^1] \mid X_F^*(q_L^n) = X\}.$$

For $X < X_F^*(0)$, the value of $\hat{q}_1$ is assumed to be zero. In other words, if one defines

$$X_1^{det} = \min\{X \in \mathbb{R} \mid q_L^{det}(X) = \hat{q}_1\},$$

then deterrence is considered by the leader for $X \geq X_1^{det}$. One can prove[10] that q_L^{det} hits the upper boundary $\tilde{q}_L^1$ at the same moment, that is, for the same value of X, when the segregation strategy becomes feasible. All together, it holds that $q_L^{det} \in \mathcal{D}$ for $X \in [X_1^{det}, X_1^{seg})$ and $q_L^{seg} \in \mathcal{S}$ for $X \in [X_1^{seg}, \infty)$. This leads to the following proposition.

Proposition 4 *Let $q_F^o \in \mathcal{R}_1$. Assume $\nu - \omega q_F^o - 2\eta\tilde{q}_L^1 > 0$. Then the investment is considered by the leader for $X > X_1^{det}$. Moreover, there exists a unique value $X_1^{seg} > X_1^{det}$, such that the follower's investment is*

[8]Both sets $\mathcal{A}$ and $\mathcal{D}$ can be written as the union of two sets, that is, $\mathcal{D} = \mathcal{D}_L \cup \mathcal{D}_U$ where $\mathcal{D}_L$ includes all values such that investment takes place in the Lower Region. Similarly $\mathcal{D}_U$ includes all points in the Upper Region. Consistently, $\mathcal{A}$ can be written as $\mathcal{A}_L \cup \mathcal{A}_U$.

[9]See proof of Lemma 2.

[10]See proof of Lemma 2.

delayed for $X \in [X_1^{det}, X_1^{seg})$ *and the leader blocks the follower on the new market for* $X \geq X_1^{seg}$*. There, moreover, exists a pair* (q_L^{opt}, X_L^{det}) *such that*

(i) for $X \geq X_L^{det}$ *the leader makes an immediate investment and obtains the value function defined by*

$$V_L^{det}(X, q_L^n; q_F^o) = \begin{cases} \frac{X}{r-\mu} q_L^n(\nu - \eta q_L^n - \omega q_F^o) + \frac{X_F}{r-\mu} q_L^n(\omega q_F^o - \eta q_F^{opt}) \left(\frac{X}{X_F}\right)^\beta - \delta q_L^n & \text{if } X \in (X_1^{det}, X_1^{seg}) \\ \frac{X}{r-\mu} q_L^n(\nu - \eta q_L^n - \omega q_F^o) - \delta q_L^n & \text{if } X \in [X_1^{seg}, \infty) \end{cases}$$

and sets capacity $q_L^n = q_L^{seg}(X; q_F^o)$ *in case* $X \geq X_1^{seg}$ *and* $q_L^n = q_L^{det}(X; q_F^o)$ *otherwise;*

(ii) for $X < X_L^{det}$ *the leader postpones investment until* x *reaches* X_L^{det}*. This yields*

$$F_L^{det}(X; q_L^o, q_F^o) = A_L^{det} X^\beta + \frac{X}{r-\mu} q_L^o(1 - \eta(q_L^o + q_F^o)).$$

The leader's investment quantity equals $q_L^{opt} = q_L^{seg}(X_L^{det}; q_F^o)$ *in case* $X_L^{det} \in [X_1^{seg}, \infty)$ *and equals* $q_L^{opt} = q_L^{det}(X_L^{det}; q_F^o)$ *otherwise.*

For the complementary case, if $\nu - \omega q_F^o - 2\eta \tilde{q}_L^1 \leq 0$, it holds that it is never possible to prevent the follower from investing in the new market. In that case the leader always considers the deterrence strategy for all X.

Investment trigger The investment trigger is defined as the value of X below which the leader finds waiting optimal and above which immediate investment is preferred. If this trigger X_L is smaller than X_1^{seg}, i.e. if it falls in the Segregation Region $\mathcal{S}$, optimal investment is undertaken while blocking the follower. However is the trigger falls in the Deterrence Region $\mathcal{D}$ the follower's adoption is merely delayed.

Proposition 5 *Let* $q_F^o \in \mathcal{R}_1$*. Assume* $\nu - \omega q_F^o - 2\eta \tilde{q}_L^1 > 0$*. Then the segregation investment trigger is given by*

$$X_L^{seg} = \frac{\delta(r-\mu)}{\beta - 1} \frac{\beta(\nu - \omega q_F^o) + \sqrt{(\nu - \omega q_F^o)^2 + (\beta^2 - 1)4\eta q_L^o(1 - \eta(q_L^o + q_F^o))}}{(\nu - \omega q_F^o)^2 - 4\eta q_L^o(1 - \eta(q_L^o + q_F^o))}$$

and exists for all different values of q_L^o *and* q_F^o*. The investment trigger under the deterrence strategy is the solution with respect to* X *of*

$$\frac{X}{r-\mu} \frac{\beta - 1}{\beta} \left[q_L^{det}(\nu - \eta q_L^{det} - \omega q_F^o) - q_L^o(1 - \eta(q_L^o + q_F^o))\right] + \tfrac{1}{2} \frac{\delta}{X} q_L^{det} \left(\frac{X}{X_F^*}\right)^{\beta - 1} = \delta q_L^{det},$$

where $q_L^{det} = \text{argmax}_{q_L^n \in \mathcal{D}} V_L^{det}$.

The value of X_L^{seg} is very much dependent on the value of q_L^o. Notice that the value of $\tilde{q}_L^1$ and $\hat{q}_1$ are only related to q_F^o and not to q_L^o. This means that the value of the leader on the old market can determine whether $X_L^{seg} > X_1^{seg}$ or not.

However, not only the value of q_L^o bares a great influence on the relative position of the segregation trigger. Also value of β plays an important role. The parameter β is influenced by the degree of uncertainty in the market, captured by σ, the discount rate r and the trend parameter μ. As an example, look at the case where $q_L^o = 0$. Then, $X_L^{seg} > X_1^{seg}$ if and only if

$$\frac{\beta+1}{\beta-1}(\nu - \omega q_F^o - 2\eta\tilde{q}_L^1) > \nu - \omega q_F^o.$$

Obviously, it depends on the value of β whether the inequality holds or does not hold. Notice that $\tilde{q}_L^1$ is not related to β or any of its components.

4.4.2 Region 2

For larger values of q_F^o, in $\mathcal{R}_2$, optimal investment can take place in three regions, the Lower Region, the Segregation Region and the Upper Region, depending on the parameterization. In a similar way as in Region 1, one can define,

$$X_2^{det} = \{X \in \mathbb{R} \mid q_L^{det}(X) = \hat{q}_2\},$$
$$X_2^{seg} = \frac{\delta(r-\mu)}{\nu - \omega q_F^o - 2\eta\tilde{q}_L^2},$$

where

$$\hat{q}_2(X) = \left\{ q_L^n \in \left[\tilde{q}_L^2, \tfrac{\nu-\omega q_F^o}{\eta}\right] \mid X_F^*(q_L^n; q_F^o) = X \right\}.$$

Suppose $\nu - \omega q_F^o - 2\eta\tilde{q}_L^2 > 0$, i.e. suppose for sufficiently large X optimal investment q_L^{n*} takes place in the Upper Region. Then we have that, under the deterrence strategy, the follower's investment is delayed for $X \in (X_1^{det}, X_1^{seg}) \cup (X_2^{seg}, X_2^{det})$ and eternally deterred for $X \in (X_1^{seg}, X_2^{seg})$. Intuitively, if $2\eta\tilde{q}_L^2 > \nu - \omega q_F^o$ then the margin on the old market is relatively too small for the follower to have no incentives to stay there.

Lemma 3 *Let $q_F^o \in \mathcal{R}_2$. If $X_2^{seg} < \infty$ then $X_2^{det} < \infty$.*

This means that if for investment becomes feasible in the Upper Region, that is, forcing the other firm to leave the old marktet, then it is also possible to have simultaneous investment.

Proposition 6 *Let $q_F^o \in \mathcal{R}_2$. Assume $\nu - \omega q_F^o - 2\eta\tilde{q}_L^2 > 0$. Then, investment under the deterrence/segragation strategy is considered by the leader for $X \in [X_1^{det}, X_2^{det}]$. There exist unique values X_1^{seg} and X_2^{seg} such that $0 < X_1^{det} < X_1^{seg} < X_2^{seg} < X_2^{det}$. Moreover, the follower's investment is delayed for $X \in (X_1^{det}, X_1^{seg}) \cup (X_2^{seg}, X_2^{det})$. The leader has no competitor on the new submarket if it invests at $X \in [X_1^{seg}, X_2^{seg}]$. Then, a pair (q_L^{opt}, X_L^*) exists such that*

(i) for $X \geq X_L^*$ the leader makes an immediate investment and obtains the value function defined by

$$V_L^{det}(X) = \begin{cases} \frac{X}{r-\mu} q_L^n(\nu - \eta q_L^n - \omega q_F^o) + \frac{X_F}{r-\mu} q_L^n(\omega q_F^o - \eta q_L^n) \left(\frac{X}{X_F}\right)^\beta - \delta q_L^n & \text{if } X \in (X_1^{det}, X_1^{seg}) \\ & \text{or } X \in (X_2^{seg}, X_2^{det}) \\ \frac{X}{r-\mu} q_L^n(\nu - \eta q_L^n - \omega q_F^o) - \delta q_L^n & \text{if } X \in [X_1^{seg}, X_2^{seg}] \end{cases}$$

and sets capacity $q_L^n = q_L^{seg}(X; q_F^o)$ in case $X \in [X_1^{seg}, X_2^{seg}]$ and $q_L^n = q_L^{det}(X; q_F^o)$ otherwise;

(ii) for $X < X_L^*$ the leader postpones investment until x reaches X_L^{det}. This yields

$$F_L^{det}(X; q_L^o, q_F^o) = A_L^{det} X^\beta + \frac{X}{r-\mu} q_L^o (1 - \eta(q_L^o + q_F^o)).$$

The leader's investment quantity equals $q_L^{opt} = q_L^{seg}(X_L^{det})$ in case $X_L^{det} \in [X_1^{seg}, X_2^{seg}]$ and equals $q_L^{opt} = q_L^{det}(X_L^{det})$ otherwise.

For the case $\nu - \omega q_F^o - 2\eta\tilde{q}_L^2 \leq 0$, one can consult the analysis for Region 1. The analysis for the leader's investment trigger is similar to the analysis for $\mathcal{R}_1$, with the only difference that $\mathcal{D}$ is split into two regions.

Investment under the accommodation strategy is not discussed here, since the main focus of this paper is the preclusion of the follower's adoption. In Appendix C, however, for the sake of completeness, there is a summary of the optimal investment decision under accommodation.

5 Endogenous firm roles

In this section the situation is studied where both firms are allowed to take the role as leader. To that purpose each firm compares the value as when it is leader to as when it is follower for each value of x. Standard in the literature (see e.g.), the intersection point is called the preemption point X_P. Let us call the two firms firm A and firm B, then the preemption trigger for firm A is defined as

$$X_{PA} := \max\{X \in \mathbb{R} \mid V_{LA}(X, q_A^n) = V_{FA}(X, q_B^n)\}.$$

where q_A^n and q_B^n are defined as the leader's capacity[11] when firm A is leader or follower respectively. Here $_{LA}$ stands for the leader's value function when firm A is leader, i.e. $q_L^o = q_A^o$ and $q_F^o = q_B^o$. These capacities are reversed in case firm B is leader in which case firm A obtains V_{FA} as a follower. It follows that for $X > X_{PA}$ firm A prefers to be a leader and for smaller values of the shock process the firm finds it optimal to take the role as follower and therefore prefers the other firm to undertake investment.

Example 1 *Suppose $\nu = 1.1$, $\eta = 0.26$, $\omega = 0.135$ and assume $q_A^o = 1$ and $q_B^o = 1.6$. Since $\tilde{q}_F^1 = 1.65$ we have that investment takes place in $\mathcal{R}_1$. First, it follows from Proposition 7 that for large values of X investment takes place in $\mathcal{S}$ and for small values the leader optimally invests in $\mathcal{D}$. Recall that since investment is feasible in $\mathcal{S}$, accommodation is never feasible (see Corollary 1). For firm A it then holds that:*

$$X_{PA} = 556 \text{ and } X_{LA}^{seg} = 392.$$

For firm B one can check that

$$X_{PB} = 517 \text{ and } X_{LB}^{seg} = 511.$$

In both cases we see that the exogenous trigger X_L lies below the preemption trigger X_P. Since it holds that $X_{1A}^{seg} = 174$ and $X_{1B}^{seg} = 265$, it can be concluded that investment takes place in $\mathcal{S}$.

[11] Since is it not known ex-ante in which of the regions $\mathcal{D}$, $\mathcal{S}$ or $\mathcal{A}$ these capacities will lie, I keep the notation open.

In the example both firms find that $X_L^{seg} < X_P$, which means that if a firm would invest at its exogenous trigger, it obtains a lower value than if the other firm invests at the same value of x. The intuition behind this result is the following. Recall that after investment of the leader, the other firm will stay on the old market permanently. In this case both firms prefer that the other firm undertakes investment so that it won't have to undertake investment itself and serve the old market on its own. However, given that the other firm will not undertake investment for all X, optimal investment is undertaken at the trigger. The situation where both firms decline to undertake the adoption is never optimal, which follows from the existence of the exogenous trigger. That is, for values of x larger than X_L, the so-called stopping region is reached, indicating that investing at that particular value of x always yields a larger value than never undertaking investment.

If both players have a second-mover advantage there is a situation where each player wants the other firm to undertake investment. Games with this structure are generally called *war of attrition* games. A game where neither of players has second-mover advantage, i.e. where $X_L > X_P$ for both firms, is called a *preemption game*. In the remainder of this section two cases are distinguished. First the case of symmetric players is studied. Here, both firms have identical capacity sizes on the old submarket. After that, the situation of asymmetric firms is discussed.

5.1 Symmetric players

Under symmetric players, capacities associated with the old submarket are of the same size. For identical firms one can clearly determine in which situations firms end up in a war of attrition or a preemption game. Moreover, this section aims to analyse under which characteristics investment is undertaken under the segregation strategy.

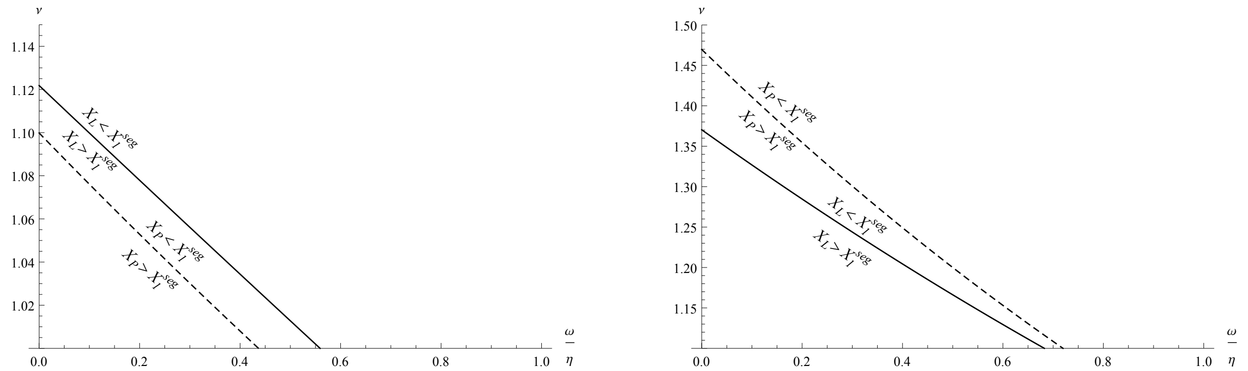
Since firms are assumed to be symmetric, define q^o to be each firm's capacity on the old market, i.e. $q^o := q_A^o = q_B^o$. Since prices are assumed to be positive it follows that it always holds that $q^o \leq \frac{1}{2\eta}$, since in that case $1 - \eta(q_A^o + q_B^o) \geq 0$[12]. Under the segregation strategy, the investment trigger can be written as,

$$X_L^{seg} = \frac{(\beta^2 - 1)\delta(r - \mu)}{\beta(\nu - \omega q^o) - \sqrt{(\nu - \omega q^o)^2 + (\beta^2 - 1)4\eta q^o(1 - 2\eta q^o)}}. \tag{12}$$

Let us first look under which circumstances the exogenous trigger X_L falls in the segregation region[13] and in the deterrence region. Figure 5 shows these different regions for X_L with respect to ν and $\frac{\omega}{\eta}$. Here, X_L either equals the trigger under the deterrence strategy or the trigger under the segregation strategy. The curves represented by the solid line are defined as the intersection point of X_L^{seg} and X_1^{seg}. For large (small) values of ν and ω the investment trigger lies in the region where the other firm also undertakes the adoption

[12]One can easily check that for ν sufficiently small to 1, $\frac{1}{2\eta} < \frac{\eta - \nu\omega}{\eta^2 - \omega^2}$. Recall that for the segregation strategy to be applicable, it is required that $q^o < \frac{\eta - \nu\omega}{\eta^2 - \omega^2}$.

[13]In slight abuse of definitions, a trigger is said to fall in the segregation region if the capacity size q_L^n resulting from investment undertaken at this trigger falls in the Segregation Region as defined in Section 4. To omit complex linguistic structures, the latter situation is described as the trigger falling in the respective region.



(a) Regions with respect to Segregation and Deterrence, $q_A^o = q_B^o = 0.8$. Equivalent to the situation with high levels of uncertainty.

(b) Regions with respect to Segregation and Deterrence, $q_A^o = q_B^o = 2$. Equivalent to the situation with small levels of uncertainty.

Figure 5: Curves distinguishing segregation and deterrence

$r = 0.1,\ \mu = 0.02,\ ,\sigma = 0.05,\ \eta = 0.2$

(stays on the old market). That is, when the quality improvement is larger or when the products are closer substitutes, it is harder to ward off the follower on the new submarket. For large ν, the new market is more profitable and it is therefore less attractive for the follower to not undertake the adoption. It is then too costly to ward off the other firm, for it would take a large investment size. This means that radical innovation, i.e. a large value of ν, induces that the follower also undertakes investment which in turn brings to existence of the old technology to an end. Incremental innovation does not, since small values of ν make sure that the follower does not adopt. For large ω, the markets are more sensitive towards the other market's production size. Since, for the same degree of vertical differentiation, the products are less differentiated, it is less attractive to stay with the old technology, as a follower.

The region where investment is optimal under the segregation region shrinks or expands for different parametrizations. The following Lemma shows that segregation is possible as long as the old submarket's capacity is sufficiently close to the optimal production level[14].

Lemma 4 *Let $q^o \in [0, \frac{1}{2\eta}]$. There exist $(\nu, \omega) \in [1, \infty) \times [0, \eta]$ such that $X_L \geq X_1^{seg}$ if and only if $q^o \geq q^*$ for some $q^* \in \left(0, \frac{1}{4\eta}\right)$. More broadly speaking, those values of (ν, ω) exist if and only if*

$$\beta - \sqrt{1 + (\beta^2 - 1)4\eta q^o(1 - 2\eta q^o)} \leq (\beta^2 - 1)(4\sqrt{\eta q^o(1 - \eta q^o)} - 1).$$

Overall, if q^o goes up, the range of parameter values (ω, ν) where firms employ the segregation strategy at X_L increases. Intuitively, the larger one's current capacity, the more one is inclined to stay on a profitable market, which implies that it becomes easier for the other firm to hold off its competitor. For smaller

[14]An optimal production level is understood to be the level one would set under a monopolist's mindset, for after the game ends the follower is the only firm present on the old market.

Region	σ	r	μ	δ
$X_L \geq X_1^{seg}$	+	-	+	0
$X_P \geq X_1^{seg}$	0	0	0	0
Attrition	-	+	-	0

Table 2: Effect of an increase in a parameter leading to a shrinking (-)/an expanding (+) size of the regions.

capacity sizes, firms can improve by replacing their capacity size.

The effect of other parameters is summarized in the first line of Table 2. Here, more market uncertainty leads to more points (ω, ν) allowing for the segregation strategy, reflected by an upward movement of the line corresponding to the cases where $X_L^{seg} = X_1^{seg}$. Larger uncertainty makes firms invest at larger values of x, which makes it less costly to install a capacity sufficiently large to hinder the follower. In this way, a broader range of parameter values emerge where segregation can be played. A similar story can be held for μ and r.

In Figure 5, a qualitatively similar figure is also shown for the regions where the preemption point appears to be in the segregation or the deterrence region respectively. The conclusions and intuition are equivalent to the analysis for the exogenous trigger X_L. The following Lemma shows that preemption through segregation is preferred over deterrence when the old product's capacity size is sufficiently large.

Lemma 5 *Let $q^o \in [0, \frac{1}{2\eta}]$. There exist $(\nu, \omega) \in [1, \infty) \times [0, \eta]$ such that $X_P \geq X_1^{seg}$ if and only if*

$$\eta q^o \geq \tfrac{1}{2} - \tfrac{1}{6}\sqrt{5}.$$

Here, the same intuition applies as before. It is only possible to block the follower's investment as long as the old market is profitable enough. Nothing else than the product ηq^o plays a role in the determination of the existence of points (ω, ν), in Figure 5. The preemption point is the value of x such that, when comparing the valuation of staying on the old market and the valuation of launching the new product, one is indifferent between the two possibilities. Here, the waiting option does not come into play. Therefore, the parameters mentioned in Table 2 have no influence on the preparedness to undertake investment.

Taken together, for small values of ω and ν both the preemption point X_P and the trigger X_L fall in the segregation region. For large values of ω and ν both fall in the deterrence region. For intermediate values two cases arise. Either the preemption point X_P is smaller than X_1^{seg} and X_L is larger or exactly the other way around. The first situation is depicted in Figure 5a and the second one in Figure 5b. Roughly speaking, the former happens for high levels of uncertainty, while the latter happens for small values. This implies that in the first case the preemption point lies below the trigger X_L which leads to a preemption run. However, in the second case, relatively, more values induce a situation where $X_P > X_L$: war of attrition games. The effect of uncertainty is further discussed in Section 6.1.

Equilibria In a preemption game, i.e. a game where $X_P < X_L$ for both players, firms end up in a preemption run. This means that, suppose one firm wants to undertake investment at X_L as a leader, then it is optimal for the other firm to invest for a value of x just before x hits X_L. In this way it obtains the leader value at $X_L - \varepsilon$ instead of the lower follower value, had the other firm invested as a leader at X_L. The same argument holds for both firms so that investment is accelerated and undertaken at X_P^- by one firm. The determination of preemptive equilibria have extensively been discussed in the literature and for this paper one can conclude that the leader undertakes investment at the preemption trigger X_P.[15]

Regarding the war of attrition games, a thorough formulation of equilibria in set-ups like this paper is done by Steg and Thijssen (2015). Here, similar to the set-up without market uncertainty by Hendricks *et al.* (1988), firms randomize their strategies in the attrition region. Translated to the context of this paper, an attrition region arises when $X_L < X_P$. Here, firms are willing to adopt before x hits X_L since waiting is always optimal, and, at the same time, firms are in the preemption context when x exceeds X_P. Therefore, one could fairly say that investment is not undertaken before x hits X_L and investment is yet undertaken when x grows beyond X_P.[16] Equilibria in a war of attrition game are also studied by e.g. Fudenberg and Tirole (1991) and Thijssen *et al.* (2006).

Taken together, if in a preemption game one finds $X_P \geq X_1^{seg}$ investment is undertaken while blocking the other firm and investment is undertaken while delaying the follower's adoption for the complementary case $X_P < X_1^{seg}$. In a war of attrition game, the same logic applies, but then for the attrition region $[X_L, X_P]$, rather than only one trigger.[17] However, since one cannot determine a priori where in the attrition region the first firm adopts, one cannot know under which strategy adoption takes place for the case $X_L < X_1^{seg} < X_P$.

Effect of horizontal differentiation Horizontal differentiation is captured by the parameter ω. The first order derivative with respect to ω of (12) shows that X_L^{seg} is positively related to ω. This means that investment is delayed when products are closer substitutes. As ω becomes larger, profits become smaller. Then, firms are only considering investment for larger values of x which compensate for the loss in profits.

[15]Formally, following Fudenberg and Tirole (1985) and Steg and Thijssen (2015), in the preemption region $\mathcal{P} = [X_P, \infty)$ firm i plays a strategy (G_i, α_i) with the distribution function G_i of investment and intensity α_i. At a stopping time $\tau \in \mathcal{P}$, this is $G_i^\tau(s) = \mathbb{I}_{s \geq \tau_\mathcal{P}}$ $\forall i$ where $\tau_\mathcal{P} = \inf\{t \geq t_0 \ : \ x(t) \geq X_P\}$ and intensity $\alpha_i^\tau(s) = \mathbb{I}_{s \geq \tau_\mathcal{P}} \frac{L(s)-F(s)}{L(s)-M(s)}$ $\forall i$. The functions L and F correspond to V_{Li} and V_{Fi} respectively where firm i is leader and sets q_{Li}^n in the first case and follower in the second. The function M corresponds to the case where firms make a coordination mistake and both invest as a leader. Since $\nu - \eta(q_{LA}^n + q_{LB}^n) \geq 0$ we don't run into any issues here. Nevertheless, since X assumed to be sufficiently small (see Section 2) the intensity equals 0, so that each firm invests with probability $\frac{1}{2}$ (see literature).

[16]In a formal analysis, again following Fudenberg and Tirole (1985) and Steg and Thijssen (2015), firms play a mixed strategy (G_i, α_i) where α_i^τ is identical to the one for the preemptive strategy and where G_i^τ is played such that firms are indifferent between investment and waiting in the attrition region $[X_L, X_P)$ and equals 1 in $\mathcal{P}$. Define $V_i(t, G_j)$ to be firm i's payoff under G_j. Then firms are indifferent if $V_i(t', G_j) = V_i(t, G_j)$ $\forall t' \geq t$ so that, as can be shown, G_j^τ solves $(F-L)\mathrm{d}G_j^\tau + (1-G_j^\tau)dL = 0$.

[17]The main focus of this paper is the presence of the second-mover advantage and under which specifications the second firm undertakes investment. Since these research questions can be answered without full specifications of the equilibria the determination and formulation is beyond the scope of this paper.

Notice that before adoption by the first firm, ω plays no role. The same holds for the preemption trigger (see e.g. (13)). A similar relation was found by Milliou and Petrakis (2011). However, in their analysis where firms precommit to their adoption strategy, a second effect is present. For values of ω above a certain reservation value they find a negative relation, i.e. competition in accelerated. In this situation, the so-called output effect becomes dominant. This happens since under their set-up firms have a cost advantage when undertaking adoption. When competition is stronger, a cost advantage is dominant over the former effect. Accelerated investment is also found in their preemption game, which is in contrast to this paper's results. This can however be explained by the difference in set-up. In their model a cost advantage makes firms less reluctant to wait when competition becomes more heavy. In this paper, waiting until the exogenous process reaches a higher value pays off, for one invests in times of a higher profitability.

5.1.1 War of attrition

In the first case, see Figure 5a and Figure 6a, we see that for large values of ω and ν one has $X_1^{seg} < X_P < X_L$, then, when decreasing their values one finds consecutively $X_P < X_1^{seg} < X_L$ and $X_P < X_L < X_1^{seg}$. Both X_P and X_L are increasing functions with respect to ω. If the new market becomes more sensitive to the other market's capacities, investment is delayed in order to wait for more profitable circumstances. However, since ω also affects profits on the old market, one finds that the preemption point does not increase as rapidly as the trigger X_L. As a result, for small values of ω, the order of the two even changes so that a war of attrition arises, i.e. $X_L < X_P < X_1^{seg}$. The intuition behind the finding that a war of attrition arises for small values of ν is straightforward. Since a small ν implies that the old market becomes relatively more profitable it follows that one is less reluctant to have the other firm invest in the new technology.

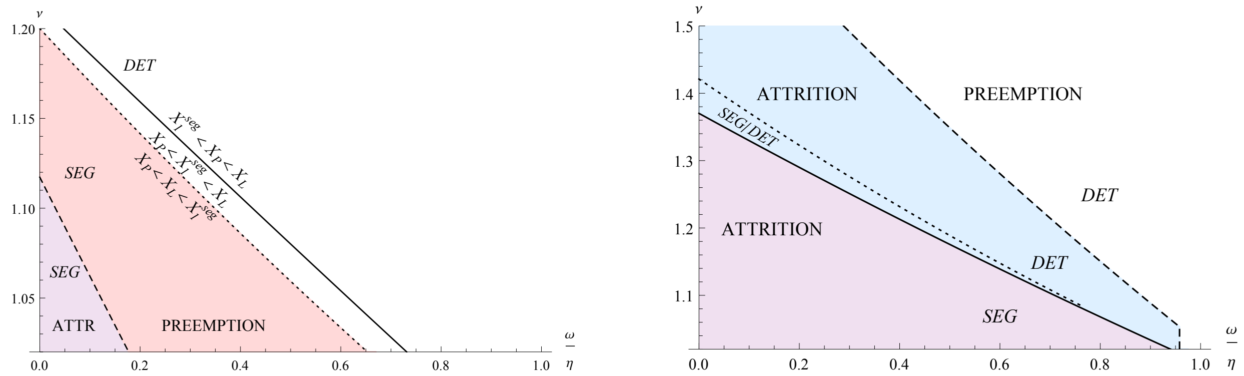
Similar as before, the size of the region with respect to (ω, ν) where a war of attrition is present depends on the parametrization. The size of the region with a war of attrition is positively related to η, q^o and r, but negatively by σ and μ, see Table 2.

Lemma 6 *Let* $q^o \in [0, \frac{1}{2\eta}]$*. There exist* $(\nu, \omega) \in [1, \infty) \times [0, \eta]$ *such that* $X_L \leq X_P \Leftrightarrow q^o \geq q^{att}$ *for some* $q^{att} \in \left(0, \frac{1}{2\eta}\right)$*. Generally, such values exist if and only if*

$$\beta - \sqrt{1 + (\beta^2 - 1)4\eta q^o(1 - 2\eta q^o)} \geq (\beta^2 - 1)(1 - 2\sqrt{\eta q^o(1 - \eta q^o)}).$$

Since a more profitable old market, i.e. a larger q^o, makes it less interesting to undertake investment as a leader, a war of attrition is more prevalent. However more market uncertainty, i.e. a larger σ, or a larger market trend, μ, makes the firm delay investment, which makes it, as explained before, less likely that firms end up in a war of attrition. Since a larger discount rate, r, excellerates investment it results in an opposite effect, i.e. firms more easily end up in a war of attrition when discounting is done more heavily.

In the second case, see Figure 5b and Figure 6b, for small values of (ω, ν) we have $X_L < X_P < X_1^{seg}$,



(a) War of attrition under segregation, $q_A^o = q_B^o = 1$.

(b) War of attrition under segregation and deterrence, $q_A^o = q_B^o = 1.7$.

Figure 6: Curves distinguishing segregation and deterrence

$r = 0.1,\ \mu = 0.02,\ ,\sigma = 0.05,\ \eta = 0.2$

where consecutively, for increasing (ω, ν), this changes into $X_L < X_1^{seg} < X_P$,[18] $X_1^{seg} < X_L < X_P$ and eventually $X_1^{seg} < X_P < X_L$. The difference with the first case is that, over there attrition only took place in the region where segregation was played. So, while having the other firm undertake investment, one stays on the old market ever after. However, in this case, a war of attrition also takes place under entry deterrence. This brings in a second type of late-mover advantage where firms do want to undertake the adoption, but only as a second adopter. In other words, the new market is sufficiently more profitable for both firms to undertake investment, but being a temporary monopolist on the old market is more beneficial than being a temporary monopolist on the new market. This happens e.g. when firms are sufficiently large on the old market so that the monopoly rents accummulated while being the only firm on the old submarket are larger than on the new submarket.

Generally, Table 3 shows under which parametrizations both cases occur. This shows that investment under deterrence, while being involved in a war of attrition, is possible in situations that are characterized by parametrizations where the leader sets a relatively small capacity on the new market. In this way the firm hopes it will trade its current market for a market where it can enter as a follower so that it scoops the profits on the old market and can set a sufficiently large capacity size.

Markets that have low uncertainty, a small market trend and faces firm discounting, i.e. under the conditions of a small value of σ and μ and a large value of r, one finds that the leader undertakes investment relatively early, i.e. for a relatively small value of x. This results in a small capacity. Similarly, a large capacity on the old market burdens the new market, while existing, in two ways. First, it reduces the price

[18] In this region it is undetermined ex-ante at which value of x investment is undertaken. More specifically, if one allows for the possibility to play a mixed equilibrium, it is unknown whether investment takes place before or after X_1^{seg}. As a result, it is unknown ex-ante whether entry deterrence or segregation will be played.

War of attrition	σ	r	μ	ηq^o	δ
One firm adopts	high	low	high	low	no effect
One or both firms adopt	low	high	low	high	no effect

Table 3: Parameter settings distinguishing war of attrition types

and secondly, it delays the follower's investment decision. As a result, the leader sets a relatively small capacity. Under these specification it becomes rewarding to be the second investor.

5.1.2 Discussion

Existence of the preemption trigger The preemption trigger for symmetric firms, under segregation, equals

$$X_P = \frac{\delta(r-\mu)}{\nu - \sqrt{4\eta q^o(1-\eta q^o) + (\omega q^o)^2}}. \tag{13}$$

Similar to the analysis in previous sections, one could notice from the specification that this value only exists if the denominator is positive. In other words, for e.g. small values of ν, the preemption point is not defined. In fact, for these parameter values the value function of the follower always lies above the leader's curve. Games with these characteristics are also defined to be a war of attrition game. Differently, let $X > X_1^{seg}$. Then

$$V_L - V_F = \frac{X}{r-\mu}\left[\tfrac{1}{4\eta}\nu^2 - \tfrac{1}{4\eta}(\omega q^o)^2 - q^o(1-\eta q^o)\right] - \tfrac{1}{2\eta}\delta\nu + \tfrac{1}{4\eta}\delta^2\frac{r-\mu}{X}$$

is the difference between the leader's value function and the follower's value function. For sufficiently large values of X, this difference can only be positive if the first term is positive. This is equivalent to a positive denominator of X_P.

The initial market conditions The equilibriums described in this paper are subject to the assumption that the initial value of the shock process is sufficiently small. In some situations this much influences the outcome of the game. In the case where a war of attrition arises, i.e. when $X_L < X_P$, one can actually speak of three different regions: a continuation region $[0, X_L)$, a war of attrition region $[X_L, X_P)$ and a preemption region $[X_P, \infty)$. This means that if $x(0) < X_P$ firms always face a second-mover advantage. However, had the state process begun at a higher level, $x(0) \geq X_P$, a first-mover advantage appears. If x is small, prices are small as well and investment is less appealing than when x is large. This shows the impact of market uncertainty by including a shock process in the model. However, saying something about the resulting equilibrium, then becomes much more involved, but one could fairly summarize it as follows. In presence of the attrition region, if the attrition region lies above X_1^{seg}, i.e. $X_1^{seg} < X_L < X_P$, there is always only a single adopter, if it lies below X_1^{seg} then multiple cases can be distinguished. If $x(0) > X_1^{seg}$ investment takes place immediatly, but only one firm adopts, which is the same for $X_P < x(0) < X_1^{seg}$, but then both firms will (eventually) adopt. For $x(0) < X_P$ firms randomize their strategies, since they find themselves in a war of attrition game. However, both firms will undertake the adoption. In no presence of the attrition region investment is always immediatly undertaken for $x \geq X_P$ and here only one firm adopts if $\max\{x(0), X_P\} \geq X_1^{seg}$.

5.2 Asymmetric players

Identical firms face the same investment problem: their the preemption points, their optimal investment triggers and boundaries X_1^{seg} are the same for both firms. As a result one finds that, for a given parametrization, either both firms have a first-mover advantage, i.e. they end up in a preemption game, or both firms face a second-mover advantage in a war of attrition game. However, if firms are asymmetric in their initial capacities, it no longer holds that both firms face the same investment problem. For a sufficient difference between the firms' capacities for the old technology one can find the situation where one firm has a second-mover advantage but the other not.

Example 2 *Under the baseline parametrization, assume in this case that $q_B^o = 1.55$. Then for firm A*

$$X_{PA} = 535 \text{ and } X_{LA}^{seg} = 398.$$

For firm B it holds that

$$X_{PB} = 513 \text{ and } X_{LB}^{seg} = 517.$$

In this game, firm A has a second-mover advantage, but firm B has a first-mover advantage. However, for $q_B^o = 1.48$ the order changes into $X_{LA} < X_{PB} < X_{PA} < X_{LB}$ and for $q_B^o = 1.45$ the order $X_{LA} < X_{PA} < X_{PB} < X_{LB}$ emerges.

Asymmetry leads to a whole range of possible cases with respect to order of X_{LA}, X_{PA}, X_{1A}^{seg}, X_{LB}, X_{PB} and X_{1B}^{seg}. Although one would expect that the firm with the first-mover advantage invests first, this is not necessarily true. In the example it was shown that for $q_B^o = 1.45$ the order of the triggers shows that firm A has a second-mover advantage and firm B has a first-mover advantage. However, firm A has no incentive to wait until x reaches any of firm B's investment triggers. As a result firm A undertakes investment first.[19]

In the example with $q_B^0 = 1.45$, firm B is the largest firm on the old market. Its preemption trigger equals $X_{PB} = 500$ and its exogenous trigger equals $X_{LB} = 523$. Since $X_{1B}^{seg} = 264 < X_{PB}$ adoption is done

[19]In this case one cannot speak of a war of attrition or preemption game. Here, firm A's triggers are below firm B's triggers so that there exists no pair $(G_A, \alpha_A) \times (G_B, \alpha_B)$ such that firm A is indifferent between investment and waiting for all 'adoption moments' in the attrition region $[X_{LA}, X_{PA}]$. Therefore, formally, since investment is not optimal for firm B at any moment before x hits X_{PB} for the first time and the arguments given above, the equilibrium strategy in this example is $G_A^\tau(s) = \alpha_A^\tau(s) = \mathbb{I}_{s \geq \tau_L}$ and $G_B^\tau = \alpha_B^\tau = 0$ for all $\tau \in [0, \tau_{\mathcal{P}})$ where $\tau_L = \inf\{t \geq t_0 \ : \ x(t) \geq X_{LA}\}$ and $\tau_{\mathcal{P}} = \inf\{t \geq t_0 \ : \ x(t) \geq X_{PB}\}$. For $\tau \geq \tau_{\mathcal{P}}$ we are back in the preemption game described in Section 5.1. Nevertheless, this framework is still under the assumption that $x(0)$ is sufficiently small, i.e. $x(0) < X_{LA}$.

under the segregation strategy[20] in case firm B undertakes it first. Similarly, for firm A, its lower bound with respect to the segregation triggers equals $X_{1A}^{seg} = 178$ so that both $X_{LA} = 409$ and $X_{PA} = 497$ fall in the segregation region. If firm A undertakes investment it switches from capacity $q_A^o = 1$ to $q_A^n = 1.36$ in which case $q_A^n(\nu - \eta q_A^n - \omega q_B^o) = 0.749$ compared to $q_A^o(1 - \eta q_A^o - \omega q_B^n) = 0.529$ in the situation where firm B undertakes investment with $q_B^n = 1.56$. So, although firm A prefers firm B to undertake investment for values of x at firm A's trigger, the firm is worse off when firm B undertakes investment at firm B's trigger. Hence, despite firm A has a second-mover advantage, its the only adopter nonetheless.

This is one example of many cases with respect to the order of the triggers. Elaborating on each of these separately does not lead to any further understanding of technology adoption in this set-up, nor does it lead to situations of extraordinary relevance. Therefore, I refrain from further studying the situation under asymmetric firms.

6 Robustness

6.1 Effect of uncertainty

6.2 Robustness

Adoption costs In this model, firms face linear adoption cost when undertaking investment. As becomes clear from e.g. (4), (12) and (13), the investment moments are positively related to the investment cost and therefore these costs delay investment. As shown before, these costs, however, have no influence on the follower's decision whether to adopt or not. One could consider to, additionally, look at fixed costs. However, these costs will have the same effect: they impose an (additional) cost, which will delay, but not block the follower's investment. Similarly, the result that a second-mover advantage is present when products are close substitutes does not qualitatively change when considering different cost structures.

Capacity choice Although firms are generally free to set their capacities, it is not a common assumption in the literature. In this paper's set-up, fixing the capacity size leads to the following result. Denote the firms' capacity sizes by K, then the follower decides to never undertake adoption if and only

$$\nu - \eta K + \omega K \leq 0.$$

Clearly, this inequality is never true and, therefore, the follower always undertakes investment. This means that the intuitive result that endogenously it is determined whether the follower also adopts does not exist in a case where the firms' capacities are assumed to be fixed. Nevertheless, if one assumes that, despite fixing them, capacities are assumed to be asymmetric, one can find cases where only one firm undertakes investment. All one then needs is to assume that the firm's capacity on the new submarket can only become

[20] Notice that we are still in $\mathcal{R}_1$.

sufficiently small, that is, sufficiently smaller relative to its capacity on the old submarket, to make adoption unattractive.

Vertical differentiation This model assumes technological development i.e. $\nu > 1$. However, it is widely observed in real life that firm enter the market with products of cheaper quality to compete for consumers of modest means. This would imply to study cases where $\nu < 1$. Although not considered in this model, it can be induced that for these cases the range of parameter values allowing for a second-mover advantage and the adoption of only one firm increases. As an example, the restriction $\eta > \omega\nu$ always hold, making that Region 1 and Region 2 always exist.

7 Conclusion

This paper has three main results. Firstly, it is shown that although the new product's quality is better than the old product's, under certain conditions, only one firms adopts. This happens when the degree of innovation is small and products are not close substitutes. In this way, sharing the same submarket with a rival firm is worse than serving your own submarket, despite the lower quality. Secondly, depending on, a.o. things, the differentiation of the products, there is a first-mover advantage or a second-mover advantage. When firms' current capacity sizes are sufficiently large, they have no incentive to leave the current technology themselves. Especially under the conditions of early investment, that is, investment when the market is relatively in an early state, firms are reluctant to undertake investment. However, both not undertaking investment is even worse. Therefore a war of attrition arises. This implies that waiting pays off, not because uncertainty is resolved, the arrival of new information or because of information spillover, but because the firms prefers to not undertake adoption all together. Third, as a result of market uncertainty, the equilibrium in this market highly depends on the level of market uncertainty. For different values of this level, firms are more or less reluctant to be the first adopter and also whether or not they want to undertake adoption in the first place. Moreover, it distinguishes two types of second-mover advantage. As explained before, firms prefer to be a late mover so that they will not undertake adoption. But under small undertainty levels, firms can be incentified to be a late mover in order to obtain temporary profits resulting from the old technology before undertaking investment. Moreover, as a second adopter, it invests at higher values of the exogenous shock process so that it might set a higher capacity level.

Furthermore, there are some smaller results. Under the case of asymmetric firms an example is shown where only one firm has a first-mover advantage. In equilibrium howerever, the firm with the second-mover advantage undertakes the adoption and the other firm decides not adoption at all. This also comes as a result of the timing decisions combined with capacity choice. Preferably the firm with the second-mover advantage prefers the other firm to undertake the adoption. However, it is worse off when the other firm adopts much later in the process, setting a large capacity size with the new technology. A second result worth mentioning here is the fact showing that simultaneous investment and segregation are two mutually exclusive strategies. This is, when first movers are able block the second firm from adopting - even when its not optimal - simultaneous investment is never a feasible strategy. Only when either the quality improvement is moderate and when the initial capacity level of the follower is very large, simultaneous investment can be feasible. In that case the follower is forced to undertake adoption since the profit margins are nil. This brings us to a next small result, where this paper shows that for sufficiently large initial capacity levels the profits associated with the new technology are marginalized, while keeping the profits on the new submarket sufficiently positive to make this a feasible strategy. Here, the quality improvement is large enough to set a capacity level that reduces the price on the old submarket. Another minor result is that, although this paper assumes to start with an initially low value of the state process, it does influence the type of equilbrium. For large values of the state process firms always end up in a preemption game, even despite a war of attrition would have arisen when the state process started with an inially small value.

References

Amir, R. and A. Stepanova (2006). Second-mover advantage and price leadership in bertrand duopoly. *Games and Economic Behavior*, 55, 1–20.

Anand, K. and K. Girotra (2007). The strategic perils of delayed differentiation. *Management Science*, 53, 697–712.

Bagwell, K. (1995). Commitment and observability in games. *Games and Economic Behavior*, 8, 271280.

Bárcena-Ruiz, J. C. and N. Olaizola (2008). Choice of flexible production technologies under strategic delegation. *Japan and World Economy*, 20, 395–414.

Chod, J. and N. Rudi (2005). Resource flexibility with responsive pricing. *Operations Research*, 53, 532–548.

Choi, J. P. (1997). Herd behavior, the penguin effect and the suppression of informational diffusion: an analysis of informational externalities and payoff interdependency. *RAND Journal of Economics*, 28, 407–425.

Dawid, H., M. Kopel, and P. M. Kort (2013). New product introduction and capacity investment by incumbents: Effects of size on strategy. *European Journa of Operational Research*, 230, 133–142.

Deneckere, R., H. P. Marvel, and J. Peck (1997). Demand uncertainty and price maintenance: markdowns as destructive competition. *American Economic Reviews*, 87, 919–641.

Dixit, A. K. and R. S. Pindyck (1994). *Investment Under Uncertainty*. Princeton University Press, Princeton, New Jersey, United States of America.

DUTTA, P. K., S. LACH, AND A. RUSTICHINI (1995). Better late than early: Vertical differentiation in the adoption of a new technology. *Journal of Economics & Management Strategy*, 4, 563–589.

FERREIRA, F. A., F. FERREIRA, M. FERREIRA, AND A. A. PINTO (2015). Flexibility in a stackelberg leadership with differentiated goods. *Optimization*, 64, 877–893.

FRISELL, L. (2003). On the interplay of informational spillover and payoff externalities. *RAND Journal of Economics*, 34, 582–592.

FUDENBERG, D. AND J. TIROLE (1985). Preemption and rent equalization in the adoption of new technology. *The Review of Economic Studies*, 52, 383–401.

FUDENBERG, D. AND J. TIROLE (1991). *Game Theory*. The MIT Press, Cambridge, Massachusetts, United States of America.

GAL-OR, E. (1985). First mover and second mover advantages. *International Economic Review*, 26, 649–653.

GOLDER, P. N. AND G. J. TELLIS (1993). Pioneer advantage - marketing logic or marketing legend. *Journal of Marketing Research*, 30, 158–170.

GÖTZ, G. (1999). Monopolistic competition and the diffusion of new technology. *The RAND Journal of Economics*, 30, 679–693.

GÖTZ, G. (2000). Strategic timing of adoption of new technologies under uncertainty: A note. *International Journal of Industrial Organization*, 18, 369–379.

GOYAL, M. AND S. NETESSINE (2007). Strategic technology choice and capacity investment under demand uncertainty. *Management Science*, 53, 192–207.

HARSANYI, J. C. AND R. SELTEN (1988). *A General Theory of Equilibrium Selection in Games*. MITPress, Cambridge, MA.

HENDRICKS, K. (1992). Reputation in the adoption of a new technology. *International Journal of Industrial Organization*, 10, 663–677.

HENDRICKS, K., A. WEISS, AND C. WILSON (1988). The war of attrition in continuous time with complete information. *International Economic Review*, 29, 663–680.

HOPPE, H. C. (2000). Second-mover advantages in the strategic adoption of new technology under uncertainty. *International Journal of Industrial Organization*, 18, 315–338.

HOPPE, H. C. (2002). The timing of new technology adoption: Theoretical models and empirical evidence. *Manchester School, Special Issue on Industrial Organization*, 70.

Hoppe, H. C. and U. Lehmann-Grube (2001). Second-mover advantages in dynamic quality competition. *Journal of Economics & Management Strategy*, 10, 419–434.

Huisman, K. J. M. and P. M. Kort (2015). Strategic capacity investment under uncertainty. *RAND Journal of Economics*, 46, 376–408.

Jensen, R. (1982). Adoption and diffusion of an innovation of uncertain profitability. *Journal of Economic Theory*, 27, 182–193.

Jensen, R. (1992). Dynamic patent licensing. *International Journal of Industrial Organization*, 10, 349–368.

Jensen, R. (2003). Innovative leadership: First-mover advantages in new product adoption. *Economic Theory*, 21, 97–116.

Karshenas, M. and P. Stoneman (1995). Technological diffusion. In *Handbook of the Economics of Innovation and Technological Change*, edited by P. Stoneman, 265–297. Basil Blackwell, Oxford, United Kingdom.

Kopel, M. and C. Löffler (2008). Commitment, first-mover-, and second-mover advantage. *Journal of Economics*, 94, 143–166.

Liu, Z. (2005). Stackelberg leadership with demand uncertainty. *Managerial and Decision Economics*, 26, 345–350.

Meza, S. and M. Tombak (2009). Endogenous location leadership. *International Journal of Industrial Organization*, 27, 687–707.

Milliou, C. and E. Petrakis (2011). Timing of technology adoption and product market competition. *International Journal of Industrial Organization*, 29, 513–523.

Mulligan, J. G. and E. Llinares (2003). Market segmentation and the diffusion of quality-enhancing innovations: The case of downhill skiing. *The Review of Economics and Statistics*, 85.

Reinganum, J. (1989). The timing of innovation: Research, development, and diffusion. In *Handbook of Industrial Organization, Vol. 1*, edited by R. Schmalensee and R. D. Willig, 850–908. Amsterdam, The Netherlands.

Reinganum, J. F. (1981). On the diffusion of new technology: A game theoretic approach. *The Review of Economic Studies*, 48, 395–405.

Reinganum, J. F. (1983). Technology adoption under imperfect information. *The Bell Journal of Economics*, 14, 57–69.

Riordan, M. H. (1992). Regulation and preemptive technology adoption. *The RAND Journal of Economics*, 23, 334–349.

Steg, J.-H. and J. Thijssen (2015). Quick or persistent? on the feedback effects between first and second mover advantages in a stochastic investment game. Working Paper, Bielefeld University, Germany and University of York, United Kingdom.

Stenbacka, R. and M. M. Tombak (1994). Strategic timing of adoption of new technologies under uncertainty. *International Journal of Industrial Organization*, 12, 387–411.

Thijssen, J. J. J., K. J. M. Huisman, and P. M. Kort (2006). The effects of information on strategic investment and welfare. *Economic theory*, 28, 399–424.

Tran, V. D., D. S. Sibley, and S. Wilkie (2012). Second mover advantage and entry timing. *The Journal of Industrial Economics*, 60, 517–535.

Vardy, F. (2004). The value of commitment in stackelberg games with observation costs.

Yoon, Y. (2009). Endogenous timing of actions under conflict between two types of second mover advantage. *International Journal of Industrial Organization*, 27, 728–738.

Appendix A: Proofs

Proof of Proposition 1 If the follower undertakes investment it obtains $V_F(X, q_L^F; q_F^o, q_L^n)$ as given in the text. The first order conditions lead to the optimal capacity size:

$$\frac{X}{r-\mu}(\nu - 2\eta q_F^n - \eta q_L^n) - \delta = 0.$$

It follows from the second order conditions that this is indeed a maximum: $\frac{X}{r-\mu}(-2\eta) < 0$. The value matching and smooth pasting conditions lead to the values of X_F^* and A_F,

$$\begin{aligned}\frac{X}{r-\mu}q_F^n(\nu - \eta(q_F^n + q_L^n)) - \delta q_F^n &= A_F X^\beta + \frac{X}{r-\mu}q_F^o(1 - \eta q_F^o - \omega q_L^n),\\ \frac{X}{r-\mu}q_F^n(\nu - \eta(q_F^n + q_L^n)) &= A_F \beta X^\beta + \frac{X}{r-\mu}q_F^o(1 - \eta q_F^o - \omega q_L^n),\end{aligned}$$

where β is the positive root (see e.g. Dixit and Pindyck (1994)) of

$$\sigma^2\beta^2 + (2\mu - \sigma^2)\beta = 2r.$$

This, together with (3) leads to (4) and

$$A_F = (X_F^*)^{-\beta}\left[\frac{X_F^*}{r-\mu}(q_F^{opt}(\nu - \eta(q_F^{opt} + q_L^n)) - q_F^o(1 - \eta q_F^o - \omega q_L^n)) - \delta q_F^{opt}\right].$$

The existence of X_F^* is treated in Proposition (2). In case the follower does not undertake investment, it stays on the old market and obtains

$$\mathbb{E}\left[\int_0^\infty q_F^o(1 - \eta q_F^o - \omega q_L^n)\ e^{-rt}\ \mathrm{d}t \mid x(0) = X\right] = \frac{X}{r-\mu}q_F^o(1 - \eta q_F^o - \omega q_L^n).$$

□

Proof of Proposition 2 The follower's trigger is the solution of $f(X) = 0$ where $f(X)$ is defined by the following equation:

$$f(X) = X(\nu - \eta q_L^n)^2 - \delta(r-\mu)(\nu - \eta q_L^n)\frac{2\beta}{\beta - 1} - 4\eta q_F^o X(1 - \eta q_F^o - \omega q_L^n) + \frac{\beta + 1}{\beta - 1}\frac{\delta^2(r-\mu)^2}{X},$$

which can be rewritten as $f(X) = bX + c + \dfrac{d}{X}$, where,

$$b = (\beta - 1)\left[(\nu - \eta q_L^n)^2 - 4\eta q_F^o(1 - \eta q_F^o - \omega q_L^n)\right] \tag{14}$$

$$c = -2\beta\delta(r-\mu)(\nu - \eta q_L^n) \tag{15}$$

$$d = \delta^2(r-\mu)^2(\beta + 1). \tag{16}$$

It follows that $f(X)$ is a convex function since $d \geq 0$. Since prices remain positive, $c < 0$ is always true. The function then only has a feasible root if $b \geq 0$.[21] Since b is convexly parabolic in q_L^n we conclude that between the roots of $b = 0$ there is no solution to $f(X) = 0$. This leads one to equations (5) and (6). Moreover, b is decreasing for $q_L^n < \tilde{q}_L^1$ and increasing for $q_L^n > \tilde{q}_L^2$. Then, overall, the largest root of $f(X)$ is increasing for $q_L^n < \tilde{q}_L^1$ and decreasing for $q_L^n > \tilde{q}_L^2$.

The parabolic formulation of b only has two roots as long $q_F^o < \frac{\eta - \omega\nu}{\eta^2 - \omega^2}$, one root in case $q_F^o = \frac{\eta - \omega\nu}{\eta^2 - \omega^2}$ and the parabola lies above the x-axis in case q_F^o exceeds this level. This concludes that Region 2 ends at

$$(\tilde{q}_F^2, q_L^n) = \left(\frac{\eta - \omega\nu}{\eta^2 - \omega^2}\ ,\ \frac{\nu(\eta^2 + \omega^2) - 2\omega\eta}{\eta(\eta^2 - \omega^2)}\right).$$

For $\nu - \eta q_L^n$ to remain positive it is required that $q_L^n < \frac{\nu}{\eta}$. It can be calculated that $\tilde{q}_L^2$ exceeds $\frac{\nu}{\eta}$ when q_F^o is below

$$\tilde{q}_F^1 = \frac{\eta - \omega\nu}{\eta^2},$$

which concludes the boundary between Region 1 and 2 as depicted in Figure 1.

For larger values of q_L^n the entire function $f(X)$ moves upwards, until at a certain point $f(X) > 0$ for all values of X. The largest value of q_L^n at which $f(X) = 0$ leads to a solution can be calculated in the following way. In this point the horizontal axis is tangent to the curve, which means that we need to solve for q_L^n and X^* such that $f(X^*) = 0$ and $f'(X^*) = 0$. This is equivalent to $c^2 = 4db$, i.e.

$$\tilde{q}_L^3 = \{q_L^n\ :\ (\beta^2 - 1)4\eta q_F^o(1 - \eta q_F^o - \omega q_L^n) + (\nu - \eta q_L^n)^2 = 0\}, \tag{17}$$

which serves as an upperbound to the leader's capacity in Region 2 and Region 3. It is easily checked that the intersection of $\tilde{q}_L^2$ and $\tilde{q}_L^3$ takes place at $\tilde{q}_F^1$. The end to the third region is then defined as

$$\tilde{q}_F^3 = \{q_F^o\ :\ \tilde{q}_L^3 = 0\} = \frac{1}{2\eta}\left(1 + \sqrt{1 + \frac{\nu}{\beta^2 - 1}}\right) > \frac{1}{\eta}.$$

[21] Technically we find 2 roots, where the smallest one also exists in cases where $b < 0$. However, since this point corresponds to a negative option value, this point is ruled out as a feasible solution to the follower's threshold.

However, to find a solution to equation (17) it is required that $q_F^o(1-\eta q_F^o-\omega q_L^n)<0$, which is equivalent to a negative price on the old market. This concludes that q_L^n is bounded from above by

$$\{q_L^n \ : \ p^o \geq 0\} = \frac{1-\eta q_F^o}{\omega},$$

where $p^o = X(1-\eta q_F^o - \omega q_L^n)$. One can easily check that this curve intersects with both $\tilde{q}_L^3$ and $\tilde{q}_L^2$ at $\tilde{q}_F^1$. $\square$

Similarly one can define an upperbound to q_L^n dictated by the price of the new product: $p^n = X(1-\eta q_L^n - \omega q_F^o)$. It holds that

- this upperbound starts at $(q_F^o, q_L^n) = (0, \frac{\nu}{\eta})$ which coincides with $\tilde{q}_L^1$ at $q_F^o = 0$;
- this upperbound intersects with the upperbound dictated by p^o exactly at $\tilde{q}_F^2$, here $q_L^n = \frac{\nu\eta-\omega}{\eta^2-\omega^2}$;
- these remarks are sufficient to show that the upperbound dictated by p^n intersects $\tilde{q}_L^2$ exactly between $\tilde{q}_F^1$ and $\tilde{q}_F^2$. This happens at:

$$(q_F^o, q_L^n) = \left(\frac{4(\eta-\omega\nu)}{4\eta^2-3\omega^2} \ , \ \frac{4\eta\nu+\nu\omega^2-4\eta\omega}{4(4\eta^2-3\omega^2)}\right).$$

Proof of Lemma 1 We want to know when it is possible to have $q_L^{n*} > \tilde{q}_L^2$. Since $\lim_{q_L^n \to \tilde{q}_L^i} X_F^* = \infty$, for both $i \in \{1,2\}$ one has $V_L^{det} = V_L^{seg}$ for $q_L^n = \tilde{q}_L^1$ and $q_L^n = \tilde{q}_L^2$. Moreover, it holds that, for $i \in \{1,2\}$,

$$\lim_{q_L^n \to \tilde{q}_L^i} \frac{\mathrm{d}}{\mathrm{d}q_L^n} V_L^{det} = \lim_{q_L^n \to \tilde{q}_L^i} \frac{\mathrm{d}}{\mathrm{d}q_L^n} V_L^{seg}. \tag{18}$$

It is therefore sufficient to look at cases where $q_L^{seg} > \tilde{q}_L^2$. Since at $q_F^o = \tilde{q}_F^2$ it holds that

$$\tilde{q}_L^1 = \tilde{q}_L^2 = \frac{\nu(\omega^2+\eta^2)-2\omega\eta}{\eta(\eta^2-\omega^2)},$$

it should at least hold that

$$\lim_{X\to\infty} q_L^{seg} = \frac{1}{2\eta}\left[\nu - \omega q_F^o\right] > \frac{\nu(\omega^2+\eta^2)-2\omega\eta}{\eta(\eta^2-\omega^2)}.$$

Rewriting gives (9). $\square$

Proof of Lemma 2 We want to know when it is possible to have $q_L^{n*} > \tilde{q}_L^1$. It is sufficient to look for intersection points between q_L^{seg} and $\tilde{q}_L^1$. One can check that

$$q_F^o = \frac{16\eta - 10\omega\nu - 8\sqrt{D}}{32\eta^2 - 14\omega^2},$$
$$q_F^o = \frac{16\eta - 10\omega\nu + 8\sqrt{D}}{32\eta^2 - 14\omega^2},$$

yield the two intersection points, where $D = 4\eta^2 + 2\omega^2\nu^2 - 5\eta\omega\nu - \nu^2\eta^2$. It is concluded that $D \geq 0$ is a sufficient condition. The equation $D = 0$ has two intersection points with respect to ν,

$$\nu = \frac{5\eta\omega \pm \eta\sqrt{16\eta^2 - 7\omega^2}}{4\omega^2 - 2\eta^2}. \tag{19}$$

If $2\omega^2 > \eta^2$ then $D > 0$ is satisfied for all values outside the roots and one can check that (??) is sufficient since the other root is larger than $\frac{\eta}{\omega}$ in which case $\eta < \omega\nu$. For the case $2\omega^2 < \eta^2$ all values between the roots are considered to satisfy $D > 0$. In this case the smallest root is negative and again (??) is sufficient. For $2\omega^2 = \eta^2$ we find that D is linearly decreasing function with a root at $\frac{4\eta}{5\omega}$. However, when rewriting of (19) one obtains $\frac{8\eta}{5\omega+\sqrt{16\eta^2-7\omega^2}}$ which has a positive denominator for all parameter values. □

Proof of Proposition 3 Since q_L^{seg} is an increasing function with respect to X, it holds that the segregation strategy is feasible, i.e. $q_L^{seg} \geq \tilde{q}_L^1$, if and only if X is larger than some threshold. Rewriting $q_L^{seg} = \tilde{q}_L^1$ with respect to X leads to X_1^{seg}. Moreover, the segregation strategy, and therefore the deterrence strategy, is considered for all $X > X_1^{seg}$. Further arguments are given in the text.

Suppose the leader invests such that the follower's investment is not blocked. Then, if the leader sets a capacity level sufficiently large, $q_L^n > \hat{q}_1(X; q_F^o)$, it delays the follower's investment and it obtains,

$$\begin{aligned}
V_L^{det} &= \mathbb{E}\left[\int_{t=0}^{t_F} q_L^n x(t)(\nu - \omega q_F^o - \eta q_L^n))e^{-rt}\mathrm{d}t + \int_{t=t_F}^{\infty} q_L^n x(t)(\nu - \eta(q_L^n + q_F^{opt})e^{-rt}\mathrm{d}t \middle| x(0) = X\right] - \delta q_L^n \\
&= \mathbb{E}\left[\int_{t=0}^{\infty} q_L^n x(t)(\nu - \omega q_F^o - \eta q_L^n)e^{-rt}\mathrm{d}t + \int_{t=t_F}^{\infty} q_L^n x(t)(\omega q_F^o - \eta q_F^{opt})e^{-rt}\mathrm{d}t \middle| x(0) = X\right] - \delta q_L^n \\
&= \frac{X}{r-\mu} q_L^n(\nu - \omega q_F^o - \eta q_L^n) + \frac{X_F^*}{r-\mu} q_L^n(\omega q_F^o - \eta q_F^{opt}) \left(\frac{X}{X_F^*}\right)^{\beta} - \delta q_L^n
\end{aligned}$$

where t_F is defined as the moment the follower undertakes investment. Optimal capacity is determined via the following first order conditions,

$$\begin{aligned}
\frac{X}{r-\mu}(\nu - 2\eta q_L^n - \omega q_F^o) + \frac{X_F^*}{r-\mu}(\omega q_F^o - \eta q_F^{opt}) \left(\frac{X}{X_F^*}\right)^{\beta} - \frac{\delta}{X_F^*}\frac{1}{2} q_L^n \left(\frac{X}{X_F^*}\right)^{\beta} \frac{\mathrm{d}X_F^*}{\mathrm{d}q_L^n} \\
- \frac{\beta - 1}{r-\mu} q_L^n(\omega q_F^o - \eta q_F^{opt}) \left(\frac{X}{X_F^*}\right)^{\beta-2} \frac{\mathrm{d}X_F^*}{\mathrm{d}q_L^n} - \delta = 0.
\end{aligned}$$

It is now shown that the value of X for which $q_L^{det} = 0$ lies in $\mathcal{D}$. Notice that since one can easily construct an example for which this is true, it is sufficient to show that it is not possible to have that $\{X \,:\, q_L^{det} = 0\} = \{X \,:\, \hat{q}_1 = 0\}$ for all parameter sets. The first order conditions can be rewritten as,

$$\frac{X}{r-\mu}(\nu - \omega q_F^o) + \frac{X_F^*}{r-\mu}(\omega q_F^o - \eta q_F^{opt}) \left(\frac{X}{X_F^*}\right)^{\beta} = \delta.$$

Then if $\{X \,:\, q_L^{det} = 0\} = \{X \,:\, \hat{q}_1 = 0\} = X_F^*(0; q_F^o)$, we have

$$X_F^*(0, q_F^o) = \frac{\delta(r-\mu)}{\nu},$$

which is equivalent to

$$(-1)(\nu^2 - 4\eta q_F^o(1 - \eta q_F^o)) = \nu\sqrt{\nu^2 + (\beta^2 - 1)4\eta q_F^o(1 - \eta q_F^o)}.$$

Since the left hand side is negative (see (14)) and the right hand side positive, one has a contradition and therefore we have that the curve of the leader's capacity starts in $\mathcal{D}$.

From (18) it follows that $q_L^{det} = q_L^{seg}$ if $X = X_1^{seg}$. From this it follows that $q_L^{seg} > \tilde{q}_L^1$ if and only if $X > X_1^{seg}$ and $q_L^{det} < \tilde{q}_L^1$ if and only if $X < X_1^{seg}$.

Proof of Proposition 4 From the smooth pasting and value matching conditions one obtains the descriptions of the triggers.

The segregation trigger only exists if $(\nu - \omega q_F^o)^2 > 4\eta q_L^o(1 - \eta(q_L^o + q_F^o))$. Fix q_F^o, then the inequality becomes an equality for

$$q_L^o = \frac{(1 - \eta q_F^o) \pm \sqrt{(1 - \eta q_F^o)^2 - (\nu - \omega q_F^o)^2}}{2\eta}.$$

However, since $(1 - \eta q_F^o)^2 < (\nu - \omega q_F^o)^2$ we always have that $(\nu - \omega q_F^o)^2 > 4\eta q_L^o(1 - \eta(q_L^o + q_F^o))$. $\square$

Proof of Lemma 3 First notice that $\tilde{q}_L^2 < \hat{q}_2$ and notice that q_L^{det} is an increasing function. By definition, for $X \to \infty$ one has $\hat{q}_2 \to \tilde{q}_L^2$. So if, for some X, $q_L^{det} > \tilde{q}_L^2$, then by the definition of the convergence of functions, one has that there exists an $\tilde{X}$ such that $q_L^{det} > \hat{q}_2$ for all $X > \tilde{X}$. Hence, if $q_L^{det} \in \mathcal{D}_U$ for some X then also $q_L^{det} \in \mathcal{A}_U$ for some X. $\square$

Proof of Proposition 5 The main part of the proof is identical to the proof of Proposition 4. The same proof applies to the intersection point between q_L^{seg} and q_L^{det} respectively with $\tilde{q}_L^2$. Moreover, since

$$\lim_{X \to \infty} \hat{q}_2 = \tilde{q}_L^2$$

and since $\hat{q}_2$ is monotonically decreasing, it must hold that if for some X it holds that $q_L^{det} \in \mathcal{D}_U$ then there must exist an X_2^{det} such that $q_L^{det} \in \mathcal{A}_U$ for $X \geq X_2^{det}$. Further arguments are given in the text. $\square$

Proof of Proposition 6 First it is shown that that the accommodation curve starts in the region where the follower is deterred. In that case we have that $\{X \mid q_L^{acc}(X) = 0\} = \frac{\delta(r-\mu)}{\nu} \in \mathcal{D}$, i.e. $\frac{\delta(r-\mu)}{\nu} < X_F^*(0; q_F^o)$. One can show that

$$X_F^*(0; q_F^o) = \frac{\delta(r - \mu)}{\beta - 1} \frac{\beta\nu + \sqrt{\nu^2 + (\beta^2 - 1)4\eta q_F^o(1 - \eta q_F^o)}}{\nu^2 - 4\eta q_F^o(1 - \eta q_F^o)},$$

which leads to inequality

$$\beta\nu^2 + \nu\sqrt{\nu^2 + (\beta^2 - 1)4\eta q_F^o(1 - \eta q_F^o)} > (\beta - 1)(\nu^2 - 4\eta q_F^o(1 - \eta q_F^o)).$$

Since this inequality always holds, it is concluded that $q_L^{acc}\left(X = \frac{\delta(r-\mu)}{\nu}\right) \in \mathcal{D}$. The value of the intersection point with respect to X between q_L^{acc} and $\hat{q}_1$, X_1^{acc}, can be calculated by substituting $q_L^n = q_L^{acc}$ in (4) and solve for X. Rewriting leads to (11).

Finally, we need to show that $\nu > \frac{2\eta}{\eta+\omega}$ is a sufficient condition. Accommodation is always feasible as long as the tail of q_L^{seg} remains in the deterrence/accommodation part as depicted in Figure 1, i.e.

$$\lim_{X \to \infty} q_L^{acc} = \frac{\nu}{2\eta} < \frac{\nu - 1}{\eta - \omega}.$$

The righthandside of this equation is the minimum of $\tilde{q}_L^1$. Rewriting this inequality gives $\nu > \frac{2\eta}{\eta+\omega}$. $\square$

Proof of Proposition 7 The accommodation strategy is considered to be feasible as long as

$$\lim_{X\to\infty} q_L^{acc}(X) = \frac{\nu}{2\eta} < \tilde{q}_L^1.$$

Using (5), one can rewrite this condition. This gives $q_F^o < \tilde{q}_1^{acc}$ and $q_F^o > \tilde{q}_2^{acc}$. Similarly, the accommodation strategy is feasible for $\lim_{X\to\infty} q_L^{acc}(X) = \frac{\nu}{2\eta} > \tilde{q}_L^2$. One can check that it comes down to the same equations.

The value with respect to q_L^n at which the Upper Region starts (see, e.g., Figure 1) equals

$$q_L^n = \frac{\nu(\eta^2+\omega^2) - 2\omega\eta}{\eta(\eta^2-\omega^2)}.$$

This means that for $\frac{\nu}{2\eta} < \frac{\nu(\eta^2+\omega^2)-2\omega\eta}{\eta(\eta^2-\omega^2)}$ one has that accommodation can only take place in the Lower Region. However, the complementary case leads to the situations where the Upper Region is reached for sufficiently large q_F^o. Rewriting leads to $\frac{4\eta\omega}{\eta^2+3\omega^2}$.

Finally, it needs to be shown that $\frac{4\eta\omega}{\eta^2+3\omega^2} \leq \frac{2\eta}{\eta+\omega}$. Rewriting gives $(\eta-\omega)^2 \geq 0$ which is always true. □

Proof of Corollary 1 Assume there exists an $X \in \mathbb{R}$ such that $q_L^{n*} \in \mathcal{S}$. Since $q_L^{acc} > q_L^{seg}$ and since we have that q_L^{acc} starts in the Lower Region, it cannot be true that $q_L^{acc} \in \mathcal{A}_1$. Accommodation is then only possible if $q_L^{acc} \in \mathcal{A}_2$. Following the same logic as in Lemma 2, we find that rewriting $\lim_{X\to\infty} q_L^{acc} \leq \tilde{q}_L^2$ leads to $\nu \geq \frac{4\omega\eta}{\eta^2+3\omega^2}$. Since this holds for $q_F^o = \tilde{q}_F^2$ this implication is true for all q_F^o. □

Proof of Proposition 8 The investment trigger follows from the smooth pasting and value matching conditions. It follows from (26) that the investment trigger exists if and only if

$$\nu^2 > 8\eta q_L^o(1-\eta(q_L^o+q_F^o).$$

Rewriting leads to the conditions with respect to q_L^o. Then these values only exist if

$$\nu^2 \leq 2(1-\eta q_F^o)^2.$$

Notice that for $\sqrt{2}(1-\eta q_F^o) \geq \nu$ the inequality always holds. Notice that for $\sqrt{2}(1-\eta q_F^o) = \nu$ the investment trigger exists for all q_L^o. □

Proof of Lemma 4 Notice that if segregation is possible, it is at least possible for $\nu = 1$ and $\omega = 0$. Under those specifications one finds

$$\begin{aligned} X_1^{seg} &= \frac{\delta(r-\mu)}{4\sqrt{\eta q^o(1-\eta q^o)} - 1} \\ X_L^{seg} &= \frac{\delta(r-\mu)(\beta^2-1)}{\beta - \sqrt{1+(\beta^2-1)4\eta q^o(1-2\eta q^o)}}. \end{aligned}$$

Then $X_L \geq X_1^{seg}$ if and only if

$$(\beta^2-1)(4\sqrt{\eta q^o(1-\eta q^o)}-1) - \beta + \sqrt{1+(\beta^2-1)4\eta q^o(1-2\eta q^o)} \geq 0. \tag{20}$$

Since for $q^o = 0$ this equation is negative and positive for $q^o = \frac{1}{4\eta}$, one can conclude that the lemma is true if the first order conditions are positive. Then q^* is defined as the value of q^o such that the inequality is binding. One can show that differentiating with respect to q^o gives,

$$\frac{1-2\eta q^o}{\sqrt{\eta q^o(1-\eta q^o)}} + \frac{1-4\eta q^o}{\sqrt{1+(\beta^2-1)4\eta q^o(1-2\eta q^o)}},$$

which is positive for all $q \in \left(0, \frac{1}{4\eta}\right]$. In order to conclude that the lemma is true for all $q \geq q^*$, perform the substitution $q^o = \frac{\gamma}{2\eta}$. To see that the inequality in (20) holds for all $\gamma \in [\frac{1}{2}, 1]$, notice that a minimum to the equation is equal to

$$(\beta^2-1)(\sqrt{3}-1) - \beta + 1.$$

Since this is an increasing function in β and since it is nonnegative for $\beta = 1$ one can conclude that indeed for all $q \geq q^*$ the lemma holds. □

Proof of Lemma 5 The same reasoning, as in the previous lemma, applies here, where

$$X_P = \frac{\delta(r-\mu)}{1-2\sqrt{\eta q^o(1-\eta q^o)}}.$$

Then $X_P \geq X_1^{seg}$ is and only if

$$4\sqrt{\eta q^o(1-\eta q^o)} - 1 \geq 1 - 2\sqrt{\eta q^o(1-\eta q^o)},$$

which is equivalent to $\eta q^o(1-\eta q^o) \geq \frac{1}{9}$. Since $q^o \leq \frac{1}{2\eta}$, the condition in the Lemma is sufficient. □

Proof of Lemma 6 Following the proof of the previous lemmas one can show that a war of attrition is present if and only if

$$\beta - \sqrt{1+(\beta^2-1)4\eta q^o(1-2\eta q^o)} + (\beta^2-1)(2\sqrt{\eta q^o(1-\eta q^o)} - 1) \geq 0.$$

Since the equation is not true for $q^o = 0$ but is for $q^o = \frac{1}{2\eta}$ we study the first order conditions,

$$\frac{1-2\eta q^o}{2\sqrt{\eta q^o(1-\eta q^o)}} - \frac{1-4\eta q^o}{\sqrt{1+(\beta^2-1)4\eta q^o(1-\eta 2q^o)}}.$$

Obviously, the derivative is positive for $q^o \geq \frac{1}{4\eta}$. For $q^o < \frac{1}{4\eta}$ we perform the following trick. Let us first substitute $q^o = \frac{\gamma}{4\eta}$ so that we are interested in the value of the derivative for all $\gamma \in [0, 1)$. This gives,

$$\frac{1-\frac{1}{2}\gamma}{\sqrt{\gamma(1-\frac{1}{4}\gamma)}} - \frac{1-\gamma}{\sqrt{1+(\beta^2-1)\gamma(1-\frac{1}{2}\gamma)}}.$$

Obviously, this term is increasing with β. Since it is positive for $\beta = 1$, one can conclude that this derivative is positive for all $\beta \geq 1$ and $\gamma \in [0, 1)$. Hence, there exists a q^{att} such that the inequality is true for all $q^o \geq q^{att}$. □

Proof of Proposition 9 Suppose firm B does not undertake investment, then firm A's best reply is to undertake investment at its trigger X_{LA}. Moreover, if firm A invests at its trigger X_{LA}, then firm B's best strategy would be to withhold investment: there is no alternative such that it could improve its position given firm A's investment at X_{LA}. As a follower firm B optimally retains and for becoming a leader firm B needs firm A to be willing to become follower at X_{LA}. However, this is never subgame perfect for firm B. $\square$

Proof of Proposition 10 Assume $X_{PA} < X_{PB}$. Suppose $x_A^* = X_{PB} - \varepsilon$, then $x_B^* = X_{PB} \in \mathfrak{R}_B(S_A)$ is an optimal reaction for firm B. Moreover,

$$x_A^* = R_A \in \mathfrak{R}_A(x_B^*) \in \mathfrak{R}_A(\mathfrak{R}_B(x_A^*)) = \mathfrak{R}_A(\mathfrak{R}_B(X_{PB} - \varepsilon)) = \mathfrak{R}_A(X_{PB}) = X_{PB} - \varepsilon = x_A^*.$$

For the case that $X_{LB} < X_{PA}$ one finds that $x_A^* = \infty$ and $x_B^* = X_{LB}$ form a Nash equilibrium,

$$x_A^* = R_A \in \mathfrak{R}_A(x_B^*) \in \mathfrak{R}_A(\mathfrak{R}_B(x_A^*)) = \mathfrak{R}_A(\mathfrak{R}_B(\infty)) = \mathfrak{R}_A(X_{LB}) = [X_{PA}, \infty).$$

And a similar proof works for the case where $X_{LA} < X_{PB}$. $\square$

Proof of Theorem 1 Arguments given in text. $\square$

8 Appendix B: Robustness

9 Appendix C

9.1 Additional analyses

The segregation strategy is feasible for the following cases, they are illustrated in Figure 8.

Proposition 7 *Let $q_F^o \in \mathcal{R}_1 \cup \mathcal{R}_2$. Define*

$$\tilde{q}_1^{seg} = \frac{8\eta - 5\omega\nu - 4\sqrt{(3\eta - 2\omega\nu)(\eta - \omega\nu) - \eta^2(\nu^2 - 1)}}{16\eta^2 - 7\omega^2}, \tag{21}$$

$$\tilde{q}_2^{seg} = \frac{8\eta - 5\omega\nu + 4\sqrt{(3\eta - 2\omega\nu)(\eta - \omega\nu) - \eta^2(\nu^2 - 1)}}{16\eta^2 - 7\omega^2}. \tag{22}$$

Then, for sufficiently large X, the leader's value function is optimized in the Segregation Region for $q_F^o > \tilde{q}_1^{seg}$ if

$$\nu \leq \frac{3\omega\eta}{\eta^2 + 2\omega^2}.$$

Moreover, the segregation strategy is feasible for $\tilde{q}_1^{seg} < q_F^o < \tilde{q}_2^{seg}$ if

$$\frac{3\omega\eta}{\eta^2 + 2\omega^2} < \nu < \frac{8\eta}{5\omega + \sqrt{16\eta^2 - 7\omega^2}}. \tag{23}$$

The segregation strategy is never feasible if

$$\nu \geq \frac{8\eta}{5\omega + \sqrt{16\eta^2 - 7\omega^2}}.$$

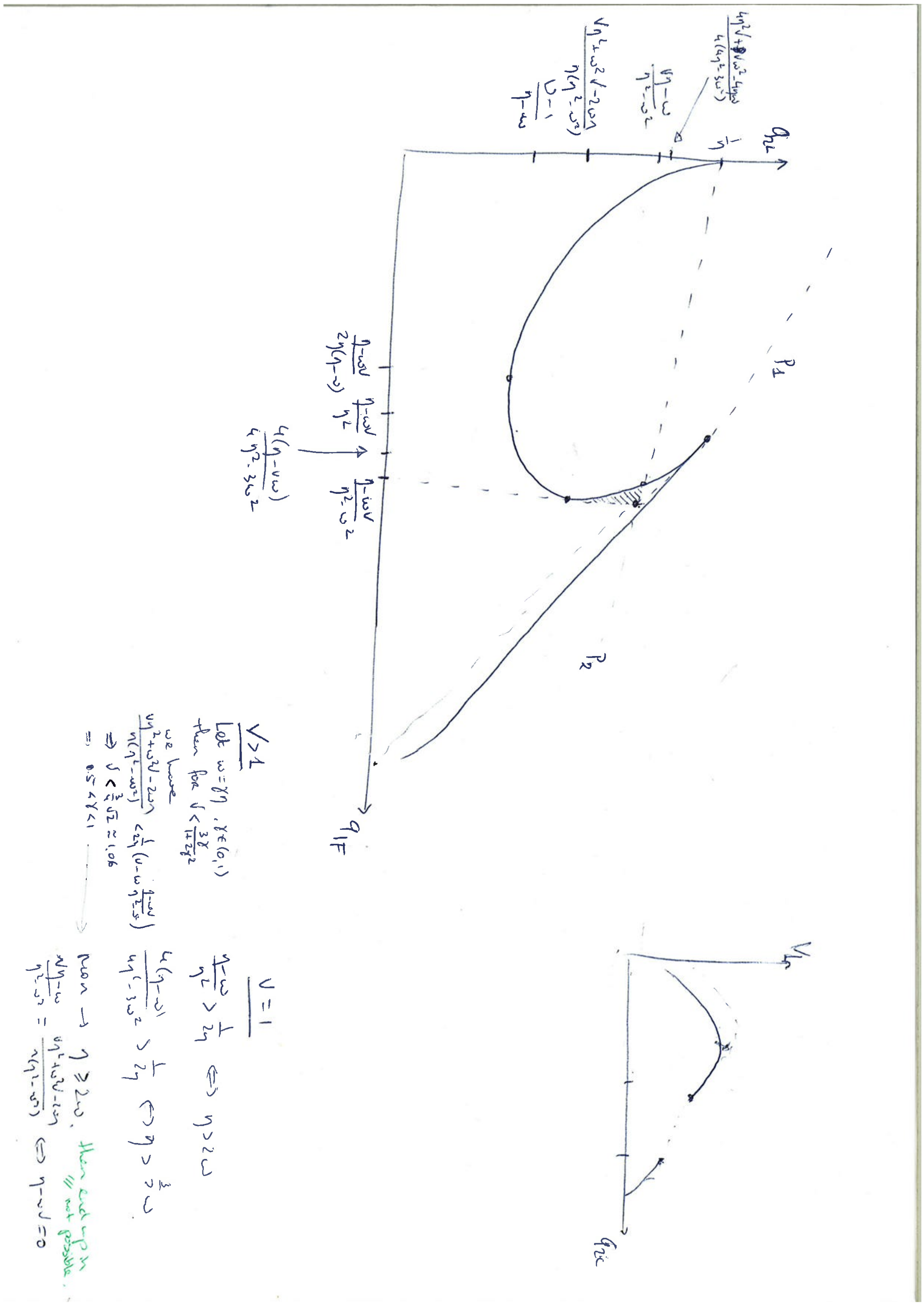


Figure 7: The information in this picture is to be incorporated in Figure 2, which will make this figure redundant.

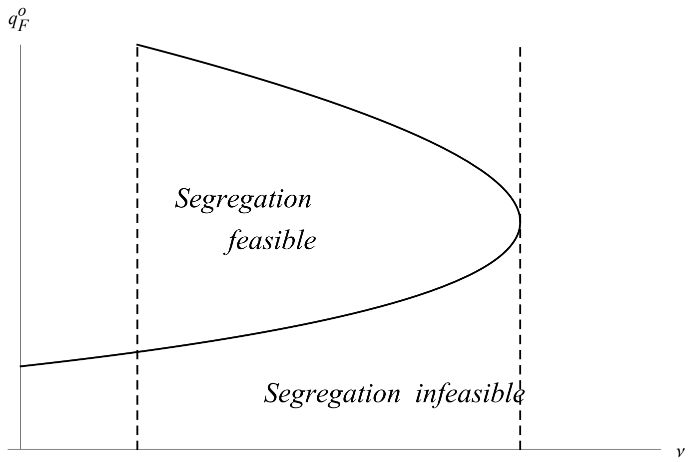


Figure 8: Regions showing where segregation is feasible, see Proposition 7.

For a large value of the vertical differentiation parameter ν, that is, for a much more profitable new product, the follower is more keen on adopting. Therefore, the leader is not able to block the second firm in that case. For values of ν close to 1, both firms are more eager to segregate since the cross-product sensitivity ω is smaller than η. However, the follower's old capacity should be sufficiently large to eliminate the situation where the follower prefers adoption simply because it currently faces very small profits due to a small capacity size. For intermediate values of ν the segregation strategy is only interesting if the follower's capacity is not too large, so that it would like to change it's capacity for a smaller size, not too small, so that it would like to adopt to increase profits.

For the accommodation strategy the following Proposition holds, Figure 9 illustrates these values.

Proposition 8 *Let $q_F^o \in \mathcal{R}_1 \cup \mathcal{R}_2$. Define*

$$\tilde{q}_1^{acc} = \frac{1}{4\eta}\left[2 - \frac{\omega\nu}{\eta} - \sqrt{\left(2 - \frac{\omega\nu}{\eta}\right)^2 - \nu^2}\right] > 0, \tag{24}$$

$$\tilde{q}_2^{acc} = \frac{1}{4\eta}\left[2 - \frac{\omega\nu}{\eta} + \sqrt{\left(2 - \frac{\omega\nu}{\eta}\right)^2 - \nu^2}\right]. \tag{25}$$

Then the accommodation strategy is feasible for $X > X_1^{acc}$ in the Lower Region for $q_F^o < \tilde{q}_1^{acc}$ and $q_F^o > \tilde{q}_2^{acc}$ if

$$\frac{4\eta\omega}{\eta^2 + 3\omega^2} < \nu \leq \frac{2\eta}{\eta + \omega}.$$

Moreover, the accommodation strategy is feasible for $X > X_1^{acc}$ in the Lower Region for $q_F^o < \tilde{q}_1^{acc}$ and in the Upper Region for $q_F^o > \tilde{q}_2^{acc}$ if

$$\nu \leq \frac{4\eta\omega}{\eta^2 + 3\omega^2}.$$

For $\tilde{q}_1^{acc} \leq q_F^o \leq \tilde{q}_2^{acc}$ accommodation is never feasible, i.e. $X_1^{acc} \to \infty$.

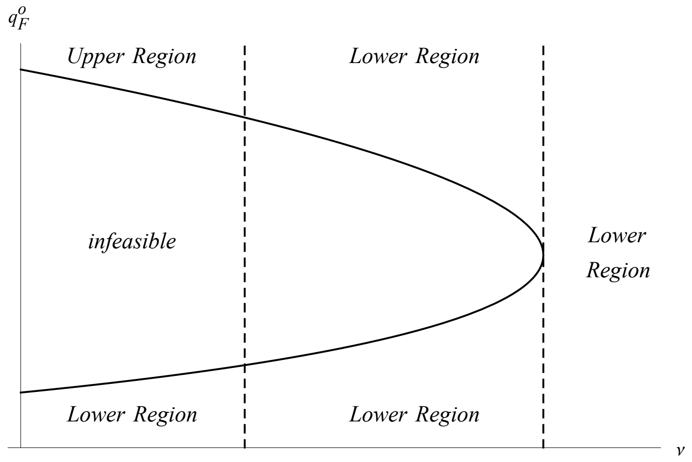


Figure 9: Regions showing where accommodation is feasible.

For small values of ν accommodation is only interesting to the leader when the follower has a large share on the old market. In that case the follower would exchange a large capacity on the old market for a smaller capacity on the new market. Conversely, when the follower is very small on the old market it prefers to quickly renew its capacity by undertaking the investment. This makes the investment trigger of the follower relatively small and it is then harder for the leader to deter the other firm.

As a side note, one can show that $\tilde{q}_2^{acc} \leq \frac{\eta-\omega\nu}{\eta^2-\omega^2}$ if and only if $\nu \leq \frac{4\eta\omega}{\eta^2+3\omega^2}$. This means that all cases presented in the Proposition exist.

9.2 Optimal investment under accommodation

At investment, the leader obtains value V_L^{acc}. The value before investment is given by

$$F_L^{acc}(X; q_L^o, q_F^o) = A_L^{acc} X^\beta + \frac{X}{r-\mu} q_L^o (1 - \eta(q_L^o + q_F^o)).$$

The optimal investment trigger is then given by,

$$X_L^{acc} = \frac{\delta(r-\mu)}{\beta-1} \frac{\beta\nu + \sqrt{\nu^2 + (\beta^2-1)8\eta q_L^o(1-\eta(q_L^o+q_F^o))}}{\nu^2 - 8\eta q_L^o(1-\eta(q_L^o+q_F^o))}. \tag{26}$$

Proposition 9 *Let $q_F^o \in \mathcal{R}_1 \cup \mathcal{R}_2$ such that $\sqrt{2}(1-\eta q_F^o) > \nu$. Under the accommodation strategy, the leader invests at X_L^{acc} if and only if*

$$q_L^o < \frac{1-\eta q_F^o - \sqrt{(1-\eta q_F^o)^2 - \frac{1}{2}\nu^2}}{2\eta} \quad or \quad q_L^o > \frac{1-\eta q_F^o + \sqrt{(1-\eta q_F^o)^2 - \frac{1}{2}\nu^2}}{2\eta}.$$

For $\sqrt{2}(1-\eta q_F^o) \leq \nu$ the investment trigger exists for all q_L^o.

For small values of ν the trigger does not exist and investment under accommodation is always delayed. Small ν imply a relatively less profitable market. It would require a larger value of X to undertake investment. However, for larger values of X one finds that the optimal investment size increases sufficiently to delay the follower's investment. In the end, the optimal moment to undertake simultaneous investment is always delayed.

Not only the existence of the trigger depends on the capacities on the old market, also the relative position of the trigger with respect to X_1^{acc} is fully dependent on q_L^o. Notice that X_1^{acc} is not dependent on the value of q_L^o, which is not the case for X_L^{acc}.