ANALYTICAL INVESTMENT CRITERIA

FOR SUBSIDIZED ENERGY FACILITIES

Roger Adkins*

Bradford University School of Management

Dean Paxson**

Manchester Business School

Submitted to the Real Options Conference, Norway, 2016

25 Jan 2016

Acknowledgements: we thank Alcino Azevedo, Trine Boomsma, Michael Flanagan, Kristin Linnerud, Paulo Pereira, and Artur Rodrigues for valuable comments on earlier versions.

*Bradford University School of Management, Emm Lane, Bradford BD9 4JL, UK.

r.adkins@bradford.ac.uk

+44 (0)1274233466.

**Manchester Business School, University of Manchester, Manchester, M15 6PB, UK.

dean.paxson@mbs.ac.uk

+44(0)1612756353. Corresponding author.

ANALYTICAL INVESTMENT CRITERIA

FOR SUBSIDIZED ENERGY FACILITIES

Abstract

We derive the optimal investment timing and real option value for an investment opportunity in a subsidized energy facility with possible price, quantity and subsidy uncertainty. The general model is for three stochastic variables, from which several other models can be easily derived, where some (or all) of the variables are constant or deterministic. Simple analytical solutions are suggested for all of the models. Sensitivity of the thresholds justifying immediate investment and the real option values to changes in the important variables are also provided. Notable findings are that the initial subsidies justifying immediate investment) are lowered by subsidies unlikely to be withdrawn, but decreasing over time , and by reducing the output price volatility, and interest rates.

JEL Classifications: D81, G31, Q42, Q48

Keywords: government or customer subsidies, investment incentives, real option values, thresholds.

1 Introduction

What type of subsidy ("S") will have the greatest effect on reducing the subsidy threshold that justifies immediate investment? We assume that in evaluating a perpetual opportunity to invest in an energy facility possibly subsidized, an investor uses modern investment criteria, allowing for volatility and drift over time of the market price of energy ("P"), and the quantity produced ("Q"). She may then consider what volatility and drift characteristics of proposed subsidies (or other government guarantees) justify commencing an investment expenditure, given the physical characteristics of the energy facility.

Our approach is consistent with other multi-factor models in solving several equations simultaneously to derive the solutions, usually given the assumption that the price and/or quantity threshold is equal to the current price or quantity. However, we provide novel analytical solutions using a simple quadratic equation (with complex but identifiable ingredients like expected volatility, drift and correlations for the energy factors). We also illustrate that the analytical solution actually solves the partial differential equation which governs the optimal investment timing, showing the deltas and gammas for the price, quantity and subsidy elements. Finally we identify some interesting aspects of the sensitivity of both the thresholds and the real option value of investment opportunities, which deserve the focus of both investors and governments.

There are numerous examples of government subsidies provided to encourage early investments in renewable energy, see Lesser and Su. (2008), Yang et al. (2008), Blyth et al. (2009), Couture and Gagnon (2009), Kettunen et al.(2011), Borenstein (2012), Mosiño (2012), Linerrud et al. (2014), and Abolhosseini and Heshimati (2014).

Lesser and Su (2008) suggest a two-part FiT (so called Feed-in-Tariffs) scheme where potential investors specify what level of FiT they will accept through an auction process, along with the acceptable duration of fixed payments, both possibly adjusted for an actual capacity factor. Also some FiT reduce payments over time. Couture and Gagnon (2009) review some seven types of FiTs, divided into market independent and dependent policies. The common independent scheme is a fixed, minimum S (replacing P), perhaps with a full or partial inflation adjustment. A variant on this scheme is a front-loaded model with specified reductions over time, "stepped tariff designs". The "spot gap tariff" is a variable subsidy equal to the difference between P and S threshold, so a fixed revenue is obtained by the investor. Three market dependent schemes are: (i) a constant premium over or under P, (ii) a "corridor" premium, which is a subsidy declining as P increases, and (iii) S= %P, where the % could exceed or be below one. Abolhosseini and Heshimati (2014) review some 55 articles on subsidies, divided into FiT, tax incentives, and RPS (so-called renewable portfolio standard). RPS is quantity-based with tradeable certificates awarded for qualifed units of renewable energy produced. We discuss how some of these subsidy design varieties can be incorporated into our analytical models at the end.

The nearest papers apparently to ours offering some models are Boomsma et al. (2012), Abadie and Chamorro (2013), Adkins and Paxson (2014), Ritzenhofen and Spinler (2015), and Boomsma and Linerrud (2015), but their solutions are either numerical (and often more realistic) or based on somewhat different assumptions and objectives. We note in the appropriate model section, or the numerical illustrations, some incidents where a solution for the models of some of these authors can be analytical.

We consider that the instantaneous cash flow from a facility is the respective commodity price of the output times the quantity produced, and either there is no operating cost, or there is a fixed operating cost that can be incorporated into the investment cost. There are no other options embedded in the facility such as expansion, contraction, suspension or abandonment. We assume that the opportunity for making or acquiring the investment is perpetual, but the lifetime of the facility is finite, there are no taxes or competition, and facility construction or acquisition is instantaneous. Moreover, the typical assumptions of real options theory apply, with drifts, interest rates, convenience yields, volatilities and correlations constant over time, ignoring the seasonality and unreliability of prices and quantities. Many of these strong assumptions may be required for an analytical solution. Relaxation of some of these assumptions may lead to greater realism, but may then require much more complex analytical solutions or numerical solutions[1].

We assume the primary government objective is to reduce the subsidy threshold that justifies making an irreversible, instantaneous investment, given the market price of energy and the physical aspects of the facility including output and construction cost, instead of creating a high real option value for any allowable prospective facility or concession. There are some eight different combinations of P, S, Q being constant or stochastic with the most general model assuming all $\tilde{P}, \tilde{Q}, \tilde{S}$ are stochastic, Model I, and the net present value model, assuming all $\bar{P}, \bar{Q}, \bar{S}$ are constant or deterministic.

[1] For instance, a positive or negative abandonment value would require another set of equations.

$\tilde{P},\tilde{S},\tilde{Q}, MI$

$\tilde{P},\tilde{S},\bar{Q}, MII$

$\tilde{P},\bar{S},\tilde{Q}, MIII$

$\tilde{P},\bar{S},\bar{Q}, MIV$

$\bar{P},\tilde{S},\tilde{Q}$

$\bar{P},\bar{S},\tilde{Q}$

$\bar{P},\tilde{S},\bar{Q}$

$\bar{P},\bar{S},\bar{Q}, NPV$

The next section suggests an analytical solution for the general model. Then easy analytical solutions are given for the other combinations where P is stochastic, and where there is no subsidy, excluding the three scenarios where P is constant and S and/or Q are not. The third section compares the subsidy or price thresholds and real option values using comparable base parameter values, and illustrates the sensitivity of these models to changes in some important variables such as the level, volatility and drifts of the price, quantity and subsidy, and interest rates. The final section concludes.

2 Models

2.1 Model I Stochastic Price, Subsidy and Quantity

We consider a perpetual opportunity to construct a renewable energy facility, such as a hydro-electric plant or a solar PV farm or another renewable energy facility at a fixed investment cost K. This investment cost is treated as irreversible or irrecoverable once incurred. The value of this investment opportunity, denoted by F_1, depends on the amount of output sold per unit of time, denoted by Q, the market price per unit of output, denoted by P [2], and the subsidy per output unit, S. In the general model, all of these variables are assumed to be stochastic and are assumed to follow geometric Brownian motion processes (gBm):

$$\mathrm{d}X = \alpha_X X\mathrm{d}t + \sigma_X X\mathrm{d}Z \tag{1}$$

for $X \in \{P, S, Q\}$, where α denotes the instantaneous drift parameter, σ the instantaneous volatility, and $\mathrm{d}Z$ the standard Wiener process. Potential correlation between the variables is represented by ρ. It may be reasonable to assume the price per unit of output follows such a stochastic process if it is a traded commodity, while treating the amount of output generated per unit of time as stochastic may reflect the random nature of demand or supply.

Assuming risk neutrality and applying Ito's lemma, the partial differential equation (PDE) representing the value to invest for an inactive firm with an appropriate perpetual investment opportunity (based on perhaps approval for the facility or a concession for infrastructure) is:

$$\begin{aligned}
&\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_1}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_1}{\partial Q^2} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_1}{\partial S^2} \\
&+ PQ\rho_{PQ}\sigma_P\sigma_Q \frac{\partial^2 F_1}{\partial P \partial Q} + PS\rho_{PS}\sigma_P\sigma_S \frac{\partial^2 F_1}{\partial P \partial S} + QS\rho_{QS}\sigma_Q\sigma_S \frac{\partial^2 F_1}{\partial Q \partial S} \\
&+ \theta_P P \frac{\partial F_1}{\partial P} + \theta_Q Q \frac{\partial F_1}{\partial Q} + \theta_S S \frac{\partial F_1}{\partial S} - rF_1 = 0.
\end{aligned} \tag{2}$$

where θ_X denote the risk-neutral drift rates and r the risk-free rate, (θ=r-α)[3]. Following Adkins and Paxson (2011), when P,Q, or S are below $\hat{P}, \hat{Q}, \hat{S}$ that justify immediate investment, the solution to (2) is:

[2] Output could be electricity, biodiesel, ethanol or directly useful energies (like heat), each of which are likely to follow somewhat different diffusion processes. Sometimes these specific diffusion processes will not enable analytical solutions, see Mosiño (2012).

$$ROV_1 = F_1 = A_1 P^{\beta_1} Q^{\gamma_1} S^{\eta_1}. \tag{3}$$

where β_1 , γ_1 and η_1 are the power parameters for this option value function. Since there is an incentive to invest when P, Q and S are sufficiently high but a disincentive when these are sufficiently low, we expect that all power parameter values are positive. Also, the parameters are linked through the characteristic root equation found by substituting (3) in (2):

$$\begin{aligned} &Q(\beta_1,\gamma_1,\eta_1) = \tfrac{1}{2}\sigma_P^2\beta_1(\beta_1-1)+\tfrac{1}{2}\sigma_Q^2\gamma_1(\gamma_1-1)+\tfrac{1}{2}\sigma_S^2\eta_1(\eta_1-1)+ \\ &\rho_{PQ}\sigma_P\sigma_Q\beta_1\gamma_1+\rho_{PS}\sigma_P\sigma_S\beta_1\eta_1+\rho_{QS}\sigma_Q\sigma_S\gamma_1\eta_1 \\ &+\theta_P\beta_1+\theta_Q\gamma_1+\theta_S\eta_1-r=0 \end{aligned}. \tag{4}$$

We assume that there is no operational flexibility once the investment to construct the plant has been made. After the investment, the plant generates revenue equaling $PQ + SQ$, with the present value factor of parts of this net revenue denoted $k_{P,}$ k_Q and k_S (no operating costs or taxes) (life assumed to be T=20 years in the base case)[4].

$$k_P = \frac{1-e^{-(r-\theta_P)*T}}{(r-\theta_P)}, k_{PQ} = \frac{1-e^{-(r-\theta_P-\theta_Q)*T}}{(r-\theta_P-\theta_Q)} \tag{5}$$

$$k_Q = \frac{1-e^{-(r-\theta_Q)*T}}{(r-\theta_Q)} \tag{6}$$

$$k_S = \frac{1-e^{-(r-\theta_S)*T}}{(r-\theta_S)}, k_{SQ} = \frac{1-e^{-(r-\theta_S-\theta_Q)*T}}{(r-\theta_S-\theta_Q)}, \tag{7}$$

The value matching relationship, when the real option value upon exercise is equal to the net present value of the investment (NPV), is:

$$A_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} = k_{PQ}\hat{P}\hat{Q} + k_{SQ}\hat{S}_1\hat{Q} - K \tag{8}$$

The three associated smooth pasting conditions can be expressed as:

$$\beta_1 A_1\hat{P}^{\beta_1}\hat{Q}^{\gamma_1}\hat{S}_1^{\eta_1} = k_{PQ}\hat{P}\hat{Q} \tag{9}$$

---

[3] In sensitivity analysis, we assume θs are not affected by changes in the specific parameter value considered.
[4] This is the methodology in Boomsma and Linnerud (2015).

$$\gamma_1 A_1 \hat{P}^{\beta_1} \hat{Q}^{\gamma_1} \hat{S}_1^{\eta_1} = k_{PQ} \hat{P} \hat{Q} + k_{SQ} \hat{S}_1 \hat{Q} \tag{10}$$

$$\eta_1 A_1 \hat{P}^{\beta_1} \hat{Q}^{\gamma_1} \hat{S}_1^{\eta_1} = k_{SQ} \hat{S}_1 \hat{Q} \tag{11}$$

A quasi-analytical solution to the set of five equations 4-8-9-10-11 for 7 unknowns $\hat{P}, \hat{Q}, \hat{S}_1, \beta_1, \gamma_1, \eta_1, A_1$ is obtained by assuming $\hat{P} = P, \hat{Q} = Q$ as in Adkins and Paxson (2014), and then finding $\hat{S}_1, \beta_1, \gamma_1, \eta_1, A_1$. An analytical solution is obtained by recognizing that:

$$A_1 = k_{PQ} \hat{P} \hat{Q} / \beta_1 \hat{P}^{\beta_1} \hat{Q}^{\gamma_1} \hat{S}_1^{\eta_1} \tag{12}$$

and

$$\hat{S}_1 = \eta_1 k_{PQ} \hat{P} / \beta_1 k_{SQ} \tag{13}$$

This implies that

$$\gamma_1 = \beta_1 + \eta_1 \tag{14}$$

Eliminating A from (8) yields:

$$\beta_1 = k_{PQ} \hat{P} \hat{Q} / (k_{PQ} \hat{P} \hat{Q} + k_{SQ} \hat{S}_1 \hat{Q} - K) \tag{15}$$

So

$$\eta_1 = 1 + \beta_1 \left(\frac{K}{k_{PQ} \hat{P} \hat{Q}} - 1 \right) \tag{16}$$

Eliminating γ_1 and η_1 from the characteristic root equation (4) yields the quadratic equation:

$$Q(\beta_1) = \beta_1^2 \{a\} + \beta_1 \{b\} - \{c\} = 0 \tag{17}$$

$$\begin{aligned} a = \Big\{ & \tfrac{1}{2}\sigma_P^2 - \rho_{PS}\sigma_P\sigma_S + \tfrac{1}{2}\sigma_S^2 \\ & + \frac{K^2}{2\hat{P}^2\hat{Q}^2 k_{PQ}^2}[\sigma_Q^2 + 2\rho_{QS}\sigma_Q\sigma_S + \sigma_S^2] \\ & + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\rho_{PQ}\sigma_P\sigma_Q + \rho_{PS}\sigma_P\sigma_S - \rho_{QS}\sigma_Q\sigma_S - \sigma_S^2] \Big\} \end{aligned}$$

$$\begin{aligned} b = \Big\{ & \theta_P - \theta_S - \tfrac{1}{2}\sigma_P^2 - \tfrac{1}{2}\sigma_S^2 + \rho_{PQ}\sigma_P\sigma_Q + \rho_{PS}\sigma_P\sigma_S - \rho_{QS}\sigma_Q\sigma_S \\ & + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\theta_Q + \theta_S + \frac{\sigma_Q^2}{2} + 2\rho_{QS}\sigma_Q\sigma_S + \frac{\sigma_S^2}{2}] \Big\} \end{aligned}$$

$$c = -\left\{ r - \theta_Q - \theta_S - \rho_{QS}\sigma_Q\sigma_S \right\}$$

This equation has the simple quadratic solution:

$$\beta_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \tag{18}$$

2.2 Model II

Stochastic Price and Subsidy with a Deterministic Quantity

We now modify the analysis to consider the impact on the investment decision of a permanent but uncertain government subsidy, denoted by S, but where the output Q sold per unit of time is deterministic.

The PDE is:

$$\begin{aligned}&\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_2}{\partial P^2} + \frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 F_2}{\partial S^2} \\ &+PS\rho_{PS}\sigma_P\sigma_S \frac{\partial^2 F_2}{\partial P \partial S} + \theta_P P \frac{\partial F_2}{\partial P} + \theta_Q Q \frac{\partial F_2}{\partial Q} + \theta_S S \frac{\partial F_2}{\partial S} - rF_2 = 0.\end{aligned} \tag{19}$$

where θ_X denote the risk-neutral drift rates and r the risk-free rate, (θ=r-α). The solution to (19) is:

$$ROV_2 = F_2 = A_2 P^{\beta_2} Q^{\gamma_2} S^{\eta_2}\ . \tag{20}$$

where β_2 , γ_2 and η_2 are the power parameters for this option value function (allowing for a deterministic quantity). We expect that all power parameter values are positive. Also, the parameters are linked through the characteristic root equation found by substituting (20) in (19):

$$\begin{aligned}&Q(\beta_2,\gamma_2,\eta_2) = \tfrac{1}{2}\sigma_P^2\beta_2(\beta_2-1) + \tfrac{1}{2}\sigma_S^2\eta_2(\eta_2-1) + \\ &+\rho_{PS}\sigma_P\sigma_S\beta_2\eta_2 + \theta_P\beta_2 + \theta_Q\gamma_2 + \theta_S\eta_2 - r = 0 \qquad .\end{aligned} \tag{21}$$

The value matching relationship becomes:

$$A_2\hat{P}^{\beta_2}\hat{Q}^{\gamma_2}\hat{S}_2^{\eta_2} = k_{PQ}\hat{P}\hat{Q} + k_{SQ}\hat{S}_2\hat{Q} - K \tag{22}$$

Eliminating γ_2 and η_2 from the characteristic root equation (21) yields the quadratic equation:

$$Q(\beta_2) = \beta_2^2\{a\} + \beta_2\{b\} - \{c\} = 0 \tag{23}$$

$$a = \left\{\tfrac{1}{2}\sigma_P^2 - \rho_{PS}\sigma_P\sigma_S + \tfrac{1}{2}\sigma_S^2 + \frac{K^2}{2\hat{P}^2\hat{Q}^2k_{PQ}^2}[\sigma_S^2] + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\rho_{PS}\sigma_P\sigma_S - \sigma_S^2]\right\}$$

$$b = \left\{\theta_P - \theta_S - \tfrac{1}{2}\sigma_P^2 - \tfrac{1}{2}\sigma_S^2 + \rho_{PS}\sigma_P\sigma_S + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\theta_Q + \theta_S + \frac{\sigma_S^2}{2}]\right\}$$

$$c = -\left\{r - \theta_Q - \theta_S\right\}$$

The solution to this equation is again:

$$\beta_2 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{24}$$

The difference between (17) and (23) is that the Q volatility has been eliminated, but not the θ_Q.

2.3 Model III

Stochastic Price and Quantity with a Permanent Deterministic Subsidy

We modify the analysis to consider the impact on the investment decision of a permanent deterministic government subsidy, denoted by S, but where the output Q and market price P are stochastic.

The PDE is:

$$\frac{1}{2}\sigma_P^2P^2\frac{\partial^2 F_3}{\partial P^2} + \frac{1}{2}\sigma_Q^2Q^2\frac{\partial^2 F_3}{\partial Q^2} + PQ\rho_{PQ}\sigma_P\sigma_Q\frac{\partial^2 F_3}{\partial P\partial Q} + \theta_P P\frac{\partial F_3}{\partial P} + \theta_Q Q\frac{\partial F_3}{\partial Q} + \theta_S S\frac{\partial F_3}{\partial S} - rF_3 = 0. \tag{25}$$

The solution to (25) is:

$$ROV_3 = F_3 = A_3 P^{\beta_3} Q^{\gamma_3} S^{\eta_3}. \tag{26}$$

where β_3 , γ_3 and η_3 are the power parameters for this option value function. The parameters are linked through the characteristic root equation found by substituting (26) in (25):

$$\begin{aligned} &Q(\beta_3,\gamma_3,\eta_3) = \tfrac{1}{2}\sigma_P^2\beta_3(\beta_3-1) + \tfrac{1}{2}\sigma_Q^2\gamma_3(\gamma_3-1) + \\ &\rho_{PQ}\sigma_P\sigma_Q\beta_3\gamma_3 + \theta_P\beta_3 + \theta_Q\gamma_3 + \theta_S\eta_3 - r = 0 \end{aligned} \quad . \tag{27}$$

Eliminating γ_3 and η_3 from the characteristic root equation yields the quadratic equation:

$$Q(\beta_3) = \beta_3^2\{a\} + \beta_3\{b\} - \{c\} = 0 \tag{28}$$

$$a = \left\{ \tfrac{1}{2}\sigma_P^2 + \frac{K^2}{2\hat{P}^2\hat{Q}^2 k_{PQ}^2}[\sigma_Q^2] + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\rho_{PQ}\sigma_P\sigma_Q] \right\}$$

$$b = \{\theta_P - \theta_S - \tfrac{1}{2}\sigma_P^2 + \rho_{PQ}\sigma_P\sigma_Q + \frac{K}{\hat{P}\hat{Q}k_{PQ}}[\theta_Q + \theta_S + \frac{\sigma_Q^2}{2}]\}$$

$$c = -\left\{ r - \theta_Q - \theta_S \right\}$$

The solution to this equation is again: $$\beta_3 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{29}$$

2.4 Model IV

Stochastic Price, Deterministic Quantity with a Permanent Deterministic Subsidy

We modify the analysis to consider the impact on the investment decision of a permanent deterministic government subsidy, denoted by S, with a deterministic output Q, when only the market price P is stochastic.

The PDE is: $$\frac{1}{2}\sigma_P^2 P^2\frac{\partial^2 F_4}{\partial P^2}+\theta_P P\frac{\partial F_4}{\partial P}+\theta_Q Q\frac{\partial F_4}{\partial Q}+\theta_S S\frac{\partial F_4}{\partial S}-rF_4=0. \qquad (30)$$

The solution to (30) is:

$$ROV_4=F_4=A_4P^{\beta_4}Q^{\gamma_4}S^{\eta_4}. \qquad (31)$$

where β_4 , γ_4 and η_4 are the power parameters for this option value function. The parameters are linked through the characteristic root equation found by substituting (31) in (30):

$$Q(\beta_4,\gamma_4,\eta_4)=\tfrac{1}{2}\sigma_P^2\beta_4(\beta_4-1)+\theta_P\beta_4+\theta_Q\gamma_4+\theta_S\eta_4-r=0. \qquad (32)$$

Eliminating γ_4 and η_4 from the characteristic root equation (32) yields the quadratic equation:

$$Q(\beta_4)=\beta_4^2\{a\}+\beta_4\{b\}-\{c\}=0 \qquad (33)$$

$$a=\{\frac{\sigma_P^2}{2}\}\quad b=\{\theta_P-\tfrac{1}{2}\sigma_P^2\}\qquad c=-\left\{r-\theta_Q-\theta_S\right\}$$

The solution to this equation is: $$\beta_4=\frac{-b+\sqrt{b^2-4ac}}{2a} \qquad (34)$$

2.5 Model V

No Subsidy, Stochastic P and Q

We consider now the case where there is no possibility ever of a subsidy, as in Adkins and Paxson (2014), (their Model I), following Paxson and Pinto (2005). The PDE is:

:

$$
\begin{aligned}
&\frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 F_5}{\partial P^2} + \frac{1}{2}\sigma_Q^2 Q^2 \frac{\partial^2 F_5}{\partial Q^2} \\
&+ PQ\rho_{PQ}\sigma_P\sigma_Q \frac{\partial^2 F_5}{\partial P \partial Q} + \theta_P P \frac{\partial F_5}{\partial P} + \theta_Q Q \frac{\partial F_5}{\partial Q} - rF_5 = 0.
\end{aligned} \tag{35}
$$

The solution to (35) is:

$$ROV_5 = F_5 = A_5 P^{\beta_5} Q^{\gamma_5}\ . \tag{36}$$

where β_5 and γ_5 are the power parameters for this option value function. The parameters are linked through the characteristic root equation found by substituting (36) in (35):

$$
\begin{aligned}
&Q(\beta_5, \gamma_5) = \tfrac{1}{2}\sigma_P^2 \beta_5(\beta_5 - 1) + \tfrac{1}{2}\sigma_Q^2 \gamma_5(\gamma_5 - 1) \\
&+ \rho_{PQ}\sigma_P\sigma_Q \beta_5\gamma_5 + \theta_P\beta_5 + \theta_Q\gamma_5 - r = 0 \qquad .
\end{aligned} \tag{37}
$$

The value matching relationship becomes:

$$A_5 \hat{P}_5^{\beta_5} \hat{Q}^{\gamma_5} = k_{PQ} \hat{P}_5 \hat{Q} - K \tag{38}$$

The two associated smooth pasting conditions can be expressed as:

$$\beta_5 A_5 \hat{P}_5^{\beta_5} \hat{Q}^{\gamma_5} = k_{PQ} \hat{P}_5 \hat{Q} \tag{39}$$

$$\gamma_5 A_5 \hat{P}_5^{\beta_5} \hat{Q}^{\gamma_5} = k_{PQ} \hat{P}_5 \hat{Q} \tag{40}$$

This implies that

$$\gamma_5 = \beta_5 \tag{41}$$

$$A_5 = k_{PQ} \hat{P}_5^{1-\beta_5} \hat{Q}^{1-\gamma_5} \,/\, \beta_5 \tag{42}$$

$$\hat{P}_5 \hat{Q} = \frac{\beta_5 K k_{PQ}}{(\beta_5 - 1)} \tag{43}$$

Eliminating γ_5 from the characteristic root equation (37) yields the quadratic equation:

$$Q(\beta_5) = \beta_5^2\{a\} + \beta_5\{b\} - \{c\} = 0 \tag{44}$$

$$a=\left\{\tfrac{1}{2}\sigma_P^2+\tfrac{1}{2}\sigma_Q^2+2\rho_{PQ}\sigma_P\sigma_Q\right\}$$
$$b=\{\theta_P+\theta_Q+\rho_{PQ}\sigma_P\sigma_Q\}$$
$$c=-\left\{r\right\}$$

The solution to this equation is: $$\beta_5=\frac{-(b-\frac{1}{2}a^2)+\sqrt{(b-\frac{1}{2}a^2)^2-2ac}}{a} \qquad (45)$$

as in Paxson and Pinto (2005).

2.6 Model VI

Stochastic Price and Quantity with a Retractable Stochastic Subsidy

We now suggest an extension of the Adkins and Paxson (2014) (their Model III) for a subsidy that can be withdrawn at any time, and determine its effects on the threshold levels for S, where the facility is finite. We assume that once the subsidy is withdrawn, it will never again be provided.

The value of the investment option in the presence of a subsidy, but when there is a possibility of an immediate withdrawal, is F_6, and in the absence of a subsidy by F_5, as in the previous sub-section. We assume that the subsidy withdrawal is well explained by a Poisson process with a constant intensity factor, denoted by λ. The change in the option value conditional on the subsidy withdrawal occurring is $F_5\left(P,Q\right)-F_6\left(P,Q,S\right)$, so the expected change is given by:

$$\left\{F_5\left(P,Q\right)-F_6\left(P,Q,S\right)\right\}\lambda \mathrm{d}t+\left\{0\right\}\left(1-\lambda \mathrm{d}t\right)=\lambda\left\{F_5\left(P,Q\right)-F_6\left(P,Q,S\right)\right\}\mathrm{d}t\,.$$

The risk-neutral valuation relationship for F_6 is:

$$\begin{aligned}&\frac{1}{2}\sigma_P^2P^2\frac{\partial^2F_6}{\partial P^2}+\frac{1}{2}\sigma_Q^2Q^2\frac{\partial^2F_6}{\partial Q^2}+\frac{1}{2}\sigma_S^2S^2\frac{\partial^2F_6}{\partial S^2}\\&+PQ\rho_{PQ}\sigma_P\sigma_Q\frac{\partial^2F_6}{\partial P\partial Q}+PS\rho_{PS}\sigma_P\sigma_S\frac{\partial^2F_6}{\partial P\partial S}+QS\rho_{QS}\sigma_Q\sigma_S\frac{\partial^2F_6}{\partial Q\partial S}\\&+\theta_PP\frac{\partial F_6}{\partial P}+\theta_QQ\frac{\partial F_6}{\partial Q}+\theta_SS\frac{\partial F_6}{\partial S}-(r+\lambda)F_6=0.\end{aligned}\tag{46}$$

The solution to (46) adopts the form:

$$F_6=A_6P^{\beta_6}Q^{\gamma_6}S^{\eta_6}+A_5P^{\beta_5}Q^{\gamma_5}\,,\tag{47}$$

where the parameters β_5 and γ_5 are specified by $Q\left(\beta_5,\gamma_5\right)=0$, with $\beta_5=\gamma_5$ (41), while β_6,γ_6 and η_6 are related through the characteristic root equation:

$$\begin{aligned}&Q\left(\beta_6,\gamma_6,\eta_6\right)=\tfrac{1}{2}\sigma_P^2\beta_6\left(\beta_6-1\right)+\tfrac{1}{2}\sigma_Q^2\gamma_6\left(\gamma_6-1\right)+\tfrac{1}{2}\sigma_S^2\eta_6\left(\eta_6-1\right)+\\&\rho_{PQ}\sigma_P\sigma_Q\beta_6\gamma_6+\rho_{PS}\sigma_P\sigma_S\beta_6\eta_6+\rho_{QS}\sigma_Q\sigma_S\gamma_6\eta_6\\&+\theta_P\beta_6+\theta_Q\gamma_6+\theta_S\eta_6-(r+\lambda)=0\end{aligned}\tag{48}$$

For any feasible values of P, Q and S, the valuation function F_6 exceeds F_5 because the coefficient A_6 is positive. This implies that the option value to invest is always greater in the presence of a government subsidy that may be withdrawn unexpectedly than in its absence, which suggests that a subsidy, even one having an unexpected withdrawal, comparatively hastens the investment commitment, while it is comparatively deferred in its absence.

If the subsidy is present, then the present value of the plant is $k_{PQ}\hat{P}\hat{Q}+k_{SQ}\hat{S}_6\hat{Q}$, and if absent, then $k_{PQ}\hat{P}\hat{Q}$, so the net present value following the investment commitment is:

$$k_{PQ}\hat{P}\hat{Q}+(1-\lambda)k_{SQ}\hat{S}_6\hat{Q}. \tag{49}$$

The thresholds signaling investment for a retractable subsidy are denoted by $\hat{P}_6=P$, $\hat{Q}_6=Q$ (see assumptions below) and $\hat{S}_6$.

The value matching condition becomes:

$$A_6\hat{P}^{\beta_6}\hat{Q}^{\gamma_6}\hat{S}_6^{\eta_6}+A_5\hat{P}^{\beta_5}\hat{Q}^{\gamma_5}=k_{PQ}\hat{P}\hat{Q}+(1-\lambda)k_{SQ}\hat{S}_6\hat{Q}-K. \tag{50}$$

The two associated smooth pasting conditions are, respectively:

$$\beta_6A_6\hat{P}^{\beta_6}\hat{Q}^{\gamma_6}\hat{S}_6^{\eta_6}+\beta_5A_5\hat{P}^{\beta_5}\hat{Q}^{\gamma_5}=k_{PQ}\hat{P}\hat{Q}\ , \tag{51}$$

$$\gamma_6A_6\hat{P}^{\beta_6}\hat{Q}^{\gamma_6}\hat{S}_6^{\eta_6}+\gamma_5A_5\hat{P}^{\beta_5}\hat{Q}^{\gamma_5}=k_{PQ}\hat{P}\hat{Q}\ +(1-\lambda)k_{SQ}\hat{S}_6\hat{Q}\ . \tag{52}$$

$$\eta_6A_6\hat{P}^{\beta_6}\hat{Q}^{\gamma_6}\hat{S}_6^{\eta_6}=(1-\lambda)k_{SQ}\hat{S}_6\hat{Q} \tag{53}$$

The parameter values A_5, β_5 and η_5 are known from the solution to Model V.

A quasi-analytical solution to the set of five equations 48-50-51-52-53 for 7 unknowns $\hat{P},\hat{Q},\hat{S}_6,\beta_6,\gamma_6,\eta_6,A_6$ is obtained by assuming $\hat{P}=P,\hat{Q}=Q$ as in Adkins and Paxson (2014), and then finding $\hat{S}_6,\beta_6,\gamma_6,\eta_6,A_6$.

3. Numerical Illustrations

It is interesting to compare the apparent incentive effectiveness of subsidies given different assumptions regarding the stochastic or deterministic nature of P, Q or S.

<u>Model I</u>

The general Model I assumes that P, Q and S are stochastic, with a base case of zero drifts and zero correlations. A numerical illustration that this analytical solution has the same result as solving the five equations is shown in Figure 1 compared to Figure 2. Figure 1 also shows that the PDE (2) is also solved assuming correlations are zero, given the Deltas and Gammas for the ROV_1 (3),

The PDE is solved, but since the drifts are zero, the deltas are not relevant.

Insert Figures 1 and 2 Here

Figure 3 shows that the ROV and NPV as a function of the subsidy S almost form a conventional picture of a call option as a function of the “underlying asset”, with a “NPV exercise” price equivalent to S=.1085 (at which point the NPV just >0). The ROV equals the NPV at the S threshold of .2243.

Insert Figures 3 and 4 Here

One way to reduce the S threshold that justifies immediate investment is to length the facility and subsidy life. Figure 4 shows that reducing T, the finite life of the facility and subsidy (assumed to be coeval), raises the S thresholds significantly, and lowers the ROV. Note that much of this influence is through the sharp reduction of the PV factors (5-6-7), which then reduces the S and P power parameter values, also consistent with (15) and (16). However, a subsidy which continues for a longer time does not benefit the government, and it is questionable whether a government can influence the physical life of a facility.

Insert Figures 5 and 6 Here

Figure 5 shows that as the correlation between S and Q increases (greater project volatility), the S threshold that justifies investment and the ROV increases, which is logical. There is hardly any relation between the correlation of S and P and the S threshold or ROV.

$\hat{S}$ is a linear function of $\hat{P} = P$ as shown in Figure 6 so at a high P no subsidy is required. Increasing P also increases the ROV of the opportunity to invest, which perhaps is a case for considering government support of the market price, say of electricity, if there were not other consequences.

Insert Figures 7 and 8 Here

Figure 7 illustrates that both the S threshold and ROV_1 increase as the S drift increases. (There is a constraint that $\theta_S<r$.) Note that as θ_S increases k_S (7) increases, and all the power parameter values increase. Delta S increases, but delta P and delta Q decrease. So apparently one of the most effective subsidy arrangements which has the objective of reducing the S threshold is a subsidy that declines even at a constant rate over time. A subsidy that reduces over time is also beneficial for the government.

Reducing the interest rate is reasonably effective in reducing the S threshold, while at the same time increasing the ROV_1, as shown in Figure 8. This appears to be a “winner” on both fronts, costing the government little (supposedly) while benefitting both investors and those welcoming early investments in renewable energy facilities. But exactly how the government is supposed to lower interest rates for specific energy projects (low interest rate loans, rate swaps for the current environment of very low interest rates?) is an important consideration.

Insert Figure 9 Here

Reducing P volatility through price guarantees, or intervention in the energy market, might reduce the S thresholds as shown in Figure 9, but also reduces the ROV, which may not be welcomed by investors in concessions for future facilities. Generally this could be an expensive way of reducing the S threshold, since a reduction of P volatility from say 12% to 6%, results in a reduction of the S threshold from .27 to only .22 (a 21% reduction).

Model II

Figure10 is the simple spreadsheet for Model II with a Q volatility of zero, but allowing for Q drift. Note that Q gamma is eliminated from the PDE and solution, Row 44, and from the Q function (21) which allows for θ_Q. The primary effect is to raise the P power parameter value, while lowering the S and Q power parameter values. With less overall volatility, the S_2 threshold and ROV_2 are slightly lower. This base case is similar to Boomsma and Linnerud (2015), who assume that Q is constant rather than deterministic. (As a cross-check, using their methodology of solving four equations for four unknowns arrives at exactly the same results as Model II.)

Insert Figures 10 and 11 Here

Figure 11 shows the S_2 threshold and ROV_2 as a function of changing levels of a constant Q. So if the solar efficiency Q for the same K is increased, eventually no subsidy is required to justify immediate investment. Incidentally the last column shows that if Q=1.10 $\hat{S} = .1012$ when the real option should be exercised which is close to S=.10. So if subsidies remain the same, as K declines and/or Q for the same K increases due to technological innovation, investors using modern financial models will have sufficient incentives to build immediately energy facilities without subsidies. Perhaps this case holds currently for some solar facilities as K has declined.

Model III

Figure 12 is a simple spreadsheet for Model III with a permanent subsidy (FiT) that is deterministic, but allowing for stochastic P and Q. Note in solving the PDE the θ_S is considered but not σ_S. Note that reducing S volatility from 8% to zero reduces the S threshold less than 4%. Although the correlation with P and Q is zero naturally, it should be possible to design a subsidy that has an opposite trend from P, so compensating if market energy prices fall, and reducing government support if P increases over time.

Insert Figure 12 Here

Model IV

Figure 13 is a simple spreadsheet for Model IV with a deterministic subsidy and quantity of production, so this ROV_4 is driven by the volatility of the market price of energy. $\hat{S}_4$ is the lowest of all the modeled thresholds, but at the cost of eliminating both the S and Q volatility. Note the PDE is basic, consisting primarily of P gamma, since the basic parameter drifts are zero. Still this model allows for design variability, for the subsidy and quantity drifts could be altered (declining FiT, and guaranteed production) in order to further reduce the S threshold.

Insert Figures 13 and 14 Here

As the level of S increases, one would expect the ROV_4 of the opportunity to invest in a facility with a permanent subsidy would increase, as shown in Figure 14 with a deterministic subsidy and quantity of production. In Figure 14 the presumed actual subsidy is increased up to the S threshold, when then the ROV_4 equals the NPV_4, and the investment option is exercised.

Model V

One comparison of the function of permanent subsidies is with an environment without any possible subsidies, which is Model I of Adkins and Paxson (2014), solved by applying the principle of similarity, see Paxson and Pinto (2005) assuming a facility with infinite life. Figure 15 shows a finite facility life with equal power parameter values ($\beta_5 = \gamma_5$) (41). Since in this case, there is no subsidy, the objective is to find the P threshold that justifies immediate investment. The additional P is greater than the S threshold because the P volatility is different from the S volatility even in Model II. Model V with no subsidy (so no subsidy volatility or drift is considered), forms the reversionary case for a retractable subsidy, where a FiT could be withdrawn with no possibility of a subsidy ever again being provided.

Insert Figures15, 16 and 17 Here

Model VI

Figure 16 adapts the Adkins and Paxson (2014) retractable subsidy Model III, but provides a subsidy threshold rather than an output price threshold that would justify immediate investment, Model VI. Note the S_6 threshold result is between the Model II and Model III thresholds, as is the ROV_6 indicating that even a retractable subsidy is worthwhile. However, Figure 17 shows that increasing the probability of withdrawal (logically) raises the S threshold, and reduces the real option value.

Figure 18

SUBSIDY INCENTIVE EFFECT UNDER DIFFERENT MODELS				
	S	**S^**	**ROV**	**Δ P**
MODEL I	0.1000	0.2243	0.3336	2.8806
MODEL II	0.1000	0.1942	0.2623	3.0613
MODEL III	0.1000	0.2158	0.3148	2.9318
MODEL IV	0.1000	0.1848	0.2376	3.1126
		+P		
MODEL V	0.0000	0.2557	0.2243	2.4974
MODEL VI	0.1000	0.1992	0.3021	3.6658

Model I is the solution to EQs17-18 with ROV EQ 3, Model II is the solution to EQs 23-24 with ROV EQ 20, Model III is the solution to EQs 28-29 with ROV EQ 26, Model IV is the solution to EQs 33-34 with ROV EQ 31, Model V is the solution to EQs 44-45 and ROV EQ 36, Model VI is the solution to EQs 48-50-51-52-53 and ROV EQ 47, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, S subsidy=€.10, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=.04, subsidy volatility σ_S=.08, correlations $\rho_{PS}=\rho_{PQ}=\rho_{QS}=0$, $\theta_P=\theta_Q=\theta_S=0$, and riskless interest rate r=.04. λ=.10 reflects the possibility of a subsidy being withdrawn. S^ indicates the S threshold that justifies commencing the investment, +P indicates an additional P threshold (over P=.40) that justifies commencing the investment if there is no possibility of a subsidy.

Figure 18 is a comparison of the thresholds, ROV (and ΔP) using the six models with common parameter values except as specified. Model I considers P, Q and S stochastic, Model II Q is deterministic, indicating either a physical context where the sun always shines, the wind always blows equal to Q=1, or alternatively a government or purchaser guarantees a specified Q (or compensates if Q<1). Model III assumes S is deterministic, which is indicated for some FiT, even those with an amount that constantly declines over time. Model IV assumes both S and Q are deterministic. Clearly a constant S and Q provide the greatest incentive to invest immediately (lowest $\hat{S}$) but also result in the lowest ROV for the concession. A constant S is less effective than a constant Q alone in motivating immediate investment. But given the relatively low volatility figures for both S and Q, reducing the volatility does not result in dramatically lower S thresholds. Model V shows the high P threshold required to justify an immediate investment in the absence of any subsidy. But a retractable subsidy with only a modest probability of withdrawal lowers the S threshold as in Model VI.

Many of these conclusions are specific to the assumed parameter values, which can be easily changed as illustrated. Lessons for developers and designers of subsidy arrangements are [A] consider $\rho_{S,Q} < 0$ as in Figure 5, [B] specify negative S drifts as in Figure 7, [C] lower interest rates as in Figure 8, [D] lower P volatility if it doesn't cost much Figure 9, [E] relate S to K/Q as in Figure 11, [F] lower S volatility if it doesn't cost much, [G] consider both low S and Q volatility if it doesn't cost much Figure 13, and [H] lower the probability of subsidy withdrawal as in Figure 17.

Some of these lessons might enlighten the consideration of different subsidy arrangements as outlined in Lesser and Su (2008) In terms of the factors we consider, possibly the S threshold could be adjusted for a particular project by $\varepsilon K/\omega Q$, or a mixture of P and S assuming the correlation of that S and P is negative. "Stepped tariff designs"could be incorporated into our models through the S drift term, and/or k_{SQ} with T=5, 10, 15 or 20. It may be hard to consider some of the non-linear arrangements like the Couture and Gagnon (2009) "corridor" premium, (a subsidy declining as P increases) in an analytical solution. RPS (quantity-based with tradeable certificates awarded for qualifed units of renewable energy produced) may be appropriately modelled as S with volality and drift, with a correlation with P and Q empirically measured.

4. CONCLUSION

We derive the optimal investment timing and real option value for a subsidized renewable energy finite facility with possible price, quantity and subsidy uncertainty. The general model is for three stochastic variables, from which several other models can be easily derived, where some (or all) of the variables are constant or deterministic. Simple analytical solutions are suggested for most of the models. Sensitivity of the thresholds justifying immediate investment and the real option values to changes in the important variables are also provided.

We illustrate a hierarchy of S thresholds with the highest allowing for volatility in two or three variables $\hat{S}_5 > \hat{S}_1 > \hat{S}_3 > \hat{S}_6 > \hat{S}_2 > \hat{S}_4$. There is a different hierarchy for the real option values, $ROV_1 > ROV_3 > ROV_6 > ROV_2 > ROV_4 > ROV_5$ all with common parameter values, except for volatility. Within each model there are interesting variations. Typically the S threshold and ROV increase with the volatility of the appropriate stochastic variable in each model. So price, subsidy and quantity guarantees (reduction of volatility) are likely to result in earlier facility investments. Similarly the longer time frame for FiT reduces the S threshold, as do reductions in interest rates. The effects of declining S also increases the S threshold, and reduces the ROV.

We suggest that a wide variety of support schemes can be evaluated using our framework, although eventually complicated arrangements require numerical solutions. Perhaps subsidies which grow or shrink at non constant rates can be incorporated into these models, along with stochastic investment costs and stochastic renewable energy certificates.

Calibrating price and sometimes quantity volatilities and drifts might be feasible for some outputs, but calibrating the convenience yields or drifts of some quantities and correlations of price and quantities is likely to be a challenge. Left for future research are uncertain operating costs along with technological developments.

References

Abadie, L. and J. Chamorro. “Valuation of Wind Energy Projects: A Real Option Approach.” Real Options Conference, Tokyo (2013), doi 10.3390/en 7053218.

Abolhosseini, S. and A. Heshmati, “The Main Support Mechanisms to Finance Renewable Energy Development.” *Renewable and Sustainable Energy Reviews*, 40 (2014), 876-885.

Adkins, R., and D. Paxson. "Renewing Assets with Uncertain Revenues and Operating Costs." *Journal of Financial and Quantitative Analysis*, 46 (2011), 785-813.

Adkins, R., and D. Paxson. "Subsidies for Renewable Energy Facilities under Uncertainty" *The Manchester School* (2014), DOI 10.1111.

Blyth, W., D. Bunn, S. Kettunen and T. Wilson. "Policy Interactions, Risks and Price Fomation in Carbon Markets", *Energy Policy*, 37 (2009), 5192-5207.

Boomsma, T., N.Meade and S-E Fleten. "Renewable Energy Investments under Different Support Schemes: A Real Options Approach", *European Journal of Operational Research* 220 (2012), 225-237.

Boomsma, T., and K. Linnerud. "Market and Policy Risk under Different Renewable Electricity Support Schemes", *Energy* 89 (2015), 435-448.

Borenstein, S. "The Private and Public Economics of Renewable Energy Generation", *Journal of Economic Perspectives*, 26 (2012), 67-92.

Couture, T., and Y. Gagnon. "An Analysis of Feed-in-Tariff Remuneration Models: Implications for Renewable Energy Investment", *Energy Policy* 38 (2009), 955-65.

Kettunen, J., D. Bunn and W. Blyth. "Investment Propensities under Carbon Policy Uncertainty", *The Energy Journal*, 32 (2011), 77-117.

Lesser, J.A., and X. Su. "Design of an Economically Efficient Feed-in Tariff Structure for Renewable Energy Development", *Energy Policy* 36 (2008), 981-990.

Linnerud, K., A.M.Andersson and S.-E. Fleten. "Investment Timing Under Uncertain Renewable Energy Policy: An Empirical Study of Small Hydropower Projects", *Energy* (2014), DOI 10.1016.

Mosiño, A. "Producing Energy in a Stochastic Environment: Switching from Non-renewable to Renewable Resources", *Resource and Energy Economics* 34 (2012), 413-430.

Paxson, D., and H. Pinto. "Rivalry under Price and Quantity Uncertainty." *Review of Financial Economics*, 14 (2005), 209-224.

Ritzenhofen, I., and S. Spinler. "Optimal Design of Feed-in-Tariffs to Stimulate Renewable Energy Investments under Regulatory Uncertainty—A Real Options Analysis", *Energy Economics* (2015).

Yang, M., W. Blyth, R. Bradley, D. Bunn, C. Clarke and T. Wilson. "Evaluating the Power Investment Options with Uncertainty in Climate Policy". *Energy Economics*, 20 (2008), 1933-1950.

Figure 1

	A	B	C	D
1			**SUBSIDIES MODEL I**	EQS
2	INPUT	**Stochastic P & Q & S**		
3	P	0.40		
4	Q	1.00		
5	S	0.10	per kwh	
6	R	0.50	B3*B5+B4*B5	
7	K	7.00		
8	σ_P	**0.06**		
9	σ_Q	**0.04**		
10	σ_S	**0.08**		
11	ρ_{PQ}	0.00		
12	ρ_{PS}	0.00		
13	ρ_{SQ}	0.00		
14	r	0.04		
15	θ_P	0.00		
16	θ_Q	0.00		
17	θ_S	0.00		
18	OUTPUT			
19	a1	0.0033	0.5*(B8^2)+0.5*(B10^2)-B12*B8*B10+((B7^2)/(2*B34))*((B9^2)+2*B13*B9*B10+(B10^2))+B35	17
20	b1	0.0001	B15-B17-0.5*(B8^2)-0.5*(B10^2)+B11*B8*B9+B12*B8*B10-B13*B9*B10+B36	17
21	β1	3.4542	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17+B13*B9*B10)))/(2*B19)	18
22	η1	1.9367	1+B21*((B7/(B28*B30*B29))-1)	16
23	γ1	5.3908	B21+B22	14
24	A1	683.0290	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))	12
25	**S^1**	**0.2243**	(B22*B28*B30)/(B21*B31)	13
26	**F1(P,Q,S)**	**0.3336**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)	3
27	F1(P,Q,S)	1.5942	(B30*B28*B29)+(B32*B25*B29)-B7	RHS 8
28	P^	0.4000		
29	Q^	1.0000		
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)	5
31	Q PV rQ	13.7668	(1-EXP(-(B14-B16)*20))/(B14-B16)	6
32	S PV rS	13.7668	(1-EXP(-(B14-B17)*20))/(B14-B17)	7
33	PQrPQ	5.5067	B28*B29*B30	
34	P^2Q^1rPQ^2	30.3239	(B28^2)*(B29^2)*(B30^2)	
35	a2	-0.0081	(B7/B33)*(B11*B8*B9+B12*B8*B9-B13*B9*B10-(B10^2))	17
36	b2	0.0051	(B7/B33)*(B16+B17+0.5*(B9^2)+2*(B13*B9*B10)+0.5*(B10^2))	17
37	β1	3.4542	B33/(B33+B32*B25*B29-B7)	15
38				
39	PDE	0.0000	0.5*(B8^2)*(B3^2)*B43+0.5*(B9^2)*(B4^2)*B44+0.5*(B10^2)*(B5^2)*B45+B15*B3*B40+B16*B4*B41+B17*B5*B42-B14*B26	2
40	ΔROV1,P	2.8806	B21*B24*(B3^(B21-1))*(B4^B23)*(B5^B22)	
41	ΔROV1,Q	1.7983	B23*B24*(B3^B21)*(B4^(B23-1))*(B5^B22)	
42	ΔROV1,S	6.4605	B22*B24*(B3^B21)*(B4^B23)*(B5^(B22-1))	
43	ΓROV1,P	17.6738	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B23)*(B5^B22)	
44	ΓROV1,Q	7.8961	B23*(B23-1)*B24*(B3^B21)*(B4^(B23-2))*(B5^B22)	
45	ΓROV1,S	60.5144	B22*(B22-1)*B24*(B3^B21)*(B4^B23)*(B5^(B22-2))	

Figure 2

	A	B	C	D
48	VM	0.0000	B53*(B28^B54)*(B29^B56)*(B57^B55)-(B30*B28*B29)-(B32*B57*B29)+B7	8
49	SP1	0.0000	B53*B54*(B28^B54)*(B29^B56)*(B57^B55)-B30*B29*B28	9
50	SP2	0.0000	B53*B56*(B28^B54)*(B29^B56)*(B57^B55)-(B30*B28*B29)-(B32*B57*B29)	10
51	SP3	0.0000	B55*B53*(B28^B54)*(B29^B56)*(B57^B55)-(B32*B29*B57)	11
52	Q	0.0000		10
53	A	683.0289		4
54	β1	3.4542		
55	η1	1.9367	1+B54*((B7/(B28*B30*B29))-1)	1.9367
56	γ1	5.3908		
57	S^	0.2243	Find	
58	Revenue	0.6243		
59	SOLVER	0.0000	SUM(ABS)(B48:B52) CHANGE B53:B57	
60	Q	0.5*(B8^2)*B54*(B54-1)+0.5*(B9^2)*B56*(B56-1)+0.5*((B10^2)*(B55*(B55-1)))+B54*B56*B11*B8*B9+B54*B55*B12*B8*B10+B56*B55*B13*B9*B10+B54*B15+B56*B16+B55*B17-B14		

Model I is the solution to EQs17-18 with ROV EQ 3, and also the solution to EQs 4-8-9-10-11 as shown in Figure 2, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, S subsidy=€.10, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=.04, subsidy volatility σ_S=.08, correlations $\rho_{PS}=\rho_{PQ}=\rho_{QS}=0$, $\theta_P=\theta_Q=\theta_S=0$, and riskless interest rate r=.04. S^ indicates the S threshold that justifies commencing the investment.

Figure 3

PDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ΔROV1,P	0.0000	0.1276	0.4884	1.0711	1.8698	2.8806	4.1005	5.5270	7.1582	8.9923	11.0278	13.2633
ΔROV1,Q	0.0000	0.0796	0.3049	0.6687	1.1673	1.7983	2.5598	3.4504	4.4687	5.6136	6.8843	8.2799
ΔROV1,S	0.0000	1.4307	2.7385	4.0037	5.2419	6.4605	7.6636	8.8540	10.0337	11.2040	12.3662	13.5209
ΓROV1,P	0.0000	0.7828	2.9967	6.5717	11.4722	17.6738	25.1582	33.9105	43.9184	55.1712	67.6598	81.3758
ΓROV1,Q	0.0000	0.3497	1.3388	2.9360	5.1254	7.8961	11.2398	15.1501	19.6212	24.6486	30.2281	36.3559
ΓROV1,S	0.0000	67.0059	64.1289	62.5035	61.3754	60.5144	59.8198	59.2388	58.7401	58.3037	57.9161	57.5676
S	**0.00**	**0.02**	**0.04**	**0.06**	**0.08**	**0.10**	**0.12**	**0.14**	**0.16**	**0.18**	**0.20**	**0.22**
ROV1	0.0000	0.0148	0.0566	0.1240	0.2165	0.3336	0.4748	0.6400	0.8289	1.0413	1.2770	1.5359
NPV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1587	0.4341	0.7094	0.9847	1.2601	1.5354

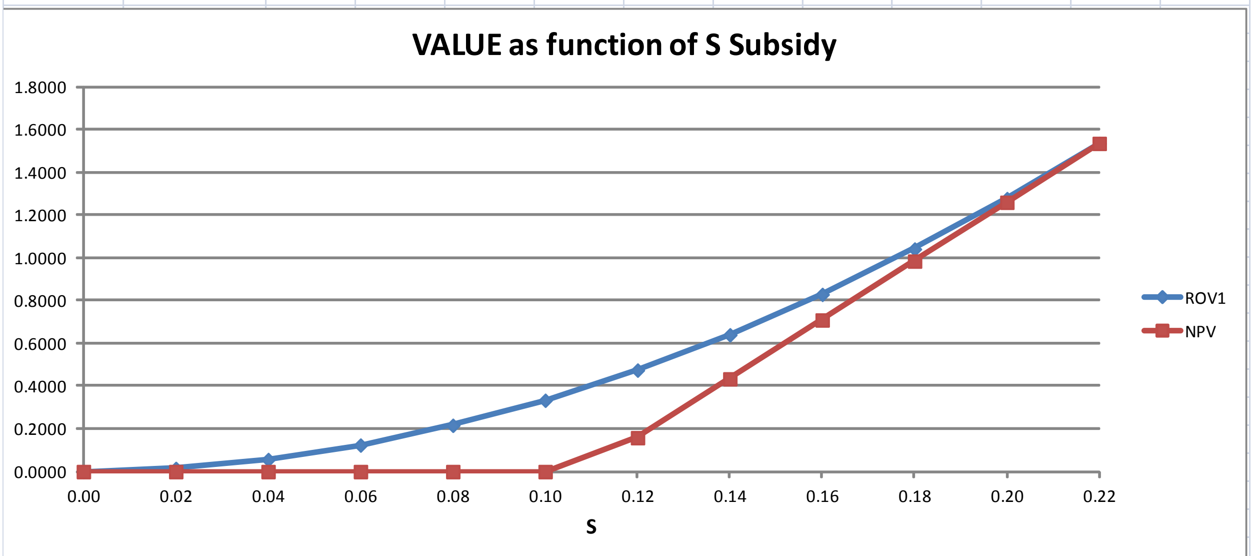

Figure 4

T	4	6	8	10	12	14	16	18	20
S^1	2.0756	1.2843	0.8931	0.6619	0.5108	0.4052	0.3281	0.2697	0.2243
ROV_1	0.0000	0.0003	0.0013	0.0049	0.0149	0.0394	0.0908	0.1846	0.3336

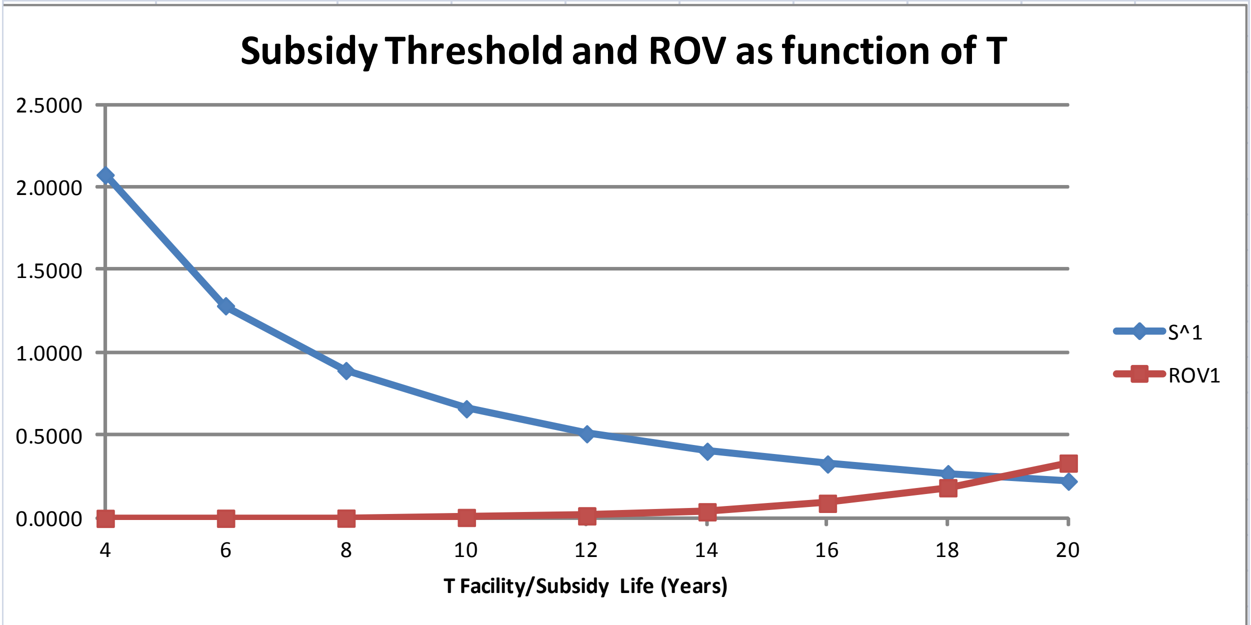

Figure 5

ρ_{SQ}	−100%	−80%	−60%	−40%	−20%	0%	20%	40%	60%	80%	100%
ROV1	0.2230	0.2459	0.2684	0.2905	0.3122	0.3336	0.3546	0.3752	0.3955	0.4154	0.4351
S^1	0.1795	0.1879	0.1966	0.2055	0.2147	0.2243	0.2341	0.2444	0.2549	0.2659	0.2772

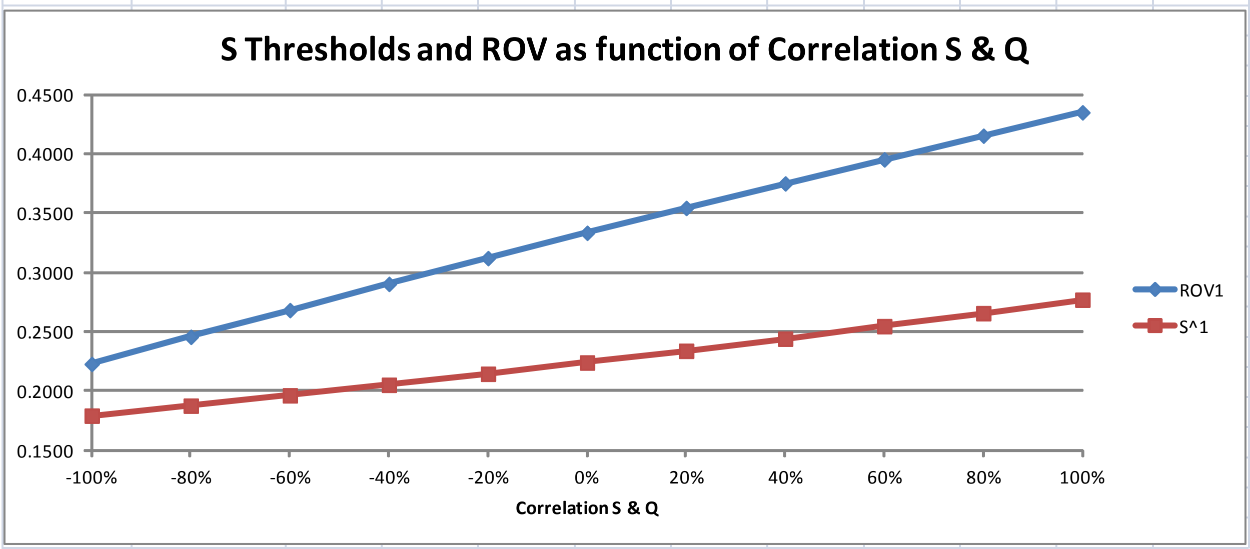

Figure 6

PDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ΔROV1,P	0.0164	0.0276	0.0507	0.1021	0.2250	0.5288	1.2638	2.8806	5.8882	10.3194
ΔROV1,Q	0.0111	0.0184	0.0331	0.0656	0.1427	0.3321	0.7895	1.7983	3.6887	6.5109
ΔROV1,S	0.1032	0.1563	0.2552	0.4522	0.8642	1.7347	3.4720	6.4605	10.3898	13.5125
ΓROV1,P	-0.2324	-0.1015	0.0108	0.2493	0.8905	2.6606	7.2590	17.6738	36.8233	63.6202
ΓROV1,Q	0.0328	0.0591	0.1159	0.2484	0.5773	1.4125	3.4529	7.8961	15.8974	27.0316
ΓROV1,S	2.7418	4.0356	6.2941	10.3798	17.7758	30.2489	47.3245	60.5144	51.4912	9.3456
P	**0.05**	**0.10**	**0.15**	**0.20**	**0.25**	**0.30**	**0.35**	**0.40**	**0.45**	**0.50**
S^1	**0.631**	**0.567**	**0.504**	**0.443**	**0.384**	**0.328**	**0.275**	**0.224**	**0.176**	**0.131**
ROV1	**0.003**	**0.004**	**0.007**	**0.014**	**0.028**	**0.063**	**0.147**	**0.334**	**0.695**	**1.264**

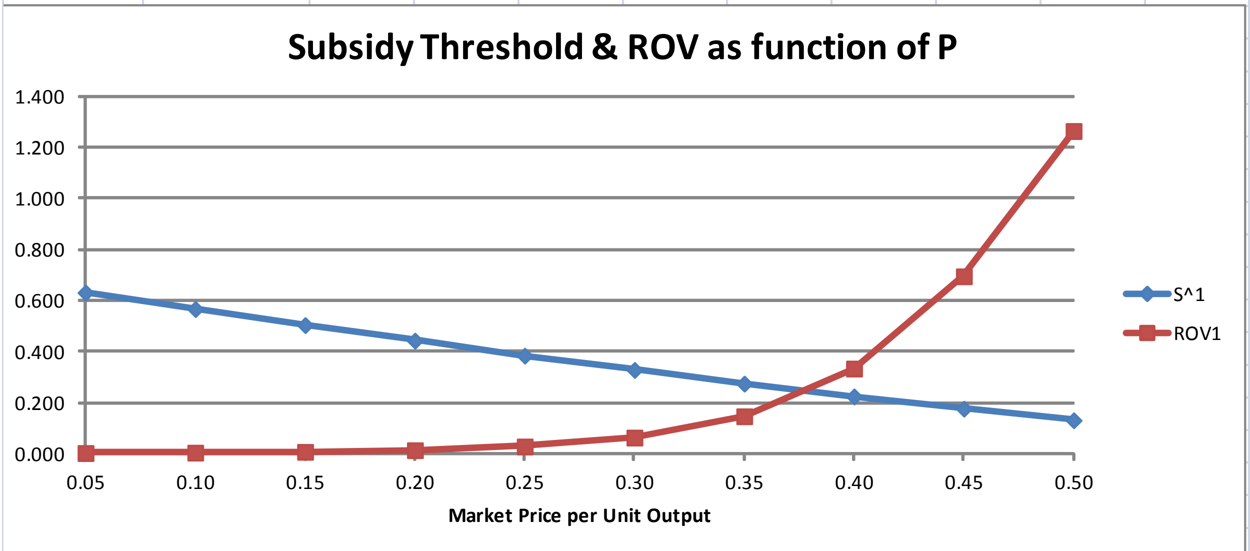

Figure 7

θs	**-0.025**	**-0.02**	**-0.015**	**-0.01**	**-0.005**	**0**	**0.005**	**0.01**	**0.015**	**0.02**	**0.025**
S^1	**0.1807**	**0.1865**	**0.1934**	**0.2017**	**0.2117**	**0.2243**	**0.2403**	**0.2616**	**0.2914**	**0.3358**	**0.4095**
ROV1	0.2263	0.2423	0.2605	0.2813	0.3054	0.3336	0.3672	0.4078	0.4583	0.5228	0.6089
NPV	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0107	0.0806	0.1551	0.2346

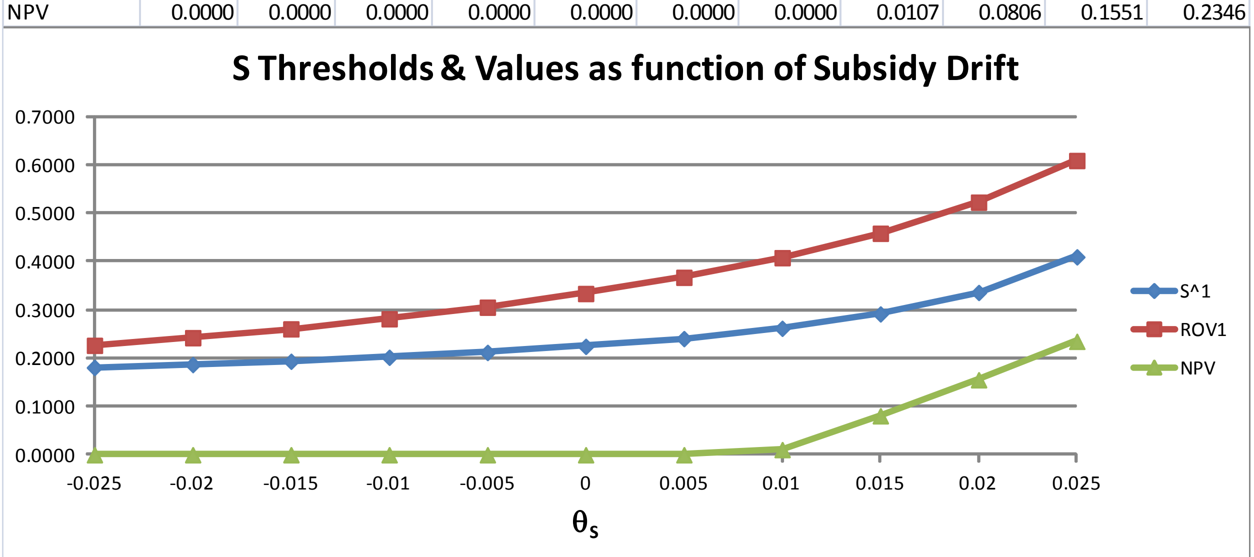

Figure 8

r	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	11%	12%
ROV1	2.0415	1.2837	0.7018	0.3336	0.1403	0.0537	0.0192	0.0066	0.0022	0.0007	0.0002	0.0001
S^1	0.1666	0.1647	0.1890	0.2243	0.2662	0.3129	0.3635	0.4171	0.4734	0.5321	0.5927	0.6552

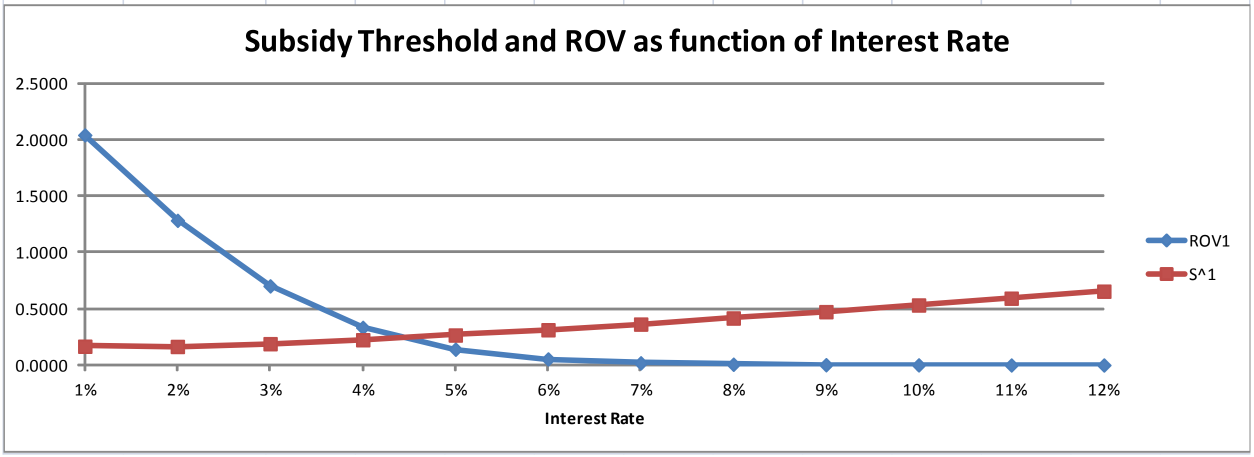

Figure 9

σ_P	0%	2%	4%	6%	8%	10%	12%	14%	16%	18%	20%	22%
ROV1	0.2687	0.2782	0.3026	0.3336	0.3657	0.3961	0.4238	0.4487	0.4709	0.4906	0.5082	0.5238
S^1	0.1966	0.2005	0.2105	0.2243	0.2396	0.2552	0.2706	0.2854	0.2994	0.3126	0.3250	0.3365

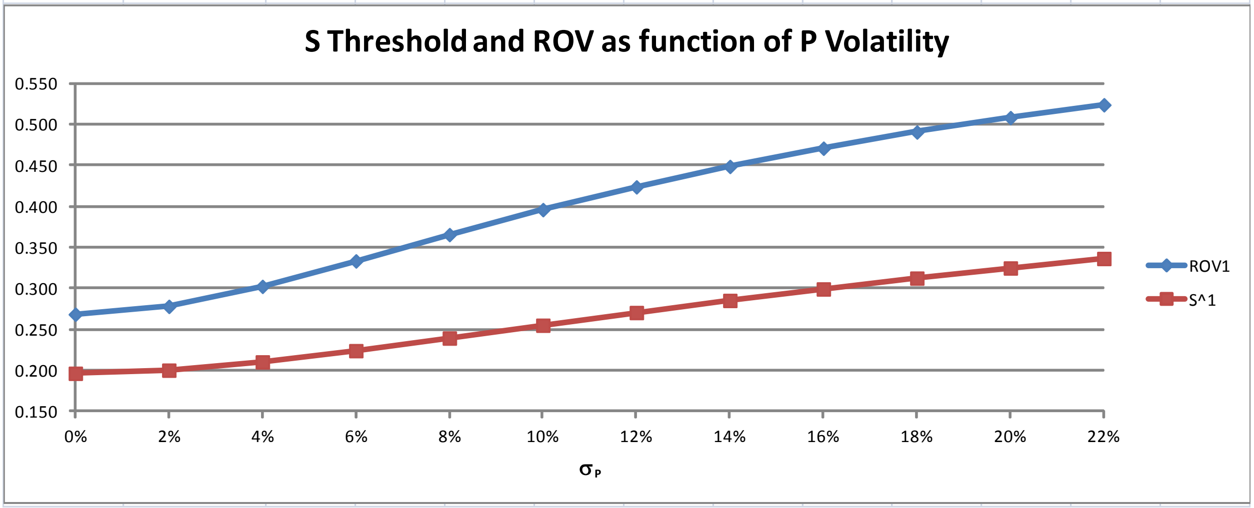

Figure 10

	A	B	C	D
1			**SUBSIDIES MODEL II**	EQS
2	INPUT	**Stochastic P & S**		
3	P	0.40		
4	Q	1.00		
5	S	0.10	per kwh	
6	R	0.50	B3*B5+B4*B5	
7	K	7.00		
8	σ_P	0.06		
9	σ_Q	**0.00**		
10	σ_S	0.08		
11	ρ_{PQ}	0.00		
12	ρ_{PS}	0.00		
13	ρ_{SQ}	0.00		
14	r	0.04		
15	θ_P	0.00		
16	θ_Q	0.00		
17	θ_S	0.00		
18	OUTPUT			
19	a1	0.0020	0.5*(B8^2)+0.5*(B10^2)-B12*B8*B10+((B7^2)/(2*B34))*((B10^2))+B35	23
20	b1	-0.0009	B15-B17-0.5*(B8^2)-0.5*(B10^2)+B12*B8*B10+B36	23
21	β2	4.6681	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17)))/(2*B19)	24
22	η2	2.2659	1+B21*((B7/(B28*B30*B29))-1)	16
23	γ2	6.9340	B21+B22	14
24	A2	3485.9562	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))	12
25	**S^2**	**0.1942**	(B22*B28*B30)/(B21*B31)	13
26	**F2(P,Q,S)**	**0.2623**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)	20
27	F2(P,Q,S)	1.1796	(B30*B28*B29)+(B32*B25*B29)-B7	RHS 22
28	P^	0.4000		
29	Q^	1.0000		
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)	5
31	Q PV rQ	13.7668	(1-EXP(-(B14-B16)*20))/(B14-B16)	6
32	S PV rS	13.7668	(1-EXP(-(B14-B17)*20))/(B14-B17)	7
33	PQrPQ	5.5067	B28*B29*B30	
34	P^2Q^1rPQ^2	30.3239	(B28^2)*(B29^2)*(B30^2)	
35	a2	-0.0081	(B7/B33)*(-(B10^2))	23
36	b2	0.0041	(B7/B33)*(B16+B17+0.5*(B10^2))	23
37	β2	4.6681	B33/(B33+B32*B25*B29-B7)	15
38				
39	PDE	0.0000	0.5*(B8^2)*(B3^2)*B43+0.5*(B10^2)*(B5^2)*B45+B15*B3*B40+B16*B4*B41+B17*B5*B42-B14*B26	19
40	ΔROV2,P	3.0613	B21*B24*(B3^(B21-1))*(B4^B23)*(B5^B22)	
41	ΔROV2,Q	1.8189	B23*B24*(B3^B21)*(B4^(B23-1))*(B5^B22)	
42	ΔROV2,S	5.9438	B22*B24*(B3^B21)*(B4^B23)*(B5^(B22-1))	
43	ΓROV2,P	28.0727	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B23)*(B5^B22)	
44				
45	ΓROV2,S	75.2407	B22*(B22-1)*B24*(B3^B21)*(B4^B23)*(B5^(B22-2))	

Model II is the solution to EQs 23-24 with ROV EQ 20, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, S subsidy=€.10, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=0, subsidy volatility σ_S=.08, correlations $\rho_{PS}=\rho_{PQ}=\rho_{QS}=0$, $\theta_P=\theta_Q=\theta_S=0$, and riskless interest rate r=.04. S^ indicates the S threshold that justifies commencing the investment.

Figure 11

PDE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ΔROV1,P	0.0017	0.0091	0.0462	0.2181	0.9037	3.0613	8.0642	16.2705
ΔROV1,Q	0.0042	0.0153	0.0565	0.2028	0.6621	1.8189	3.9781	6.7953
ΔROV1,S	0.0140	0.0554	0.2106	0.7502	2.3445	5.9438	11.5028	16.4613
ΓROV1,P	0.0038	0.0336	0.2412	1.4685	7.3074	28.0727	80.1477	169.4107
ΓROV1,S	0.4108	1.5348	5.3077	16.3162	40.9428	75.2407	89.0613	50.4284
Q	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20
S^2	**0.8273**	**0.6090**	**0.4559**	**0.3439**	**0.2594**	**0.1942**	**0.1426**	**0.1012**
ROV_2	**0.0004**	**0.0015**	**0.0060**	**0.0236**	**0.0854**	**0.2623**	**0.6483**	**1.2601**

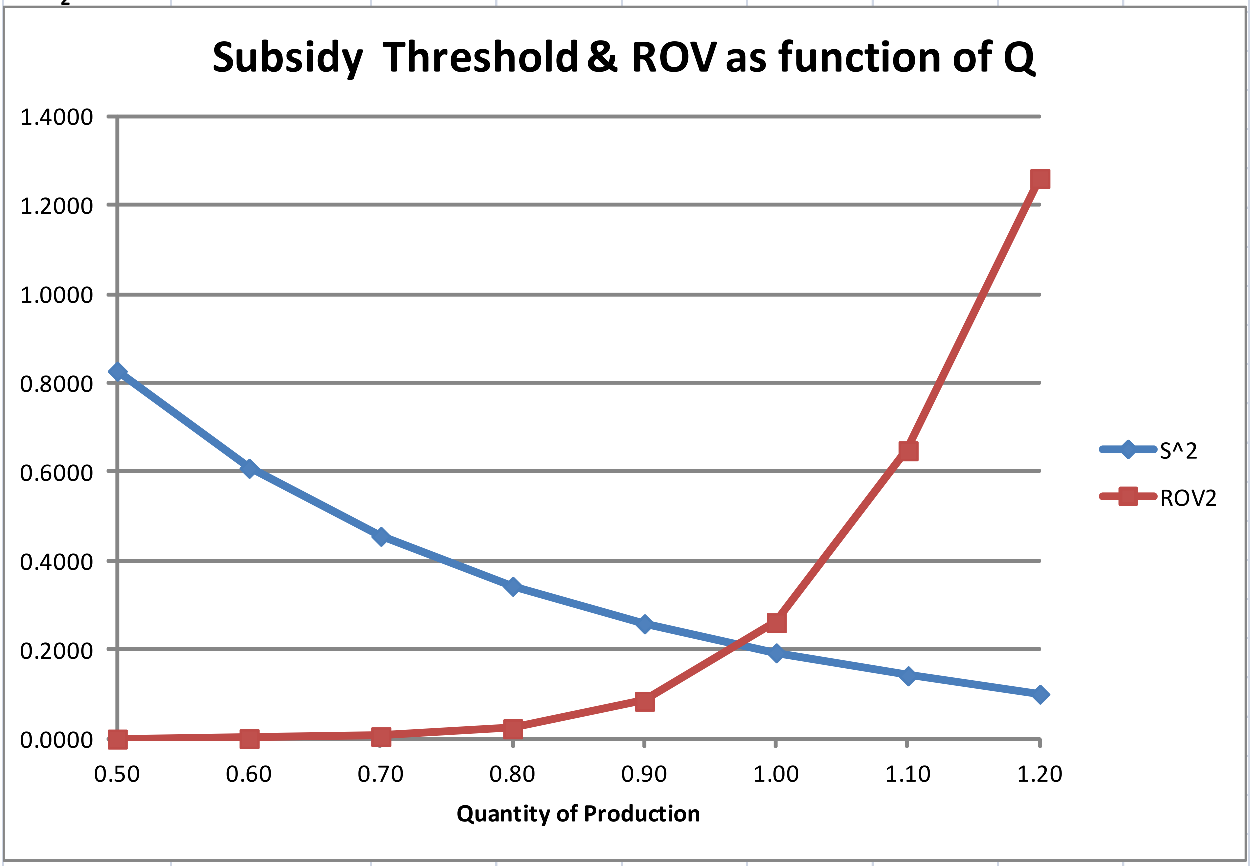

Figure 12

	A	B	C
1			**SUBSIDIES MODEL III**
2	INPUT	Stochastic P & Q	
3	P	0.40	
4	Q	1.00	
5	S	0.10	per kwh
6	R	0.50	B3*B5+B4*B5
7	K	7.00	
8	σ_P	0.06	
9	σ_Q	0.04	
10	σs	**0.00**	
11	ρ_{PQ}	0.00	
12	ρ_{PS}	0.00	
13	ρ_{SQ}	0.00	
14	r	0.04	
15	θ_P	0.00	
16	θ_Q	0.00	
17	θs	0.00	
18	OUTPUT		
19	a1	0.0031	0.5*(B8^2)+((B7^2)/(2*B34))*((B9^2))+B35
20	b1	-0.0008	B15-B17-0.5*(B8^2)+B11*B8*B9+B36
21	β3	3.7252	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17)))/(2*B19)
22	η3	2.0102	1+B21*((B7/(B28*B30*B29))-1)
23	γ3	5.7353	B21+B22
24	A3	978.6232	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))
25	**S^3**	**0.2158**	(B22*B28*B30)/(B21*B31)
26	**F3(P,Q,S)**	**0.3148**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)
27	F3(P,Q,S)	1.4782	(B30*B28*B29)+(B32*B25*B29)-B7
28	P^	0.4000	
29	Q^	1.0000	
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)
31	Q PV rQ	13.7668	(1-EXP(-(B14-B16)*20))/(B14-B16)
32	S PV rS	13.7668	(1-EXP(-(B14-B17)*20))/(B14-B17)
33	PQrPQ	5.5067	B28*B29*B30
34	P^2Q^1rPQ^2	30.3239	(B28^2)*(B29^2)*(B30^2)
35	a2	0.0000	(B7/B33)*(B11*B8*B9+B12*B8*B9)
36	b2	0.0010	(B7/B33)*(B16+B17+0.5*(B9^2))
37	β3	3.7252	B33/(B33+B32*B25*B29-B7)
38			
39	PDE	0.0000	0.5*(B8^2)*(B3^2)*B43+0.5*(B9^2)*(B4^2)*B44+B15*B3*B40+B16*B4*B41+B17*B5*B42-B14*B26
40	ΔROV3,P	2.9318	B21*B24*(B3^(B21-1))*(B4^B23)*(B5^B22)
41	ΔROV3,Q	1.8055	B23*B24*(B3^B21)*(B4^(B23-1))*(B5^B22)
42	ΔROV3,S	6.3282	B22*B24*(B3^B21)*(B4^B23)*(B5^(B22-1))
43	ΓROV3,P	19.9741	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B23)*(B5^B22)
44	ΓROV3,Q	8.5499	B23*(B23-1)*B24*(B3^B21)*(B4^(B23-2))*(B5^B22)

Model III is the solution to EQs 28-29 with ROV EQ 26, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, S subsidy=€.10, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=.04, subsidy volatility σ_S=0, correlations $\rho_{PS}=\rho_{PQ}=\rho_{QS}=0$, $\theta_P=\theta_Q=\theta_S=0$, and riskless interest rate r=.04. S^ indicates the S threshold that justifies commencing the investment.

Figure 13

	A	B	C
1			**SUBSIDIES MODEL IV**
2	INPUT	**Stochastic P**	
3	P	0.40	
4	Q	1.00	
5	S	0.10	per kwh
6	R	0.50	B3*B5+B4*B5
7	K	7.00	
8	σ_P	0.06	
9	σ_Q	**0.00**	
10	σs	**0.00**	
11	ρ_{PQ}	0.00	
12	ρ_{PS}	0.00	
13	ρ_{SQ}	0.00	
14	r	0.04	
15	θ_P	0.00	
16	θ_Q	0.00	
17	θs	0.00	
18	OUTPUT		
19	a1	0.0018	0.5*(B8^2)+B35
20	b1	-0.0018	B15-B17-0.5*(B8^2)+B36
21	β4	5.2405	(-B20+SQRT((B20^2)-4*B19*(-B14+B16+B17)))/(2*B19)
22	η4	2.4211	1+B21*((B7/(B28*B30*B29))-1)
23	γ4	7.6616	B21+B22
24	A4	7626.2185	B33/(B21*(B28^B21)*(B29^B23)*(B25^B22))
25	**S^4**	**0.1848**	(B22*B28*B30)/(B21*B31)
26	**F4(P,Q,S)**	**0.2376**	IF(B5<B25, B24*(B3^B21)*(B4^B23)*(B5^B22),B27)
27	F4(P,Q,S)	1.0508	(B30*B28*B29)+(B32*B25*B29)-B7
28	P^	0.4000	
29	Q^	1.0000	
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)
31	Q PV rQ	13.7668	(1-EXP(-(B14-B16)*20))/(B14-B16)
32	S PV rS	13.7668	(1-EXP(-(B14-B17)*20))/(B14-B17)
33	PQrPQ	5.5067	B28*B29*B30
34	P^2Q^1rPQ^2	30.3239	(B28^2)*(B29^2)*(B30^2)
35	a2	0.0000	(B7/B33)*(B11)
36	b2	0.0000	(B7/B33)*(B16+B17)
37	β4	5.2405	B33/(B33+B32*B25*B29-B7)
38			
39	PDE	0.0000	0.5*(B8^2)*(B3^2)*B43+B15*B3*B40+B16*B4*B41+B17*B5*B42-B14*B26
40	ΔROV4,P	3.1126	B21*B24*(B3^(B21-1))*(B4^B23)*(B5^B22)
41	ΔROV4,Q	1.8203	B23*B24*(B3^B21)*(B4^(B23-1))*(B5^B22)
42	ΔROV4,S	5.7521	B22*B24*(B3^B21)*(B4^B23)*(B5^(B22-1))
43	ΓROV4,P	32.9975	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B23)*(B5^B22)

Model IV is the solution to EQs 33-34 with ROV EQ 31, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, S subsidy=€.10, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=0, subsidy volatility σ_S=0, correlations $\rho_{PS}=\rho_{PQ}=\rho_{QS}=0$, $\theta_P=\theta_Q=\theta_S=0$, and riskless interest rate r=.04. S^ indicates the S threshold that justifies commencing the investment.

Figure 14

S^4	**0.1848**																	
S	**0.100**	**0.105**	**0.110**	**0.115**	**0.120**	**0.125**	**0.130**	**0.135**	**0.140**	**0.145**	**0.150**	**0.155**	**0.160**	**0.165**	**0.170**	**0.175**	**0.180**	**0.185**
ROV$_4$	**0.2376**	**0.2674**	**0.2992**	**0.3332**	**0.3694**	**0.4078**	**0.4484**	**0.4913**	**0.5365**	**0.5841**	**0.6341**	**0.6865**	**0.7413**	**0.7987**	**0.8585**	**0.9209**	**0.9859**	**1.0508**
NPV$_4$	0.0000	0.0000	0.0211	0.0899	0.1587	0.2276	0.2964	0.3652	0.4341	0.5029	0.5717	0.6406	0.7094	0.7782	0.8471	0.9159	0.9847	1.0508

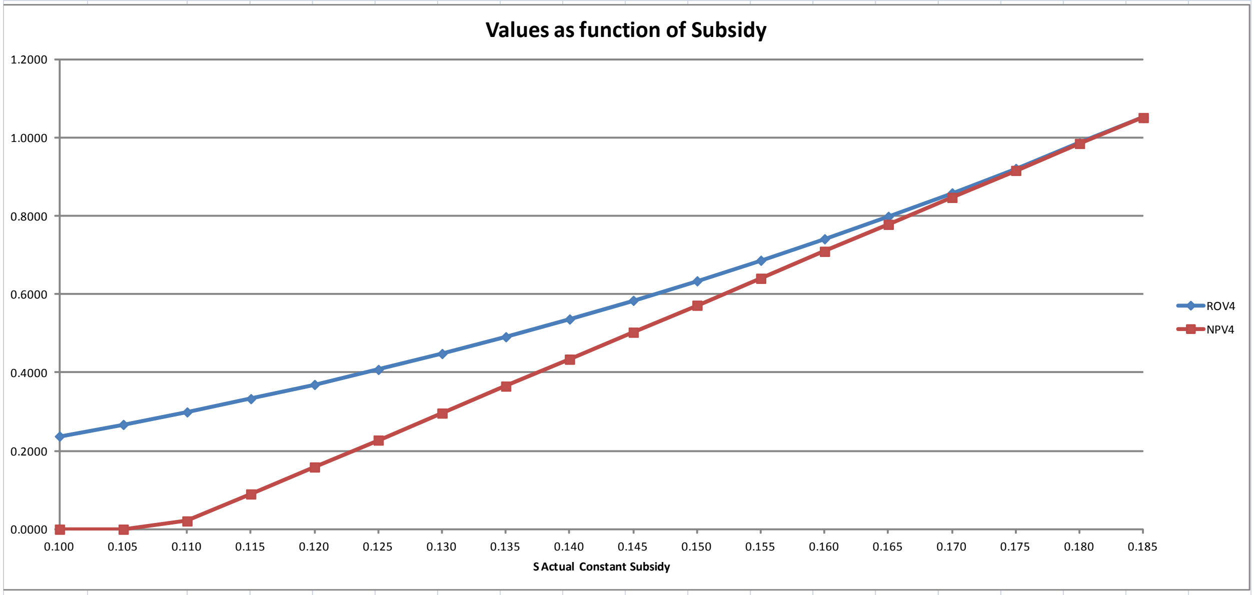

Figure 15

	A	B	C	D
1	k_{PQ}		**SUBSIDIES MODEL V**	EQS
2	INPUT	**Stochastic P & Q**		
3	P	0.40		
4	Q	1.00		
5				
6	R	0.40	B3*B4	
7	K	7.00		
8	σ_P	**0.06**		
9	σ_Q	**0.04**		
10				
11	ρ_{PQ}	0.00		
12				
13				
14	r	0.04		
15	θ_P	0.00		
16	θ_Q	0.00		
17				
18	OUTPUT			
19	a5	0.0052	0.5*(B8^2)+0.5*(B9^2)+2*B11*B8*B9	44
20	b5	0.0000	B11*B8*B9+B15+B16	44
21	β5	4.4541	(-(B20-0.5*B19)+SQRT((((B20-0.5*B19)^2))-2*(-B14)*B19))/B19	45
22	γ5	4.4541	B21	41
23	+ P	0.2557	Addition P required if no subsidy, to justify immediate investment.	
24	A5	13.2811	B30*(B28^(1-B21))*(B29^(1-B22))/B21	42
25				
26	**F5(P,Q)**	**0.2243**	**IF(B3<B28, B24*(B3^B21)*(B4^B22),B27)**	36
27	F5(P,Q)	2.0266	(B30*B28*B29)-B7	RHS 38
28	P^	0.6557	(B21/(B21-1))*B7/B30*B29	43
29	Q^	1.0000		
30	P PV rP	13.7668	(1-EXP(-(B14-B15)*20))/(B14-B15)	5
31	Q PV rQ	13.7668	(1-EXP(-(B14-B16)*20))/(B14-B16)	6
32	PDE	0.0000	0.5*(B8^2)*(B3^2)*B36+0.5*(B9^2)*(B4^2)*B37+B15*B3*B33+B16*B4*B34-B14*B26	35
33	ΔROV5,P	2.4974	B21*B24*(B3^(B21-1))*(B4^B22)	
34	ΔROV5,Q	0.9989	B22*B24*(B3^B21)*(B4^(B22-1))	
35				
36	ΓROV5,P	21.5650	B21*(B21-1)*B24*(B3^(B21-2))*(B4^B22)	
37	ΓROV5,Q	3.4504	B22*(B22-1)*B24*(B3^B21)*(B4^(B22-2))	

Model V is the solution to EQs 44-45 and ROV EQ 36, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=.04, correlations ρ_{PQ}=0, θ_P=θ_Q=0, and riskless interest rate r=.04. +P indicates an additional P threshold (over P=.40) that justifies commencing the investment if there is no possibility of a subsidy.

Figure 16

	A	B	C	D
1			**RETRACTABLE SUBSIDIES MODEL VI**	
2	INPUT		**Stochastic P & Q & S**	
3	P	0.40		
4	Q	1.00		
5	S	0.100		
6	R	0.50		
7	K	7.00		
8	σ_P	0.06		
9	σ_Q	0.04		
10	σs	0.08		
11	ρ_{PQ}	0.00		
12	ρ_{PS}	0.00		
13	ρ_{SQ}	0.00		
14	r	0.04		
15	θ_P	0.00		
16	θ_Q	0.00		
17	θs	0.00		
18	λ	0.1000	POSSIBILITY OF SUBSIDY WITHDRAWAL	
19	OUTPUT			
20	F6(P,Q,S)	0.3021	IF(B5<B31, B27*(B3^B28)*(B4^B29)*(B5^B30)+B42*(B3^B43)*(B4^B44),B21)	47
21	F6(P,Q,S)	0.9745	B35*B32*B33+(1-B18)*B37*B31*B33-B7	RHS 50
22	VM	0.0000	B27*(B32^B28)*(B33^B29)*(B31^B30)+B42*(B32^B43)*(B33^B44)-B35*B32*B33-(1-B18)*B37*B31*B33+B7	50
23	SP1	0.0000	B28*B27*(B32^B28)*(B33^B29)*(B31^B30)+B43*B42*(B32^B43)*(B33^B44)-B35*B32*B33	51
24	SP2	0.0000	B29*B27*(B32^B28)*(B33^B29)*(B31^B30)+B44*B42*(B32^B43)*(B33^B44)-B35*B32*B33-(1-B18)*B37*B31*B33	52
25	SP3	0.0000	B30*B27*(B32^B28)*(B33^B29)*(B31^B30)-(1-B18)*B37*B31*B33	53
26	Q	0.0000		48
27	A6	37278.9426		
28	β6	6.0087		
29	γ6	9.2982		
30	η6	3.2895		
31	S^	**0.1992**		
32	P^	0.4000		
33	Q^	1.0000		
34	SOLVER	0.0000	Set B34=0, changing B27:B31	
35	P PV rP	13.7668		
36	Q PV rQ	13.7668		
37	S PV rS	13.7668		
38	PQrPQ	5.5067		
39	P^2Q^1rPC	30.3239		
40	T	20.0000		
41	Q	0.5*(B8^2)*B28*(B28-1)+0.5*(B9^2)*B29*(B29-1)+0.5*((B10^2)*(B30*(B30-1)))+B11*B8*B9*B28*B29+B12*B8*B10*B28*B30+B12*B9*B10*B30*B29+B28*B15+B29*B16+B30*B17-(B14+B18)		
42	A5	13.2811	13.28	
43	β5	4.4541		
44	γ5	4.4541		
45	P^5	0.6557	MV: No subsidy	
46	Q^	1.00		
47	ΔROV6,P	3.67	B28*B27*(B3^(B28-1))*(B4^B29)*(B5^B30)+B43*B42*(B3^(B43-1))*(B4^B44)	

Model VI is the solution to EQs 48-50-51-52-53 and ROV EQ 47, with the parameter values as follows: price P=€.40, quantity Q=1.0 KWh, S subsidy=€.10, investment cost K=€7.0, price volatility σ_P=.06, quantity volatility σ_Q=.04, subsidy volatility σ_S=.08, correlations $\rho_{PS}=\rho_{PQ}=\rho_{QS}=0$, $\theta_P=\theta_Q=\theta_S=0$, and riskless interest rate r=.04. λ=.10 reflects the possibility of a subsidy being withdrawn. S^ indicates the S threshold that justifies commencing the investment.

Figure 17

λ	0.10	0.15	0.20	0.25	0.30	0.35	0.40
ROV_6	0.3021	0.2732	0.2548	0.2429	0.2353	0.2314	0.2287
S^6	0.1992	0.2017	0.2078	0.2167	0.2280	0.2441	0.2627