Compete or Cooperate? Firms' Interactive Investment Decisions

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Abstract

This paper analyzes the optimal capital investment scale and timing in the context of real options, cooperative bargaining and network effects. Firms in the same industry often have related and interacting investment opportunities, such as the construction of a shared production facility. Each firm has a real option to delay investment, driven by price and quantity uncertainty. Accelerating investment is a first mover advantage arising from the ability of the first mover to customize the common facility to its own specification. Mitigating the desire to move first is the desire to capture network effects arising from the ability to spread the industry's public costs over a larger user base. The first mover (the leader) has to decide on the scale and timing of construction, as well as the optimal economic rent to charge the second mover for use of the common facility. The second mover (the follower) has to decide whether to use the first mover's facility or build its own facility, and if it decides to build its own, the optimal scale and timing of construction.

The analysis demonstrate that (i) the leader can improve its enterprise value by being cooperative, i.e., building excess production capacity and allowing joint usage. (ii) there is a non-monotonic concave relationship between each firms' reservation lease rate and the commodity price. This provides testable implications for understanding firms investment behavior under competition.

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JEL Classification: D43, G31, L13, L14

1 Introduction

Firms' optimal investment decisions under uncertainty has been a controversial topic for a long time due to the observed deviation from zero NPV threshold. The standard real options literature asserts that investments should be delayed until uncertainty is resolved or wait for the optimal threshold. The competitive real options literature argues that competition diminishes the real option values and mitigates investment delays, thus, with sufficient competition, firms' investment threshold may be pushed back to zero NPV.

Standard real option literature (Brennan and Schwartz (1985); Dixit and Pindyck (1994); Dixit (1995)) shows that firms should optimally delay the investment under uncertainty until a suitable threshold for price, demand or other stochastic variable is met. Myopic firms simply apply this standard model to decide the optimal time of investment without contemplating future ramifications of their current investment decisions. However, strategic firms will deliberate the interaction of real option investments among firms when their inputs or outputs are substitutable or complementary. There may be market power, patents, proprietary expertise or location that cause these interactions. In such settings, one firm's investment decision may influence the other firm's investment decision through various factors such as the first mover advantage and the economies of scale. For example, in the petroleum industry and the real estate industry, we often observe that firms compete to become the first mover by building significant excess production capacity even when the commodity price is fairly low, and even though they realize there is a real-option value to wait. The preemptive real options literature (Fudenberg and Tirole (1985); Smit and Ankum (1993); Grenadier (1996, 2002); Mason and Weeds (2010); Garlappi (2004); Boyer et al. (2004); Murto and Keppo (2002); Lambrecht and Perraudin (2003); Murto et al. (2004); Huisman and Kort (2004); Smit and Trigeorgis (2004)) explain this as a tradeoff between the real option value to delay and the first-mover advantage. They use the intersection of real options and industrial organization theory to analyze firms' strategic preemptive investment decisions. Most of these articles develop a Bertrand, a Cournot, or a Stackelberg equilibrium depending on the type of competition assumed. They argue that, in extreme case, sufficient competition may deteriorate the real option value and push firms' investment threshold back to zero NPV. However, Novy-Marx (2007) shows that supply side heterogeneity can reduce the competition effect and leads to an investment threshold even later than the standard real option threshold.

However, despite the substantial development of this literature, little attention has been paid to the firms' another critical choice, cooperation during strategic investment. Cooperation often creates positive externalities which can yield operating synergy. These operating synergy may be introduced as a form of shared infrastructure, a network effect, or a lower production cost structure and so on. A single firm may not have enough production volume to make the construction or the purchase of the production facility economically viable. If it can induce others to participate, the unit costs will fall and it will face a lower cost structure including the saved fixed cost of repetitive construction or purchase of certain production facility, lower unit production costs, lower marketing costs, or lower transportation costs paid to a third party. Alternatively, the operating synergy may come in the form of higher overall revenue because the cooperative investment may generate larger market demand or improve the quality of goods.

The positive externalities is modelled as the network effect in this paper and it is benefiting all the participating firms including the first mover and second mover. Another offsetting factor is the first-mover advantage that encourages early investment and can only be accrued to the first mover that builds or purchases a production facility, because it can build or purchase the facility based on its own specifications, and locational or functional preference. Moreover, once the facility is built, it can engage in a bargaining game with later movers in which it offers to lease access to its facility. The first mover has a trade-off between the rents it can earn on a high lease rate and the opportunity to capture network benefits by having the second mover enter early. Therefore, the strategic firms not only choose the optimal investment time, but also make decisions about the optimal investment size, whether to cooperate with the competitor by sharing the facility, and how much to charge the competitor for using the facility. Decisions on these investment issues can either create or destroy significant value, which makes them important for management. Such investment opportunities share similar characteristics and will be analyzed using real option theory and cooperative game theory in this paper.

This article studies the effect of interaction between firms' flexible investment decisions — the size (capacity choice) and timing of investment for certain industries by recognizing firms' capability of making strategic capacity choice, extracting rents from competitors, and taking the advantage of economical positive externalities (network effect) depending on the level of industry concentration. In fact, this article demonstrates an equilibrium real options exercise game in which the investment cash flows are not purely exogenous (solely relying on the market demand) to the firm, but somewhat endogenous in the sense that it is affected by the firm's capacity choice and the competitor reactions.

One typical application of this is the investment decisions of two adjacent gas producers. Their decisions consist of two stages. In the first stage, the natural gas price and the unproven initial reserves will determine who develops the land first and becomes the first mover. The first mover then has to decide on the optimal size and timing of construction depending on whether it plans to be cooperative or non-cooperative. As defined in later sections of this article, the cooperative producer recognizes the economies of scale and the positive network effect and thus will construct a larger production facility for sharing, whereas the non-cooperative producer will construct a smaller facility optimal for its own reserves. In the second stage, the first mover and the second mover play a sequential bargaining game to decide the optimal economic rent paid by the second mover to the first mover for use of the common facility. The second mover has to decide whether to use the first mover's facility or build its own facility, and if it decides to build its own, the optimal scale and timing of construction. By analyzing firms' behavior under a general setting of a sequential bargaining game of incomplete information in the presence of the positive externality, this article demonstrates that firms sometimes invest earlier than optimal and build excess production capacity not only for the preemptive effect or the first mover advantage, but also for being able to extract rent from the follower. Specifically, it shows that the leader can improve its enterprise value by being cooperative, i.e., building excess production capacity and leasing the excess capacity to the follower.

The objective of this article is to extend the current literature on real option exercise games by allowing size and timing decisions, as well as by incorporating the network effect into a dynamic bargaining game of incomplete information in terms of cooperation v.s. competition. It addresses the dynamics of optimal economic rents and capacity choice, given network effects, real options and incentives for preemption.

2 The Model for Oil and Gas Industry

2.1 ASSUMPTIONS OF THE MODEL

An important feature of oil and gas industry is that producers often own adjacent lands from which they may produce in the future. Therefore, we assume there two gas explorers, A and B, who have adjacent properties for gas exploration and production. This provides for an opportunity for joint use of infrastructure to exploit the resource. Our paper focuses on two such types of infrastructure, which typically have different ownership structures.

1. Gas processing plants remove liquids and hydrogen sulfide from the gas at the field before it can be safely shipped by pipeline. Gas plants are typically owned and operated by the first company to drill in a particular field, and they may build excess capacity and lease out that capacity to other producers in the same area.

2. Pipeline gathering systems are needed to ship the gas to central hubs, where they join the main line pipelines that distribute gas to consuming areas. These are typically owned by a company that specializes in pipelines, and it usually isn't a producer.

There are fixed costs in both of these types of infrastructure, which generates a network effect. A single gas producer may not have enough reserves to make a gas plant or pipeline connection economically viable. Also, if it can induce others to participate in the infrastructure, the unit costs will fall and it will tend to face a lower overall cost structure to produce its reserves. The first mover advantage accrues to the first company (the leader) that builds a gas processing plant to serve the field. The advantage arises because the leader can locate the plant nearest its part of the field and can customize the construction of the plant to be most efficient with the type of gas it owns. In the first stage, firms having similar size of initial reserves will invest simultaneously whereas if one firm has larger initial reserve, it will develop first and becomes the leader. In the second stage, once the plant is built, the leader can extract rents from the follower because of the fixed costs of building a competitive plant. However, the leader's efforts to extract rents are offset by its desire to have the follower agree to produce, thereby enabling the pipeline to be built or reducing the toll charges it has to pay the pipeline owner to induce it to build the pipeline. Also, there is a trade-off between the first-mover advantage for building the gas plant and the real options incentive to delay construction until more uncertainty about volumes and prices can be resolved. The leader decides the optimal plant capacity and the leasing fee. The follower decides whether to accept the leader's offer or wait to build its own processing plant.

$K(q_i^c)$ is the cost of constructing a gas plant with capacity of q_i^c which has fixed and variable components: $K(q_i^c) = a + bq_i^c$ where $i \in \{A, B\}$. The producers have the same construction parameters $a, b > 0$. There are two kinds of uncertainty, price uncertainty and production uncertainty which are going to affect firms' optimal investment scale and timing.

Price Uncertainty. The price of gas P is a source of economic uncertainty. We model it as $dP = \mu(P)dt + \sigma(P)dz_P$, where we assume the correlation between technical and economic uncertainty is zero: $\mathrm{corr}(dz_P, dz_A) = \mathrm{corr}(dz_P, dz_B) = 0$.

Production Uncertainty. The first is the technical uncertainty of the estimated quantity of reserves on the property. Let $Q_i(t)$ be producer i's expected remaining reserves conditional on information gathered to time t and production up to time t, and it evolves according to: $dQ_i = \mu_i(Q_i)dt + \sigma_i(Q_i)dz_i$, where $i \in \{A, B\}$ and the correlation is $\rho_Q = \mathrm{corr}(dz_A, dz_B)$. We can model the exponentially declining production volume as $q_i = \alpha_i Q_i$, where α_i is the production rate. Production does two things: (i) depletes the reservoir at rate q_i; (ii) provides information that causes revised information about total reserves. So $\sigma_i(q_i)$ is non-decreasing in q_i. Therefore, once the production starts, the remaining reserve quantity shall be modified as $dQ_i = -q_i dt + \sigma_i(q_i)dz_i$

In addition, there are two constraints on production. One is a regulatory or technical upper bound on the production rate, $\overline{\alpha}_i$. Regulators often restrict the production rate based on the natural maximum flow rate for the field depending on the porosity of the rock to avoid damaging the rock formation and having water floods, which could reduce the ultimate production from the field. The other is the capacity of the processing plant, q_i^c. Therefore, if there is one producer and one plant only, the production rate q_i must satisfy the constraint $q_i = \min\{q_i^c, \overline{q_i}$, where $\overline{q_i} = \overline{\alpha}_i Q_i$. At the start of production, the capacity is binding: $\frac{dQ_i(t)}{dt} = -q_i^c$, so $Q_i(t) = Q_i(\tau_i) - q_i^c t$ when $\tau_i \leq t \leq \theta_{i,\mathrm{trans}}$. τ_i is player i's production starting time, and $\theta_{i,\mathrm{trans}}$ is the transition time from the capacity constraint to the technology/regulatory constraint. After $\theta_{i,\mathrm{trans}}$, the regulatory constraint (α_i) is binding, so the remaining reserve will evolve as: $\frac{dQ_i(t)}{dt} = -\overline{\alpha}_i Q_i(t)$ Thus, producer

i's production function is

$$q_i(t) = \begin{cases} q_i^c & t \in [\tau_i, \theta_{i,\text{trans}}] \\ \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} & t \in [\theta_{i,\text{trans}}, \theta_i] \end{cases} \tag{1}$$

The remaining reserves continue to drop once the production starts. After producing for certain period of time, the remaining reserves will drop below a critical level at which it may be optimal to shut down the production when the profit is not enough to cover the variable production cost.

2.2 THE ISOLATED PLAYERS' INVESTMENT DECISIONS

Suppose that neither producer initially has a gas processing facility. If the producers' properties are not adjacent, the problem for each producer would be a classic two dimensional real option problem. The real option decisions are those that would be made by a monopolist owner of the project, without any consideration of interaction with the other producer. The optimal development option for producer $i \in \{A, B\}$ has a threshold $\{(P^*(Q_i), Q_i) \,|\, Q_i \in \mathbb{R}^+\}$ where $P_i^* : \mathbb{R}^+ \to \mathbb{R}^+$ is the threshold development price if the estimated reserves are Q_i. That is, producer i develops the first time $(P_t, Q_{i,t})$ are such that $P_t \geq P_i^*(Q_{i,t})$.

The cash flow for producer i at time t is $\pi_{i,t} : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}$ given by: $\pi_{i,t} = (P_t - C)q_{i,t}$ where C is the variable production cost. The expected payoff from an investment made by player i at time τ_i, is:

$$W_i(P, Q_i, \tau_i) = \widehat{E}_{\tau_i} \int_{\tau_i}^{\theta_i} e^{-rt} \pi_{i,t} \, dt - K(q_i^c) \tag{2}$$

This evolves according to geometric Brownian motion. It may be the case that the threshold can be simplified to a threshold level of cash flow π^*. Lambrecht and Perraudin (2003) discuss the possibility of a sufficient statistic to determine the threshold. But this is not necessarily the case, since the uncertainty and risk neutral growth rates in Q and P may not be the same, so that the (threshold) level of profit may vary over the threshold. These isolated producers are non-cooperative in the sense that they do not have to consider the strategic effect from the investments by the competitors. As P and Q are assumed uncorrelated, generally, these non-cooperative firms' value of the investment opportunity(real option values) $V\big(W(P,Q),t\big)$ must satisfy the valuation PDE:[1]

$$\begin{gathered}\frac{1}{2}\left[\sigma^2(Q)V_{QQ}(P,Q)+\sigma^2(P)V_{PP}(P,Q)\right]+\\ V_Q(P,Q)\mu(Q)+V_P(P,Q)\left[\mu(P)-\lambda_P\beta(P)\right]+V_t=rV(P,Q)\end{gathered}\tag{3}$$

and the value-matching and smooth pasting boundary conditions:[2]

$$\begin{aligned}V(P^*,Q^*)&=W(P^*,Q^*)\\ V_P(P^*,Q^*)&=\frac{\partial W_i}{\partial P_{\tau_i}}=\widehat{E}_{\tau_i}\Bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}}e^{(\hat{\mu}(P)-r)(t-\tau_i)}q_i^c dt\\ &\qquad+\int_{\theta_{i,\text{trans}}}^{\theta_i}e^{(\hat{\mu}(P)-r)(t-\tau_i)-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}\overline{\alpha}_iQ_i(\theta_{i,\text{trans}})dt\Bigg]-K(q_i^c)\\ V_Q(P^*,Q^*)&=\frac{\partial W_i}{\partial Q_i}=\widehat{E}_{\tau_i}\left[\int_{\theta_{i,\text{trans}}}^{\theta_i}e^{-r(t-\tau_i)}(P_t-C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\right]-K(q_i^c)\end{aligned}\tag{4}$$

Notice here risk-neutral drift of P is: $\mu(P,t)-\lambda_P\beta(P)=rP-\delta(P,t)$, where $\beta(P)=\frac{\text{cov}(dP,d\tilde{f})}{\sqrt{\text{var}(dP)\text{var}(d\tilde{f})}}$, r is the risk free interest rate, δ is the convenience yield of the underlying asset. λ is the risk premium for the systematic risk factor $\tilde{f}$, and $\tilde{f}$ is some systematic risk factor such that the investment asset is expected to earn a risk premium in proportion to the covariance between asset price changes and the risk factor. Similarly, the risk-neutral drift of Q is: $\mu(Q)-\lambda_Q\beta(Q)=-q_t$, where $\beta(Q)=\frac{\text{cov}(dQ,d\tilde{f})}{\sqrt{\text{var}(dQ)\text{var}(d\tilde{f})}}=0$ because the production rate $q_t=0$ is zero before the initial investment. Equation (3) with the boundary conditions, equation (4) can be easily solved numerically as Section 5 will demonstrate.

2.3 THE ADJACENT PLAYERS' INVESTMENT DECISIONS

The cooperative producers will follow a symmetric, subgame perfect equilibrium entry strategies in which each producer's exercise strategy maximizes value conditional upon the other's exercise strategy, as in Kreps and Scheinkman (1983); Garlappi (2004). The solutions have two different exercise models: simultaneous and sequential exercise.

[1]This is an extension of the classic model of operating real options by Brennan and Schwartz (1985) to finite reserves.

[2]See appendix A for detailed derivation of the two smooth-pasting conditions.

2.3.1 *Equilibrium under simultaneous exercise*

Suppose both producers exploration reveal that their initial reserve quantity are same. Denote F as the follower, and L as the leader, $F, L \in \{A, B\}$. In this case, $P_A^*(Q_A) = P_B^*(Q_B) = P_F^*(Q_F) = P_L^*(Q_L)$, producers' have the same trigger price. Therefore once the price hits the trigger, they both want to exercise the real option and build their own plant immediately. Whoever moves faster becomes the natural leader. Given that the prices P and quantities Q_A, Q_B are continuously distributed and not correlated, this is a knife-edge condition that only occurs with probability zero if the producers do not interact.

However, when their properties are adjacent, they can interact. The leader can build a plant large enough to process both producers' gas and offer a processing lease rate to the follower. The follower can accept the offer and process its gas in the leader's plant, or build its own processing plant right away. If they are cooperative, they would exercise simultaneously and play a bargaining game at that time to determine the lease rate l and plant capacity q_L^c. We define the follower in this simultaneous exercise case as a *big follower*, denoted as F_b. For simplicity, we assume that they both commit not to renegotiate the lease later.

2.3.2 *Equilibrium under sequential exercise*

Suppose the leader has a larger initial reserve and therefore lower optimal trigger price $P^*(Q_L)$. In this case, $P_L^*(Q_L) < P_F^*(Q_F)$ for $L, F \in \{A, B\}, L \neq F$. The leader will enter alone, building a gas processing plant to cover its own production only. Once its production volumes decline, it will offer excess capacity to the follower at a lease rate l to be negotiated, bearing in mind the follower's reservation cost of building its own plant.

Thus, there is a bargaining game played at and after the time the leader decides to build the plant. This game determines whether the follower starts production at the same time or delays. If the follower accepts the lease, both producers start production simultaneously and the game ends. If the follower rejects the lease, they play the same sequential bargaining game at subsequent dates, where the leader offers a lease rate and capacity, and the follower decides whether to accept the offer, build its own plant or delay further. We define the follower in this sequential exercise case as a *small follower*, denoted as F_s.

3 The Sequential Bargaining Game under Incomplete Information for Adjacent Players

One significant difference between our paper and other option exercise game papers is that we model the expected payoff $W(P, Q, q_i^c, l, t; N)$ as a result of lease vs. build (exercise the real option of investment) bargaining game when the two producers have adjacent properties in the presence of the network effect N. This bargaining game is a dynamic game of incomplete information as the leader does not have the information about the follower's payoffs function.

Denote τ_L as the first time (P, Q_L) hits the threshold $\big(P^*(Q_L), Q_L\big)$. The follower also solves for a threshold trigger price $P^*(Q_F)$ that determines the optimal condition under which it would build its own plant and start production. Denote the first hitting time to the threshold $\big(P^*(Q_F), Q_F\big)$ by the stopping time $\tau_F \in [\tau_L, \infty)$. Hence the big follower exercises at τ_{F_b} and $\tau_{F_b} = \tau_L$ because the big follower's initial reserve is of the same size as the leader's. The small follower exercises at $\tau_{F_s} > \tau_{F_b}$ because the small follower's initial reserve is smaller than the big follower. The lease will start at $\tau_{\text{lease}} \in [\tau_L, \tau_{F_s}]$. The leader's maximum production time is θ_L. The big or small follower's maximum production time is θ_{F_b} or θ_{F_s} respectively.

We now formally construct this sequential bargaining game under incomplete information. There are two players in the game, the leader and the follower. The product to be traded is the leader's (the seller) excess processing capacity. The quantity of product to be traded is the contracted fixed lease production capacity per unit of time q_{FL}. The network effect is the gain from cooperation. The transfer is the leasing fee l from the follower to the leader. The leader knows its cost of providing the excess capacity $K(\cdot)$. The follower has private information about its valuation $l_F \in \{\underline{l}_F, \bar{l}_F\}$. As shown in Section 3.3, the benefit from bargaining with the leader is smaller for the big follower than for the small follower. Hence, there are two types of buyers, the low type buyer (the big follower, F_b) who values the lease at $\underline{l}_F$ and the high type buyer (the small follower, F_s) who values the lease at $\bar{l}_F$. The leader does not know what type of buyer the follower is. Therefore, there is a conflict between efficiency (the realization of the gain from cooperation) and rent extraction in mechanism design. The leader's strategy space is to offer the lease at either $\underline{l}_F$, or $\bar{l}_F$. The follower's strategy space is to either accept or reject the leader's offer. (Note that one could certainly argue there is another way of designing the bargaining mechanism where the leader

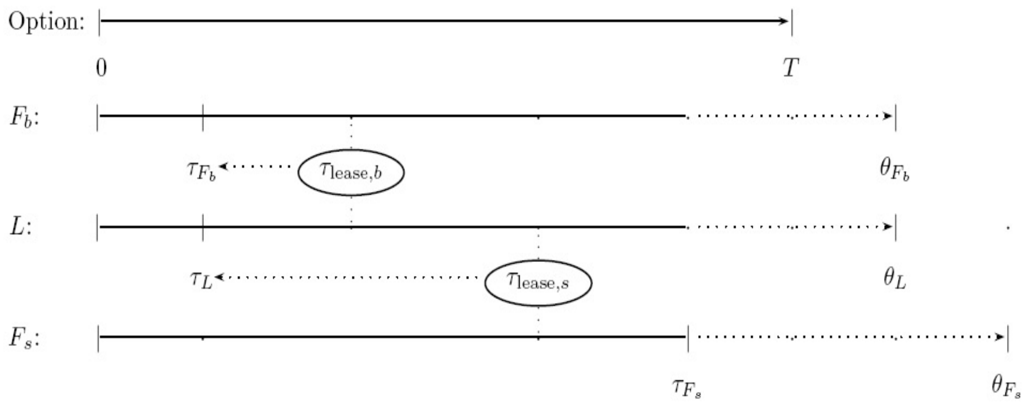


Figure 1: **The leader and the follower's timeline**

provide two types of contracts, $\{\underline{l}_F, q_{F_BL}\}$ and $\{\bar{l}_F, q_{F_SL}\}$, where $\underline{l}_F < \bar{l}_F$ and $q_{F_BL} > q_{F_SL}$. This is a bargaining game on both the lease rate and the lease quantity. However, for the sake of simplicity and without loss of generality, we decide to focus on the bargaining on the lease rate l only, in which $q_{F_BL} = q_{F_SL} = q_{\text{FL}}$, which leads to $\int_{\tau_{F_b}}^{\theta_{F_b}} t dt > \int_{\tau_{F_s}}^{\theta_{F_s}} t dt$ since the F_b has larger initial reserve.) If the follower accepts, the game ends. If the follower rejects, the leader will make another offer in the next period. The decision variables are the leasing rate l, the cooperative and non-cooperative plant capacity choices q_L^Ω, or q_L^c and q_F^c, which determine the construction costs $K(q_L^\Omega)$, or $K(q_L^c)$, $K(q_F^c)$ and production volumes q_L and q_F. The exogenous variables are the stochastic gas price P, the expected reserve quantities at the time of construction, Q_L and Q_F as assumed in Section 2.1, and the network effect N. The timing of the game is illustrated in Figure 1.

3.1 THE ADJACENT PLAYERS' BELLMAN EQUATION

In this game, the adjacent players will maximize their own total enterprise values by optimally controlling their respective capacity choices q_L^c, q_F^c and the lease rate l, given the two stochastic variables P and Q_i that evolve over time, and the exogenous network effect N. The adjacent player i's Bellman equation can be stated as:

$$U_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N) = \max_{\{P_{\tau_i}, Q_{\tau_i}\}} \left\{ E_{i,t}\Big[V_{i,t+1}(P_{t+1}, Q_{i,t+1})\Big], \quad \max_{\{q_{i,t}^c, l_t\}} W_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N) \right\} \quad (5)$$

$U_{i,t}(P_t, Q_{i,t}, q_{i,t}^c, l_t; N)$ is the total enterprise value for player i and has two components, the real option value for the investment opportunity $V_{i,t}$ and the pure asset value of the property $\text{AV}_{i,t}$.

The maximization of $V_{i,t}$ is a mixture of deterministic and stochastic optimal control problem for $\{P_{\tau_i}, Q_{\tau_i}, q^c_{i,t}, l_t\}$. The maximization of $\mathrm{AV}_{i,t}$ is a deterministic optimal control problem for $\{q^c_{i,t}, l_t\}$. The real option value $V_{i,t}$ still has to satisfy the stochastic PDE equation (3), but the boundary conditions are different because the optimal trigger $W^*(P_t, Q_{i,t}, q^c_{i,t}, l_t; N)$ will be determined by the equilibrium of the game. The real option will expire at time T.

The optimal trigger threshold $(P^*(Q_i), Q_i)$ which solves equation (5) for the non-cooperative producers can be achieved by different combinations of the price and expected reserves $(P^*(Q_i), Q_i)$. The players enter when (P, Q_i) first moves above the threshold $P^*(Q_i)$ so that $P_{\tau_i} \geq P^*(Q_i)$. The optimization of player i's non-cooperative enterprise value, $U_{i,\mathrm{nc}}(P, Q_i, q^c_i; N_{\tau_i})$ is done in two steps.

- Step 1: Solve for $q^{c*}_i = \mathrm{argmax}_{q^c_i} U_{i,\mathrm{nc}}(P, Q_i, q^c_i; N_{\tau_i})$. The solution is $q^{c*}_i(P, Q_i)$
- Step 2: Use $U_{i,\mathrm{nc}}\big(P, Q_i, q^{c*}_i(P, Q_i); N_{\tau_i}\big)$ to solve for the threshold $P^*_{\tau_i}(Q_i)$

In the presence of strategic effect from the competitor, equation (5) for player $i \in \{L, F\}$ have to be solved jointly because the bargaining game and the real option to invest mutually affect each other. The equilibria of the game affect the expected payoff $W(P, Q, q^c_i, l, t; N)$, which affects the optimal exercise trigger of the option. Conversely, the exercise of option which determines the value of P^* and Q^*_i, will affect the expected payoff $W(P^*, Q^*, q^{c*}_i, l^*, t; N)$ which further affects the refinement of the players' strategy space and hence the equilibria of the game.

3.2 THE NETWORK EFFECT — GAINS FROM COOPERATION

The network effect N is modeled as the reduction in pipeline tolls, one component of the production cost that affects players' cash flow. Economy of scale and network effect of pipeline arise because the average cost of transporting oil or gas in a pipeline decreases as total throughput increases. There are two categories of costs for pipelines which generate network effect.

1. Long-run fixed operating costs: The cost of monitoring workers is a long-run fixed cost due to the indivisibility of workers – a minimum number of monitoring workers is required. This cost is fixed as it is independent of throughput.

2. Capital investment cost

- *Setup costs:* The expenses associated with the planning, design and installation of pipeline, and the right of way are fixed setup costs.
- *Volumetric returns to scale:* The costs of steel are proportionate to the surface area. The capacity of the pipeline depends on its volume and the amount of horsepower required. The amount of horsepower required is determined by resistance to flow, which is decreasing in the diameter of the pipeline.

Among these two cost categories, if the total throughput increasing, the long-run fixed operating costs per unit of throughput will decrease, which generates the category 1 network effect N^1. N^1 is monotonic increasing when the total throughput increasing. Hence, producers will get N^1 only when they both produce. In addition, setup costs and volumetric return to scale will generate the category 2 network effect N^2 if the pipeline company is strategic and can anticipate the future exercise of both players. If the pipeline company observes a higher probability that players will be producing together for a certain period of time, it may build a larger pipeline to accommodate both of them. Thus, the producers will get N^2 if the producers can make a commitment to a larger throughput volume.

The pipeline company has to decide whether to build and, if it builds, what the capacity and toll rate should be. For simplicity, we will assume that, based on the information about both producer's initial reserve Q_L, Q_F and production rate q_L, q_F, the pipeline company can estimate and build a pipeline to accommodate the non-cooperative total transportation throughput, $(q_{L,\text{nc}} + q_{F,\text{nc}})$, for the leader and the follower.

$$\text{The actual non-cooperative pipeline throughput} \\ = \begin{cases} q_{L,\text{nc}}(t) & \text{when } t < \tau_F; \\ q_{L,\text{nc}}(t) + q_{F,\text{nc}}(t) & \text{when } t \geq \tau_F. \end{cases}$$

This results a higher pipeline toll rate for the leader before τ_F,[3] and a lower pipeline toll rate (category 1 network effect N^1) for both producers after τ_F as the total throughput transported increases. If the lease contract is negotiated successfully at $\tau_{\text{lease}} < \tau_F$ or even simultaneously at τ_L, the pipeline company sees the producers' commitment, and it will construct a larger pipeline to

[3] We will suppress the subscript B and S for F if we are not differentiating the F_b from the F_s in the context.

accommodate this larger cooperative total throughput, $q_{L,\text{coop}}(\tau_{\text{lease}}) + q_{F,\text{coop}}$, which will generate the category 2 network effect, N^2.

$$\begin{aligned} &\text{The actual cooperative pipeline throughput} \\ &= \begin{cases} q_{L,\text{coop}}(t) & \text{when } t < \tau_{\text{lease}}; \\ q_{L,\text{coop}}(t) + q_{F,\text{coop}}(t) & \text{when } t \geq \tau_{\text{lease}}. \end{cases} \end{aligned}$$

3.3 THE FOLLOWER'S PARTICITPATION CONSTRAINT

For a small follower, it can either lease the capacity from the leader at τ_{lease}, or delay further until τ_{F_s} to build its own plant. The small follower gets the network effect in both cases. The difference is that if it chooses to build its own plant, the benefit of network effect comes only after τ_{F_s} and will end at θ_L when the leader's production ends. Denote this network benefit for small follower that builds its own plant as $N^{\theta_L}_{\tau_{F_s}} = N \cdot \int_{\tau_{F_s}}^{\theta_L} q_{Lt}\, dt$. If it chooses to lease, the lease contract may allow the small follower to start production earlier than τ_{F_s} and small follower will get the network effect in the interval $[\tau_{\text{lease}}, \theta_L]$. Denote this network benefit for the small follower who leases the plant as $N^{\theta_L}_{\tau_{\text{lease}}} = N \cdot \int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\, dt$. Clearly, $N^{\theta_L}_{\tau_{\text{lease}}} > N^{\theta_L}_{\tau_{F_s}}$ as $\tau_{\text{lease}} < \tau_{F_s}$. Therefore, for small follower, the lease contract not only saves its capital investment,[4] but also increases the total amount of network effect received. The small follower will make the comparison of $U_{F_s,\text{nc}}$ and $U_{F_s,\text{coop}}$ at the date after τ_L whenever the leader offers a lease at rate l. Thus, we have the small follower's *participation constraint*:

$$U_{F_s,\text{coop}}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) \geq U_{F_s,\text{nc}}(P, Q_{F_s}, q^{c*}_{F_s}; N^{\theta_L}_{\tau_{F_s}}) \tag{6}$$

which defines the high type buyer's valuation of lease:

$$\bar{l}_F \equiv \sup\Big\{ l_F \in \mathbb{R}^+ : U_{F_s,\text{coop}} \geq U_{F_s,\text{nc}}|_{q^c_{F_s} = q^{c*}_{F_s}} \Big\} \tag{7}$$

[4]The annual cost of owning an asset over the its entire life is calculated as $\text{EAC}\big(K(q^c_F)\big) = \frac{K(q^c_F)r}{1-(1+r)^{-n}}$.

Similarly, the big follower's *participation constraint* is derived as:

$$U_{F_b,\text{coop}}(P, Q_{F_b}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}=\tau_{F_b}}) \geq U_{F_b,\text{nc}}(P, Q_{F_b}, q^{c*}_{F_b}; N^{\theta_L}_{\tau_{F_b}}) \tag{8}$$

For the big follower, $N^{\theta_L}_{\tau_{\text{lease}}} = N^{\theta_L}_{\tau_{F_b}}$, the lease does not increase its total amount of network effect received, only saves its capital cost. Hence, the low type buyer's valuation of lease:

$$\underline{l}_F \equiv \sup\left\{ l_F \in \mathbb{R}^+ : U_{F_b,\text{coop}} \geq U_{F_b,\text{nc}}|_{q^c_{F_b}=q^{c*}_{F_b}} \right\} \tag{9}$$

Notice the right hand sides of equation (7) and (9) are optimized over q^c_F, which means $U_{F,\text{coop}}$ has to be greater than $U_{F,\text{nc}}$ when the follower builds the optimal capacity for itself. Since $U_{F,\text{coop}}$ is decreasing in l, when equation (7) and (9) are binding, they determine a reservation lease rate $\bar{l}_F$ or $\underline{l}_F$ for the small follower or the big follower respectively.

3.4 THE LEADER'S PARTICIPATION CONSTRAINTS

At τ_L, the leader has a non-cooperative optimal capacity q^c_L which maximizes its total non-cooperative enterprise value $U_{L,\text{nc}}(P, Q_L, q^c_L; N^{\theta_L}_{\tau_F})$, where $N^{\theta_L}_{\tau_F} = N \cdot \int_{\tau_F}^{\theta_L} q_{Lt}\ dt$.

$$q^{c*}_L = \underset{q^c_L}{\text{argmax}}\, U_{L,\text{nc}}(P, Q_L, q^c_L; N^{\theta_L}_{\tau_F})$$

Here a non-cooperative leader is a leader who does not consider the future possibility of leasing excess capacity to the follower. Thus $U_{L,\text{nc}}$ function does not involve a lease rate l. The network effect $N^{\theta_L}_{\tau_F}$ occurs when the follower's production starts at τ_F and ends at θ_L. This is different from the leader's cooperative enterprise value $U_{L,\text{coop}}(P, Q_L, q^{\Omega}_L, l; N^{\theta_L}_{\tau_{\text{lease}}})$ as defined in equation (5), where $N^{\theta_L}_{\tau_{\text{lease}}} = N \cdot \int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\ dt$. This early network effect $N^{\theta_L}_{\tau_{\text{lease}}}$ occurs when the follower's production starts at τ_{lease} and ends at θ_L. We now define the leader's cooperative optimal capacity as:

$$q^{\Omega*}_L = \underset{q^{\Omega}_L}{\text{argmax}}\, U_{L,\text{coop}}(P, Q_L, q^{\Omega}_L, l; N^{\theta_L}_{\tau_{\text{lease}}}) \qquad \text{st.} \quad \tau_{\text{lease}} \leq \tau_F \tag{10}$$

The leader will build cooperative capacity if the following *participation constraint I* ($\mathrm{IR_I}$) is satisfied:

$$U_{L,\mathrm{coop}}(P, Q_L, q_L^{\Omega*}, l; N^{\theta_L}_{\tau_{\mathrm{lease}}}) \geq U_{L,\mathrm{nc}}(P, Q_L, q_L^{c*}; N^{\theta_L}_{\tau_F}) \tag{11}$$

The leader's reservation lease rate is defined as

$$\underline{l}_L \equiv \inf\left\{ l_L \in \mathbb{R}^+ : U_{L,\mathrm{coop}} \geq U_{L,\mathrm{nc}}\big|_{q_L^{\Omega}=q_L^{c*}} \right\} \tag{12}$$

Moreover, the leader's additional cost of building extra capacity $(q_L^{\Omega} - q_L^{c})$ has to be compensated by the present value of all future leasing fees, plus the benefit difference between $N^{\theta_L}_{\tau_{\mathrm{lease}}}$ and $N^{\theta_L}_{\tau_F}$, i.e., the leader's *participation constraint II* ($\mathrm{IR_{II}}$):

$$\begin{aligned}
&\int_{\tau_{\mathrm{lease}}}^{\theta_F} e^{-rt}(q_{\mathrm{FL}} \cdot l)\ dt + \left(N^{\theta_L}_{\tau_{\mathrm{lease}}} - N^{\theta_L}_{\tau_F}\right) \geq b \cdot (q_L^{\Omega} - q_L^{c}) \\
\Rightarrow &\int_{\tau_{\mathrm{lease}}}^{\theta_F} e^{-rt}(q_{\mathrm{FL}} \cdot l)\ dt + N \cdot \int_{\tau_{\mathrm{lease}}}^{\tau_F} q_{Lt}\ dt \geq b(q_L^{\Omega} - q_L^{c})
\end{aligned} \tag{13}$$

If inequalities (11) and (13) are binding, they determine the leader's cooperative capacity q_L^{Ω} and the lease rate l. Otherwise, they set the upper bound for q_L^{Ω} and lower bound for l.

If the follower is the high type F_s, the leader obtains an increase in network effect. Equation (13) then becomes:

$$\int_{\tau_{\mathrm{lease}}}^{\theta_{F_s}} e^{-rt}(q_{\mathrm{FL}} \cdot \bar{l}_F)\ dt + N \cdot \int_{\tau_{\mathrm{lease}}}^{\tau_{F_s}} q_{Lt}\ dt \geq b(q_L^{\Omega} - q_L^{c}) \tag{14}$$

If the follower is the low type F_b, the leader obtains no increase in network effect by encouraging F_b to lease because $\tau_{F_b} = \tau_L$, and $\tau_{\mathrm{lease}} = \tau_L \Longrightarrow N^{\theta_L}_{\tau_{\mathrm{lease}}} = N^{\theta_L}_{\tau_F}$. If F_b accepts the lease, it saves the capital cost of $K(q^c_{F_b})$. Equation (13) then becomes:

$$\int_{\tau_{\mathrm{lease}}=\tau_L=\tau_{F_b}}^{\theta_{F_b}} e^{-rt}(q_{\mathrm{FL}} \cdot \underline{l}_F)\ dt \geq b(q_L^{\Omega} - q_L^{c}) \tag{15}$$

In other words, $\bar{l}_F$ and $\underline{l}_F$ defined in equation (7) and (9) have to satisfy equation (14) and (15) respectively, in order to give the leader enough motivation to build extra capacity.

Also, it makes no sense for the leader to build cooperative capacity that cannot be used when production is at a maximum, so $q_L^\Omega \le \overline{q}_{L,\tau_L} + \overline{q}_{F,\tau_L} = \overline{\alpha}_L Q_{L,\tau_L} + \overline{\alpha}_F Q_{F,\tau_L}$. If this inequality is strict, the joint production is constrained until the leader and follower have produced enough so that their combined maximum production rate is below the plant capacity. The leader's cooperative capacity has to be at least as large as its own maximum production rate, i.e., $q_L^\Omega \ge q_{L,\tau_L}$.

3.5 THE LEADER'S CONTROL SET $\{q_L^\Omega, l\}$

Recall that q_L and q_F[5] are defined as the leader's and the follower's production volume respectively, q_L^c is the leader's non-cooperative capacity and $\overline{\alpha}_L$ and $\overline{\alpha}_F$ are the maximum production rates that are set by a regulator or technological constraints. From equation (1) we have the non-cooperative leader and follower's production function as:

$$q_{L,\mathrm{nc}}(t) = \begin{cases} q_L^c & t \in [\tau_L, \theta_{L,\mathrm{trans}}] \\ \overline{\alpha}_L Q_L(\theta_{L,\mathrm{trans}}) e^{-\overline{\alpha}_L (t-\theta_{L,\mathrm{trans}})} & t \in [\theta_{L,\mathrm{trans}}, \theta_L] \end{cases} \tag{16}$$

and

$$q_{F,\mathrm{nc}}(t) = \begin{cases} q_F^c & t \in [\tau_L, \theta_{F,\mathrm{trans}}] \\ \overline{\alpha}_F Q_F(\theta_{F,\mathrm{trans}}) e^{-\overline{\alpha}_F (t-\theta_{F,\mathrm{trans}})} & t \in [\theta_{F,\mathrm{trans}}, \theta_F] \end{cases} \tag{17}$$

After $\theta_{L,\mathrm{trans}}$, the non-cooperative leader's capacity is not binding, and it can offer the follower its excess processing capacity $q_L^c - q_L$ providing the follower has not built its own plant yet.

Thus, we have the cooperative follower's production volume under leasing:

$$q_{F,\mathrm{coop}} = \min\{q_L^c - q_L,\ \overline{\alpha}_F Q_F\}$$

Suppose that there is asymmetric information about the leader's and follower's initial reserves. The leader can only make an estimation about the follower's expected initial reserve quantity Q_F and maximum production rate and $\overline{\alpha}_F$. Based on this estimation, the leader builds a gas plant which can process the amount $q_L^\Omega \ge q_{L,\mathrm{coop}} + q_{F,\mathrm{coop}}$ per unit of time. The results of the bargaining game depend on the amount information of available to the leader and the follower. The cooperative leader will estimate both producers' needs and builds a gas plant with capacity $q_L^\Omega \ge q_L^c$. Therefore,

[5] For notation simplicity, we suppress the subscripts S and B for F in this subsection as F_b and F_s's production functions share the same functional form.

the above production functions becomes:

$$
\begin{aligned}
q_{F,\text{coop}} &= \min\{q_L^{\Omega} - q_{L,\text{coop}},\ \overline{\alpha}_F Q_F\} \\
0 \leq q_{L,\text{coop}} &\leq \min\{q_L^{\Omega}, \overline{\alpha}_L Q_L\}
\end{aligned}
\tag{18}
$$

The cooperative leader has an excess capacity of $q_L^{\Omega} - q_{L,\text{coop}}$, which will increase as the leader's production volume $q_{L,\text{coop}}$ falls over time. Assume that the cooperative follower will use all the capacity offered in the lease until reserves drop to constrain the production rate. That is, $q_{F,\text{coop}} = \min\{q_{\text{FL}}, \overline{\alpha}_F Q_F\}$. Once excess capacity reaches the contracted leasing capacity q_{FL} at τ_{lease}, the lease can start. The cooperative production function is:

$$
q_{L,\text{coop}}(t) = \begin{cases} q_L^{\Omega} & t \in [\tau_L, \theta_{L,\text{trans}}] \\ \overline{\alpha}_L Q_L(\theta_{L,\text{trans}}) e^{-\overline{\alpha}_L (t - \theta_{L,\text{trans}})} & t \in [\theta_{L,\text{trans}}, \theta_L] \end{cases}
\tag{19}
$$

and

$$
q_{F,\text{coop}}(t) = \begin{cases} q_{\text{FL}} & t \in [\tau_{\text{lease}}, \theta_{F,\text{trans}}] \\ \overline{\alpha}_F Q_F(\theta_{F,\text{trans}}) e^{-\overline{\alpha}_F (t - \theta_{F,\text{trans}})} & t \in [\theta_{F,\text{trans}}, \theta_F] \end{cases}
\tag{20}
$$

The cooperative leader's choices about q_L^{Ω} and l will have opposite effects on τ_{lease}. On one hand, the cooperative leader can control an early or late τ_{lease} by controlling the size of its cooperative capacity q_L^{Ω}. When q_L^{Ω} is larger, the lease can happen earlier. The earlier lease will allow the cooperative leader to benefit from the network effect earlier than τ_F. The incremental benefit of this earlier network effect is calculated as $N\big(\int_{\tau_{\text{lease}}}^{\theta_L} q_{Lt}\ dt - \int_{\tau_F}^{\theta_L} q_{Lt}\ dt\big)$ in equation (13). On the other hand, the cooperative leader wants to charge the follower the highest leasing rate up to $\overline{l}_F$ for a small follower or $\underline{l}_F$ for a big follower as defined by equation (7) and (9).[6] Thus, the lease offer is inversely related to the time the lease is accepted. The cooperative leader's objective is to find a balance among the incremental network effect benefit, the earlier leasing fee, and the extra construction costs of $q_L^{\Omega} - q_L^c$, bearing in mind the fact that a high lease rate will cause the follower to delay. Denote this equilibrium leader's cooperative capacity as $q_L^{\Omega *}$ which gives the leader largest total enterprise value and also ensures $\tau_{\text{lease}}^* \leq \tau_F$, as defined in equation (10).

[6]In fact, this is the standard way of extracting rents through price discrimination without losing the efficiency.

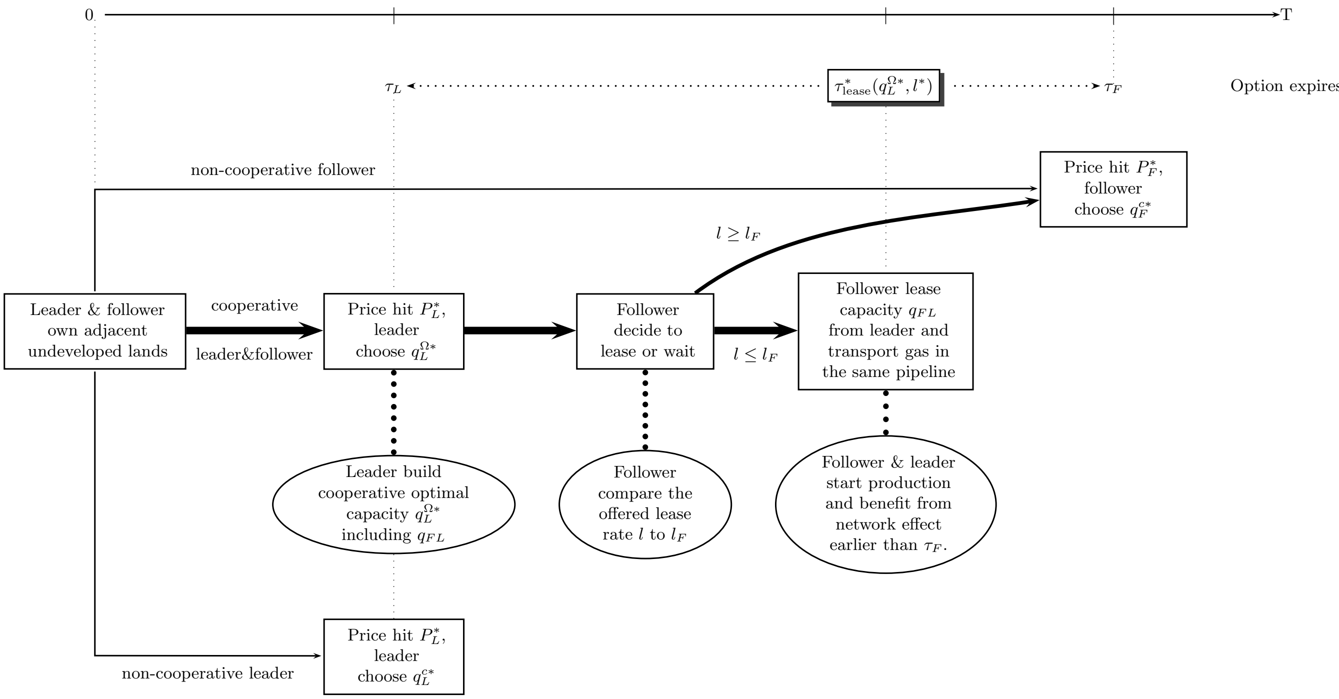


Figure 2: **The leader and the follower's strategy map with timeline**

In addition, both the leader and the follower will have to consider how much pipeline space to request and the term of the request. If the producer(s) commit(s) to a larger volume or longer-term contract, the pipeline toll rates will be even smaller, generating a category 2 network effect as discussed in Section 3.2. The leader and the follower's strategy map is shown in Figure 2.

4 The Perfect Bayesian Equilibrium

We will extend the backward induction solution for a real option to this game theory setting as in Grenadier (1996); Garlappi (2004); Murto et al. (2004). This provides a simple computation of a subgame-perfect Nash equilibrium. After explicitly analyzing the player's beliefs, i.e., ruling out incredible threats and promises, we develop a perfect Bayesian equilibrium for this dynamic bargaining game under incomplete information using Coasian Dynamics as discussed in Fudenberg and Tirole (1991, Ch 10). We assume the leader is chosen exogenously, because one of the two companies has a comparative advantage for entering early (e.g. has a larger reserve[7] or a reserve that has lower drilling costs), and that it naturally moves first.

The enterprise value of the leader (plant lessor or "seller") is common knowledge. The incomplete information aspect of the sequential bargaining is limited to the uncertainty the leader faces about the reservation lease rate of the follower (buyer). As defined in equation (7) and (9), the high type buyer F_s has a reservation lease rate of $\bar{l}_F$ and the low type buyer F_b has a reservation lease rate of $\underline{l}_F$. If the high type buyer tells the truth, its total enterprise value is $U_{F_s}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$. If the high type buyer lies successfully, its total enterprise value is $U_{F_s}(P, Q_{F_s}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$. Since $\bar{l}_F > \underline{l}_F$ and U_{F_s} decreases on l we have

$$U_{F_s}(P, Q_{F_s}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) < U_{F_s}(P, Q_{F_s}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}}) \tag{21}$$

Thus, the high type buyer F_s is motivated to pretend to be the low type buyer F_b. In addition, notice that the follower's valuation is correlated with the leader's cost. A larger plant will allow the lease to start earlier, because of the extra capacity as noted in the discussion after equation (18). This makes a more valuable network effect, which increases the follower reservation lease rates $\bar{l}_F$ and $\underline{l}_F$. But a larger plant also incurs larger construction costs. The leader's objective is to extract maximum rents through price discrimination without losing efficiency. The leader wants the follower to accept the lease offer so that the network effect is larger.

We now consider the equilibrium of this game in a two period case. Let $t \in \{t, t+1\}$. The ex ante unconditional probability that the follower is high type (F_s) is $\overline{p}$, and $\underline{p} = 1 - \overline{p}$ is the probability that the follower is low type (F_b).

The leader offers lease rates l_t and l_{t+1} at time t and time $t+1$, respectively. Let $\overline{\eta}(l_t)$ denote the leader's posterior probability belief that the follower is high type (F_s) conditional on the rejection of offer l_t in period t, and define $\underline{\eta}(l_t) \equiv 1 - \overline{\eta}(l_t)$. The extensive form representation of this sequential bargaining game is shown in Figure 10 in Appendix B.

Definition 1. *Define the leader's critical probability as* $\chi \equiv \frac{U_L(\underline{l}_F)}{U_L(\bar{l}_F)}$.

In the last period $t+1$, the leader with probability belief $\overline{\eta}(l_t)$ makes a "take it or leave it" offer

[7] In Section 5, we shall see that larger reserve quantity will subsidize the trigger price, which gives a smaller trigger value $P^*_i(Q_i)$ and $i \in \{A, B\}$.

l_{t+1}. The follower will accept if and only if this l_{t+1} is not greater than its reservation lease rate.

Theorem 1. *The followers' optimal strategies at date $t+1$ is given by:*

$$\text{If } l_{t+1} = \begin{cases} \underline{l}_F, & \text{then } F_s, F_b \text{ both accept} \\ \bar{l}_F, & \text{then } F_s \text{ accepts}, F_b \text{ rejects} \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \text{then } F_s \text{ accepts}, F_b \text{ rejects} \end{cases} \tag{22}$$

If the leader offers $l_{t+1} = \underline{l}_F$, both type followers will accept, the leader obtains the enterprise value of $U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \underline{l}_F; N_{\tau_{\text{lease}}}^{\theta_L})$, simplified as $U_L(\underline{l}_F)$. If the leader offers $l_{t+1} = \bar{l}_F$, only the high type follower accepts, so the leader has second period enterprise value of $\overline{\eta} \cdot U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \bar{l}_F; N_{\tau_{\text{lease}}}^{\theta_L})$, simplified as $\overline{\eta} \cdot U_L(\bar{l}_F)$.[8]

Theorem 2. *The leader's optimal strategy at date $t+1$ is given by:*

$$l_{t+1} = \begin{cases} \underline{l}_F, & \text{if } \overline{\eta} < \chi \\ \bar{l}_F, & \text{if } \overline{\eta} > \chi \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \text{if } \overline{\eta} = \chi \end{cases} \tag{23}$$

At time t, if the leader offers a lease rate at $l_t = \underline{l}_F$, both type followers will accept. If the leader offers a lease rate at $l_t > \underline{l}_F$, the followers' decisions are more complex.

Definition 2. *Let $y(l_t)$ be the probability that a high type follower F_s accepts l_t. According to the Bayes rule, the leader's posterior probability belief that the follower is high type conditional on the rejection is given by:*

$$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y(l_t)\big)}{\overline{p}\big(1 - y(l_t)\big) + \underline{p}}$$

If the leader offers a lease rate at $l_t > \underline{l}_F$, the high type follower F_s should not reject this l_t with probability 1, because that will make the leader's posterior probability belief $\overline{\eta}(l_t)$ greater than χ and the leader will offer a higher second period lease rate at $l_{t+1} = \bar{l}_F$, so the high type F_s would

[8]Since all other variables are the same, we shall simplify the cooperative leader and follower's total enterprise value function as $U_L(\underline{l}_F)$, $U_L(\bar{l}_F)$ and $U_F(\underline{l}_F)$ and $U_F(\bar{l}_F)$ throughout this subsection. The non-cooperative leader and follower do not participate in this game and their total enterprise values only helps to define the reservation lease rate.

be better off accepting l_t. On the other hand, the high type follower F_s should not accept that l_t with probability 1 either, because that will make the leader's posterior probability belief $\overline{\eta}(l_t)$ less than χ and the leader will offer a lower second period lease rate at $l_{t+1} = \underline{l}_F$, so the high type F_s would be better off rejecting l_t.

Lemma 1. *In equilibrium, when $l_t > \underline{l}_F$, the high type follower has a mixed strategy of randomizing between accept and reject in order to make the leader's posterior belief satisfy $\overline{\eta}(l_t) = \chi$. The leader will offer the second period price l_{t+1} to be any randomization between $\underline{l}_F$ and $\bar{l}_F$. Let $y^*(l_t)$ denote the equilibrium probability with which the high type F_s accepts l_t. Then*

$$y^*(l_t) = 1 + \frac{\chi \underline{p}}{\overline{p}(\chi - 1)} \in [0, 1] \tag{24}$$

which satisfies the equilibrium condition $\overline{\eta}(l_t) = \chi$.

Since the equilibrium has to be Pareto efficient, in order for the high type follower F_s to be indifferent between accepting and rejecting l_t, we need

Definition 3. *Let $x(l_t)$ to be the conditional probability that the high type follower receives the lowest price $\underline{l}_F$ at time $t+1$ if it rejects l_t. Then*

$$x(l_t) = \frac{U_{F_s}(l_t) - U_{F_s}(\bar{l}_F)}{e^{-r}\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big)} \,. \tag{25}$$

Definition 4. *Let $\tilde{l}_F$ be the lease rate at which the high type follower is indifferent between accepting l_t and rejecting l_t in order to wait for $l_{t+1} = \underline{l}_F$ at time $t+1$. It is defined implicitly by*

$$U_{F_s}(\tilde{l}_F) = (1 - e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)$$

Since the follower's enterprise value function, $U_F(l)$ decreases in l, we now summarize the optimal strategy for the follower at time t.

Theorem 3. *The low type follower only accepts $\underline{l}_F$. The high type follower always accepts an offer $l_t \in [\underline{l}_F, \tilde{l}_F]$, and accepts an offer $l_t \in [\tilde{l}_F, \bar{l}_F]$ with probability y^*.*

Suppose the leader's one period discount factor is e^{-r}. The next theorem provides the equilibrium strategy for the leader at time t.

Theorem 4. *If there is a preponderance of low type followers, defined as $\overline{p} < \chi$, then the leader is pessimistic and its optimal strategy is one of the following:*

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \end{cases} \tag{26}$$

If $\overline{p} > \chi$, the leader is optimistic and the leader's first period optimal strategy is given by one of the following.

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \frac{1-e^{-r}\underline{p}}{\overline{p}}, \text{ and } \frac{U_L(\bar{l}_F)}{U_L(\underline{l}_F)} < \frac{1-A}{B} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \frac{1-e^{-r}\underline{p}}{\overline{p}}, \text{ and } BU_L(\bar{l}_F) + (A - e^{-r}\underline{p})U_L(\underline{l}_F) < \overline{p}U_L(\tilde{l}_F) \\ \bar{l}_F, & \text{if } \frac{U_L(\bar{l}_F)}{U_L(\underline{l}_F)} > \frac{1-A}{B}, \text{ and } BU_L(\bar{l}_F) + (A - e^{-r}\underline{p})U_L(\underline{l}_F) > \overline{p}U_L(\tilde{l}_F) \end{cases} \tag{27}$$

where $A = e^{-r}\overline{p}(1-y)x + e^{-r}x\underline{p} > 0$ and $B = \overline{p}y + e^{-r}\overline{p}(1-y)(1-x) > 0$

The proof of Theorems 1 to 4 are given in Appendix B. The conclusion is thus that there exists a unique perfect Bayesian equilibrium, and that this equilibrium exhibits Coasian dynamics — that is, $\overline{\eta}(l_t) \leq \overline{p}$ for all l_t, so the leader becomes more pessimistic over time, and $l_{t+1} \leq l_t$, so the leader's lease rate offer decreases over time.

5 Simulation of the Bargaining Game: The Comparative Statics and the Equilibrium Region

This real option game problem has three stochastic variables: commodity price P and expected reserves for the two producers, Q_A, Q_B. Such a three-dimensional problem is not well-suited to numerical solution of the fundamental differential equations, so we will use the least-squares Monte Carlo method to determine the optimal policy. It has been implemented in a real options settings by Broadie and Glasserman (1997); Longstaff and Schwartz (2001); Murto et al. (2004). The essence of the technique is to replace the conditional risk-neutral one-step expectation of a binomial lattice model with a conditional expectation formed by regressing realized simulation values on observable variables (price and quantity) known at the start of the time step. With the conditional expectation, one can use the Bellman equation to determine the (approximately) optimal policy at each step. Then, given the optimal policy, the simulation can be run again (or recycled) to calculate the risk-neutral expected values arising from the policy.

The model also generates a sequential game between the two players. Sequential games often generate a large number of equilibria that have to be distinguished by a variety of refinements. However, in this setting, we can impose sequential play by the two players, except at the point where they may develop simultaneously. Even at this point, one of the players will be a natural leader, because one will have larger reserves expectation than the other. Thus, we can reduce the sequential game with simultaneous moves to one with sequential moves. Choosing the Nash Bargaining equilibrium at each point (typically a dominant strategy) will result in a unique solution with subgame-perfect strategies. This point has been established by Garlappi (2004); Murto et al. (2004).

With the solution to the game, we propose to explore the sensitivity of the threshold boundary manifolds to the parameters faced by the players, compare the results to those of an isolated monopolist making a real options decision and assess the probability of the various game scenarios that can unfold. More specifically, we are going to investigate the dynamics of the following three enterprise values: (i) the follower's enterprise value if lease, $U_{F,\text{coop}}(P, Q_F, l; N)$; (ii) the follower's enterprise value if build, $U_{F,\text{nc}}(P, Q_F, q_F^c; N)$; (iii) the leader's enterprise value $U_{L,\text{coop}}(P, Q_L, q_L^\Omega, l; N)$.

5.1 THE FOLLOWER'S OPTIMAL DECISIONS

Figure 3 is the graph showing the exercise of the non-cooperative follower's enterprise value with embedded real option to invest. Consistent with the standard real option value, the non-cooperative follower's enterprise value is increasing on the initial price P in all three graphs. As indicated in the left and right graphs, the follower's initial reserve and the network effect N both have positive effects on the real option value. The smooth-pasting condition $\frac{\partial U_F}{\partial P} = 1$ will be reached at a lower price, hence an earlier exercise of real options is optimal under larger initial reserve or network effect. The middle graph shows that the effects of follower's capacity choice q_F^c on real option value and optimal exercise threshold are unclear. The reason for this is the follower's capacity choice are supposed to be optimized based on the level of initial price and reserve quantity, which will be illustrated in the next figure. Figure 4 reveals the non-monotonic effect of q_F^c on the non-

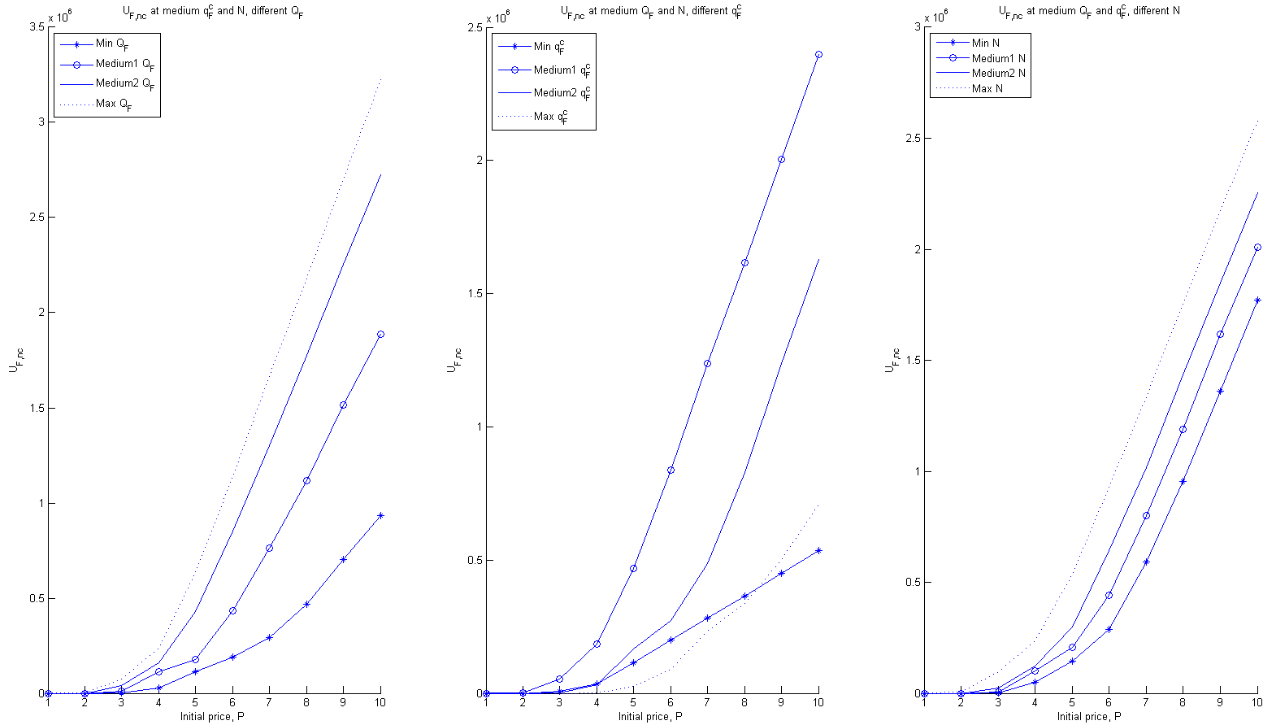


Figure 3: Comparative statics for the follower's enterprise value $U_{F,\text{nc}}$ with respect to initial price P for different initial reserve Q_F, capacity choice q_F^c, and network effect levels N. The left graph varies Q_F, the middle graph varies q_F^c, and the right graph varies N

cooperative follower's enterprise value $U_{F,\text{nc}}$, which suggests that the follower could and probably should choose the optimal plant capacity to maximize its enterprise value. The left, middle and right graphs indicate that the optimal plant capacity shall increase as initial price, initial reserve or the network effect increases, respectively.

Figures 5 shows a comparison of the follower's non-cooperative enterprise values $U_{F,\text{nc}}$ and cooperative enterprise values $U_{F,\text{coop}}$, assuming plant capacity has been optimally chosen. The $U_{F,\text{nc}}$ is represented by the solid line and has more curvature. The $U_{F,\text{coop}}$ is represented by the dotted line and monotonically increasing on the initial price because it is the follower's enterprise value under lease contract with the leader. By comparing the $U_{F,\text{nc}}$ line with the $U_{F,\text{coop}}$ line, the follower can decide whether to accept the lease offer (cooperative) for various commodity price levels. In the top two sub-graphs, where the lease rate and network effect are set at medium level, even for maximum initial reserve, the $U_{F,\text{nc}}$ line is always below the $U_{F,\text{coop}}$ line for the entire varying range of commodity price. However, when the initial reserve, the lease rate and (or) network effect are set to maximum in the bottom sub-graphs, there are some range of initial price

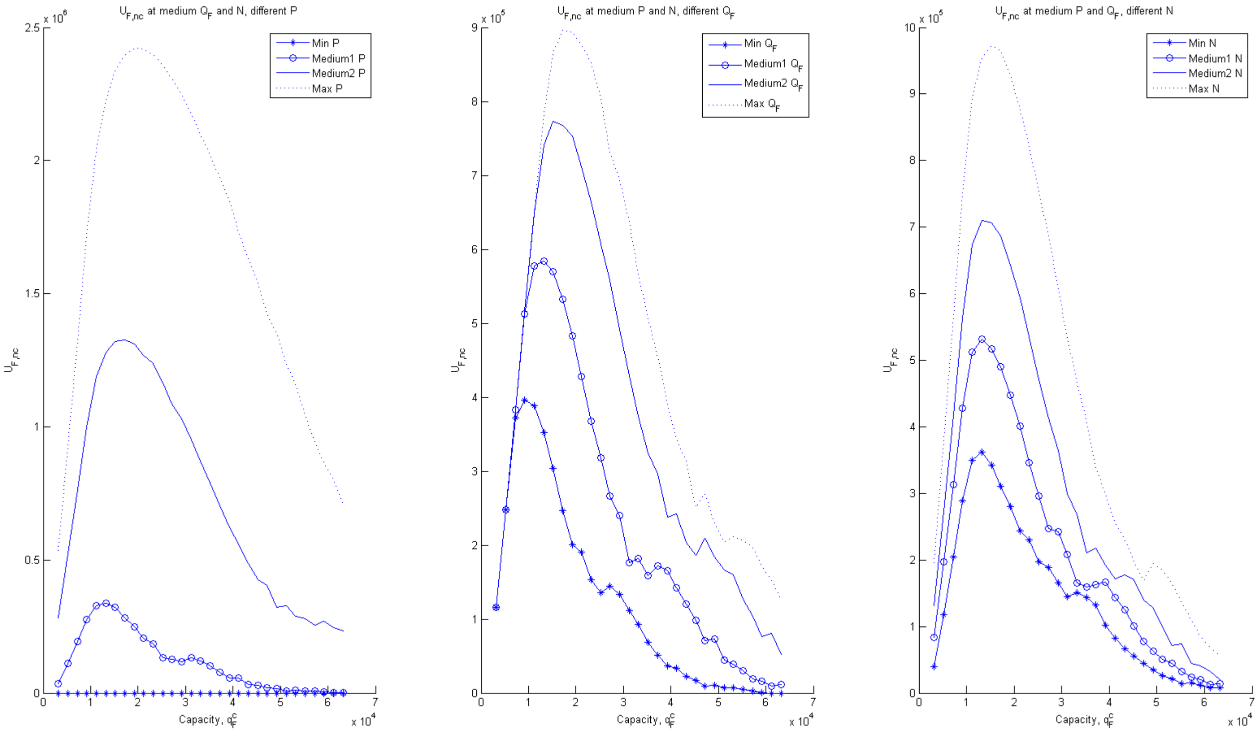


Figure 4: Comparative statics for the follower's enterprise value $U_{F,\mathrm{nc}}$ with respect to capacity choice q_F^c for different initial price P, initial reserve Q_F, , and network effect levels N. The left graph varies P, the middle graph varies Q_F, and the right graph varies N.

where $U_{F,\mathrm{coop}}$ increases much faster than $U_{F,\mathrm{nc}}$ and eventually surpasses it. This indicates that for lower commodity price, smaller initial reserve, lower network effect, it is better for the follower to choose to lease the plant from the leader. For higher commodity prices, initial reserves, and network effect, the follower is better off being non-cooperative or building-own-plant.

If we look at the sub-graphs in Figure 5 individually, we find that $U_{F,\mathrm{coop}}$ increases linearly in P, holding other variables fixed. The $U_{F,\mathrm{nc}}$ grows non-linearly (convex upward) because it contains the follower's real option value which increases as commodity price increasing. When the commodity price is below the trigger threshold, $U_{F,\mathrm{coop}}$ grows faster. After the trigger, $U_{F,\mathrm{nc}}$ grows faster. Therefore, as commodity price get higher, $U_{F,\mathrm{nc}}$ will finally exceed $U_{F,\mathrm{coop}}$. Hence, the follower's benefit from lease decreases in increasing commodity price P because it loses the real option to delay if it leases. In Figure 5, the bottom left graph is similar to the bottom right graph, except that now the network effect level increases to its maximum. The region where $U_{F,\mathrm{coop}}$ line is above $U_{F,\mathrm{nc}}$ line become larger as the network effect level increases. This means there is a larger probability for the follower to accept the lease if the network effect level is higher. Also, the intersection of $U_{F,\mathrm{coop}}$ line and $U_{F,\mathrm{nc}}$ line shifts up as the network effect level increases. This shows that higher network

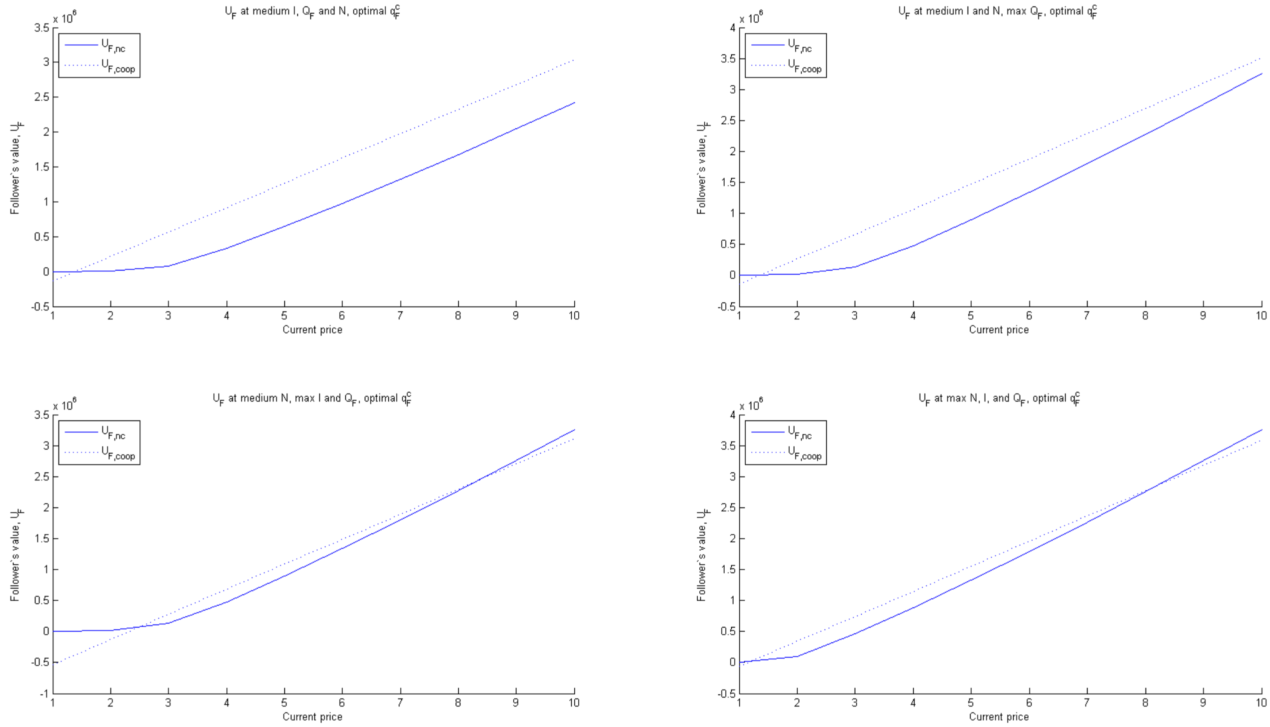


Figure 5: Comparison of the follower's enterprise values $U_{F,\text{nc}}$ and $U_{F,\text{coop}}$ with respect to initial price P for different initial reserve Q_F, capacity choice q_F^c, lease rate l and network effect levels N. The top left graph uses medium l, medium Q_F, medium N and optimal q_F^c. The top right graph uses medium l, medium N, maximum Q_F and optimal q_F^c. The bottom left graph uses medium N, maximum l, maximum Q_F and optimal q_F^c. The bottom right graph uses maximum N, maximum l, maximum Q_F and optimal q_F^c.

effect level increases the follower's benefit from lease, and thus $U_{F,\text{nc}}$ needs a higher commodity price level to exceed the $U_{F,\text{coop}}$. Figure 6 shows the reservation lease rates the follower is willing to accept corresponding to different initial reserves and initial prices. Firstly, we analyze how the follower's reservation lease rate changes as the commodity price and initial reserve change. Since these four lines have similar shape, we can focus on one of them. As commodity price increases, the follower's reservation lease rate first increases then decreases. To understand this, recall that the follower's reservation lease rate is define as $\bar{l}_F \equiv \sup\{l_F \in \mathbb{R}^+ : U_{F,\text{coop}} \geq U_{F,\text{nc}}\}$. In other words, it is equivalent to the distance between $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$. As we observe in Figure 5, initially $U_{F,\text{coop}} > U_{F,\text{nc}}$, as commodity price increases, both $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ increase, and $U_{F,\text{coop}}$ increases faster than $U_{F,\text{nc}}$. The distance gets larger. However, above the follower's trigger threshold, $U_{F,\text{nc}}$ increases faster than $U_{F,\text{coop}}$, and the distance becomes smaller, eventually, $U_{F,\text{nc}}$ will catch up (intersect) with and then exceed $U_{F,\text{coop}}$. That is why the follower's reservation lease rate first increase then decrease. Furthermore, one can infer that the follower's peak reservation lease rate is achieved when the distance between $U_{F,\text{coop}}$ and $U_{F,\text{nc}}$ is largest, i.e., the neighbor area below

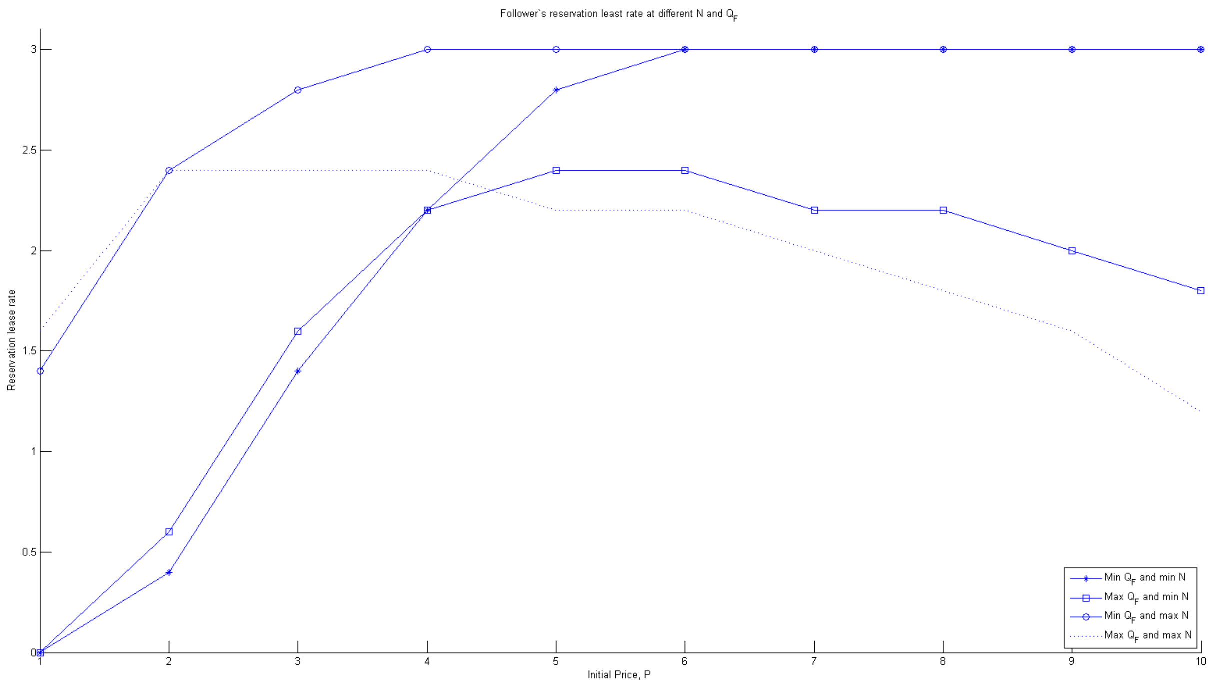


Figure 6: Comparative statics of the follower's reservation lease rate with respect to initial price P for minimum or maximum initial reserve Q_F, and minimum or maximum network effect levels N.

the trigger threshold. Secondly, the comparison of these four lines (starred, squared, circled, and dotted) shows that the initial reserve and network effect both have positive effects on the follower's reservation lease rate for relatively low commodity price level, holding everything else constant. However, this effect becomes negative for relatively higher commodity price. More specifically, we find that before the peak, the high network effect gives the follower a larger reservation lease rate if holding the price level fixed. But after the peak, the high network effect gives the follower a smaller reservation lease rate if holding the price level fixed. This is because before real option being exercised, the network effect is favoring $U_{F,\mathrm{coop}}$ more (making $U_{F,\mathrm{coop}}$ increase faster), whereas after real option being exercised, the network effect is favoring $U_{F,\mathrm{nc}}$ more (making $U_{F,\mathrm{nc}}$ increase faster). The implication for the leader from this observation is that for extremely low commodity price, larger network effect increases the follower's willingness to pay for the lease. For high commodity price, larger network effect decreases the follower's willingness to pay for the lease, certeris paribus.

5.2 THE LEADER'S OPTIMAL DECISIONS

The leader has two options: (i) Build a plant with optimal non-cooperative capacity q_L^{c*} to process its own gas only for a construction cost $K(q_L^c)$. The effect of building this small plant on the optimal

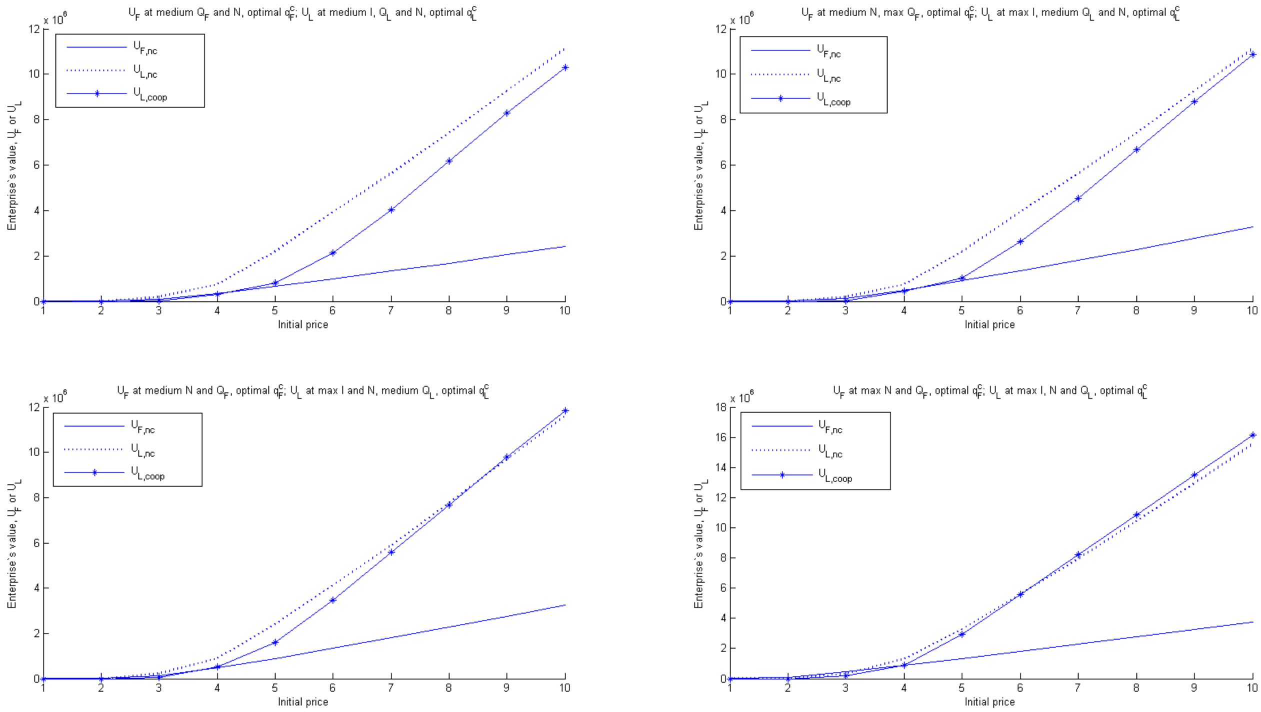


Figure 7: The leader's cooperative enterprise value (the solid line) $U^*_{L,\text{coop}}$ crossing with its non-cooperative enterprise value (the dotted line) $U^*_{L,\text{nc}}$ at various levels of lease rate, initial reserve Q_L and network effect N.

exercise point is mixed: it could be earlier or later than if a large plant is built. (ii) Build a plant with optimal cooperative capacity ($q_L^{\Omega *}$) to process his (q_L) and the follower's gas q_{FL}. The larger plant has a construction cost $K(q_L^{\Omega}) > K(q_L^c)$. The cash flow from this option is larger because leasing gives a lower toll rate (network effect) and the leasing fee is a cash inflow to the leader.

As discussed in Section 3.5, the leader wants to find a balance among the incremental network effect benefit, the earlier leasing fee, and the extra construction costs, $K(q_L^{\Omega}) - K(q_L^c)$, bearing in mind the fact that a high leasing rate will cause the follower to delay. The leader's enterprise value with respect to the initial commodity price is of similar pattern to the follower's non-cooperative enterprise value illustrated in Figure 3. Basically, $U^*_{L,\text{coop}}$ monotonically increase on the initial price P. And all other factors including initial reserve, lease rate and network effect, have positive effect on this relationship. That is higher levels of initial reserve, lease rate and network effect will reduce the optimal exercise threshold for the leader, i.e., an earlier exercise. For the benefit of saving space, we will skip the leader's graphs for simple cooperative or non-cooperate enterprise values here and readers shall be able to derive same comparative statics using the follower's graphs.

We now turn our focus to the question of whether the leader and follower are competing to become the first mover. As illustrated in Figure 7, in all four graphs, the follower's non-cooperative enterprise value $U^*_{F,\text{nc}}$ (the solid line) is always under the leader's enterprise value (the dotted and starred lines), and the optimal exercise threshold P^*_F for the follower is obviously higher, which means, the follower will exercise its real option later than the leader even if the lease rate and the network effect are both set to their maximum level as long as we assume the follower has smaller initial reserve than the leader. This is consistent with the proposition in Novy-Marx (2007) that the competition effect will not push the real option optimal exercise threshold back to the point of zero NPV, because it is going to be mitigated by the the heterogeneity of these producers, e.g., the different initial reserve levels of two firms.

Figure 7 also plots the leader's cooperative v.s. non-cooperative enterprise values assuming plant capacity has been optimally chosen. In the top left graph in which all three parameters are set at their medium levels, the leader's non-cooperative enterprise value dominates its cooperative enterprise value for the entire range of commodity price. The top right graph sets the lease rate to its maximum and the other two parameters at their medium level, the leader's cooperative value is able to increase much faster than its non-cooperative value as the commodity price increases, but still below it. The bottom two graph gradually sets the lease rate, the network effect and(or) the initial reserve at their maximum. In these two graphs, we start to observe leader's cooperative enterprise value catching up with and eventually exceed its non-cooperative value. This shows that higher levels of commodity price, lease rate, network effect and initial reserve will motivate the leader to be cooperative, i.e., build a bigger plant, lease the excess capacity to the follower and collect the rent.

Figure 8 graphs the leader's reservation lease rates (the smallest lease rate that the leader is willing to offer and still able to keep $U^*_{L,\text{coop}}$ bigger than $U^*_{L,\text{nc}}$) against the commodity price for various initial reserve levels. The leader will accept a lower lease rate if it has larger reserves, because it has a lower marginal cost of production with larger volume. For low commodity price the leader's reservation lease rate is maximum. There are two possible causes for this. One is the leader has not exercise the real option yet and hence can not provide the lease. The other is that even if the leader has exercised its real option, the lease rate and network effect are not enough to boost the leader's cooperative enterprise value above its non-cooperative value, and therefore wants to charge the highest lease rate. The left graph shows when the network effect is low, for leaders with three different initial reserve levels (minimum, medium and maximum), they are not

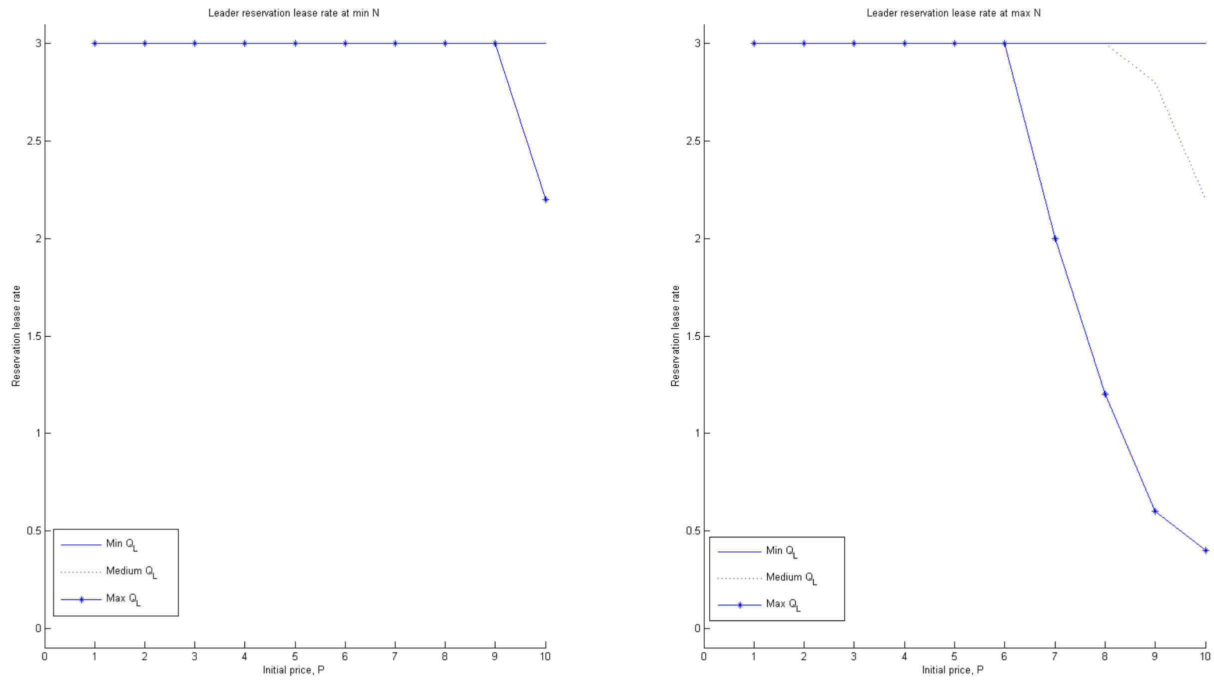


Figure 8: The leader's reservation lease rates for minimum (left graph) and maximum (right graph) network effect levels. The solid, dotted and starred lines correspond to leaders with minimum, medium and maximum initial reserve levels, respectively

willing to drop their reservation lease rate until a very high commodity price (9). The right graph shows when the network effect is high, the leaders are more willing drop their reservation lease rate. For instance, the leader with a maximum initial reserve is willing to drop its reservation lease rate after the commodity price of 6. The implication for the leader from this observation is that, once it exercises the real option, larger network effect will reduce the leader's reservation lease rate, i.e., the leader is willing to set a lower lease rate to capture the larger network effect, certeris paribus.

5.3 THE POSSIBLE EQUILIBRIUM REGION FOR THE LEASE RATE

Figure 9 really is a combination of Figure 6 and Figure 8. It indicates the possible region of the equilibrium lease rate for bargaining. Apparently, the equilibrium region is increasing on the commodity price level. In the left graph, within the commodity price range of \$9 and \$10, the leader with maximum initial reserve (the dotted line) has a lower reservation lease rate than the follower with minimum initial reserve, given a minimum level of network effect. In the right graph, within the commodity price range of \$6 and \$10, the leader with maximum initial reserve has a lower reservation lease rate than the follower with minimum initial reserve, given a maximum level of network effect. This significant increase in the size of the equilibrium region from the left

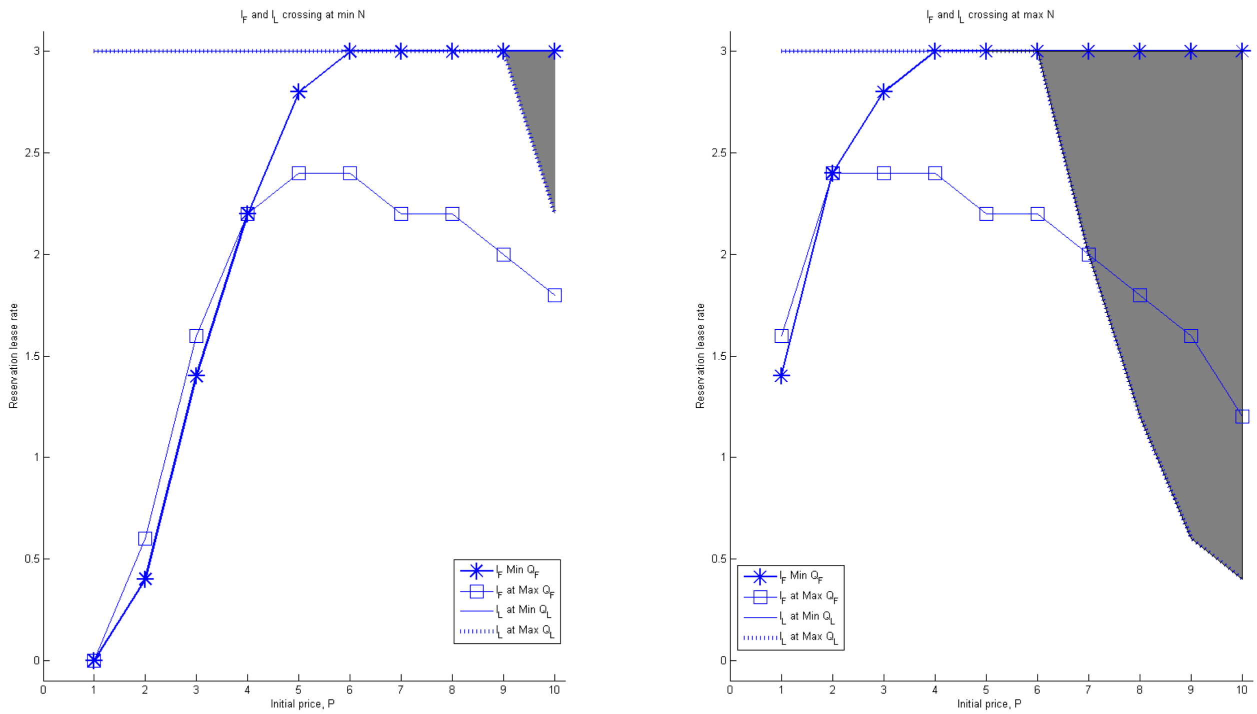


Figure 9: The leader's reservation lease rate compared with the follower's reservation lease rate for minimum (left graph) and maximum (right graph) network effect levels. The starred and squared lines correspond to the followers with minimum and maximum initial reserves, respectively. The solid and dotted lines correspond to the leaders with minimum and maximum initial reserves, respectively. The grey shaded area represents the possible equilibrium region where the leader's reservation lease rate is below the follower's reservation lease rate ($\underline{l}_L \leq \bar{l}_F$)

graph to the right graph suggests that the network effect has a positive impact on the likelihood of reaching an equilibrium.

As compared to the follower with minimum initial reserve (the starred line), the follower with maximum initial reserve (the squared line) has a lower (less than \$2) reservation lease rate, which leads to a smaller equilibrium price range of \$7 to \$10. This suggests that the follower's initial reserve level has a negative impact on the likelihood of reaching an equilibrium. It is also clear that the leaders with minimum initial reserve level (the solid line) always set its reservation lease rate at the maximum, hence an equilibrium is not possible, which suggests that the leader's initial reserve level has a positive impact on the likelihood of reaching an equilibrium.

6 Conclusions

This article presents an equilibrium model reconciling the contradiction in previous literature, which helps to explain firms' investment behavior – sometimes delaying the investment until the standard real option exercise threshold, sometimes later than that, sometimes don't delay at all. The analysis presented in this article shows that firms should monitor the benefit of delaying the investment and the benefit of exercising the real option. When the former is larger, firms should keep delaying the investment. When the latter is larger, firms should invest immediately. The benefit of delaying is the real option value. The benefit of exercising includes the earlier cash flows from the investment, the first mover advantage and the ability of extracting economic rents from the competitors if exercising before the competitors. Particularly, in industries with lots of positive externalities (network effect), firms may still invest later than the zero NPV threshold to keep the real option value, but earlier than the standard real options exercise threshold in order to obtain the first mover advantage and the ability of extracting rents. Even if the competition is sufficiently fierce in these industries, it will not force firms to invest at zero NPV threshold because heterogeneity in firms will determine whichever firm invest first. Instead, sufficient competition will push firms to consider an alternative strategy – cooperation, which allows firms to benefit from the positive externalities.

The equilibrium of investment threshold is formed where firms can successfully negotiate a cooperation contract and it is positively affected by the commodity price, the level of network effect and the leader's initial reserve, but negatively affected by the follower's initial reserve. We observe that the follower tends to accept the lease contract if the commodity price is high and its initial reserves are low, and rejects the lease contract if the commodity price low and its initial reserves are high. With high commodity price and initial reserves, the follower has more bargaining power, so the leader should charge a relatively low lease rate to encourage the follower's immediate start of production. If the commodity price and the leader's initial reserves are high, the leader should lower the lease rate, which coincides with the behavior of its reservation lease rate. Furthermore, the network effect positively affects equilibrium region, which creates a larger space for cooperative bargaining.

References

Boyer, M., Lasserre, P., Mariotti, T., and Moreaux, M. (2004). Preemption and rent dissipation under bertrand competition. *International Journal of Industrial Organization*, 22(3):309328.

Brennan, M. J. and Schwartz, E. (1985). Evaluating natural resource investments. *Journal of Business*, 58(2):135–157.

Broadie, M. and Glasserman, P. (1997). Pricing american-style securities using simulation. *Journal of Economic Dynamics and Control*, 21(8-9):1323–1352.

Dixit, A. (1995). Irreversible investment with uncertainty and scale economies. *Journal of Economic Dynamics and Control*, 19(1–2):327–350.

Dixit, A. K. and Pindyck, R. S. (1994). *Investment under Uncertainty.* Princeton University Press, Princeton, N.J.

Fudenberg, D. and Tirole, J. (1985). Preemption and rent equalization in the adoption of new technology. *Review of Economic Studies*, 52(170).

Fudenberg, D. and Tirole, J. (1991). *Game Theory.* MIT Press, Cambridge (MA).

Garlappi, L. (2004). Risk premia and preemption in R&D ventures. *Journal of Financial and Quantitative Analysis*, 39:843–872.

Grenadier, S. R. (1996). The strategic exercise of options: Development cascades and overbuilding in real estate markets. *Journal of Finance*, 51(5):1653 – 1679.

Grenadier, S. R. (2002). Option exercise games: An application to the equilibrium investment strategies of firms. *Review of Financial Studies*, 15(3):691–721.

Huisman, K. J. M. and Kort, P. M. (2004). Strategic technology adoption taking into account future technological improvements: A real options approach. *European Journal of Operational Research*, 159(3):705–728.

Kreps, D. M. and Scheinkman, J. (1983). Quantity precommitment and bertrand competition yield cournot outcomes. *Bell Journal of Economics*, 14(2):326–337.

Lambrecht, B. and Perraudin, W. (2003). Real options and preemption under incomplete information. *Journal of Economic Dynamics and Control*, 27(4):619–643.

Longstaff, F. A. and Schwartz, E. S. (2001). Valuing american options by simulation: a simple least-squares approach. *The Review of Financial Studies*, 14(1):113–147.

Mason, R. and Weeds, H. (2010). Investment, uncertainty and pre-emption. *International Journal of Industrial Organization*, 28(3):278–287.

Murto, P. and Keppo, J. (2002). A game model of irreversible investment under uncertainty. *International Game Theory Review*, 40(2):127–140.

Murto, P., Näsäkkälä, E., and Keppo, J. (2004). Timing of investments in oligopoly under uncertainty: A framework for numerical analysis. *European Journal of Operational Research*, 157:486–500.

Novy-Marx, R. (2007). An equilibrium model of investment under uncertainty. *The review of Financial Studies*, 20(5):1461–1502.

Smit, H. T. J. and Ankum, L. A. (1993). A real options and game-theoretic approach to corporate investment strategy under competition. *Financial Management*, 22(3):241–250.

Smit, H. T. J. and Trigeorgis, L. (2004). *Strategic Investment: Real Options and Games*. Princeton University Press.

A Appendix: The Derivation of the Smooth-Pasting Conditions for Non-Cooperative Player's Investment Decision

In order to derive the smooth-pasting conditions for non-cooperative player's investment decision, we need to partially differentiate the expected payoff function W_i with respect to P_{τ_i} and Q_i. This requires differentiating a definite integral with respect to a parameter that appears in the integrand and in the limits of the integral. The following formula is discovered by Gottfried Wilhelm von Leibniz. Let f be a differentiable function of two variables, let a and b be differentiable functions of a single variable, and define the function F by

$$F(t) = \int_{a(t)}^{b(t)} f(t,x)dx \qquad \forall\ t.$$

Then

$$F'(t) = f\big(t, b(t)\big)b'(t) - f\big(t, a(t)\big)a'(t) + \int_{a(t)}^{b(t)} f_t(t,x)dx$$

We now apply this Leibniz formula to differentiate W_i with respect to P_{τ_i} and Q_i separately, where τ_i is the first time the manifold (P(Q),Q) hits the threshold (P*(Q),Q). Producer i's cash flow function:

$$\pi_{i,t} = (P_t - C)q_{i,t}$$

Producer i's production function:

$$q_i(t) = \begin{cases} q_i^c & t \in [\tau_i, \theta_{i,\text{trans}}] \\ \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i (t-\theta_{i,\text{trans}})} & t \in [\theta_{i,\text{trans}}, \theta_i] \end{cases}$$

The production transition time is defined as:

$$\theta_{i,\text{trans}} = \frac{Q_i(\tau_i)}{q_i^c} - \frac{1}{\overline{\alpha}_i} \implies \frac{\partial \theta_{i,\text{trans}}}{\partial P_{\tau_i}} = 0 \quad \text{and} \quad \frac{\partial \theta_{i,\text{trans}}}{\partial Q_i} = \frac{1}{q_i^c}$$
$$\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) = q_i^c$$

Therefore, we have

$$\begin{aligned} W_i \equiv \widehat{E}_{\tau_i}\Bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}} & e^{-r(t-\tau_i)}(P_t - C)q_i^c dt \\ + \int_{\theta_{i,\text{trans}}}^{\theta_i} & e^{-r(t-\tau_i)}(P_t - C)\overline{\alpha}_i Q_i(\theta_{i,\text{trans}})e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\Bigg] - K(q_i^c) \end{aligned}$$

Also, to simplify the notation, we denote $\hat{\mu}$ as the risk-neutral drift rate of the price P, i.e., $\hat{\mu}(P) = \mu$. Since the price is assumed to follow the GBM, we have

$$P_t = P_{\tau_i} e^{\hat{\mu}(t-\tau_i)}$$

Therefore, the first smooth-pasting condition is:

$$\begin{aligned} V_P(P^*, Q^*) &= \frac{\partial W_i}{\partial P_{\tau_i}} \\ &= \widehat{E}_{\tau_i}\Bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{-r(t-\tau_i)} e^{\hat{\mu}(t-\tau_i)} q_i^c dt \\ &\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)} e^{\hat{\mu}(t-\tau_i)} \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} dt\Bigg] - K(q_i^c) \\ &= \widehat{E}_{\tau_i}\Bigg[\int_{\tau_i}^{\theta_{i,\text{trans}}} e^{(\hat{\mu}-r)(t-\tau_i)} q_i^c dt \\ &\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{(\hat{\mu}-r)(t-\tau_i)-\overline{\alpha}_i(t-\theta_{i,\text{trans}})} \overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) dt\Bigg] - K(q_i^c) \end{aligned}$$

The second smooth-pasting condition is

$$
\begin{aligned}
V_Q(P^*, Q^*) &= \frac{\partial W_i}{\partial Q_i} \\
&= \widehat{E}_{\tau_i}\Bigg[\frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)q_i^c - 0 + 0 \\
&\quad 0 - \frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) e^{-\overline{\alpha}_i(\theta_{i,\text{trans}}-\theta_{i,\text{trans}})} \\
&\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\Bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\Bigg[e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C) - \frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)\overline{\alpha}_i Q_i(\theta_{i,\text{trans}}) \\
&\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\Bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\Bigg[e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C) - \frac{1}{q_i^c} e^{-r(\theta_{i,\text{trans}}-\tau_i)}(P_t - C)q_i^c \\
&\quad + \int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\Bigg] - K(q_i^c) \\
&= \widehat{E}_{\tau_i}\Bigg[\int_{\theta_{i,\text{trans}}}^{\theta_i} e^{-r(t-\tau_i)}(P_t - C)e^{-\overline{\alpha}_i(t-\theta_{i,\text{trans}})}dt\Bigg] - K(q_i^c)
\end{aligned}
$$

B Appendix: The Derivation of the Perfect Bayesian Equilibrium for the Leader and Follower Bargaining Game

The extensive form representation of this sequential bargaining game is shown in Figure 10 of Appendix B, which will be used throughout the discussion of the perfect Bayesian equilibrium.

B.1 THE LEADER AND FOLLOWER OPTIMAL STRATEGIES AT TIME $t+1$

In period $t+1$, the leader with beliefs $\overline{\eta}(l_t)$ makes a "take it or leave it" offer l_{t+1} so as to maximize that period's profit. Because period $t+1$ is the last period, the leader's threat of offering no other contract in the future is credible, so the follower will accept if and only if his reservation is at least l_{t+1}. The follower's optimal strategy at date $t+1$ is defined as: [9]

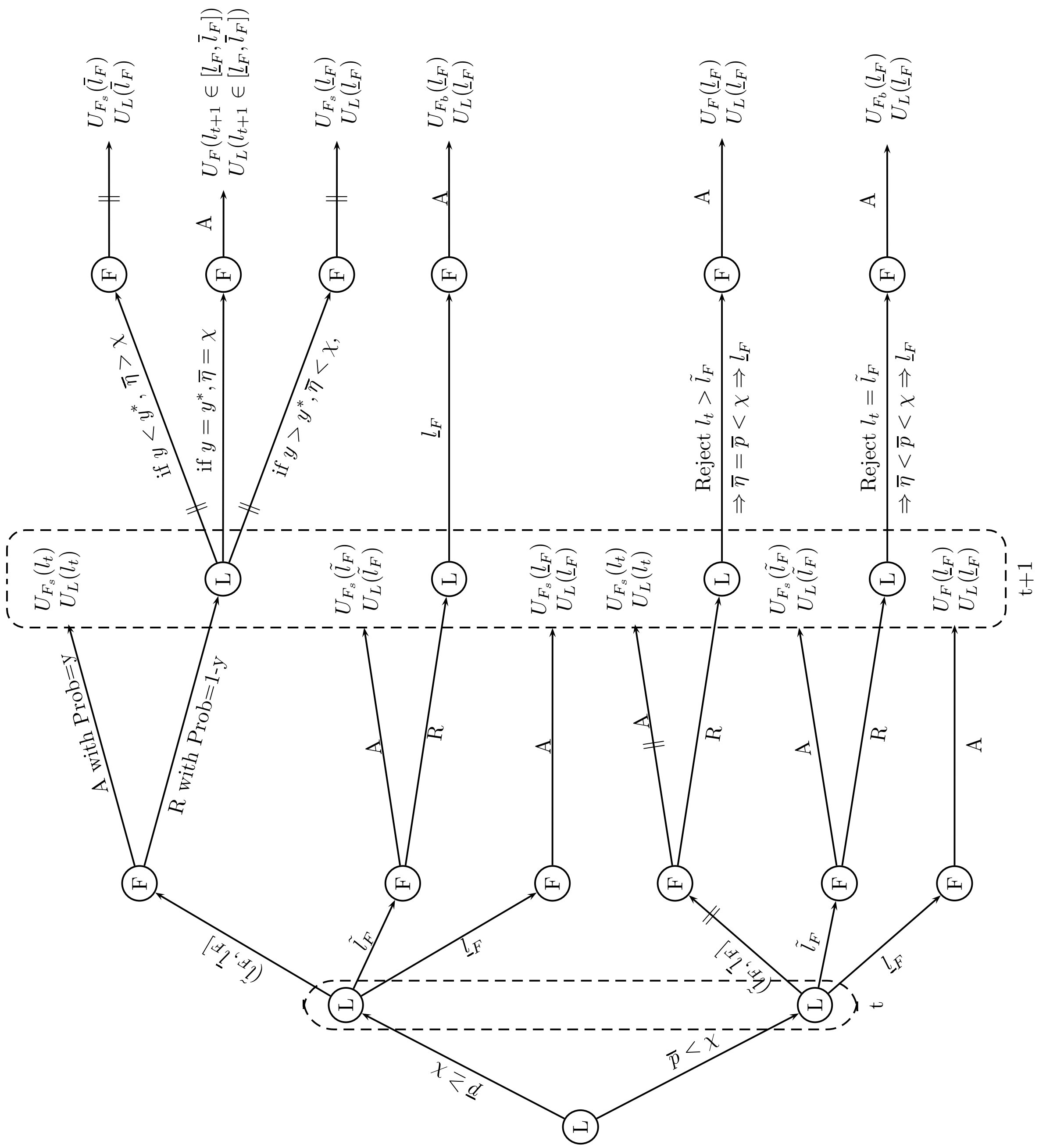
$$\text{If } l_{t+1} = \begin{cases} \underline{l}_F, & F_s, F_b \text{ both accept} \\ \bar{l}_F, & F_s \text{ accepts}, F_b \text{ rejects} \\ \text{Random}[\underline{l}_F, \bar{l}_F], & F_s \text{ accepts}, F_b \text{ rejects} \end{cases} \tag{28}$$

The leader's offer l_{t+1} ranges from $\underline{l}_F$ to $\bar{l}_F$. If offering $l_{t+1} = \underline{l}_F$, the leader sells for sure and obtains the enterprise value of $U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \underline{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$, simplified as $U_L(\underline{l}_F)$. If offering $l_{t+1} = \bar{l}_F$, the leader sells with probability $\overline{\eta}$ and has second period enterprise value of $\overline{\eta} \cdot U_{L,\text{coop}}(P, Q_L, q_L^{\Omega}, \bar{l}_F; N^{\theta_L}_{\tau_{\text{lease}}})$, simplified as $\overline{\eta} \cdot U_L(\bar{l}_F)$.Therefore, there exists a unique critical probability $\chi \equiv \frac{U_L(\underline{l}_F)}{U_L(\bar{l}_F)}$, and the leader's optimal strategy at date $t+1$ is defined as:

$$l_{t+1} = \begin{cases} \underline{l}_F, & \text{if } \overline{\eta} < \chi \\ \bar{l}_F, & \text{if } \overline{\eta} > \chi \\ \text{Random}[\underline{l}_F, \bar{l}_F], & \text{if } \overline{\eta} = \chi \end{cases} \tag{29}$$

[9] Each type follower is actually indifferent between accepting and rejecting a lease rate of l_{t+1} that exactly equals that type's reservation rate. However, as long as the supremum of the leader's total enterprise value is achieved in the limit of lease rate $l_{t+1} = l - |\varepsilon|$ as $\varepsilon \to 0$, we could assume, without loss of generality, the existence of an equilibrium given the leader's beliefs requires that type l accept $l_{t+1} = l$, and whether the other type accepts a lease rate equal to its reservation rate is irrelevant.

Figure 10: **The extensive form of the bargaining game**



B.2 THE LEADER AND THE FOLLOWER'S OPTIMAL STRATEGY AT TIME t

At time t, the leader and the follower's decisions are more complex. Ideally, the leader would want to offer the high type follower at $\bar{l}_F$ and the low type follower at $\underline{l}_F$. But we have already shown that the high type follower is motivated to lie. Therefore, the leader's task is to differentiate the high type follower from the low type follower by testing them with different lease rate. At time t, the low type follower F_b will accept if and only if $l_t = \underline{l}_F$ since it will never obtains a surplus at next period. Of course, the high type follower F_s accepts $l_t = \underline{l}_F$ too. The high type follower, however, if offered $l_t > \underline{l}_F$, has to consider how its rejection might affect the leader's posterior belief about the follower's type. High type follower F_s obtains a surplus only if the leader is sufficiently convinced that it is the low type follower, i.e., $\overline{\eta} < \chi$.

B.2.1 *The consequence of the follower's rejection on the leader's posterior belief*

We now discuss how the follower's rejection might affect the leader's posterior belief.

1. Choice of mixed and pure strategy. Suppose the rejection of $l_t > \underline{l}_F$ generates "optimistic posterior beliefs": $\overline{\eta} > \chi$. From equation (23) the leader charges $l_{t+1} = \bar{l}_F$. High type F_s has no second period surplus from rejecting (continue lying) that $l_t > \underline{l}_F$. Therefore, the high type F_s is better off accepting $l_t > \underline{l}_F$. And since l_t is rejected by the low type F_b, Bayes' rule yields $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 0}{\overline{p}\cdot 0 + \underline{p}} = 0$, a contradiction. Thus neither of the pure strategies, accept or reject, is optimal here. In the following subsections, we will develop a mixed strategy for the follower and the leader in the case of the rejection generating optimistic posterior, and we will also elaborate the leader and follower's pure strategy in the case of the rejection generating "pessimistic posterior beliefs".

   Let $y(l_t)$ denote the probability that the high type F_s accepts l_t. Then the high type follower consider how its probability of rejection will affect the leader's posterior according to the following formula:

   $$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y(l_t)\big)}{\overline{p}\big(1 - y(l_t)\big) + \underline{p}}$$

   (a) If F_s accept with probability of 1, then $y = 1 \Rightarrow 1 - y = 0$, then $\overline{\eta}(l_t) = \frac{\overline{p}\cdot 0}{\overline{p}\cdot 0 + \underline{p}\cdot 1} = 0 < \chi$.

According to equation (23), the leader with posterior $\overline{\eta} < \chi$ will offer $l_{t+1} = \underline{l}_F$. So F_s who anticipates this lower second period price l_{t+1} should not accept l_t with probability of 1. A contradictory.

(b) If F_s reject with probability of 1, then $y = 0 \Rightarrow 1 - y = 1$, then $\overline{\eta}(l_t) = \frac{\overline{p} \cdot 1}{\overline{p} \cdot 1 + \underline{p} \cdot 1} = \overline{p}$. Now since in the top branch of the extensive form of game, we have $\overline{p} > \chi$. Therefore, $\overline{\eta} = \overline{p} > \chi$. According to equation (23), the leader with posterior $\overline{\eta} > \chi$ will offer $l_{t+1} = \bar{l}_F$. So F_s who anticipates this higher second period price l_{t+1} should not reject l_t with probability of 1. A contradictory.

In equilibrium the high type F_s should not reject l_t with probability 1, because in that case we would have $\overline{\eta}(l_t) = \overline{p} > \chi$ and the leader charging $l_{t+1} = \bar{l}_F$, so the high type F_s would be better off accepting l_t. But we already saw that the high type F_s cannot accept such an l_t with probability 1 either. Hence, the high type follower needs a mixed strategy here by randomizing between accept and reject, i.e., controlling the y so that the leader's posterior is $\overline{\eta}(l_t) = \chi$.

2. Rejection deteriorates the leader's ex ante belief. According to the Bayes rule, for any rejection of $l_t > \underline{l}_F$, the leader's posterior belief is calculated as:

$$\overline{\eta}(l_t) = \frac{\text{Prob}(\text{type} = F_s \text{ \& reject } l_t > \underline{l}_F)}{\text{Prob}(\text{reject } l_t > \underline{l}_F)} = \frac{\overline{p} \cdot \text{Prob}(l_t > \tilde{l}_F)}{\overline{p} \cdot \text{Prob}(l_t > \tilde{l}_F) + \underline{p}} = \frac{\overline{p}}{\overline{p} + \dfrac{\underline{p}}{\text{Prob}(l_t > \tilde{l}_F)}} \leq \overline{p} \tag{30}$$

which means the posterior is always less than or equal to the prior conditional on the rejection of $l_t > \underline{l}_F$.

B.2.2 *The follower's indifference lease rate* $\tilde{l}_F$

To analyze the high type follower's behavior at t when offered price $l_t \in (\underline{l}_F, \bar{l}_F]$, we have to define a critical indifference lease rate $\tilde{l}_F$. The high type follower F_s should accept l_t only if

$$
\begin{aligned}
U_{F_s}(l_t) - U_{F_s}(\bar{l}_F) &\geq e^{-r}\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big) \\
\Rightarrow \qquad U_{F_s}(l_t) &\geq (1 - e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)
\end{aligned} \tag{31}
$$

To see this, note that $U_{F_s}(l_t) - U_{F_s}(\bar{l}_F)$ is the realized gain from lying at time t and $U_{F_s,\text{coop}}(\underline{l}_F) - U_{F_s,\text{coop}}(\bar{l}_F)$ is the maximum possible gain from continuing lying at time $t+1$. Denote $\tilde{l}_F$ as the l_t which makes the above inequality equal. That is

$$
U_{F_s}(\tilde{l}_F) = (1 - e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)
$$

Obviously, when $l_t = \tilde{l}_F$, the high type follower F_s is indifferent between accepting this l_t and getting $l_{t+1} = \underline{l}_F$ at time $t+1$ by rejecting this l_t. As the high type follower's enterprise value function, $U_{F_s}(l)$ decreases in l, we have the optimal strategy for the high type follower when facing the lease offer at $l_t > \underline{l}_F$.

- If $\underline{l}_F < l_t \leq \tilde{l}_F \Rightarrow U_{F_s}(l_t) \geq U_{F_s}(\tilde{l}_F) = (1 - e^{-r})U_{F_s}(\bar{l}_F) + e^{-r}U_{F_s}(\underline{l}_F)$. Equation (31) is satisfied. High type F_s accepts this $l_t \in (\underline{l}_F, \tilde{l}_F]$.
- If $l_t > \tilde{l}_F$, rejecting l_t is optimal for the high type F_s as it is for the low type F_b, and therefore Bayes' rule yields

$$
\overline{\eta}(l_t > \tilde{l}_F) = \frac{\overline{p} \cdot 1}{\overline{p} \cdot 1 + \underline{p} \cdot 1} = \overline{p}
$$

  which means the posterior beliefs coincide with the prior beliefs. In other words, the follower is safe to reject any offer $l_t > \tilde{l}_F$ at time t without improving the leader's information about the follower's type.

B.2.3 The strategy of the pessimistic leader $\overline{p} < \chi$

Equation (30) shows $\overline{\eta} \leq \overline{p}$, combined with $\overline{p} < \chi$, we have $\overline{\eta} < \chi$. This means no matter what the first period offer is, the follower's rejection always makes the leader pessimistic. Therefore the leader's second period strategy is limited to $l_{t+1} = \underline{l}_F$ whenever it observes a rejection at time

t. We now compare the leader's expected total enterprise values from three different first period strategies, as illustrated in the bottom branch of Figure 10.

1. Bottom-Bottom strategy (BB): $l_t = \underline{l}_F$. Both type followers will accept this l_t as they knows this is the most favorable price. BB therefore leads to a pooling equilibrium. The leader has an enterprise value of $U_L(\underline{l}_F)$.

2. Bottom-Middle strategy (BM): $l_t = \tilde{l}_F$. The high type F_s would accept this l_t because it is indifferent as discussed in Section B.2.2. The low type F_b rejects this offer because leasing would give him a negative surplus, i.e., $U_{F_b,\mathrm{coop}}(\tilde{l}_F) < U_{F_b,\mathrm{nc}}$ according to equation (9). Thus if the leader observes a rejection, it knows the follower is low type and will set $l_{t+1} = \underline{l}_F$. BM therefore leads to a separating equilibrium. The leader's expected enterprise value from BM strategy is: $\overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F)$.

3. Bottom-Top strategy (BT): $l_t \in (\tilde{l}_F, \bar{l}_F]$. Again, the low type follower F_b rejects this offer because $U_{F_b,\mathrm{coop}}(\tilde{l}_F +) < U_{F_b,\mathrm{nc}}$. The high type follower F_s would rather reject this l_t since it knows that the consequence of rejecting the leader's offer is $\overline{\eta} = \overline{p} < \chi$ and the leader will offer a lower lease rate next period, $l_{t+1} = \underline{l}_F$. BT therefore leads to a pooling equilibrium as both type followers reject. BT strategy will give the leader a total enterprise value of $e^{-r} \cdot U_L(\underline{l}_F)$.

The leader's valuation function $U_L(l)$ increases at l. Hence, $\overline{p}U_L(\tilde{l}_F) > \overline{p}U_L(\underline{l}_F)$. Therefore, $\overline{p}U_L(\tilde{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) > \overline{p}U_L(\underline{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) > \overline{p}e^{-r}U_L(\underline{l}_F) + e^{-r}\underline{p}U_L(\underline{l}_F) = e^{-r}U_L(\underline{l}_F)$. Clearly, BB is better than BT and BM is better than BT. Either BB or BM can give the leader higher value depending on the generic values of parameters. Thus, we summarize the pessimistic leader's optimal strategy as:

$$l_t = \begin{cases} \underline{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} < \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \\ \tilde{l}_F, & \text{if } \frac{U_L(\tilde{l}_F)}{U_L(\underline{l}_F)} > \dfrac{1 - e^{-r}\underline{p}}{\overline{p}} \end{cases} \tag{32}$$

B.2.4 *The strategy of the optimistic leader* $\overline{p} > \chi$

1. Top-Bottom strategy (TB): $l_t = \underline{l}_F$. The TB strategy is same as the BB strategy. Both type followers accept the lease and the leader's enterprise value is $U_L(\underline{l}_F)$, a pooling equilibrium.

2. Top-Middle strategy (TM): $l_t = \tilde{l}_F$. This is also similar to BM strategy. The high type F_s accepts whereas the low type F_b rejects this offer, a separating equilibrium. The leader's expected enterprise value from TM strategy is: $\overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F)$.

3. Top-Top strategy (TT): $l_t \in (\tilde{l}_F, \bar{l}_F]$. The low type follower F_b rejects this offer. The high type follower F_s has a more complex decision because it has to consider the consequence of rejecting the leader's offer, i.e., whether the leader is going to charge a higher or lower l_{t+1}. In equilibrium the high type F_s cannot reject l_t with probability 1, because in that case we would have $\overline{\eta}(l_t) = \overline{p} > \chi$ and the leader charging $l_{t+1} = \bar{l}_F$, so the high type F_s would be better off accepting l_t. But we already saw that the high type F_s cannot accept such an l_t with probability 1 either. In fact, the offer of $l_t \in (\tilde{l}_F, \bar{l}_F]$ is a dilemma for the high type because if it rejects, the leader will charge an even higher $l_{t+1} = \bar{l}_F$; if it accepts, it gets the smallest expected enterprise value.

   Hence, the high type follower needs a mixed strategy here by randomizing between accept and reject. In equilibrium the high type F_s must randomize in order to make the leader's posterior belief satisfy $\overline{\eta}(l_t) = \chi$ so that the leader will offer the price l_{t+1} to be any randomization between $\underline{l}_F$ and $\bar{l}_F$. Let $y(l_t)$ denote the probability that the high type F_s accepts l_t. Then $\overline{\eta}(l_t) = \chi$ will give:

$$\overline{\eta}(l_t) = \frac{\overline{p}\big(1 - y^*(l_t)\big)}{\overline{p}\big(1 - y^*(l_t)\big) + \underline{p}} = \chi \;\Rightarrow\; y^*(l_t) = 1 + \frac{\chi \underline{p}}{\chi \overline{p} - \overline{p}}$$

   which defines a unique $y^*(l_t) = y^* \in [0, 1]$. Note that $y^*(l_t)$ is independent of l_t. Any $y < y^*$ will make the leader's posterior belief $\overline{\eta} > \chi$, which leads to $l_{t+1} = \bar{l}_F$. Any $y > y^*$ will make the leader's posterior belief $\overline{\eta} < \chi$, which leads to $l_{t+1} = \underline{l}_F$. Since the equilibrium has to be Pareto efficient, in order for the high type F_s to be indifferent between accepting and rejecting l_t, we need to define another probability $x(l_t)$ for the high type follower to realize its maximum second period gain.

$$U_{F_s}(l_t) - U_{F_s}(\bar{l}_F) = e^{-r} x(l_t)\big(U_{F_s}(\underline{l}_F) - U_{F_s}(\bar{l}_F)\big)$$

   which defines a unique probability $x(l_t)$ for $l_{t+1} = \underline{l}_F$. The leader's expected enterprise value can be calculated as:

$$\overline{p}yU_L(\bar{l}_F) + e^{-r}\left[\overline{p}(1-y)(1-x)U_L(\bar{l}_F) + \overline{p}(1-y)xU_L(\underline{l}_F) + x\underline{p}U_L(\underline{l}_F)\right] \tag{33}$$

Any of those strategies, TT, TM and TB can generates the highest total enterprise value for the leader depending on the parameter values. We summarize the optimistic leader's optimal strategy and expected enterprise value in the first period as one the following:

$$l_t = \begin{cases} \underline{l}_F, & \text{which generates value } U_L(\underline{l}_F); \\ \tilde{l}_F, & \text{which generates value } \overline{p} \cdot U_L(\tilde{l}_F) + e^{-r}\underline{p} \cdot U_L(\underline{l}_F); \\ \bar{l}_F, & \text{which generates value } \overline{p}y \cdot U_L(\bar{l}_F) + e^{-r}\big(\overline{p}(1-y) + \underline{p}\big)U_L(\underline{l}_F). \end{cases}$$

where the third enterprise value is computed using the fact that, for posterior beliefs $\overline{\eta} = \chi$, $l_{t+1} = \underline{l}_F$ is an optimal price in the second period for the seller as $x(\underline{l}_F) = 1$. Note that if the third value is highest, the leader never sells to the low type F_b as $x(\bar{l}_F) = 0$.

The conclusion is thus that there exists a unique perfect Bayesian equilibrium, and that this equilibrium exhibits Coasian dynamics — that is, $\overline{\eta}(l_t) \leq \overline{p}$ for all l_t, so the leader becomes more pessimistic over time, and $l_{t+1} \leq l_t$, so the leader's offer decreases over time.