The Odd Notion of “Reversible Investment”

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Abstract: A special place is occupied in the analysis of investment under uncertainty by the cases of complete reversibility and complete irreversibility. Irreversible investment, with its notion of option value, has been well discussed. Formally, complete reversibility has been less studied, but it has been considered by some to correspond to the long run in economics. Here we use a modification of Brennan and Schwartz’s (1985) model of partially reversible natural resource investment to examine the reversible stopping problem. For a completely reversible investment option the optimal stopping rule is to invest when the project generates enough cash flow to cover the fixed opportunity cost of investment, and to disinvest when it does not. Given the static nature of this rule, net present value as a timing rule under reversibility is rejected. We find that investments that are even slightly irreversible have much in common with completely irreversible investments but nothing in common with completely reversible investments. The case of reversible investment provides a foil for understanding that all investment entails some irreversibility.

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1. Introduction

Real options theory has had much to say about the timing of irreversible investment under uncertainty. Partial reversibility is also of interest given the active secondary markets in capital equipment (Chirinko 1996). Completely reversible investment was studied by Jorgenson (1963) for the case of certainty, and by Abel and Eberly (1996) and Hartman and Hendrickson (2002) for the case of uncertainty. These papers assume that investment decisions are over a continuous flow, yielding rules for intensity of investment at the margin. In this note we investigate the case of a deferrable, completely reversible, lumpy investment. We focus on lumpy investment because lumpiness is a frequent assumption in the analysis of investment, and it yields explicit timing rules such as embodied in the orthodox net present value rule, which has been suggested to apply to reversible investment. Our interest is also drawn to the case of reversibility because often the long run in economics is considered to be a situation of perfect or complete reversibility, a state before investment is irreversibly sunk.

Our point is brought out using a modification of the Brennan and Schwartz (1985) model of deferrable, partially reversible investment. The modified model is the simplest model of investment that we can devise. The simplification allows transparency: the degree of reversibility or irreversibility is summarized in a single parameter. Timing decisions are relatively easily analyzed. Valuation is also transparent. Statics, certainty, variability, and reversibility are intimately related in the analysis.

We demonstrate that the orthodox NPV rule, which has been suggested as applying to completely reversible investment, is not applicable. Instead, a simple static rule comparing current revenues against the current opportunity cost of capital is appropriate. Under reversibility, uncertainty does not affect the timing rule.
2. The Model

Brennan and Schwartz (1985) provide an early paper on partially reversible decision making under uncertainty. Because of its clear presentation of the issues it has been seminal to the development of the economics of optimal stopping under uncertainty. The simplest version of their model, in Section II of their paper, studies the production of a homogeneous good with repeated options to invest and to partially or completely reverse that investment. The essence of their model is as follows:

1. A firm can produce a good or service at exogenous rate $q > 0$ or it can remain idle. In order to move from producing nothing to producing at $q$, an exogenous cash cost, indicative of investment in capital or capacity, is incurred.

2. There is no constraint on how long the firm can remain idle before investing.

3. The unit price of the good, $p > 0$, follows a geometric Brownian motion

$$dp = \eta pdt + \sigma pdz.$$ The risk-adjusted rate of drift of price, $\eta$, is less than the riskless rate of interest, $r$, such that there is an observable rate-of-return shortfall $\kappa = r - \eta > 0$.\(^1\)

4. Should the firm elect to switch from positive production to being idle it incurs an instantaneous cash flow which may be negative (e.g., dismantling costs) or positive (e.g., salvage value).

5. The firm maximizes the expected present value of production by optimally switching production on and off. It may switch between producing and being closed as often as it likes, incurring the relevant cash flow each time it changes policy.

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\(^1\) Brennan and Schwartz (1985) use the concept of a storable commodity that produces an observable net rate of convenience, $\kappa$. McDonald and Siegel (1984) introduce the less restrictive notion of rate of return shortfall, which can apply to non-storable commodities. One can alternatively assume, as in most of the literature on stopping, that investors are risk-neutral.
We emphasize investment timing decisions by making the following modifications to the model:

1. There are no operating costs, making capital the only factor input.\(^2\)

2. The cash flows for changes of policy are \(k < 0\) for opening and \(-\phi k\) for closing the project, where \(\phi \in [0,1]\).\(^3\) Full irreversibility is induced via \(\phi = 0\), and reversibility via \(\phi = 1\).\(^4\)

Partial reversibility obtains for \(0 < \phi < 1\).\(^5\) When \(\phi = 1\) (a possibility not considered by Brennan and Schwartz), reversing the investment resets the program to its original state with no diminishment of the number of investment or reversal options at hand.\(^6\)

3. There are no holding costs while the project is in an idle state. This obviates the need for the separate option to scrap the project.

These modifications produce a very simple lumpy reversible investment problem.

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\(^2\) This is also the approach taken in Dixit and Pindyck’s (2000) analysis of reversibility.

\(^3\) Dixit and Pindyck (2000) consider the case in which the receipt from disinvesting is declining with time, perhaps due to physical deterioration. They also consider the possibility of adding capacity over time.

\(^4\) Under the assumption of no operating costs, no firm would disinvest from a positive, perpetual income stream if there was no compensation for doing so. This is proven by Merhi and Zervos (2007) for the continuous investment case. See their Example 1, with \(\alpha = 1\) and \(\beta = 0\), along with \(n > 1\): Corollary 10 (a) holds, and disinvestment never occurs.

\(^5\) Bell (1995) uses this same representation of reversibility given that operating costs in his model are zero.

\(^6\) Discussions of the reversibility of lumpy investment are sometimes confined to considerations of the absolute amount of investment recovered by disinvesting. Wong (2010) allows for complete recovery of the investment amount at the end of the planning period, and Abel et al. (1996) allow for recovery and reinvestment over two periods. Arguably, full reversibility should reset the program to its initial state, as we model here. In that sense, a single option to reverse an investment is partial reversibility. Keswani and Shackleton (2006) mention what we call full reversibility with a view to estimating its value compared with irreversible investment.
3. The Optimal Timing Solution

The solution to this problem yields two unit price triggers, $p_{open}$, at which it is optimal to open a closed project, and $p_{close}$, at which it is optimal to close an open project, $0 \leq p_{close} \leq p_{open}$. There are also two value functions, one for an open project and one for a closed project. The solution technique is well known and is presented in Chapter 7 of Dixit and Pindyck (1994). The ordinary differential equations for project value, combined with boundary conditions, produce the two analytic value functions for the project. The project value when closed is

$$ w(p) = \beta_1 p^{\gamma_1} > 0, \quad \beta_1 > 0 \quad (1) $$

where

$$ \gamma_1 = \alpha_1 + \alpha_2 > 1, \quad \alpha_1 = \frac{1}{2} \frac{r - \kappa}{\sigma^2}, \quad \alpha_2 = \left[ \alpha_1^2 + \frac{2r}{\sigma^2} \right]^{\frac{1}{2}}, $$

and $\beta_1$ is a constant to be determined by the optimality conditions. The project value when open is

$$ v(p) = \beta_2 p^{\gamma_2} + \frac{qp}{\kappa}, \quad \beta_2 > 0, \quad \gamma_2 = \alpha_1 - \alpha_2 < 0 \quad (2) $$

where $\beta_2$ is another constant to be determined by the optimality conditions. The expected value of the open project at current unit price $p$ in the absence of the option to close is the present value of a perpetual income stream, $\frac{qp}{\kappa}$. Given this, the term $\beta_2 p^{\gamma_2}$ in (2) is the value of reversibility of the investment decision. It reflects an infinitely repeated compound put option, since the put has embedded within it a call to reopen, which in turn contains a put to reclose, and so on.

An optimal solution for $p_{open}$ and $p_{close}$ includes the value-matching conditions

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7 Dixit and Pindyck (1994) set $q = 1$ in their analysis of the problem.
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\[ w(p_{open}) = \beta_1 p_{open}^{\gamma_1} = v(p_{open}) + k = \beta_2 p_{open}^{\gamma_2} + \frac{qp_{open}}{\kappa} + k, \]  

(3)

\[ v(p_{close}) = \beta_2 p_{close}^{\gamma_2} + \frac{qp_{close}}{\kappa} = v(p_{close}) - \phi k = \beta_1 p_{close}^{\gamma_1} - \phi k. \]  

(4)

An optimal solution for \( p_{open} \) and \( p_{close} \) also includes the so-called smooth-pasting conditions from (3) and (4):

\[ w'(p_{open}) = \gamma_2 \beta_2 p_{open}^{\gamma_2-1} + \frac{q}{\kappa} = v'(p_{open}) = \gamma_1 \beta_1 p_{open}^{\gamma_1-1}, \]  

(5)

\[ v'(p_{close}) = \gamma_1 \beta_1 p_{close}^{\gamma_1-1} = v'(p_{close}) = \gamma_2 \beta_2 p_{close}^{\gamma_2-1} + \frac{q}{\kappa}. \]  

(6)

Rearranging these four equations,

\[-\beta_1 p_{open}^{\gamma_1} + \beta_2 p_{open}^{\gamma_2} + \frac{qp_{open}}{\kappa} = -k, \]  

(7)

\[-\gamma_1 \beta_1 p_{open}^{\gamma_1-1} + \gamma_2 \beta_2 p_{open}^{\gamma_2-1} + \frac{q}{\kappa} = 0, \]  

(8)

\[-\beta_1 p_{close}^{\gamma_1} + \beta_2 p_{close}^{\gamma_2} + \frac{qp_{close}}{\kappa} = \phi k, \]  

(9)

\[-\gamma_1 \beta_1 p_{close}^{\gamma_1-1} + \gamma_2 \beta_2 p_{close}^{\gamma_2-1} + \frac{q}{\kappa} = 0. \]  

(10)

These four equations that can be solved iteratively for the four unknowns \( \beta_1, \beta_2, p_{open}, \) and \( p_{close}. \)

Dixit (1989) shows that a solution exists and is unique. The model can be solved for any amount of reversibility \( 0 \leq \phi \leq 1. \) Analytic solutions are possible only when \( \phi = 0 \) or \( \phi = 1. \) By inspection, for \( \phi = 1 \) the solution to (7) through (9) has \( p_{open} = p_{close}. \) For \( 0 < \phi < 1 \) Brennan and Schwartz (1985) provide an algorithm for an iterated solution:

\[ p_{open} = \frac{\gamma_2 (\phi k - \kappa x^{\gamma_1})}{(x^{\gamma_1} - x^{\frac{q}{\kappa}} (\gamma_2 - 1))}. \]  

(11)
where \( x(\phi) = \frac{P_{close}}{P_{open}} < 1 \) is the solution to the nonlinear equation

\[
\frac{(x^{\gamma_2} - x)(\gamma_1 - 1)}{\gamma_1 (\phi k - kx^{\gamma_2})} = \frac{(x^{\gamma_1} - x)(\gamma_2 - 1)}{\gamma_2 (\phi k - kx^{\gamma_1})}.
\] (12)

Then,

\[
\beta_1 = \frac{q}{\kappa} P_{open} (\gamma_2 - 1) + k \gamma_2
\]

\[
(y_2 - y_1)p_{\gamma_1}^{\text{open}}
\] (13)

and

\[
\beta_2 = \frac{q}{\kappa} P_{close} (\gamma_1 - 1) + \phi k \gamma_1
\]

\[
(y_2 - y_1)p_{\gamma_2}^{\text{close}}
\] (14)

In the rest of the paper we use these results to contrast the case of completely reversible investment with partially reversible or completely irreversible investment.

4. Stopping Under Complete Irreversibility

Though we are concerned with complete reversibility we first present the case of complete irreversibility to illustrate in a simple way the usual rejection of the orthodox NPV rule for timing investment under uncertainty. When the initial investment is irreversible, \( \phi = 0 \). Since \( \phi k = 0 \) upon disinvesting, \( p_{\text{close}} = 0 \). Closure to avoid positive cash flows given \( p > 0 \) will not take place if there is no disinvestment payoff from doing so. The value of the put option \( \beta_2 p_{\gamma_2} \) in equation (2) is also zero.

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8 There is a typographical error in the Brennan and Schwartz paper; the correct general expression for their \( \beta_4 \), which we have labeled \( \beta_2 \), is \( \beta_4 = \frac{d_s (\gamma_1 - 1) + e \gamma_1}{(\gamma_2 - \gamma_1) s_{\gamma_1}^{\gamma_2}} \) given their notation.
since a price of zero is never realized. Dixit and Pindyck (1994, pp. 181-184) solve this problem and find the timing solution\(^9\)

\[ P_{\text{open}} = \frac{-k \kappa}{1} \frac{\gamma_1}{(\gamma_1 - 1)} . \tag{15} \]

Project net present value conditional on current price \( p \) and conditional on immediate investment that that has no put options associated with it is, \( NPV(p) = \frac{qP}{\kappa} + k . \) \tag{16} \]

This is our interpretation of the calculation that leads to the *orthodox NPV rule*: invest when

\[ P_{\text{open}} = \frac{\kappa (-k)}{q} . \tag{17} \]

Since \( \gamma_1 > 1 \), the orthodox NPV timing rule causes premature initiation of the irreversible project, a well-publicized result.

5. Stopping Under Complete Reversibility

Investment is completely reversible when there is no intertemporal price difference between the initial investment cost, the net proceeds from dismantling that capital, and the cost of reinstalling it. This condition disallows switching costs, and also requires that capital not melt during use. It is clearly a fiction, the limit of the more reasonable case of partially irreversible investment.

Mathematically, when investment is completely reversible \( \phi = 1 \) and \( x(1) = 1 \). From (11), and given \( x'(\phi) > 1 \) the solution for \( P_{\text{open}} \) for this case becomes analytically solvable:

\[
\lim_{\phi \to 1} P_{\text{open}} = \lim_{\phi \to 1} \frac{\gamma_2 \left( \phi k - k x(\phi) \gamma_1 \right)}{x(\phi) \gamma_1 - x(\phi) \gamma_1 - 1} = \lim_{x \to 1} \frac{\gamma_2 \left( k - k x \gamma_1 \right)}{x^{\gamma_1} - x \gamma_1 - 1} = \frac{k \kappa \gamma_1}{q} \frac{\gamma_1}{(\gamma_1 - 1)(\gamma_2 - 1)} = \frac{-rk}{q} . \tag{18}
\]

\(^9\) They solve the problem for \( q = 1 \), and so \( q \) does not show up explicitly in their solution.
the last equality following algebraic simplification. As predicted by the advantage of having a put option to reverse the investment, this stopping point is to the left of the stopping point under irreversibility, since \( 0 < \frac{\gamma_2}{(\gamma_2 - 1)} < 1 \) given \( \gamma_2 < 0 \) (see equation (15)).

The economic results from reversibility are revealing. The rule for opening and closing,

\[
p_{\text{close}} = \frac{-rk}{q} = p_{\text{open}},
\]

(19)
is independent of the rate of risk-adjusted rate of drift of price, risk preferences, or any parameter related to the stochastic nature of the problem. The only variables of relevance to the timing of the application of capital to production are the current spot price, the riskless interest rate and the production function linking the application of lumpy investment cost to corresponding output level. Most notably, the uncertainty-investment relationship has disappeared. Nor does the rule require dynamic analysis: multiplication by \( q \) in equation (18) shows that the rule is a direct comparison of current cash flows \( pq \) and current fixed opportunity costs \( rk \). For this reason it is a static timing rule akin to the short-run analysis in microeconomic textbooks. Reversibility can thus be viewed as rendering the stopping problem trivial. Project value is not calculated at each instant; only the instantaneous return \( pq + rk \) from employing reversible capital is calculated, and decisions are taken point by point. In managing the project there is no need for dynamic planning, no need for present value analyses, and no need for a net-present-value timing rule.

6. The Implications of Reversibility for the Concept of Investment

The stopping rule in (19) parallels Jorgenson’s rule for continuous, reversible investment flows. Under Jorgenson’s rule, the capital stock continually adjusts such that the marginal revenue product
of capital is kept equal to the user cost of capital (its rental cost), instant by instant. If the return should fall below \((r + \delta)\) for a period, the capital can be sold for its full value, net of any deterioration, and bought back if returns cover the cost of capital, until in equilibrium it earns exactly \((r + \delta)\). Hartman and Hendrickson (2002) show that the same result obtains under uncertainty – reversibility renders the problem unaffected by uncertainty and risk preferences.

Applications with continuous reversible investment flows cause capital to be a variable input, indistinguishable from labor. It is continually applied to the problem as a rental good, at varying intensity. Investment is usually distinguished from factor flows because of lumpiness. Lumpiness invokes notions of timing since the marginal optimality conditions associated with continuous flows cannot be met. Investment timing decisions are difficult because they require intertemporal decision making given the persistence of investments in lumpy capital. Reversible lumpy investment, though requiring timing via (19), requires no intertemporal comparisons. “Reversible investment” has no persistence, and for that reason is not really investment at all, even when applied in lumps. Irreversibility, rather than lumpiness, is the defining attribute of investment.

7. The Implications of Reversibility for the Theory of the Firm

We have shown that the average cost of production under lumpy reversible investment is \(rk/q\). It is fixed and via its reversibility implies an opportunity cost. This is the short-run case in microeconomic theory, associated with certainty but often without explicit recognition that a positive opportunity cost of capital implies reversibility; the capital cannot be sunk. The requirement that investment be

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10 Lau (2000) provides a succinct account of Jorgenson’s analysis of capital. If there are adjustment costs (if there is not perfect reversibility) Jorgenson’s rule is modified. See, for example, Hayashi (2000).

11 Irreversibility also injects timing considerations into the problem of applying continuously variable capital, where actions to continuously invest in or disinvest in capital happen over discrete intervals of time (Dixit and Pindyck 2000).
reversible means that the certainty case carries over to the case of uncertainty. The textbook approach is more general than it appears.

The long-run case of no fixed costs is often associated with increased flexibility of investment. Our example shows that reversibility of investment, the ultimate flexibility, does not accord with the long-run if it is lumpy. Instead, reversibility must be combined with continually variable capital to remove the notion of fixed costs. Textbook analyses therefore always assume reversibility of capital, with the long-run and short-run simply being distinguished by whether capital is available continuously or in lumps. The analysis eschews investment, as we have defined it, since there is no consideration of irreversibility.

8. The Implications of Reversibility for the Orthodox NPV Rule of Investment

Reversible investment has been previously linked to the orthodox NPV rule of investment, which is presented by Dixit and Pindyck (1994, 4) as follows: “First, calculate the present value of the expected stream of profits that [the lumpy investment] will generate. Second, calculate the present value of the stream of expenditures required to [make the investment]. Finally, determine whether the difference between the two—the net present value (NPV) of the investment—is greater than zero. If it is, go ahead and invest.”\(^{12}\) This is the timing rule presented in (17) above. The literature on real options emphasizes the suboptimality of using the orthodox rule as a basis for decisions on investment because “…it assumes that either the investment is reversible, that is, it can somehow be undone and the expenditures recovered should market conditions turn out worse than anticipated, or,

\(^{12}\) The orthodox rule could also be interpreted as suggesting investment as soon as the present value index, the ratio of the present value of project net cash flows to the present value of investments, equals one. The orthodox rule is the mainstay of analysis in engineering economy.
if the investment is irreversible, it is a now or never proposition…” (Dixit and Pindyck 1994, 6). That the orthodox rule assumes now-or-never decision making is well accepted. Authors from a wide array of disciplines have repeated the claim that it is optimal for deferrable, reversible investments.\textsuperscript{13} It has appeared in articles aimed at the practitioner (e.g., Dixit and Pindyck 1995). The NPV timing rule for a now-or-never investment, (17), is different from the instantaneous, degenerate rule under reversibility, (19). As we have found, the application of reversible capital does not require present value analysis. The NPV rule applies to a sunk (irreversible) input, not to a reversible input. The situations represented in (17) and (19) are not manifestations of the same phenomenon or even comparable, but completely different.

While there is no need to calculate a net present value, or indeed conduct any dynamic analysis in managing a reversible stopping problem, the value functions in (1) and (2) nevertheless allow for additional insights. Davis and Cairns (2012) show that there are two reasons for waiting to act, pure postponement flow, where the rate of gain in the underlying asset’s net present value exceeds the rate of discount, and quasi-option flow, which is the value of information from delayed decision making. Pure postponement flow can occur under both certainty and uncertainty, and quasi-option flow only under uncertainty. Using these concepts we can present stopping rule (19) in the notional equivalent to a general NPV timing rule. A general NPV rule is optimal when all available choices of the decision maker are taken into account, i.e. when (a) mutually exclusive investments at alternative dates are considered to be mutually exclusive projects and (b) the rule prescribes investing in the project with the highest NPV as of the current date. This general rule is a simple extension of the rule

\textsuperscript{13} See, for example, Holland et al. (2000, 34); Adner and Levinthal (2004, 76); Demont, Wesseler, and Tollens (2005, 116); and Huisman (2010, 30). In 2003 testimony to the FCC Pindyck (2003) asserts that the orthodox NPV timing rule is optimal under complete reversibility.
used in cost-benefit analysis for analysis involving mutually exclusive projects at a given point in
time, usually the current date.

Since the stopping rule in (19) does not involve the level of uncertainty (does not involve $\sigma^2$),
there can be no quasi-option flow associated with the solution. Mathematically, quasi-option flow is
(Davis and Cairns, 2012)

$$\hat{\alpha}(p_{\text{open}}) = \frac{1}{2} \sigma^2 p_{\text{open}}^2 \left( w''(p_{\text{open}}) - y''(p_{\text{open}}) \right) \frac{1}{y''(p_{\text{open}})} ,$$  (20)

where $y$ is defined as the returns from immediate stopping,

$$y(p_{\text{open}}) = \beta_2 p_{\text{open}}^{\gamma_2} + \frac{q p_{\text{open}}}{\kappa} + k. $$  (21)

From (1) and (3),

$$w''(p_{\text{open}}) = \frac{\gamma_1 (\gamma_1 - 1) \beta_1 p_{\text{open}}^{\gamma_1}}{p_{\text{open}}^2}$$  (22)

and

$$y''(p_{\text{open}}) = \frac{\gamma_2 (\gamma_2 - 1) \beta_2 p_{\text{open}}^{\gamma_2}}{p_{\text{open}}^2}$$  (23)

If $w''(p_{\text{open}}) = y''(p_{\text{open}})$ then there is no quasi-option flow from waiting. Some tedious algebra shows
that $w''(p_{\text{open}}) = y''(p_{\text{open}})$ at the stopping point $p_{\text{open}} = \frac{-r k}{q}$.

In stopping problems where only pure postponement flow is enjoyed by waiting, the myopic r-
percent stopping condition (Davis and Cairns 2012) obtains;

$$\frac{E[dy(p_{\text{open}})]}{y(p_{\text{open}})dt} = r,$$  (24)
where the expectation is taken over the risk-adjusted measure and equated to the risk-free rate.

Waiting longer would cause the rate of growth in net project value to fall below the interest rate, indicating that such additional waiting is suboptimal. From (21) and making use of Ito calculus,

\[
E[dy(p_{\text{open}})] = \eta \left( \gamma_2 \beta_2 p_{\text{open}}^{\gamma_2} + \frac{p_{\text{open}} q}{\kappa} \right) + \frac{1}{2} \sigma^2 \gamma_2 (\gamma_2 - 1) \beta_2 p_{\text{open}}^{\gamma_2} - \beta_2 p_{\text{open}}^{\gamma_2} + \frac{p_{\text{open}} q}{\kappa} + k_{\text{open}}.
\]

More tedious algebra shows that the right-hand side of (24) equals \( r \) at the stopping point

\[
p_{\text{open}} = \frac{-rk}{q}.
\]

9. Numerical Example

We use the parameter values in Brennan and Schwartz, Table 1, and the simplifications in our model (no operating costs, no taxes, etc.) to show a numerical example for a reversible cash flow of -$1 million yielding 10 million units of output per period (see the caption to Figure 1 below for the other parameter values). Stopping rule (18) yields \( p_{\text{open}} = 0.01 \). Substituting this into (24) shows that at stopping the expected rate of growth in net realizable project value is \( r = 0.10 \).

The value of the project conditional on spot price is shown in Figure 1. The difference in value between open and closed projects at spot price 0.01 is the reversible investment cost. The project value on each side of the opening price is strictly convex in price, as indicated in equations (1) and (2). Of note, at the optimal investment trigger the open project value, at $10.1 million, is 10.1 times the investment cost, giving a PVI of 10.1. Via equation (21) the NPV at entry is $9.1 million, of which $0.1 million arises from reversibility.
Figure 1: The value of the project ($ million) given investment cash flow $k = -$1 million, shut-down cash flow of $-k = $1 million, output rate $q = 10$ million units/period, average production cost $a = $0, shut-down maintenance cost $f = $0, inflation rate $\pi = 0\%$, unit price rate of net convenience $\kappa = 1\%$, price volatility $\sigma^2 = 8\%$, no taxes, and risk-free rate of return $r = 10\%$. The optimal open/shut trigger is at unit price $p_{open} = p_{close} = 0.01$.

Introducing even slight irreversibility into this example changes the unit price trigger quite substantially from that associated with reversibility: for $\phi = 0.98$, $p_{open} = 0.012$ and $p_{close} = 0.008$.

Figure 2 provides the opening and closing unit price triggers for the range of investment reversibility. The nonlinearity of price triggers to degree of reversibility has previously been noted by Abel and Eberly (1996, p. 587), who compute that the slope of the curve at $\phi = 1$ is infinite. Of interest in this case is the sensitivity of the closing trigger and insensitivity of the opening trigger to reversibility.$^{14}$

$^{14}$ This in contrast to a numerical example shown in Dixit and Pindyck (1994), where the opening and closing price triggers “do not rise and fall by very much” as reversibility varies (p. 226).
Figure 2: Open and close price triggers for varying degrees of reversibility, $\phi$, for the parameter values given in Figure 1. $\phi = 0$ is complete irreversibility and $\phi = 1$ is complete reversibility.

10. Conclusions

Reversibility is not a benchmark or idealization in the theory of capital or in the measurement of capital. Rather, the distinguishing attributes of capital are purged through reversibility:

1. Investing in capital is often said to entail a sacrifice, and to have persistence. Reversible capital does not entail a sacrifice beyond the extent to which any variable input entails an opportunity cost. There is no rule of timing because decisions are static, not dynamic.
2. There is no impact of uncertainty when capital is reversible. As under certainty, the gap between the investment price and the disinvestment price vanishes.
3. Information required for decisions are reduced to the observation of current economic conditions. There is no need to compute project present value.

4. Present value is not relevant to decisions.

The stopping problem under reversibility is the limit of the dynamic stopping problem as the level of irreversibility goes to zero. This limit is qualitatively different: it is static, not dynamic. The conceptual discontinuity attested to by the infinite slopes of the curves for the trigger prices in Figure 2.

The notions of net present value and of optimal timing are inapplicable in the case of reversible investment. The orthodox NPV rule does not apply. Instead, under reversibility the project manager should use a static rule that activates the project when the instantaneous net-cash-flow rate of return exceeds the riskless opportunity cost of capital. The manager should deactivate the project when they do not. With even an infinitesimal level of irreversibility, the above four properties no longer apply and the usual attributes of capital re-assert themselves – in particular, net present value, risk and timing emerge. Reversibility is the smile on the Cheshire cat of investment.

The purging of the four properties through the assumption of reversibility indicates that irreversibility is the defining quality of capital under both certainty and uncertainty. If capital is reversible, it is a riskless input comparable to materials. The lumpiness that we have modeled here causes that input to have fixed, as opposed to variable, opportunity costs, which in microeconomic theory distinguishes the short run from the long run. That the consideration of reversible capital accords to the static microeconomic theories of the firm signals reversibility as a limit representing the nullity, the non-substantiality of capital.
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