Generation Investments under Uncertainty

Roger Adkins*

Bradford University School of Management

Dean Paxson**

Manchester Business School

28 Jan 2015

JEL Classifications: D81, G31, H25

Keywords: Real Options, Generation Investments, Old Product Retention, New Product Enhancement, Investment Cost Reduction.

Acknowledgements: we thank Alcino Azevedo, Michael Flanagan, Paulo Pereira, and Artur Rodrigues for valuable comments on earlier versions.

*Bradford University School of Management, Emm Lane, Bradford BD9 4JL, UK. r.adkins@bradford.ac.uk

+44 (0)1274233466.

**Manchester Business School, University of Manchester, Manchester, M15 6PB, UK. dean.paxson@mbs.ac.uk

+44(0)1612756353.

Generation Investments under Uncertainty

Abstract

We develop a general model for generation investments under uncertainty that considers replacing an incumbent product with a new generation product. We allow for partial (retentions) or complete replacement, but also for the possibility that the existence of an incumbent product may enhance the value of the new product, or reduce the investment required to develop and promote that new product, as is characteristic of movie and book sequels, and other new product developments. We provide quasi-analytical solutions for the thresholds that justify new product introduction and the real option value of the investment opportunity, considering both incumbent and new product value uncertainty, and possible correlation. The new product thresholds are more or less a linear function of the value of the incumbent product sacrificed, even with partial retention, but not linear with regard to any enhancement of the new product due to the incumbent, or a investment cost reduction. The real option value sensitivity to changes in these factors is far from linear.

Generation Investments under Uncertainty

A generational investment is an ordered sequence of individual but associated investment opportunities that can be conceptualized as a chain of successive links. Each link represents a stage in the chain depicting a single investment opportunity. The links are organized according to a particular order that reflects their logically meaningful position in the chain. It specifies which one of the various investment opportunities has to be started first, which one completes the sequence and is last, and the ordering for those remaining. Except for the two labeled first and last, each individual investment opportunity has a single predecessor stage and a single successor stage. A chain of ordered links is a useful conception for a set of individual investment opportunities whenever there are real economic gains to be made by completing the set as an ordered sequence rather than treating each one as independent. Some of the gains from treating a set of opportunities as generational may be realized by redeploying any of the assets created by a predecessor stage at a successor stage for a cost below the original.

The redeployable asset can be tangible, such as equipment or real estate acquired at a predecessor stage that can be redeployed economically at the successor stage owing to scope economies. Alternatively, they can be intangibles, such as reputation generated through marketing and usage that is leveraged at a successor stage to enhance market volume or reduce marketing expenditure, or distinctive competences and proprietary knowledge that are deployed in completing the successor stage having been specifically developed at a predecessor stage. Illustrations of generational investments abound in several diverse industries. Supermarkets and hotel chains typically treat their outlets as generational investments as their operating and marketing expertise is shared with new ventures, leading to investment cost reduction and subsequent product enhancements, although seldom incumbent replacements due to locational advantages. Often, film-makers produce a generation of film creations that achieve economies from a shared marketing expense. Software developers continually upgrade their offerings that explicitly exploit previously developed code and expertise. Firms in R&D significant industries redeploy their organizational competences and past reputation in their endeavours to sustain their leading-edge advantage.

Bollen (1999) provides a real option model for the demand in a product life cycle. Hitsch (2006) looks at product launch and exit under uncertainty. Chao and Kavadis (2008) suggest strategic buckets of R&D for particular products, based on product maturity and specific requirements. Rodrigues (2009) allows for demand to vary by stages, where investment can alter expected demand at a particular stage. Rubera et al. (2012) consider technological innovation effects in new products, which result in value enhancement over stages.

Sequels are not uncommon in the movie, publication (and indeed academic) industries, where there is "value enhancement" over stages, with also possible high value retention for earlier stages, as viewers seek to read (or re-read), view or view again, earlier series when the latest sequel appears. Elberse and Eliashberg (2003) examine the dynamics for motion picture sequels, Chance et al. (2008) provide a option pricing model for box office revenue, and Gong et al. (2011) separate the movie production into four distinct investment stages, where there is the possibility of failure reducing over the stages.

Adkins and Paxson (2011) provide a real option model for equipment replacements, where revenues decline and operating costs increase over time, but do not allow for any incumbent equipment retention, or value enhancement for subsequent stages. Adkins and Paxson (2013) allow for technological progress in new equipment, which does not necessarily depend on the incumbent or investment cost at any stage. Adkins and Paxson (2014) allow for a type of incumbent value retention in terms of a stochastic abandonment value, which might be in the second-hand market, or alternative use, or scrap value, but do not consider a successor value.

The next section presents a general model, allowing for incumbent retention, successor value enhancement, and investment cost reduction. The middle section provides numerical illustrations for base case parameter values, and shows the sensitivity of thresholds and real option values to changes in parameter values over ranges. The final section is a conclusion, suggesting some future research.

1 The Model

1.1 General Two Stage Generation Model

A firm in a monopoly position is considering a generational investment opportunity that offers the possibility for investing in and benefiting from the next generational offering while simultaneously relinquishing part or all of the incumbent. We can conceive of two successive stages, where the primary benefits for an incumbent occur only during the current stage, whilst those for next generational offering only during the next subsequent stage. The firm is motivated to actively pursue a research policy for developing the next generational offering. The benefits of such a policy are only obtainable by sacrificing the present value rendered by the incumbent and then expending an investment cost in order to receive contemporaneously the net cash flow stream accruing to the next generational offering. In an uncertain world, the present value for any generational offering would normally behave as stochastic. If we confine our attention to only the current and next stage, then the resulting model for determining the optimal policy for terminating the incumbent and its subsequent replacement by the next generational offering would be formulated based on two stochastic factors.

The present value for the incumbent during the current stage, stage-1, is denoted by V_1, while that for the next generation offering during the subsequent stage, stage-2, by V_2. Both V_1 and V_2 are stochastic, described by distinct geometric Brownian motion (gBm) processes:

$$\mathrm{d}V_I = \mu_I V_I \mathrm{d}t + \sigma_{V_I} V_I \mathrm{d}z_I \text{ for } I = 1, 2, \tag{1}$$

where μ_I denotes the instantaneous drift term per unit of time[1], σ_{V_I} the instantaneous volatility per unit of time, and $\mathrm{d}z_I$ is an increment of the standard Wiener process. Dependence between the two stochastic factors is described by the instantaneous covariance term $\rho\sigma_{V_1}\sigma_{V_2}$ where $\mathrm{Cov}\left[V_1, V_2\right] = \rho\sigma_{V_1}\sigma_{V_2}V_1V_2\mathrm{d}t$ with $\left|\rho\right| \le 1$.

The decision to "terminate" the incumbent in favour of the next generational offering is partly decided by the value of the option to switch between stage-1 and stage-2. This embedded option arises from owning the incumbent with value V_1 during stage-1, but its value is also determined

[1] We assume μ=θ for convenience, and these values are not affected by other parameter value changes.

by the value V_2 rendered during stage-2. Denoting the option value by F_{12}, then $F_{12} = F_{12}\left(V_1,\, V_2\right)$, and its valuation relationship is specified by applying Ito's Lemma to (1):

$$\tfrac{1}{2}\sigma_{V_1}^2 V_1^2 \frac{\partial^2 F_{12}}{\partial V_1^2} + \tfrac{1}{2}\sigma_{V_2}^2 V_2^2 \frac{\partial^2 F_{12}}{\partial V_2^2} + \rho\sigma_{V_1}\sigma_{V_2} V_1 V_2 \frac{\partial^2 F_{12}}{\partial V_1 \partial V_2} + \theta_{V_1} V_1 \frac{\partial F_{12}}{\partial V_1} + \theta_{V_2} V_2 \frac{\partial F_{12}}{\partial V_2} - rF_{12} = 0.$$

where the parameters θ_{V_1} and θ_{V_2} denote the respective risk neutral drift terms, and r the risk-free rate. From McDonald and Siegel (1986) and Adkins and Paxson (2011), the generic valuation function for the switch option is:

$$F_{12}\left(V_1,\, V_2\right) = A_1 V_1^{\beta_{11}} V_2^{\beta_{21}}, \tag{2}$$

where A_1 is a generic coefficient, and β_{11} and β_{21} are the respective generic power parameters for V_1 and V_2. While $A_1 > 0$, since an option value is always non-negative, the power parameters can be of either sign contingent on the particular context. For a generational opportunity, there is an incentive to switch between stage-1 and stage-2 when simultaneously V_1 is relatively low and V_2 is relatively high, since the net gain from the switch more than compensates any loss. Moreover, the incentive intensifies as V_1 decreases and V_2 increases. Accordingly, we conjecture that $\beta_{11} < 0$ and $\beta_{21} > 0$.

The option value (2) satisfies the valuation relationship with characteristic root equation Q:

$$\begin{aligned} Q\left(\beta_{11}, \beta_{21}\right) = \tfrac{1}{2}\sigma_{V_1}^2 \beta_{11}\left(\beta_{11} - 1\right) + \tfrac{1}{2}\sigma_{V_2}^2 \beta_{21}\left(\beta_{21} - 1\right) + \rho\sigma_{V_1}\sigma_{V_2}\beta_{11}\beta_{21} \\ + \theta_{V_1}\beta_{11} + \theta_{V_2}\beta_{21} - r = 0. \end{aligned} \tag{3}$$

The switch decision depends on the values V_1 and V_2, so we denote by $\hat{V}_{11}$ and $\hat{V}_{21}$ the respective threshold levels that these values have to attain for an optimal switch event to take place. For given threshold levels, $\hat{V}_{11}$ and $\hat{V}_{21}$, an optimal switch event occurs provided that the prevailing V_1 and V_2 satisfy the joint condition $V_1 \le \hat{V}_{11}$ and $V_2 \ge \hat{V}_{21}$. As a trade-off exists between the optimal threshold levels, since any increase in $\hat{V}_{11}$ can be compensated by a commensurate increase in $\hat{V}_{21}$, we need to identify the boundary separating the region indicating a possible viable optimal switch event from that indicating no viable optimal switch event. The boundary that we need to identify is characterized by a relationship between $\hat{V}_{11}$ and $\hat{V}_{21}$.

The switch from stage-1 to stage-2 is predicated under value conservation for values V_1 and V_2 at their respective threshold levels. The value immediately prior to making the switch is that rendered by the incumbent V_1 together with its embedded switch option F_{12}. Immediately following the switch, the created value is derived from the next generational offering less the assumed constant investment cost, denoted by K_{12}, required to obtain the cash flow stream. At the exercise event, the incumbent value $\hat{V}_{11}$ and switch option value $F_{12}\left(\hat{V}_{11}, \hat{V}_{21}\right)$ are sacrificed, and compensated by the net present value for the next generational offering, $\hat{V}_{21} - K_{21}$.

If an allowance is made for partial retention (γ) of the incumbent, for enhancement (α) of stage-2 value due to existence of an incumbent, and reduction (ϕ) of investment cost at stage-2[2], the thresholds and investment cost are:

$$(1-\gamma)\hat{V}_{11}, (1+\alpha)\hat{V}_{21}, \phi K_{21}$$

Accordingly, the value matching relationship becomes:

$$A_1 \hat{V}_{11}^{\beta_{11}} \hat{V}_{21}^{\beta_{21}} = (1+\alpha)\hat{V}_{21} - \phi K_{21} - (1-\gamma)\hat{V}_{11} \tag{4}$$

The smooth pasting conditions associated with (4), for first order optimality, one for each factor V_1 and V_2, respectively, can be expressed as:

$$(1-\gamma) + \beta_{11} A_1 \hat{V}_{11}^{\beta_{11}-1} \hat{V}_{21}^{\beta_{21}} = 0, \tag{5}$$

$$\beta_{21} A_1 \hat{V}_{11}^{\beta_{11}} \hat{V}_{21}^{\beta_{21}-1} - (1+\alpha) = 0. \tag{6}$$

From (5) and (6), our conjecture on the signs of the two power parameters is corroborated. By combining (4), (5) and (6), if γ=0, ϕ=1, α=0, we obtain:

$$\hat{V}_{21} = \frac{\beta_{21}}{\beta_{21} + \beta_{11} - 1} K_{21}, \tag{7}$$

[2] We assume that γ,α and ϕ are not dependent on time, or on the level of or changes in V_1, V_2 or K_2.

$$\hat{V}_{11} = \frac{-\beta_{11}}{\beta_{21} + \beta_{11} - 1} K_{21}. \quad (8)$$

It is observed from (7) and (8) that the value thresholds signifying an optimal switch from stage-1 to -2 are positively, linearly dependent on the investment cost, so an increase in K_{21} produces increases in both $\hat{V}_{11}$ and $\hat{V}_{21}$ while their ratio $\hat{V}_{21} / \hat{V}_{11} = -\beta_{21} / \beta_{11}$ depends on only β_{11} and β_{21}. Further, the stage-2 value V_2 has to exceed the stage-1 value V_1 whenever an optimal switch between stages is being contemplated, since from (4) the switching gain $V_2 - K_{21}$ must compensate the foregone value $V_1 + A_1 V_1^{\beta_{11}} V_2^{\beta_{21}}$. This entails that $\beta_{21} \geq -\beta_{11}$.

The stage-2 generation model is composed of 4 equations, 3, 4, 5 and 6, or alternatively,
3 equations: (i) and (ii) two reduced form value matching relationships (7) and (8), and (iii) the Q equation (3). From these, it is possible in principle to eliminate the power parameters β_{11} and β_{21} to construct the threshold boundary linking $\hat{V}_{11}$ and $\hat{V}_{21}$. In practice, this is achieved numerically.

1.2 Specific Two Stage Generation Models

Retention

If γ does not equal 0, but ϕ=1, α=0, allowing for some retention of the incumbent value upon introduction of a new produce, the revised value matching relationship is:

$$\hat{V}_{11} + A_1 \hat{V}_{11}^{\beta_{11}} \hat{V}_{21}^{\beta_{21}} = \gamma \hat{V}_{11} + \hat{V}_{21} - K_{21}$$

where $0 \leq \gamma$ denotes the proportion of the stage-1 value retained. Then, similarly:

$$\hat{V}_{11}(1-\gamma) = \frac{-\beta_{11}}{\beta_{21} + \beta_{11} - 1} K_{21}, \ \hat{V}_{21} = \frac{\beta_{21}}{\beta_{21} + \beta_{11} - 1} K_{21} \text{ and } \frac{\hat{V}_{11}(1-\gamma)}{-\beta_{11}} = \frac{\hat{V}_{21}}{\beta_{21}}.$$

Since $\beta_{11} + \beta_{21} = \beta_2$ and $K_{21} = K_2$, then:

$$\frac{\beta_{21}}{\beta_{21} + \beta_{11} - 1} K_{21} + \frac{\beta_{11}}{\beta_{21} + \beta_{11} - 1} K_{21} = \frac{\beta_2}{\beta_2 - 1} K_2$$

$$\hat{V}_{21} - \hat{V}_{11}(1-\gamma) = \hat{V}_2$$

Less value to be sacrificed lowers $\hat{V}_{21}$ but not sufficiently for $\hat{V}_{21} < \hat{V}_2$.

Enhancement

If γ=0, ϕ=1, α does not equal 0, allowing for enhancement of the new product due to the existence of an incumbent, the revised value matching relationship is:

$$\hat{V}_{11} + A_1 \hat{V}_{11}^{\beta_{11}} \hat{V}_{21}^{\beta_{21}} = \left(1+\alpha\right)\hat{V}_{21} - K_{21}$$

where $0 \le \alpha$ denotes the proportional increase in stage-2 value due to the generation effect. Then as before:

$$\hat{V}_{11} = \frac{-\beta_{11}}{\beta_{21}+\beta_{11}-1} K_{21},\ \hat{V}_{21}\left(1+\alpha\right) = \frac{\beta_{21}}{\beta_{21}+\beta_{11}-1} K_{21} \text{ and } \frac{\hat{V}_{11}}{-\beta_{11}} = \frac{\hat{V}_{21}\left(1+\alpha\right)}{\beta_{21}}.$$

Since $\beta_{11}+\beta_{21}=\beta_2$ and $K_{21}=K_2$, then:

$$\frac{\beta_{21}}{\beta_{21}+\beta_{11}-1} K_{21} + \frac{\beta_{11}}{\beta_{21}+\beta_{11}-1} K_{21} = \frac{\beta_2}{\beta_2 - 1} K_2$$

$$\hat{V}_{21}\left(1+\alpha\right) - \hat{V}_{11} = \hat{V}_2$$

More value to be created lowers $\hat{V}_{21}$, $\hat{V}_{21} < \hat{V}_2$ provided $\hat{V}_{11} < \alpha\,\hat{V}_2$.

Investment Cost Reduction

If γ=0, and α=0, but ϕ does not equal 1, allowing for a reduction in the second stage investment cost due to the existence of an incumbent (proprietary knowledge, learning effect, economies of scale), the revised value matching relationship is:

$$\hat{V}_{11} + A_1 \hat{V}_{11}^{\beta_{11}} \hat{V}_{21}^{\beta_{21}} = \hat{V}_{21} - \phi K_{21}$$

Then as before:

$$\hat{V}_{11} = \frac{-\beta_{11}}{\beta_{21}+\beta_{11}-1} \phi K_{21},\ \hat{V}_{21} = \frac{\beta_{21}}{\beta_{21}+\beta_{11}-1} \phi K_{21} \text{ and } \frac{\hat{V}_{11}}{-\beta_{11}} = \frac{\hat{V}_{21}}{\beta_{21}}.$$

Since $\beta_{11}+\beta_{21}=\beta_2$ and $K_{21}=\phi K_2$, where $0 \le \phi$ measures the proportional investment cost reduction, then lower K lowers $\hat{V}_{21}$.

2 Numerical Illustrations

Although the model analysis has revealed some useful properties, further insights into the behaviour of the models can be gained through the application of numerical simulations. The numerical analyses are founded on the base case values presented in Table 1 with variations to reflect the parameter of interest. The base case values assume $\sigma_{V_1} = \sigma_{V_2}$, $\rho_{V1,V2} = 1$ and $\theta_{V_1} = \theta_{V_2} = 0$.

**** Table 1 about here ****

Based on the base case values, the investment threshold boundary is numerically constructed as the locus relating $\hat{V}_{21}$ evaluated from (3), (4) (5) and (6) for incremental variations in $\hat{V}_{11} = V_1, \hat{K}_{21} = K_2$. This reveals a linear threshold boundary linking $\hat{V}_{11}$ and $\hat{V}_{21}$, such that $\hat{V}_{21} = \hat{V}_2 + \hat{V}_{11}$ with $\hat{V}_{21} = \hat{V}_2 = 3.0277$ for $\hat{V}_{11} = 1$. Figure 1 shows that there is a positively increasing boundary. For any increase in the sacrifice of the stage-1 value $\hat{V}_{11}$, there is a commensurate and equal increase in the stage-2 threshold $\hat{V}_{21}$. Incidentally, Figure 6 illustrates that there the same linear relationship of the stage-2 threshold $\hat{V}_{21}$ to the proportional retention γ of some of the stage-1 value upon introduction of the stage-2 product. However, the real option value of the opportunity to make the second stage investment declines as the incumbent value V_1 increases as in Figure 1, and naturally increases as the retention proportion increases as in Figure 6, which considers also the possibility that the V_1 value might increase as V_2 is introduced, as in some movie sequels.

****Figures 1 and 6 about here****

2.1 Sensitivity of Thresholds and ROV to Changes in Standard Inputs

The numerical sensitivities to changes in the basic input parameter values below are assumed not to affect the basic V_1, V_2 and K_2 values, which is not entirely realistic. For instance, an increase in the interest rate should result in a decline of the present values of future cash flows, but this possibility is ignored, and changes in r are only considered in equation 3, the Q function. Similarly it is assumed that changes in expected volatility and correlation of V_1 and V_2 do not affect the risk neutral drift rates, which is a heroic assumption.

Typically the thresholds and real option values are expected to increase as the V_1 and/or V_2 volatilities increase (positive "vegas") if V_1 and V_2 correlations are positive, but this is not always the result as shown in Figure 2. Both the optimal threshold and the real option value appear to decline as the V_2 volatility is increased from a very low 5% to 10% but then eventually increase as volatility increases. This appears to be a peculiar case of a very high correlation between the two stage product values. The usual case where correlation is less than perfect shows a more or less continuous increase in both thresholds and real option values as stage-2 product volatility increases, with the relationship becoming more linear the less positive the correlation.

Figure 3 shows that thresholds and real option values decrease with the increase of correlation between the products over the two stages, with very high V_2 thresholds at very negative correlations. It seems logical if high increases in V_2 values are accompanied by decreases in V_1 values, and vice versa, V_2 would have to be very high before new product introduction is justified. Such a high negative correlation would indicate that these products are completely different, which is not exactly in the spirit of examining "generation" investments.

Figure 4 is not easy to understand, since it is apparent that large negative drifts in V_1 values results in a high $\hat{V}_2$ value that justifies introducing that new product (which is assumed in the base case to have a nil drift rate). However, at the other end of high positive drift for V_1, there is logically little real option value in the opportunity to introduce a new product with a nil drift, if the incumbent product drift is so positive, indicating demand is still strong.

Figure 5 is also not easy to understand, since normally a high interest rate results in a high call option value. Note that it is assumed that changes in r are not accompanied by changes in V_1 or V_2, or in the risk neutral drifts, which is not realistic. Higher interest rates result in both lower thresholds and also real option values, but at a decreasing rate.

****Figures 2, 3, 4 and 5 about here****

2.2 Sensitivity of Thresholds and ROV to Changes in Value Enhancement & Cost Reduction

The greater the enhancement of V_2 due to the existence of V_1 (reputation, brand image, quality perception, habit), the higher the V_2 value that justifies immediate new product introduction, and the lower the real option value of that generation investment opportunity as shown in Figure 7. It is not clear that α depends on the continued existence of V_1, but the implicit assumption is that V_2 as an independent product may be worth less than as a sequel to a successful incumbent product. The assumption that this relationship is proportional and constant, especially over time, is simple, and could be supplemented by a complex, perhaps quadratic, relationship. But a time varying α would no longer be consistent with the quasi-analytical solution proposed herein.

The greater the investment cost reduction (lower ϕ) due to the existence of V_1 (experience, learning effect, or investment efficiency), the lower the $\hat{V}_2$ value that justifies immediate new product introduction, and the lower the real option value of that generation investment opportunity as shown in Figure 8. It is clear that ϕ does not depend on the continued existence of V_1. The assumption that ϕ is proportional and constant, especially over time, is simple, and could be supplemented by a complex, perhaps quadratic, relationship. But a time varying ϕ would no longer be consistent with the quasi-analytical solution proposed herein.

****Figures 7 and 8 about here****

2.3 Lessons for the Chief Real Options Manager and Investors

The critical activity for the Chief Real Options Manager (CROM) advising the marketing department on the introduction of a new generation product is to view V_1, and estimate V_2, the drifts and volatilities, and correlation, standard real option inputs along with K_2. While there might be some historical experience on V_1, θ_{V1} and σ_{V1}, the inputs for a new product have no history, naturally. Also it may well be that θ_{V1} and σ_{V1} are time varying, even mean reverting, in which case gBm is not an appropriate process as the basis for a model. In addition, the CROM must estimate the retention, value enhancement and investment cost reduction factors, which herein are assumed to be proportional and constant.

Investors may not have sufficient information to estimate whether V_2 at the time of a new product introduction exceeds $\hat{V}_2$ given V_1 and K_2, or whether such an introduction is premature and kills the real option value of the introduction opportunity. Also investors will want to consider the real option value embedded as it were in the current incumbent product state, and whether the stock market value of a generation investment firm reflects V_1+ ROV2 plus other net assets/liabilities.

3 Conclusion

We develop a general model for generation investments under uncertainty that considers replacing an incumbent product with a new generation product. We allow for partial (γ) incumbent product replacement, but also for the possibility that the existence of an incumbent product may enhance the value of the new product (α), or reduce (ϕ) the investment required to develop and promote that new product, as is characteristic of movie and book sequels, and some other new products.

We provide quasi-analytical solutions for the thresholds that justify new product introduction and the real option value of the investment opportunity, considering both incumbent and new product value uncertainty, and possible correlation. The new product thresholds are a linear function of the value of the incumbent product sacrificed, even at various levels of partial retention, but not linear with regard to any enhancement of the new product due to the incumbent, or a investment cost reduction. The real option effects are far from linear. The ROV increases at an increasing rate with regard to increases in γ, and α, in a form similar to the effect of increased new product volatility. Perhaps clever scholars or CROMs will create new names for these effects (NPP super deltas, Cost reduction kappas?).

This basic model requires several broad assumptions, and is currently limited to a very basic environment. A major critical extension is to consider several stages, with a first new product stage, and a final stage beyond which no further products are envisioned (new horse carts, or perhaps laptop computers may eventually have no immediate similar successors). Stochastic investment cost is possibly a relatively easy addition to the basic model, resulting in solving additional equations simultaneously. Slightly complex functions for γ, α, ϕ might be designed which are still compatible with these quasi-analytical solutions. Competition has been completely ignored so far, which is not realistic for most new products. Finally, we anticipate that calibrating the parameter values will be an interesting challenge for the future.

References

Adkins, R., and D. Paxson. "Renewing Assets with Uncertain Revenues and Operating Costs." *Journal of Financial and Quantitative Analysis*, 46 (2011), 785-813.

Adkins, R., and D. Paxson. "Stochastic Equipment Capital Budgeting with Technological Progress." *European Financial Management,* (2013), doi:10.1111/eufm.12000.

Adkins, R., and D. Paxson. "The Effects of an Uncertain Abandonment Value on the Investment Decision." PFN Vilamoura (2014).

Bollen, N.P. “Real Options and Product Life Cycles.” *Management Science*, 45 (1999), 670-684.

Chao, R.O. and S. Kavadis. “A Theoretical Framework for Managing the New Product Development Portfolio: When and How to Use Strategic Buckets.” *Management Science,* 54 (2008), 907-921.

Chance, D.M., E. Hillebrand, and J. Hilliard. “Pricing an Option on Revenue from an Innovation: An Application to Movie Box Office Revenue.” *Management Science*, 54 (2008), 1015-1028.

Elberse, A., and J. Eliashberg. “Demand and Supply Dynamics for Sequential Released Products: The Case of Motion Pictures.” *Marketing Science*, 22 (2003), 329-354.

Gong, J.H., W.A.Van der Stede, and S.M. Young. “Real Options in the Motion Picture Industry: Evidence from Film Marketing and Sequels.” *Contemporary Accounting Research*, 28 (2011), 1438-1466.

Hitsch, G. J. “An Empirical Model of Optimal Dynamic Product Launch and Exit Under Demand Uncertainty.” *Marketing Science*, 25 (2006), 25-50.

McDonald, R. L., and D. R. Siegel. "The Value of Waiting to Invest." *Quarterly Journal of Economics*, 101 (1986), 707-728.

Rodrigues, A. “Market Segmentation under Uncertainty.” Real Options Conference, Braga and Santiago, 2009.

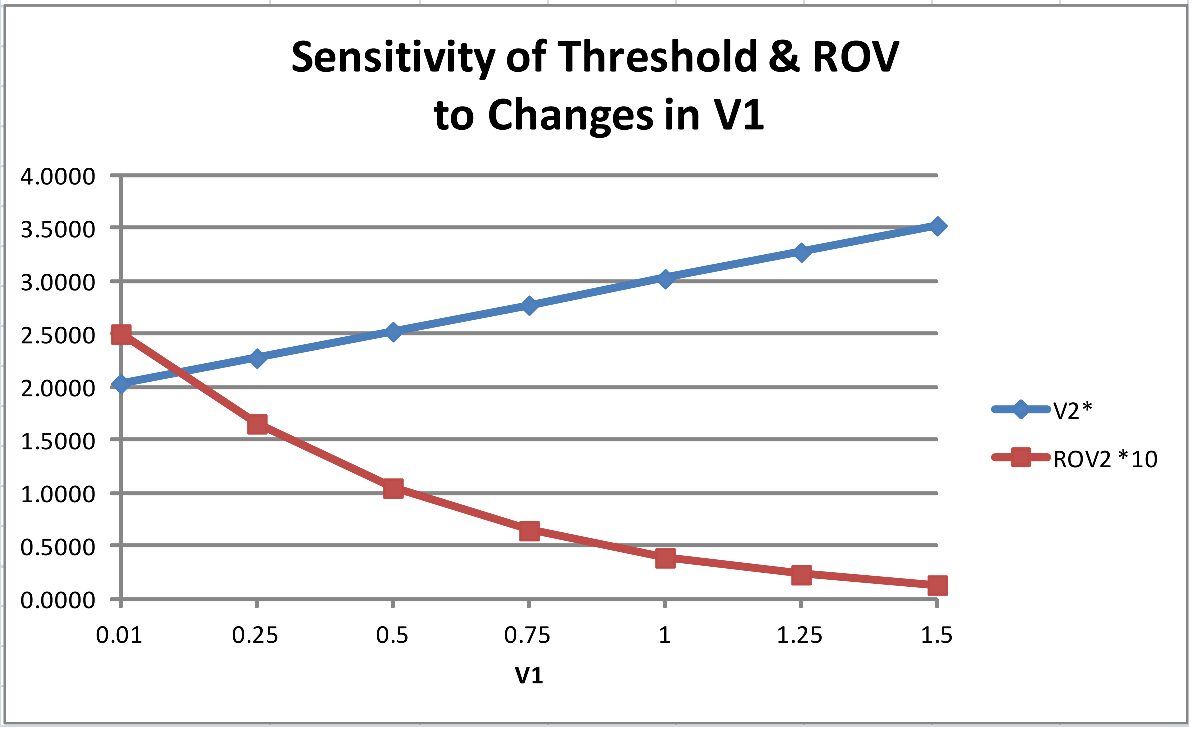
Rubera, G., D.A. Griffith, and G. Yalcinkaya. “Technological and Design Innovation Effects in Regional New Product Rollouts: A European Illustration.” *Journal of Product Innovation Management,* 29 (2012), 1047-1060.

Table 1

	A	B	C	D
1		GENERATION INVESTMENTS		
2	**INPUT**		EQ	
3	V2	1.00		
4	V1	1.00		
5	K2	1.00		
6	σV1	0.25		
7	σV2	0.25		
8	σK2	0.00		
9	ρV1V2	1.00		
10	r	0.06		
11	θV1	0.00		
12	θV2	0.00		
13	θK2	0.00		
14	γ	**0.00**		Portion of V1 retained
15	α	**0.00**		Portion of V2 extra value added by V1
16	ϕ	**1.00**		Portion of lower investment cost due to V1
17	**OUTPUT**			
18	Q(β,η)	0.0000	3	0.5*(B6^2)*B23*(B23-1)+0.5*(B7^2)*B24*(B24-1)+B9*B6*B7*B23*B24+B23*B11+B24*B13-(B10)
19	SP1	0.0000	5	(1-B14)+B23*B25*(B27^(B23-1))*(B26^B24)
20	SP2	0.0000	6	B24*B25*(B27^B23)*(B26^(B24-1))-(1+B15)
21	VM1	0.0000	4	(1-B14)*B27+B25*(B27^B23)*(B26^B24)-(1+B15)*B26+B16*B28
22	SOLVER SUM	0.0000		B22=0, CHANGING B23:B26
23	β1	-0.9731		
24	β2	2.9462		
25	A1	0.0393		
26	V2*	**3.0277**		
27	V1*	**1.0000**		Assume V1*=V1
28	K2*	**1.0000**		Assume K2*=K2
29	ROV2	0.0393	2	IF(B3<B26,B25*(B4^B23)*(B3^B24),(1+B15)*B26-B16*B28-(1-B14)*B27)
30	NPV	-1.0000		(1+B15)*B3-B16*B5-(1-B14)*B3
31	V1*	1.0000	7	(B23/(B23+B24-1))*B16*B5/(1-B14)
32	V2*	3.0277	8	(B24/(B24+B23-1))*B16*B5/(1+B15)
33	V1*(1-γ)/V2*(1+α)+β1/β2	0.0000		(B31*(1-B14)/-B23)-(B32*(1+B15))/B24

Figure 1

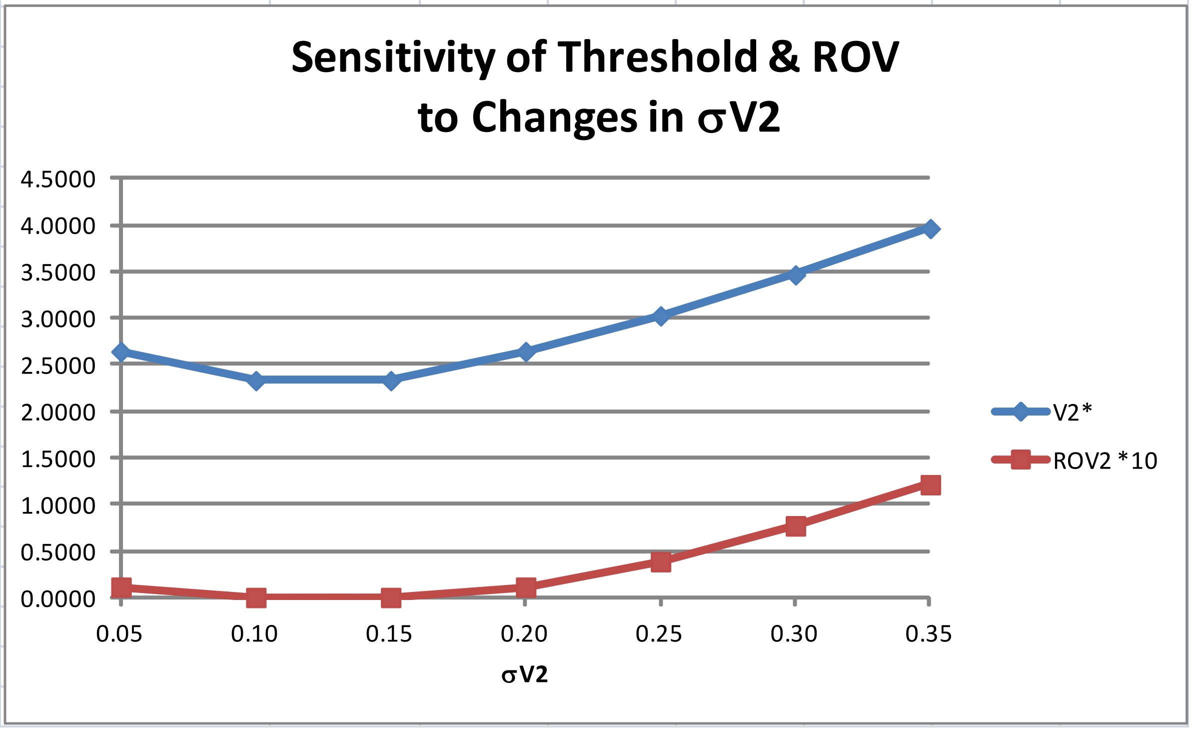
V1*	0.01	0.25	0.5	0.75	1	1.25	1.5
V2*	2.0377	2.2777	2.5277	2.7777	3.0277	3.2777	3.5277
ROV2 *10	2.5055	1.6578	1.0503	0.6496	0.3930	0.2331	0.1357



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1=σV2=.25, ρV1V2=1, drifts are nil, r=.06, γ=0, α=0, ϕ=1, ROV2 is from Equation 2, scaled by 10, and V1*=V1.

Figure 2

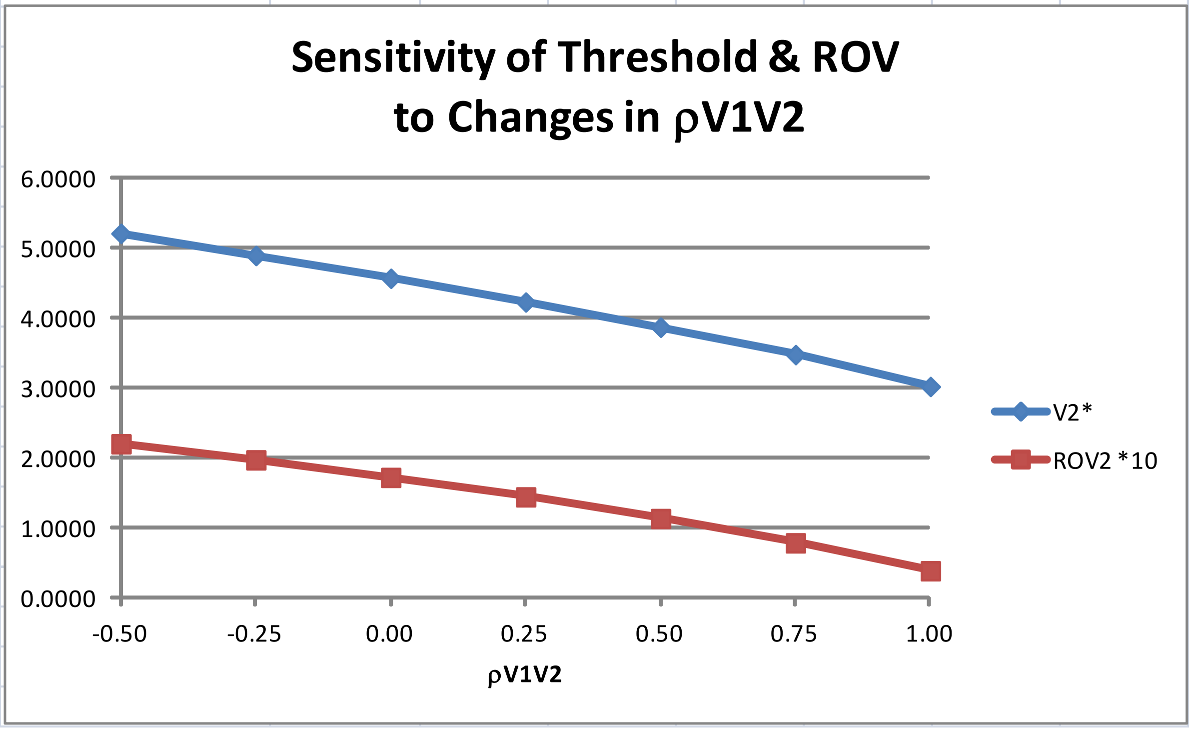
σV2	0.05	0.10	0.15	0.20	0.25	0.30	0.35
V2*	2.6449	2.3333	2.3333	2.6449	3.0277	3.4668	3.9636
ROV2 *10	0.1194	0.0089	0.0089	0.1194	0.3930	0.7767	1.2184



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1=.25, ρV1V2=1, drifts are nil, r=.06, γ=0, α=0, ϕ=1, ROV2 is from Equation 2, scaled by 10, σV2 is as specified, and V1*=V1=1.

Figure 3

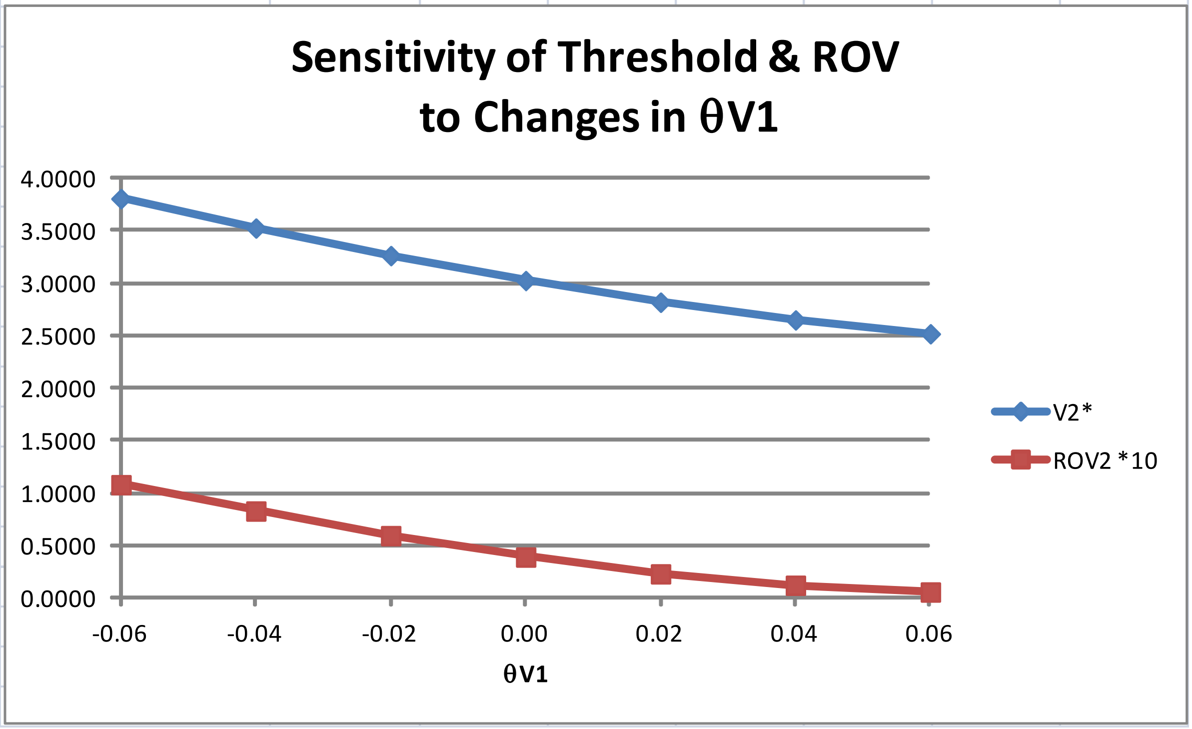
ρV1V2	-0.50	-0.25	0.00	0.25	0.50	0.75	1.00
V2*	5.2167	4.9004	4.5742	4.2345	3.8750	3.4834	3.0277
ROV2 *10	2.2079	1.9782	1.7270	1.4499	1.1409	0.7916	0.3930



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1= σV2=.25, ρV1V2 as specified, drifts are nil, r=.06, γ=0, α=0, ϕ=1, ROV2 is from Equation 2, scaled by 10, and V1*=V1=1.

Figure 4

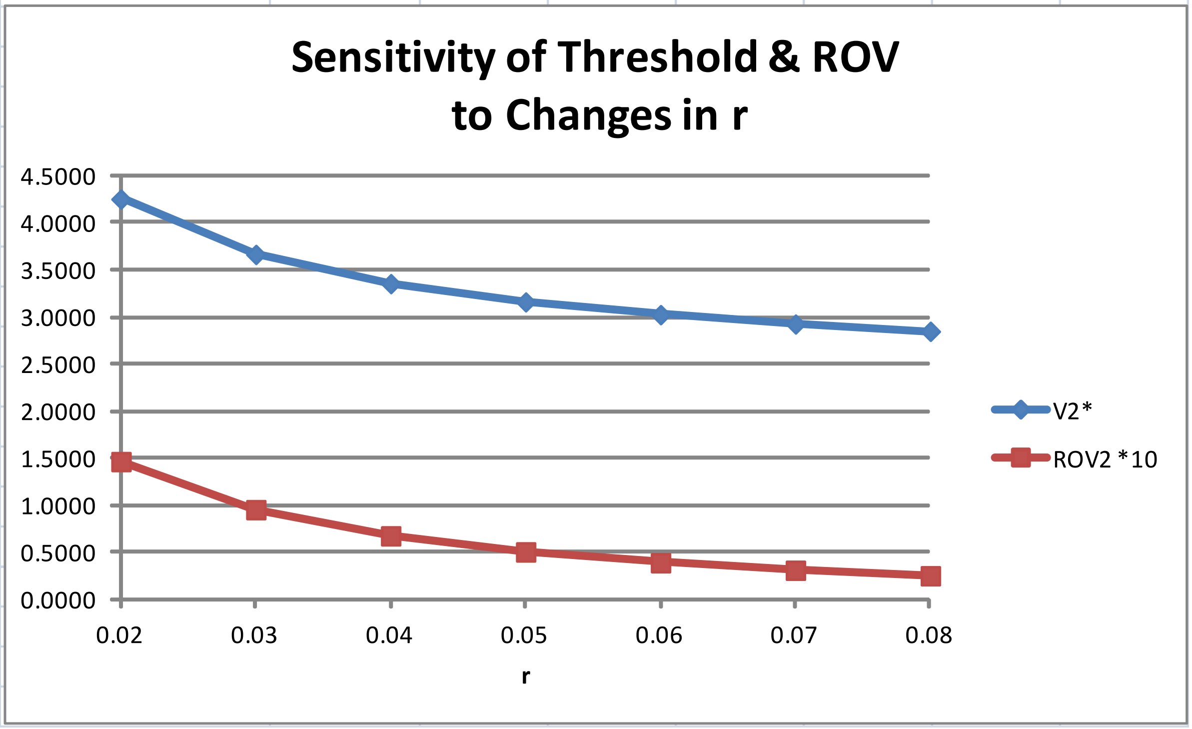
θV1	-0.06	-0.04	-0.02	0.00	0.02	0.04	0.06
V2*	3.8088	3.5283	3.2657	3.0277	2.8215	2.6524	2.5208
ROV2 *10	1.0825	0.8319	0.5973	0.3930	0.2330	0.1237	0.0593



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1= σV2=.25, ρV1V2=1, V1 drifts are as specified, V2 drifts are nil, r=.06, γ=0, α=0, ϕ=1, ROV2 is from Equation 2, scaled by 10, and V1*=V1=1.

Figure 5

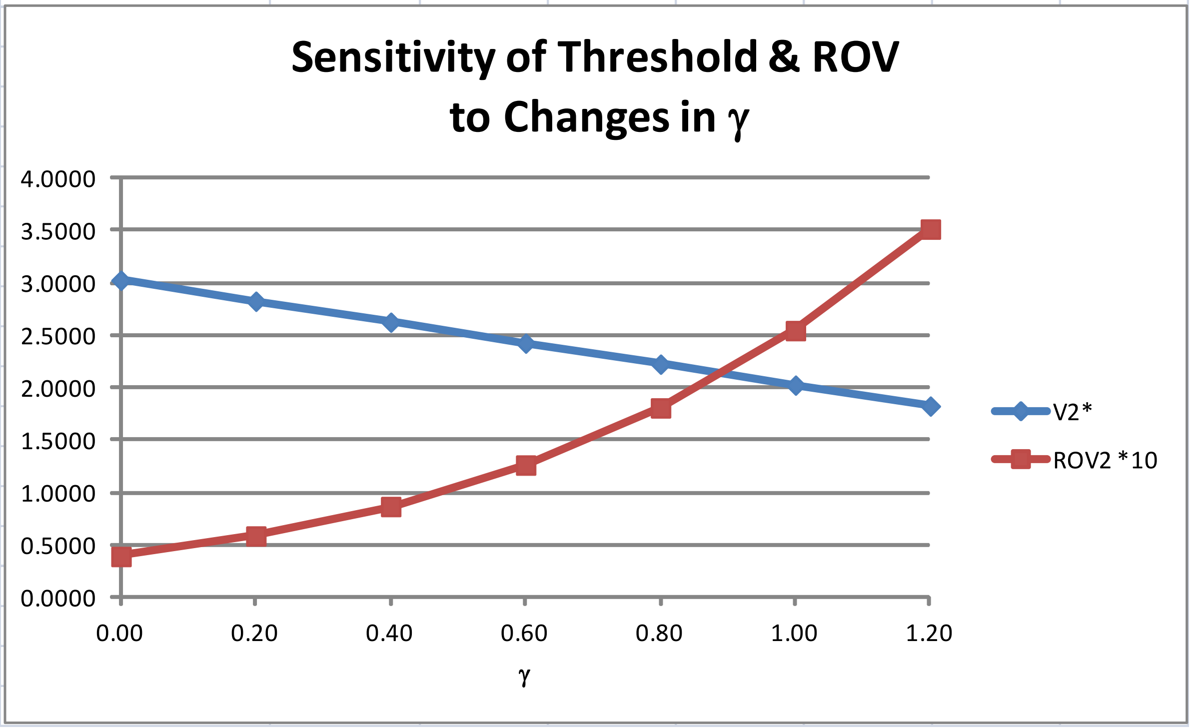
r	0.02	0.03	0.04	0.05	0.06	0.07	0.08
V2*	4.2553	3.6667	3.3570	3.1626	3.0277	2.9277	2.8501
ROV2 *10	1.4674	0.9560	0.6783	0.5072	0.3930	0.3127	0.2538



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1= σV2=.25, ρV1V2=1, drifts are nil, r as specified, γ=0, α=0, ϕ=1,ROV2 is from Equation 2, scaled by 10, and V1*=V1=1.

Figure 6

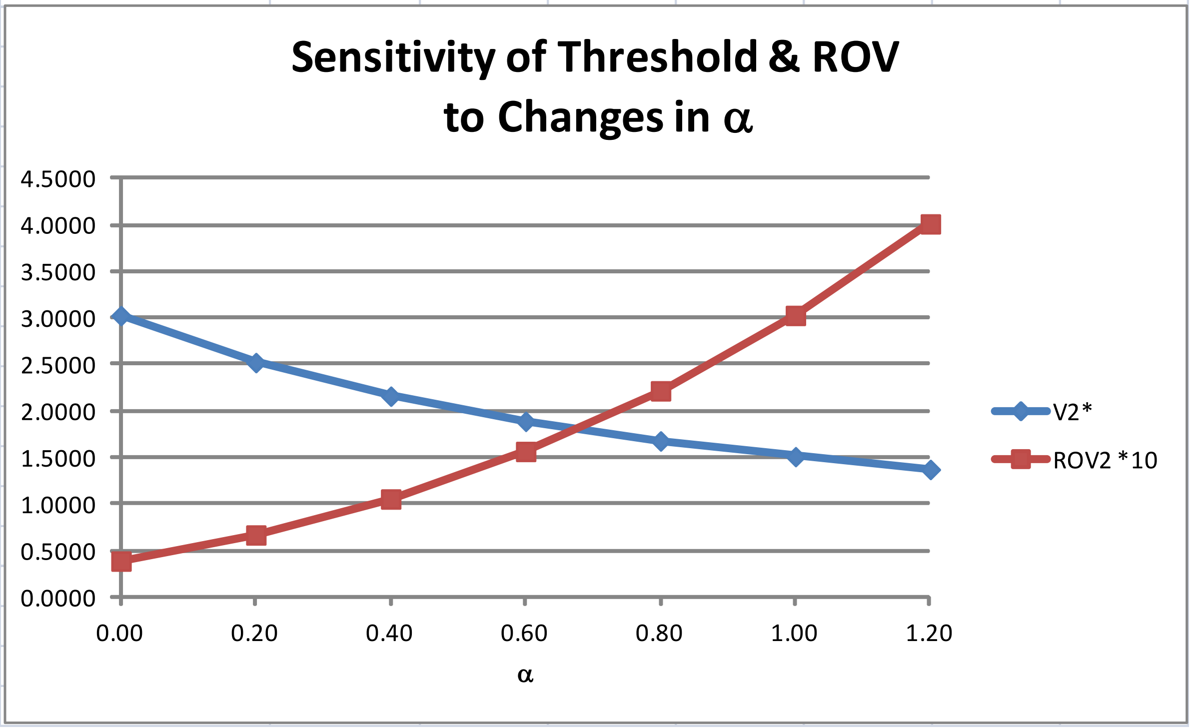
NPV	-1.0000	-0.8000	-0.6000	-0.4000	-0.2000	0.0000	0.2000
γ	0.00	0.20	0.40	0.60	0.80	1.00	1.20
V2*	3.0277	2.8277	2.6277	2.4277	2.2277	2.0277	1.8277
ROV2 *10	0.3930	0.5885	0.8690	1.2645	1.8106	2.5475	3.5162



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1= σV2=.25, ρV1V2=1, drifts are nil, r=.06, γ={0, 1.2}, α=0, ϕ=1, ROV2 is from Equation 2, scaled by 10, and V1*=V1=1.

Figure 7

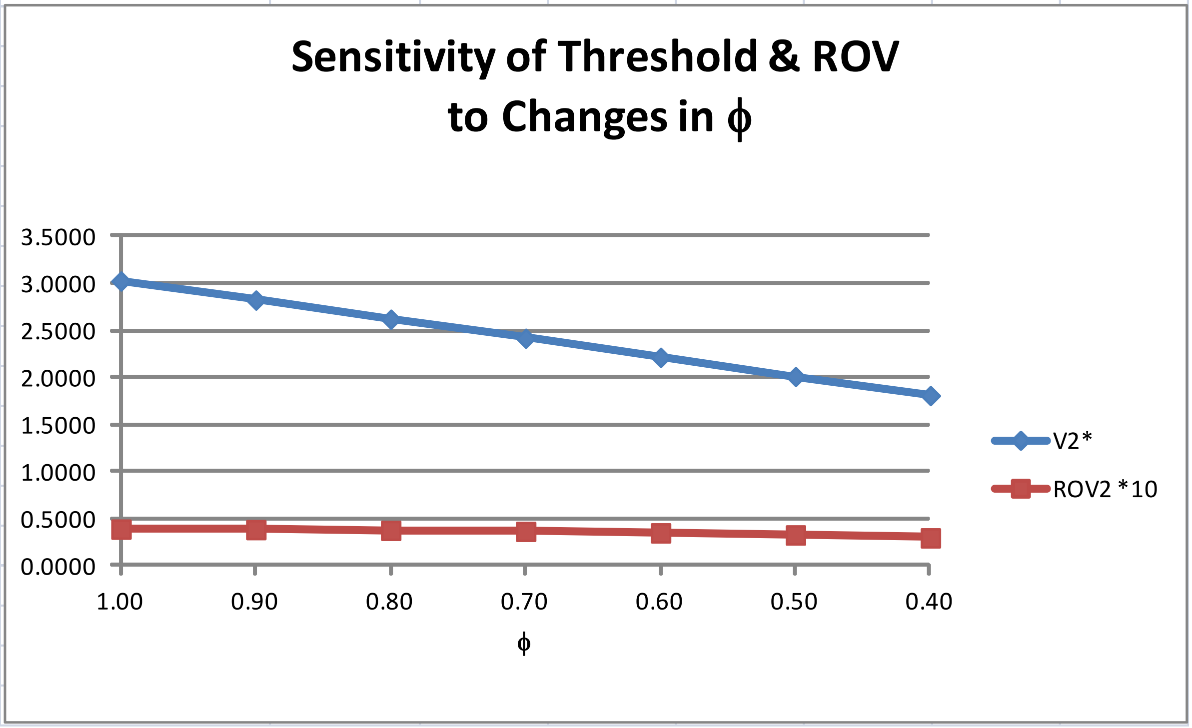
NPV	-1.0000	-0.8000	-0.6000	-0.4000	-0.2000	0.0000	0.2000
α	0.00	0.20	0.40	0.60	0.80	1.00	1.20
V2*	3.0277	2.5230	2.1626	1.8923	1.6820	1.5138	1.3762
ROV2 *10	0.3930	0.6725	1.0591	1.5696	2.2207	3.0291	4.0111



V2* is the solution to Equations 3,4,5,6, assuming V2=1, K2=1=K2*, σV1= σV2=.25, ρV1V2=1, drifts are nil, r=.06, γ=0, α={0, 1.2}, ϕ=1, ROV2 is from Equation 2, scaled by 10, and V1*=V1=1.

Figure 8

NPV	-1.0000	-0.9000	-0.8000	-0.7000	-0.6000	-0.5000	-0.4000
ϕ	1.00	0.90	0.80	0.70	0.60	0.50	0.40
V2*	3.0277	2.8249	2.6221	2.4194	2.2166	2.0138	1.8111
ROV2 *10	0.3930	0.3878	0.3799	0.3685	0.3526	0.3306	0.3003



$\hat{V}_2$ is the solution to Equations 3,4,5,6, assuming $\hat{V}_1$=V_1=1, V_2=1, K_2=1=$\hat{K}_2$, σ_{V1}= σ_{V2}=.25, ρ_{V1V2}=1, drifts are nil, r=.06, γ=0, α=0, ϕ={1, .4}, ROV_2 is from Equation 2, scaled by 10.