A theory on merger timing and announcement returns

Paulo J. Pereira‡ and Artur Rodrigues§∗
‡CEF.UP and Faculdade de Economia, Universidade do Porto.
§NIPE and School of Economics and Management, University of Minho.

February 2015

Abstract

This paper develops a dynamic model for the timing and terms of mergers and acquisitions. Differently from previous models, we show that the firms agree about the timing, independently from how the merger surplus is shared. Firms always agree on the timing and discuss the sharing rule of the merger surplus according to their bargaining power or some other exogenous factor. We also show that, under asymmetry of information, the combination of surprises regarding merger timing and merger terms, can produce either negative or positive abnormal returns for the merging firms.
A theory on merger timing and announcement returns

1 Introduction

The motivations that govern merger and acquisition (M&A) decisions are well established in the literature. Typically, when firms merge, they look for gains related to operating efficiencies (e.g.: economies of scale), market power conditions, economies by vertical integration, technology transfers, among others. These gains are generally presented in the form of synergies.

However, since the final outcome of a merger process is usually uncertain, companies may have an incentive to delay the decision, waiting for a given optimal timing. Recent literature addresses this topic, following the real options theory. For instance, [Lambrecht (2004)] studies the timing of mergers motivated by economies of scale, and [Thijssen (2008)] addresses the timing when both efficiency gains and diversification benefits are considered. The optimal timing (or strategic timing) appears also in [Alvarez and Stenbacka (2006)], [Lambrecht and Myers (2007)], [Hackbarth and Morellec (2008)], and in [Bernile et al. (2012)].

Also an important topic regarding M&A is how the merger potential gains are split between firms. The split of potential gains is established by the exchange terms of the merger, which can be proposed by the acquiring firm or negotiated by both firms. Some literature discusses what drives the definition of the terms, how they can be determined and the effects on the value of merging firms.

For instance, [Lambrecht (2004)] defines a two-round process where firstly the firms agree about the timing of the merger, maximizing the overall merger gains, and then the merger terms are defined as those that induce both firms to merge at this efficient threshold. A similar approach appears in [Morellec and Zhdanov (2005)] and [Hackbarth and Morellec (2008)]. The terms of the merger play an important role in these papers, since they are the unique solutions that ensure the firms merge in the efficient timing. Accordingly, the timing of the merger and the exchange terms seem to be closely related.

However, differently from the related literature, our paper suggests that the timing and the terms are not necessarily linked. In fact, our approach shows that the firms agree about the timing independently from the way merger synergies are split. The merger terms are not needed to align firms behavior with the efficient threshold, suggesting that some other factors may explain how the surplus is shared.

In fact, the determinants of the merger terms need to hold on different arguments, namely, the relative negotiating power (bargaining power) of firms, that can be influenced by takeover defenses, termination fees, and stock ownership, among others. If managers are considered, also agency problems may arise.
This paper also shows that some of the mixed results regarding the announcement effects of merger and acquisitions can be explained in the context of asymmetry of information, as suggested by Moeller et al. (2007). In efficient markets, abnormal returns can only be explained by new information conveyed by the announcement. The announcement reveals information to the market about on the merger gains or synergies and the merger terms, i.e. how synergies will be split between the firms (Barraclough et al. 2013). Under asymmetry of information, the market can only form expectations about these parameters, adjusting share prices depending on the (lack of) occurrence of a merger announcement. This process of incorporating information into share prices, can explain all sorts of abnormal returns, as computed in the event studies methodology. Incomplete information has also been considered before a dynamic mergers model by Morellec and Zhdanov (2005), who suggest it plays a role in explaining positive announcement returns and a price run-up prior to the announcement. However, they can only explain negative abnormal returns for target firms introducing multiple competing bidders in the model. In our model, imperfect information is the only ingredient necessary to produce negative and positive abnormal returns, for both firms. Furthermore, and contrary to their proposition, the combined returns of firms in our model can only be explained by an early than expected announcement, meaning higher than expected synergies.

The paper unfolds as follows. Section 2 presents a dynamic model where merger timing and merger terms are derived in the context of Lambrecht and Myers (2007) model. Section 3 generalizes the analysis for any merger payoff. Section 4 discusses the determinants of mergers terms. Section 5 shows that asymmetric information can explain the empirical mixed results on the announcement abnormal returns. Section 6 concludes.

2 The timing and terms of mergers

This section presents a dynamic model of the timing of mergers and acquisitions and discusses how the merger terms - the merger surplus accruing to the bidder and target firm shareholders - can be obtained.

We build on Lambrecht (2004) setting and show that the merger terms cannot be endogenously obtained. According to Lambrecht (2004) the merger terms are unique and are obtained endogenously, as part of the optimization process. We argue that, contrary to what he suggests, there are multiple Nash equilibriums, and his model is a particular case of a more general model, under the restrictive assumption of constant merger terms across the state variable.

According to the Lambrecht (2004) model, two firms have the irreversible option to merge into a single firm benefiting from economies of scale produced by their production functions. The firms are price takers producing an output with a Cobb-Douglas profit function.
function, with $p$, the output price, following a geometric Brownian motion (gBm):

$$\frac{dp}{p} = \mu dt + \sigma dz \quad (1)$$

where $\mu < r$ is the drift rate, $\sigma > 0$ is the instantaneous volatility, and $dz$ is the standard increment of a Wiener process. Throughout the text we assume risk neutrality and a constant risk-free interest rate, $r$.

Following Dixit and Pindyck (1994) the value of a firm without the option to merge, or the stand-alone firm, is given by:

$$V_i(p) = \alpha_i p^\eta \quad (2)$$

where

$$\alpha_i = \frac{\Omega K_i^\theta}{r - g(\eta)} \quad (3)$$

$$g(\eta) = r - \mu \eta - 0.5\sigma^2 \eta(\eta - 1) \quad (4)$$

$$\eta = \frac{1}{1 - a} \quad (5)$$

$$\theta = \frac{b}{1 - a} \quad (6)$$

$$\Omega = \left(a^{-\frac{a}{1-a}} - a^{\frac{1}{1-a}}\right) w^{-\frac{a}{1-a}} \quad (7)$$

Equation (2) denotes the value of firm $i$ ($i \in \{1, 2\}$), both the stand-alone firms and the merged firm ($i = M$). $K_i$ is the fixed input (capital), and $K_M = K_1 + K_2$. The instantaneous profit function, determined by a Cobb-Douglas profit function is:

$$pL^a_i K_i^b - wL_i \quad (8)$$

where $L$ stands for the variable input, $K$ for the fixed input, and $w$ is the cost per unit of variable input. This function is assumed to have decreasing returns of scale with respect to $L$ ($a < 1$). However, if both inputs are variable, which is the case when a merge takes place combining firms fixed inputs, increasing returns of scale are assumed ($a + b > 1$).

When firms merge, they combine their fixed inputs ($K_M = K_1 + K_2$), incurring in some fixed sunk costs, $X_i > 0$. Therefore, the payoff of merging for firm $i$ is:

$$\Pi_i(p) = \Gamma_i(p)V_M(p) - V_i(p) - X_i = (\Gamma_i(p)\alpha_M - \alpha_i) p^\eta - X_i \quad (9)$$

where $\Gamma_i(p)$ is the fraction of the merged firm owned by firm $i$, and $\Gamma_1(p) + \Gamma_2(p) = 1$. Denoting the fraction of firm 1 as $\gamma(p) = \Gamma_1(p)$, the fraction of firm 2 becomes $\Gamma_2(p) = 1 - \gamma(p)$.

All previous models derive the merge terms, $\gamma(p)$, assuming that they are constant.
across $p$, i.e. $\gamma(p) = \bar{\gamma}$, which implies that the terms are independent of the value of firms prior to merge. This assumption allowed them to obtain endogenously the merger terms.

We conjecture that merger terms should depend on the relative value of firms or the value of the state variable ($p$). This is more perceivable in the case of Morelec and Zhdanov (2005) model, where the state variable is precisely the stochastic relative value of the firms. This means that when a firm is considering to merge for a certain level $p$, and compares the payoff with the so-called continuation value, she has to consider that merging later can change the merger terms, i.e. $\gamma$ could be a function of $p$.

Following standard procedures any perpetual contingent claim on $p$, $F_i(p)$, must satisfy the following ordinary differential equation:

$$0.5\sigma^2 p^2 F''_i(p) + \mu p F'_i(p) - r F_i(p) = 0$$

yielding the following general solution:

$$F_i(p) = A_i p^\beta_1 + B_i p^\beta_2$$

where $\beta_1$ and $\beta_2$ are the solutions to the fundamental quadratic equation: $0.5\sigma^2 \beta(\beta - 1) + \mu \beta - r = 0$.

When analyzing the option to merge, each firm must consider the optimal behavior of the other firm, i.e. what are the terms that the other firm would require in order to agree merging for different values of $p$.

Let us start by analyzing how firm 1 incorporates the optimal behavior of the firm 2. She must know what are the merger terms that make firm 2 indifferent to merge for any $p$ ($\gamma_2(p)$). $\gamma_2(p)$ is firm 1 share of the merged firm that firm 2 is willing to concede, such that every $p$ is an optimal merger trigger. The value of the option to merge for firm 2 ($O_2$) must satisfy the following boundary conditions:

$$O_2(p) = A_2 p^{\beta_1} = [(1 - \gamma_2(p)) \alpha_M - \alpha_2] p^n - X_2$$

$$O'_2(p) = \beta_1 A_2 p^{\beta_1 - 1} = \eta [(1 - \gamma_2(p)) \alpha_M - \alpha_2] p^{n-1} - \gamma'_2(p) \alpha_M p^n$$

Please notice that we require every $p$ to be an optimal trigger, and so these boundary conditions must be valid to every $p$, an not only for a single trigger ($p^*$) as in standard models. Given that the payoff is determined by $\gamma_2(p)$, this function can be arranged in order to allow every $p$ to become a trigger for investment.

**Proposition 1.** The value of the option to merge for firm 2, that makes her indifferent to merge for any $p$ is:

$$O_2(p) = C_2 \alpha_M p^{\beta_1}$$

---

1 All proofs can be found in the Appendix.
where \( C_2 > 0 \) is a constant yet to be determined, and

\[
\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left( \frac{1}{2} + \frac{\alpha}{\sigma^2} \right)^2 + \frac{2\eta}{\sigma^2}} > 1 \tag{15}
\]

The share of the bidder that the target is willing to concede is:

\[
\gamma_2(p) = 1 - \frac{C_2\alpha_Mp^{\beta_1} + \alpha_2p^\eta + X_2}{\alpha_Mp^\eta} \tag{16}
\]

The other firm (firm 1) must now consider \( \gamma_2(p) \) in her decision to merge. Taking into consideration \( \gamma_2(p) \) (i.e. replacing \( \Gamma_1(p) \) or \( \gamma(p) \) by \( \gamma_2(p) \) in equation (9)), firm 1 receives the following payoff on merging at \( p = p_1 \):

\[
\Pi_{12}(p_1) = (\alpha_M - \alpha_1 - \alpha_2)p_1^\eta - C_2p_1^{\beta_1} - (X_1 + X_2) \tag{17}
\]

where 12 denotes the case when firm 1 incorporates firm 2 required terms.

**Proposition 2.** The value of the option to merge for firm 1, taking into account the merger terms required by firm 2 is:

\[
O_{12}(p) = \begin{cases} 
\Pi_{12}(p_1^a) \left( \frac{p}{p_1^a} \right)^{\beta_1} & \text{for } p < p_1^a \\
\Pi_{12}(p) & \text{for } p_1^a \leq p < p_1^b \\
\Pi_{12}(p_1^b) \left( \frac{p}{p_1^b} \right)^{\beta_2} & \text{for } p \geq p_1^b
\end{cases} \tag{18}
\]

where

\[
p_1^a = \left( \frac{\beta_1}{\beta_1 - \eta \alpha_M - \alpha_1 - \alpha_2} \right)^{\frac{1}{\eta}} \tag{19}
\]

and

\[
\Pi_{12}(p) = (\alpha_M - \alpha_1 - \alpha_2)p^\eta - C_2p^{\beta_1} - (X_1 + X_2) \tag{20}
\]

and \( p_1^b \) is the solution to this nonlinear equation:

\[
(\beta_2 - \eta) (\alpha_M - \alpha_1 - \alpha_2) (p_1^b)^\eta - (\beta_2 - \eta) C_2(p_1^b)^{\beta_1} - \beta_2(X_1 + X_2) = 0 \tag{21}
\]

Firm 1 will be willing to merge when \( p \) crosses \( p_1^a \) from below. If, for any circumstance, the first observation of \( p \) occurs at a higher level, merging will only be optimal for \( p < p_1^b \).

This second trigger is relevant if the starting value of \( p \) is above \( p_1^b \), and merging becomes optimal when \( p \) crosses \( p_1^b \) from above. However it is possible to show that at this trigger mergers are not possible, because the two firms will not agree on merger terms.
Following similar steps for firm 1, we can obtain the value of the option to merge and the terms that firm 1 requires in order to make she indifferent to merge for any \( p (\gamma_1(p)) \).

**Proposition 3.** The value of the option to merge for firm 1, that makes her indifferent to merge for any \( p \) is:

\[
O_1(p) = C_1\alpha M p^{\beta_1}
\]

where \( C_1 > 0 \) is a constant yet to be determined.

The share that firm 1 requires is:

\[
\gamma_1(p) = \frac{C_1\alpha M p^{\beta_1} + \alpha_1 p^n + X_1}{\alpha M p^l}
\]

Incorporating \( \gamma_1(p) \) in the decision of firm 2 (i.e. replacing \( \Gamma_2(p) \) by \( 1 - \gamma_1(p) \) in equation [9]), the payoff accruing to firm 2 on merging at \( p = p_2 \) is:

\[
\Pi_{21}(p_2) = (\alpha_M - \alpha_1 - \alpha_2) p_2^n - C_1 p_2^{\beta_1} - (X_1 + X_2)
\]

**Proposition 4.** The value of the option to merge for firm 2, taking into account the merger terms by required firm 1 is:

\[
O_{21}(p) = \begin{cases} 
\Pi_{21}(p_2) \left( \frac{p}{p_2} \right)^{\beta_1} & \text{for } p < p_2^a \\
\Pi_{21}(p) & \text{for } p_2^a \leq p < p_2^b \\
\Pi_{21}(p_2) \left( \frac{p}{p_2} \right)^{\beta_2} & \text{for } p \geq p_2^b 
\end{cases}
\]

where

\[
p_2^a = \left( \frac{\beta_1}{\beta_1 - \eta} \left( \frac{X_1 + X_2}{\alpha M - \alpha_1 - \alpha_2} \right) \right)^{\frac{1}{\eta}}
\]

and

\[
\Pi_{21}(p) = (\alpha_M - \alpha_1 - \alpha_2) p^n - C_1 p^{\beta_1} - (X_1 + X_2)
\]

and \( p_2^b \) is the solution to this nonlinear equation:

\[
(\beta_2 - \eta) (\alpha_M - \alpha_1 - \alpha_2) (p_2^b)^n - (\beta_2 - \eta) C_1 (p_2^b)^{\beta_1} - \beta_2 (X_1 + X_2) = 0
\]

As mentioned before the second trigger is not relevant, since mergers are not possible at \( p_2^b \).

From equations [19] and [26] we find that \( p_1^a = p_2^a (= p_M) \), making mergers optimal for that level of \( p \). The most relevant characteristic of this solution is that the timing of mergers is independent of the constants \( C_1 \) and \( C_2 \), meaning that it is also independent
of the terms of the merger ($\gamma(p)$). Both firms agree to merge at $p_M$, but the terms of the merger are not determined endogenously by the parameters that explain the trigger for merging.

Interestingly enough, this solution is the central planner solution, that has the advantage of ensuring the maximization of the overall merger gain. In fact, a central planner would maximize the following payoff:

$$\Pi_C(p_M) = (\alpha_M - \alpha_1 - \alpha_2) p_M^B - (X_1 + X_2)$$

which yields the same trigger, $p_M$.

The timing of mergers found in our model is the same obtained by Lambrecht (2004).

### 3 Generic payoff

In the previous section it was show that for Lambrecht (2004) model the merger timing is independent of the merger terms and that these terms are not unique, and must be explained by some exogenous factor. In this section we show that these propositions hold for any general merger payoff.

Let us assume a generic merger surplus $\pi(p)$, and that this surplus is shared by both firms: $\gamma(p)$ to firm 1 and $1 - \gamma(p)$ for firm 2. The merge payoffs for each firm are:

$$\Pi_1(p) = \gamma(p)\pi(p) - X_1$$  
$$\Pi_2(p) = (1 - \gamma(p))\pi(p) - X_2$$

Firm 2 is willing to merge for any $p$ if the following boundary conditions are met:

$$A_2p_1^B = (1 - \gamma_2(p))\pi(p) - X_2$$  
$$\beta_1A_2p_1^{\beta_1-1} = (1 - \gamma_2(p))\pi'(p) - \gamma'_2(p)\pi(p)$$

where $\gamma_2(p)$ is firm 1 share of the merger surplus that firm 2 is willing to concede. These boundary conditions can be reduced to the following differential equation, which $\gamma_2(p)$ must solve:

$$p\pi(p)\gamma'_2(p) + p\pi'(p)\gamma_2(p) - \beta_1\pi(p)\gamma_2(p) - p\pi'(p) + \beta_1\pi(p) - \beta_1X_2 = 0$$

When firm 1 considers $\gamma_2(p)$ in her decision, the following boundary conditions apply:

$$A_1p_1^{\beta_1} = \gamma_2(p_1^B)\pi(p_1^a) - X_1$$  
$$\beta_1A_1p_1^{\beta_1-1} = \gamma_2(p_1^{a1})\pi'(p_1^a) + \gamma'_2(p_1^{a1})\pi(p_1^a)$$
where \( p^*_1 \) is the trigger value. These conditions can also be reduced to the following differential equation:

\[
p^*_1 \pi'(p^*_1) \gamma_2(p^*_1) + p\pi'(p^*_1) \gamma_2(p^*_1) - \beta_1 \pi(p^*_1) \gamma_2(p^*_1) - \beta_1 X_1 = 0
\]  

Combining equations (34) and (37), and considering that, for firm 2, \( p \) is always a trigger, we are able to find the trigger value for firm 1. The same approach, starting for firm 1 and considering for firm 2 the optimal \( \gamma(p) \) for firm 1 \((\gamma_1(p))\), leads to the solution for the trigger of firm 2.

**Proposition 5.** Two firms considering a merger with a shared surplus of any general payoff \( \pi(p) \) will agree to merge at:

\[
p_M = \frac{\beta_1 (\pi(p) - (X_1 + X_2))}{\pi'(p)}
\]

and their trigger is the central planner solution.

As in the previous section, the trigger is independent of the merger terms \((\gamma(p))\).

Proposition 5 shows that if our approach is applied to other real option models of mergers and acquisitions, it produces the same result. For example, using Morellec and Zhidanov (2005) setting we would find the same solution trigger. However, we show that the merger terms are not unique as in their model.

### 4 Finding merger terms

In the related literature the terms of the merger are usually endogenously determined, i.e. the terms result as part of the global solution.

Lambrecht (2004) defines a two round procedure where parties first negotiate and agree about the timing, and then decide how to share the new company. In the first round the author assumes that it is in the best interest of each firm to merge at the central planner trigger. In the second round they agree on the terms that induce both to exercise the merger option at that optimal timing. In doing so, he assumes that those terms are constant over time and are independent of the state variables. In this model, the post-merging shareholding is unique and is a function of the stock of capital of each firm and the merger costs. Our model differs from this model in two important aspects. First, we do not need to impose the restriction that the firms must ex ante agree to merge on the central planner trigger. The central planner solution arises as the result of the equilibrium strategies of both firms. Second, we show that there are multiple acceptable sharing rules.

The approach of Morellec and Zhidanov (2005) is different. First, firms compute their own optimal merger timing, and then the sharing rule is found as the unique solution that ensures both firms agree on the timing. They conclude that the timing is the same as
the central planner solution. Although their model is different from Lambrecht’s (2004) model, if their procedure is used for the same model setting, it is straightforward to show that the solution is exactly the same. Differently from our model, the merger terms are constant and independent of the state variable, which produces an unique sharing rule, in contrast to the multiple viable sharing rules in our model.

Thijssen (2008) is closely related with our paper. He assumes that the share of the merged firm is a function of the state variables. However, in his model the timing and terms are obtained endogenously, being a function of each other. In our model they are independent. His model produces a single sharing rule, while in our model there are multiple solutions. Similarly to our paper, he assumes that the bidder makes an offer that makes the target shareholder indifferent between accepting and rejecting the bid. However he assumes that this only occurs when the target payoff is set to zero, which corresponds in our setting to the case of a null bargaining power for the target. When he allows both firms to enter in a strategic merger game with the roles endogenously determined, he suggests that the option value completely disappears and the timing is independent of the bargaining power. This results from the assumption that the unique indifference rule for the other firm is to have a null payoff. This is in contrast with our results, since we show that the option value does not vanish, making worth delaying the merger until the optimal timing. Similarly to his model, we show that the bargaining power determines the merger terms and does not influences the timing of mergers.

Alvarez and Stenbacka (2006) make use of an exogenous bargaining power, and show that it determines the merger terms. However, contrary to our results, they suggest that the merger timing is influenced by firms bargaining power.

The outcome of the merger is, as we show, determined by the bargaining power of each firm or any other exogenous factor. The determinants of that bargaining power are a relevant question. The observation of the merger terms is however extremely difficult, if not impossible. In perfect markets, the shares in the merged firm, will be exactly the relative values of the firms before merging. The value of each firm before merging includes already the merger option value, i.e. a given expectation about the sharing rule of the merger surplus. The occurrence of any abnormal return can only be explained as the result of a surprise to the market, either in terms of timing or the sharing rule of the merger surplus.

5 Explaining announcement abnormal returns

Literature on mergers and acquisitions traditionally reports empirical evidence showing positive returns for target firms and negative (or zero) returns for the shareholders of the acquirers (Jensen and Ruback (1983), Jarrell and Poulsen (1989), and Andrade et al. (2001)). Value-loss for the acquiring shareholder is also reported by Moeller et al. (2005)
for large deals in the late 90’s.

Recent studies suggest mixed results. Martynova and Renneboog (2011) show evidence of positive abnormal returns for both parties, being however significant for the targets, and only slightly positive for the bidder (previously, similar results appear in Bradley et al. (1988)). Barraclough et al. (2013) shows mixed signs for the returns, depending on the base price used in the analysis. However, the results indicate that the returns of the bidding firms are relatively small when compared with the returns of the target. In a different approach, opposite evidence is reported by Ahern (2012), where the average gains for target firms are only modestly larger than the gains obtained by the acquirers.

Different explanations for the realized abnormal returns have been suggested in the literature. Some of the arguments are: the relative size of the firms (e.g. Moeller et al. (2004)), the existence information asymmetry (e.g. Moeller et al. (2007)), the success of offer (e.g. Barraclough et al. (2013)), the rivalry of the merging firms (e.g. Song and Walkling (2000)), the form of payment (e.g. Gao (2011) and Barraclough et al. (2013)), and the nature (public or private) of the target firm (e.g. Fuller et al. (2002)), among others.

In the context of the model herein presented, we follow the information asymmetry argument of Moeller et al. (2007). The abnormal returns are driven by the adjustment of prices to the information revealed by the announcement. In line with Barraclough et al. (2013), the announcement informs the market about both the merger gains (synergies) and the merger terms (how the synergies will be split between the firms shareholders). Under asymmetry of information, the market can only form expectations about this relevant piece of information, adjusting immediately the share prices once it becomes public. This adjustment can produce all types of abnormal returns, depending on the prior assessment, made by each firm shareholders, about the merger gains and the bargaining power the their own managers.

5.1 Asymmetry of information between firms and the market

Let us assume that managers have private information about the merger synergies, not yet released to the market. Shareholders can only form expectations on the value of the merged firm \(E(V_M)\) and the expected merger terms \(E(\gamma)\). We assume furthermore that managers of both firms negotiate, and shareholders agree on the terms and timing of the merge, excluding any revision of the terms negotiated by the managers. In this setting, the true \(V_M\) is only observable by the managers of the merging firms.

The announcement of the merger reveals the timing and terms negotiated by the managers, and the market reacts adjusting the share prices to the information revealed by the announcement. If the announcement information is different from market expectations, an abnormal return occurs. Since the merger timing is independent from the terms, the timing of the announcement reveals the value of the merger synergies (or equivalently the
value of $V_M$).

Let us study the impact of a surprise in each of the two variables, starting with the expected value of the synergies, assuming perfect information about the merger terms, leaving for the next section the analysis of the combined effect.

The expectations of the market on $V_M$ can predict, under or overestimate the true value revealed by the announcement. If expectations are exceeded ($V_M > E(V_M)$), the announcement occurs early than expected, producing a positive abnormal return. On the contrary, if expectations are not met by an announcement, the market will revise the expectations on the value (timing) of the merger until an announcement occurs. This late announcement produces a negative abnormal return.

The announcement effects are traditionally computed using the event study methodology, under which the return ($R$) during the event window around the announcement date is measured against the expected return $E(R)$, that is calculated using a given estimation window prior to the event. The abnormal return ($AR$) is simply $R - E(R)$.

The diffusion process of the option to merge, prior to the expected merger timing, is given by

$$
\frac{dOC}{OC} = rdt + \beta_1 \sigma dz \tag{39}
$$

and therefore the expected option return is:

$$
E(R) = \frac{E[dOC]}{OC} = r \tag{40}
$$

For early announcements, the event window return is positive and greater than the expected (option) return, producing a positive abnormal return:

$$
AR = R - E(R) > 0 \tag{41}
$$

Consider now the case where $p$ reaches $E(p_M)$ and the announcement does not occur. The absence of an announcement reveals that the synergies are lower than expected by the market participants. Without any additional information, the market will revise the expected trigger to the next infinitesimal increment of $p$. The diffusion of the price will depend on what parameter explains the late announcement. For example, in the context of the model it can be either the value of synergies (the alphas)

\footnote{Please note, that using Itô’s lemma:

$$
\frac{dOC}{dp} = \frac{\partial OC}{\partial p} dp + \frac{1}{2} \frac{\partial^2 OC}{\partial p^2} (dp)^2 = \left( \mu \beta_1 OC + \frac{1}{2} \sigma^2 \beta_1 (\beta_1 - 1) OC \right) dt + \sigma \beta_1 OC dz.
$$

From the fundamental quadratic equation

$$
\frac{1}{2} \sigma^2 \beta_1 (\beta_1 - 1) = r - \mu \beta_1,
$$

we obtain equation (39).}
or the merging costs. If we assume that the market updates the expectations on synergy related parameters, let us assume $\alpha_M$, the value of the option will remain at $O_C(E(p_M)) = \frac{\eta}{\beta - \eta} (X_1 + X_2)$ until the announcement occurs (figure 1). A normal movement of $p$, increasing and decreasing over time, will always produces a positive average return in the estimation window. The window will capture days of null returns and positive returns. This is illustrated by figure 1. If the initial market expectation is that the merger occurs at $E[p_M]$, share prices will remain along the line $0A$, with average positive returns. If the announcement is delayed until $E[p'_M]$, the share prices can move along the line $AB$, with null returns, or along any option value function, from 0 to the segment $AB$, if the prices move down, with positive returns.

For late announcements, given that the expected return is positive and the realized return on the event date is zero, the abnormal return is negative:

$$\ AR = R - E(R) < 0 \tag{42}$$

If the surprise of the lack of an announcement leads to a revision of the merging costs $X_i$, the movement of prices is illustrated by figure 2. A late announcement will move prices

---

Only for unlikely case where $p$ always increases during the estimation window, and it begins after the first expected $p_M$, the return during the estimation window will be zero.
along $AB$, which means positive but lower returns than along $0A$. The realized return on the event date will be also lower than the expected return, producing a negative abnormal return.

Furthermore it is possible that even if information asymmetry disappears, abnormal returns can persist. Let us suppose that that all information is released to the market after $E[p_M]$, meaning that all market participants and firms agree that a merger should occur, let us assume at $E[p'_M]$. As long as the estimation window captures any period during which there is imperfect information the event window return return will be lower than the expected return, producing a negative abnormal return.

As mentioned before, the announcement also reveals how the firms will split the synergies. The merging terms can differ from those expected by the market. This can also produce positive or negative returns in the announcement event window. If the fraction of the merged firm accruing to firm 1 announced is higher than expected ($\gamma > E(\gamma)$), firm 1 will have a positive AR, while the other firm will have a negative return. On the contrary, if $\gamma < E(\gamma)$, firm 1 obtains a negative AR, and firm 2 obtains a positive AR.
5.2 Summary of announcement returns

Combining the effects of a surprise in the merger timing and the merger terms, it is possible to show that for either firm the announcement can produce negative, null or positive abnormal returns. Table 1 presents the summary of those effects.

<table>
<thead>
<tr>
<th></th>
<th>Early announcement</th>
<th>Timely announcement</th>
<th>Late announcement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smaller $\gamma$</td>
<td>$+/-$</td>
<td>$-$</td>
<td>$-$</td>
</tr>
<tr>
<td>Expected $\gamma$</td>
<td>$+$</td>
<td>$0$</td>
<td>$-$</td>
</tr>
<tr>
<td>Larger $\gamma$</td>
<td>$+$</td>
<td>$+/-$</td>
<td>$-/+-$</td>
</tr>
</tbody>
</table>

$\gamma$ is the share of the merged firm accruing to firm 1.

Early announcements produce positive returns for both firms that can be offset by the negative effect of a smaller than expected share of the merged firm. Late announcements produce negative abnormal returns for both firms. A higher than expected fraction on the merged firm reduces this negative effect. For timely mergers, the only effect that can arise comes from a surprise in the merger terms. Since the roles of bidder and target are not endogenously determined in this model, it can explain positive, null and negative abnormal returns for each firm.

The combined abnormal return of both firms can only be explained by a surprise in the timing, since that a surprise in the terms can only transfer the gains from one firm to the other. Therefore, early announcements produce positive returns and late announcements negative returns for both firms.

Morellec and Zhdanov (2005) have also proposed a real options model where incomplete information of the market regarding the parameters of the merger plays a role in explaining positive announcement returns and a price run-up prior to the announcement. However, they can only explain negative abnormal returns for target firms introducing multiple competing bidders in the model. In our model, imperfect information is the only ingredient necessary to produce negative and positive abnormal returns, for both firms. Furthermore, and contrary to their proposition, the combined returns in our model can only be explained by an early than expected announcement, meaning higher than expected synergies.

6 Conclusion

This paper develops a dynamic real options model for the timing and terms of mergers and acquisitions. Under perfect information, we show that firms always agree on the merger timing independently from how the surplus is shared between firms. Contrary to most of
the previous related models, the terms are shown not to be unique, and must depend on some exogenous factor, namely the bargaining power of each firm.

Under asymmetry of information between managers and the market, the merger announcement can arrive as a surprise to the market, and produce abnormal returns. We show that the combination of surprises in the merger timing and merger terms can produce negative or positive abnormal returns for either firm.

Further research could also consider the effect asymmetric information between firms and the effect of managers compensation along with takeover incentives and defences.

References


Appendix

Proof of Proposition[4] Since the merger payoff must be positive, and given that \( X_2 > 0 \), \( \eta > 0 \), and \( \gamma_2(p) < 1 \), then \( \lim_{p \to +\infty} O_2(p) = +\infty \), and \( \lim_{p \to 0} O_2(p) = 0 \). Therefore, \( B_2 \)
must be set to 0, and so $O_2(p) = A_2 p^{\beta_1}$. Replacing in the boundary conditions, $\gamma_2(p)$ is the solution to the following differential equation:

$$\gamma'_2(p)p\alpha_M p^\eta + (\beta_1 - \eta) \left( (1 - \gamma_2(p)) \alpha_M - \alpha_2 \right) p^\eta - \beta_1 X_2 = 0 \quad (43)$$

yielding:

$$\gamma_2(p) = 1 - \frac{C_2 \alpha_M p^{\beta_1} + \alpha_2 p^\eta + X_2}{\alpha_M p^\eta} \quad (44)$$

where $C_2$ is a constant yet to be determined.

Replacing in Equation (12), the value of the option to merge for the target firm becomes:

$$O_2(p) = C_2 \alpha_M p^{\beta_1} \quad (45)$$

and, therefore, $C_2 > 0$.

Proof of Proposition 2. Given that $\alpha_M - \alpha_1 - \alpha_2 > 0$, $\beta_1 > \eta > 0$, $\gamma_2(p) < 1$, and $C_2 > 0$, $\lim_{p \to +\infty} \Pi_{12}(p) = -\infty$, and $\lim_{p \to 0} \Pi_{12}(p) = -(X_1 + X_2)$. The option value $O_{12}(p)$ must be non-negative, and therefore $\lim_{p \to +\infty} O_{12}(p) = 0$, and $\lim_{p \to 0} O_{12}(p) = 0$. Therefore, both constants $A_{12}$ and $B_{12}$ in the general solution (11) can not be set to 0, and $O_{12}(p)$ is a concave function, producing two possible merger triggers:

$$O_{12}(p) = \begin{cases} 
A_{12} p^{\beta_1} & \text{for } p < p_1^a \\
(\alpha_M - \alpha_1 - \alpha_2) p^\eta - C_2 p^{\beta_1} - (X_1 + X_2) & \text{for } p_1^a \leq p < p_1^b \\
B_{12} p^{\beta_2} & \text{for } p \geq p_1^b 
\end{cases} \quad (46)$$

The first trigger $p_1^a$ is obtained with the usual boundary conditions:

$$A_{12} p_1^{a\beta_1} = (\alpha_M - \alpha_1 - \alpha_2) (p_1^a)^\eta + C_2 (p_1^a)^{\beta_1} - (X_1 + X_2) \quad (47)$$

$$\beta_1 A_{12} (p_1^a)^{\beta_1 - 1} = \eta (\alpha_M - \alpha_1 - \alpha_2) (p_1^a)^{\eta - 1} + \beta_1 C_2 (p_1^a)^{\beta_1 - 1} \quad (48)$$

Solving these two equations, we obtain $p_1^a$:

$$p_1^a = \left( \frac{\beta_1}{\beta_1 - \eta \alpha_M - \alpha_1 - \alpha_2} \right)^\frac{1}{\eta} X_1 + X_2 \quad (49)$$

The second trigger $p_1^b$ is obtained with the following boundary conditions:

$$B_{12} (p_1^b)^{\beta_2} = (\alpha_M - \alpha_1 - \alpha_2) (p_1^b)^\eta + C_2 (p_1^b)^{\beta_1} - (X_1 + X_2) \quad (50)$$

$$\beta_2 B_{12} (p_1^b)^{\beta_2 - 1} = \eta (\alpha_M - \alpha_1 - \alpha_2) (p_1^b)^{\eta - 1} + \beta_1 C_2 (p_1^b)^{\beta_1 - 1} \quad (51)$$
$p_1^b$ is the solution to this nonlinear equation:

$$\left(\beta_2 - \eta\right) \left(\alpha_M - \alpha_1 - \alpha_2\right) (p_1^b)^n - \left(\beta_2 - \eta\right) C_2 (p_1^b)^{\beta_1} - \beta_2 (X_1 + X_2) = 0 \quad (52)$$

Proof of Proposition 3: Following similar steps for the firm 1, the share that she requires in order to make she indifferent to merge for any $p$, $\gamma_1(p)$, is the solution to the following differential equation:

$$\gamma_1'(p) p_1^{\alpha_M p^n - \left[\beta_1 - \eta\right] (\alpha_M - \alpha_2) p^n + \beta_1 X_1 = 0 \quad (53)$$

yielding:

$$\gamma_1(p) = \frac{C_1 \alpha_M p^{\beta_1} + \alpha_1 p^n + X_1}{\alpha_M p^n} \quad (54)$$

where $C_1$ is a constant yet to be determined.

The value of the option to merge for firm 1 is:

$$O_1(p) = C_1 \alpha_M p^{\beta_1} \quad (55)$$

and, therefore, $C_1 > 0$.

Proof of Proposition 4: Given that $\alpha_M - \alpha_1 - \alpha_2 > 0$, $\beta_1 > \eta > 0$, $\gamma_1(p) < 1$, and $C_1 > 0$, $\lim_{p \to +\infty} O_{21}(p) = -\infty$, and $\lim_{p \to 0} O_{21}(p) = -(X_1 + X_2)$. The option value $O_{21}(p)$ must be non-negative, and therefore $\lim_{p \to +\infty} O_{21}(p) = 0$, and $\lim_{p \to 0} O_{21}(p) = 0$. $B_{21}$ can not be set to 0, and $O_{21}(p)$ is a concave function, producing two possible merger triggers, and the option to merge for the target, $O_{21}(p)$, is a concave function:

$$O_{21}(p) = \begin{cases} A_{21} p^{\beta_1} & \text{for } p < p_2^a \\ (\alpha_M - \alpha_1 - \alpha_2) p^n - C_1 p^{\beta_1} - (X_1 + X_2) & \text{for } p_2^a \leq p < p_2^b \\ B_{21} p^{\beta_2} & \text{for } p \geq p_2^b \end{cases} \quad (56)$$

The first trigger $p_2^a$ is obtained with the usual boundary conditions:

$$A_{21} p_2^{a \beta_1} = (\alpha_M - \alpha_1 - \alpha_2) (p_2^a)^n - C_1 p_2^{a \beta_1} - (X_1 + X_2) \quad (57)$$

$$\beta_1 A_{21} (p_2^a)^{\beta_1 - 1} = \eta (\alpha_M - \alpha_1 - \alpha_2) (p_2^a)^{\eta - 1} - \beta_1 C_1 (p_2^a)^{\beta_1 - 1} \quad (58)$$

Solving these two equations, we obtain $p_2^a$:

$$p_2^a = \left(\frac{\beta_1}{\beta_1 - \eta \alpha_M - \alpha_1 - \alpha_2}\right)^{1/n} \quad (59)$$
The second trigger $p_2^b$ is obtained with the following boundary conditions:

$$A_21(p_2^b)^{\beta_1} = (\alpha_M - \alpha_1 - \alpha_2) (p_2^b)^{\eta} - C_1(p_2^b)^{\beta_1} - (X_1 + X_2) \quad (60)$$

$$\beta_1 A_21(p_2^b)^{\beta_1 - 1} = \eta (\alpha_M - \alpha_1 - \alpha_2) (p_2^b)^{\eta - 1} - \beta_1 C_1(p_2^b)^{\beta_1 - 1} \quad (61)$$

$p_2^b$ is the solution to this nonlinear equation:

$$(\beta_2 - \eta) (\alpha_M - \alpha_1 - \alpha_2) (p_2^b)^{\eta} - (\beta_2 - \eta) C_1(p_2^b)^{\beta_1} - \beta_2 (X_1 + X_2) = 0 \quad (62)$$

Proof of Proposition

Combining equations (34) and (37) ans simplifying produces the following trigger:

$$p_1^q = \frac{\beta_1 (\pi(p) - (X_1 + X_2))}{\pi'(p)} \quad (63)$$

Following similar steps form firm 2 produces the trigger:

$$p_2^q = \frac{\beta_1 (\pi(p) - (X_1 + X_2))}{\pi'(p)} \quad (64)$$

It is straightforward to show that this is the same trigger of the central planner that has the following payoff:

$$\Pi_C(p) = \Pi_1(p) + \Pi_2(p) = \pi(p) - (X_1 + X_2). \quad (65)$$