Capacity choice under uncertainty in a duopoly with endogenous exit

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Abstract

Applying the real options framework, this article investigates the investment decision of the entrant given that an incumbent is already active. Both firms have an option to exit this market if the demand level falls too low. The combination of three decision components, capacity choice, entry and exit timing, results into multiple trigger strategies for the entrant and generates a hysteresis region. In particular, in the presence of a large incumbent it can either choose to coexist with its rival in a duopoly or (eventually) monopolize the market by installing a sufficiently large capacity. The former scenario is realized when the market is large, while the latter occurs when the market is small. When the market is of intermediate size, a hysteresis region emerges where the entrant does not take any actions and prefers to postpone investment.

1 Introduction

Traditional real options models address the question of the investment timing in uncertain markets applying the optimal stopping technique. Most of these models associate stopping with the decision to enter the new market by undertaking an irreversible investment. The common assumption in such models is that firms can temporarily suspend their operations in the case of negative profit flow and later resume it at no cost if the market profitability increases. This means that after investment firms stay in the market irrespectively of the realized demand patterns. In reality resumption of the firms' operations is rarely costless and sometimes even impossible.[1] As a result the negative demand shock may trigger their decision to exit the market forever. Irreversibility of exit decisions in uncertain markets allows to treat them as real options.

Exit options have received limited attention in the literature. Ghemawat and Nalebuff (1985) and Fudenberg and Tirole (1986) analyze the exit game in a duopoly with asymmetric firms in the deterministic setting. The early literature on stochastic monopoly models includes Dixit (1989), that focuses on combined entry and exit strategies, and Alvarez (1998), Alvarez (1999) studying optimal exit strategy of a firm operating with a fixed capacity.

The continuous time duopoly setting with the option to exit was investigated among others by Lambrecht (2001), who presents a model of strategic interactions of firms that have both entry and exit options. He explicitly derives entry and exit thresholds and investigates how the exit order is influenced by different economic factors. He shows that consistent with earlier findings the firm that has a lower monopoly exit threshold leaves the market last. Additionally, he modifies the model by assuming that financially distressed firms can decrease their debt through debt exchange offers. As a result a reversed bankruptcy order of the firms may appear. Murto (2004) examines the exit decisions under uncertainty in a duopoly game with asymmetric firms in a declining market. He shows that when market uncertainty is sufficiently low, there is a unique equilibrium where the larger firm exits the market earlier. However, in a highly uncertain environment there exists an empty span between the exit regions of the firms, within which neither of them leaves the market and a reversed exit order may appear. As a result, the equilibrium is no longer unique and it is not clear which firm is first to exit the market. Ruiz-Aliseda (2006) studies an entry/exit game in a duopoly market that first expands until some random moment in time and then starts declining. He finds that the monopolist does not exit as long as the market grows. After the market matures and starts declining no firm enters the market anymore and the incumbent ultimately leaves. In case both players are active when the market reaches maturity the firm with higher sunk costs exits first. Bayer (2007) presents a model where firms consider an option to increase their capacity in order to stimulate sooner exit of the

[1] For example due to the loss of technology or the team of professionals.

opponent. He shows that such predatory behavior occurs in a more competitive and a less uncertain market.

Similarly to Lambrecht (2001) we investigate the combination of firms exit and entry decisions in a duopoly. The main difference, however, is that in our model capacity choice is considered. Namely, in order to become active in the market firms can freely choose the scale of their investment. This, in turn affects not only their investment decision, but also the exit order. We adopt the approach for capacity optimization used in the monopoly model of Dangl (1999) and later extended by Huisman and Kort (2015) to a duopoly scenario. In the setting where firms are able to choose both timing and size of their investment, Huisman and Kort (2015) show that the firm with a larger capacity invests at a higher investment threshold. Moreover, they demonstrate that the market leader overinvests in capacity in order to ensure that its rival enters the market later and installs smaller capacity.

Here we extend Huisman and Kort (2015) by incorporating the exit option into the model. In that way there exists a second mover advantage for the firm that enters the market last, the entrant, as it can influence the exit game. In this paper we focus on the analysis of the investment strategies of the entrant given that the first investor is already active in the market with a certain capacity.

We demonstrate that the firm with the larger capacity level exits the market first. As a result, in the presence of a sufficiently large incumbent the entrant has an incentive to drive the incumbent out of the market by installing a relatively large capacity. This may result in a non-monotonicity in the entrant's threshold with respect to the size of the incumbent. In particular, the entrant's threshold first increases as a result of the decrease in the output price similarly to Huisman and Kort (2015), yet then starts declining as the entrant anticipates sooner exit of the incumbent.

In addition, we show that the introduction of an exit option leads to a multiple trigger strategies of the entrant. This result is associated with the existence of so called region of hysteresis. This region corresponds to a gap between the investment regions of the entrant. In particular, if the market is large enough, or in other words the exit is far away, the entrant chooses to coexist with his opponent in a duopoly. In a small market, however, given that it is already optimal for the monopolist to enter, the entrant has an incentive to monopolize the market by driving its rival out. For the case of an intermediate market size it is optimal to wait until either of the scenarios is profitable. As a result, the entrant, does not take any actions and prefers to postpone investment.

The hysteresis region can be related to the gap equilibrium of Murto (2004), where exit in a reversed order may occur, or the inaction region of Decamps *et al.* (2006). The latter extends Dixit (1993) and studies a single firm's decision to invest in alternative projects with an uncertain cash flow. For each project there is a certain region of the output price that triggers the firm's investment. The main similarity with our finding is associated with the fact that the optimal investment intervals of two projects do not intersect, creating the inaction region. In an inaction region the firm does not invest yet and it is unknown in which of the two projects it will eventually invest.

The remainder of this paper is organized as follows. Section 2 is devoted to the analysis of the investment decisions of the monopolist that has an option to exit the market. Section 3 discusses the exit order of the firms and specifies the solution for the entrant's entry-exit problem. Section 4 summarizes the main results and concludes the paper. The proves of the propositions are presented in the Appendix.

2 Monopoly

Consider the investment problem of a monopolist, that faces a possibility to undertake an irreversible investment in a plant with a certain capacity. Once the investment is made the firm becomes active on the market and launches the production process. The market for the final output is characterized by uncertain demand, specified by a multiplicative inverse demand function:

$$P_t = X_t(1 - \eta Q_t), \tag{1}$$

with $\eta > 0$, Q_t total market output and X_t the stochastic shock which follows a Geometric Brownian Motion:

$$dX_t = \alpha X_t dt + \sigma X_t dZ_t, \tag{2}$$

where α and σ are the drift and volatility parameters respectively, and Z_t is a Wiener process. The firm is assumed to be risk neutral with a discount rate r. Moreover, it should hold that $r > \alpha$, otherwise the discounted value of the future revenue stream is infinite and the firm always prefers to invest in the future.

We assume that the firm that becomes active on the market always produces up to capacity and, thus, henceforth we will refer to Q as the capacity level. The investment costs the firm bears are proportional to the capacity and are given by δQ, where δ is the sensitivity parameter. Apart from the investment costs that are incurred only at the moment of investment the fixed production costs proportional to capacity, cQ, are paid by the firm in each period.[2] Moreover, once invested the firm faces the possibility to exit the market at no cost when the demand level is too low. The exit decision is assumed to be irreversible, i.e. the production cannot be resumed once being shut down. Hence, the problem of the potential market entrant consists of the optimal choice of investment timing, capacity level and exit timing.

[2]Here we can think of the costs for regular maintenance of machinery or rent for production spaces, laboratories, etc.

In the presented setting the firm holds an option to exit the market when it is active, while if the firm has not entered the market yet an investment option. For the idle firm there exists an optimal investment trigger, which we denote by $X_M^I(Q)$, such that once it is equal to the current value of the stochastic process, X, the firm is indifferent between investing and waiting. Thus, for $X \geq X_M^I$ the monopolist enters the market, forgoing its investment option, V_0, for the operating project value, V_1, and pays sunk investment costs δQ. After the firm has entered the market it possesses the option to abandon the project, i.e. exit the market. The optimal level of X to exercise such an option is denoted by $X_M^E(Q)$. First, consider the situation where the capacity level of the firm is given. The value of the firm and the optimal thresholds in this case are summarized by the following proposition.

Proposition 1 *The value of the idle and active monopolist for a given level of the stochastic process, X, and capacity, Q, are given by* (3) *and* (4) *respectively:*

$$V_0^M(X,Q) = \frac{\beta_2}{\beta_2-\beta_1}\left(\frac{X}{X_M^I}\right)^{\beta 1}\left(\frac{X_M^I(1-\eta Q)Q}{r-\alpha}\left(1-\frac{1}{\beta_2}\right)-\left(\frac{c}{r}+\delta\right)Q\right), \tag{3}$$

$$V_1^M(X,Q) = \left(\frac{X}{X_M^E(Q)}\right)^{\beta_2}\frac{cQ}{r(1-\beta_2)}+\frac{X(1-\eta Q)Q}{r-\alpha}-\frac{cQ}{r}, \tag{4}$$

where exit and investment thresholds for a given capacity choice satisfy (5) *and* (6) *respectively:*

$$X_M^E(Q) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-\eta Q)}, \tag{5}$$

$$\frac{(\beta_1-\beta_2)c}{(1-\beta_2)\beta_1 r}\left(\frac{X^I}{X_M^E(Q)}\right)^{\beta_2}+\left(1-\frac{1}{\beta_1}\right)\frac{X^I(1-\eta Q)}{r-\alpha}-\frac{c}{r}-\delta = 0, \tag{6}$$

and β_1, β_2 are given by

$$\beta_1 = \frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} > 1, \tag{7}$$

$$\beta_2 = \frac{1}{2}-\frac{\alpha}{\sigma^2}-\sqrt{\left(-\frac{1}{2}+\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} < 0. \tag{8}$$

In the more realistic setting the firm is free to choose its capacity level at the moment of investment. Intuitively, the monopolist chooses its capacity such that the investment yields the highest possible value until it is optimal to exit, i.e. for $X > X_M^E(Q)$. Using (5) we can express the later condition in terms of Q, namely as $Q < \tilde{Q}_M$, defined by

$$\tilde{Q}_M(X) = \frac{1}{\eta}\left(1-\frac{\beta_2(r-\alpha)c}{r(\beta_2-1)X}\right). \tag{9}$$

This means that if the capacity of the firm is too large, $Q \geq \tilde{Q}_M(X)$, the firm is not able to bear the production costs for a given market profitability and it will exit the market immediately. Thus, the optimal capacity level of the monopolist is found maximizing the value of an operating project $V_1(X, Q)$ with respect to Q such that $Q < \tilde{Q}_M(X)$. This results in the following proposition.

Proposition 2 *The optimal capacity level of the monopolist, $Q_M^*(X)$, for a given level of X is implicitly determined by*

$$\left(\frac{X}{X_M^E(Q)}\right)^{\beta_2} \frac{c\,(1-\eta Q(1+\beta_2))}{r(1-\beta_2)(1-\eta Q)} + \frac{X(1-2\eta Q)}{r-\alpha} - \frac{c}{r} - \delta = 0. \tag{10}$$

The optimal investment trigger, $X_M^{I^*}$ satisfies

$$-\left(\frac{r\beta_1(\beta_2-1)X_M^{I^*}}{(\beta_1+1)\beta_2(r-\alpha)c}\right)^{\beta_2} \frac{(\beta_1-\beta_2)c}{\beta_1(\beta_2-1)r} + \frac{(\beta_1-1)X_M^{I^*}}{(\beta_1+1)(r-\alpha)} - \left(\frac{c}{r}+\delta\right) = 0, \tag{11}$$

and the corresponding capacity level is equal to

$$Q_M^* \equiv Q_M^*(X_M^{I^*}) = \frac{1}{\eta(\beta_1+1)}. \tag{12}$$

The optimal exit trigger $X_M^{E^*}$ given by

$$X_M^{E^*} = \frac{\beta_2(\beta_1+1)(r-\alpha)c}{\beta_1(\beta_2-1)r}. \tag{13}$$

As follows from the above proposition the production costs are crucial for the decision of the firm to exit the market. Intuitively, the larger are the production costs of the firms, the larger losses it faces when the demand level becomes low. This causes the exit threshold of the monopolist to increase with c. Note, however, that the optimal capacity level does not depend on the production costs incurred by the firm, while the optimal investment threshold increases with c. This is due to the assumption that the production costs the firm bears in each period are fixed. Therefore, given that there are no strategic effects involved in the firm's decision the firm will respond to an increase in the production costs in a same way as to an increase in the investment costs, namely, by postponing their investment decision keeping the capacity choice unaffected.

3 Duopoly

In the duopoly we define the incumbent as a firm that has made the first move in the investment game. That is the firm that has entered the market first and set its capacity level, which we denote by Q_L. When the demand is high enough, the second firm, the entrant, also becomes active on the market. It decides upon its capacity level, Q_F, given the level acquired by the market incumbent.

Once both firms have undertaken their investment the exit game starts. It continues until the demand becomes low enough to trigger the exit of one of the firms. Several papers, e.g. Lambrecht (2001), demonstrate that the firm with a lower monopoly threshold exists last in the equilibrium. In the current setting, similarly to Ghemawat and Nalebuff (1985) and Murto (2004), this is the firm with the lower capacity level.[3] Intuitively, such a firm incurs larger production costs, that in the face of declining demand induce larger losses. As a result such a firm exits earlier. Incorporating this idea into our model we specify the exit order of the firms according to the following proposition.

Proposition 3 *The firm with a larger capacity level exits a market first at the optimal duopoly exit threshold, X_D^E, determined by*

$$X_D^E(Q_L, Q_F) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2 - 1)(1 - \eta(Q_L + Q_F))}, \tag{14}$$

while the firm with a smaller capacity level exits once X hits optimal monopoly exit threshold, X_M^E, given by

$$X_M^E(Q_i) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2 - 1)(1 - \eta Q_i)}, \tag{15}$$

with $i = L$ if the smaller firm has entered the market first and $i = F$ if it has entered last.

If the firms install the same capacity level, $Q_L = Q_F$, the game is is characterized by two Nash equilibria, where either of the firms can exit first.

The crucial feature of the model with both exit and investment option is that the entrant gains a second mover advantage. Namely, in the case of sufficiently large capacity level of the incumbent for specific values of the market size the entrant can install a capacity large enough to force the incumbent out of the market. Hence, in contrast with the model without the option to exit, the incumbent no longer has ultimate control over the outcome. Thus, the optimization problem of the entrant should incorporate that the exit order depends on its capacity choice. In particular, the entrant can reply to a certain capacity level of the incumbent, Q_L, with either larger, $Q_F > Q_L$, smaller $Q_F < Q_L$, or equal capacity level, $Q_L = Q_F$. Each of these cases leads to different exit order scenarios. This, in turn, affects the value functions, because now they incorporate the possibility that in case of the market decline one firm is going to exit the market, while another becomes a monopolist.

As mentioned earlier, in what follows we concentrate on the entrant's investment and exit decision for a given capacity choice of the incumbent, which is already operating in the market. This corresponds to the case of the investment game with exogenous roles. In this setting the entrant determines both timing of its investment, or in other words its investment threshold, which we denote

[3] Bayer (2007), however, concludes that the firm with larger capacity exits last. This is because he imposes the assumption that the production costs are fixed and, unlike in our model, do not depend on capacity level.

by X_F^I, and its capacity size, Q_F. Before the entrant has become active on the market it holds an option to invest in the future. The value of this option corresponds to the value of an idle firm, F_0, which together with the investment threshold, X_F^I, is defined by the following proposition.

Proposition 4 *The value of the idle entrant is given by*

$$F_0(X, Q_L, Q_F) = A_F(Q_L, Q_F)X^{\beta_1}, \tag{16}$$

where

$$A_F(Q_L, Q_F) = \frac{\beta_2}{\beta_2 - \beta_1}\left(\frac{1}{X_F^I}\right)^{\beta 1}\left(\frac{X_F^I(1-\eta(Q_L+Q_F))Q_F}{r-\alpha}\left(1-\frac{1}{\beta_2}\right) - \left(\frac{c}{r}+\delta\right)Q_F \right.$$
$$\left. - \frac{X_F^I}{\beta_2}\frac{\partial F_1(X_F^I, Q_L, Q_F)}{\partial Q_F}\frac{\partial Q_F}{\partial X}\bigg|_{X=X_F^I}\right), \tag{17}$$

X_F^I is implicitly determined by $h(X_F^I, Q_L, Q_F) = 0$, where

$$h(X, Q_L, Q_F) = \left(1-\frac{\beta_2}{\beta_1}\right)\frac{B_F(Q_L, Q_F)}{Q_F}X^{\beta_2} + \left(1-\frac{1}{\beta_1}\right)\frac{X(1-\eta(Q_L+Q_F))}{r-\alpha} - \frac{c}{r} - \delta$$
$$- \frac{X}{\beta_1 Q_F}\frac{\partial F_1(X, Q_L, Q_F)}{\partial Q_F}\frac{\partial Q_F}{\partial X} = 0, \tag{18}$$

and $B_F(Q_L, Q_F)$ is

$$B_F(Q_L, Q_F) = B_F^B\chi_{\{Q_F>Q_L\}} + (\lambda B_F^B + (1-\lambda)B_F^S)\chi_{\{Q_F=Q_L\}} + B_F^S\chi_{\{Q_F<Q_L\}}, \tag{19}$$

with λ being the probability that the entrant exits first in a symmetric game, $\chi_{\{true\}} = 1$, $\chi_{\{false\}} = 0$ and

$$B_F^B(Q_L, Q_F) = \left(\frac{1}{X_D^E(Q_L, Q_F)}\right)^{\beta_2}\frac{cQ_F}{r(1-\beta_2)}, \tag{20}$$

$$B_F^S(Q_L, Q_F) = \left(\frac{1}{X_M^E(Q_F)}\right)^{\beta_2}\frac{cQ_F}{r(1-\beta_2)} + \left(\frac{1}{X_D^E(Q_L, Q_F)}\right)^{\beta_2}\frac{X_D^E(Q_L, Q_F)\eta Q_L Q_F}{r-\alpha}. \tag{21}$$

Once the entrant has invested, which is when $X > X_F^I$, the exit game starts. Clearly, if the entrant observes that the incumbent is not active on the market, it directly infers that the incumbent has already exited given that the roles in the investment game are assigned exogenously and the exit is assumed to be irreversible. In this case strategic factors do not influence its exit decision as the entrant ends up being a monopolist and exist at X_M^E. In the complementary case, when the incumbent is active in the market, the location of the entrant's investment threshold with respect to the monopoly and duopoly exit triggers may vary. If the entrant is a larger player in the market, it invests only if $X_F^I > X_D^E$, otherwise it is forced to exit immediately. Being a small firm, entrant can in principle enter before X_D^E. In this case, however, the incumbent, being the larger firm, is immediately displaced from the market and the entrant becomes a monopolist. As a result, again, there are no strategic effects involved. In the symmetric case where $Q_L = Q_F$ either of the above situations can occur. Since we are interested in the strategic aspects of the exit model we will concentrate only on the scenario when both players are present on the market by the time the exit game starts, which is when $X_F^I > X_D^E$.

The above observations imply that in the exit game the initial X is larger than the exit thresholds of the firms and, thus, rule out the possibility of the gap equilibrium shown by Murto (2004). Therefore, the equilibrium described by Proposition 3 is unique and the entrant's strategy is specified as follows. When the entrant enters a market as a larger firm, i.e. $Q_F > Q_L$, it exits last at the duopoly exit threshold X_D^E. Once this threshold is hit the entrant leaves the market. Intuitively, the larger is the capacity that the entrant acquires the earlier will it exit the market, which is confirmed by (14). This brings us to the capacity level, $\tilde{Q}_F(X, Q_L)$, that leads to the immediate displacement of the entrant from the market:

$$\tilde{Q}_F(X, Q_L) = \frac{1}{\eta}\left(1 - \frac{\beta_2 c(r-\alpha)}{r(\beta_2 - 1)X}\right) - Q_L. \tag{22}$$

When the entrant is a smaller firm, i.e. $Q_F < Q_L$, the incumbent exits first at X_D^E and the entrant enjoys monopoly profits until X hits the monopoly exit threshold, X_M^E. In this case $\tilde{Q}_F(X, Q_F)$ has a different interpretation, namely, it is the capacity level such that once installed by the entrant the incumbent is forced out of the market.

If $Q_F = Q_L$ the game is symmetric, and as shown by Murto (2004), the exit order is not identified.

The described strategies of the entrant are illustrated in Table 1, where we demote by F_1^B is the value larger entrant (B stands for "big"), by F_1^S the value of the smaller entrant (S stands for "small") and by V_1^M the value of the monopolist, defined in the previous section.

Conditions	$Q_F < \tilde{Q}_F$ *Current X is above X_D^E*	$Q_F \geq \tilde{Q}_F$ *Current X is below X_D^E*
$Q_F > Q_L$ *entrant exits first*	$F_1^B - \delta Q_F$	$-\delta Q_F$
$Q_F < Q_L$ *entrant exits last*	$F_1^S - \delta Q_F$	$V_1^M - \delta Q_F$
$Q_F = Q_L$ *Exit order unclear*	$\lambda F_1^B + (1-\lambda)F_1^S - \delta Q_F$	$(1-\lambda)V_1^M - \delta Q_F$

Table 1: Value of the entrant for the different capacity levels of the incumbent, where λ the probability that the entrant exits first in a symmetric game.

The above mentioned entrant values are characterized in Proposition 5.

Proposition 5 *The value of the active entrant is given by*

$$\begin{aligned} F_1(X, Q_L, Q_F) &= \chi_{\{Q_F<\tilde{Q}_F\}} \left(F_1^B \chi_{\{Q_F>Q_L\}} + (\lambda F_1^B + (1-\lambda) F_1^S) \chi_{\{Q_F=Q_L\}} + F_1^S \chi_{\{Q_F<Q_L\}} \right) \\ &+ \chi_{\{Q_F \geq \tilde{Q}_F\}} \left((1-\lambda) V_1^M \chi_{\{Q_F=Q_L\}} + V_1^M \chi_{\{Q_F<Q_L\}} \right) - \delta Q_F, \end{aligned} \tag{23}$$

with λ and χ introduced earlier and where F_1^B is the value of the large entrant defined by

$$F_1^B(X, Q_L, Q_F) = \left(\frac{X}{X_D^E(Q_L, Q_F)} \right)^{\beta_2} \frac{cQ_F}{r(1-\beta_2)} + \frac{XQ_F(1-\eta(Q_L+Q_F))}{r-\alpha} - \frac{cQ_F}{r}, \tag{24}$$

and F_1^S is the value of the smaller entrant determined by

$$\begin{aligned} F_1^S(X, Q_L, Q_F) = \left(\frac{X}{X_M^E(Q_F)} \right)^{\beta_2} \frac{cQ_F}{r(1-\beta_2)} + \left(\frac{X}{X_D^E(Q_L, Q_F)} \right)^{\beta_2} \frac{X_D^E(Q_L, Q_F)\eta Q_L Q_F}{r-\alpha} \\ + \frac{XQ_F(1-\eta(Q_L+Q_F))}{r-\alpha} - \frac{cQ_F}{r}, \end{aligned} \tag{25}$$

and V_1^M is a value of the active monopolist defined earlier.

Note that both $F_1^B(X, Q_L, Q_F)$ and $F_1^S(X, Q_L, Q_F)$ contain the duopoly revenue net of production costs, reflected by the last two terms in (24) and (25). When the entrant is the larger firm this revenue is adjusted by the stochastic discount factor $\left(\frac{X}{X_D^E(Q_L,Q_F)}\right)^{\beta_2}$ once, due to the fact that it leaves the market at X_D^E, while in the complementary case of the entrant being the small firm it is corrected twice. First, for becoming a monopolist at X_D^E, using the same stochastic discount factor, and second, for leaving the market at X_M^E by means of $\left(\frac{X}{X_M^E(Q_F)}\right)^{\beta_2}$.

Moreover, it can be shown that if the capacities in the market are given and are not restricted it is always better for the entrant to exit last[4], i.e. it holds that $F_1^S(X, Q_L, Q_F) > F_1^B(X, Q_L, Q_F)$ for $X < \infty$. Note that in the symmetric game the entrant gets a weighted average of the values under different exit orders. This implies that the entrant prefers to take a chance over exit first with certainty, while leaving the market last is still its most preferable option. Given that the value functions are continuous it will be never optimal to choose the exact same capacity as the incumbent, because given a non-zero probability of being first to exit in a symmetric game the entrant can always improve by setting an ε-smaller capacity and exit last. Therefore, the symmetric game will never occur. The value of the active entrant can be rewritten given that $\chi_{\{Q_F=Q_L\}} = 0$ in the following way

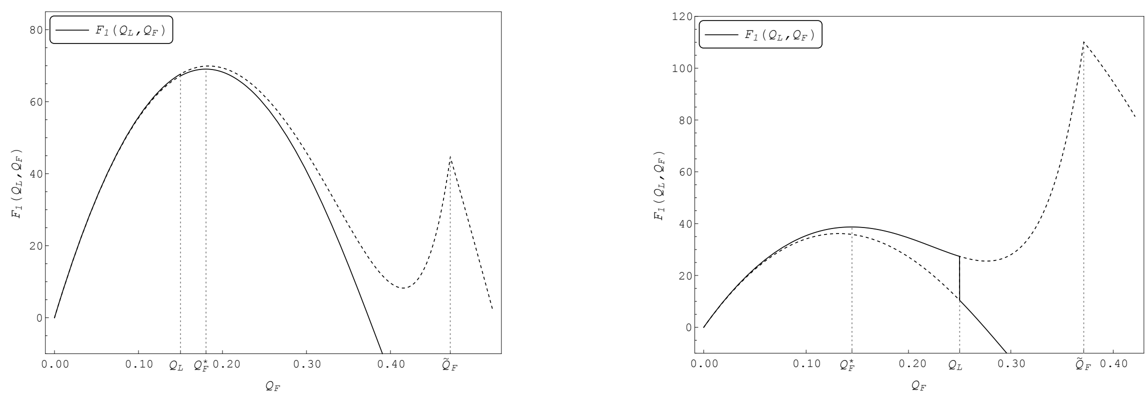
$$\begin{aligned} F_1(X, Q_L, Q_F) &= \chi_{\{Q_F<\tilde{Q}_F\}} \left(F_1^B \chi_{\{Q_F>Q_L\}} + F_1^S \chi_{\{Q_F<Q_L\}} \right) \\ &+ \chi_{\{Q_F \geq \tilde{Q}_F\}} \left(V_1^M \chi_{\{Q_F<Q_L\}} \right), \end{aligned} \tag{26}$$

[4]See poof of Proposition 6.

In order to find the optimal response of the entrant to a given capacity level acquired by the incumbent, Q_L, we maximize $F_1(X, Q_L, Q_F) - \delta Q_F$ with respect to Q_F.

$$\underset{Q_F}{\text{maximize}} \quad \{F_1(X, Q_L, Q_F) - \delta Q_F\}$$

Even though in the case when the capacity choice is not restricted by the exit order condition the entrant always prefers to be the last firm to exit the market, it is not always possible. This is because in our model the exit order is endogenously determined by the firms' relative capacity size, namely that the firm with the smaller capacity exits first. Given this capacity restriction the strategy of being the last firm to exit is not always preferable, as it requires a relatively low capacity level. We illustrate this situation below.



(a) Maximum is at Q_F^B for $Q_L = 0.15$.

(b) Maximum is at Q_F^S for $Q_L = 0.25$

Figure 1: The value function of the entrant given the capacity level of the incumbent for the parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\eta = 1$, $\delta = 100$, $c = 50$ and $X = 66$.

Figure 1 shows the values function of the entrant for different values of the incumbent's capacity. The lower curves in both figures correspond to the value that the entrant gets if it exits first, while the upper curves – if it exits last. Note that the later value is not a unimodal function of the capacity, Q_F. Instead, it has a spike for the large values of Q_F. This is because given that the entrant exits first it becomes a monopolist as soon as Q_F reaches $\tilde{Q}_F(X, Q_F)$ by construction. Thus, anticipation of a sooner monopoly position causes the entrant's value to increase as Q_F approaches $\tilde{Q}_F(X, Q_F)$. This value, however, can only be reached for the larger values of incumbent's capacity. Due to the exit order constraint the entrant can end up on the upper curve only if it becomes the smallest firm in the market, $Q_F < Q_L$. The complementary case of $Q_F > Q_L$ yields the value that corresponds to the lower curve. Thus, the solid parts of the curves represent the actual value of the entrant. As it can bee seen in Figure 1a the capacity of the incumbent is small, meaning that the entrant is relatively close to the monopoly situation. In this case the value of becoming the last firm to exit is smaller than the value of installing a larger capacity and exiting last. On the contratry, when the capacity of the firm that already operates in the market is large, the exit order becomes more important. Figure 1b shows that for larger Q_L the entrant prefers to install smaller capacity and exit last. Thus, we can conclude that the firm faces a trade-off between staying longer in the market and installing a larger capacity. In fact, if Q_L is small it brings larger value to extract rents from installing a larger capacity and the entrant is willing to forgo its potential monopoly position. Proposition 6 shows the entrant's capacity choice for each level of the incumbent's capacity.

Proposition 6 *The optimal capacity level of the entrant depending on the capacity level of the incumbent is given by*

$$Q_F^*(X, Q_L) = Q_{F,D}^*(X, Q_L)\chi_{\{Q_L \leq \bar{Q}_4(X)\}} + Q_{F,M}^*(X, Q_L)\chi_{\{Q_L > \bar{Q}_4(X)\}}, \tag{27}$$

where $Q_{F,D}^*$ denotes the optimal capacity of the entrant when it enters as a duopolist, while $Q_{F,M}^*$ – when it becomes a monopolist upon entry. These capacity levels are defined as follows:

$$Q_{F,D}^*(X, Q_L) = \begin{cases} Q_F^B(X, Q_L) & \text{if } Q_L < \bar{Q}_1(X), \\ Q_L - \varepsilon & \text{if } Q_L \in (\bar{Q}_1(X), \bar{Q}_2(X)] \cup (\bar{Q}_3(X), \bar{Q}_4(X)], \\ Q_F^S(X, Q_L) & \text{if } \bar{Q}_L \in (\bar{Q}_2(X), \bar{Q}_3(X)], \end{cases} \tag{28}$$

with capacity levels of the entrant, $Q_F^B(X, Q_L)$ and $Q_F^S(X, Q_L)$ implicitly determined by the first order conditions (29) *and* (30) *respectively:*

$$\frac{\partial(F_1^B(X, Q_L, Q_F) - \delta Q_F)}{\partial Q_F} = 0, \tag{29}$$

$$\frac{\partial(F_1^S(X, Q_L, Q_F) - \delta Q_F)}{\partial Q_F} = 0, \tag{30}$$

and

$$Q_{F,M}^*(X, Q_L) = \begin{cases} \tilde{Q}_F(X, Q_L) & \text{if } \bar{Q}_L \in (\bar{Q}_4(X), \bar{Q}_5(X)], \\ Q_M(X) & \text{if } \bar{Q}_L > \bar{Q}_5(X). \end{cases} \tag{31}$$

The expressions for $\bar{Q}_1(X)$, $\bar{Q}_2(X)$, $\bar{Q}_3(X)$, $\bar{Q}_4(X)$, $\bar{Q}_5(X)$ are given in the Appendix.

A numerical example illustrating the optimal capacity choice of the entrant for a given X is presented in the Figure 2.

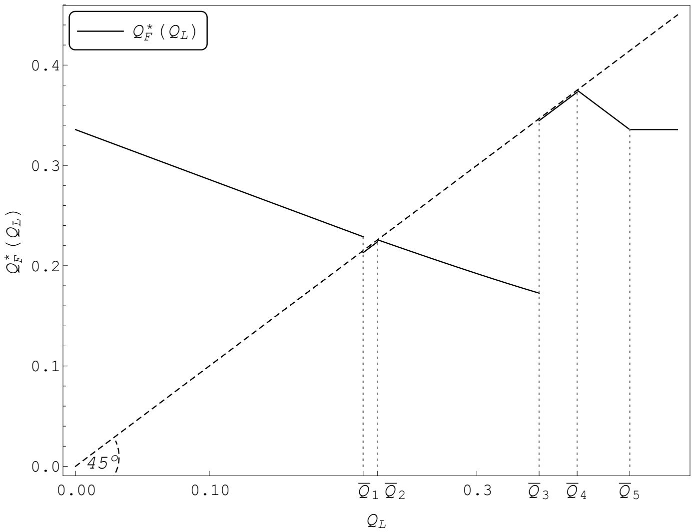


Figure 2: *The optimal reaction of the entrant given the capacity level of the incumbent for the parameter values:* $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\eta = 1$, $\delta = 100$, $c = 20$ *and* $X = 100$.

Apart from illustrating the optimal capacity choice of the entrant, Figure 2 also helps to infer which exit order corresponds to a certain capacity choice of the incumbent. Namely, the entrant will choose to leave the market first when its optimal capacity level is above 45° line, otherwise it acquires a market share smaller than the incumbent and exits last.

As it can be seen, the entrant prefers to exit first only when the capacity level of the incumbent is relatively small, i.e. $Q_L < \bar{Q}_1(X)$. Then the entrant can obtain a large revenue by installing a larger capacity. The large revenue outweighs the advantage of leaving the market last and, consequently, the entrant behaves as a large duopolist.

On the contrary, if the entrant observes that the incumbent has installed capacity of a considerable size, it is not possible to obtain such a large revenue that it would be still profitable to leave the market first. As a result, the entrant chooses to be a small duopolist in order to stay longer on the market for $Q_L \in (\bar{Q}_2(X), \bar{Q}_3(X)]$.

Note that for both the large and small duopolist the optimal capacity level decreases with Q_L, because in general the larger capacity taken by the first investor reduces the output price for a given capacity of the entrant However, Figure 2 shows that at some intervals entrant's capacity increases with incumbent's capacity.

The increasing parts of the curves correspond to the scenarios, where the entrant chooses to mimic the incumbent's behavior and acquires capacity $Q_F = Q_L - \varepsilon$. When $Q_L \in (\bar{Q}_1(X), \bar{Q}_2(X)]$, the share of the incumbent is large enough to stimulate the entrant to leave last. However, leaving last would require that the capacity of the entrant satisfies the constraint, $Q_F < Q_L$. As a result the optimum of the small duopolist cannot be reached. In this case the entrant maximizes its revenue in a constrained duopoly using a mimicking strategy, with the optimal choice $Q_F = Q_L - \varepsilon$.

When Q_L hits $\bar{Q}_3$ we observe a relatively large discontinuous upward jump in the entrant's optimal capacity. This result corresponds to the findings in Kwon and Zhang (2015), namely, that if the capacity of the one firm is large enough, it becomes optimal for its rival to increase the capacity to force such a firm out of the market. Thus, in the regions where $Q_L \in (\bar{Q}_3(X), \bar{Q}_4(X)]$ and $Q_L \in (\bar{Q}_4(X), \bar{Q}_5(X)]$ the capacity of the incumbent is so large, and, consequently, the output price is so low, that the entrant acts to force a soon or immediate exit of the incumbent respectively. Anticipating the incumbent's (almost) immediate exit, the entrant behaves as a constrained monopolist, i.e. it installs a capacity level such that on the one hand the incumbent exits first, and, on the other hand, that it gets the largest possible monopoly value for itself. As a result it chooses either $Q_L - \varepsilon$ or $\tilde{Q}_F$ for the two regions respectively[5]. Note that mimicking strategy arises here for a different reason than in the case of smaller levels of the incumbent's capacity. Namely, for $Q_L \in (\bar{Q}_3(X), \bar{Q}_4(X)]$ the duopoly exit threshold is relatively close, yet due to the capacity constraint for a small firm the immediate exit of its rival cannot be triggered. Thus, the entrant installs the largest capacity available to ensure that this threshold is hit as soon as possible.

In the last region, i.e. where $Q_L > \bar{Q}_5(X)$, the entrant becomes an unconstrained monopolist as acquiring the monopoly capacity level is enough to ensure the incumbent leaves the market given its capacity choice.

In the above formulation we concentrated on the capacity strategies of the entrant for a given X. Naturally, the problem can be reversed and the boundaries of the strategic regions can be defined in terms of X for a given level of Q_L using the inverse function of $\bar{Q}_i(X)$ with $i = 1, .., 5$. The illustration of the capacity choice of the entrant in two dimension, thus a a function of both X and Q_L, is illustrated below.

[5] The difference between these regions arises only due to exit order restriction on the capacity level. For $Q_L \in (\bar{Q}_3(X), \bar{Q}_4(X)]$ the level $\tilde{Q}_F$ is not available for the entrant that is willing to exit first, therefore, it chooses the second best alternative.

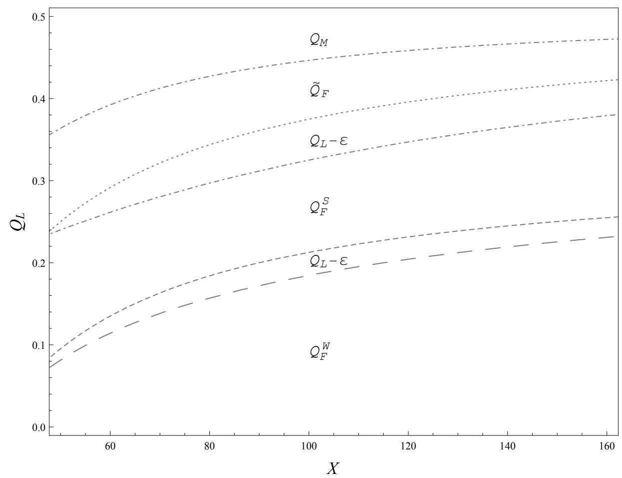
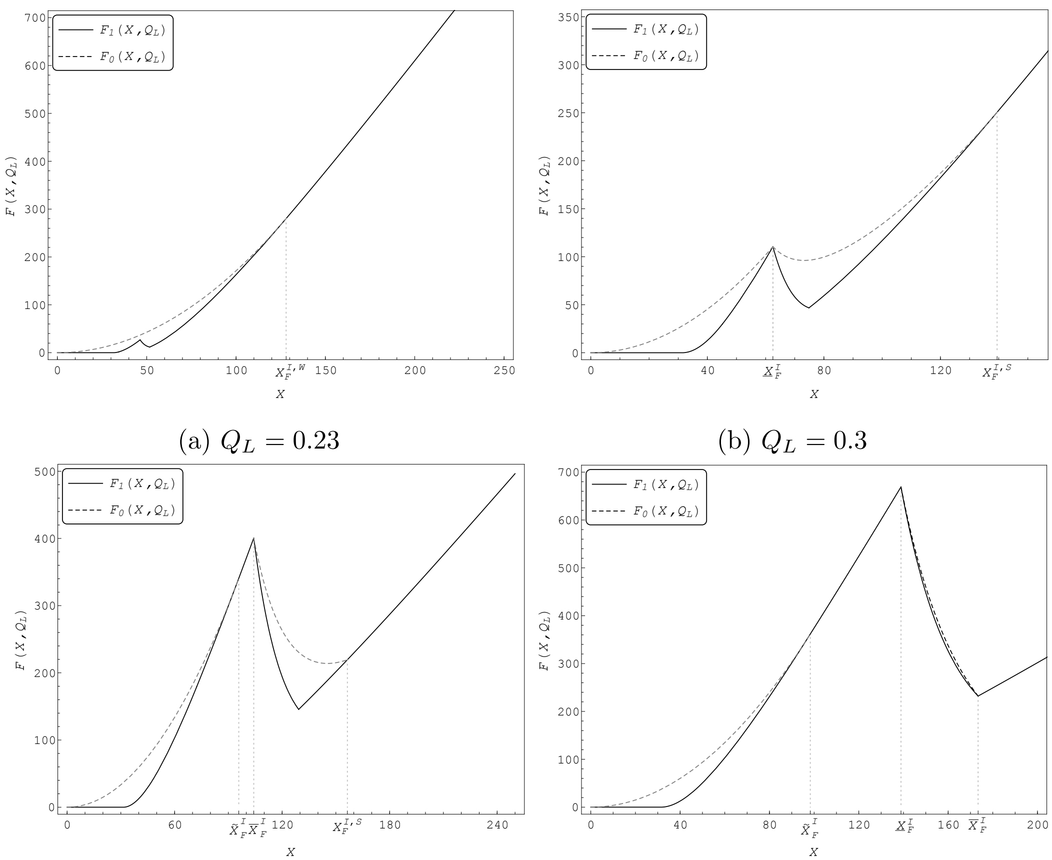


Figure 3: *The optimal capacity strategies of the the entrant for the set of parameter values:* $r = 0.05$, $\alpha = 0.01$, $\sigma = 0.17$, $\eta = 1$, $\delta = 100$, *and* $c = 20$.

As we can see, for a given level of Q_L, where Q_L is relatively large, for X small enough the entrant chooses capacity to maximize monopoly profits, because the rival leaves the market anyhow. For X a little larger the entrant overinvests to force the exit of the incumbent. As X increases even further the market moves away from the duopoly exit trigger and the entrant prefers to capture larger immediate profits. Thus, it gradually increases its capacity as exit becomes further away and thus less crucial to take into account in its investment decision. Consequently, apart from a direct effect on revenues, the market profitability also indirectly influences the firms' exit strategies through the firm's capacity choice. In particular, exit decisions are hastened in less profitable markets and delayed in more profitable markets.

The piecewise structure of the entrant's optimal capacity strongly affects its optimal investment timing. In particular, one of the main implications of the above findings for the entrant value function is that it can obtain different values corresponding to different capacity strategies. For example, it can end up either being a duopolist or a monopolist upon entry. Naturally, these cases correspond to two different value curves with the latter scenario resulting in higher profits. As a result multiple investment thresholds corresponding to different value functions may be available to the entrant as long they are consistent with the boundaries for the capacity choice, i.e. capacity at the moment of entry should fall into the specific regions defined in Proposition 6. In order to find the optimal strategy of the entrant for each level of X, we first need to establish whether it is optimal to invest immediately given the available capacity strategy for the particular value of X. If this is not the case we need to determine until what moment is it optimal to wait with investment, or in other words, the threshold corresponding to which capacity strategy will eventually trigger the investment. We do this by comparing the values of the the options to enter the market with capacities given in Proposition 6 at the corresponding optimal investment thresholds. Figure 4 presents entrant values as functions of market profitability for different values of the incumbent's capacity choice.



(a) $Q_L = 0.23$ (b) $Q_L = 0.3$

(c) $Q_L = 0.38$ (d) $Q_L = 0.41$

Figure 4: *The value functions of the entrant for the set of parameter values:* $r = 0.05$*,* $\alpha = 0.2$*,* $\sigma = 0.1$*,* $\eta = 1$*,* $\delta = 100$*,* $c = 20$ *and different values of* Q_L*.*

The solid line in Figure 4 represent the value of an active entrant. As it can be seen, if X is relatively small the entrant is able to obtain the monopoly value, which corresponds to the increasing part of the entrant curve before the spike. It is possible because current output price is low, so it is easier to force the incumbent out of the market. In a large market this strategy is too costly for the entrant, as a larger capacity is needed to stimulate the incumbent's immediate exit. Thus, the entrant operates in a duopoly. The declining part of the duopoly value after the spike is associated with the mimicking strategy. As stated earlier, under this strategy the entrant's capacity is given by $Q_F = Q_L - \varepsilon$. Thus, for a given level of Q_L the capacity of the entrant is also fixed in such a way that it always exits last. An increase in market profitability has two effects on the entrant value in this case. On the one hand, a larger X means that exit of its rival is farther in the future, affecting the firm's value negatively. On the other hand, a larger X increases its revenues, leading to an increase in value. The mimicking strategy enters the optimal capacity of the entrant twice – for small $Q_L \in (\bar{Q}_1, \bar{Q}_2]$ and for large $Q_L \in (\bar{Q}_3, \bar{Q}_4]$. In the first case an increasing effect prevails as the exit is so far that the firms care more about higher revenues. In the latter case the exit of the large firm is relatively close, and therefore the negative effect of an increase in market size dominates. Due to the latter we observe a decline in the entrant value in the region connecting the strategies of monopolist and duopolist.

The dashed lines in Figure 4 correspond to the values of the idle entrant, or in other words to the value of waiting until the optimal investment threshold. Intuitively, each capacity strategy results in a different investment timing. For example, in Figure 4a it is optimal for the entrant to invest as a large firm. The investment threshold of a large duopolist is denoted by $X_F^{I,W}$. In this case the capacity of the incumbent is so small, that waiting for a larger market and capturing a larger market share yields a higher value. Once the capacity of the incumbent increases, taking into account its exit becomes a more valuable strategy. Denote the maximal market size such that for a given Q_L the entrant can enter as a monopolist by $\underline{X}_F^I$. Since each strategy is possible only in a particular region in terms of Q_L and X it may happen that the optimal investment moment given a particular strategy lies outside the admissible boundaries.

First consider the case of low initial X , i.e. $X \in [0, \underline{X}_F^I]$. If the monopoly threshold lies beyond $\underline{X}_F^I$, given that it aims at the monopoly position, the entrant chooses to enter the market at the next best alternative, namely, at $\underline{X}_F^I$. This happens if the capacity of the incumbent is relatively small, as in this case the entrant has more incentives to wait for a larger market, as shown in Figure 4b. If the capacity of the incumbent increases further, it hastens the entrant's investment decision, making the optimal thresholds first leading to a constrained and then leading to an unconstrained monopoly available in the corresponding strategic regions. Thus, for low initial X the entrant waits either until X_M^I in the unconstrained monopoly region, or until $\tilde{X}_F^I$ – in the case of constrained monopoly. The latter situation is illustrated in Figures 4c and 4d.

If initial value of X is so large that forcing the incumbent out of the market immediately upon entry is not possible anymore, i.e. $X > \underline{X}_F^I$, the entrant has an option to wait either until the market is large enough to enter as a duopolist or until the market is low enough to enter as a monopolist. Thus, in Figures 4b and 4c waiting also pays off if $X \in (\underline{X}_F^I, X_F^{I,S})$, where $X_F^{I,S}$ is the optimal investment threshold of a small entrant. If the capacity of the incumbent is larger, the optimal investment threshold of the small entrant in a duopoly declines and it may occur below the starting point of the corresponding strategy, which we denote by $\overline{X}_F^I$, see Figure 4d. Clearly, in this case for $X \in (\underline{X}_F^I, \overline{X}_F^I)$ the entrant can either enter with a mimicking capacity, or wait until either $\underline{X}_F^I$ or $\overline{X}_F^I$ is hit. It turns out that in this case waiting for either of the thresholds is a preferable strategy as it brings a larger value.

Therefore, the introduction of the exit option together with being able to choose the capacity has the following result concerning the entrant's investment behavior. If the capacity of the incumbent is relatively large the entrant has three investment thresholds. Two of them trigger investment providing a monopoly position for the entrant. The first threshold occurs if the initial market size is so low that the entrant waits until it is profitable enough to enter as a monopolist. The second one is present for intermediate market size, where the entrant has an option to wait until the incumbent's exit threshold is close enough, so that the incumbent is expected to exit soon. The last investment threshold corresponds to the standard case in the real options models, i.e. when the market is large enough for the two firms to operate together in a duopoly. Hence, in the presence of a large incumbent in a small market the entrant waits until the monopoly threshold is hit and then invests immediately as long as the monopoly strategy is available. In the case of an intermediate market size it waits until either monopoly or duopoly threshold is hit, i.e. until the exit is close enough to force the incumbent out or until the market is large enough to coexist with the incumbent. Following the recent literature, we will refer to the region between these thresholds as inaction region or hysteresis region (Decamps *et al.* (2006)). Lastly, in a large market the entrant invests immediately.

If the incumbent sets its capacity at an intermediate level, the only difference with the previous case is that the thresholds that lead to the monopoly situation merge in one. Thus, the entrant waits for the same moment to invest both in the cases of small or intermediate market. At the threshold the entrant overinvests to trigger immediate exit of the incumbent.

If the capacity of the incumbent is small, stimulating its exit becomes so costly that the entrant prefers to wait until a duopoly is profitable and we are back in the situation of one investment threshold.

The investment thresholds of the entrant described above are summarized by the following proposition.

Proposition 7 *The threshold which leads to the immediate investment in a duopoly is given by*

$$X_{F,D}^{I}(Q_L) = \begin{cases} \left\{ X \middle| h(X, Q_L, Q_{F,D}^{*}) = 0 \right\} & \textit{if } Q_L \leq \bar{\bar{Q}}_1, \\ \overline{X}_F^I(Q_L) & \textit{if } Q_L > \bar{\bar{Q}}_1. \end{cases} \tag{32}$$

The entrant monopolization strategy becomes available only if the capacity of the incumbent satisfies $Q_L \geq \widehat{Q}_L$. *In this case the entrant's investment threshold that leads to the monopoly once being hit by* X *from below is*

$$X_{F,\underline{M}}^{I}(Q_L) = \begin{cases} \underline{X}_F^I(Q_L) & \textit{if } Q_L \leq \bar{\bar{Q}}_2, \\ \left\{ X \middle| h(X, Q_L, Q_{F,M}^{*}) = 0 \right\} & \textit{if } Q_L > \bar{\bar{Q}}_2, \end{cases} \tag{33}$$

while the situation when monopoly is triggered from above corresponds to the threshold

$$X^I_{F,\overline{M}}(Q_L) = \underline{X}^I_F(Q_L). \tag{34}$$

$\bar{\bar{Q}}_1$, $\bar{\bar{Q}}_2$ and $\widehat{Q}_L$ defined in the Appendix.

The investment thresholds of the entrant described above as well as the entrant's strategies for different capacity levels of the incumbent are illustrated in the following Figure 5. The dark gray areas in this figure represent the combinations of X and Q_L such that the entrant only waits for an increase in X and then enters with the capacity level, indicated below the figure. The light gray area corresponds to hysteresis region, where the the entrant waits either for a decline in market profitability or an increase and enters at either of the two investment thresholds. In the white parts of the graph where the entrant invests immediately with the capacity level indicated above the figure.

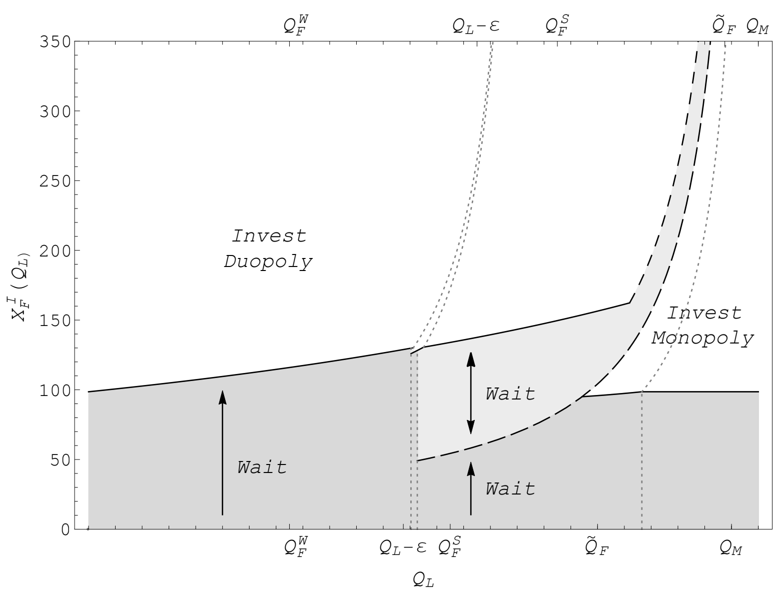


Figure 5: *The optimal investment strategy of the the entrant for the set of parameter values: $r = 0.05$, $\alpha = 0.01$, $\sigma = 0.17$, $\eta = 1$, $\delta = 100$, and $c = 20$.*

Interesting result arises when we allow α, a constant drift in the Brownian motion, to be negative. In this case we observe non-monotonicity in the entrant's duopoly threshold. If the capacity level of the incumbent is low and, thus exit is still far away, the duopoly threshold increases with the incumbent's capacity as in Figure 5. However, once the incumbent's capacity becomes sufficiently large the entrant chooses to be a smaller firm in order to exit last. At the same this means that the exit of the larger firm, the incumbent, becomes closer. Anticipating sooner bankruptcy of its rival the entrant as an incentive to enter the market sooner and its investment threshold decreases. This situation is illustrated in the figure below.

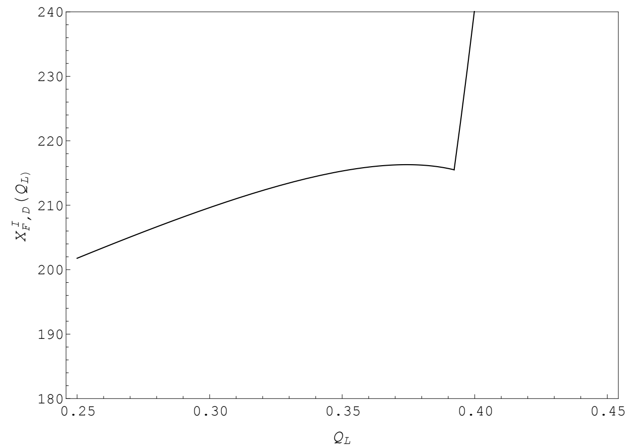
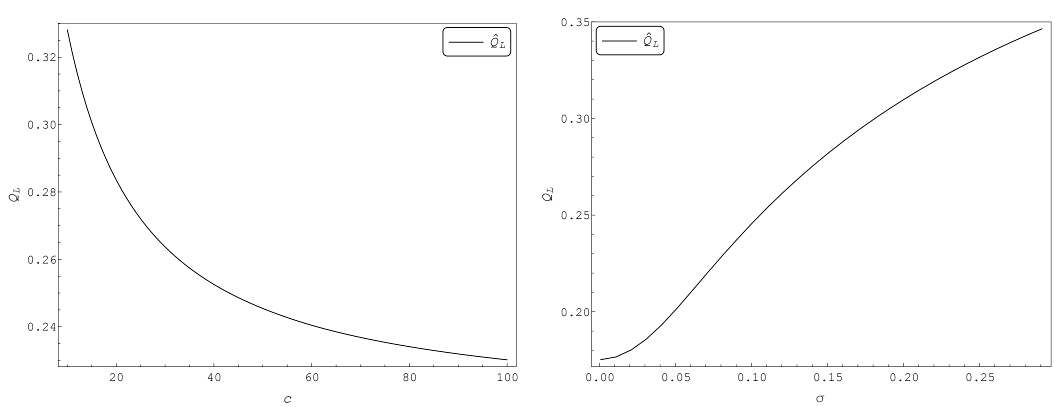


Figure 6: *The optimal investment strategy of the the entrant for the set of parameter values:* $r = 0.05$, $\alpha = -0.03$, $\sigma = 0.17$, $\eta = 1$, $\delta = 100$, *and* $c = 20$.

In general, the entrant considers monopolization strategy only if the capacity of the incumbent exceeds a certain threshold. As stated in Proposition 7 the entrant never has an incentive to become a monopolist if the incumbent's capacity is such that $Q_L < \widehat{Q}_L$. This is because given that the market share of the opponent is small, in order to induce monopoly scenario the entrant either needs to wait until the market is is low enough or to install a large enough capacity. In both cases the entrant exits rather soon itself. Therefore, for the small Q_L the entrant prefers to extract greater duopoly rents to becoming a monopolist for a short period. The capacity level $\widehat{Q}_L$, thus, represents the minimal value of the incumbent's capacity for which in the small market benefits of monopolization outweigh the disadvantages of the sooner exit, or in other words, the first $\widehat{Q}_L$ for which the hysteresis region occurs. Figures 7 and 8 illustrate how the capacity level $\widehat{Q}_L$ changes with respect to different parameter values.

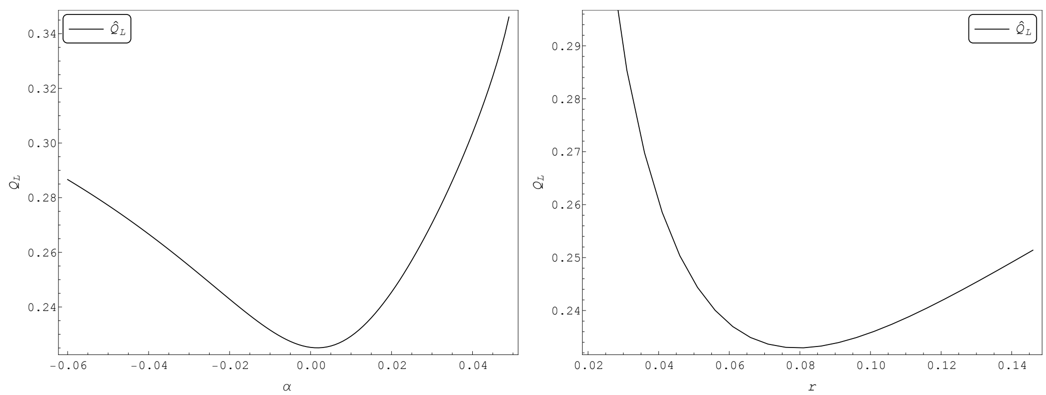


(a) $\sigma = 0.1$.

(b) $c = 50$.

Figure 7: *The capacity level* $\widehat{Q}_L$ *for the set of parameter values* $r = 0.05$, $\alpha = 0.02$, $\eta = 1$, $\delta = 100$ *and different values of* c *and* σ.

As it can be seen, an increase in production costs, c, results in a smaller level of $\widehat{Q}_L$. This is because larger production costs increase the exit triggers of the firms, consequently smaller capacity of the incumbent is needed to ensure that monopolization is profitable. An increase in market uncertainty, σ has an opposite effect. The standard result in the real options literature (see e.g. Dixit and Pindyck (1994)) is that the firms delay their decisions for higher uncertainty. In particular, the decision to exit the market is delayed, that is why larger σ implies that the larger capacity of the incumbent is needed to trigger the entrant's monopolization strategy. The effects of α and r are non-monotonic as it can be see from the next figure.



(a) $r = 0.05$.

(b) $\alpha = 0.02$.

Figure 8: *The capacity level* $\widehat{Q}_L$ *for the set of parameter values* $\sigma = 0.1$, $\eta = 1$, $c = 50$, $\delta = 100$ *and different values of* r *and* α.

Consider now the effect of a change in a drift constant in Brownian motion, α. When α is positive the firms expect the market to grow in the future and, as a result, to move away from exit threshold. Therefore, as α increases the monopolization strategy brings less benefits for the entrant and $\widehat{Q}_L$ increases. When α is negative, this means on the one hand that both firms expect to exit the market soon, while the less negative α becomes, the longer the monopoly period of the entrant is anticipated. Thus, we can see a decline in $\widehat{Q}_L$ for the negative α, because the monopolization strategy becomes more attractive. The similar type of non-monotonicity is to be observed considering the effect of the discount rate if α is positive. On the one hand, r is large the firms discount their future payoffs more heavily, or put differently, carry more about the present rather than the exit decisions in the future. Thus, the entrant needs a larger capacity installed by the incumbent to consider the monopoly scenario. On the other hand, for relatively small α another effect comes into the picture. Namely, the exit trigger of the incumbent increases and it becomes easier to drive it out of the market, i.e. less capacity is needed. Note, that for negative α the latter effect disappears as that would mean that the entrant expects to exit sooner itself and to stay for a shorter period in the monopoly. As a result, for negative α the capacity level $\widehat{Q}_L$ as a function of the discount rate exhibits only increasing behavior.

4 Conclusion

The paper examines the entry and exit decisions of the firm in the existing market under uncertainty. In the presence of an incumbent the entrant launches its market operations by undertaking an investment in a certain capacity. In our model the entrant decides not only upon its optimal investment threshold but also upon the exit threshold and its optimal capacity level. Thus, the duopoly model with capacity optimization was modified to incorporate an option to exit the market. In this way the entrant, while observing the existing quantity in the market installed by the incumbent, can use the second mover advantage in choosing capacity to influence the exit order. We show that the firm with the larger capacity exits last. Thus, in order to stay longer in the market the entrant has to choose a capacity below the incumbent's level. As a result, new strategies are available for the entrant in terms of its capacity choice. In particular, it can chose to mimic the behavior of the incumbent and install capacity that is just an ε below the incumbent's capacity. This happens when extracting higher rents by installing larger capacity is not possible due to the capacity constrained induced by the exit order. On the other hand, when the incumbent's capacity is large enough the constraint becomes redundant and the entrant is able to boost its capacity such that the incumbent exits immediately. The latter is crucial for the main result of our paper. Namely, that in contrast with the basic model the entrant has multiple investment thresholds. Now it not only has an option to enter as a duopolist but also to monopolize the market by forcing the incumbent out. The first situation appears when the market is big enough for the firms to coexist. The second scenario occurs when the market is sufficiently small, so that it is relatively easy to drive the competitor out by installing a large enough capacity. However, for an intermediate market size a gap between the two strategies is generated. We call this gap the hysteresis region. Intuitively, within this region the market is too small to coexist in a duopoly, yet too big to make monopolization of the market profitable for the entrant. Furthermore, for negative market growth prospects the entrant's investment trigger exhibits a non-monotonicity with respect to the capacity of the incumbent. At first the investment trigger increases with the incumbent's capacity, however, once the incumbent's capacity becomes sufficiently large, it starts declining, as the entrant anticipates sooner exit of the incumbent and is eager to invest sooner.

Lastly, it is important to indicate the possibilities for further research. This paper is focused only on the decisions of the entrant entering an existing market. However, it is interesting to examine the case of a new market where both firms have an option to invest. In this way we include the decision of the incumbent in the analysis. In addition, the obtained results are derived for the specific case when firms produce up to capacity. This assumption can be relaxed by allowing the firms to leave some capacity idle when the demand level decreases. Moreover, different demand functions could be considered. The firms facing the multiplicative demand function cannot freely increase their capacity as market profitability grows because it will result in negative prices. Therefore, the presence of an exit option may affect optimal investment decisions differently in the markets described by alternative demand structures with unlimited capacity expansion.

5 Appendix

Proof of Proposition 1 Following Dixit and Pindyck (1994) we define the values of active firm, $V_0^M(X,Q)$, and idle firm, $V_1^M(X,Q)$, as follows

$$V_0^M(X,Q) = A_1 X^{\beta_1} + A_2 X^{\beta_2}, \tag{35}$$

$$V_1^M(X,Q) = B_1 X^{\beta_1} + B_2 X^{\beta_2} + \frac{X(1-\eta Q)Q}{r-\alpha} - \frac{cQ}{r}, \tag{36}$$

with A_1, A_2, B_1 and B_2 being constants, β_1, β_2 – the roots of the fundamental quadratic equation [6]. To rule out the possibility of speculative bubbles it should hold that $A_2 = 0$, $B_1 = 0$, thus, we can rewrite (35) and (36) as

$$V_0^M(X,Q) = A_1 X^{\beta_1}, \tag{37}$$

$$V_1^M(X,Q) = B_2 X^{\beta_2} + \frac{X(1-\eta Q)Q}{r-\alpha} - \frac{cQ}{r}. \tag{38}$$

Consider optimal exit threshold, X_M^E, and optimal investment threshold, X_M^I. The following boundary conditions must hold

$$\begin{cases} V_1^M(X^I,Q)) - \delta Q = V_0^M(X^I,Q), \\ \left.\dfrac{\partial V_1^M(X,Q)}{\partial X}\right|_{X=X_M^I} + \dfrac{\partial V_1(X_M^I,Q)}{\partial Q}\left.\dfrac{\partial Q(X)}{\partial X}\right|_{X=X_M^I} = \left.\dfrac{\partial V_0^M(X,Q)}{\partial X}\right|_{X=X_M^I}, \\ V_1^M(X^E,Q) = 0, \\ \left.\dfrac{\partial V_1^M(X,Q)}{\partial X}\right|_{X=X_D^E} = 0. \end{cases} \tag{39}$$

Plugging in the values of the idle and active monopolist from (37) and (38) we get[7]

$$\begin{cases} -A_1 X_M^{I\beta_1} + B_2 X_M^{I\beta_2} + \dfrac{X_M^I(1-\eta Q)Q}{r-\alpha} - \dfrac{cQ}{r} - \delta Q = 0, \\ -\beta_1 A_1 X_M^{I\beta_1 - 1} + \beta_2 B_2 X_M^{I\beta_2 - 1} + \dfrac{(1-\eta Q)Q}{r-\alpha} = 0, \\ B_2 X_M^{E\beta_2} + \dfrac{X_M^E(1-\eta Q)Q}{r-\alpha} - \dfrac{cQ}{r} = 0, \\ \beta_2 B_2 X_M^{E\beta_2 - 1} + \dfrac{(1-\eta Q)Q}{r-\alpha} = 0. \end{cases} \tag{40}$$

[6] $\frac{1}{2}\sigma^2\beta(\beta-1) + \alpha\beta - r = 0$

[7] Given that $\frac{\partial V_1(X,Q)}{\partial Q} = 0$ for the optimal Q. See proof of Proposition 2.

Solving for A_1, B_2, X_M^I and X_M^E leads to

$$\begin{cases} \frac{(\beta_1-\beta_2)c}{(1-\beta_2)\beta_1 r}\left(\frac{X^I}{X_M^E(Q)}\right)^{\beta_2} + \left(1-\frac{1}{\beta_1}\right)\frac{X^I(1-\eta Q)}{r-\alpha} - \frac{c}{r} - \delta = 0, \\ A_1(Q) = \frac{\beta_2}{\beta_2-\beta_1}\left(\frac{1}{X_M^I}\right)^{\beta 1}\left(\frac{X^I(1-\eta Q)Q}{r-\alpha}\left(1-\frac{1}{\beta_2}\right) - \left(\frac{c}{r}+\delta\right)Q\right), \\ X_M^E(Q) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-\eta Q)}, \\ B_2(Q) = \left(\frac{1}{X_M^E(Q)}\right)^{\beta_2}\frac{cQ}{r(1-\beta_2)}, \end{cases} \tag{41}$$

and the corresponding values of the idle and active monopolist:

$$V_0^M(X,Q) = A_1(Q)X^{\beta_1}, \tag{42}$$

$$V_1^M(X,Q) = \left(\frac{X}{X_M^E(Q)}\right)^{\beta_2}\frac{cQ}{r(1-\beta_2)} + \frac{X(1-\eta Q)Q}{r-\alpha} - \frac{cQ}{r}. \tag{43}$$

Proof of Proposition 2 The monopolist maximizes the value of being active on the market:

$$\begin{aligned} \underset{Q}{\text{maximize}} \quad & V_1^M(X,Q) - \delta Q \\ \text{s.t.} \quad & Q < \tilde{Q}_M(X). \end{aligned}$$

We now want to show that this function has a single maximum in the feasible region of the investment problem. We start by considering the first and the second order conditions defined below:

$$\frac{\partial(V_1^M(X,Q)-\delta Q)}{\partial Q} = \left(\frac{X}{X_M^E(Q)}\right)^{\beta_2}\frac{c\,(1-\eta Q(1+\beta_2))}{r(1-\beta_2)(1-\eta Q)} + \frac{X(1-2\eta Q)}{r-\alpha} - \frac{c}{r} - \delta, \tag{44}$$

$$\frac{\partial^2(V_1(X,Q)-\delta Q)}{\partial Q^2} = \left(\frac{r(\beta_2-1)}{\beta_2 c}\right)^{\beta_2-1}\left(\frac{X}{r-\alpha}\right)^{\beta_2}\frac{\eta(2-(\beta_2+1)\eta Q)}{(1-\eta Q)^{2-\beta_2}} - \frac{2\eta X}{r-\alpha}. \tag{45}$$

In order for the firm to enter the market it should hold that $X > X_M^E(Q)$ otherwise it will immediately exit. Thus, the necessary condition for this problem is $X > X_M^E(0) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)}$. Note that given this and the fact that $\beta_2 < 0$ it holds that $\left.\frac{\partial^2(V_1(X,Q)-\delta Q)}{\partial Q^2}\right|_{Q=0} = \frac{2\eta X}{r-\alpha}\left(\left(\frac{r(\beta_2-1)X}{\beta_2 c(r-\alpha)}\right)^{\beta_2-1} - 1\right) < 0$. Moreover, $\lim_{Q\to\frac{1}{\eta}}\frac{\partial^2(V_1(X,Q)-\delta Q)}{\partial Q^2} = \infty$.[8] As $\frac{\partial^3(V_1(X,Q)-\delta Q)}{\partial Q^3} = \left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{\beta_2}\frac{c\eta^2\beta_2(-3+(\beta_2+1)\eta Q)}{r(1-\eta Q)^{3-\beta_2}} > 0$, we can conclude that the second order condition is an increasing function in Q with a single root. This means that the first order condition is first declining with Q, reaches it's minimum and then starts increasing. Translating the condition $X > X_M^E(Q)$ in terms of capacity level it should hold

[8] Capacity is defined such that $0 \le Q \le \frac{1}{\eta}$ so that the prices cannot be negative.

that $0 \le Q < \tilde{Q}_M(X)$. Note that by construction $\left.\frac{\partial(V_1(X,Q)-\delta Q)}{\partial Q}\right|_{Q=\tilde{Q}_M(X)} = -\delta < 0$. This implies that there exist two possibilities depending on the sign of $\left.\frac{\partial(V_1(X,Q)-\delta Q)}{\partial Q}\right|_{Q=0}$: either the function $V_1^M(X,Q)-\delta Q$, which takes the value of zero for zero capacity, is first increasing and then decreasing or is strictly decreasing for the considered range of Q. In the latter scenario the optimal capacity choice is 0, meaning that the firm will forgo its investment option. In the former scenario there exists a single maximum defined by the first order condition.[9]

The resulting optimal capacity level $Q_M^*(X)$ is determined by[10]

$$\left(\frac{X}{X_M^E(Q)}\right)^{\beta_2} \frac{c\,(1-\eta Q(1+\beta_2))}{r(1-\beta_2)(1-\eta Q)} + \frac{X(1-2\eta Q)}{r-\alpha} - \frac{c}{r} - \delta = 0. \tag{46}$$

Optimal capacity level at the investment threshold can be found by solving the following system:

$$\begin{cases} \dfrac{(\beta_1-\beta_2)c}{(1-\beta_2)\beta_1 r}\left(\dfrac{X}{X_M^E(Q)}\right)^{\beta_2} + \left(1-\dfrac{1}{\beta_1}\right)\dfrac{X(1-\eta Q)}{r-\alpha} - \dfrac{c}{r} - \delta = 0, \\ \left(\dfrac{X}{X_M^E(Q)}\right)^{\beta_2} \dfrac{c\,(1-\eta Q(1+\beta_2))}{r(1-\beta_2)(1-\eta Q)} + \dfrac{X(1-2\eta Q)}{r-\alpha} - \dfrac{c}{r} - \delta = 0. \end{cases} \tag{47}$$

$$\frac{\beta_2-1}{\beta_2-\beta_1}\frac{X(1-\eta Q(1+\beta_1))}{r-\alpha} + \frac{\beta_2}{\beta_1-\beta_2}\left(\frac{c}{r}+\delta\right)\left(\frac{1-\eta Q(1+\beta_1)}{1-\eta Q}\right) = 0. \tag{48}$$

From (48) it holds that either $Q_M^* = \dfrac{1}{\eta(\beta_1+1)}$ or $X^I(Q) = \dfrac{\beta_2(r-\alpha)}{(\beta_2-1)(1-\eta Q)}\left(\dfrac{c}{r}+\delta\right)$. Plugging the latter back into (47) and solving for Q gives $Q^* = \dfrac{1}{\eta(\beta_2+1)}$. If $\beta_2 < -1$ this gives negative capacity, whereas the complementary case when $\beta_2 > -1$, leads to the negative prices, $P = \dfrac{X\beta_2}{\beta_2+1} < 0$. Therefore, we conclude that

$$Q_M^* = \frac{1}{\eta(\beta_1+1)}. \tag{49}$$

The corresponding X_M^{I*} is implicitly defined by[11]

$$-\left(\frac{\beta_1(\beta_2-1)rX}{(\beta_1+1)\beta_2(r-\alpha)c}\right)^{\beta_2}\frac{(\beta_1-\beta_2)c}{\beta_1(\beta_2-1)r} + \frac{(\beta_1-1)X}{(\beta_1+1)(r-\alpha)} - \left(\frac{c}{r}+\delta\right) = 0. \tag{50}$$

[9] This also ensures that the smooth pasting condition is correctly specified as $\frac{\partial V_1(X_M^I,Q)}{\partial Q} = 0$ for the optimal Q.

[10] Given that the second order condition for maximum, $\left.\frac{\partial^2(V_1(X,Q)-\delta Q)}{\partial Q^2}\right|_{Q=Q_M^*(X)} < 0$, is satisfied.

[11] We choose the root such that $X > \frac{(\beta_1+1)\beta_2(r-\alpha)}{\beta_1(\beta_2-1)}\left(\frac{c}{r}+\delta\right)$, otherwise A is negative.

The optimal exit threshold $X_M^{E^*}$ is given by

$$X_M^{E^*} = \frac{\beta_2(\beta_1+1)(r-\alpha)c}{\beta_1(\beta_2-1)r}. \tag{51}$$

Moreover,

$$X^{I^*} = X^{E^*}\phi(r,\alpha,\sigma,c,u,\delta), \tag{52}$$

where $\phi(r,\alpha,\sigma,c,u,\delta)$ is implicitly determined by

$$-\phi^{\beta_2}\frac{(\beta_1-\beta_2)}{\beta_2} + (\beta_1-1)\phi - \frac{\left(\frac{c}{r}+\delta\right)\beta_1(\beta_2-1)}{\frac{c}{r}\beta_2} = 0. \tag{53}$$

Proof of Proposition 3 Similarly to the monopoly case, the value of the firm i, which constitutes a duopoly with firm j and possesses an exit option is given by

$$V_1^D(X,Q_i,Q_j) = B_iX^{\beta_2} + \frac{X(1-\eta(Q_i+Q_j))Q_i}{r-\alpha} - \frac{cQ_i}{r}, \tag{54}$$

Applying the boundary conditions

$$\begin{cases} V_1^D(X_D^E,Q_i,Q_j) = 0, \\ \left.\dfrac{\partial V_1^D(X,Q_i,Q_j)}{\partial X}\right|_{X=X_D^E} = 0, \end{cases} \tag{55}$$

we obtain the following duopoly exit threshold

$$X_D^E(Q_i,Q_j) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-\eta(Q_i+Q_j))}. \tag{56}$$

As it can be seen the capacity levels of the firms i and j enter the expression for the exit threshold only as a sum. Thus, we can conclude that both firms have the same duopoly exit threshold. Yet the monopoly thresholds are different if $Q_i \neq Q_j$. This can be seen from the expression for the monopoly threshold (57), which is derived using (41):

$$X_M^E(Q_i) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-\eta Q_i)}. \tag{57}$$

Moreover, if $Q_i > Q_j$, then $X_M^E(Q_i) > X_M^E(Q_j)$ and visa versa. It is now easy to show that if $Q_i > Q_j$ the firm i will always exit first at the duopoly threshold. This scenario is indeed an equilibrium, as if j exits at the duopoly threshold, $X_D^E(Q_i,Q_j)$, it is optimal for firm i to leave once X hits $X_M^E(Q_i)$. The opposite scenario, however, is not an equilibrium. This is because if j exits at its monopoly threshold, $X_M^E(Q_j)$, there is still the region where firm i prefers to stay on the market and get monopoly profits, namely, when $X \in (X_M^E(Q_i), X_M^E(Q_j)]$. Hence, the firm with a larger capacity exits market first. Applying this result for the incumbent-entrant setting we obtain (14) and (15). Note that when the firms are of the same size, $Q_L = Q_F$, both strategies are equilibrium strategies, and as a result, in a symmetric game it is unclear which firm exits first.

Proof of Proposition 4 Assuming that the incumbent has already entered the market, we obtain the values of active and idle entrant similarly to the monopoly case:

$$F_0(X) = A_F X^{\beta_1}, \tag{58}$$

$$F_1(X, Q_L, Q_F) = B_F(Q_L, Q_F) X^{\beta_2} + \frac{X(1 - \eta(Q_F + Q_L))Q_F}{r - \alpha} - \frac{cQ_F}{r}. \tag{59}$$

For the investment problem the following boundary conditions must hold

$$\begin{cases} F_1(X_F^I, Q_L, Q_F) - \delta Q_F = F_0(X_F^I, Q_L, Q_F), \\ \left(\dfrac{\partial F_1(X, Q_L, Q_F)}{\partial X} + \dfrac{\partial \left(F_1(X, Q_L, Q_F) - \delta Q_F\right)}{\partial Q_F} \dfrac{\partial Q_F(X)}{\partial X} \right) \Bigg|_{X = X_F^I} = \dfrac{\partial F_0(X, Q_L, Q_F)}{\partial X} \Bigg|_{X = X_F^I}, \end{cases} \tag{60}$$

which can be written as

$$\begin{cases} -A_F X_F^{I\beta_1} + B_F X_F^{I\beta_2} & + \dfrac{X_F^I(1 - \eta(Q_F + Q_L))Q_F}{r - \alpha} - \dfrac{cQ_F}{r} - \delta Q_F = 0, \\ -\beta_1 A_F X_F^{I\beta_1 - 1} + \beta_2 B_F X_F^{I\beta_2 - 1} & + \dfrac{(1 - \eta(Q_F + Q_L))Q_F}{r - \alpha} \\ & + \left(\dfrac{\partial F_1(X_F^I, Q_L, Q_F)}{\partial Q_F} - \delta \right) \dfrac{\partial Q_F(X)}{\partial X} \Bigg|_{X = X_F^I} = 0, \end{cases} \tag{61}$$

with $\dfrac{\partial F_1(X_F^I, Q_L, Q_F)}{\partial Q_F} = \dfrac{\partial B_F(Q_L, Q_F)}{\partial Q_F} + \dfrac{X_F^I(1 - \eta(2Q_F + Q_L))}{r - \alpha} - \dfrac{c}{r}.$

Solving the above system we obtain

$$\begin{aligned} A_F(Q_L, Q_F) = \frac{\beta_2}{\beta_2 - \beta_1} \left(\frac{1}{X_F^I} \right)^{\beta_1} \Bigg(& \frac{X_F^I(1 - \eta(Q_L + Q_F))Q_F}{r - \alpha} \left(1 - \frac{1}{\beta_2} \right) - \left(\frac{c}{r} + \delta \right) Q_F \\ & - \frac{X_F^I}{\beta_2} \left(\frac{\partial F_1(X_F^I, Q_L, Q_F)}{\partial Q_F} - \delta \right) \frac{\partial Q_F(X)}{\partial X} \Bigg|_{X = X_F^I} \Bigg), \end{aligned} \tag{62}$$

and the investment threshold of the entrant, X_F^I, is found by solving

$$\begin{aligned} \left(1 - \frac{\beta_2}{\beta_1} \right) \frac{B_F(Q_L, Q_F)}{Q_F} X_F^{I\beta_2} + \left(1 - \frac{1}{\beta_1} \right) \frac{X_F^I(1 - \eta(Q_L + Q_F))}{r - \alpha} - \frac{c}{r} - \delta & \\ - \frac{X_F^I}{\beta_1 Q_F} \left(\frac{\partial F_1(X_F^I, Q_L, Q_F)}{\partial Q_F} - \delta \right) \frac{\partial Q_F(X)}{\partial X} \Bigg|_{X = X_F^I} & = 0, \end{aligned} \tag{63}$$

where B_F can be inferred from the exit problem.

If the entrant is a larger firm it will leave the market once X hits $X_D^E(Q_L, Q_F)$, which is the optimal exit threshold for a large firm in a duopoly. Thus, it must hold that

$$F_1(X_D^E(Q_L, Q_F), Q_F) = 0, \tag{64}$$

which after plugging in (69) and solving for B_F yields

$$B_F^B(Q_L, Q_F) = \left(\frac{1}{X_D^E(Q_L, Q_F)}\right)^{\beta_2} \frac{cQ_F}{r(1-\beta_2)}. \tag{65}$$

In the complementary case, when the entrant is a smaller firm, it becomes a monopolist as X hits $X_D^E(Q_L, Q_F)$, and then exits at $X_M^E(Q_F)$ in case X declines further. Hence, the following condition must be satisfied

$$F_1(X_D^E(Q_L, Q_F), Q_F) = V_1(X_D^E(Q_L, Q_F), Q_F), \tag{66}$$

where $V_1(X, Q)$ is given by (43). This gives the following expression for B_F

$$B_F^S(Q_L, Q_F) = \left(\frac{1}{X_M^E(Q_F)}\right)^{\beta_2} \frac{cQ_F}{r(1-\beta_2)} + \left(\frac{1}{X_D^E(Q_L, Q_F)}\right)^{\beta_2} \frac{X_D^E(Q_L, Q_F)\eta Q_L Q_F}{r-\alpha}. \tag{67}$$

Let λ be the probability that the entrant exits first in a symmetric game, then

$$B_F(Q_L, Q_F) = B_F^B \chi_{\{Q_F > Q_L\}} + (\lambda B_F^B + (1-\lambda) B_F^S)\chi_{\{Q_F = Q_L\}} + B_F^S \chi_{\{Q_F < Q_L\}}, \tag{68}$$

where $\chi_{\{\text{true}\}} = 1$ and $\chi_{\{\text{false}\}} = 0$.

Proof of Proposition 5 As it follows from Proposition 4 the value of the active entrant is

$$F_1(X, Q_F, Q_L) = B_F X^{\beta_2} + \frac{X(1-\eta(Q_F+Q_L))Q_F}{r-\alpha} - \frac{cQ_F}{r}. \tag{69}$$

To to obtain the value of the large entrant we plug in B_F^B from (65) in the above equation, for the value of the stong floower – and B_F^S from (67), which gives

$$F_1^B(X, Q_L, Q_F) = \left(\frac{X}{X_D^E(Q_L, Q_F)}\right)^{\beta_2} \frac{cQ_F}{r(1-\beta_2)} + \frac{X(1-\eta(Q_L+Q_F))Q_F}{r-\alpha} - \frac{cQ_F}{r}, \tag{70}$$

$$\begin{aligned} F_1^S(X, Q_L, Q_F) = {} & \left(\frac{X}{X_M^E(Q_F)}\right)^{\beta_2} \frac{cQ_F}{r(1-\beta_2)} + \left(\frac{X}{X_D^E(Q_L, Q_F)}\right)^{\beta_2} \frac{X_D^E(Q_L, Q_F)\eta Q_L Q_F}{r-\alpha} \\ & + \frac{XQ_F(1-\eta(Q_L+Q_F))}{r-\alpha} - \frac{cQ_F}{r}. \end{aligned} \tag{71}$$

The value of the symmetric game, when the incumbent and the entrant are of the same size, depends on the probability that either of the firms will leave the market first. For the entrant in this case it is a weighted average of the two scenarios, F_1^B and F_1^S, with respect to the probability of leaving the market first given that the incumbent has already invested (denoted earlier by λ). This yields the value for the active entrant determined by

$$F_1(X,Q_L,Q_F)=\begin{cases} F_1^B(X,Q_L,Q_F) & \text{if } Q_F>Q_L,\\ \\ F_1^S(X,Q_L,Q_F) & \text{if } Q_F<Q_L,\\ \\ \lambda F_1^B(X,Q_L,Q_F)+(1-\lambda)F_1^S(X,Q_L,Q_F) & \text{if } Q_F=Q_L,\end{cases} \tag{72}$$

which is equivalent to

$$F_1(X,Q_L,Q_F)=F_1^B\chi_{\{Q_F>Q_L\}}+(\lambda F_1^B+(1-\lambda)F_1^S)\chi_{\{Q_F=Q_L\}}+F_1^S\chi_{\{Q_F<Q_L\}}, \tag{73}$$

with χ and λ defined earlier.

Proof of Proposition 6 First, it is possible to show that $F_1^S(X,Q_L,Q_F)=F_1^B(X,Q_L,Q_F)$ if $X\to\infty$. Otherwise it holds that

$$\begin{aligned} F_1^S(X,Q_L,Q_F)-F_1^B(X,Q_L,Q_F)=&\frac{cQ_F}{r(1-\beta_2)}\left(\frac{X}{X_D^E(Q_L,Q_F)}\right)^{\beta_2}\Bigg[\left(\frac{1-\eta Q_F}{1-\eta(Q_L+Q_F)}\right)^{\beta_2}-1\\ &-\frac{\eta Q_L\beta_2}{1-\eta(Q_L+Q_F)}\Bigg]. \end{aligned} \tag{74}$$

For $\beta_2<0$ and $Q_F>0$, the first two multipliers in the difference are positive, thus, it has the same the sign as the expression in the square brackets, which we denote by $g(Q_L)$. Note first that $g(0)=0$. Moreover, taking a derivative with respect to Q_L gives

$$\frac{\partial g(Q_L)}{\partial Q_L}=\frac{-\beta_2\eta(1-\eta Q_F)}{(1-\eta(Q_F+Q_L))^2}\left(1-\left(\frac{1-\eta Q_F}{1-\eta(Q_F+Q_L)}\right)^{\beta_2-1}\right)>0, \tag{75}$$

because $\beta_2-1<-1$ and $\frac{1-\eta Q_F}{1-\eta(Q_L+Q_F)}>1$. Given this the value of the symmetric game, being a weighted average of $F_1^B(X,Q_L,Q_L)$ and $F_1^S(X,Q_L,Q_L)$, is always smaller or equal than the value of the small firm. Thus, for some positive probability to exit first in a symmetric game, λ, it is always possible to find an ε small enough to ensure that installing a capacity $Q_L-\varepsilon$ and, hence, becoming a small firm, brings a larger value.

Case 1: big follower. Analogous to the monopoly case the follower curve $F_1^B(X,Q_L,Q_F)$ as a function of its capacity level, Q_F, for $Q_F<\tilde{Q}_F(X,Q_L)$[12] and can be proved to have a single maximum[13], which is defined by the following first order condition

$$\frac{\partial(F_1^B(X,Q_L,Q_F)-\delta Q_F)}{\partial Q_F}=0, \tag{76}$$

[12]The case $Q_F\geq\tilde{Q}_F(X,Q_L)$ is not relevant for $F_1^B(X,Q_L,Q_F)$, because then the large firm exits the market and its value is equal to 0.

[13]The proof is completely analogous to the proof in Proposition 2 for the monopoly case.

or rewritten

$$\left(\frac{X}{X_D^E(Q_L,Q_F)}\right)^{\beta_2}\frac{c}{r(1-\beta_2)}\left(1-\frac{\beta_2\eta Q_F}{1-\eta(Q_F+Q_L)}\right)+\frac{X(1-\eta Q_L-2\eta Q_F)}{r-\alpha}-\frac{c}{r}-\delta=0. \quad (77)$$

We denote the capacity level corresponding to (77) by Q_F^B.

Case 2: small follower. The value of the small follower, $F_1^S(X,Q_L,Q_F)-\delta Q_F$, is no longer a unimodal function of its capacity choice. Its shape and, as a result, the location of the maximum may change depending on the parameter values. In order to describe the behavior of this function we need to determine the signs of its first and second order derivatives with respect to Q_F defined below:

$$\begin{aligned}\frac{\partial(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{\partial Q_F} &= \left(\frac{X}{X_M^E(Q_F)}\right)^{\beta_2}\frac{c(1-\eta Q_F(\beta_2+1))}{r(\beta_2-1)(1-\eta Q_F)}+\frac{X\,(1-\eta Q_L-2\eta Q_F)}{r-\alpha}-\frac{c}{r}-\delta\\ &- \left(\frac{X}{X_D^E(Q_L,Q_F)}\right)^{\beta_2}\frac{c\beta_2\eta(1-\eta(\beta_2 Q_F+Q_L))}{r(\beta_2-1)(1-\eta(Q_F+Q_L))^2}. \end{aligned} \quad (78)$$

$$\begin{aligned}\frac{\partial^2(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{(\partial Q_F)^2} &= \left(\frac{X}{X_M^E(Q_F)}\right)^{\beta_2}\frac{\beta_2 c\eta(2-(\beta_2+1)\eta Q_F)}{r(\beta_2-1)(1-\eta Q_F)^2}-\frac{2\eta X}{r-\alpha}\\ &- \left(\frac{X}{X_D^E(Q_L,Q_F)}\right)^{\beta_2}\frac{\beta_2 c\eta^2 Q_L(2-2\eta Q_L-\beta_2 Q_F)}{r(1-\eta(Q_F+Q_L))^3}. \end{aligned} \quad (79)$$

The sign of the above expressions cannot be uniquely determined. However, it is possible to describe the behavior of these two functions for the different parameter values. First, we can show that (79) is a strictly increasing function of Q_F. Consider the derivative $\frac{\partial^3(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{(\partial Q_F)^3}$ given by expression (80). It is always positive for $X>0$ and $Q_L>0$, because $\beta_2<0$ and the price is non-negative, $1-\eta(Q_L+Q_F)\geq 0$:

$$\frac{c\beta_2\eta^2}{r}\left(\frac{X}{X_M^E(Q_F)}\right)^{\beta_2}\left(\frac{(\beta_2+1)\eta Q_L-3}{(1-\eta Q_L)^3}+\frac{(2-\beta_2)\eta Q_L(\beta_2\eta Q_L-3(1-\eta Q_L))}{(1-\eta(Q_L+Q_F))^{4-\beta_2}(1-\eta Q_L)^{\beta_2}}\right)\geq 0. \quad (80)$$

Moreover, rewriting (79) as

$$\left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{\beta_2}\frac{\beta_2 c\eta}{(\beta_2-1)r}\left(\frac{2-(\beta_2+1)\eta Q_F}{(1-\eta Q_F)^{2-\beta_2}}-\frac{(\beta_2-1)\eta Q_L(2-\beta_2\eta Q_F-2\eta Q_L)}{1-\eta Q_F-\eta Q_L)^{3-\beta_2}}\right)-\frac{2\eta X}{r-\alpha}, \quad (81)$$

it can be seen that $\lim_{Q_F\to-\infty}\frac{\partial^2(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{(\partial Q_F)^2}=-\infty$ and $\lim_{Q_F\to\frac{1}{\eta}}\frac{\partial^2(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{(\partial Q_F)^2}=\infty$. This means the second order derivative (79) always has a single root in the interval $(-\infty,\frac{1}{\eta}]$. Thus, we can conclude that the first order derivative (78) is a convex function of Q_F with a single minimum reached in $Q_F\in(-\infty,\frac{1}{\eta}]$. Such function in general may have either two roots (when its minimum is smaller than zero), one root (when its minimum value is exactly zero) or none (when the minimum value is positive). On the other hand, the negative roots will not affect the shape of the value function as it is only defined for $Q_F \geq 0$. Thus, the sign of the value function to a large extent depends on the location of the minimum of its fist order derivative. We will now demonstrate how it changes as we increase Q_L and/or X.

First, we can show that the first order derivative (78) evaluated at its minimum is an increasing function of Q_L for $Q_F \geq 0$ and decreasing for $Q_F < 0$. Consider first the derivative of $\frac{\partial(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{\partial Q_F}$ with respect to Q_L, which equals to

$$\frac{\eta X}{r-\alpha}\left(1-\frac{(\beta_2-1)\eta\left(Q_F(1-\eta Q_F-\beta_2\eta Q_L)+Q_L(1-\eta Q_L)\right)}{(1-\eta Q_F-\eta Q_L)^2}\right)\left(\frac{X}{X_D^E(Q_L,Q_F)}\right)^{\beta_2-1}-1. \tag{82}$$

At the minimum point the second order condition should be satisfied, i.e. (79) should be equal to zero. Dividing (79) by (-2) and adding it to (82) yields

$$\left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{\beta_2-1}\frac{\eta X((\beta_2+1)\eta Q_F-2)}{2(r-\alpha)(1-\eta Q_F)^{2-\beta_2}}+\frac{2(1-\eta(Q_F+Q_L))^2+(\beta_2-1)\eta Q_F(2\eta Q_F+\beta_2\eta Q_L-2)}{(1-\eta(Q_F+Q_L))^{3-\beta_2}}. \tag{83}$$

The sign of (83) is the same as (82) given that the latter is evaluated at $\underset{Q_F}{\operatorname{argmin}}\frac{\partial(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{\partial Q_F}$. We can demonstrate now that for $Q_F \geq 0$ the derivative of (83) with respect to Q_L is positive:

$$\frac{(\beta_2-2)(\beta_2-1)\eta\left(3\eta Q_F(1-\eta Q_F)-\beta_2\eta^2 Q_F Q_L\right)}{(1-\eta(Q_F+Q_L))^{4-\beta_2}}+\frac{2(1-\beta_2)\eta}{(1-\eta(Q_F+Q_L))^{2-\beta_2}}>0, \tag{84}$$

and that (83) evaluated at minimum $Q_L = 0$ is non-negative:

$$(\beta_2-1)\eta(-Q_F)(1-\eta Q_F)^{\beta_2-2}\geq 0. \tag{85}$$

This means that for $Q_F \geq 0$ the expression given by (83) is always positive and so is (82) evaluated at $\underset{Q_F}{\operatorname{argmin}}\frac{\partial(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{\partial Q_F}$. Note however that for $Q_F < 0$ the sign of the both (84) and (85) changes and the derivative of (83) with respect to Q_L becomes negative. Thus, we can conclude that as Q_L increases y-coordinate of the minimum of (78) as a function of Q_F is increasing for $Q_F \geq 0$ and decreases for $Q_F < 0$.

In order to determine how a change in Q_L affect the x-coordinate of the minimum we set the second order condition (81) to zero and apply the implicit function theorem. This gives

$$\frac{dQ_F}{dQ_L}=\frac{-\beta_2\eta Q_F(1-\eta Q_F)+\left((\beta_2-1)^2+1\right)\eta^2 Q_F Q_L+2(1-\eta Q_L)(1-\eta Q_F-(\beta_2-1)\eta Q_L)}{(1-\eta(Q_F+Q_L))^{4-\beta_2}\frac{(\beta_2+1)\eta Q_F-3}{(1-\eta Q_F)^{3-\beta_2}}+(\beta_2-2)\eta Q_L(3(1-\eta Q_L)-\beta_2\eta Q_F)}<0. \tag{86}$$

Hence, an increase in Q_L results into a decrease in the x-coordinate of the minimum of the first order derivate (78) together with an increase in its y-coordinate. This allows us to conclude that the largest possible value of the minimum of the first order derivative (78) is reached for $Q_F = 0$. This allows to determine the number of roots that the first order derivative has for different values of Q_L and X. Namely, if the second order condition is satisfied for $Q_F = 0$, i.e. $\frac{\mathrm{d}^2}{\mathrm{d}Q_F^2} F_1^S(X, Q_L, Q_F)\big|_{Q_F=0} = 0$, and at the same time

1. $\frac{\mathrm{d}}{\mathrm{d}Q_F} F_1^S(X, Q_L, Q_F)\big|_{Q_F=0} \leq 0$, then the first order derivative always has two roots (or one when it touches the x-axis),
2. $\frac{\mathrm{d}}{\mathrm{d}Q_F} F_1^S(X, Q_L, Q_F)\big|_{Q_F=0} > 0$, then the first order derivative has two roots for small Q_L (or one when it touches the x-axis) and none for large Q_L.

Now it is possible to find a specific value of market size, X, to distinguish these two scenarios. First, consider the derivative of $\left.\frac{\partial F_1^S(X,Q_L,Q_F)}{\partial Q_F}\right|_{Q_F=0}$ with respect to Q_L:

$$\frac{\mathrm{d}}{\mathrm{d}Q_L}\left(\left.\frac{\partial F_1^S(X, Q_L, Q_F)}{\partial Q_F}\right|_{Q_F=0}\right) = \frac{\eta X}{r-\alpha}\left(-1 + \left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{\beta_2-1}\frac{(1-\beta_2\eta Q_L)}{(1-\eta Q_L)^{2-\beta_2}}\right) \tag{87}$$

This function is clearly increasing in Q_L. Moreover, if $Q_L = 0$ its value is negative, while for $Q_L \to \frac{1}{\eta}$ it becomes infinitely large. Thus, $\left.\frac{\partial F_1^S(X,Q_L,Q_F)}{\partial Q_F}\right|_{Q_F=0}$ has a single minimum, that can be found by setting (87) to zero. This gives us the following expression $X = \left(\frac{1-\beta_2\eta Q_L}{1-\eta Q_L}\right)^{\frac{1}{1-\beta_2}}\frac{\beta_2 c(r-\alpha)}{(\beta_2-1)r(1-\eta Q_L)}$, which we can plug into $\frac{\mathrm{d}}{\mathrm{d}Q_F} F_1^S(X, Q_L, Q_F)\big|_{Q_F=0}$ and get its value at the minimum Q_L. Setting the obtained result to zero will give us the value of Q_L such that this function exactly touches the x-axis and, thus, has a single root:

$$\frac{\beta_2 c}{(\beta_2-1)r}\left(\frac{\eta Q_L}{1-\beta_2\eta Q_L} - \frac{1}{\beta_2(1-\beta_2\eta Q_L)(1-\eta Q_L)^{\beta_2-1}} + 1\right)\left(\frac{1-\beta_2\eta Q_L}{1-\eta Q_L}\right)^{\frac{1}{1-\beta_2}} - \frac{c}{r} - \delta = 0. \tag{88}$$

Note that the above expression is zero for $Q_L = 0$ and goes to infinity as $Q_L \to \frac{1}{\eta}$. Together with the fact that its derivative with respect to Q_L is positive, $\frac{\beta_2 c\eta(2-\beta_2\eta Q_L)\left(1-(1-\eta Q_L)^{1-\beta_2}\right)\left(\beta_2-\frac{\beta_2-1}{1-\eta Q_L}\right)^{\frac{1}{1-\beta_2}}}{(\beta_2-1)r(1-\eta Q_L)(1-\beta_2\eta Q_L)^2} > 0$, it allows to conclude that the solution of (88) is unique. As a result there exists a unique $X = \left(\frac{1-\beta_2\eta Q_L}{1-\eta Q_L}\right)^{\frac{1}{1-\beta_2}}\frac{\beta_2 c(r-\alpha)}{(\beta_2-1)r(1-\eta Q_L)}$, such that for the values above it the condition 2. is satisfied for all Q_L, i.e. $\frac{\mathrm{d}}{\mathrm{d}Q_F} F_1^S(X, Q_L, Q_F)\big|_{Q_F=0}$ is always positive.

Numerical experiments show that at the moment of investment X is large enough to satisfy this constraint for the most of the parameter values. The intuition behinds this is that for smaller X the option value exceeds the value of investing immediately and the firm waits till the market is large enough.

Moreover, evaluating (79) at $Q_F = 0$ we get

$$\left.\frac{\partial^2 F_1^S(X, Q_L, Q_F)}{(\partial Q_F)^2}\right|_{Q_F=0} = \frac{2\eta X}{r-\alpha}\left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{\beta_2-1}\left(1 - \left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{1-\beta_2} - \frac{(\beta_2-1)\eta Q_L}{(1-\eta Q_L)^{2-\beta_2}}\right), \tag{89}$$

which has the same sign as the last expression in the brackets. Note that its last part, $\frac{(\beta_2-1)\eta Q_L}{(1-\eta Q_L)^{2-\beta_2}}$, is monotonically decreasing with Q_L, because its derivative is negative $(\beta_2-1)\eta\left(\frac{1-(\beta_2-1)\eta Q_L}{(1-\eta Q_L)^{3-\beta_2}}\right) < 0$. The other part, $1-\left(\frac{(\beta_2-1)rX}{\beta_2 c(r-\alpha)}\right)^{1-\beta_2}$, is a constant with respect to Q_L. Therefore, there exists unique Q_L, wihch is denoted by $\breve{Q}_L$ such that if $Q_L < \breve{Q}_L$ it holds that $\left.\frac{\partial^2 F_1^S(X,Q_L,Q_F)}{(\partial Q_F)^2}\right|_{Q_F=0} < 0$, and if $Q_L \geq \breve{Q}_L$ then $\left.\frac{\partial^2 F_1^S(X,Q_L,Q_F)}{(\partial Q_F)^2}\right|_{Q_F=0} \geq 0$.

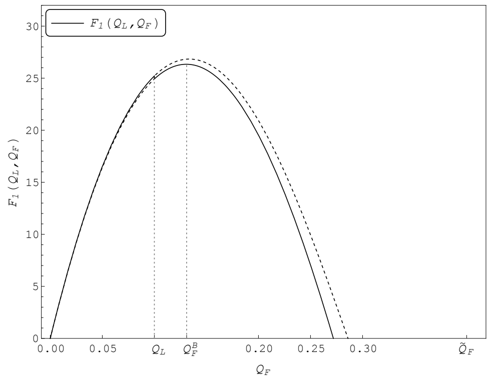
This means that for $Q_L \geq \breve{Q}_L$ the function $F_1^S(X,Q_L,Q_F)-\delta Q_F$ is convex. Its first order derivative with respect to Q_F is strictly increasing and positive for the considered parameter values. Hence, keeping in mind that in order to be a small follower the firms capacity should be smaller that the capacity of the leader, $Q_F < Q_L$, for $Q_L \geq \breve{Q}_L$ the maximum of $F_1^S(X,Q_L,Q_F)-\delta Q_F$ will be always reached at $Q_L-\varepsilon$.

If $Q_L < \breve{Q}_L$ the sign of the second order derivative changes from negative to positive, so that the $F_1^S(X,Q_L,Q_F)-\delta Q_F$ is convex for small values of Q_F and concave for large values of Q_F. As showed earlier the first order derivative may have either two or (one) roots (for small Q_L) or none (for large Q_L). In the latter case $F_1^S(X,Q_L,Q_F)-\delta Q_F$ is again a strictly increasing function of Q_F. Thus, there exists a critical value of Q_L such that for the values above it the first order derivative is always positive. This Q_L can be determined by simultaneously setting to zero the first and second derivatives of $F_1^S(X,Q_L,Q_F)-\delta Q_F$, given by 78 and 79 respectively. Finally, for Q_L smaller that the critical value $F_1^S(X,Q_L,Q_F)-\delta Q_F$ is decreasing for the intermediate values of Q_F and increasing if Q_F is either small or large.

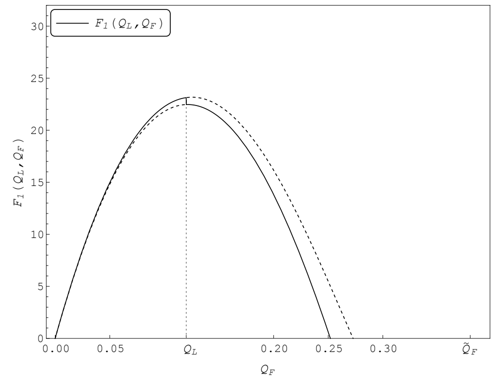
To summarize, the value function of the small follower starts from the origin and has a polynomial shape. Depending on the combination of the parameter values it can either have two turning points (e.g. small Q_L) or none (e.g. for large Q_L). In the latter case the value function is strictly increasing and given the constraint $Q_F < Q_L$, its maximum is located at $Q_F = Q_L-\varepsilon$. If Q_L is relatively small the follower curve increases until the first turning point, then coming to its minimum and from there on it starts increasing again reaching the boundary at $\tilde{Q}_F(X,Q_F)$. For this case the location of the maximum can be determined differently depending on the level of Q_L, which is crucial for the constraint in the optimization problem. In particular, the follower's optimum can be reached either at the first turning point, i.e. at the boundary $\tilde{Q}_F(X,Q_F)$ or according to the following first order condition:

$$\frac{\partial(F_1^S(X,Q_L,Q_F)-\delta Q_F)}{\partial Q_F}=0. \tag{90}$$

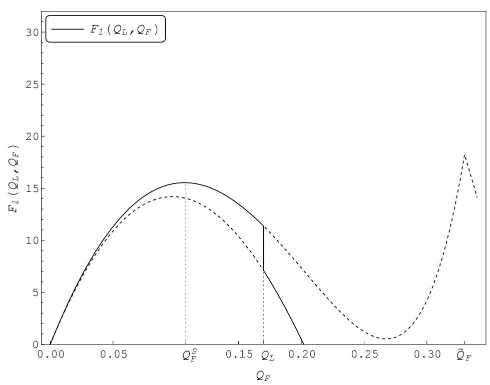
The capacity level that is defined by the above first order condition is denoted in this case by Q_F^S. It is is implicitly determined by setting (78) to zero and choosing the solution such that (79) is negative.



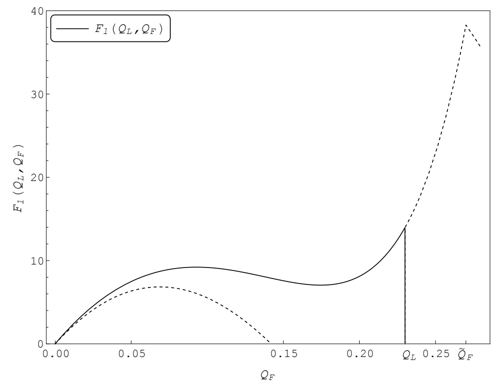
(a) Maximum is at Q_F^B for $Q_L = 0.1$.



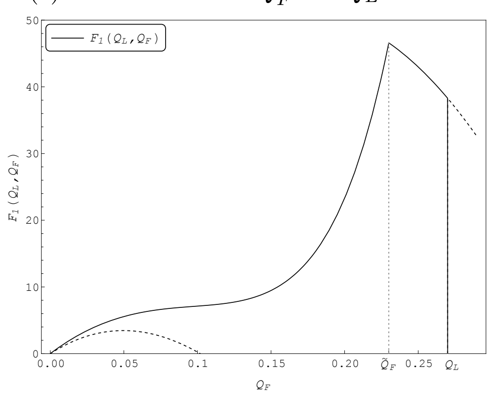
(b) Maximum is at $Q_L - \varepsilon$ for $Q_L = 0.12$



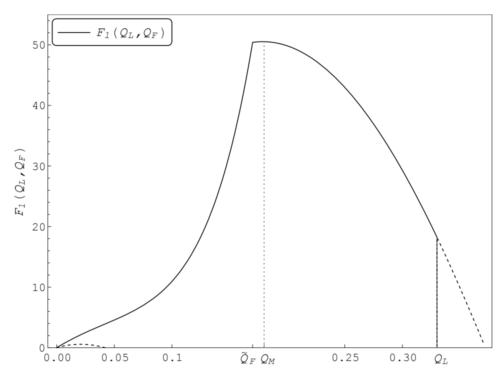
(c) Maximum is at Q_F^S for $Q_L = 0.17$



(d) Maximum is at $Q_L - \varepsilon$ for $Q_L = 0.23$



(e) Maximum is at $\tilde{Q}_F$ for $Q_L = 0.27$



(f) Maximum is at Q_M for $Q_L = 0.33$

Figure 9: Illustration of possible locations of the maximum for $X = 50$.

In the cases (b), (c) and (d) the entrant always replies with a smaller capacity and exits last.

$$\bar{Q}_1(X) = \left\{ Q_L < Q_F^B(X, Q_L) \middle| F_1^B(X, Q_L, Q_F^B(X, Q_L)) = F_1^S(X, Q_L, Q_L) \right\}, \tag{91}$$

$$\bar{Q}_2(X) = \left\{ Q_L \middle| Q_F^S(X, Q_L) = Q_L \right\}, \tag{92}$$

$$\bar{Q}_3(X) = \left\{ Q_L > Q_F^S(X, Q_L) \middle| F_1^S(X, Q_L, Q_F^S(X, Q_L)) = F_1^S(X, Q_L, Q_L) \right\}, \tag{93}$$

$$\bar{Q}_4(X) = \frac{1}{2\eta}\left(1 - \frac{\beta_2 c(r-\alpha)}{r(\beta_2 - 1)X}\right), \tag{94}$$

$$\bar{Q}_5(X) = \left\{ Q_L \middle| Q_M(X) = \tilde{Q}_F(X, Q_L) \right\}. \tag{95}$$

Proof of Proposition 7 The threshold which leads to the immediate investment in a duopoly is given by

$$X_{F,D}^I(Q_L) = \begin{cases} \left\{ X \middle| h(X, Q_L, Q_{F,D}^*(X, Q_L)) = 0 \right\} & \text{if } Q_L \leq \bar{\bar{Q}}_1, \\ \overline{X}_F^I(Q_L) & \text{if } Q_L > \bar{\bar{Q}}_1, \end{cases} \tag{96}$$

where $\overline{X}_F^I(Q_L)$ is the inverse function of the last point in terms of Q_L where it is still optimal to invest as a strong entrant instead of mimicking the incumbent's strategy:

$$\overline{X}_F^I(Q_L) = (\bar{Q}_3(X))^{-1}, \tag{97}$$

while $h(X_F^I, Q_L, Q_F)$ is defined by (18) and $\bar{\bar{Q}}_1$ is found by solving

$$h\left(\overline{X}_F^I(Q_L), Q_L, Q_{F,D}^*\left(\overline{X}_F^I(Q_L), Q_L\right)\right) = 0. \tag{98}$$

The entrant monopolization strategy becomes available only if the capacity of the incumbent satisfies $Q_L \geq \widehat{Q}_L$. In this case the entrant's investment threshold that leads to the monopoly once being hit by X from below is

$$X_{F,\underline{M}}^I(Q_L) = \begin{cases} \underline{X}_F^I(Q_L) & \text{if } Q_L \leq \bar{\bar{Q}}_2, \\ \left\{ X \middle| h(X, Q_L, Q_M^*(X)) = 0 \right\} & \text{if } Q_L > \bar{\bar{Q}}_2, \end{cases} \tag{99}$$

where

$$\overline{X}_F^I(Q_L) = \frac{\beta_2 c(r-\alpha)}{r(\beta_2 - 1)(1 - 2\eta Q_L)}, \tag{100}$$

and $\bar{\bar{Q}}_2$ is found by solving

$$h\left(\frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-2\eta Q_L)}, Q_L, Q_M^*\left(\frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-2\eta Q_L)}\right)\right)=0. \tag{101}$$

The situation when monopoly is triggered from above corresponds to the threshold

$$X^I_{F,\overline{M}}(Q_L)=\underline{X}^I_F(Q_L). \tag{102}$$

The capacity level $\widehat{Q}_L$ can be found by solving the following system

$$\begin{cases} V_M\left(\frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-2\eta Q_L)}, Q_L\right)-\delta Q_L = F_0\left(\frac{\beta_2 c(r-\alpha)}{r(\beta_2-1)(1-2\eta Q_L)}, Q_L, Q^*_{F,D}\left(X, Q_L\right)\right), \\ h(X, Q_L, Q^*_{F,D}(X, Q_L))=0. \end{cases} \tag{103}$$

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