Stepwise Investment Value under Stage Specific Parameters

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Abstract

We provide a general model for comparing stepwise and lumpy investments, considering stage specific volatilities, drifts and possibility of project failure. Stepwise investments allow for interim project value realizations, instead of considering only a final project value as in sequential investments. We conceive of an environment in which stepwise investment costs exceed lumpy investments, even if the total combined project value of the stages equals the lumpy project value. We find there are tight conditions on the parameter values required in order to compare the two strategies. Also that increased uncertainty does not necessarily reduce the relative value of stepwise investments. We evaluate the tradeoffs between the proportion of project value in each stage, and the relative investment costs. Our model could be extended to allow for inhibited or enhanced second stage project values, or even reduced investment costs, due to a learning effect, in arriving at those optimal trade-offs.

Stepwise Investment Value under Stage Specific Uncertainty

1 Introduction

Does the flexibility of stepwise investments with interim (but incomplete) project values result in greater investment attractiveness compared to a lumpy once-for-all investment? Kort et al. (2010) argues that following real option intuition, the flexibility of the stepwise strategy makes it more attractive especially with greater uncertainty, but their analytical results do not endorse this view. Higher uncertainty makes lumpy investment relatively attractive.

Investment attractiveness may be defined in terms of the real option value (or the option value coefficients) of the investment opportunity, or alternatively in terms of the value threshold that justifies immediate investment. We use both measures of investment attractiveness, and examine the sensitivity of both thresholds and real option values to changes in project value uncertainty.

We extend the Adkins and Paxson (2014a) sequential investment model, which does not consider interim realized project values, but only a completed project value at the end. However, we allow for stage specific project volatilities as in Cassimon et al. (2011) and also for stage specific value drifts or for other uncertainties regarding project failure as in Adkins and Paxson (2014b).

Rodriques (2009) uses differential segment demand volatilities, investment costs, and some other measures, to evaluate optimal timing among segments, under an endogenous regime-switching process. Kort et al. (2010) propose that the American perpetuity option value for a two-stage sequential investments is equal to the sum of the separate option values, but this formulation suffers the defect of a lack of compoundedness in the sense that the first and second stage option values are independent.

Some authors eschew the reputed merits of closed-form European compound options and solve the sequential investment opportunity through the power of numerical techniques. A trinomial lattice formulation is used by Childs and Triantis (1999) to solve a multiple sequential investment model having cash-flow interaction. Schwartz and Moon (2000) provide a numerical solution for complex R&D options, with project failure that does not always decline as the project approaches completion, but with constant asset volatility, drifts and investment cost volatility over four stages. Cortazar et al. (2003) consider four "exploration" stages with success probability increasing as the stages near completion (production) with investment cost almost always increasing near completion. The early (pure exploration) stage has primarily geological-technical risk (represented by a zero-drift constant Brownian motion) independent of the production stage commodity price risk. An implicit finite-difference numerical solution provides a value without options, and with operating, development and exploration options as a function of expected copper mine size. Koussis et al. (2013) provide numerical solutions for multi-stages with multiple options. The shortcomings of these solution methods are the possibly onerous and not always transparent calculations.

Our analysis is founded on the idea that the betas for the two stages may be different because the information set has altered between the times of exercising the stage-1 and stage-2 investments since the investor benefits from operating the first (incomplete) part of the project. The model is reworked with different betas for stage-1 and stage-2. This makes the current formulation to be richer in scope and wider in interpretation. Different betas for different stages may be due to different stage specific project value yields, volatilities or the possibility of project failure.

Based on value maximization, the stepwise strategy is assessed to be more attractive than the lumpy strategy whenever it is both feasible and viable. The stepwise strategy is feasible provided its stage-1 project threshold level is less than that for the lumpy strategy, indicating that the stepwise option is always exercised first. The stepwise strategy is viable provided its stage-1 option value is greater. By treating the contexts underpinning the stage-1 stepwise strategy and the lumpy strategy as identical, the concepts of feasibility and viability are formulated as a discriminatory condition that differentiates between the two strategies, given that the context underpinning the stage-2 stepwise strategy is different. Because of the absence of a closed-form solution for this condition, we use numerical simulation to identify the effect of variations in key model parameters on the condition. Essentially, any changes enhancing the property of the stepwise strategy results in a less restrictive condition. Finally, we consider the role of differences between the stage-1 and -2 parameters. A less restrictive condition results from a gain in the stage-2 volatility relative to its stage-1 value but a decline in stage-2 failure probability relative to its stage--1 value. The stepwise strategy is more attractive whenever the range of stage-2 possibilities enlarges or its failure probability declines.

We believe that the stepwise strategy is more attractive than lumpy investment if both stage option value coefficients (A1, A2) are greater than for the lumpy option value coefficients (A0), or the project value thresholds (V1, V2) are both lower (V0). The results are often ambiguous, or dependent on the level of the relative betas. At very low beta 2, A0<A1, A2, but V1<V0<V2, but at high beta 2 $A_0 > \{A_1, A_2\}$ and $V_0 < \{V_1, V_2\}$. These beta differentials could be due to volatility levels (high volatility results in low betas) or when there is a greater probability of the stage-1 option failing than for stage-2, which is common.

2 The Model

A firm, assumed to be without a current earnings stream, is considering two alternative strategies for investing in a project. Either the firm invests in the project at a single stage, referred to as "lumpy", or alternatively at two consecutive distinct stages, referred to as "stepwise". The two stages for the stepwise strategy are labeled 1 and 2, respectively. The project value realized by the lumpy strategy is exactly equal to that generated by the two stages combined under the stepwise strategy. This means that the project is infinitely divisible and that by completing a ¥ percentage of the project produces a ¥ percentage of the project's total value , which implies that the project value is linear in both revenues and operating cost. In contrast, the total investment costs for each of the two strategies are different. The combined investment cost for stages-1 and -2 under the stepwise strategy is taken to be greater than that under the lumpy strategy. This represents the additional cost of separating the investment over stages-1 and -2 as well as reflecting scale economy efficiencies available under the lumpy strategy.

The optimal timing and strategy (lumpy versus stepwise) are determined from the principle of firm value maximization based on a stochastic formulation. The continuous time model treats the project value, denoted by V as uncertain, following a geometric Brownian motion (gBm) process, so it follows that the value for the investment option, denoted by F, is also stochastic. Further, there exists a possibility that during a time interval $\mathrm{d}t$ the option value may collapse to zero with probability ª $\mathrm{d}t$, since it is assumed that potentially, project tenability can be overwhelmingly undermined by emergent factors such as new technologies or changes in taste, or by the unanticipated arrival of a challenger's offering having more robust first mover advantages.

In the model design, variables and parameters may be additionally labeled by a subscript index to distinguish between the two strategies and the two stages for the stepwise strategy. Variables and parameters defined for the lumpy strategy model are labeled 0, while those for the stepwise strategy model are 1 and 2, respectively for stages-1 and -2. The optimal time to invest is denoted by $\hat{t}$ and the project value threshold signaling an optimal investment by $\hat{V}$. Since the stage-1 stepwise solution must always occur before the step-2 stepwise solution, we require that $\hat{t}_1 < \hat{t}_2$ and $\hat{V}_1 < \hat{V}_2$. We now need to explain the method for deciding between the two alternative investment strategies. First, the preferred solution must have an earlier optimal start time to ensure that investment under the preferred strategy is the first to begin. If the stepwise strategy is preferred to the lumpy strategy, then $\hat{t}_1 < \hat{t}_0$ and $\hat{V}_1 < \hat{V}_0$. Second, and given this requirement, the option value to invest under the preferred strategy at its optimal time must be greater to ensure value maximization. Again, for the stepwise strategy to be preferred, then $F_1\left(\hat{V}_1\right) > F_0\left(\hat{V}_1\right)$. So, establishing that the stepwise strategy is the preferred alternative requires identifying the full conditions for $F_1\left(\hat{V}_1\right) > F_0\left(\hat{V}_1\right)$ to be satisfied given that $\hat{V}_1 < \hat{V}_0$.

The project value threshold signaling the optimal time to invest is influenced by several key model parameters. Besides the probability ª, these include the risk-free rate, denoted by r, the dividend yield associated with the project, ¢, and the project value volatility, ±. All their magnitudes are determined from the information set held by the decision makers of the firm at the relevant time. Under the stepwise strategy, the state of the information set may change between the two stages because of the time difference. During stage-1, the decision makers are relying on their experiences of similar projects implemented in the past in assessing the magnitudes of the various parameters. During stage-2, the decision makers not only have access to this information set but acquire information due to the passage of time and also from the actualization of the part project. Between the two, the information obtained from actually operating the part project in the market environment may be thought to be more significant because it is gained experientially, from the active participation in the market rather than arising from being a passive player. The change in the state of the information set may be sufficient to effect a revision in the firm's estimation of the key parameters at stage-2 relative to stage-1, which needs to be captured in the model design. In this formulation, the impact of being a passive player, or the effect of the mere passage of time, on the information set is treated as negligible and can be ignored. Consequently, the information set facing the decision maker during stage-1 while deliberating on the stepwise strategy can be treated as identical to that while deliberating on the lumpy strategy. This means that $ª_1 = ª_0$, $r_1 = r_0$, $¢_1 = ¢_0$ and $±_1 = ±_0$. If the risk-free rate is expected to remain constant over time, then $r_2 = r_1$ as well.

For $J = 0,1,2$, the risk-neutral valuation relationship defining the investment option is given by:

$$\tfrac{1}{2}±_J^2 V^2 \frac{\partial^2 F_J}{\partial V^2} + \left(r_J - ¢_J\right) V \frac{\partial F_J}{\partial V} - \left(r_J + ª_J\right) F_J = 0 .$$

Following standard theory, the solution to the valuation function for $V \le \hat{V}_J$ is:

$$F_J(V) = A_J V^{\ \ J}\,, \tag{1}$$

where $\ \ _J > 1$ is the positive root of the quadratic equation:

$$Q_J(\ \ _J) = \tfrac{1}{2}\pm_J^2\ \ _J(\ \ _J - 1) + (r_J - ¢_J)\ \ _J - (r_J + ª_J) = 0\,. \tag{2}$$

We note that:

$$\frac{\partial\ \ _J}{\partial \pm_J} < 0, \frac{\partial\ \ _J}{\partial ¢_J} > 0, \frac{\partial\ \ _J}{\partial r} < 0, \frac{\partial\ \ _J}{\partial ª_J} > 0\,. \tag{3}$$

Since we assume that the states for the information sets facing the decision maker when deliberating on the lumpy and stepwise strategies means are identical, then $\ \ _0 = \ \ _1$.

Finally, the investment cost is denoted generically by K, with $K_1 + K_2 > K_0$ by assumption. The additional cost incurred by the stepwise strategy is represented by $\Delta = K_1 + K_2 - K_0 > 0$.

2.1 *One-Stage Lumpy Model*

Since the single-stage lumpy investment strategy corresponds to the standard model of investment under certainty, see Dixit and Pindyck (1994), the standard findings apply. We merely state the main findings. The optimal project value threshold $\hat{V}_0$ is given by:

$$\hat{V}_0 = \frac{\ \ _0}{\ \ _0 - 1} K_0 \tag{4}$$

and for $V \le \hat{V}_0$, the investment option value by:

$$ROV_0 = F_0 = A_0 V_0^{\ \ _0} \tag{5}$$

where $A_0 = \ \ _0^{-\ \ _0}(\ \ _0 - 1)^{\ \ _0 - 1} K_0^{1-\ \ _0}$.

2.2 *Two-Stage Stepwise Model*

We now consider the model that splits the project investment into two consecutive distinct stages. The stage-1 investment option is conceived as a compound option, since its exercise produces not only the value rendered by acquiring the part project but also the stage-2 investment option as well. This implies that the value for the stage-1 option may be dependent on the stage-2 option value, so when evaluating the stage-1 option value, it is essential to have already evaluated the stage-2 option value. We adopt the backwardation principle to accomplish this by first determining the stage-2 option value and then proceed to find the stage-1 option value.

Between stages-1 and -2, the firm has acquired not only the part project value, but the embedded stage-2 investment option as well. The firm through its investment at stage-1 is said to acquire a part project with a value $¥V$, where $0 < ¥ < 1$ denotes the acquired proportion of the entire project. Then, the value generated by exercising the stage-1 option is the sum of the part project value $¥V$ and the stage-2 investment option value $F_2\left(V\right)$. In determining the value rendered between stages-1 and -2, the stage-1 investment cost K_1 can be safely ignored as it is a sunk cost. By the conservation principle, the stage-2 investment option is exercised whenever the full value foregone by exercising the option $¥V + F_2\left(V\right)$ is exactly balanced by the net value generated by its exercise, which equals the value rendered by the entire project value V less the incurred stage-2 investment cost K_2. Optimal stage-2 exercise occurs for $V = \hat{V}_2$, so the value matching relationship specified as the exact balance between the value foregone instantaneously prior to exercise, $¥\hat{V}_2 + F_2\left(\hat{V}_2\right)$, and the net value generated instantaneously after exercise, $\hat{V}_2 - K_2$, becomes:

$$A_2 \hat{V}_2^{\;2} = \left(1 - ¥\right)\hat{V}_2 - K_2 . \tag{6}$$

Equation (6) adopts the standard form, so:

$$\hat{V}_2 = \frac{\ _2}{\ _2 - 1}.\frac{K_2}{1-\yen} \tag{7}$$

and

$$F_2 = A_2 V_2^{\ _2} \tag{8}$$

where $A_2 = \ _2^{-\ _2}\left(\ _2 - 1\right)^{\ _2 - 1}\left(1-\yen\right)^{\ _2} K_2^{1-\ _2}$.

The stage-1 investment option value can now be determined from the stage-2 option value. The project value threshold signaling the optimal stage-1 exercise is denoted by $\hat{V}_1$. Due to value conservation, the value foregone by exercising the option has to exactly balance the net value rendered by the exercise, which equals the part project value, $\yen V$, net of the stage-1 investment cost, K_1, and the stage-2 investment option value, $F_2\left(V\right)$, all evaluated at the stage-1 threshold, $\hat{V}_1$. The value matching relationship becomes:

$$A_1 \hat{V}_1^{\ _1} = \yen \hat{V}_1 - K_1 + A_2 \hat{V}_1^{\ _2}. \tag{9}$$

The smooth pasting condition associated with (9) can be expressed as:

$$\ _1 A_1 \hat{V}_1^{\ _1} = \yen \hat{V}_1 + \ _2 A_2 \hat{V}_1^{\ _2}. \tag{10}$$

Since A_2 is obtainable from (8), $\hat{V}_1$ is determined after eliminating A_1 from (10):

$$\hat{V}_1 = \frac{\ _1}{\ _1 - 1}\frac{K_1}{\yen} + \frac{\ _2 - \ _1}{\ _1 - 1}\frac{A_2 \hat{V}_1^{\ _2}}{\yen}, \tag{11}$$

which enables A_1 to be found using (10) from:

$$A_1 = \frac{\yen \hat{V}_1^{1-\ _1}}{\ _1} + \frac{\ _2 A_2 \hat{V}_1^{\ _2 - \ _1}}{\ _1}. \tag{12}$$

Generally, there is no closed-form solution for $\hat{V}_1$, which has to be evaluated numerically. However, two special cases do exist. The trivial if $\ _2 = 2$, and the significant if the information sets at both stages-1 and -2 are identical, since the decision makers have not gained any relevant additional information from active participation in the market. For the latter case, $\ _1$ and $\ _2$ are equal so (11) simplifies to the solution produced by Kort et al. (2010):

$$\hat{V}_{1,KMP} = \frac{\ _1}{\ _1 - 1}\frac{K_1}{¥}. \tag{13}$$

Also from (12):

$$A_1 = \frac{¥\hat{V}_1^{1-\ _1}}{\ _1} + A_2 = \ _2^{-\ _2}\left(\ _2 - 1\right)^{\ _2 - 1}\left\{¥^{\ _1}K_1^{1-\ _1} + \left(1-¥\right)^{\ _1}K_2^{1-\ _1}\right\}.$$

If $\ _2 > \ _1$, then $\hat{V}_1$ equals $\hat{V}_{1,KMP}$ plus a positive amount so $\hat{V}_1 > \hat{V}_{1,KMP}$, while if $\ _2 < \ _1$, then $\hat{V}_1$ equals $\hat{V}_{1,KMP}$ minus a positive amount so $\hat{V}_1 < \hat{V}_{1,KMP}$, which implies that $\hat{V}_1$ is an increasing function of $\ _2$ around $\ _2 = \ _1$. Although $\partial\hat{V}_1/\partial\ _2 > 0$, $\hat{V}_2$ is a decreasing function of $\ _2$ as prescribed by the standard model. Further, when $\ _2 = \ _1$ there are simplified expressions for ordering $\hat{V}_0$, $\hat{V}_1$ and $\hat{V}_2$, which are $\hat{V}_1 < \hat{V}_2$, $\hat{V}_1 < \hat{V}_0$ and $\hat{V}_0 < \hat{V}_2$ provided $¥ > K_1/\left(K_1 + K_2\right)$, $¥ > K_1/K$ and $¥ > \left(K_1 + \Delta\right)/K_0$, respectively. Amongst these, the first is the least restrictive whilst the last is the most restrictive.

It is crucial to recognize the impact of the difference in the information sets between stages-1 and -2 if it is reflected in unequal $\ _1$ and $\ _2$. Under the Kort et al. (2010) formulation, the states of the information sets at the two consecutive stages are identical, which implies the actualization of the part project plays no role in deciding between the two strategies. This assumes that the actualization following the stage-1 investment does not contribute incrementally to the stage-2 information set and that the acquired experiential knowledge is uninformative. This absence in information gain between the two stages is reflected in the equal values for $_1$ and $_2$. When $_1 = \ _2$, the stage-1 option value is simply the sum of the stage-1 option value in the absence of a stage-2 and the stage-2 option value. The formulation treats the stage-1 option as a combination of two discrete and independent options that do not manifest any compoundedness. For the stage-1 option to become compounded, the parameters $_1$ and $_2$ have to be unequal. If compoundedness is crucial in identifying the characteristics conducive to making the stepwise strategy more attractive and in discriminating between the two strategies, then the formulation has to allow the possibility that $_1$ and $_2$ may differ.

The stepwise strategy is judged to be more attractive whenever its stage-2 option value exceeds that for the lumpy strategy for $V \leq \hat{V}_1 < \hat{V}_0$. Because of (1), a comparative assessment of the option values reduces to a size comparison of their coefficients. Although the lumpy option coefficient is specified by $A_0 = \hat{V}_0^{1-\ _1} \big/ \ _1$, there exists no equivalent closed-form solution for A_1, but from (12) it can expressed as:

$$A_1 = \frac{\hat{V}_1^{1-\ _1}}{\ _1} \left\{ ¥ + (1-¥) \left(\frac{\hat{V}_1}{\hat{V}_2} \right)^{\ _2 - 1} \right\} \tag{14}$$

Since the expression within the curly brackets of (14) is always less than 1 because $\hat{V}_1 < \hat{V}_2$ by assumption, for $A_1 > A_0$ then $\hat{V}_1$ has be less than $\hat{V}_0$. Although $\hat{V}_1 < \hat{V}_0$ for $F_1(V) > F_0(V)$, tighter conditions on the parameter ranges are required for making more insightful conclusions on the comparable attractiveness of the two strategies.

3 Numerical Illustrations

In the absence of a closed-form solution, we resort to numerical illustrations to reveal more precise conditions capable of producing a greater investment option value for the stepwise than for the lumpy strategy. The illustrations that we present use various ranges of values for the different parameters in order to construct an overall picture of the way that the stepwise strategy is preferential. The analytical focus applied by Kort et al. (2010) is based on comparing the relative desirability of the two strategies under parameter changes affecting them in identical ways. In contrast, the approach adopted here is dynamic and investigates the choice between the two strategies by considering the impact of parameter changes between stages-1 and -2 for the stepwise strategy while maintaining identical parameter values for the lumpy and the stage-1 stepwise strategy. In this way, we are exploring whether parameter dynamics are crucial in deciding the choice between the two strategies. The various comparisons of the two strategies are made by first strictly imposing the condition $\hat{V}_1 < \hat{V}_0 < \hat{V}_2$ and ignoring any non-compliant cases, and secondly by making conclusions based on the relative magnitudes of A_0 and A_1, the option coefficients for the lumpy and the stage-1 stepwise strategy, respectively, after setting $ª_1 = ª_0$, $r_1 = r_0$, $¢_1 = ¢_0$, $±_1 = ±_0$ and $r_2 = r_1$.

The various simulations allow $_2$ to vary but assume $_0$ remains the same. This convenience means that the threshold level and option value for the lumpy strategy both remain constant while those for the stepwise strategy vary according with the variations in the specific parameters. Throughout our investigations, we maintain a constant lumpy strategy investment cost by setting $K_0 = 1$ but allow the two stepwise investment costs to vary within $K_1 + K_2 > K_0$. The base case simulation values are recorded in Table 1, where $¥ > K_1/K$, the Kort et al. (2010) condition for $\hat{V}_1 < \hat{V}_0$. The simulations are run by computing the various thresholds and option values for variations in $_2$.

--- *Table 1 about here* ---

Figures 1a and 1b illustrate the profiles for the project value threshold and option value, respectively, for each of the two strategies and for the two stages of the stepwise strategy, due to variations in $_2$. The profiles are evaluated by setting the power parameter $_0 = 2$, the stage-1 and -2 investment costs $K_1 = 0.23$ and $K_2 = 0.8$, respectively, and the stage-1 proportional part of the project value ¥ $= 0.3$. This means that the investment cost condition is met and the stage-1 proportional part ¥ exceeds $K_1/(K_1 + K_2)$ as stipulated by Kort et al. (2010), although this is subsequently relaxed. The figure, and subsequent figures, are only illustrated for a limited range, in this case $1.4 \le \ _2 \le 2.3$, since the solution is infeasible at the upper limit, as explained shortly, while for $\ _2 < 1.4$, the stepwise project value threshold and option value increase in line with the standard findings. Figure 1a reveals that for the lumpy strategy, the threshold level and option coefficient and thereby the option value are constant for variations in $\ _2$, respectively $\hat{V}_0 = 2.0$ and $A_0 = 0.25$. The condition $\hat{V}_1 \le \hat{V}_0 = 2.0$ is satisfied for $\ _2 \le 2.2328$, so $\ _2$ values exceeding this upper limit are infeasible, and since $\hat{V}_1 < \hat{V}_0$ is more restrictive than $\hat{V}_1 < \hat{V}_2$, these values are consequently ignored. As expected, the stage-1 and stage-2 project value thresholds are respectively increasing and decreasing for variations in $\ _2$ while their option values displayed in Figure 1b are both decreasing. When $\ _2$ is set to equal $\ _0 = 2.0$, A_1 is slightly greater than A_0 indicating a marginal preference for the stepwise strategy. Indifference between the two strategies occurs for $\ _2 = 2.0161$, when $\hat{V}_1 = 1.5530$ and $\hat{V}_2 = 2.2677$. Below this limit, the stepwise strategy is preferred and increasingly so as $\ _2$ decreases while the lumpy strategy dominates for $2.0161 < \ _2 \le 2.2328$.

--- *Figure 1 about here* ---

3.1 Proportional Part Level

Kort et al. (2010) stipulate that the proportional part parameter ¥ is constrained to be greater than the stage-1 proportional investment cost $K_1/(K_1 + K_2)$ to ensure that $\hat{V}_1 < \hat{V}_2$ and a feasible solution results. In the current formulation, this constraint is not incorporated nor do we deduce a similar condition, possibly owing to the absence of a closed-form solution. It is therefore interesting to investigate the impact of proportional part parameter changes to determine whether the magnitude of ¥ is crucial for obtaining a feasible solution. This is achieved by maintaining the remaining parameters according to Table 1 but setting ¥ at a level around $K_1/(K_1+K_2)=0.2233$; specifically we allow ¥ to take on the levels 0.24, 0.22 and 0.20. These are illustrated, respectively, in Figures 2a-f.

---- *Figures 2a-f and Table 2 about here* ----

As Figures 2a-b show that the stepwise option values are decreasing functions of $\ _2$ while the stage-1 and -2 project value thresholds are, respectively, increasing and decreasing functions, the profiles adopt a similar shape to those presented in Figure 1. However, differences do exist in the range of $\ _2$ values that make the stepwise strategy more attractive. For ¥ $=0.24$, Figures 2a-b, although a feasible solution is obtainable for $\ _2 = \ _0 = 2.0$ the lumpy strategy is more attractive owing to a smaller proportion of the value realized by the stage-1 stepwise strategy compared with that for Figure 1. In contrast, for both ¥ $=0.22$ and ¥ $=0.20$, Figures 2c-f, there is no feasible solution for the stepwise strategy when $\ _2 = \ _0 = 2.0$ because the condition $\hat{V}_1 \le \hat{V}_0 \le \hat{V}_2$ is violated. The values of $\ _2$ and the thresholds for varying ¥ signifying indifference between the stepwise and lumpy strategies are exhibited in Table 2. This shows that as the proportional part realized by the stage-1 stepwise strategy declines, the value of $\ _2$ accordingly has to decline to ensure that the stepwise strategy remains feasible and more attractive. Accompanying this decline in $\ _2$, there are increases in the stage-1 and -2 project value threshold levels. This is in line with the standard finding of a negative relationship between the option value power parameter and the threshold.

The interesting feature of Figure 2 is the occurrence of the stepwise strategy being the more attractive of the two strategies despite a ¥ value violating the condition ¥ $> K_1/(K_1+K_2)$. For a particular ¥, the stepwise strategy is said to be the more attractive when $\ _2$ is less than some upper limit, denoted by $\tilde{\ }_2$. For $\ _2 = \tilde{\ }_2$, when the lumpy and the stage-1 stepwise strategy options are equal, the corresponding values of $\hat{V}_1$ and $\hat{V}_2$ are denoted by $\tilde{\hat{V}}_1$ and $\tilde{\hat{V}}_2$, respectively. To be at least attractive, the option value for the stepwise strategy has to be at least greater than that for the lumpy strategy, so:

$$F_1\left(V \le \tilde{\hat{V}}_1\right) \ge F_0\left(V \le \hat{V}_0\right),$$

with the requirement that $\tilde{\hat{V}}_1 < \hat{V}_0 < \tilde{\hat{V}}_2$ for feasibility. Indifference between the stepwise and lumpy strategies occurs at $\;_2 = \tilde{\;}_2$. Figure 2 reveals that the stepwise strategy is both more attractive yet feasible even when the condition ¥ $> K_1/\left(K_1 + K_2\right)$ is violated. This means that by reformulating the model to allow the option power parameter to vary between stages owing to a possible information set change, a broader representation is developed that is not confined to the shortcomings of requiring ¥ $> K_1/\left(K_1 + K_2\right)$.

These findings are of practical significance because for a studied context having different valued stage option power parameters, the restriction on ¥ may be inappropriate. The condition on ¥ implies that $1-$¥ $< K_2/\left(K_1 + K_2\right)$, so the value realized per unit investment cost declines as we progress from stage-1 to stage-2, which creates a comparative disincentive to start stage-2 having completed stage-1. The concept of decreasing returns is acceptable and commonly applied in assessing the optimal investment level in a world of limitless investment. Within the current framework, the overall optimal investment level is not contestable, since it is fixed as K_0 and $K_1 + K_2$ for the lumpy and the stepwise strategies, respectively. Instead, the enquiry focuses on determining the circumstances favouring a stepwise compared with a lumpy investment strategy given fixed overall investment costs. For a divisible investment that is executable in two consecutive stages, it is credible that the value realized per unit investment cost would be less at stage-1 due to the need to finance upfront, essential and significant investments in R&D and marketing before production revenues can flow. At stage-2, a need to continue investing in these essential activities may persist, but because of the previous investments the level can be assumed to be appreciably less. The restriction on ¥ confines the model outcomes in a similar way as the condition $_2 = {}_1$, while their relaxation opens up new possibilities by widening the model scope and representation.

Figure 2 reveals that ¥ influences the three indifference factors, $\tilde{}_2$, $\tilde{\hat{V}}_1$ and $\tilde{\hat{V}}_2$. Their numerical values are reported in Table 2. A decreasing ¥ level produces a decline in the proportional part project value realized at stage-1 for the stepwise strategy, and if the stage-1 investment cost is kept constant, then the realized value per unit investment cost also declines. Table 2 reveals that a ¥ decline leads to a fall in $\tilde{}_2$, so a reduced ¥ can only be justified provided that the incremental information acquired during the between-stage actualization lowers the $_2$ value. Further, accompanying the decline in ¥, there are expected increases in the levels of both $\tilde{\hat{V}}_1$ and $\tilde{\hat{V}}_2$. As the standard model predicts, any $_2$ decrease is accompanied by a rise in $\hat{V}_2$. Also, a ¥ decline makes the stepwise strategy comparatively less attractive and inhibits its take-up, so $\tilde{\hat{V}}_1$ rises as a consequence.

3.2 *Stage Investment Cost Impact*

The effect of stage-1 and -2 investment cost changes on the levels of the indifference factors. The results are presented in Table 3. They record the various indifference levels evaluated from variations in the investment cost levels, K_1 and K_2, and in ¥. These results need to be evaluated in conjunction with those in Table 2. The variations considered all comply with the condition $K_1 + K_2 > K_0$.

The illustrations reveal that the effect of an investment cost increase, whether at stage-1 or -2, is to produce a decrease in $\tilde{}_2$, while a decrease leads to an increase in $\tilde{}_2$. Whenever the stage-1 or -2 investment cost increases, the stepwise strategy is at a disadvantage compared to the lumpy strategy so for the indifference between the two strategies to prevail, the stepwise strategy has to be redeemed through a change in the between stage information set, which causes β_2 to fall. Accompanying the change in $\tilde{\beta}_2$, there are corresponding changes in the other two indifference factors, $\tilde{\hat{V}}_1$ and $\tilde{\hat{V}}_2$. A decrease (increase) in $\tilde{\beta}_2$ due to an increase (decrease) in either stage investment cost is accompanied by a $\tilde{\hat{V}}_1$ decrease (increase) and a $\tilde{\hat{V}}_2$ increase (decrease). Although the negative association between β_2 and $\hat{V}_2$ is well recognized, the positive association between β_2 and $\hat{V}_1$ arises from (11) and is due to the impact of the stage-2 option value on the stage-2 project value threshold.

---- *Table 3 about here* ----

3.3 Impact of β_1

The parameter β_1 reflects the project's underlying characteristics for the lumpy and stage-1 stepwise strategies. While variations in β_2 due to an information set change between the two stages identify the conditions favouring the stepwise strategy, we now consider the extent that the indifference factor levels are influenced by a change in the value of β_1. The effect of β_1 changes on the threshold levels and option coefficients with varying ¥ levels are illustrated in Figure 3. Table 4 reports the numerical impact on the three indifference factors. The base case values specified in Table 1 are used in the evaluations except that β_1 is increased to 2.5 and 3.0 while ¥ takes the values 0.24 or 0.20. The two ¥ values imply that the Kort et al. (2010) restriction is just obeyed and violated, respectively.

The profiles illustrated in Figure 3 display for identical ¥ levels a similar shape to those in Figure 2. The lumpy option threshold is constant while those for the stage-1 and -2 stepwise strategy are increasing and decreasing with respect to $_{2}$, respectively. Again, the option value coefficient is constant for the lumpy strategy, but is decreasing for both stage-1 and -2 of the stepwise strategy. Now, since for a constant ¥, a $_{1}$ change does not exert any influence over the stage-2 stepwise threshold and its option value because of backwardation, $\hat{V}_2$ and A_2 remain unchanged for varying $_{1}$ but constant $_{2}$. As expected, the $_{1}$ increase produces a fall in both the project value threshold and option coefficient for the lumpy strategy. It also leads to a rise in the $_{2}$ level signifying indifference between the two strategies, mainly because of the decrease in the lumpy strategy option value. The $\tilde{}_{2}$ levels all lie below the corresponding $_{1}$ level either because ¥ violates the Kort et al. (2010) restriction or because the investment cost increment Δ is too high. The $_{1}$ increase is similarly reflected in falls in the indifference factor levels, $\tilde{\hat{V}}_1$ and $\tilde{\hat{V}}_2$. Provided that the $_{2}$ level as specified by the project does not exceed the indifference level $\tilde{}_{2}$, then the rise in the $_{1}$ level has the effect of lowering the indifference level $\tilde{\hat{V}}_1$ and increasing the attractiveness of the stepwise strategy. Finally, a lower ¥ level disfavours the stepwise relative to the lumpy strategy, which is reflected in slightly higher $\tilde{\hat{V}}_1$ and $\tilde{\hat{V}}_2$ indifference levels.

---*Figure 3 and Table 4 about here* ---

3.4 Discussion

Allowing ¥ to take on values violating the Kort et al. (2010) restriction and the parameters, $_{1}$ and $_{2}$, not to be necessarily equal creates a more versatile formulation that is richer in scope and interpretation. The model, however, rests on the assumption that following the exercise of the stage-1 stepwise option, the project actualization does contribute to an enhanced information set that produces a value of $_{2}$ different from $_{1}$. Although a range of $_{2}$ levels conducive to preferring the stepwise strategy was determined numerically, we did not address the plausibility of obtaining a $_{2}$ value below its indifference level $\tilde{}_{2}$. Our attention now turns to considering the range of values for the constituent elements that combine to make a particular $_2$ value plausible. From (2), $_2$ depends on four elements, being the risk-free rate r, the dividend yield associated with the project ¢, the underlying project volatility ± and the probability ª that the investment option collapses to zero. We continue to defend a constant risk-free rate over the option duration, $\hat{t}_0$, $\hat{t}_1$ and $\hat{t}_2$. This assumption is not only typical for many analytical real option studies, but a risk-free rate change due to an information set change between $\hat{t}_1$ and $\hat{t}_2$ is also unlikely. An additional assumption is an identical underlying context at $\hat{t}_0$ and $\hat{t}_1$, so $_0 = {}_1$. Although dependence between the three parameters, ¢, ± and ª, is acknowledge, each is treated singly.

Amongst these parameters, volatility is considered to be the most important. Volatility, or the uncertainty level underlying market conditions and the project's future cash flow stream, is associated with managerial flexibility, since as new information emerges and uncertainty is resolved, decision makers have the potential to adjust their investment strategies and timings. Since the lumpy and stage-1 stepwise strategies share identical information sets, the significant emergence of information occurs during the project actualization following a stage-1 exercise. Now, $_2$ must be less than an upper limit $\tilde{}_2$ for the stepwise strategy to be preferred. The numerical simulations reveal that $\tilde{}_2$ is positively related to ¥ but negatively to Δ, and only exceeds $_1$ provided ¥ is significantly greater than $K_1/(K_1+K_2)$. Even so, the preponderance of viable $_2$ values lie below $_1$. Since the option power parameter is negatively related to volatility, obtaining $_2 < {}_1$ requires $\pm_2 > \pm_1$. An increase in stage-2 volatility raises its option value with a consequential increase in the stage-1 option and improved attractiveness for the stepwise strategy.

The second parameter considered is ¢, the dividend yield associated with the project. For most values of ¥ and Δ, the stepwise compared with the lumpy strategy becomes increasingly attractive and viable as $_1$ increasingly exceeds $_2$. Since the option power parameter and ¢ are positively related, obtaining $_2 < {}_1$ is only obtainable if the information gain from project actualization following the stage-1 exercise leads to a downward revision of the dividend yield.

Finally, we consider the role of the probability ª. Since the option power parameter is positively related with ª, the stage-2 probability has to be less than that for stage-1, ª$_2 <$ ª$_1$, for $_2 < {}_1$. The probability ª is the chance that the option value collapses to zero before exercise owing to a radical change in market preferences, the advent of disruptive technology or the arrival of combative rivals having stronger first mover advantages. During the run-up period until the exercise of the lumpy and stage-1 stepwise strategies, the decision maker is preparing to launch his offering but aware of competing forces which if manifested would gain first-mover advantages and completely erase his competitive advantage. In these circumstances, the probability of a collapsing option value for the lumpy or the stage-1 stepwise strategy is significant. However, as soon as the decision-maker’s option is exercised, and the project is actualized, the threat from competing forces is allayed and the probability ª$_2$ of a stage-2 option collapse recedes. Under this scenario, the credible presence of adverse competing forces as well as their mitigation during actualization are sufficient to plausibly explain the decline in the option failure probability between stage-1 and -2, which engenders a commensurate decrease in $_2$ relative to $_1$ and enhances the attractiveness of the stepwise strategy. Firms engage in stepwise investment strategies in order to gain early first-mover advantages, moderate competing forces and deter rival entry.

4 Conclusion

We provide a general model for comparing the lumpy and stepwise investment strategies, which permits differential stage specific volatilities, drift rates and the possibility of project failure. The contexts for the lumpy and stage-1 stepwise strategies are treated as identical. A strict condition on the option power parameters is established that indicates a preference for the stepwise strategy. This condition is formulated on the stepwise strategy being feasible since it has a lower project threshold, $\hat{V}_1 < \hat{V}_0$, with $\hat{V}_0 < \hat{V}_2$ and viable since its stage-1 option value is greater. The indifference point between the two strategies occurs for $_2 = \tilde{\ }_2$ and $\hat{V}_1 = \tilde{\hat{V}}_1$. Then, the stage-1 option value is greater than that for the lumpy strategy provided $F_1\left(V < \tilde{\hat{V}}_1\right) > F_0\left(V < \tilde{\hat{V}}_1\right)$.

The values for the indifference factors, $\tilde{\ }_2$ and $\tilde{\hat{V}}_1$, vary according to the model parameters. Numerical simulations reveal that $\tilde{\ }_2$ increases and $\tilde{\hat{V}}_1$ decreases as a result of either ¥ increases or Δ decreases. As the stepwise strategy becomes increasingly more attractive due to a greater proportion of the value realized at stage-1, ¥, or a lower investment cost differential, Δ, the project value indifference threshold level declines, reflecting a relatively enhanced attractiveness. An increase in $_1$, which is common to both the lumpy and the stage-1stepwise strategies, also produces a rise in $\tilde{\ }_2$ and a consequential fall in $\tilde{\hat{V}}_1$ along with the fall in $\hat{V}_0$.

The preponderance of the indifference $\tilde{\ }_2$ solutions lies below the $_1$ value and any exceptions only occur for significantly high ¥ values. Because of this, the focus is identifying the underlying parameter values conducive to achieving a sufficiently low $_2$ value that results in the stepwise strategy being more greatly favoured. Achieving a $_2$ value below $_1$ requires either a greater stage-2 volatility or a lower stage-2 failure probability. A volatility increase between stages-1 and -2, with $\pm_2 > \pm_1$, arises whenever the part project actualization following the stage-1 exercise reveals greater market or country acceptability for the project offering than was originally envisaged. Since stage-2 creates this additional value, the stepwise strategy becomes more attractive. Similarly, a lower stage-2 failure probability implies that the competitive or operating conditions during actualization becoming more favourable so learning becomes a source of value creation.

Our model could be extended to allow for inhibited or enhanced second stage project values, or even reduced investment costs, due to a learning effect, in arriving at those optimal trade-offs. Further extensions are to n stages, or allowing for time for investment completion, stochastic K, and considering competition.

Figure 1a
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$

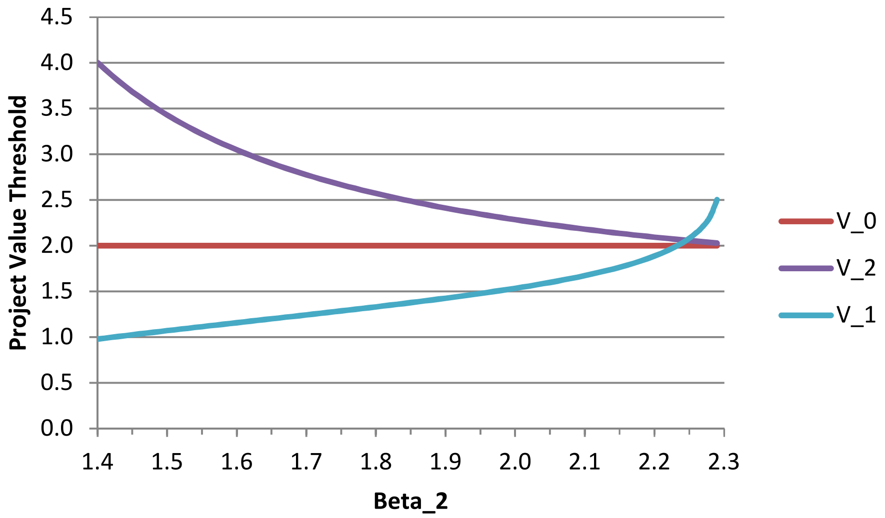
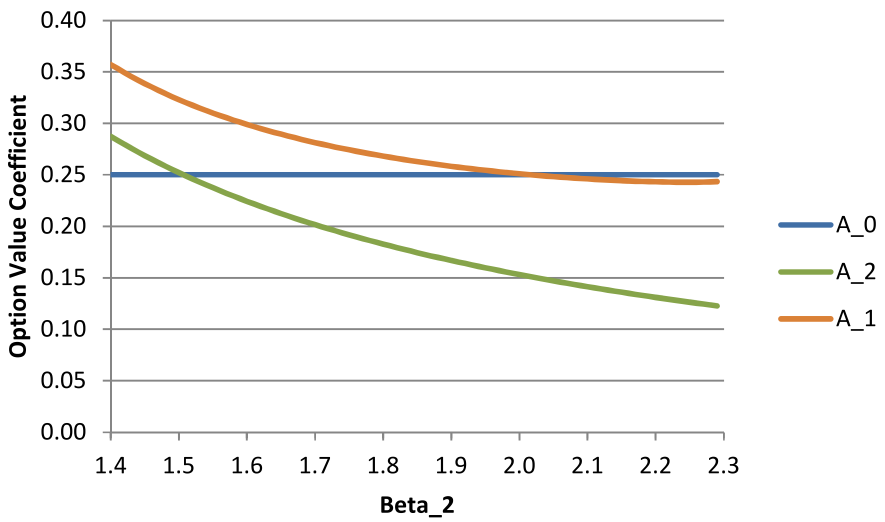


Figure 1b
Option Value Coefficients for the Lumpy and Stepwise Models
for Variations in $_2$



Variations in $_2$ have no effect on the lumpy strategy solution, and calculated values for the project threshold and option coefficient are $\hat{V}_0 = 2.00$ and $A_0 = 0.25$. Solution values for the stepwise strategy given some representative $_2$ variations are presented in the following table.

$_2$	$\hat{V}_1$	$\hat{V}_2$	A_1	A_2
1.40	0.97722	4.00000	0.35732	0.28717
1.50	1.06904	3.42857	0.32313	0.25203
1.60	1.15626	3.04762	0.29896	0.22418
1.70	1.24204	2.77551	0.28127	0.20152
1.80	1.32988	2.57143	0.26809	0.18268
1.90	1.42446	2.41270	0.25822	0.16676
2.00	1.53333	2.28571	0.25095	0.15313
2.10	1.67195	2.18182	0.24592	0.14131
2.20	1.88591	2.09524	0.24310	0.13098

It can be shown that for the stepwise strategy to be more attractive than the lumpy strategy, when $A_1 > A_0$, $_2 \leq 2.01606$, with indifference occurring at the equality where $\hat{V}_1 = 1.55302$ and $\hat{V}_2 = 2.26767$.

Figure 2a
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ = 0.24

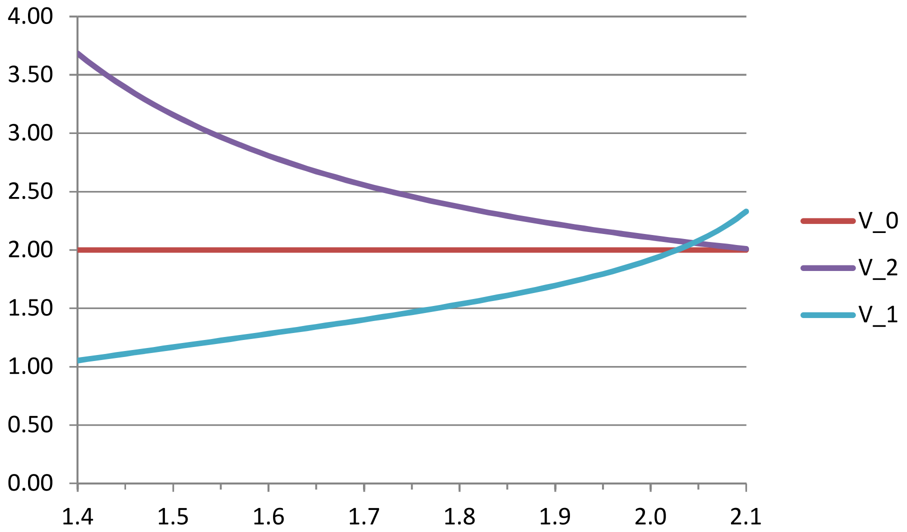


Figure 2b
Option Value Coefficients for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ = 0.24

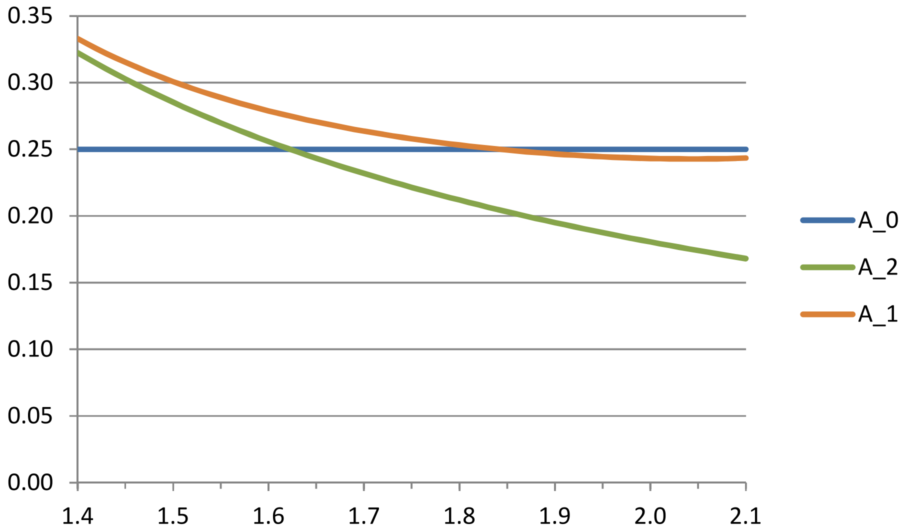


Figure 2c
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ = 0.22

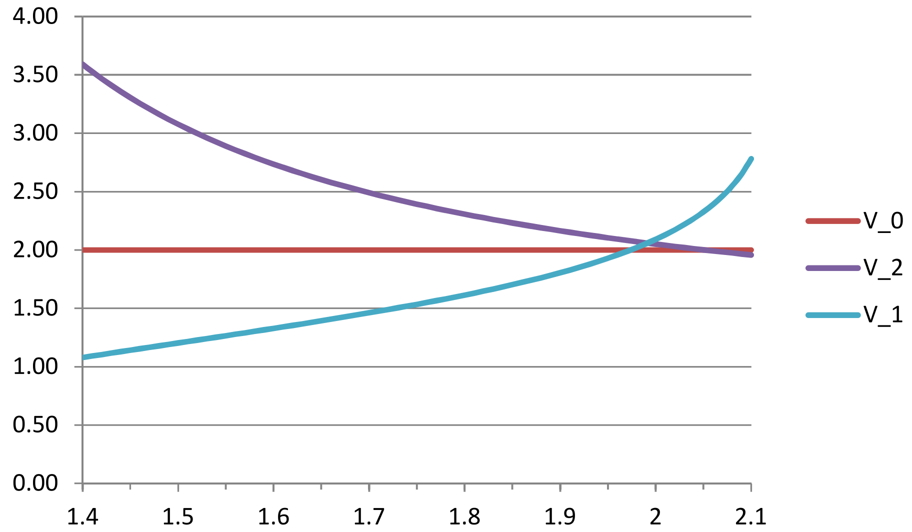


Figure 2d
Option Value Coefficients for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ = 0.22

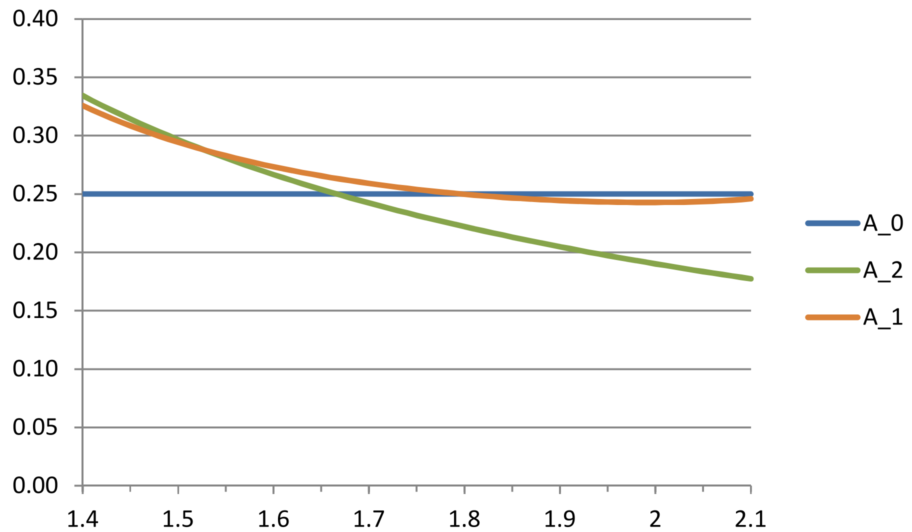


Figure 2e

Project Value Thresholds for the Lumpy and Stepwise Models

For Variations in $_{2}$ with ¥ $= 0.20$

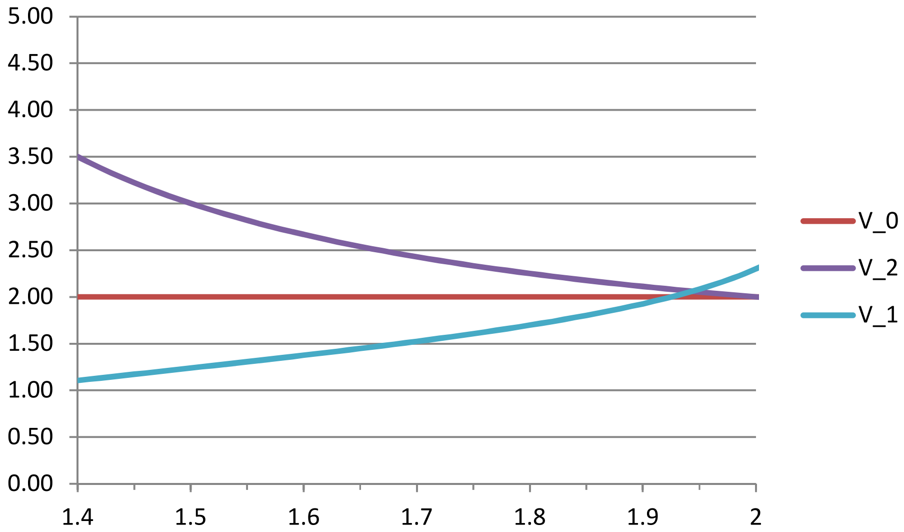


Figure 2f

Option Value Coefficients for the Lumpy and Stepwise Models

For Variations in $_{2}$ with ¥ $= 0.20$

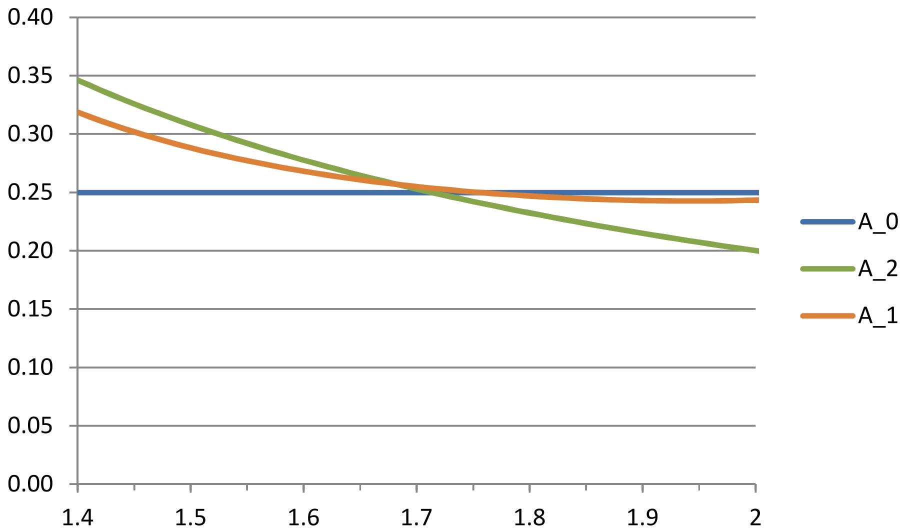


Figure 3a
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.24$ and $_1 = 2.5$

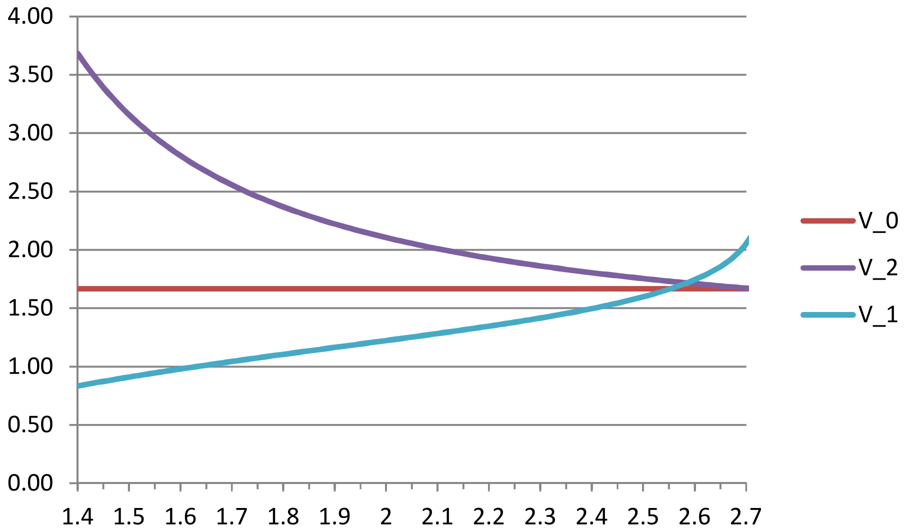


Figure 3b
Option Value Coefficients for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.24$ and $_1 = 2.5$

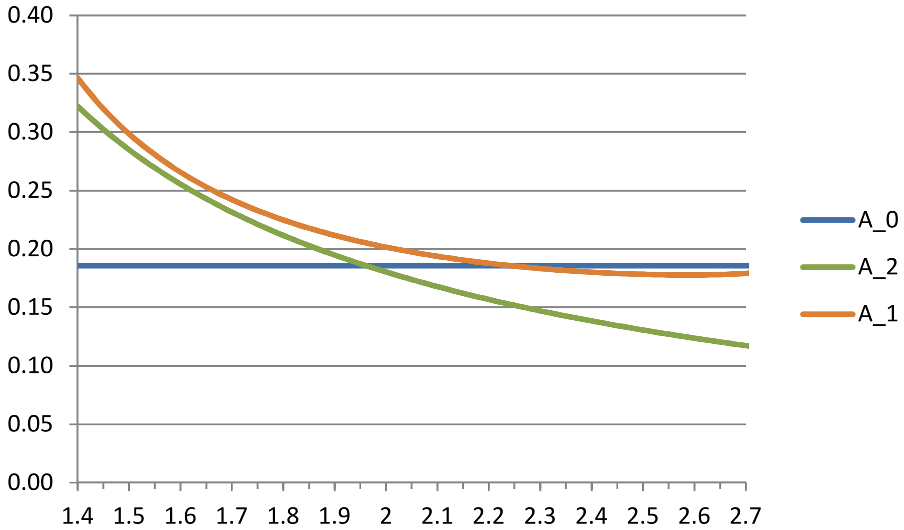


Figure 3c
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.20$ and $_1 = 2.5$

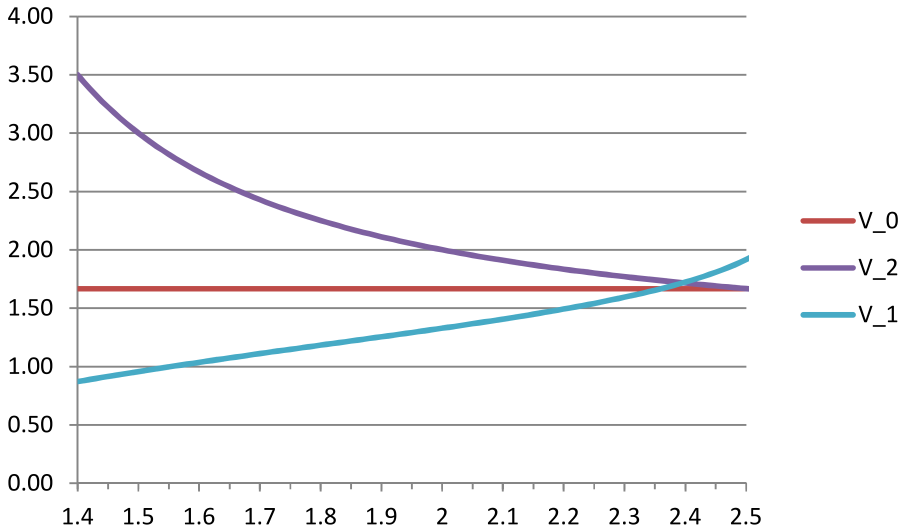


Figure 3d
Option Value Coefficients for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.20$ and $_1 = 2.5$

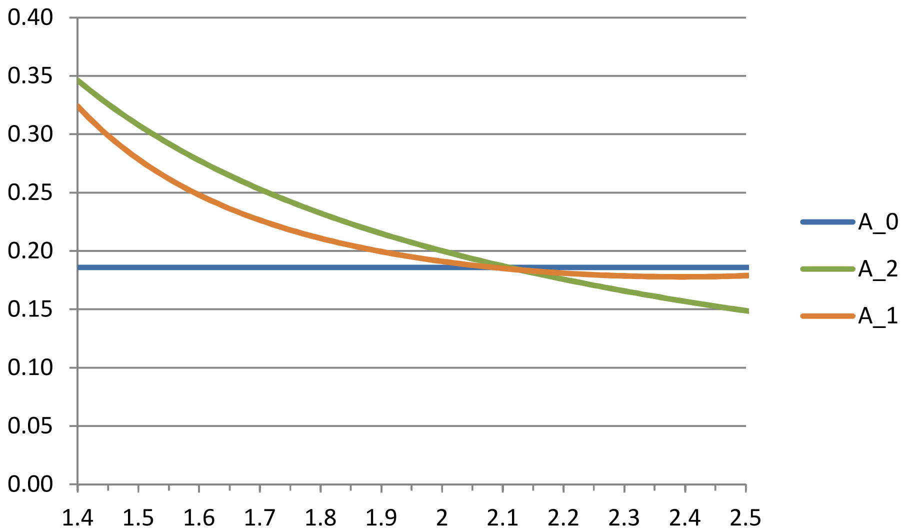


Figure 3e
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.24$ and $_1 = 3.0$

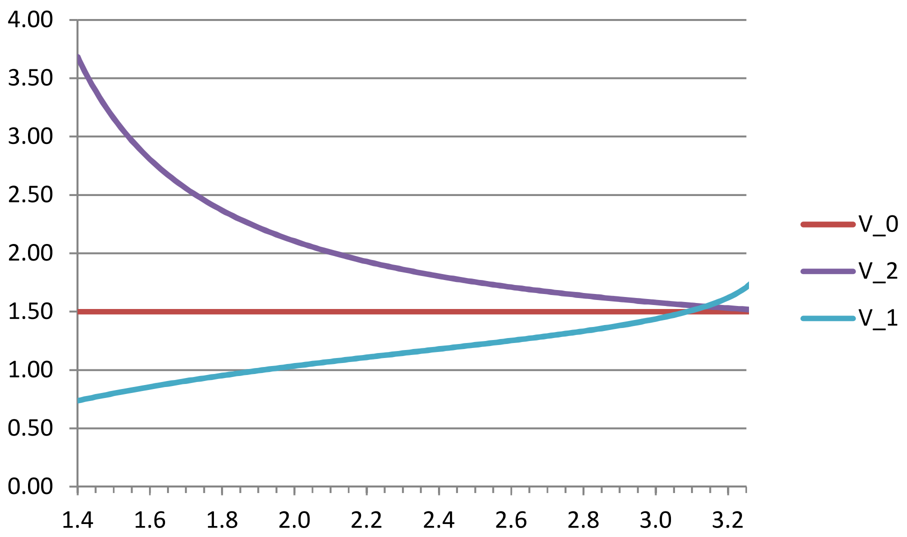


Figure 3f
Option Value Coefficients for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.24$ and $_1 = 3.0$

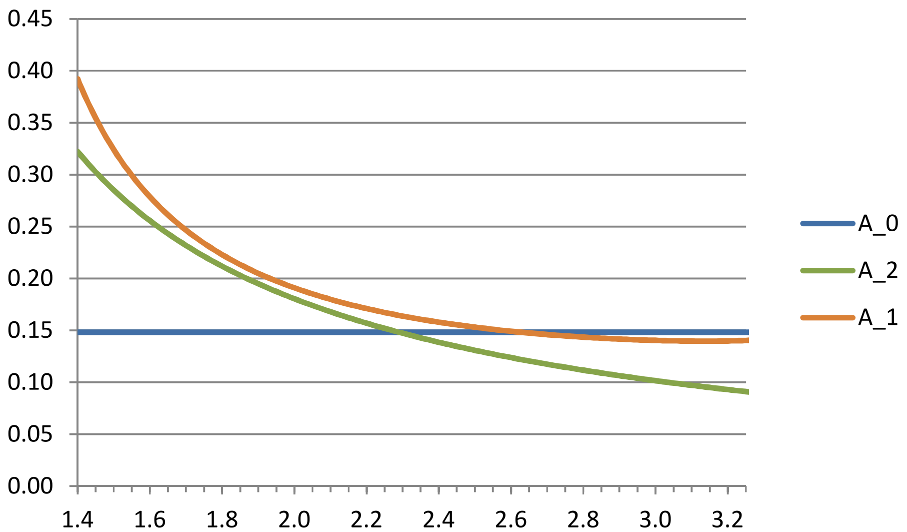


Figure 3g
Project Value Thresholds for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.20$ and $_1 = 3.0$

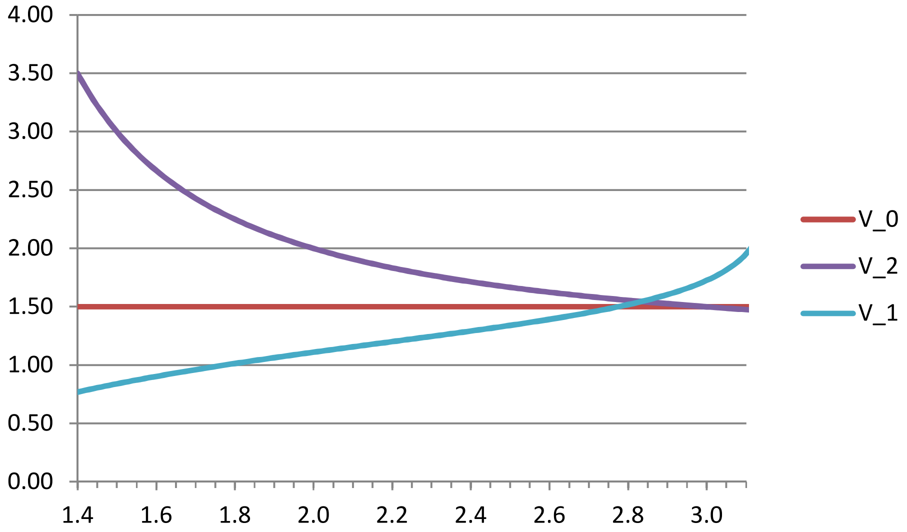


Figure 3h
Option Value Coefficients for the Lumpy and Stepwise Models
For Variations in $_2$ with ¥ $= 0.20$ and $_1 = 3.0$

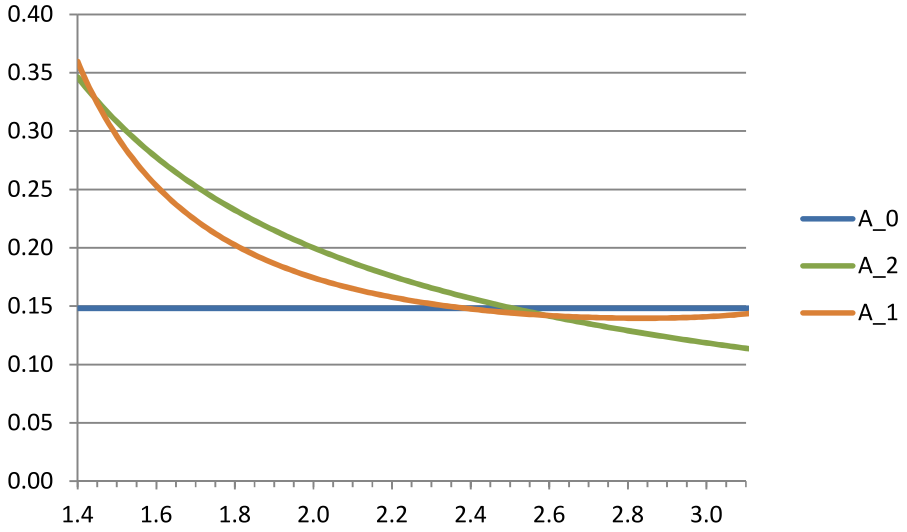


Table 1

Base Case Specification

Lumpy option power parameter	$_1$	2.0
Stage-1 stepwise proportional project value	¥	0.3
Stage-1 stepwise investment cost	K_1	0.23
Stage-2 stepwise investment cost	K_2	0.80
Lumpy investment cost	K_0	1.00

Table 2
The Effect of Changing ¥ on the Indifference Factors
between the Lumpy and the Stepwise Strategy

¥	$\tilde{\ }_2$	$\tilde{\hat{V}}_1$	$\tilde{\hat{V}}_2$
0.20	1.7558	1.6165	2.3231
0.22	1.7961	1.6071	2.3140
0.24	1.8411	1.5964	2.3041
0.30	2.0161	1.5530	2.2677

Table 3

The Effect of Changing ¥ and Stage-1 and -2 Investment Costs
on the Indifference Factors between the Lumpy and the Stepwise Strategy

K_1	K_2	¥	$\tilde{\ }_2$	$\tilde{\hat{V}}_1$	$\tilde{\hat{V}}_2$
0.21	0.80	0.20	1.8741	1.7191	2.1441
0.21	0.80	0.22	1.9217	1.7086	2.1384
0.25	0.80	0.22	1.7028	1.5695	2.4852
0.25	0.80	0.24	1.7410	1.5607	2.4732
0.23	0.78	0.22	1.8685	1.7327	2.1515
0.23	0.78	0.24	1.9169	1.7233	2.1456
0.23	0.82	0.20	1.7120	1.5501	2.4647
0.23	0.82	0.22	1.7501	1.5406	2.4529

Table 4

Variations in the Indifference Factors Levels due to changes in $_1$ and ¥

$_1$	¥	$\tilde{}_2$	$\hat{V}_0$	$\tilde{\hat{V}}_1$	$\tilde{\hat{V}}_2$
2.0	0.24	1.8411	2.0000	1.5964	2.3041
2.5	0.24	2.2395	1.6667	1.3723	1.9019
3.0	0.24	2.6278	1.5000	1.2628	1.6993
2.0	0.20	1.7505	2.0000	1.6028	2.3283
2.5	0.20	2.0837	1.6667	1.3932	1.9228
3.0	0.20	2.3852	1.5000	1.2841	1.7219

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