Entry deterrence by timing rather than overinvestment in a strategic real options framework

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Abstract

This paper considers an incumbent-entrant framework, where the incumbent has the option to extend his current capacity and where the entrant has the option to enter the market by a capacity investment. Our model explicitly considers both optimal timing of the investment and setting the capacity level at the moment of investment. Where in the literature entry deterrence is done by overinvestment, we find instead that entry deterrence takes place by timing: the presence of a potential entrant gives the incumbent the incentive to invest first. The incumbent only invests a small amount, which is, however, large enough to delay a larger investment by the entrant.

We also consider the situation where the investment decision involves only timing, i.e. the capacity decision is given. In such a case we find that the investment order changes, i.e. now the entrant invests before the incumbent does.

Keywords: Incumbent/Entrant, Capacity choice, Investment under Uncertainty, Oligopoly, Game Theory

JEL classification: C73, D92, L13

1 Introduction

In a traditional set-up, where a two period model is considered, entry deterrence leads to eternal absence of the competitor on the market. After installing a sufficiently large capacity by the incumbent, the potential entrant finds the market not profitable enough to undertake an investment. In a continuous time model with an infinite planning horizon, however, it cannot be expected that potential entrants are perpetually deterred from the market. Huisman and Kort (2015) studied this problem for two symmetric entrants on a new market. In our case an established market exists on which an incumbent operates that faces a potential entrant.

This paper considers a dynamic model where both an incumbent and an entrant have the option to acquire some (additional) production capacity. Both firms are free to choose the size of their installment. As a first result, we find that the incumbent is most eager to undertake the investment first. In this way the incumbent accomplishes that it delays the investment of the entrant and it extends its monopoly period. The entrant reacts by waiting until the demand on the market has grown in order to install a larger capacity than it would install if it preempts the incumbent. This result is always true, except when the incumbent is initially present on the market with only an extraordinarily small production size and/or when quantity hardly affects price.

A second important result is that entry deterrence is not achieved via overinvestment, but via timing. The incumbent invests soon in order to precede investment of the entrant. Since the incumbent's investment increases the quantity on the market the output price is reduced, which reduces the profitability of entering this market, and thus delays entry. Overinvestment is defined as the difference in capacity size between a firm that expands as a monopolist and a firm that expands due to threat of entry. Where other papers find that a monopolist sets a smaller capacity than a duopolist facing a threat of entry, we find the opposite result. Since the incumbent invests early, i.e. in a market with a still relatively small demand, it will only want to acquire a small capacity expansion. The monopolist, however, could easily wait for a market with a higher demand and invest in a larger capacity. In other words, when considering deterrence, timing is of greater importance than overinvesting.

A crucial aspect of these results is that the size of the investment is flexible. We consider an additional model in this paper where investment sizes are fixed. As a result, the incumbent no longer has the possibility to undertake a small investment in a small market in order to preempt the entrant. Interestingly, we find that now the investment order is reversed; the entrant undertakes an investment first. In this situation the incumbent suffers a larger cost than the entrant, because, while the investment size and thus investment costs are equal, the entrant suffers no cannibalization. Being able to choose the investment size is thus of determinative importance for obtaining these results.

This paper has qualitative new results in two main literature streams. In the first place, it contributes to the studies where incumbent firms try to ward off entrants, i.e. the incumbent-entrant literature. In the second place, it supplies the literature on strategic real option theory.

A substantial part of the incumbent-entrant literature follows the "eat your own lunch before someone else does" theory (Deutschman, 1994), stating that incumbent firms better cannibalize their own demand to the benefit of staying monopolist in the market. To that extend, incumbents expand their capacity to deter other firms from the market. Contributions are made in this area by Nault and Vandenbosch (1996), Robles (2011), Liu *et al.* (2006), Akcura *et al.* (2013), Crampes and Hollander (1993) and others. Nault and Vandenbosch (1996) show that incumbent firms expand earlier compared to the case where competition is absent. The main difference between their work and ours is that our model includes an exogenous shock process. Moreover, in their formulation expansion is done as a result of technological development. Next-generation advantages are created by launching a new product on the market that is vertically differentiated from the current product. In our model we assume homogenous products. Robles (2011) develops a 2-period game where incumbents raise excess capital to deter other firms from the market. This study makes clear that under certain assumptions there exist sets of parameter values where incumbents deter entry by maintaining idle capacity. Liu *et al.* (2006) consider online markets where an incumbent retailer deters online entry of an e-tailer by either abstaining from participation in this new market or by entering the online market while differentiating oneself as an independent profit center. The same type of market is studied by Akcura *et al.* (2013) where an incumbent firm only expands in case of an impending threat of entry by preempting the entrant and deterring it from the market through cannibalization. However, when the entrant's product differentiation is qualitatively below a certain threshold, entry is accommodated. The incumbent firm defined by Crampes and Hollander (1993) holds off entrants' investment by setting sufficiently low prices. In Swinney *et al.* (2011) established firms accommodate start-up firms in case of high demand uncertainty and when costs do not decrease rapidly in future.

Another related analysis is performed by Yang and Zhou (2007). They study the entry deterrence strategy is when applied by the incumbent firm. They do not consider the capacity choice problem, nor do they consider the possibility that the entrant preempts the incumbent. Entry accommodation is not taken into consideration either. They show that the entrant can successfully be deterred by means of excess capacity. Maskin (1999) carries out a study similar to Yang and Zhou (2007), but then looks at a static model.

Our paper extends, in the second place, the literature on strategic real option models, where firms have to decide about investing in a stochastic oligopolistic environment. Early work includes Smets (1991) and Grenadier (1996). Like most of the papers in this field the investment decision only involves the timing of investment. However, we study a problem where firms are free to choose their capacity levels. To our knowledge, within a strategic real options framework, investment decisions involving both capacity choice and timing have first been implemented by Huisman and Kort (2015). Our paper differs from their analysis by assuming one of the players to have an initial capacity on the market, by which we have obtained an incumbent-entrant framework.

This paper is organized in the following way. Section 2 explains the model and discusses its assumptions. Section 3.1 looks at the case of exogenous firms roles, i.e. an individual firm knows beforehand whether it will be the first or second investor. Then the other firm can choose to invest at the same time or later. This is followed by Section 3.2 studying the game when endogenizing investment roles, i.e. both firms are allowed to become the first investor. Section 4 presents some additional studies regarding investment timing and capacity choice. Robustness checks are performed in Section 5. The last section, Section 6, considers the problem from the point of view of the social planner. The paper is concluded in Section 7.

2 Model

Consider an industry where one firm is actively producing. This firm is called the incumbent and is denoted as firm I. It now faces threat of an entrant, through which it might loose its monopoly position. The inactive firm is denoted as firm E. Both firms have a one-off investment opportunity in this market. For firm I this means an expansion of its current capacity, for the entrant, for firm E, an investment means starting up production and entering this market. Both firms are assumed to be rational and risk neutral and they are assumed to be value maximizing. The firm that invests first is called the leader. The firm that makes the final investment is called the follower. The inverse demand function on this market is chosen to be multiplicative[1] and equals

$$p(t) = x(t)(1 - \eta Q(t)),$$

where $Q(t)$ equals the total aggregate quantity made available at time t and where η is a fixed price sensitivity parameter. $x(t)$ is the value of an exogenous shock process, it follows a geometric Brownian motion, i.e.

$$\mathrm{d}x(t) = \alpha x(t)\mathrm{d}t + \sigma x \mathrm{d}z(t).$$

Here, α and σ are the fixed trend and volatility parameters, $z(t)$ is a Wiener process. It is assumed that, at present, the market demand is too small to invest for either of the firms, i.e. $x(0)$ is small.[2] Discounting takes place under a fixed rate $r > \alpha$. The investment costs are linearly related to the investment size, where the marginal cost parameter equals δ. The inverse demand function is chosen to be in line with Huisman and Kort (2015), giving a linear relation between the production size and the output price. This relation is also used by e.g. Pindyck (1988), He and Pindyck (1992), Aguerrevere (2003) and Wu (2007). As of now, the subscript t shall be omitted unless necessary. In this model firms are fully commited, that is, firms always produce the amount their capacity allows. This assumption is used throughout the literature (e.g. Deneckere *et al.* (1997), Chod and Rudi (2005), Anand and Girotra (2007), Goyal and Netessine (2007) and Huisman and Kort (2015)).

The investment comprises two decisions: timing and capacity size. The game is solved backwards, first determining the reaction curves of the final investing firm and then determining the optimal strategies of the firm that invests first. In this way all subgame perfect equilibria are determined.

Initial capacity size

The incumbent firm is currently active on the market with capacity q_{1I}. We will call this capacity the initial capacity. The current market, that is, the market before the incumbent expands and the entrant undertakes the investment, is referred to as the old market or the initial market. In principal, q_{1I} can take any value, for it is considered to be a parameter. However, one can impose an upper bound to this value.

[1] In Section 5.2 the robustness of our results will be tested by analyzing a different demand function.
[2] Throughout the paper the initial value of the process is denoted as $x(0) = X$.

Myopic investment is defined as the investment decision when not taking into account any future investment activity. Here, one assumes that the project will last eternally and no other investment is undertaken by any firm. Following the analysis from Huisman and Kort (2015), one finds that, when investing myopically, one ends up with an initial capacity that equals the monopolist's optimal capacity level. We denote this as q_{1I}^{myop}. This value can be considered to be an upper bound to the initial capacity, for firms have no incentives to invest at a larger capacity when taking into account potential future investment. In this paper we assume the initial capacity to be equal to the value under myopic investment, unless stated otherwise.

3 Capacity choice

This section starts with the study where investment roles are taken exogenously. That means that the order in which the firms invest is fixed. This is done by first looking at the optimal decisions of the follower. Next, the leader's strategies are studied. There are multiple ways the leader's capacity influences the follower's decision. Setting a large capacity as the leader, will result in a low market price, so that immediate investment is certainly not optimal for the follower. In this way the follower's investment is temporarily deterred. Likewise, when setting a sufficiently low leader's capacity, the follower enters immediately after the leader's investment. The tipping point considering the leader's capacity, i.e. the leader's capacity below which the follower invests immediately and above which the follower is deterred, can only be found by first examining the follower's strategies. The main difference with the situation where firm roles are endogenized is that the other firm is obliged to wait for the leader to invest. The firms do not compete to become leader which leads to different investment triggers.

3.1 Exogenous firm roles

First the general case is studied, where the following firm is denoted as firm F and similarly, the leading firm is denoted as firm L. The follower's and leader's initial capacities are denoted by q_{1F} and q_{1L} respectively. Expansion of the market size is done by installing extra quantities q_{2F} and q_{2L}. There are two cases possible. In one case the incumbent is the firm that expands before the entrant undertakes an investment. In that situation the incumbent takes the role of the leader and the entrant takes the role of the follower, with $q_{1L} = q_{1I}$, $q_{1F} = 0$, $q_{2L} = q_{2I}$ and $q_{2F} = q_E$. In the other case the entrant undertakes an investment before the incumbent expands, then $q_{1L} = 0$, $q_{1F} = q_{1I}$, $q_{2L} = q_E$ and $q_{2F} = q_{2I}$. In this section, both cases are analyzed simultaneously.

Follower's decision

Consider the situation where one firm, the leader, has already invested. Suppose the market has grown sufficiently large for the follower to make an investment, i.e. the current value of the process x, defined as $X = x(0)$, is sufficiently large. One then obtains the following value function reflecting the follower's aggregate expected total payoff,

$$\begin{aligned}V_F(X, q_{1L}, q_{1F}, q_{2L}, q_{2F}) &= \mathbb{E}\left[\int_{t=0}^{\infty}(q_{1F}+q_{2F})P(t)e^{-rt}\mathrm{d}t - \delta q_{2F} \;\middle|\; x(0)=X\right] \\ &= \frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) - \delta q_{2F}.\end{aligned}$$

The follower's value function consists of two terms. The total expected discounted future payoffs are reflected by the first term. The involved costs, when making the investment, are captured by the second term. The optimal quantity is found by optimizing the value function.

To determine the optimal moment of investment we derive the investment threshold $X_F^*(q_{1L}, q_{1F}, q_{2L})$. Investment takes place at the moment the stochastic process X reaches this level for the first time (see, e.g., Dixit and Pindyck (1994)). Thereto one first needs the value function of the follower $F_F(X, q_{1L}, q_{1F}, q_{2L})$ before it invests. The elaborations can be found in the appendix. One then finds,

$$F_F(X, q_{1L}, q_{1F}, q_{2L}) = A_F(q_{1L}, q_{1F}, q_{2L})X^{\beta} + \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})),$$

where

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}. \tag{1}$$

The value function F_F consists of two terms. The second term represents the current profit stream. In case the incumbent is follower, this stream is positive whith $q_{1F} = q_{1I}$. When the entrant is the follower one has $q_{1F} = 0$ leading to zero current profits. The first term is the current value of the option to invest.

The following proposition presents the follower's optimal investment strategy.[3]

Proposition 1 *Let the current value of the stochastic demand process be denoted by X, and let the initial production capacity be denoted by q_{1L} and q_{1F} respectively for the leader and the follower. Let the capacities associated with the investments be denoted by q_{2L} and q_{2F} respectively for the leader and the follower. Then the value function of the follower can be partitioned into two regions: for small X the firm waits until it reaches the investment trigger X_F^* and for $X \geq X_F^*$ the firm invests immediately. As a result, the follower's value function $V_F^*(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ is given by*

$$V_F^* = \begin{cases} A_F(q_{1L}, q_{1F}, q_{2L})X^{\beta} + \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})) & \text{if } X < X_F^*, \\ \frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) - \delta q_{2F} & \text{if } X \geq X_F^*, \end{cases} \tag{2}$$

where the optimal capacity level for the follower q_{2F}^*, the investment trigger X_F^* and A_F are defined by

$$q_{2F}^*(X, q_{1L}, q_{1F}, q_{2L}) = \frac{1}{2\eta}\left(1-\eta(q_{1L}+2q_{1F}+q_{2L}) - \frac{\delta(r-\alpha)}{X}\right), \tag{3}$$

$$X_F^*(q_{1L}, q_{1F}, q_{2L}) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}, \tag{4}$$

[3]Note that all of these formulas are analogous to the ones as obtained in Huisman and Kort (2015), where one assumes $q_{1L} = q_{1F} = 0$.

$$A_F(q_{1L}, q_{1F}, q_{2L}) = \frac{\delta^2(r-\alpha)}{\eta(\beta-1)^2}\left(\frac{1-\eta(q_{1L}+2q_{1F}+q_{2L})}{\delta(r-\alpha)}\frac{\beta-1}{\beta+1}\right)^{\beta+1}. \tag{5}$$

The follower's capacity in case the follower invests at the investment trigger equals

$$q^*_{2F}(X^*_F, q_{1L}, q_{1F}, q_{2L}) = \frac{1-\eta(q_{1L}+2q_{1F}+q_{2L})}{\eta(\beta+1)}. \tag{6}$$

There exists, as can be inferred from equation (4), a positive relation between the leader's investment quantity q_{2L} and the follower's investment threshold. The leader thus deters the follower by setting q_{2L} in such a way that the follower's trigger X^*_F exceeds the current value of x. To that extend, there exists a $\hat{q}_{2L}$ such that for $q_{2L} > \hat{q}_{2L}$ it holds that $X < X^*_F$. This complies with the deterrence strategy as described in the introductory section. The accommodation strategy, i.e. the scenario where the follower immediately joins the leader in making an investment, only applies when $q_{2L} \leq \hat{q}_{2L}$. So, if the current value of x falls below X^*_F, in the deterrence case, the follower sets a capacity equal to the quantity given by equation (6). In any other case, the follower's capacity equals the amount given by equation (3) evaluated at the value of X at the investment moment.

Leader's decision

As stated, the leader's quantity tipping point is denoted by $\hat{q}_{2L}$ and can be found as the point for which $X = X^*_F$, when $q_{2L} = \hat{q}_{2L}$. Rewriting $X = X^*_F$ from equation (4), gives that the value of $\hat{q}_{2L}$ equals

$$\hat{q}_{2L}(X, q_{1L}, q_{1F}) = \frac{1}{\eta}\left[1-\eta(q_{1L}+2q_{1F}) - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right].$$

It follows that for $q_{L,2} > \hat{q}_{2L}$ the deterrence strategy is played. Conversely, when $q_{L,2} \leq \hat{q}_{2L}$, the accommodation strategy is played. In this section, first the deterrence strategy is examined, followed by the accommodation strategy.

Entry deterrence

The value function corresponding to investment under the deterrence strategy consists of two separate integrals. The first one denotes the expected discounted revenue stream obtained by the leader before the follower has invested. Then, at moment $t_F \geq 0$ the follower decides to make an investment, where,

$$t_F = \inf\{t \geq 0 \mid x(t) \geq X^*_F\}.$$

The second integral reflects the leader's expected discounted revenue stream from the moment the follower starts (additional) production. One then obtains $V^{det}_L(X, q_{1L}, q_{1F}, q_{2L})$ defined by

$$\begin{aligned} V^{det}_L &= \mathbb{E}\Bigg[\int_{t=0}^{t_F}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}))e^{-rt}\mathrm{d}t \\ &\quad + \int_{t=t_F}^{\infty}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}+q^*_{2F}))e^{-rt}\mathrm{d}t\Big|x(0)=X\Bigg] - \delta q_{2L} \\ &= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L})) - \frac{\delta}{\beta-1}(q_{1L}+q_{2L})\left(\frac{X}{X^*_F}\right)^{\beta} - \delta q_{2L}. \end{aligned}$$

Evaluating the second equation, one can interpret the terms as follows. The first term reflects the total expected future cash flows in case the follower will never invest. The second term corrects the first term in that it negatively adapts the cash flow stream from the moment the second firm makes an investment. The second term includes the stochastic discount factor $\mathbb{E}\left[e^{-rt_F}\right] = \left(\frac{X}{X_F^*}\right)^\beta$ where the investment moment for the follower equals $t = t_F$. The final term, as before, consists of the investment costs.

Similar to the case of the follower, one can find the optimal capacity by optimizing the leader's value function. The capacity for the deterrence strategy has to satisfy the constraint that the firm invests in a capacity that is sufficiently large to deter entry, $q_{2L} > \hat{q}_{2L}$.

Considering the shock process x, there exists a region in which the deterrence strategy is considered to be feasible. As explained below, this region is denoted by lower bound X_1^{det} and upper bound X_2^{det}.

In principle, the optimal entry deterrence capacity level, q_L^{det}, can be determined for all different values of $x(0)$. However, considering the lower bound $\hat{q}_{2L}$ on the capacity one can infer that for some values of x the optimum does not lie within the feasible region, i.e. for some $x(0)$ one has $q_L^{det} < \hat{q}_{2L}$. Consequently, for these values of x, the optimum is then attained at the boundary, $q_L^{det} = \hat{q}_{2L}$, leading already to immediate entry of the follower. Thus, for these scenarios the deterrence strategy does not apply. One can prove that there exists a boundary level X_2^{det} such that for $x(0) < X_2^{det}$ it holds that $q_L^{det} > \hat{q}_{2L}$ and vice versa for $x(0) > X_2^{det}$. The deterrence region, hence, does not exist for $x(0) > X_2^{det}$. It would be too costly for the leader to deter investment, for the demand has grown so large that the incentive for the follower to invest at the same time as the leader is very high. The gain from temporary deterrence does not weigh out the involved costs, since the required capacity level is too high to delay the follower's investment.

The value X_2^{det} is the value of x such that for $x(0) < X_2^{det}$, one has that $q_L^{det} > \hat{q}_{2L}$, or put differently, one has that $x(0) < X_F^*$. It should then hold that for $x(0) = X_2^{det}$ one obtains $x(0) = X_F^*$ when evaluating the latter at the leader's optimal capacity given the leader invests at that moment. One can conclude that X_2^{det} can be determined by solving

$$X_F^*(q_{1L}, q_{2L}, q_L^{det}(X, q_{1L}, q_{1F})) = X.$$

The lower bound to the region of the shock process x for which the deterrence strategy can be utilized is defined as the smallest value of x such that the optimal capacity q_L^{det}, corresponding to this value of $x(0)$, is non-negative. In other words, X_1^{det} is determined as X such that $q_L^{det}(X, q_{1L}, q_{1F}) = 0$. One can prove that this value is unique. The following proposition summarizes the entry deterrence strategy.

Proposition 2 *Let the production capacities be defined as in Proposition 1 and let the current value of the shock process be defined as X. Then the deterrence strategy leads to value function $V_L^{det}(X, q_{1L}, q_{1F}, q_{2L})$,*

$$V_L^{det} = \frac{X}{r-\alpha}(q_{1L} + q_{2L})(1 - \eta(q_{1L} + q_{1F} + q_{2L})) - \frac{\delta}{\beta - 1}(q_{1L} + q_{2L})\left(\frac{X}{X_F^*}\right)^\beta - \delta q_{2L}, \tag{7}$$

where $X_F^*(q_{1L}, q_{1F}, q_{2L})$ is defined as equation (4).

For large initial values of X the leader invests immediately and chooses optimal capacity

$$q_L^{det}(X, q_{1L}, q_{1F}) = \text{argmax}\{V_L^{det}(X, q_{1L}, q_{1F}, q_{2L}) \mid q_{2L} > \hat{q}_{2L}\}, \tag{8}$$

where,

$$\hat{q}_{2L}(X, q_{1L}, q_{1F}) = \frac{1}{\eta}\left[1 - \eta(q_{1L} + 2q_{1F}) - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right].$$

The entry deterrence strategy is considered for $X \in (X_1^{det}, X_2^{det})$, where

$$X_1^{det} = \{X \mid q_2^{det}(X, q_{1L}, q_{1F}) = 0\},$$
$$X_2^{det} = \frac{\beta+1}{\beta-1}\frac{2\delta(r-\alpha)}{1-(\beta+3)\eta q_{1F}}.$$

For low initial values of x, that is $x(0) < X_L^{det}$, the leader invests at the moment x reaches the investment threshold value X_L^{det}. The value of the investment threshold and the associated capacity level q_L^{det} are determined as the solution of the set of equations determined by equation (8) and

$$X_L^{det}(q_{1L}, q_{1F}, q_{2L}) = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta q_{1L} - \eta q_{1F} - \eta q_{2L}}.$$

The value function before investment is defined as

$$F_L^{det}(X, q_{1L}, q_{1F}, q_L^{det}) = \frac{X}{r-\alpha}q_{1L}(1-\eta(q_{1L}+q_{1F})) + \left(\frac{X}{X_L^{det}}\right)^\beta \frac{\delta q_L^{det}}{\beta-1} - (q_{1L}+q_L^{det})\left(\frac{X}{X_F^*}\right)^\beta \frac{\delta}{\beta-1}. \tag{9}$$

Entry accommodation

The value function of the leader, when applying the accommodation strategy, considers immediate investment of both firms where the leader becomes the Stackelberg capacity leader. The value function contains two terms, the accumulated future discounted payoffs resulting from investment and the investment cost,

$$\begin{aligned} V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L}) &= \mathbb{E}\left[\int_{t=0}^{\infty}(q_{1L}+q_{2L})x(t)(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^*))e^{-rt}\mathrm{d}t \middle| x(0) = X\right] - \delta q_{2L} \\ &= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^*)) - \delta q_{2L}. \end{aligned}$$

When applying the accommodation strategy the firm chooses its capacity $q_L^{acc}(X, q_{1L}, q_{1F})$ in such a way that it optimizes $V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L})$, given the restriction $q_L^{acc} < \hat{q}_{2L}$. The latter makes that the accommodation strategy is restricted to a certain region of x. When the shock process attains a relatively large value, the optimal quantity q_L^{acc} meets the restriction $q_L^{acc} < \hat{q}_{2L}$. However, for small values of x, the market is too small for two firms to enter at the same time and one observes that $q_L^{acc} > \hat{q}_{2L}$. Therefore, there exists a $X_0^{acc}(q_{1L}, q_{1F})$ such that for $x(0) < X_0^{acc}$ the accommodation strategy can not be applied for $q_L^{acc} > \hat{q}_{2L}$. The Appendix includes the derivation of X_0^{acc}. There is no upper bound to the accommodation strategy considering the shock process.

There is a second lower bound to the accommodation region. Similar to the deterrence strategy, accommodation can only be considered to be feasible when capacity takes a non-negative value. As shown in the Appendix, q_L^{acc} is negative for $x(0) < \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$ and positive for the complementing region of x.[4]

[4]Note that q_{1F} cancels out when substituting the expression for q_{2F}^* in V_L^{acc}, leaving only q_{1L} to appear in q_L^{acc} and V_L^{acc}.

This means that investment only takes place for $x(0) \geq \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}$. Therefore, the overall lowerbound to the accommodation X_1^{acc} region is defined as the maximum of the two previously specified bounds.

Proposition 3 *Let the production capacities be defined as in Proposition 1 and let the current value of the shock process be defined as X. Then the accommodation strategy is considered for* $X \in (X_1^{acc}, \infty)$, *where,*

$$X_1^{acc} = \max\left\{\frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}}\ ,\ \frac{\delta(r-\alpha)}{1-2\eta q_{1L}}\right\}. \tag{10}$$

The accommodation strategy leads to value function $V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L})$,

$$V_L^{acc} = \frac{X}{r-\alpha}\tfrac{1}{2}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{2L})) - \tfrac{1}{2}\delta(q_{2L}-q_{1L}). \tag{11}$$

For large initial values of X the leader invests immediately and chooses optimal capacity

$$q_L^{acc}(X, q_{1L}) = \max\left\{\frac{1}{2\eta}\left[1-2\eta q_{1L}-\frac{\delta(r-\alpha)}{X}\right], 0\right\}. \tag{12}$$

For low values of $x(0) = X$, *that is* $X < X_L^{acc}$, *the leader will invest when x reaches investment threshold value* X_L^{acc}. *The value of the investment threshold and the associated capacity level* q_L^{acc} *are determined as the solution of the set of equations determined by equation (12) and*

$$X_L^{acc}(q_{1L}, q_{1F}, q_{2L}) = \frac{\delta(r-\alpha)\beta}{\beta-1}\frac{q_{2L}-q_{1L}}{(q_{2L}-q_{1L})(1-\eta q_{1L})-\eta q_{2L}(q_{2L}+q_{1L}))}. \tag{13}$$

The value function before investment is defined as

$$F_L^{acc}(X, q_{1L}, q_{1F}) = \frac{X}{r-\alpha}q_{1L}(1-\eta(q_{1L}+q_{1F})) + \left(\tfrac{X}{X_L^{acc}}\right)^{\beta}\frac{\delta q_L^{acc}}{\beta-1}.$$

Optimal timing

The deterrence and accommodation strategy are both to be considered by the leader. Deterrence can be considered for $X \in [X_1^{det}, X_2^{det}]$, and the accommodation region is defined for $X \in [X_1^{acc}, \infty)$. As a result, there is a region where $X \in [X_1^{acc}, X_2^{det}]$. In this, ever non-empty, region both strategies are available and the leader chooses the one yielding the largest project value. Then, there exists a point $\hat{X} \in [X_1^{acc}, X_2^{det}]$ that indicates the point where deterrence no longer yields a larger value and where the region starts in which accommodation is preferred, i.e. deterrence is optimal for $X_1^{det} \leq X < \hat{X}$ and accommodation is chosen for $X \geq \hat{X}$.

Considering the different possible values for $q_{1I} \leq q_{1I}^{myop} = \frac{1}{\eta(\beta+1)}$ a single strategy is found for the leader. Optimally, one waits in the region $0 \leq X < X_L^{det}$, plays deterrence in the region $X_L^{det} \leq X < \hat{X}$ and finds accommodation the best strategy for $X \geq \hat{X}$. Analogous to Huisman and Kort (2015), despite the accommodation trigger X_L^{acc} is an increasing function of the initial capacity size, it holds that $X_L^{acc} < \hat{X}$. Figure 1 illustrates the conclusion that

$$V_L(X, q_{1L}, q_{1F}) = \begin{cases} F_L^{det}(X, q_{1L}, q_{1F}) & \text{if } X \in (0, X_L^{det}) \\ V_L^{det}(X, q_{1L}, q_{1F}) & \text{if } X \in [X_L^{det}, \hat{X}) \\ V_L^{acc}(X, q_{1L}, q_{1F}) & \text{if } X \in [\hat{X}, \infty), \end{cases} \tag{14}$$

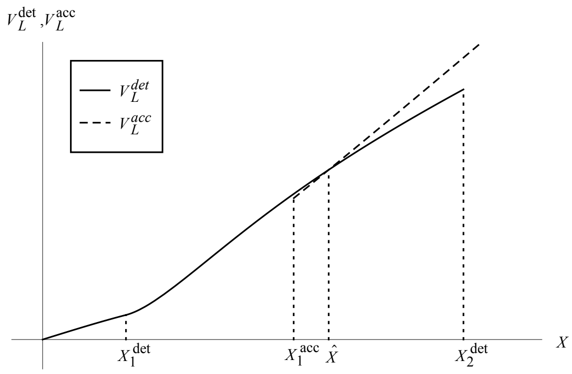


Figure 1: The leader's curves.

where

$$\hat{X}(q_{1L}, q_{1F}) = \{X \in [X_1^{acc}, X_2^{det}] \mid V_L^{acc} = V_L^{det}\}.^5$$

Optimal capacity equals q_L^{det} when investment is made in the deterrence region and q_L^{acc} when investment is made in the accommodation region. In the region where the firm waits, optimal investment takes place when x reaches X_L^{det} and for that reason the (expected) investment size in the waiting region is the capacity evaluated at the investment trigger. Taken together, this means that the (eventual) capacity the firm invests in, viewed for the current value of x, $x(0) = X$, equals

$$q_{2L}^*(X, q_{1L}, q_{1F}) = \begin{cases} q_L^{det}(X_L^{det}(q_{1L}, q_{1F}), q_{1L}, q_{1F}) & \text{if } X \in (0, X_L^{det}) \\ q_L^{det}(X, q_{1L}, q_{1F}) & \text{if } X \in [X_L^{det}, \hat{X}) \\ q_L^{acc}(X, q_{1L}, q_{1F}) & \text{if } X \in [\hat{X}, \infty). \end{cases}$$

Hence, for exogenous firm roles, assuming $x(0) = X$ to be sufficiently small, the firm waits until it reaches X_L^{det} and invests. Then the follower is deterred from the market until x reaches X_F^*, at which point in time the follower invests.

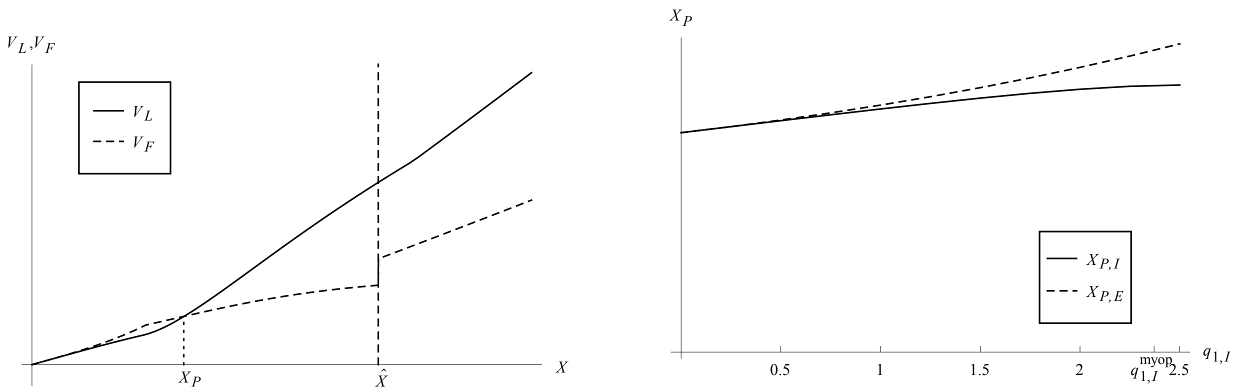
3.2 Endogenous firm roles

In the previous section the game was analyzed when the investment order is fixed ex-ante. This section aims to study a second type of game, where the investment order is endogenized, that is, firms compete in obtaining the best position in the investment order.

Preemptive investment

Figure 2a shows two curves as functions of the current value of state variable x. The solid curve corresponds to the outcome where firms take the leader role, equivalent to equation (14). When taking the position of the

[5] All examples in this paper use the following parameterization: $\alpha = 0.02,\ r = 0.1,\ \sigma = 0.1,\ \eta = 0.1,\ \delta = 1000,\ q_{1I} = \frac{1}{\eta(\beta+1)}$.



(a) Curves that correspond to becoming leader (solid curve) and follower (dashed curve). The follower is deterred for $X < \hat{X}$ and accommodated for $X \geq \hat{X}$.

(b) Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with free capacity choice for different values of q_{1I}.

Figure 2: Preemption points.

follower, one arrives at the dashed curve, coming from equation (2). For small values of X firms, evidently, prefer to wait, suggested by the preference for the follower's investment role. For larger values, though, it prefers to become leader. The curves are qualitatively similar to both firms. That means that when X is large enough both firms prefer to become leader. To prevent that the competitor undertakes an investment, and making the firm gain the follower value instead of the higher leader value, it is best to preempt the other firm by making an investment just a bit earlier. For this strategy is optimal to both firms, investment is made as early as possible, provided the leader's payoff exceeds the follower's payoff. Hence, the first moment for firms to invest, that is, when investment as a leader is interesting, is at the point where the leader curve no longer yields a larger value than the follower curve. This point is called the preemption point X_P, which is illustrated by Figure 2a. Formally, the preemption points of the incumbent and entrant are defined in the following way,

$$X_{PI} = \min\{X > 0 \mid V_L^{det}(X, q_{1I}, 0, q_L^{det}(X, q_{1I}, 0), q_{2,E}^*) = V_F(X, 0, q_{1I}, q_L^{det}(X, 0, q_{1I}), q_{2I}^*)\},$$
$$X_{PE} = \min\{X > 0 \mid V_L^{det}(X, 0, q_{1I}, q_L^{det}(X, 0, q_{1I}), q_{2I}^*) = V_F(X, q_{1I}, 0, q_L^{det}(X, q_{1I}, 0), q_{2,E}^*)\}.$$

For the two firms are asymmetric, their preemption points do not coincide. Assume the incumbent firm employed the myopic strategy, that is, the initial capacity on the market was chosen without consideration of future investments. The incumbent's initial capacity, following the monopoly model at Huisman and Kort (2015), equals $q_{1I}^{myop} = \frac{1}{\eta(\beta+1)} = 2.37$. Implementing this value, one obtains the following values: $X_{PI} = 134$ and $X_{PE} = 167$. This means that for $X < 134$ both firms prefer to wait, for $134 \leq X < 167$ the incumbent prefers to be leader and the entrant prefers to wait and for $X \geq 167$ both want to invest. For $x(0)$ is assumed to be small, the incumbent invests first, just before X reaches $X_{PE} = 167$. The investment trigger X_{LI}^{det} does not exist in this situation since the incumbent would not undertake an investment in case of exogenous firm roles (see Lemma 1 in the Appendix). Hence, the only reason the incumbent invests is strategic; investment delays investment by the entrant so that it can stay a monopolist for a longer time. The same exercise can be done for different values of q_{1I}. Figure 2b shows the resulting preemption points. The result stays the same: the incumbent invests first and deters the entrant.

To understand this result one must realize that any investment reduces prices, since the inverse demand function is negatively related with the total market output. Investment by the entrant thus reduces the incumbent's value. It is then better for the incumbent to do the, so called, cannibalization itself than let the entrant eat its demand. To do so, the incumbent will install a small capacity level: small in order not to make the cannibalization effect too large, but large enough to delay investment of the entrant. To conclude, the incumbent installs a small additional capacity, protect its demand, and prolongs the period where it can profit from its monopoly position. The entrant will invest later when demand is higher so it can set a larger quantity on the market. This leads to the result that the incumbent invests first and expands to delay a large investment by the entrant. The latter waits until the state variable hits the follower's investment threshold.

Proposition 4 *Let the initial market be myopically determined. Then preemptive investment constitutes an unique equilibrium. Here, the incumbent undertakes investment first, just before the shock process hits the preemption trigger of the entrant. The entrant invests at the follower's trigger.*

As Proposition 4 states, preemptive investment yields a unique equilbrium. The Appendix validates the proposition by proving that other equilibria do not exist.

Sensitivity analysis

The aim of this section is to briefly study the effect of the model parameters on the equilibrium. In this model there are six parameters to be taken a closer look at. First of all, the sensitivity parameter η capturing the negative relation between prices and output. The second parameter is the fixed discount rate r. Then, the drift parameter α and the volatility parameter σ reflecting the market's uncertainty, both present in the geometric Brownian motion describing the state variable's path. Subsequently, we have the marginal investment cost δ. However, this section will continue with the initial investment by the incumbent firm q_{1I}.

The initial market size is captured by q_{1I}. When this parameter increases, expansion is delayed. A larger initial market output requires a higher price for investment and therefore the incumbent waits for a larger value of x. There are two effects on the expansion q_{2I}. A larger initial capacity, makes the expansion decrease for the cannibalization effect is larger and for the firm already acquired a larger part. However, since investment is delayed a larger market is observed at investment, which gives the incentive to increase investment size. The former, however, is dominant and one observes that a larger initial market makes the expansion decrease. The incumbent's total capacity increases when the old market is larger, since the decrease on the expansion is always smaller than the increase on the old market. Section 4.3 takes a deeper look at the realized capacities, also for the follower.

	η	r	α	δ	σ
X_{PE} $(q_{1I} = q_{1I}^{myop})$	0	+	-/+	+	+
q_{2I}^{det} $(q_{1I} = q_{1I}^{myop})$	-	+/-	+/-	0	+/-
X_{PE} (q_{1I} fixed)	+	+	-	+	+
q_{2I}^{det} (q_{1I} fixed)	-	-	+	0	+

Table 1: Effect of an increase in parameter values on triggers and capacities

When η increases the output q_{2I} decreases exactly cancelling out the increase in η, i.e. the product $\eta \cdot q_{2I}$ remains constant. Similarly $\eta \cdot q_{1I}^{myop}$ and $\eta \cdot q_E$ remain constant. In this way, when assuming $q_{1I} = q_{1I}^{myop}$, neither the investment threshold X_L^{det}, nor the preemption trigger are affected by an increase in η. However, when one assumes q_{1I} to be fixed, triggers are affected. An increase in η means an increase in ηq_{1I} and resultingly a decrease in the price, which, hence, makes firms delay investment. Nevertheless, the total effect on the investment size is negative, considering the different effects. When discounting is done under a higher rate, one values future revenues relatively less and one becomes more concerned about current profits. If the interest rate increases, one prefers current profits to be higher and therefore delays investment. In the first place, this increases the myopic capacity size on the initial market. In the second place, since there are two effects that influence the optimal investment size for the expansion - i.e. delaying increases the capacity level, but a larger old market decreases it - it is found that the change is ambiguous. For small r the installment increases, but for relatively large r it decreases. When one fixes the initial capacity, the effect of the old market dominantly influences the capacity leading to decreasing installments. As standard in literature, the drift parameter has an opposite effect: a larger α makes firms invest earlier. The main line of reasoning is the same, when the drift parameter increases. Market demand, and therefore profits, are expected to increase more rapidly; one is then prepared to invest earlier to meet the same expectations concerning expected revenues. Nevertheless, when the initial capacity also changes under a change in parameter values[6], a second effect comes in, similar to the analysis of η: a larger drift increases the initial capacity which leads to a delay of the investment. Also see Figure 10a. The effect on the optimal capacity is similar to the effect of r when the old market is taken myopically, but is, as expected, opposite to r when fixing it. The marginal investment cost has a positive effect on the investment trigger. When investing becomes more expensive, firms prefer to wait for a market where a larger output is required in order to meet the larger costs. The optimal capacities, both when fixing the initial market size and taking it myopically, are not affected. Finally, in a more uncertain market, i.e. a larger σ, future realizations become more important. Waiting gives more information. This leads to the decision to wait for a higher price, in other words, the firm is only prepared to invest for a larger value of x. This leads to an increase in the optimal capacity size. However, as in the

[6]Note that, since the initial capacity equals the myopic investment level, i.e. $q_{1I} = \frac{1}{\eta(\beta+1)}$, its level depends on the other parameter values.

case of r and α, the effect is ambiguous when assuming a myopic initial market size.

4 Timing & Capacity size

This section includes three analyses showing the importance of timing and capacity size. In the first subsection it is shown that the investment order reverses whenever the firms do not have free choice of investment size. Next, a comparison with the monopoly model makes clear that deterring the entrant is done by means of timing and not by means of overinvestment. Finally we study the effect of the size of the initial capacity.

4.1 Fixed capacity

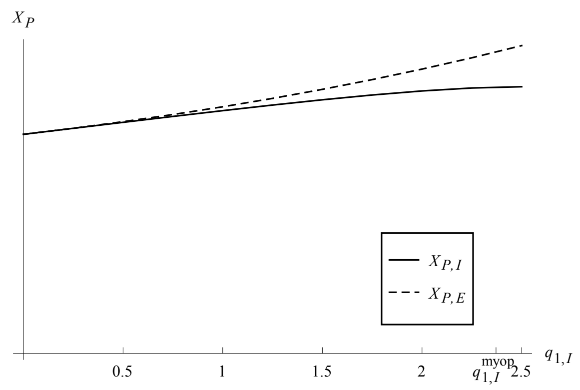
In most industries the size of an investment is variable. There are, however, industries where the capacity size is fixed as in e.g. the pharmaceutical industry. This section shows that the endogenization of investment size drives the investment order.

Consider the same model as presented in section 3.2, but assume investment size is fixed such that $q_{2I} = q_E = K$. The incumbent's value functions are then similar to what was found previously,

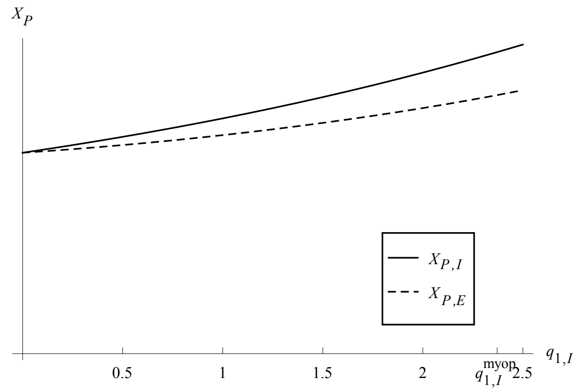
$$\begin{aligned}
V_{LI}^{det}(X, q_{1I}, 0, K) &= \frac{X}{r-\alpha}(q_{1I}+K)(1-\eta(q_{1I}+K)) - \frac{X_{FE}^*}{r-\alpha}\eta K(q_{1I}+K)\left(\frac{X}{X_{FE}^*}\right)^{\beta} - \delta K, \\
X_{FE}^* &= \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1I}+2K)}, \\
V_{FI}(X, q_{1I}, 0, K) &= \frac{X}{r-\alpha}(q_{1I}+K)(1-\eta(q_{1I}+2K)) - \delta K, \\
F_{FI}(X, q_{1I}, 0, K) &= A_{FI}^{det}X^{\beta} + \frac{X}{r-\alpha}q_{1I}(1-\eta(q_{1I}+K)), \\
A_{FI}^{det}(q_{1I}, 0, K) &= \frac{\delta K}{\beta-1}\left(X_{FI}^*\right)^{-\beta}, \\
X_{FI}^*(q_{1I}, 0, K) &= \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-2\eta(q_{1I}+K)}.
\end{aligned}$$

In a similar way one can determine the value functions of the entrant. Next, one can calculate the preemption points. In Section 3.2 it was shown that the incumbent invests first, i.e. the incumbent preempts the potential entrant using entry deterrence. It expands by an adequate amount such that the entrant's investment is temporarily hold off. Figure 3a shows the preemption points of the entrant and incumbent for the model where firms are free to choose their capacity level. Clearly, the preemption points of the incumbent occur for smaller values of x than the ones of the entrant as illustrated before. Figure 3b shows the preemption points for the model presented in this section, i.e. where capacity is fixed. Very clearly the curves are switched: the entrant's curve now lies below the incumbent's curve, signifying that in this model the entrant precedes the incumbent in undertaking an investment. Thus, the entrant takes the leader role and the incumbent becomes follower.

If firms are free to choose the size of their installment, the incumbent finds most incentives to invest first, for it can undertake a small investment in order to delay a large investment by the entrant. When fixing



(a) Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with free capacity choice for different values of q_{1I}.



(b) Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with fixed capacity for different values of q_{1I} with $K = 2$.

Figure 3: Preemption triggers

capacity for both firms, this no longer applies: the incumbent cannot make a small investment to delay a large investment. Moreover, the investment cost, δK, are equal to both firms. Then, the costs for the entrant are smaller, since it does not suffer cannibalization. As a result, the incumbent is more eager to delay investment, having the entrant ending up being investment leader. This is observed for most values of the investment size K, as illustrated by Figure 4. Only very small values of K give the same order as the one with flexible investment size. For these values the incumbent has an increased incentive to delay investment by undertaking a small investment, as can be observed in Figure 4.

4.2 Overinvestment

In the literature on entry deterrence incumbents mainly deter entrants by means of overinvestment. That is, by setting a very large quantity on the market, it becomes unprofitable for other firms to enter this market.

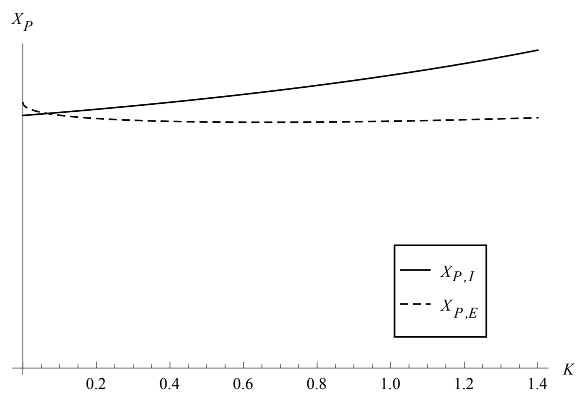


Figure 4: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with fixed capacity for different values of K with $q_{1I} = q_{1I}^{myop}$.

In many cases, the quantity put on the market exceeds the amount that would be optimal for the firm in case that there is no potential entrant. This section aims to investigate whether the notion of overinvestment also applies to the model presented in Section 3.

Overinvestment is defined as the difference between the quantity an incumbent sets on the market when there is a threat of an entrant, to the quantity it would set when this threat would not be present. In other words, the expansion of the incumbent in the duopoly setting as presented in the previous section is compared to the expansion of the incumbent in case it is a monopolist. To this end, the monopolist's model is presented and analyzed.

The value function of the monopolist, at the moment of investment, is defined in the following way,

$$\begin{aligned} V^{mon}(X, q_1) &= \mathbb{E}\left[\int_0^\infty (q_1 + q_2) X(t)(1 - \eta(q_1 + q_2)) e^{-rt} \mathrm{d}t \;\middle|\; x(0) = X \right] - \delta q_2 \\ &= \frac{X}{r - \alpha}(q_1 + q_2)(1 - \eta(q_1 + q_2)) - \delta q_2. \end{aligned}$$

Maximizing the monopolist's value function leads to the optimal expansion size,

$$q_{2M}^*(X, q_1) = \max\left\{ \frac{1}{2\eta}\left(1 - 2\eta q_1 - \frac{\delta(r-\alpha)}{X}\right), 0 \right\}.$$

Resultingly, one obtains,

$$V_M(X, q_1) = \begin{cases} \frac{X}{r-\alpha} q_1(1 - \eta q_1) & \text{if } X < \frac{\delta(r-\alpha)}{1-2\eta q_1}, \\ \frac{(X(1-2\eta q_1) - \delta(r-\alpha))^2}{4\eta(r-\alpha)X} + \frac{X q_1(1-\eta q_1)}{r-\alpha} & \text{if } X \geq \frac{\delta(r-\alpha)}{1-2\eta q_1}. \end{cases}$$

The optimal moment of expansion is defined as the value of x for which the option to wait no longer yields a larger value than immediate investment. Having found the value of the firm before it invests,

$$\begin{aligned} F_M(X, q_1) &= A_M(q_1) X^\beta + \frac{X}{r - \alpha} q_1(1 - \eta q_1), \\ A_M(q_1) &= (X_M^*)^{-\beta} \frac{\delta}{\beta - 1} q_2^{mon}, \end{aligned}$$

where β is defined as in (1), one can show that for the expansion the optimal threshold and size equal

$$\begin{aligned} X_M^* &= \frac{\beta + 1}{\beta - 1} \frac{\delta(r - \alpha)}{1 - 2\eta q_1}, \\ q_2^{mon} &= \frac{1 - 2\eta q_1}{\eta(\beta + 1)}. \end{aligned}$$

To measure overinvestment, the difference between q_2^{det} and q_2^{mon} needs to be considered. As an illustration an example is used, given by Table 2. In this table, the optimal investment moment and the optimal investment size are given for different values of the initial investment size, both with (first pair of columns) and without (second pair of columns) threat of an entrant. Overinvestment would occur if $q_2^{det} > q_2^{mon}$. However, the table illustrates the opposite. To explain this, one must realize that the investment threshold values of the monopolist are higher than the ones from the incumbent in a duopoly setting. The incumbent,

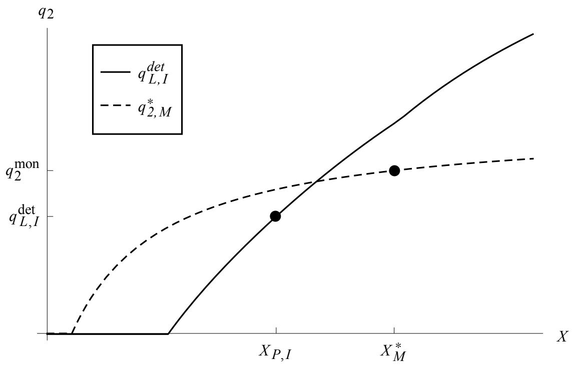


Figure 5: $q_{2I}^{det}(X, q_{1L}, q_{1F})$ (solid) and $q_2^{mon}(X, q_1)$ (dashed) for different values of X.

by all force, prefers to keep its monopoly position as long as possible and thereto it delays investment of the entrant by preempting the entrant's preferred investment moment. This leads to an investment in a market that is still small at the moment of investment. For the same reason the capacity level the firm invests in will be small as well, which is preferred to waiting and investing at a higher demand level, but having a second investor on the market. The monopolist, however, has all flexibility to wait for a price that has grown to a considerable level. We conclude that entry deterrence is not so much about the size but more about timing. Surely, this could only be concluded from a model including capacity choice, which allows for a small and early investment.

To take a closer look at the investment size, comparison between the two capacities can also be made when evaluating them at the same value of X. Figure 5 shows that for large values of X the duopolist takes a larger value. However, for smaller values of X the monopolist sets a larger capacity. In the former case, i.e. when X is large, demand is large, so that sufficient room may exist on the market to have two firms present. However, to deter the entrant the incumbent is obliged to increase its capacity size, exceeding the monopolist's optimal expansion. This explains why the duopolist sets a larger quantity. In the opposite case, i.e. when X is small, demand is small. Then the incumbent needs to take into account that eventually

	Duopolist		Monopolist	
q_{1I}	Trigger	Expansion	Trigger	Expansion
0	123	1.59	152	2.37
0.5	129	1.26	169	2.13
1	136	0.97	190	1.90
1.5	145	0.71	217	1.66
2	157	0.48	254	1.42
2.5	171	0.29	304	1.19

Table 2: Expansions made by the incumbent and a monopolist for different values of q_{1I}.

	Incumbent (Leader)		Entrant (Follower)	
q_{1I}	Trigger	Total Capac.	Trigger	Capacity
0	123	1.59	180	1.99
0.5	129	1.76	185	1.95
1	136	1.97	190	1.90
1.5	145	2.21	195	1.85
2	157	2.48	202	1.78
2.5	171	2.79	211	1.71

Table 3: Total capacities of the incumbent and the entrant for different values of q_{1I}.

the entrant will invest, which lowers the investment's profitability. Conversely, the monopolist knows there will be no other future investment, enabling him to set a larger quantity anticipating expected future market growth. Therefore, the monopolist expands with a larger amount. In the end, due to the timing effect, the monopolists always sets a larger capacity.

4.3 Market leadership

This section illustrates that the incumbent does not necessarily maintain its market leader position after the entrant's investment. To clarify this, Table 3 shows the total capacity of the incumbent firm (first pair of columns) and entrant (second pair of columns). As one can observe, for a small initial capacity the follower becomes market leader. However, when the incumbent starts with a capacity larger than a certain tipping point q_{1I}^{ML} it keeps its position as market leader after the second firm's entry. Market leadership thus depends on the initial market size.

In a framework with two potential entrants, i.e. no firm prossesses an initial capacity, Huisman and Kort (2015) point out that market leadership is dependent on uncertainty. In particular, they show that for

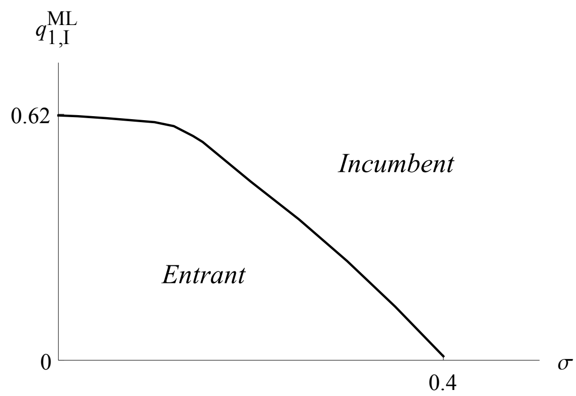


Figure 6: Market leader regions for different σ.

large demand uncertainty the first investor becomes market leader, while the second investor will invest in a larger capacity when the demand uncertainty is low. Combining this with our finding, implies that market leadership thus depends on both initial capacity and demand uncertainty. Recall that q_{1I}^{ML} is defined as the value of the initial capacity for which the total incumbent's capacity equals the amount set by the entrant. As illustrated by Figure 6, q_{1I}^{ML} decreases when uncertainty increases. Larger uncertainty makes the incumbent delay investment, which results in a larger expansion, making him market leader for smaller values of q_{1I} relative to the case of smaller uncertainty. In this figure one can clearly observe for which combinations of the initial capacity and uncertainty the incumbent is market leader and in which region the entrant becomes market leader. For very large uncertainty, the incumbent always becomes market leader. However, for small values of uncertainty a certain range $q_{1I} \in [0, q_{1I}^{ML})$ exists for which the entrant ends up with the largest market share.

5 Robustness

In this section two types of robustness checks are performed in order to verify the validity of our results. First, the effects of a change in parameter values is studied. It is shown that the investment order remains the same for different sets of parameter values, whilst one can find one exception which is concerned with the strength of the old market and the inverse demand function parameter. Second, we derive the results imposing a different demand structure.

5.1 Strength of the initial market

In order to inspect the effect of changes in parameter values on the investment order, both preemption points are drawn in a great variety of graphs in the Appendix. These pictures make clear that the incumbent takes the leader role to delay the entrant's investment. There is, however, an exception. When the sensitivity

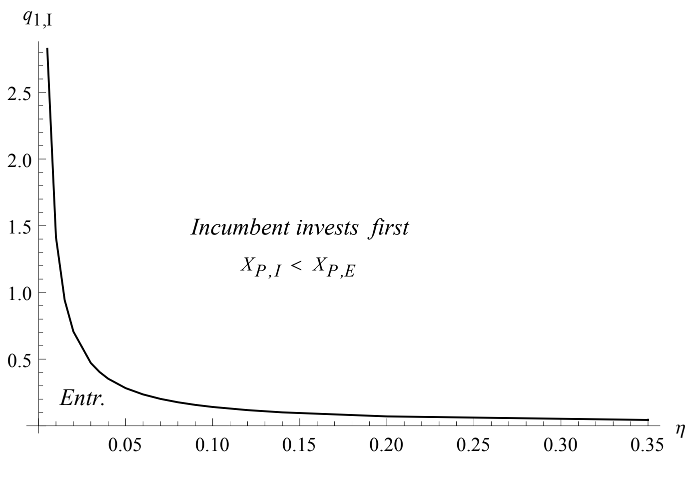


Figure 7: Regions where the incumbent invests first (upper right) and where the entrant invests first.

parameter η is taken to be substantially small while being combined with a substantially small initial capacity size, one observes that the entrant's preemption trigger falls below the investment trigger of the incumbent. This, however, is only true for a substantially small initial market. The trade-off between the initial market and the sensitivity parameter is depicted in Figure 7. This figure shows the two regions where either of the firms invest first. The line depicts all values of η and q_{1L} for which both firms' preemption triggers are identical. We see that the incumbent invests first, except for a small region close to both axes, where the entrant is the first investor. In fact, it holds that for $\eta \cdot q_{1L} > 0.01413$ the incumbent is leader and the entrant invests first for $\eta \cdot q_{1L} < 0.01413$. Intuition behind this result is that for the situation where η or q_{1L} is small, the incumbent prefers a large expansion. Setting both parameters small makes that the incumbent's presence on the initial market is barely noticable: it has little effect on the demand function and the cannibalization effect is small. The incentives to preempt the entrant vanish the moment there is not so much to protect.

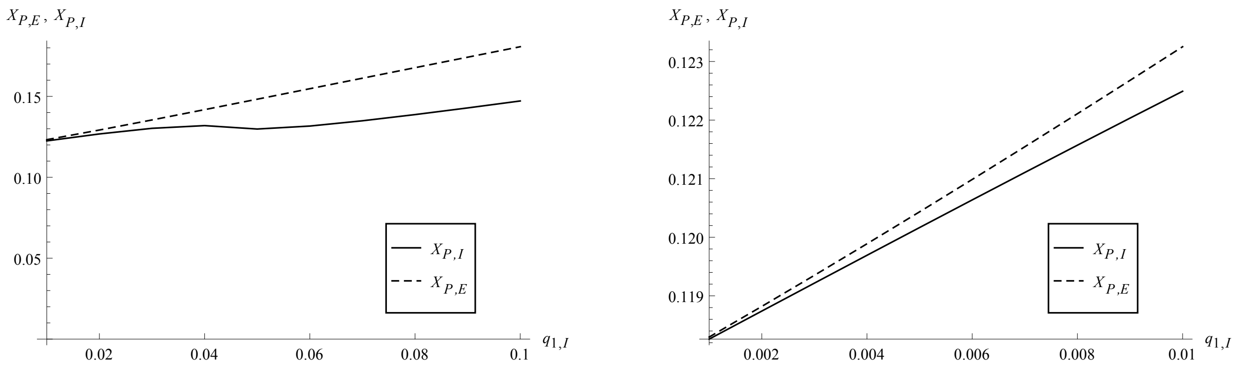
One might notice that for very small values of η the initial market is allowed to be relatively large, allowing for a large region where the entrant becomes leader. Then, remind that the optimal investment size increases for smaller η. As an example, for $\eta = 0.02$, is holds that $q_{1I}^{myop} = 11.86$ and $q_{1I}^{myop} = 23.17$ for $\eta = 0.01$.

5.2 Additive demand structure

One characterization of the multiplicative demand function, as chosen in this paper, is that the market size is bounded. In particular, price is only positive when market quantity is lower than $\frac{1}{\eta}$. In order to check that this, nor any other characterization, influences our results, we perform the same study on the case where an additive demand structure is chosen,

$$p(t) = x(t) - \eta Q(t),$$

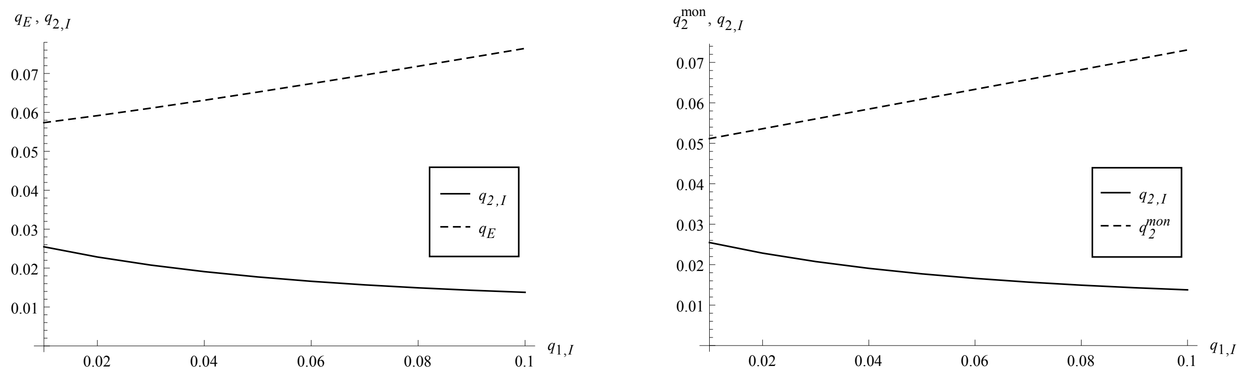
where all are defined as before.



(a) The effect of the initial market on the triggers.

(b) The effect of q_{1I} for very small values.

Figure 8: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) under additive demand.



(a) Resulting capacities for different values of q_{1I}, incumbent (solid) and entrant (dashed).

(b) Resulting capacities for different values of q_{1I}, incumbent (solid) and monopolist (dashed).

Figure 9: Preemption triggers X_{PI} (solid) and X_{PE} (dashed) under additive demand.

Similar to the findings before, it is found that the incumbent preempts the entrant and sets a smaller capacity. For different values of the initial market, Figure 8 shows the resulting preemption triggers[7]. The initial capacity under myopic investment equals $q_{1I}^{myop} = 0.0487$. In the Appendix, the sensitivity of the parameter values is tested. Here, we show that for a range of different parameter sets the incumbent always undertakes an investment first. Figure 9 shows the installed capacities of the incumbent, entrant and monopolist, illustrating that the results discussed before remain valid. We conclude that the results under additive demand are the same as under multiplicative demand.

6 Welfare analysis

This section aims to analyze the effect of the initial market from a welfare perspective. In addition, welfare is analyzed for the situation where a social planner is allowed to make an investment twice. The analysis for the latter is analogous to Section 3.1: first the final investment moment is determined, i.e. the moment of the second expansion, together with the optimal investment size. Next the investment moment of the first expansion is determined, along with the corresponding capacity size. The first part of this section studies the investment policy of the social planner. The second part of this section looks at the welfare implications.

Consider a social planner that has the option to invest twice in an existing market. It optimizes total welfare, being the sum of the total expected consumer surplus and expected producer surplus. Concerning the final investment, the output price after investment equals $p(t) = x(t)(1 - \eta(q_0 + q_1 + q_2))$. The total

[7] The parameter values are chosen differently in this example, for a different inverse demand function requires a different parametrization. Analogous to Boonman (2014) we set $\alpha = 0.01,\ r = 0.1,\ \sigma = 0.05,\ \eta = 0.5$, and $\delta = 1$.

expected consumer surplus after the final investment then equals

$$CS_2(X, q_0, q_1, q_2) = \mathbb{E}\left[\int_{t=0}^{\infty} \int_{P=p(t)}^{X} D(P)\, \mathrm{d}P\, e^{-rt}\, \mathrm{d}t \;\middle|\; x(0) = X \right] = \frac{X\eta(q_0 + q_1 + q_2)^2}{2(r-\alpha)},$$

where $D(P) = \frac{1}{\eta}\left(1 - \frac{P}{X}\right)$. The expected producer surplus equals the value of the firm,

$$PS_2(X, q_1, q_2, q_3) = \frac{X}{r-\alpha}(q_0 + q_1 + q_2)(1 - \eta(q_0 + q_1 + q_2)) - \delta q_2 - \frac{X}{r-\alpha}(q_0 + q_1)(1 - \eta(q_0 + q_1)).$$

The total expected surplus is then obtained by adding the two together, leading to

$$TS_2(X, q_0, q_1, q_2) = \frac{X}{r-\alpha} q_2(1 - \eta(q_0 + q_1 + \tfrac{1}{2}q_2)) - \delta q_2.$$

We find that, for a fixed level of q_1 cq. q_{2I}, the investment moment of the final investment equals the investment moment of the follower, but the resulting investment size is twice as large for the case of the social planner,

$$\begin{aligned} X_2^W &= \frac{\beta+1}{\beta-1} \frac{\delta(r-\alpha)}{1 - \eta(q_0 + q_1)} \\ q_2^W &= 2\frac{1 - \eta(q_0 + q_1)}{\eta(\beta+1)}. \end{aligned}$$

In a similar way, one can determine the optimal moment of the first expansion,

$$X_1^W = \frac{\beta}{\beta-1} \frac{\delta(r-\alpha)}{1 - \eta q_0 - \frac{1}{2}\eta q_1^W},$$

where the optimal investment size of the first expansion q_1^W is implicitely determined by solving the following equation for q_1,

$$1 - \frac{\beta\eta q_1}{2(1 - \eta(q_0 + q_1))} - 2\left(\frac{\beta}{\beta+1} \frac{1 - \eta(q_0 + q_1)}{1 - \eta q_0 - \frac{1}{2}\eta q_1}\right)^{\beta} = 0.$$

The total expected surplus before any expansion can be divided into two components. The first part, consisting of one term, reflects the accumulated discounted revenue stream resulting from the initial market. The second part consists of two terms, reflecting the value of the options to the future expansions,

$$TS^W(X, q_0) = \frac{X}{r-\alpha} q_0(1 - \tfrac{1}{2}\eta q_0) + \left(\frac{X}{X_1^W}\right)^{\beta} \frac{\delta}{\beta-1} q_1^W + \left(\frac{X}{X_2^W}\right)^{\beta} \frac{\delta}{\beta-1} q_2^W.$$

Let us denote the first term as $TS_0^W(X, q_0)$ and the sum of the final two terms $TS_{opt}^W(X, q_0)$, then we can rewrite $TS^W(X, q_0)$ as $TS_0^W(X, q_0) + TS_{opt}^W(X, q_0)$.

Tabel 4 shows the investment triggers, the correspondig capacities and the resulting surpluses for both the social planner and the duopoly. The table also shows the accumulated capacities Q^{duop} and Q^W. The first thing one observes is that the first and second investment moment of the social planner are later than the investment moments in the duopoly model. We moreover observe that the resulting capacities are larger in the case of a welfare maximizer. So, the preemption effect combined with the cannibalization effect forces the

	Duopoly						Social Planner							
q_0	X_P	q_L	X_F	q_F	Q^{duop}	TS^{duop}_{opt}	X^W_1	q^W_1	X^W_2	q^W_2	Q^W	TS^W_{opt}	$\frac{TS^{duop}_{opt}}{TS^W_{opt}}$	$\frac{Q^{duop}}{Q^W}$
0	123	1.59	180	1.99	3.58	0.3058	135	2.84	212	3.40	6.24	0.3790	0.8069	0.5737
0.5	129	1.26	185	1.95	3.71	0.2258	142	2.70	224	3.23	6.43	0.3052	0.7397	0.5770
1	136	0.97	190	1.90	3.87	0.1647	150	2.55	236	3.06	6.61	0.2423	0.6798	0.5855
1.5	145	0.71	195	1.85	4.06	0.1179	159	2.41	250	2.89	6.80	0.1899	0.6208	0.5971
2	157	0.48	202	1.78	4.26	0.0815	169	2.27	266	2.72	6.99	0.1469	0.5551	0.6094
2.5	171	0.29	211	1.71	4.50	0.0565	180	2.13	283	2.55	7.18	0.1126	0.5020	0.6267

Table 4: Welfare implications for duopoly and social planner.

incumbent to invest too soon in smaller capacity. The entrant also invests sooner compared to the second investment of the social planner, but this is because it invests in a smaller capacity. The social planner is interested in larger quantities because they reduce price, and this raises consumer surplus more than it reduces the producer's surplus.

Table 4 can also be used to study the effect of the initial incumbent's capacity size on the welfare. For both the duopoly model and for the social planner model, additional welfare gained by investing drops when the initial capacity is larger. Intuitively, the larger the old market, the lower the marginal value of an additional unit of capital. Additionally, a larger initial capacity is equivalent to a more severe cannibalization effect. The result that the duopoly is more affected by an increase in the old market can be explained by the presence of competition that marginalizes surplus as a result of protective behavior towards the firms' own profit. The social planner just delays investment for a larger old market, since he is not affected by a potential entrant's willingness to invest soon.

7 Conclusions

This paper shows that timing is a crucial element in firms' decision making. Where entry deterrence is generally understood to ward off entrants by doing an overinvestment, it is our understanding that entrants are delayed by accelerating the investment. The main contributions are made to the literature streams concerning incumbent-entrant models and strategic real option theory.

First, it is found that the incumbent has most incentives to invest first. By undertaking a relatively small investment first, the firm mitigates the cannibalization effect and it prolongs its monopoly period. The entrant prefers to enter late, in order to set larger capacity. The latter is valid when the incumbent is noticeably present on the installed market. It is then found that preemptive investment is the sole feasible strategy in these games and therefore constitutes the games' equilibrium. Second, despite what some other papers find, e.g. Spence (1977) and Robles (2011), we do not find any notion of overinvestment. Contrarily, incumbent firms set lower capacities in case a second firm considers entry, than when this threat is absent. This paper also includes a model where the investment size is determined exogenously. Industries where the capacity size is fixed are for example the pharmaceutical industry, the excavation of raw materials or the production of intermediate goods, or any other supplies to a fixed set of buyers. Opposite to what was found for the model with flexible capacity size, it is found that in this equilibrium the entrant makes an investment first.

Hence, these notions prove the importance to include both capacity choice and to endogenize timing. It, moreover, proves that deterrence is not related to overinvestment, but to timing.

The model could be extended in different ways. First, one can include the option to leave the market. This option may influence the strategic behavior of firms and may therefore be of interest. Secondly, the number of incumbents and/or potential entrants can be extended to investigate whether the results of the incumbents being first investors would remain valid. Third, like in reality, firms can be given multiple investment options. Fourth, one could consider innovating firms, either adopting new technologies or performing R&D themselves.

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Appendix

Proof of Proposition 1 The follower's value function with respect to the shock process x can be divided into two regions. For x sufficiently large the firm invests, that is for $x \geq X_F^*$, this region is called the stopping region. The complementary region is called the continuation region, for these values the firm waits (see e.g. Dixit and Pindyck (1994)). In the stopping region the firm realizes the following accumulated and discounted expected profits $V_F(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ at the investment moment,

$$
\begin{aligned}
V_F = \mathbb{E}\left[\int_{t=0}^{\infty} x(t)(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X\right] - \delta q_{2F} \\
= (q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})\mathbb{E}\left[\int_{t=0}^{\infty} x(t)e^{-rt}\mathrm{d}t \;\Big|\; x(0)=X\right] - \delta q_{2F} \\
= (q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})\int_{t=0}^{\infty} x(0)e^{(\alpha-r)t}\mathrm{d}t - \delta q_{2F} \\
= \frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}) - \delta q_{2F}.
\end{aligned}
$$

The firm chooses capacity such that it maximizes its profits, thereto,

$$
\begin{aligned}
\frac{\partial V_F}{\partial q_{2F}} &= \frac{X}{r-\alpha}\left[1-\eta(q_{1L}+2q_{1F}+q_{2L})-2\eta q_{2F})\right]-\delta \\
&= 0 \quad \Leftrightarrow \\
q_{2F}(X, q_{1L}, q_{1F}, q_{2L}) &= \frac{1}{2\eta}\left[1-\eta(q_{1L}+2q_{1F}+q_{2L})-\frac{\delta(r-\alpha)}{X}\right].
\end{aligned}
$$

The second order conditions reassure us that this is indeed a maximum, $-2\eta\frac{X}{r-\alpha}<0$.

In the continuation region it is optimal for the firm the delay investment, for waiting yields a larger value than investment. The waiting value is embodied by the option value. The function F_F, following standard real options analysis (see e.g. Dixit and Pindyck (1994)), equals the sum two terms reflecting the value of waiting and the value of current production,

$$
F_F(X, q_{1L}, q_{1F}, q_{2L}) = A_F X^{\beta} + \frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})),
$$

where β is the positive root following from,

$$
\sigma^2\beta^2 + (2\alpha-\sigma^2)\beta = 2r \Leftrightarrow \beta = \tfrac{1}{2}-\tfrac{\alpha}{\sigma^2}+\sqrt{\left(\tfrac{1}{2}-\tfrac{\alpha}{\sigma^2}\right)^2+\tfrac{2r}{\sigma^2}}.
$$

The investment trigger and the value of the parameter $A_F(q_{1L}, q_{1F}, q_{2L})$ can be found by applying the value matching and smooth pasting conditions,

$$
\begin{aligned}
\frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}))-\delta q_{2F} &= A_F X^{\beta}+\frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})), \\
\frac{1}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F})) &= A_F\beta X^{\beta-1}+\frac{1}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L})).
\end{aligned}
$$

Together they make

$$
\frac{X}{r-\alpha}(q_{1F}+q_{2F})(1-\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}))\left(1-\frac{1}{\beta}\right)-\frac{X}{r-\alpha}q_{1F}(1-\eta(q_{1L}+q_{1F}+q_{2L}))\left(1-\frac{1}{\beta}\right)=\delta q_{2F},
$$

which leads to

$$
X_F = \frac{\beta}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L}+q_{2F})}.
$$

Plugging in q_{2F} leads to

$$X_F^*(q_{1L}, q_{1F}, q_{1L}) = \frac{\beta+1}{\beta-1}\frac{\delta(r-\alpha)}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}.$$

Moreover,

$$\begin{aligned} A_F(X_F^*)^\beta &= \frac{X_F^*}{r-\alpha}q_{2F}^*(1-\eta(q_{1L}+2q_{1F}+q_{2L}+q_{2F}^*)) - \delta q_{2F}^* \\ &= \frac{\delta q_{2F}^*}{\beta-1}. \end{aligned}$$

Rewring leads to equation (5). □

Proof of Proposition 2 The leader's value function from equation (7) is obtained in the following way,

$$\begin{aligned} V_L^{det} &= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}) - \eta q_{2F}^*(q_{1L}+q_{2L})\frac{X_F^*}{r-\alpha}\left(\frac{X}{X_F^*}\right)^\beta - \delta q_{2L} \\ &= \frac{X}{r-\alpha}(q_{1L}+q_{2L})(1-\eta(q_{1L}+q_{1F}+q_{2L}) - (q_{1L}+q_{2L})\frac{\delta}{\beta-1}\left(\frac{X}{X_F^*}\right)^\beta - \delta q_{2L} \end{aligned}$$

The deterrence region is bounded from below and above by X_1^{det} and X_2^{det}. In this region the leader optimizes capacity at investment. The optimal amount can be found by solving the first order conditions,

$$\frac{\partial V_L^{det}}{\partial q_{2L}} = \frac{X}{r-\alpha}[1-\eta(2q_{1L}+q_{1F}+2q_{2L})] - \frac{\delta}{\beta-1}\left(\frac{X}{X_F^*}\right)^\beta\left[1-\frac{\eta\beta(q_{1L}+q_{2L})}{1-\eta(q_{1L}+2q_{1F}+q_{2L})}\right] - \delta.$$

The value of X_2^{det} then follows from first implementing X_F^* in the first order conditions,

$$\frac{\partial V_L^{det}}{\partial q_{2L}} = \frac{\delta}{\beta-1}\frac{1-2\eta q_{1L}+\eta(\beta-1)q_{1F}-2\eta q_{2L}}{1-\eta(q_{1L}+2q_{1F}+q_{2L})} = 0 \Leftrightarrow q_{2L}^R = \frac{1}{2\eta}(1-2\eta q_{1L}+(\beta-1)\eta q_{1F}). \tag{15}$$

Hence, by plugging the latter expression into X_F^* one obtains X_2^{det},

$$X_2^{det} = X_F^*(q_{1L}, q_{1F}, q_{2L}^R) = \frac{\beta+1}{\beta-1}\frac{2\delta(r-\alpha)}{1-(\beta+3)\eta q_{1F}}$$

The conditions determining X_1^{det} also follow from the first order conditions by setting $q_{2L} = 0$. To show that there exists a unique point X_1^{det}, it is sufficient to do the following. Define $\psi(X) = \frac{\partial V_L^{det}}{\partial q_{2L}}|_{q_{2L}=0}$, this function dictates the first order conditions for the value of X yielding zero capacity,

$$\psi(X) = \frac{X}{r-\alpha}[1-\eta(2q_{1L}+q_{1F})] - \frac{\delta}{\beta-1}\left(\frac{\beta-1}{\beta+1}\frac{1-\eta q_{1L}-2\eta q_{1F}}{\delta(r-\alpha)}X\right)^\beta\left[1-\frac{\eta\beta q_{1L}}{1-\eta q_{1L}-2\eta q_{1F}}\right] - \delta.$$

Then,

$$\begin{aligned} \psi(0) &= -\delta < 0, \\ \psi(X_F^*) &= \frac{\delta}{\beta-1}\frac{1-2\eta q_{1L}+(\beta-1)\eta q_{1F}}{1-\eta q_{1L}-2\eta q_{1F}}, \\ \psi'(X) &= \frac{1-\eta(2q_{1L}+q_{1F})}{r-\alpha}\left[1-\frac{\beta}{\beta+1}\left(\frac{X}{X_F^*}\Big|_{q_{2L}=0}\right)^{\beta-1}\right] + \frac{\beta}{\beta+1}\left(\frac{X}{X_F^*}\Big|_{q_{2L}=0}\right)^{\beta-1}\frac{\eta q_{1F}+(\beta-1)\eta q_{1L}}{r-\alpha}. \end{aligned}$$

From (15) it follows that $\psi(X_F^*) > 0$. Since $\psi'(X) > 0$ one can conclude that there exists a unique $X_1^{det} \in (0, X_F^*)$ such that $q_{2L}(X_1^{det}, q_{1L}, q_{1F}) = 0$.

The value before investment is similar to the one for the follower,

$$F_L^{det}(X, q_{1L}, q_{1F}) = A_L^{det} X^\beta + \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})).$$

Applying the value matching and smooth pasting conditions gives (2) and

$$A_L^{det} = (X_L^{det})^{-\beta} \frac{\delta q_L^{det}}{\beta - 1} - \frac{\delta}{\beta - 1}(q_{1L} + q_L^{det})\,(X_F^*)^{-\beta}.$$

□

We will now show that $X_L^{acc} > X_1^{acc}$ by showing that there exists no set of parameter values such that

$$\frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}} = \frac{\delta(r-\alpha)\beta}{\beta-1}\frac{\eta q_{2L} - \eta q_{1L}}{\eta q_{2L}(1-2\eta q_{1L} - \eta q_{2L}) - \eta q_{1L}(1 - \eta q_{1L} - 2\eta q_{1F})}.$$

Rewriting and substitution of $q_{2L} = \frac{2}{\eta(\beta+3)}(1 + (\beta-1)\eta q_{1F}) - q_{1L}$ gives:

$$\begin{aligned}2(1+(\beta-1)\eta q_{1F})[1 - \eta q_{1L} - \frac{2}{\beta+3}(1 + \eta q_{1F}(\beta-1))] - q_{1F}[2 + (\beta-1)\eta q_{1F} - 2(\beta+3)\eta q_{1F}]\\ = \frac{\beta}{\beta+3}(2 + 2(\beta-1)\eta q_{1F} - 2\eta(\beta+3)q_{1L})(1 - 4\eta q_{1F}).\end{aligned}$$

We distinguish 2 cases. In the first case the leader is the entrant, i.e. $q_{1L} = 0$. Then:

$$\frac{1}{\beta+3} + \eta q_{1F}\frac{2\beta+2}{\beta+3} - \eta q_{1F}\frac{2 - \eta q_{1F}(\beta+7)}{2(1+(\beta-1)\eta q_{1F})} = 0.$$

This gives a contradiction since both the first term and the second and third term combined are positive. The sum of two positive terms can never give 0. In the second case we assume the incumbent to be leader, i.e. $q_{1F} = 0$. Then,

$$1 + (\beta+3)\eta q_{1L}(\beta - 1) = 0,$$

which cannot be true either. □

Proof of Proposition 3 The leader's value function under entry accommodation is determined in the same way as before. In this case, however, one needs to substitute the follower's capacity (3) to obtain equation (11). The leader chooses capacity such that it optimizes the value function,

$$\frac{\partial V_L^{acc}}{\partial q_L^2} = \frac{X}{2(r-\alpha)}\left[1 - 2\eta(q_{1L} + q_{2L})\right] - \frac{1}{2}\delta \Leftrightarrow q_L^{acc}(X, q_{1L}, q_{1F}) = \frac{1}{2\eta}\left[1 - 2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right].$$

It is easily checked that $q_L^{acc} \geq 0$ if and only if $1 - 2\eta q_{1L} \geq \frac{\delta(r-\alpha)}{X}$, i.e. if

$$X \geq \frac{\delta(r-\alpha)}{1 - 2\eta q_{1L}}.$$

The second order conditions again make sure that we obtain a maximum, $-2\eta\frac{X}{r-\alpha} < 0$. Solving $q_L^{acc} = \hat{q}_{2L}$ leads to the first term in equation (10), i.e.,

$$q_L^{acc} = \frac{1}{2\eta}\left[1 - 2\eta q_{1L} - \frac{\delta(r-\alpha)}{X}\right] = \frac{1}{\eta}\left[1 - 2\eta q_{1F} - \eta q_{1L} - \frac{\delta(\beta+1)(r-\alpha)}{(\beta-1)X}\right] = \hat{q}_{2L}$$

$$\Leftrightarrow$$

$$1 - 4\eta q_{1F} = \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)X} \Leftrightarrow X_0^{acc} = \frac{\beta+3}{\beta-1}\frac{\delta(r-\alpha)}{1-4\eta q_{1F}}.$$

Since $\dfrac{\partial \hat{q}_L^2}{\partial X} > \dfrac{\partial q_L^{acc}}{\partial X} > 0$ we know that this point is unique. Using value function

$$F_L^{acc}(X, q_{1L}, q_{1F}) = A_L^{acc}X^\beta + \frac{X}{r-\alpha}q_{1L}(1 - \eta(q_{1L} + q_{1F}))$$

one can apply the value matching and smooth pasting conditions. This leads to the decription of the investment trigger $X_L^{acc}(q_{1L}, q_{1F}, q_{2L})$. Since $q_{1L} \cdot q_{1F} = 0$ one can simplify the terms and in this way one ends op at (13). Moreover,

$$\begin{aligned} A_L^{acc} \cdot (X_L^{acc})^\beta &= \frac{X_L^{det}}{r-\alpha}\left[(q_{1L} + q_L^{acc})(1 - \eta(q_{1L} + q_{1F} + q_L^{acc} + q_{2F}^*)) - q_{1L}(1 - \eta(q_{1L} + q_{1F}))\right] - \delta q_L^{acc} \\ &= \frac{\delta\beta}{\beta-1}q_L^{acc} - \delta q_L^{acc} = \frac{\delta q_L^{acc}}{\beta-1}. \end{aligned}$$

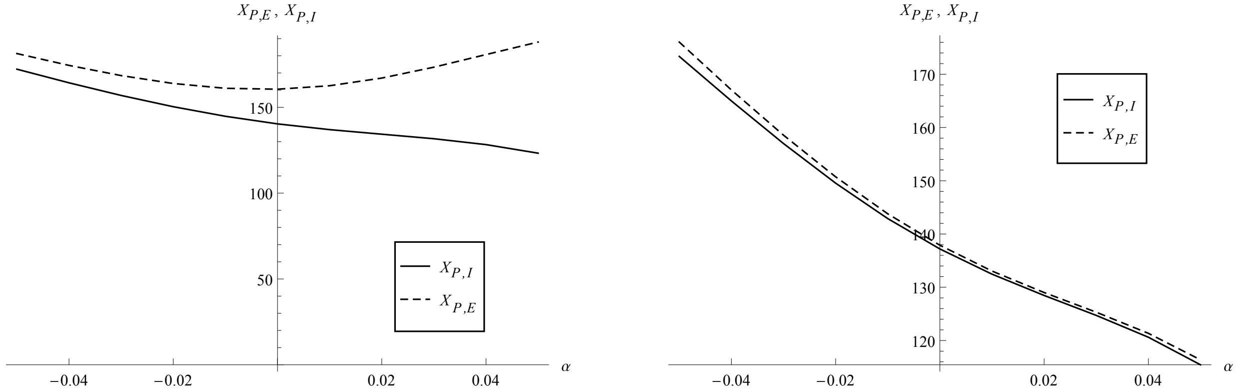
□

Endogenous firm roles In this type of games, there are four different possible strategies, which are similar to both firms (see e.g. Pawlina and Kort (2006)), each leading tot a different type of equilibrium: preemptive investment, sequential investment, simultaneous investment and joint investment.

In the first scenario both firms eagerly want to take the investment leader's role. This results in the preemptive equilibrium where firms want to preempt each other to precede the other firm's investment.

Preemptive investment yields an equilibrium.

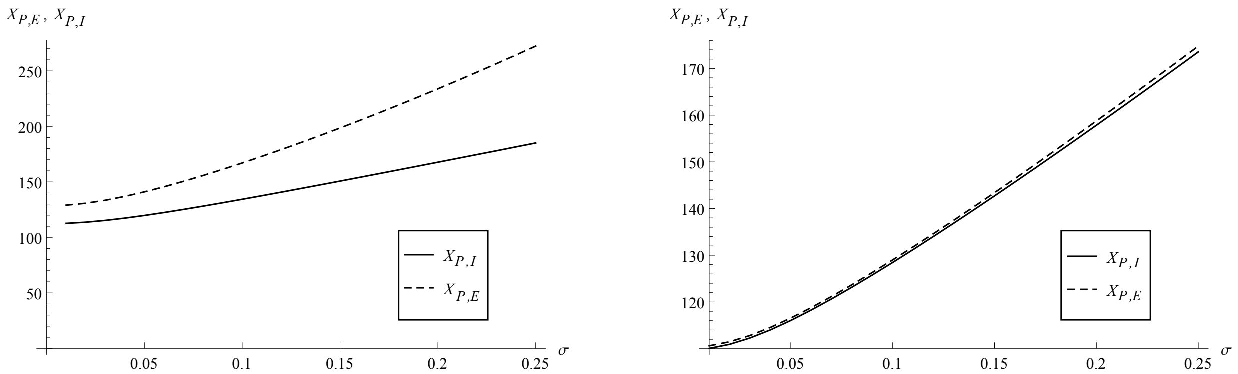
Figure 10 shows how the preemption points change if there is a change in the drift parameter α. The left graph shows this for the case in which the initial market is myopic, i.e. $q_{1I} = q_{1I}^{myop}$, the second graph for a smaller initial market, $q_{1I} = 0.5$. The solid graph reflects the preemption trigger of the incumbent, the dashed one reflects the preemption trigger of the entrant. Notice that in the former case the capacity on the old market changes when the parameters change. The same analysis is done for σ in Figure 11, δ in Figures 12 and 15a and r and η in Figures 13 and 14, 15b respectively. For α, σ, δ and r it is clearly observed that a change in the parameter values does not change the investment order. There are some concerns however about the case in which η takes a small value. It appears in Figure 14b that for small values the sensitivity parameter that the entrant invests first. Figure 15b presents both curves for small values of η. Here it becomes clear that the investment order reverses indeed for small values of η.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Investment triggers with $q_{1I} = 0.5$.

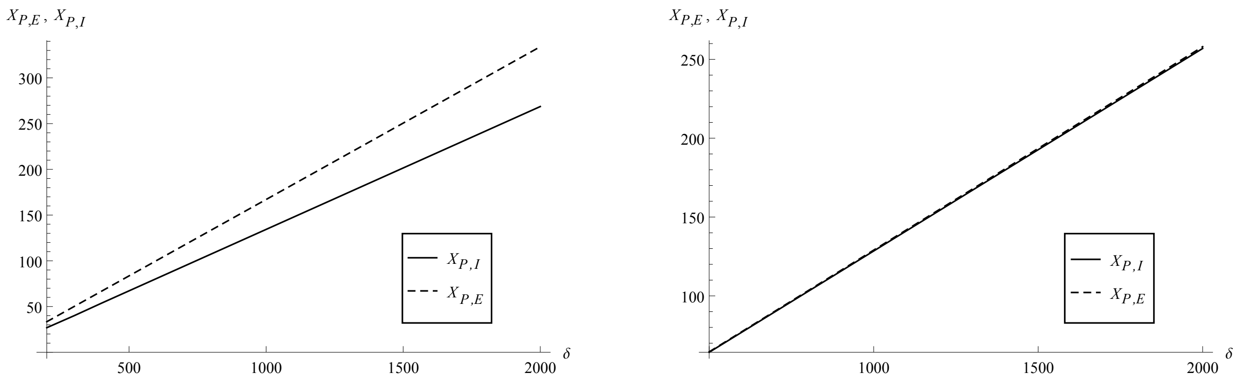
Figure 10: Preemption triggers for different values of α.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Investment triggers with $q_{1I} = 0.5$.

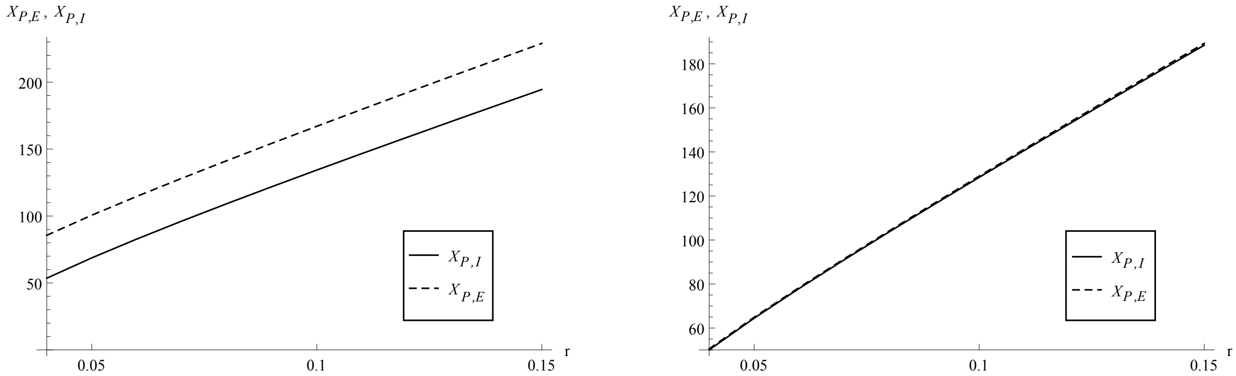
Figure 11: Preemption triggers for different values of σ.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Investment triggers with $q_{1I} = 0.5$.

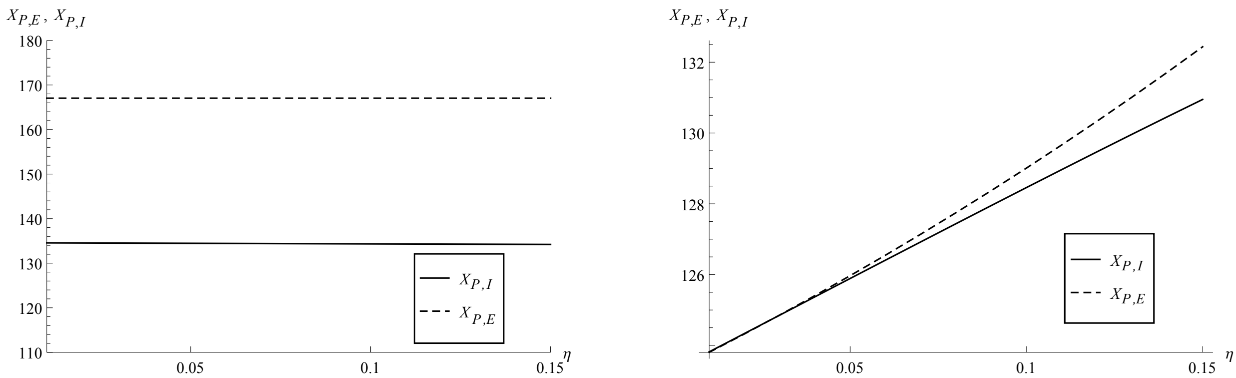
Figure 12: Preemption triggers for different values of δ.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Investment triggers with $q_{1I} = 0.5$.

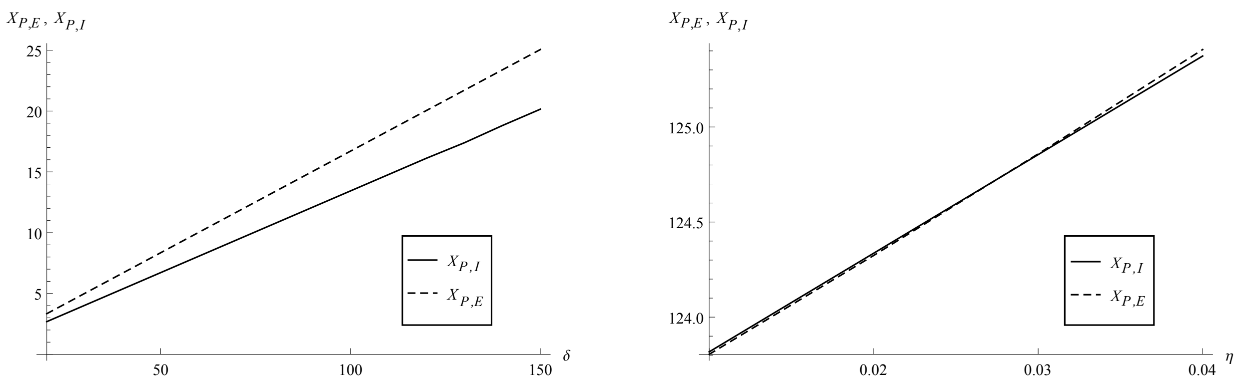
Figure 13: Preemption triggers for different values of r.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Investment triggers with $q_{1I} = 0.5$.

Figure 14: Preemption triggers for different values of η.



(a) Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with $q_{1I} = q_{1I}^{myop}$ for different values of δ.

(b) Preemption triggers X_{PI} (solid) and X_{PE} (dashed) with $q_{1I} = 0.5$ for different values of η.

Figure 15: Preemption triggers for small values of δ and η.

Lemma 1 *Sequential investment does not yield an equilibrium.*

Suppose the investment threshold X_L^{det} of the leader for the case of exogenous firm roles lies between the preemption triggers of the incumbent and entrant. Then the incumbent firm invests at the investment threshold and not just before the preemption trigger of the entrant, since it has no incentive to wait. This is called sequential investment.

Proof of Lemma 1 We show that the investment trigger does not exist, this is sufficient. Let us rewrite equation (9) as,

$$F_L^{det}(X, q_{1L}, q_{1F}) = \frac{X}{r-\alpha} q_{1L}(1 - \eta(q_{1L} + q_{1F})) + A_L^{det} X^{\beta}.$$

Then A_L^{det} reflects the net gain from investment. Let X_L^{det} and X_F^* be defined as in equations (2) and (4). Let $q_{1L} = q_{1I} = q_{1I}^{myop} = \frac{1}{\eta(\beta+1)}$ and $q_{1F} = 0$. Then,

$$\frac{X_L^{det}}{X_F^*} = \frac{\beta - 1 + \frac{1}{\beta+1} - \beta\eta q_{2I}^{det}}{\beta - 1 - (\beta+1)\eta q_{2I}^{det}} > 1.$$

Then,

$$\begin{aligned} A_L^{det} &= (X_L^{det})^{-\beta} \frac{\delta q_{2I}^{det}}{\beta - 1} - \frac{\delta}{\beta - 1}(q_{1I}^{myop} + q_{2I}^{det})(X_F^*)^{-\beta} \\ &= \frac{\delta}{\beta - 1}\left[q_{2I}^{det}\left[\left(\frac{1}{X_L^{det}}\right)^{\beta} - \left(\frac{1}{X_F^*}\right)^{\beta}\right] - q_{1I}^{myop}\left(\frac{1}{X_F^*}\right)^{\beta}\right] \\ &< 0. \end{aligned}$$

This means that investment will only lead to a lower pay-off, in which case the incumbent will never make an investment. □

Notice that in case of the entrant being leader we always have $X_F^* > X_L^{det}$, irrespective of the value of q_{1I}.

There exists an alternative strategy: tacit collusion. When firms decide to collude, they wait for the market to expand, that is, wait for a larger value of X, before investment is undertaken together at the same time. One can discriminate two types of collusion, distinguished by the order in which firms determine their capacity size. In the first type, one firm is Stackelberg capacity leader and decides upon the amount first where subsequently the second firm makes an immediate investment. The second investor sets its capacity after the first firm decided upon its investment scale. This type is called simultaneous investment. The second type, referred to as joint investment, is the category where there is no colluded investment order. Firms simultaneously decide upon capacities, leading to a Cournot type of equilibrium.

Lemma 2 *Simultaneous investment does not yield an equilibrium.*

Proof of Lemma 2 For the resulting value functions, the curves in Figure 2a should be considered. Here, the Stackelberg leader utilizes the accommodation strategy. As a result, the competitor receives the follower value, being smaller than the leader value. For this reason neither of the firms would prefer to be a follower in the outcome and they would, consequently, preempt each other in taking the leader role. This forces the firms to end up in the deterrence region and the sole resulting equilbrium is the preemptive equilibrium. Hence, simultaneous investment is not an equilibrium. □

Lemma 3 *Joint investment does not yield and equilibrium.*

Proof of Lemma 3 Let $J(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ be the firm value for joint investment, then,

$$J = \frac{X}{r-\alpha}(q_{1L} + q_{2L})(1 - \eta(q_{1L} + q_{1F} + q_{2L} + q_{2F})) - \delta q_{2L}.$$

Optimal capacities equal,

$$\begin{aligned} q_{2L}^{join} &= \frac{1}{3\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right) - q_{1L}, \\ q_{2F}^{join} &= \frac{1}{3\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right) - q_{1F}. \end{aligned}$$

This leads to,

$$\begin{aligned} V_L^{acc}(X, q_{1L}, q_{1F}, q_L^{acc}, q_{2F}^*) &= \frac{X}{r-\alpha}\frac{1}{8\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right)^2 + \delta q_{1L} \\ J(X, q_{1L}, q_{1F}, q_{2L}^{join}, q_{2F}^{join}) &= \frac{X}{r-\alpha}\frac{1}{9\eta}\left(1 - \frac{\delta(r-\alpha)}{X}\right)^2 + \delta q_{1L} \end{aligned}$$

Hence, it holds that $V_L^{acc} > J$. This is sufficient to show that joint investment does not yield an equilibrium. Intuition behind this result is that when firms are leader they can set a larger capacity which leads to a higher payoff. □

Proof of Proposition 4 It follows from Lemmas 1, 2 and 3 that preemptive investment is the sole equilibrium. Moreover, under myopic behavior on the initial market, it is shown that the incumbent always invests first.

Fixed capacity Suppose $X = X_{PE}$, then one can show that

$$V_{LE}^{det} - F_{FE}^{det} = \left[\frac{X}{r-\alpha}K(1 - \eta K - \eta q_{1I}) - \delta K\right]\left[1 - \frac{\frac{(X_{FE}^*)^{1-\beta}}{r-\alpha}K(\eta q_{1I} + \eta K) + \frac{\delta K}{\beta-1}(\frac{1}{X_{FI}^*})^\beta}{\frac{(X_{FI}^*)^{1-\beta}}{r-\alpha}\eta K^2 + \frac{\delta K}{\beta-1}(\frac{1}{X_{FE}^*})^\beta}\right].$$

One can conclude, if

$$\begin{aligned} f_1(q_{1I}, K) &= (1 - \eta q_{1I} - 2\eta K)^{\beta-1}(1 - \eta q_{1I}(\beta+1) - \eta K(\beta+2)) \\ &> \\ f_2(q_{1I}, K) &= (1 - 2\eta q_{1I} - 2\eta K)^{\beta-1}(1 - 2\eta q_{1I} - \eta K(\beta+2)), \end{aligned}$$

then $V_{LI}^{det}(X_{PE}) > F_{FI}(X_{PE})$ and as a result $X_{PI} < X_{PE}$.

Additive demand Here, we first shortly summarize all the obtained propositions. Then, we will show some graphs to check the robustness of the results.

Proposition 5 *Let the current value of the stochastic demand process be denoted by X, and let the initial production capacity be denoted by q_{1L} and q_{1F} respectively for the leader and the follower. Let the capacities associated with the investments be denoted by q_{2L} and q_{2F} respectively for the leader and the follower. Then the value function of the follower can be partitioned into two regions: for small X the firm waits until it reaches the investment trigger X_F^* and for $X \geq X_F^*$ the firm invests immediately. As a result, the follower's value function $V_F^*(X, q_{1L}, q_{1F}, q_{2L}, q_{2F})$ is given by*

$$V_F^* = \begin{cases} A_F X^\beta + q_{1F}\left(\frac{X}{r-\alpha} - \frac{\eta}{r}(q_{1L} + q_{1F} + q_{2L})\right) & \text{if } X < X_F^*, \\ (q_{1F} + q_{2F})\left(\frac{X}{r-\alpha} - \frac{\eta}{r}(q_{1L} + q_{1F} + q_{2L} + q_{2F})\right) - \delta q_{2F} & \text{if } X \geq X_F^*, \end{cases}$$

where the optimal capacity level for the follower q_{2F}^*, the investment trigger X_F^* and A_F are defined by

$$q_{2F}^*(X, q_{1L}, q_{1F}, q_{2L}) = \frac{r}{2\eta}\left(\frac{X}{r-\alpha} - \delta\right) - \tfrac{1}{2}(q_{1L} + 2q_{1F} + q_{2L}), \tag{16}$$

$$X_F^*(q_{1L}, q_{1F}, q_{2L}) = (\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r)\frac{\beta(r-\alpha)}{r(\beta-2)}, \tag{17}$$

$$A_F = \frac{\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r}{\eta(\beta-2)\beta(r-\alpha)}(X_F^*)^{1-\beta}. \tag{18}$$

The follower's capacity in case the follower invests at the investment trigger equals

$$q_{2F}^*(X_F^*, q_{1L}, q_{1F}, q_{2L}) = \frac{\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r}{\eta(\beta-2)}.$$

Proposition 6 *Let the production capacities be defined as in Proposition 5 and let the current value of the shock process be defined as X. Then the deterrence strategy leads to value function $V_L^{det}(X, q_{1L}, q_{1F}, q_{2L})$,*

$$V_L^{det} = (q_{1L} + q_{2L})\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L} + q_{1F} + q_{2L})}{r}\right) - (q_{1L} + q_{2L})\frac{\eta(q_{1L} + 2q_{1F} + q_{2L}) + \delta r}{r(\beta-2)}\left(\frac{X}{X_F^*}\right)^\beta - \delta q_{2L},$$

where X_F^* is defined as equation (17).

For large initial values of X the leader invests immediately and chooses optimal capacity

$$q_L^{det}(X, q_{1L}, q_{1F}) = \operatorname{argmax}\{V_L^{det}(X, q_{1L}, q_{1F}, q_{2L}) \mid q_{2L} > \hat{q}_{2L}\}, \tag{19}$$

where,

$$\hat{q}_{2L}(X, q_{1L}, q_{1F}) = \frac{r}{\eta}\left[\frac{X(\beta-2)}{\beta(r-\alpha)} - \frac{\delta}{r}\right] - (q_{1L} + 2q_{1F}).$$

The entry deterrence strategy is considered for $X \in (X_1^{det}, X_2^{det})$, where

$$X_1^{det} = \{X \mid q_2^{det}(X, q_{1L}, q_{1F}) = 0\},$$
$$X_2^{det} = \{X \mid q_2^{det}(X, q_{1L}, q_{1F}) = \hat{q}_{2L}(X, q_{1L}, q_{1F})\}.$$

For low initial values of x, that is $x(0) < X_L^{det}$, the leader invests at the moment x reaches the investment threshold value X_L^{det}. The value of the investment threshold and the associated capacity level q_L^{det} are determined as the solution of the set of equations determined by equation (19) and

$$X_L^{det}(q_{1L}, q_{1F}, q_{2L}) = \left[\frac{\eta}{r}(2q_{1L} + q_{1F} + q_{2L}^*) + \delta\right] \frac{\beta(r-\alpha)}{\beta - 1}.$$

The value function before investment is defined as

$$F_L^{det}(X, q_{1L}, q_{1F}, q_L^{det}) = q_{1L}\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L}+q_{1F})}{r}\right) + \left(\tfrac{X}{X_L^{det}}\right)^\beta \frac{\delta q_L^{det}}{\beta-1} - (q_{1L} + q_L^{det})\left(\tfrac{X}{X_F^*}\right)^\beta \frac{\delta}{\beta-1}.$$

Proposition 7 *Let the production capacities be defined as in Proposition 5 and let the current value of the shock process be defined as X. Then the accommodation strategy is considered for $X \in (X_1^{acc}, \infty)$, where,*

$$X_1^{acc} = \max\left\{\frac{\beta(r-\alpha)}{r(\beta-4)}(4\eta q_{1F} + \delta),\ \delta(r-\alpha)\right\}.$$

The accommodation strategy leads to value function $V_L^{acc}(X, q_{1L}, q_{1F}, q_{2L})$,

$$\begin{aligned} V_L^{acc} &= (q_{1L} + q_{2L})\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L}+q_{1F}+q_{2L}+q_{2F}^*)}{r}\right) - \delta q_{2L}, \\ &= (q_{1L} + q_{2L})\tfrac{1}{2}\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L}+q_{2L})}{r}\right) - \tfrac{1}{2}\delta(q_{2L} - q_{1L}). \end{aligned}$$

For large initial values of X the leader invests immediately and chooses optimal capacity

$$q_L^{acc}(X, q_{1L}) = \frac{r}{2\eta}\left(\frac{X}{r-\alpha} - \delta\right) - q_{1L}. \tag{20}$$

For low values of $x(0) = X$, that is $X < X_L^{acc}$, the leader will invest when x reaches investment threshold value X_L^{acc}. The value of the investment threshold and the associated capacity level q_L^{acc} are determined as the solution of the set of equations determined by equation (20) and

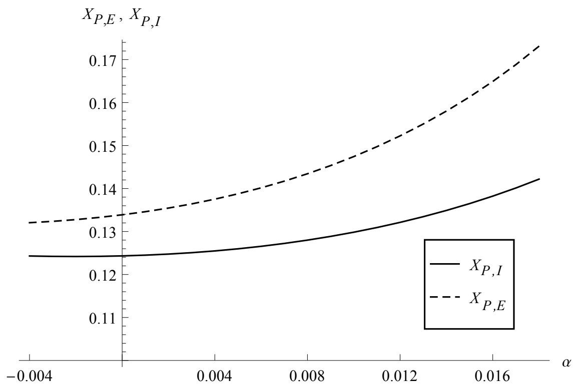
$$X_L^{acc}(q_{1L}, q_{1F}, q_{2L}) = \frac{\eta q_{2L}(q_{2L} + 2q_{1L}) + \delta r q_{2L}}{q_{2L} - q_{1L}} \frac{2\beta(r-\alpha)}{r(\beta-1)}.$$

The value function before investment is defined as

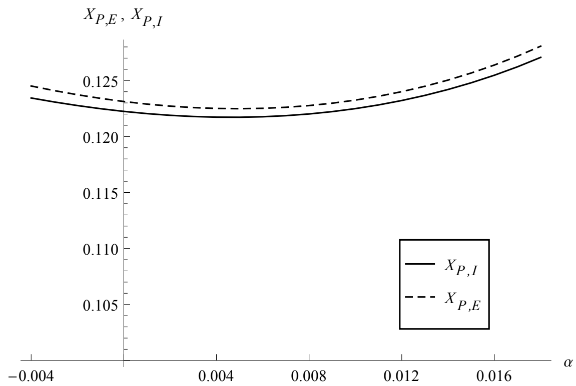
$$F_L^{acc}(X, q_{1L}, q_{1F}) = q_{1L}\left(\frac{X}{r-\alpha} - \frac{\eta(q_{1L}+q_{1F})}{r}\right) + \left(\tfrac{X}{X_L^{acc}}\right)^\beta \frac{\delta q_L^{acc}}{\beta-1}.$$

Preemptive equilibrium

The following figures show how the preemption triggers change under a change in parameter values.

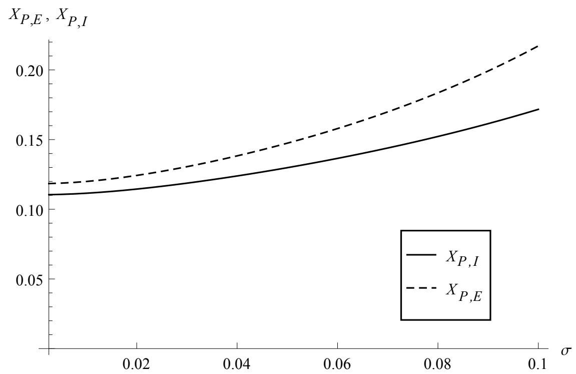


(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

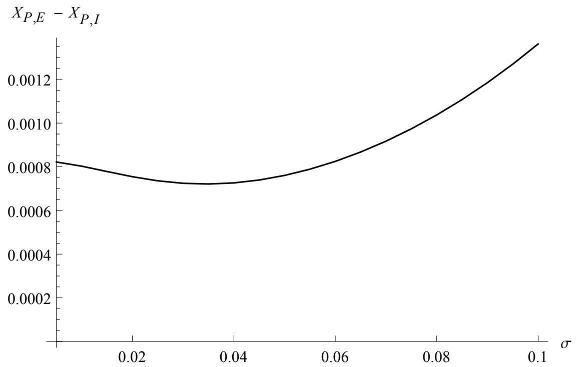


(b) Investment triggers with $q_{1I} = 0.5$.

Figure 16: Preemption triggers for different values of α.

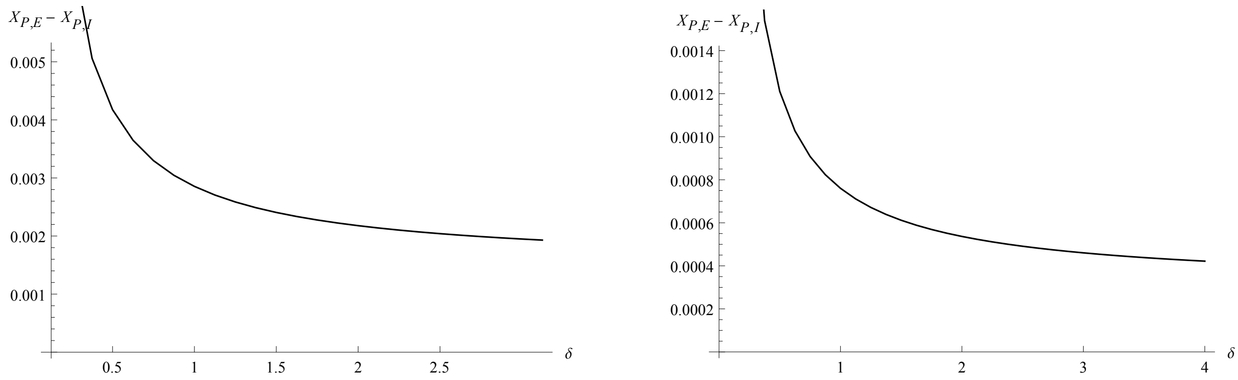


(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.



(b) Difference in investment triggers with $q_{1I} = 0.5$.

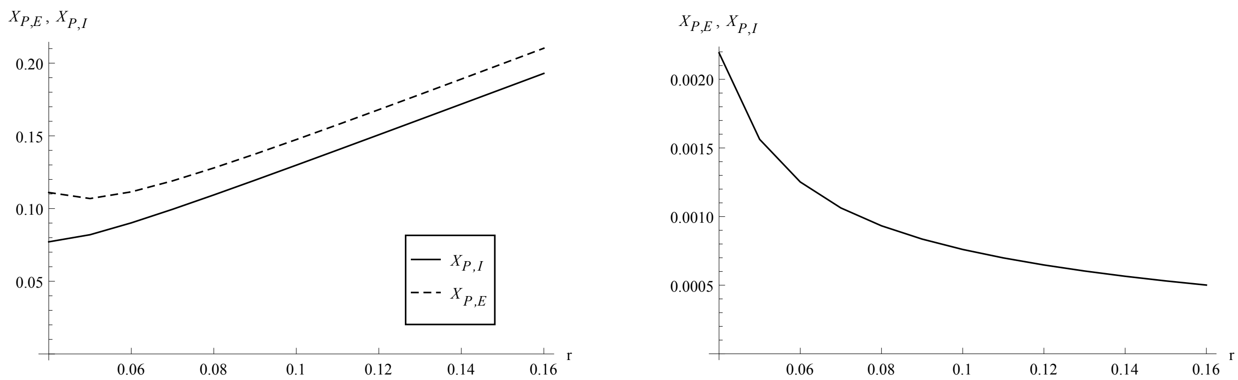
Figure 17: Preemption triggers for different values of σ.



(a) Difference in investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Difference in investment triggers with $q_{1I} = 0.5$.

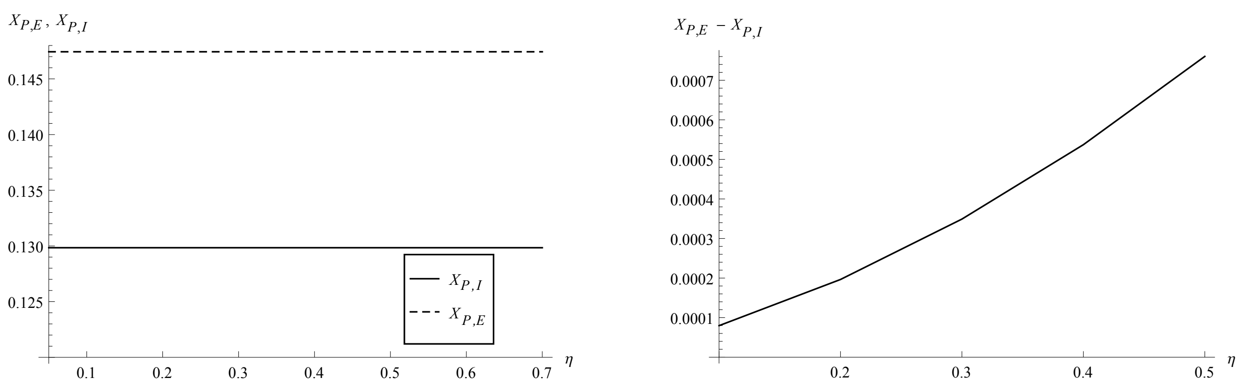
Figure 18: Preemption triggers for different values of δ.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Difference in investment triggers with $q_{1I} = 0.5$.

Figure 19: Preemption triggers for different values of r.



(a) Investment triggers with $q_{1I} = q_{1I}^{myop}$.

(b) Difference in investment triggers with $q_{1I} = 0.5$.

Figure 20: Preemption triggers for different values of η.