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AN ANALYSIS OF THE EFFECTS OF RISK BIASES ON REAL OPTIONS PRICING

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ABSTRACT

Over the past two decades, a significant amount of academic knowledge has been created on how to apply real options analysis to business investments. However, despite the many apparent advantages of using real options to value projects, the approach has not found favor with managers in practice. Some critics claim that the method is untrustworthy and might encourage too much risk taking. This paper provides an exploration of risk biases, viewed through the lens of prospect theory, as a potential cause for the mistrust toward real options. Using evidence from a survey of 67 business school students, the results showed that participants generally evaluated options in a manner consistent with prospect theory's S-shaped utility function. This research agrees with prior findings that buyers will price options at a discount and adds to the literature by confirming a new hypotheses that call options are consistently discounted more than put options of similar expected value. Additionally, evidence is provided that, in agreement with prospect theory, small probabilities cause distortions in options pricing. In general, pricing biases were found to be dependent on the framing of the scenario as either a gain or a loss and whether or not there were small probabilities involved. These findings bring into question the applicability of standard risk measures, such as discount rates derived from opportunity costs, to options scenarios.

Keywords: Real options, prospect theory, expected utility, options pricing, risk aversion

INTRODUCTION

The advantages of real options analysis (ROA) over more common techniques such as NPV, IRR, and the payback period in valuing flexible projects have been recognized in several areas including natural resource investment, competition and corporate strategies, manufacturing, real estate, international finance and business, research and development, regulated sectors, mergers and acquisitions, general finance (interest rates), inventory management, labor force strategy, venture capital, advertising, law, and environmental conservation (Lander and Pinches (1998); Schwartz (2013); Rigopoulos (2015)

However, despite the widespread advantages that it provides over other methods, ROA has not gained a strong following with managers in practice. The limited amount of empirical research on the use of real options in practice indicates that ROA is not a popular approach for valuing business projects (Block, 2007; Busby & Pitts, 1997; Hartmann & Hassan, 2006; Ryan & Ryan, 2002). Busby and Pitts (1997) found that, as of 1996, 80% of large firms in the United Kingdom did not have procedures in place to evaluate the flexibility of what could be considered a deferment option. In addition, the authors found that 86% of large United Kingdom firms provided no procedures for accounting for an ability to abandon a project; 57% had no procedures to value rescaling options; and 75% had no way to consider the effect of growth options. A decade later, Block (2007) found that about 86% of firms appearing on the Fortune 1,000 still did not make use of real options. The main reason ROA was not used in these firms was a “lack of top management support.”

Block (2007) finding of a lack of top-management support for real options raises some further questions regarding the root cause. For instance, why are managers so reluctant to accept real options analysis in practice despite the plethora of theoretical evidence that the method is superior for many common types of projects? This paper suggests that one possible reason could be that cognitive risk biases cause managers to price options in ways that may not be anticipated in normative ROA modeling.

Real Options Analysis

ROA can model a wide variety of corporate projects more accurately than traditional valuation techniques such as discounted cash flows (DCF), internal rate of return (IRR), or the payback period (Luehrman, 1998; Myers, 1977; van Putten & MacMillan, 2004). This is because these more traditional methods tend to model investments as a single stage, fixed set of cash flows discounted by a singular rate such as a weighted average cost of capital (WACC). Put another way DCF, IRR, and the payback period ignore the ability for managers to make adjustments as new information is learned throughout the project. In contrast, ROA models assume projects are managed dynamically and can accommodate multiple discount rates to reflect varying levels of risk associated with each phase of a project.

However, despite the widespread advantages that ROA provides over other methods of valuing flexible projects, the method has not gained a strong following with managers in practice. Tiwana, Wang, Keil, and Ahluwalia (2007) found that managers are only likely to explore real options when a valuation performed using one of the conventional methods is perceived as being lower than the subjective value that the manager expected. Two major theories have been proposed in the behavioral economics and finance literature to account for such subjective valuation: Friedman and Savage's (1948, 1952) expected utility theory and Kahneman and Tversky's (1979, 1992) prospect theory. The following sections provide a brief summary and review of each.

Expected Utility Theory

Expected utility theory, pioneered by (Friedman and Savage (1948), 1952)) is based on the position that when evaluating situations involving risk, such as gambling, buying insurance, or making a business investment, individuals tend to make decisions based on the utility or the subjective value that an individual assigns to an item, whether that item is an object, an experience, or even money, relative to all other alternatives, rather than on the objective value. Prior research has shown that nearly everything, including goods, services, and money, has diminishing marginal utility such that each additional item, experience, and dollar yields less satisfaction than the one before it. In the case of money, this means that the dollars one already has in one's pocket carry more utility per dollar than the dollars one has yet to gain.

Following the development of expected utility theory, several researchers set out to test its axioms empirically. The general result was that there were many types of situations in which humans could be observed in ways inconsistent with expected utility theory. One of the most well studied deviations from expected utility, and one that may be likely to affect options pricing deals with human aversion towards ambiguity (Fox & Weber, 2002). Allais (1953) observed that small probabilities are given stronger decision weight than more moderate probabilities. For example, he found that an overwhelming number of research subjects would prefer a certain gain over a nearly certain but disproportionately larger expected gain such that the expected value of the latter was significantly and obviously greater than the former.

Prospect theory

(Kahneman and Tversky (1979), 1992)) recognized the intuition of expected utility theory and sought to reconcile its shortcomings by solving the Allais paradox problem (Allais, 1953; Allais & Hagen, 1979). Their empirical research showed that people faced with risky decision not only weighed probabilities as well as outcomes but that they did so by comparing outcomes to a reference point, usually the current state of nature. Thus a major advantage of prospect theory is that it integrates many components from expected utility theory and behavioral finance literatures such as (Friedman and Savage (1948), 1952)) and Von Neumann and Morgenstern (1944) into a single framework.

In prospect theory, the slope of disutility for losses (to the southwest of the reference point at the origin in Figure 4) is steeper than the slope of utility for gains (to the northeast of the reference point at the origin in Figure 4) (Barberis, 2012; Kahneman & Tversky, 1979, 1992). Using data from surveys of students and faculty at various universities, the researchers found that people are risk-averse when choices are framed in terms of gains; however, they are risk-seeking when choices are framed in terms of losses. For example, in one survey, 84% of people exhibited risk aversion by preferring a certain gain of $500 to a 50/50 chance of winning $1000 or $0. However, 69% of the same respondents displayed risk-seeking behavior by preferring a 50/50 chance of losing $1,000 to a certain loss of $500. Although the expected gain or loss was the same in both scenarios ($500), the respondents were consistently risk-averse when the situation was framed as a gain and risk-seeking when the situation was framed as a loss (Kahneman & Tversky, 1979). Along a similar line of thought, the researchers found that for any pair of equally sized gains and losses, the loss was always associated with more disutility than the gain was associated with utility, that is, losses hurt more than gains felt good. For example, if a person were to find a sum of money and later lose it, that person would be less happy (have less utility) than he or she would have been before the money was found (Thaler, 1999). The resulting s-shaped utility function appears with gains stemming from the reference point (usually a person's current state of affairs), resembling the concave shape of Friedman and Savage's (1948, 1952) model and reflecting risk aversion, as shown in Figure 1.

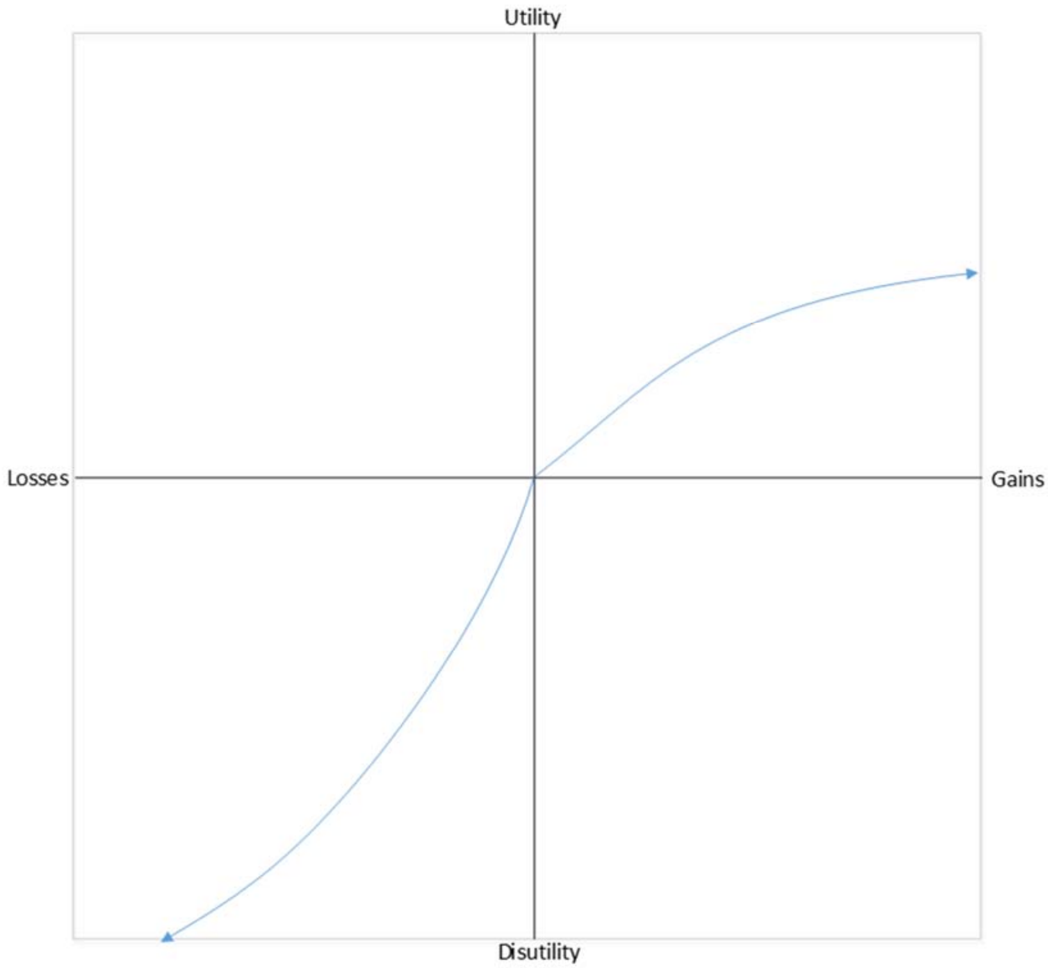

Figure 1 - Prospect theory utility curve adapted from Kahneman & Tversky (1979)

In addition to finding a difference between human perceptions of gains and losses, (Kahneman & Tversky, 1979, 1992) found that people tend to overweight small probabilities, that is, people tend to act as if improbable outcomes are more likely to occur than they really are. For example, a person would be willing to pay a proportionately (premium to expected value) higher premium for an event that had a 1% chance of occurring than they would pay for an event with a 10% chance of occurring.

Impact of Prospect Theory on Options Pricing

Given that prospect theory suggests that managers may subjectively discount real options much more than objectively warranted, a question which arises is whether or not the use of mainstay financial valuation techniques such as DCF or IRR as well as options pricing models such as Black-Scholes, which assume that investors' attitudes toward risk are linear, is appropriate. In scenarios where an investor is evaluating the purchase of an option, whether a call or a put, he or she must accept a certain and immediate loss in the present in exchange for an uncertain gain or (in the case of a put) the avoidance of an uncertain loss in the future; the option is essentially a bet on an outcome that has no actual effect on the outcome itself. This is often in contrast to other types of investments where investment represents ownership stake in an asset that is already has or is producing value. Miller and Shapira (2004) argue that option premiums may be perceived as a loss by investors and, therefore, fall into the steeper disutility domain of prospect theory. In other words, the price that must be paid for the option premium is more utility heavy than the expected value of the gain, thus the gains will be discounted at rate higher than might be expected in situations where initial outlays are not perceived as a loss.

Other option scenarios likely to affect discount rates in unexpected ways include those dealing with nearly certain outcomes. According to prospect theory, small probabilities are overweighted such that when they are evaluated by human subjects they appear to be larger than they really are.

Theoretically, these biases should manifest themselves in the form of pricing discounts or premiums relative to similar options with more central probabilities. For example, call options that are nearly certain to pay off and have a small chance to not pay off – it can be reasoned that subjects ought to price this option as if the probability of it not paying off were larger than it is, reflecting a different risk appetite than might be captured by a discount rate appropriate for other types of investments.

Empirical tests of prospect theory on option pricing

Miller and Shapira (2004) built on this intuition and hypothesized that a prospect theory utility curve should result in both buyers and sellers pricing options at discounts relative to standard valuation models. The researchers hypothesized that call option buyers, operating within the prospect theory domain of gains, would exhibit risk aversion by being willing to pay a maximum price below that of the expected value of the call option. Similarly, call option sellers would be expected to show risk aversion by being willing to accept a price less than the expected value. In other words, the expectation was for call sellers (to some extent) to prefer a smaller but certain gain to a larger but uncertain one.

Miller and Shapira (2004) also reasoned that put option buyers, aiming to exchange an uncertain loss for a certain one, would display risk-seeking behavior by being willing to pay only an amount less than the expected value of the loss, that is, they would prefer to keep the gamble than pay the expected value of the loss. At the same time, a put option seller, seeking risk exposure they did not have before would show risk-seeking behavior by being willing to accept a price that was lower than the expected value of the loss they were covering. The net effect of these biases is that options, even when mispriced according to normative models, will still clear the market because both buys and sellers end up being willing to accept prices below expected value. Miller and Shapira's (2004) results were consistent with the hypotheses: Both buyers and sellers of both calls and puts showed a clear trend toward discounting valuations, suggesting not only that subjects had an s-shaped utility curve consistent with prospect theory, but that this utility curve was affecting the subjects' option valuations.

However, a significant limitation of Miller and Shapira (2004)'s study is that they only examined call options and they assumed the utility curve was symmetric and invertible, so their model predicted that individuals were willing to pay an amount that equated to the utility associated with the expected utility of the uncertain gain Y_{1A} (via the inverse function). As Miller and Shapira (2004) pointed out, the assumption was not critical to their hypotheses because they were looking only for discounting behavior and were not attempting to compare any behavior they found. However, this means Miller and Shapira (2004) did not necessarily show prospect theory in action; a symmetric and invertible utility curve is also consistent with other diminishing marginal utility models (Friedman & Savage, 1948).

Research aim

The aim of this research was to overcome some of the gaps and limitations in Miller and Shapira's (2004) study. First, this research included both call and put options in its questionnaire to capture the duality of gains and losses in prospect theory. Second, this research did not assume a symmetric utility function for gains and losses; thus, potential effects of prospect theory's steeper disutility curve on options pricing could be explored. Third, a variety of payoff probabilities ranging from 5% to 95% were used so that the relationship between probability and options pricing, as expected by prospect theory, could be measured.

Research Model

In order to assist in the design of this research, a research model (figure 2) was built to depict the utility of options premiums and payoffs from the purchaser's point of view, although evidence indicates that similar effects should also apply to option sellers (Copeland & Antikarov, 2003; Dixit & Pindyck, 1994; Miller & Shapira, 2004).

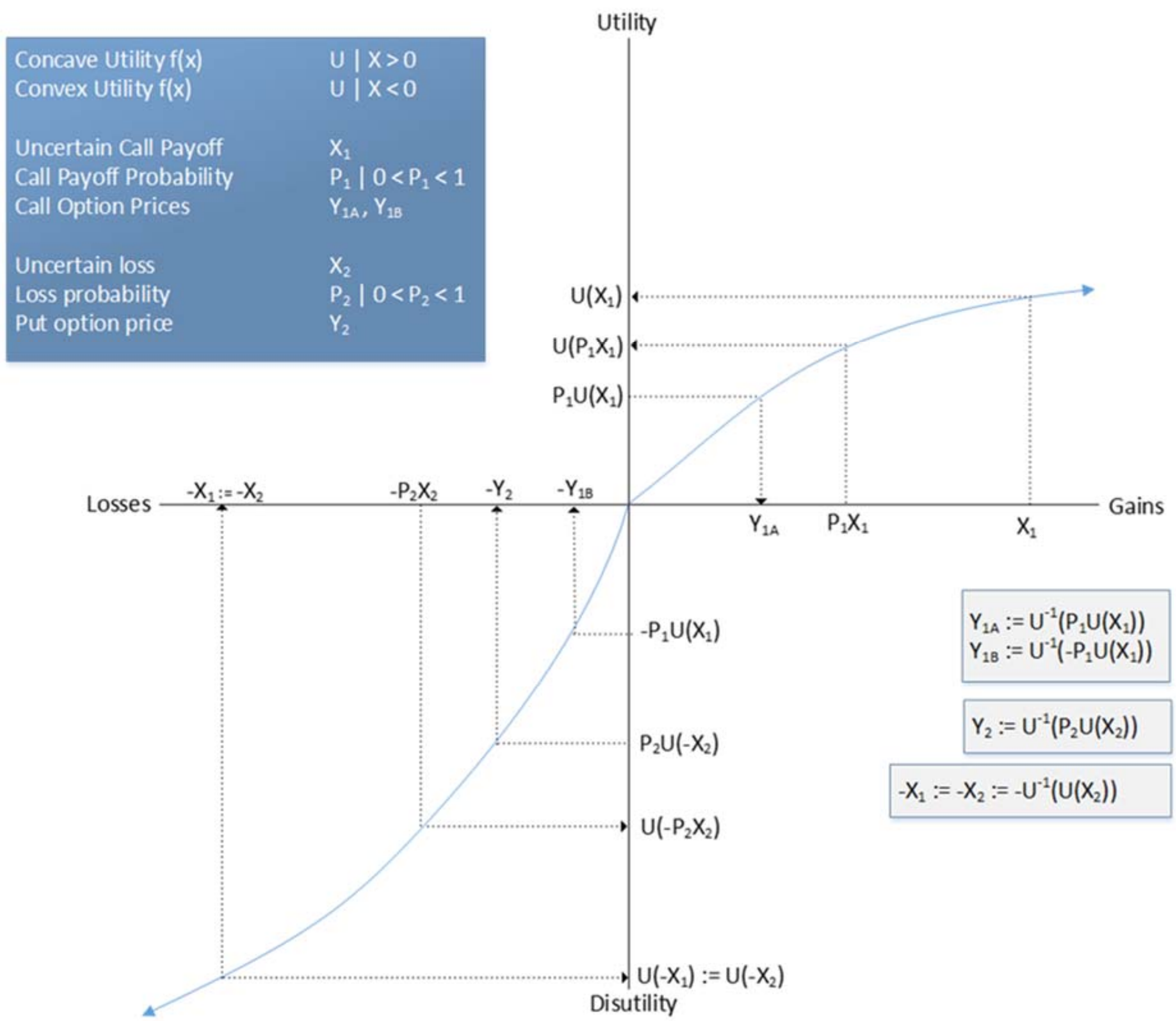

Figure 2 – Research Model

The research model relates the s-shaped utility curve to pricing expectations using a two-axis grid. The *X*-axis shows the change (gain or loss) in wealth, while the *Y*-axis represents the resulting utility gain or loss. The relationship between changes in wealth and changes in utility is illustrated by a prospect theory curve drawn over the matrix showing diminishing marginal utility for gains and diminishing marginal disutility for losses. Losses show a steeper slope than do gains. The curve is described by the equation $Y = U(X)$ where Y is utility, X is the change in wealth, and U is the utility function.

An uncertain gain scenario with a value of X_1, and with a probability P_1 of occurring is shown and mapped to the utility associated with the gain $U(X_1)$, and the probability-adjusted expected value of the gain $U(P_1X_1)$. Expected utility holds that humans will evaluate the utility of the gain first and then adjust for probability, meaning that for risk-averse individuals, the expected utility associated with the uncertain gain at $P_1U(X_1)$ will be less than the utility of gain equal to the expected value at $U(PX_1)$ (Barberis, 2012; Friedman & Savage, 1948, 1952; Kahneman & Tversky, 1979, 1992).

A hallmark feature of prospect theory is that losses have a steeper utility function than gains—losses hurt more than gains feel good. This aspect of prospect theory is reflected in the research model by emphasizing the distinction between Y_{1A}, as hypothesized by Miller and Shapira (2004), and Y_{1B}, which comes as a result of measuring the disutility of the premium against prospect theory's steeper disutility curve. The result is that larger discounts are expected for call option prices than were predicted by Miller and Shapira (2004).

Based on the research model, put options are expected to show similar price discounting behavior, with the expected utility from the loss at $P_2U(-X_2)$ linked to the hypothetical put option price at Y_2. The major difference for put options is that their purchase serves to limit losses. Thus, both the loss potentially avoided by the put and the put option premium are evaluated on the steeper disutility portion of prospect theory's utility curve. The result of this evaluation is that for any pair of call and put options with similar expected values, the call option (Y_1A) will be perceived to have a lower value than will the put option (Y_2).

Hypotheses

The following hypotheses were formed from the full research model shown in Figure 2.

Hypothesis 1: Call option buyers will price options below expected values.

Given a call option with probability *P* of paying off an amount X_1 with premium of Y_1 or Y_2:

1. A risk-averse call buyer will be willing to pay an option premium that provides less disutility than the expected utility from the payoff, assuming a symmetric and invertible utility curve for gains and losses, as in Miller and Shapira (2004).

$$-U\left(-Y_{1A|B}\right) < P_1 U(X_1)$$

2. The concavity of the utility function for gains means that expected utility of the payoff will always be less than the utility of the expected payoff, that is, utility for the payoff is evaluated first and then adjusted for probability, as opposed to evaluating the expected value first and then converting to utility (Miller & Shapira, 2004). The difference can be visualized by comparing the two values forming the inequality on the research model, as shown:

$$P_1 U(X_1) < U(P_1 X_1)$$

3. Thus, the value associated with the expected utility from the payoff is less than the value associated with the expected value of the payoff:

$$Y_{1A} := U^{-1}(P_1 U(X_1)) < P_1 X_1$$

4. Still assuming that the utility is symmetric and invertible, buyers will value calls at a discount (Miller & Shapira, 2004):

$$\frac{Y_{1A}}{P_1 X_1} < 1$$

5. Expanding on Miller and Shapira's (2004) reasoning, there will be more disutility associated with a given loss than with an equally sized gain, since the slope of utility is steeper for losses than it is for gains in prospect theory:

$$-U(-X_1) > U(X_1)$$

6. Thus, the value associated with a given amount of disutility will be less than is associated with an equal amount of utility, that is, a smaller value is placed on disutility because utility for losses is steeper than it is for gains.

$$Y_{1B} < Y_{1A} \mid |-Y_{1B}| < Y_{1A}$$

Hypothesis 2: Put option buyers will price options below expected values.

Given a put option with probability *P* of losing an amount X_1 with premium of Y_2:

1. A risk-averse put buyer will be willing to pay an option premium where the disutility associated with the put option premium is less than the expected disutility from the adverse outcome (Miller & Shapira, 2004).

$$U(-Y_2) > P_2 U(X_2)$$

2. The convexity of the utility function for losses means that expected disutility of the loss will always be less than the disutility of the expected loss. To put it another way, disutility for the loss is calculated first and then adjusted for probability (Miller & Shapira, 2004).

$$-\,P_2U(X_2) < -\,U(P_2X_2)$$

2. Thus the value change associated with the expected disutility from the loss is less than the value change associated with the expected value of the loss:

$$Y_2 \;:=\; U^{-1}\big(P_2\mathrm{U}(-X_2)\big) <\; U^{-1}(-X_2)$$

This results in a discount from normative pricing:

$$\frac{-Y_2}{P_2X_2} < 1$$

4. Since the value associated with the disutility of the option premium is less than the value associated with the expected utility of the gain, the option will be valued at an even greater discount than if the utility function was symmetrical for gains and losses.

$$\frac{Y_{1A}}{P_1X_1} < \frac{Y_{1B}}{P_1X_1} < 1$$

Hypotheses 3: Relative to expected values, call options will be priced at greater discounts than puts.

Given a pair of call and put options with equal payoffs X_1 and X_2, respectively, and probabilities P_1 and P_2, respectively:

1. The slope of the utility function is greater for gains than it is for losses. Thus, disutility (–*U*) for a loss is always greater than utility (*U*) for a gain of equal magnitude.

$$-U(-X_2) > U(X_1)$$

2. Given equal probabilities ($P_1 = P_2$) and payoffs ($X_1 = X_2$), expected disutility for a probabilistic loss will be greater than the utility for a probabilistic gain of equal magnitude:

$$-P_2U(-X_2) > P_1U(X_1)$$

3. Inverting the utility function means that a loss associated with a given amount of disutility will be smaller than the gain associated with an equal amount of utility. In general terms with Q representing a quantity of utility:

$$-U^{-1}(-X_2) < U^{-1}(X_1)$$

4. Put options, dealing with loss avoidance are expected to be evaluated in the domain of losses. While certain types of "naked" financial options allow an individual to gain from a put option by not being exposed to the underlying asset risk, this is not possible with real options. Within the scope of real options, a put buyer exchanges a probabilistic loss for a certain loss and seeks a premium with a disutility that is less than the expected disutility of the loss.

$$-U(-Y_2) < -PU(X_2)$$

5. The payoff for a call option is evaluated in the domain of gains as a quantity of expected utility while the option premium is evaluated in the domain of losses as quality of disutility. The call buyer seeks a price where expected utility exceeds the disutility:

$$PU(X_1) > -U(-Y_1)$$

6. The payoff from a call option is an uncertain gain. Thus, it results in less expected utility than an uncertain loss of similar magnitude. Because of this, the maximum price associated with a call option will be lower than a put option covering a loss of the expected value.

$$-U^{-1}(-P_1U(X_1)) < -U^{-1}(P_2U(-X_2))$$

7. Since the price a buyer will be willing to pay for a call is lower than that for a similar put, it can be said that calls will be priced at a greater discount relative to expected values:

$$1 - \frac{-U^{-1}(-P_1 U(X_1))}{P_1 X_1} > 1 - \frac{-U^{-1}(P_2 U(-X_2))}{P_2 X_2}$$

Hypothesis 4A: Call options with near-certain payoffs will be priced at larger discounts relative to calls with smaller probabilities.

Hypothesis 4B: Put options with near-certain payoffs will be priced at a larger discounts relative to puts with smaller probabilities.

As the probability of an option's payoff increases, so will its expected value. A risk-neutral individual would perceive a linear relationship between probability and expected utility, that is, doubling the probability of a payoff would also double expected utility. In prospect theory, however, small probabilities are overweighted (Barberis, 2012; Kahneman & Tversky, 1979, 1992). This means that for a call option with a nearly certain payoff, the probability of not paying off will be perceived to be larger than it actually is. Likewise, in the case of a put option with a nearly-certain payoff, the probability of the adverse outcome not occurring is also perceived to be larger than it really is. The result in both extreme cases should be that the ratio of expected utility to expected value is lower for these options than for others with less certain payoffs (reflected as a higher discount).

Hypothesis 5A: Call options with very low probabilities of paying off will be priced at smaller discounts relative to calls with larger probabilities.

Hypothesis 5B: Put options with very low probabilities of paying off will be priced at smaller discounts relative to puts with larger probabilities.

As the probability of an option's payoff decreases, so does its expected value. A risk-neutral individual would perceive a linear relationship between probability and expected utility. As a result, halving the probability of a payoff would also halve expected utility. Since small probabilities are overweighted in prospect theory, call options with a very low chance of paying off will be perceived to have a higher chance than they really do. Likewise, in the case of a put option with a very low chance of paying off, the probability of the adverse outcome actually occurring is also perceived to be larger than it really is (Barberis, 2012; Kahneman & Tversky, 1979, 1992; Tuthill & Frechette, 2002). The result in both extreme cases should be that the ratio of expected utility to expected value is higher for these options than for others with less certain payoffs (reflected as a higher discount or even a premium).

Methodology

Sample

Approximately 320 business school students at the University of Newcastle were contacted via various methods, including e-mail or in-classroom invitations, to participate in an online experiment, which asked them to value hypothetical real options. Before performing any analysis, the data were checked for spurious responses that might indicate that the respondent did not understand what was expected or that he or she may have hurried through the survey. After excluding suspect responses, the remainder contained a total of 344 responses from 67 subjects, resulting in a response rate of about 21%.

Questionnaire

To ensure that respondents comprehended the questions, a primer on options theory and vocabulary was provided prior to presenting any questions. The principal goal of the primer was to ensure that research subjects understood the options problems that would be presented to them and not to provide any advice or guidance on how to solve the problems which may have biased the results. Additionally, both net worth and a time horizon were provided, which were intended to give context and scale to the risks and rewards associated with each option presented for pricing.

The research questionnaire was divided into two sections. The first section collected respondents' demographic information, including age, gender, work experience, and status in school. The second section provided hypothetical options pricing scenarios, in which respondents were asked to value three call options and three put options. The three call options were presented in the following format:

Suppose that your net worth totals \$W. You are presented with an opportunity to buy a call option that has a P_1 percent chance to be worth \$$X_1$, and a P_{1B} percent chance to be worth \$0 by the end of the day. For this option, I would be willing to pay: __Y1

Three questions involving put options were presented in the following fashion:

Suppose that you own an investment portfolio totaling \$W, representing most of your net worth. By the end of the day, the portfolio has a P_2 percent chance to retain its current value and a P_{2B} percent chance to lose \$$X_2$. You are offered the chance to buy a put option on your portfolio which would limit your losses to zero. For the option to transfer the day's outcome to someone else (insuring against any loss), I would be willing to pay: __Y2

In order to collect a sample with a wide variety of payoff (or loss) probabilities, the questionnaire was programmed to produce random values within certain bounds. A summary of the variables used in the research model, hypotheses, data collection, and analyses is shown in Table 1.

Table 1 - Research Variables

Variable Name	Type and Bounds	Description
W	Independent, Fixed W = \$100,000	Net worth, intended to provide context and scale to the gains and losses presented.
P_1	Independent, Random $.05 <= P_1 <= .95$ Stepping: .15	Probability of call option payoff. Varies randomly between 5% and 95% at 15% increments (5%, 20%, 35%).
P_{1B}	Independent, Random $P_{1B} = 1- P_1$	Probability of call option not paying off. This was whatever quantity was needed to make total probability equal to 1.
X_1	Independent, Random $10{,}000 <= X_1 <= 100{,}000$ Stepping: 10,000	The sum that will be gained if the call option paid off. This varies randomly between \$10,000 and \$100,000 at \$10,000 increments.
Y_1	Dependent, input $0 < Y_1 > X_1$ Whole number	The amount the respondent reports he or she is willing to pay for the call option. Y_{1A} is compatible with Miller and Shapira (2004) symmetrical and invertible simplification whereas Y_{1B} is consistent with prospect theory in (Kahneman and Tversky (1979), 1992)).
P_2	Random $.05 <= P_2 <= .95$ Stepping: .15	Probability of loss occurring (put option payoff). Varies randomly between 5% and 95% at 15% increments (5%, 20%, 35%).
P_{2B}	Random $P_{2B} = 1- P_2$	Probability of put option not paying off. This was whatever quantity was needed to make total probability equal to 1.
X_2	Random $10{,}000 <= X_2 <= 100{,}000$ Stepping: 10,000	The loss that will be avoided if the call option pays off. This varies randomly between \$10,000 and \$100,000 at \$10,000 increments.
Y_2	Dependent, input $0 < Y_2 > X_2$ Whole number	The amount the respondent reports he or she is willing to pay for the put option.

As the respondents completed each question, the values for all variables generated, as well as the text displayed to the respondent and his or her response, were recorded in a database. In order to link responses from the same subject while maintaining privacy, a unique but non-identifying Internet session identifier (SID) was used as a grouping variable.

Analysis and Findings

Table 2 summarizes the general results of the hypotheses testing and a detailed description of the test and rationale for each result follows. A key dependent variable in the hypothesis testing was the discount, which reflects the percentage difference between the expected value of the option and the price survey respondents reported that they were willing to pay. Positive discounts, which are reported without a sign, indicate a price lower than the option's expected value, while a negative discounts are reported using the minus (-) symbol, reflect a price higher than the option's expected value. Where hypotheses involved testing the difference between two mean discounts, such as that between calls and puts, results are always reported with a sign; the plus (+) symbol to indicates a positive difference the first and second variable (VAR_1 - VAR_2 > 0) and the minus (-) symbol indicates a negative difference between the first and second variable (VAR_2 – VAR_1 < 0).

Table 2 - Hypotheses Testing Summary

Hypothesis Number	**Summary**	**Mean Discount or *Difference***	**Result**
H1	Discounting of call options with non-extreme probabilities	62.65%*	Confirmed
H2	Discounting of put options with non-extreme probabilities	25.82%*	Confirmed
H3	Call options priced at higher discount than put options	36.83%*	Confirmed
H4A	High probability-to-pay-off call options discounted more	*-8.60%*	Rejected
H4B	High probability-to-pay-off put options discounted more	*38.09%**	Confirmed
H5A	Low probability-to-pay-off call options discounted less	*-101.83%***	Confirmed
H5B	Low probability-to-pay-off put options discounted less	*-139.38%***	Confirmed

***Denotes statistical significance at the $p < .05$ level**
****Denotes statistical significance at the $p < .10$ level**

Hypothesis 1

Hypothesis 1 predicted that call options with nonextreme probabilities (.05 < P1 < .95) would be priced at a discount relative to expected value. In other words, the discount observed was neither zero nor negative (which would represent a premium). Testing this hypothesis was accomplished by constructing a 95% confidence interval for the mean discount observed and then determining if zero or any negative values existed within the confidence interval. The sample mean for discounts with call probabilities between .05 and .95, exclusively, was 62.65%, with a 95% confidence interval for the population mean of 53.14% to 72.17%, confirming the hypothesis. A relatively large (compared to the mean) standard deviation of 53.75% reflects the large variety of call payoffs and probabilities presented as well as a diverse set of risk preferences brought by the respondents.

Hypothesis 2

Hypothesis 2 predicted that put options with nonextreme probability (.05 < P < .95) would also be priced at a discount relative to expected value. Testing this hypothesis was completed by constructing a 95% confidence interval for the mean discount observed for these put options and then determining if zero or any negative values existed within the confidence interval. The sample mean for discounts with probabilities between .05 and .95 exclusively was 25.82%, with a 95% confidence interval for the population mean of 11.72% to 39.92%, confirming the hypothesis. As with Hypothesis 1, a relatively large standard deviation of 79.31% may reflect the large variety of inputs and risk preferences.

Hypothesis 3

Hypothesis 3 predicted that call options would be priced at a greater discount than puts. This was tested by performing an independent samples t-test comparing the mean discount observed of calls and puts, producing a likelihood that the two means are the same. The test produced a statistically significant mean difference of +36.83% for calls with a 95% confidence interval for the mean difference between +19.90% and +53.77%; $t(216.086) = 4.287$, $p = .000$. Since call options were found to have greater discounts (relative to expected values) than put options, Hypothesis 3 was confirmed. It is of importance to note that that equal variances were not assumed in this test due to a statistically significant ($p < .05$) result on Levene's test for equality of variances, that is, the variance for discount was different for call options than it was for put options.

Hypothesis 4

Hypotheses 4 predicted that because small probabilities are overweighted in prospect theory, participants ought to price options with a near-certain probability of payoff at larger discounts than they would for options with less-certain probabilities, that is, respondents were expected to perceive the small chance of not paying off to be larger than it really was and price the option lower as a result. Hypothesis 4 was separated into 4A and 4B for call and put options, respectively, primarily so that each type of option could be tested independently. Each was tested by performing an independent samples equality of means test.

The independent samples test for Hypothesis 4A, that calls which are nearly certain to pay off would be discounted relatively more than other calls, did not find a statistically significant difference of means; $t(145)$, $p = .472$. The finding was a mean difference of -8.60% for call options with near-certain payoffs with a 95% confidence interval of -32.13% to +14.95%. There was no significant difference in variance between the options with near-certain probabilities and those without (Levene's test, $p > .05$), so equal variances were assumed in the means test. Because the null hypothesis that there the two means are the same cannot be rejected, the hypothesis that the means are different is rejected.

The means test for Hypotheses 4B, that puts which were nearly certain to avoid losses would be discounted more relative to other puts, found a statistically significant, +38.09% mean difference for discount for these put options; $t(104.483)=4.025$, $p = .000$. The 95% confidence interval for the difference was with a confidence interval of +19.39% to +56.86%. Equality of variances was not assumed for this test because Levene's test for equal variances was significant ($p < .05$). Since the mean difference was positive and statistically significant, Hypothesis 4B was confirmed.

Hypothesis 5

Hypothesis 5 predicted that overweighting of small probabilities (according to prospect theory) should result in options with small probabilities of paying off being priced at a premium, compared to options with other probabilities. Hypothesis 5 was separated into 5A and 5B for call and put options, respectively, to test each with an independent samples equality of means test.

The statistical test for Hypothesis 5A, that call options with small probabilities of paying off would be priced relatively higher than others, resulted with an expected negative difference in mean discount. The mean difference was -101.83% with a 95% confidence interval of -200.07% to +3.60%, without assuming equal variances (Levene's test, $p < .05$). Although the significance was higher than the .05 threshold set before the analyses began, there is still a 94% probability that calls with a small probability of paying off were priced at a premium to other calls; $t(25.446) = 1.988$, $p = .058$. Additionally the boxplot in Figure 6 visualizes the tendency for respondents to price options with small (.05) probabilities of paying off at a premium (negative discount). The interquartile range (IQR) for options with a 5% chance of paying off extends much farther into negative territory than for the call options with other probabilities and several outliers can be seen indicating that, in some cases, respondents were willing to pay several times the options expected value.

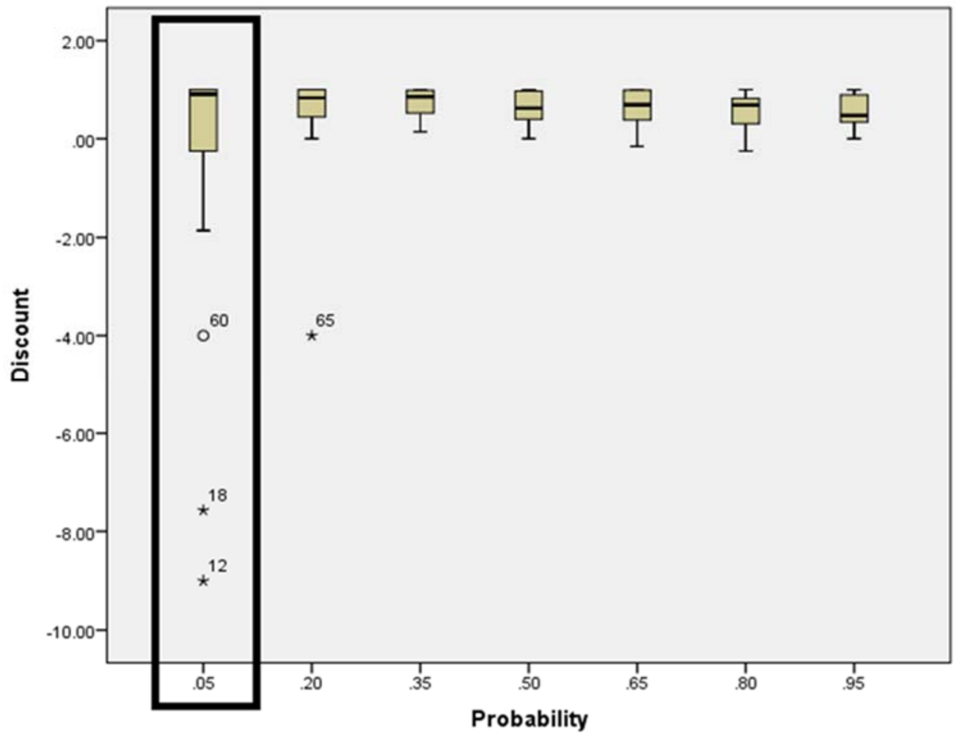

Figure 3 – Discount versus probability for call options

The statistical test for Hypothesis 5B, that put options with small probabilities of paying off will be priced at a relative premium, resulted in an expected negative mean difference. Put options with small (.05) probabilities of paying off were priced at a premium relative to call options with greater probabilities. The mean difference was 139.38% with a 95% confidence interval of 306.11% to +27.35% without assuming equal variances, because Levene's test indicated unequal variances; t(18.293), p = .096. Although once again the significance (p) value of .096 was higher than the.05 threshold, the boxplot in Figure 4 shows the tendency for puts with a small probability of paying off to be priced at a premium. The pattern is strikingly similar to that observed in the test for Hypothesis 5A, with the IQR and several outliers extending far into negative territory, indicating that respondents were willing to pay larger and in some cases, extreme, premiums for put options with small probabilities.

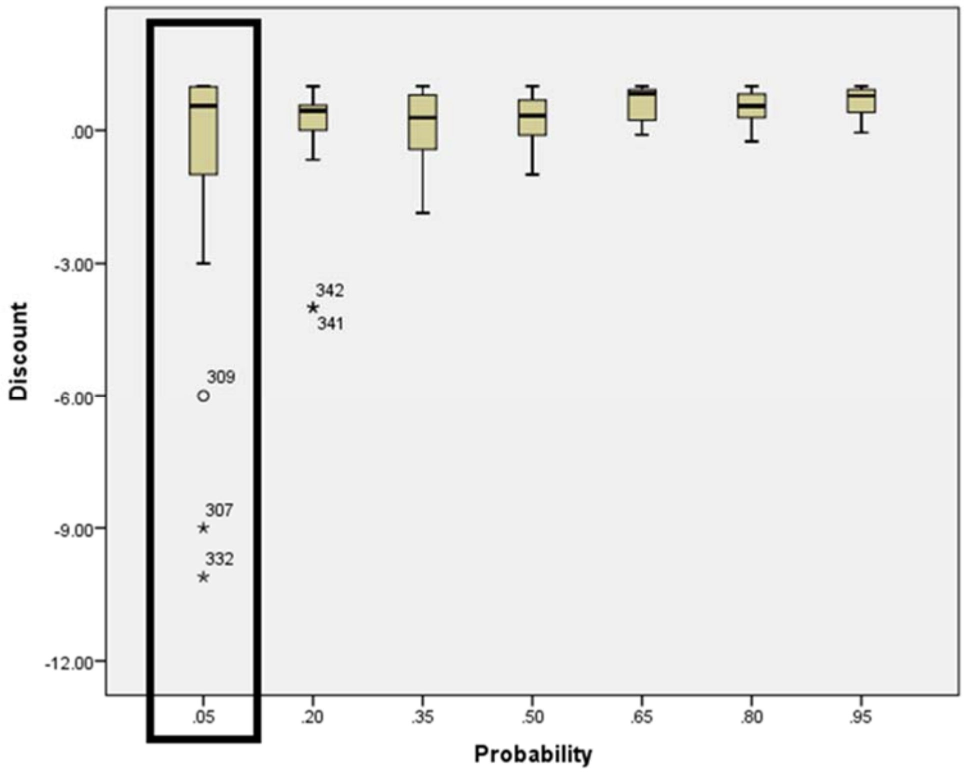

Figure 4 –Discount versus probability for puts

Demographic differences

In addition to testing the hypotheses detailed previously, the data were also explored to look for differences in discounting behavior between groups of respondents such as those based on sex, age, geography, and capital budgeting knowledge. In comparing discounting behavior between males and females, there was no significant difference with males discounting call options with non-small probabilities (greater than .05 but less than .95) with a mean discount of 59.35% for males and 69.21% for females with 95% confidence intervals of [50.80%, 67.90%] and [48.31%, 84.39%], respectively. There was a significant ($p < .05$) difference in discounting put options with non-small probabilities between males and females having mean discounts of 31.85% (n=81, SD=67.45%) and -24.77% (n=62, SD=212.66%) respectively. Perhaps more interesting than the mean discount difference however the relatively large standard deviation for female respondents' put discounting compared to that of the males'. This observation may mean that the conclusions about put discounting behavior may not apply to at least some females. This potential discovery warrants follow up research.

Capital budgeting experience

Finally, means comparisons (via one-way ANOVA) were completed for call and put option discount based on respondents' self-assessed familiarity with capital budgeting methods. There were no significant differences in discounting behavior between groups in discounting behavior for either call or put options although a potentially counterintuitive trend did reveal itself for the latter. Responses from research subjects reporting that they were "not familiar" with capital budgeting methods had the lowest mean discount at -9.94% (n=40, SD =183.70%), those reporting to be "somewhat familiar" with capital budgeting presented a mean discount of 7.4% (n=68, SD=158.43%), and finally the group reporting themselves as "very familiar" with capital budgeting methods had a mean discount of 28.60% (n=32, SD=82.46%). While the large standard deviations for each of the groups prevent the trend from having statistical significance at the $p < .05$ level, additional research using a larger sample size might be worthwhile to determine if there is anything more to be learned from this trend.

Discussion

Consistent with the earlier conclusions of Miller and Shapira (2004), this research showed that individuals priced both calls and puts at a discount. This behavior was expected and can be explained by prospect theory's uniquely shaped utility curve. Within the domain of gains, participants acting as option buyers exhibited risk aversion by reliably showing willingness to pay a maximum amount that was lower than the expected value. This finding is consistent with both prospect theory and other utility theories, such as (Friedman and Savage (1948), 1952)) and Von Neumann and Morgenstern (1944) where wealth provides decreasing marginal utility—each additional dollar gained provides individuals slightly less utility than the one before it. These decreasing marginal utility models manifest as risk aversion when humans are faced with risky situations, such as when considering a call option, where one must give up wealth that he or she already has in exchange for a chance at a gain. In other words, utility-heavy money that a person already has must be given up for the chance to gain money that is lighter on utility. Thus, for a lottery situation such as a call option to be attractive in terms of utility, the payoff will need to be comparatively large in monetary terms, resulting in pricing that is discounted relative to what it would have been if all money yielded the same utility.

In addition, as expected according to the hypotheses, subjects in this study exhibited the expected risk-seeking behavior in the domain of losses by consistently being willing to pay put option premiums that were below expected values. This finding is consistent with both prospect theory and other expected utility hypotheses where the utility curve for gains and losses is symmetric and invertible about the axis in a way similar to that assumed by Miller and Shapira (2004). In these utility models, disutility associated with monetary losses behaves in a similar fashion as utility does for gains with decreasing marginal disutility for losses, that is, the first few dollars lost result in proportionately greater utility loss than the next few, and so on.

With the first two hypotheses confirmed, the research findings at this point are consistent with Miller and Shapira (2004)'s findings and show that buyers of call and put options will bid at prices discounted relative to expected values, as predicted by prospect theory. However, if prospect theory is truly at the root of this pricing behavior, then other hallmark characteristics of prospect theory should also be evident.

In particular, a distinctive characteristic of prospect theory is the notion that losses relative to the reference point will create more disutility than a gain of equal proportion will create utility. We hypothesized this feature to appear when research participants priced call options at a greater discount than they priced put options. This discrepancy occurred because the option premium was evaluated on the steeper losses curve, while payoffs were assessed on the shallower gains curve. The results strongly supported this hypothesis: The results showed a 95% probability that calls would be priced at least 19% higher than puts, with a mean difference of more than 38%.

Another hallmark feature of prospect theory is the concept that humans will overweight small probabilities for both good and bad outcomes. Hypotheses H4 and H5 predicted that this cognitive bias would affect options pricing in such a way that options with small probabilities of paying off would be priced at a premium compared to options with larger probabilities (see H4A, H4B, H5A, and H5B). Hypotheses 4B, 5A and 5B were confirmed.

The only hypothesis that did not pan out as expected involved call options with a high probability of paying off. Hypothesis 4A predicted these call options would be priced at a smaller discount than other calls, but no significant difference or pricing signal was found. It is important to note, however, that this could be the result of the research design rather than because of a flaw in the general hypothesis. The research design included the assumptions that a 95% or greater chance of paying off would be perceived as "nearly certain" and a 5% or smaller chance of paying off would be perceived as "nearly impossible." It is possible that these effects might yet be observable with different cut-offs—for example, 99% and 1% for "nearly certain" and "nearly impossible," respectively. Follow-up research could focus directly on the topic of overweighting small probabilities.

Conclusions

In summary, the major finding of this research is that options valuations by managers are likely to be biased in a manner consistent with prospect theory. Three major areas of contribution include:

- a confirmation of the discounting behavior observed by Miller and Shapira (2004),
- a finding that put options are perceived to be more valuable than call options of similar expected value, and
- a finding that the overweighting of small probabilities causes options to be priced at a premium.

In order to provide a baseline or a more comprehensive investigation into the effects of risk biases in prospect theory on options pricing, this research began first by essentially repeating and confirming the research conducted by (Miller & Shapira, 2004). Although a similar technique was used in this study, the data were collected in a potentially more reliable way by providing respondents with context about their hypothetical decision, including offering providing contextual information such as net worth and the timeframe for the investment. Providing net worth ensured that each of the respondents were able to evaluate the options using the same scale, helping to mitigate the effect that respondent's own wealth might have on his or her valuations. Also, by explaining that the outcome of the lottery was imminent rather than at some point in the future, options valuations collected in this research are isolated from any time value of money or opportunity cost of capital effects. Although enhancements to the method were made to accommodate deeper analysis, the findings of this research are fully consistent and complimentary to Miller and Shapira (2004).

Second, by replacing Miller and Shapira's (2004) assumption of a symmetric and invertible s-shaped utility function with one more closely aligned to prospect theory where the slope of disutility for losses was steeper than that of utility for gains, we hypothesized and confirmed that put options are discounted less than call options with similar expected values.

Third, this research suggests that options with outside chances of payoff will be valued at a premium compared to options with more moderate probabilities. Although the data were not sufficient to find statistical significance in every test, there nevertheless appeared to be a noteworthy shift in pricing behavior at these low probabilities.

Implications for Real Options in Practice

A chief contribution of this research was to show how prospect theory may govern the difference between what options are worth according to normative models and what human decision makers are willing to pay for them. Evidence from this study suggests that biases toward risk, as described by prospect theory, results in decision makers that are biased by the framing of the potential outcomes (as either a gain or a loss) and by small probabilities. As a result, finance practitioners and consultants should be made aware of these effects themselves and take them into account when providing options valuations, especially to managers who may be entirely unaware of these biases.

The realization that options may be worth, at least probabilistically, more than most humans are willing to pay for them probably has little consequence for institutional options traders operating on financial markets. These firms often have departments dedicated to risk management, employing highly specialized trading professionals and computer-based trading and decision-making platforms. Indeed, as much as 84% of all equity trading uses a computer to make the final decision to buy or sell, thus there is little opportunity for arbitrage as any pricing “errors” that might occur due to risk biases will quickly corrected by computerized trading systems or individuals who rely primarily on computerized or mathematical pricing models rather than subjective judgement (Demos, 2012). On the other hand, real options are neither standardized nor traded on electronic markets. Compared to financial options, projects containing real options are likely to be far less liquid but have the potential to be much more valuable and strategic to firms. These traits, when combined with multiple sources of uncertainty make real options notoriously complicated to model. Even if managers hire a team of professionals to perform the real options analysis, the managers may still need to overcome the significant cognitive biases present in prospect theory in order to trust the results. The findings of this research show that subjective valuations of real options that can differ greatly from the expected values that a real options analysis suggests they are worth. This may provide an explanation as to why Teach (2003) observed that, despite being a theoretically sounder method for valuing flexibly business projects, real options analysis has yet to gain the trust of managers in practice.

Finally, the findings of this research may be impactful outside the realm of real options. While this research was primarily concerned with the effect that risk biases might have on real options pricing, there is no reason to expect that the discounting effects consistent with prospect theory would not also occur with humans as buyers of other types of options. The findings here suggest that agency issues may exist for both real and financial options when managers’ utility curves have a different shape or scale than that of the actual investor. For instance, a portfolio manager who manages money for clients with different levels of wealth may be more or less risk averse for calls and, similarly, more or less risk seeking for puts than his or her clients would prefer.

Limitations

A number of limitations of this research should be noted. The most serious limitation of this research involved the sample size. After amendments (as described below) the sample included 344 hypothetical options prices collected from 61 participants, with each participant providing up to six options prices, three calls and three puts, each. This sample was large enough to test for general trends in the data, such as a comparison of discounting behaviors between calls and puts. However, some of the tests required comparisons of means between small subsets of data, such as those with only a small percentage chance of paying off. Because the number of observations in these subsets was only a fraction of the entire sample, some analyses could not obtain means estimated within useful confidence intervals. In some cases where a pricing bias was found but did not quite reach the .05 threshold for statistical significance, a larger sample might have been especially beneficial. To overcome these issues, future research, perhaps not bound by time and resource constraints of a student dissertation, could include either a larger body of participants or the collection of more responses from each participant.

Another limitation pertaining to the sample was the potential that real managers facing actual investment decisions would behave differently than did the subjects of this research who were business school students. Also the sample size was not sufficient to provide a high resolution view of choices made with small probabilities. While the evidence presented here suggests an overweighting of small probabilities resulting in people being willing to pay premium for these options relative others, the small sample size and high overall volatility observed in options pricing likely contributed to being unable to show the difference with statistical significant. Future research could focus more closely on outside probabilities or include a larger sample size.

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