# Comparative statics for real options on oil: What stylized facts to use?

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**Abstract**

An important application in the real options literature has been to investments in the oil sector. Two commonly applied "stylized facts" in such applications are tested here. One is that the correlation of the returns on oil and the stock market is positive, the other that it is invariant to changes in oil price volatility. Both are rejected in data for 1993–2008 for crude oil and the S&P 500 stock market index. Based on real options theory, consequences are pointed out. Higher volatility need not imply increased value and postponed investment.

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# 1 Introduction

A major development in investment theory over the last three decades has been the theory of real options. Energy investments have been a central application, with oil as perhaps the most prominent example.[1] A central result in option theory is that option values increase with higher volatility of the price of the underlying asset. Whether this carries over to real options is an unresolved issue (Davis, 2002).[2]

A widespread approach in the real options literature is to rely on the Capital Asset Pricing Model (CAPM) to determine required expected rates of return. This would not be necessary if forward or futures contracts were traded with sufficiently long maturities. But many authors observe that these are not available, and rely instead on the CAPM.[3] Because of the prominence of this approach, the present study investigates the empirical basis for two types of assumptions ((i) and (ii) below) that are commonly applied.

The first assumption (i(a)) is that the correlation between returns on the underlying asset and the stock market is invariant to changes in the volatility of the underlying asset. Below we explain the consequences of the assumption for comparative statics. We show that some authors have used an alternative version (i(b)), that the covariance is invariant to such changes. Many authors pay little or no attention to their choice between these alternatives (or other possible assumptions). We test assumption (i(a)) empirically, using data for the period 1993–2008. We reject that the correlation is invariant.

The second assumption (ii) is that the correlation (and thus also the covariance) is positive. We show that this is an important assumption for comparative statics when it is combined with the invariance assumption (i(a)), but not with (i(b)). We show that many authors have used the assumption without empirical basis, either implicitly, or explicitly based on a priori motivation. This is surprising given the consensus in the empirical literature that in recent decades, for the U.S. and European stock markets, the correlation has been negative. We test the assumption and find non-positive correlations during 1993–2008.

Real options have received interest lately in papers trying to explain empirical failures of asset pricing models (Grullon et al., 2012; Da et al., 2012). Some authors assume that real option values are increasing in volatility. Grullon et al. (2012, p. 1500) write that "One of the main implications of real options theory is that a real option's value is increasing in the volatility of an underlying process". This does not hold in general, as shown by Davis (2002) and the present paper. Hence, care must be taken when real option valuation is introduced in asset pricing models.

For most real options, the underlying asset is not an investment asset. Its price process will exhibit what McDonald and Siegel (1984) call a rate-of-return shortfall, $\delta$. This may in some cases be observed in markets for forward or futures contracts. The questions is then whether one should assume that it is invariant to changes in volatility. In other cases the relevant contracts do not exist, in particular not for the long maturities needed. The

[1]Early applications to oil exploration and development were Tourinho (1979), Paddock et al. (1988), Ekern (1988), Jacoby and Laughton (1992), and Pickles and Smith (1993). Oil is also used as example in the real options textbook by Dixit and Pindyck (1994). Additional references are found in the overview by Dias (1994) and more recent papers, e.g., Laughton (1998) and Smith and Thompson (2008).

[2]For financial options the result holds for both puts and calls. For real options, the question that follows from theory is whether it holds for calls, cf. Berg et al. (2009, p. 9).

[3]This is found in, e.g., McDonald and Siegel (1986), Dixit and Pindyck (1994), Salahor (1998), Laughton (1998), Sarkar (2000), Davis (2002), Lund (2005), Wong (2007), Gutiérrez (2007), Berg et al. (2009), Kanniainen (2009), Guthrie (2009), and Brooks and Chance (2014).

present study does not use data for futures contracts.[4] Many authors then recommend to estimate $\delta$ as

$$\delta = \mu - \alpha. \tag{1}$$

These two can be estimated separately, the required expected rate of return, $\mu$, and an actual expected rate of return, $\alpha$. The question is then whether these are invariant to changes in volatility. Typically it is assumed that $\alpha$ is unaffected, and the question is then whether $\mu$ is.

This study clarifies the issues and suggests that empirical studies are needed. We consider data for crude oil spot prices. In line with most of the literature, the price process is assumed to be a geometric Brownian motion (GBM) with drift.

The existing literature is reviewed in section 2. Section 3 presents the model and discusses the use of comparative statics results for volatility. Section 4 presents an empirical specification which corresponds to the prevailing practice of comparative statics. Then the data are presented. Section 5 contains the empirical results. Section 6 concludes.

# 2 The previous literature

In their seminal paper on real options, McDonald and Siegel (1986) suggest to use the CAPM[5] to estimate required expected rates of return. Regarding invariance, they acknowledge, in their footnote 14, that their results "ignore the possibility that changes in" [volatilities] "affect the required rates of return". Furthermore, "This assumption would be valid if the uncertainty is uncorrelated with the market portfolio or if investors are risk neutral." They state that the opposite case "can lead to ambiguity in the comparative static results." In the following, the assumption of uncorrelated uncertainty will be interpreted, not as the underlying asset being uncorrelated with the market portfolio, but that the additional uncertainty, the increase to be analyzed, has this property.[6]

Dixit and Pindyck (1994, p. 148, p. 178) also introduce the CAPM,

$$\mu = r + \phi\sigma_p\rho_{pm}, \tag{2}$$

where $r$ is the discount rate for riskless cash flows, $\rho_{pm}$ is the correlation between the rates of return of the market portfolio, $r_m$, and "the $P$ asset" (an investment asset perfectly correlated with the underlying asset), $\sigma_p$ is the volatility of the underlying asset, and $\phi$ is the market price of risk. They regard $r$ and $\phi$ as exogenous in their analysis, which seems reasonable. They state that "when the $\sigma$ of the $P$ asset increases, $\mu$ must increase."[7] This is at odds with the assumption in McDonald and Siegel (1986) of an unchanged covariance, which would imply that $|\rho_{pm}|$ is reduced as a consequence of the higher volatility.

[4] Most real options relate to investment projects lasting for several years. Most researchers on futures contracts refrain from using data for maturities longer than one year. According to Alquist and Kilian (2010, p. 544), "the market remains illiquid at horizons beyond 1 year even in recent years. Trading volumes fall sharply at longer maturities." Guo and Kliesen (2005, footnote 5) also restrict their study to maturities of 12 months and shorter. Schwartz (1997) uses proprietary data with up to nine years maturity.

[5] The Capital Asset Pricing Model (Sharpe, 1964; Lintner, 1965; Mossin, 1966) was extended by Merton (1973) to continuous time. The single-beta version of this extension is used in the real options literature.

[6] This interpretation is different from the one in Davis (2002, p. 217).

[7] There is an implicit assumption that $\rho_{pm} > 0$, in contrast with the empirical results here and elsewhere, see below.

The presentation of the CAPM equation in (2) may have mislead many to assume that $\rho_{pm}$ is unchanged when $\sigma_p$ changes, and, perhaps, vice versa. A simple example shows that such a procedure is not necessarily correct. Consider what happens to $\mathrm{cov}(r_m, r_p)$ if $r_p$ is multiplied by a noise factor. If a stochastic "multiplicative noise" variable $X$ has $E(X) = 1$, $\mathrm{var}(X) > 0$, and is stochastically independent of $(r_m, r_p)$, then

$$\mathrm{cov}(r_m, Xr_p) = E(r_m X r_p) - E(r_m)E(Xr_p) = E(X)\,\mathrm{cov}(r_m, r_p) = \mathrm{cov}(r_m, r_p). \qquad (3)$$

This is one natural way to think about increased volatility of $r_p$, and it does not increase the covariance of $r_p$ with another variable $r_m$. Instead, $|\rho_{pm}|$ has been reduced. Whether this is what actually happens, is an empirical question.

The question can be broken into several parts, depending on which of the CAPM parameters are allowed to change. Equation (2) can be rewritten as

$$E(r_p) \equiv \mu = r + \phi\sigma_p\rho_{pm} = r + \frac{\sigma_{pm}}{\sigma_m^2}\left[E(r_m) - r\right], \qquad (4)$$

since $\phi$ is defined as $[E(r_m) - r]/\sigma_m$, and $\rho_{pm} = \sigma_{pm}/\sigma_p\sigma_m$, where $\sigma_{pm}$ is the covariance between $r_p$ and $r_m$. The CAPM beta for the return $r_p$ is the fraction in the final expression. Even when it is maintained that $\alpha$ in (1) is invariant to changes in $\sigma_p$, one can ask whether $\rho_{pm}$ is invariant, whether $\rho_{pm}\sigma_p$ is invariant, and the same question for $\rho_{pm}\sigma_p\sigma_m$ (which is the covariance), $\phi\sigma_p\rho_{pm}$, and even $r + \phi\sigma_p\rho_{pm}$. These successive extensions allow for the possibility that several parameters change simultaneously with a change in volatility.

Extensions to $\phi\sigma_p\rho_{pm}$, and in particular to $r + \phi\sigma_p\rho_{pm}$, would be beyond the comparative statics analysis which has motivated this study. The risk-free interest rate is a separate argument in any formula for real option values. A comparative-statics analysis of a change in volatility assumes that $r$ is constant, implying that an extension to $r+\phi\sigma_p\rho_{pm}$ would include more possible variation than allowed for by the original purpose of the analysis. Something similar can be said about $\phi \equiv [E(r_m) - r]/\sigma_m$, although it is conceivable that $r$ is constant while $E(r_m) - r$ changes. Due to these considerations, the question to be analyzed here is whether the correlation is invariant.

Bradley (1998, p. 59) states explicitly that "we vary the level of price uncertainty, but keep the forward prices the same across models. This is done so that we may examine the direct effects of uncertainty when cash-flow models are nonlinear." The assumption is in line with McDonald and Siegel (1986), and corresponds to the constant covariance assumption when the CAPM is applied. On the other hand, in the same special issue, Laughton (1998), using the CAPM, does not seem to keep the forward price constant when $\sigma$ is varied, see his equation (A-1).

Davis (2002) has an extensive non-empirical discussion of the problem, with a general formulation indicating that both $\delta$, $\mu$, and $\alpha$ may be functions of $\sigma_p$. After introducing the CAPM equation identical to (2), Davis (2002) treats the correlation as a constant. This is also found in Berg et al. (2009) and Kanniainen (2009). All these studies point out the implication that increased volatility now affects the option value both directly and indirectly via the rate-of-return shortfall. For a call option these effects go in opposite directions when $\rho_{pm} > 0$. The latter effect may outweigh the former, so that the total effect is negative.[8]

[8] These authors assume that the underlying asset follows a GBM. McDonald and Siegel (1986) has a jump process as an alternative, and Dixit and Pindyck (1994) discuss several alternative processes, while maintaining the GBM as the main assumption. Bradley (1998) has two alternatives price processes, one of which is mean reverting. Other discussions on the type of process are found in Lund (1993), Laughton and Jacoby (1993), Baker et al. (1998), Sarkar (2003), Tourinho (2013).

It is interesting to note that there is a related literature that asks a related question: In real option models, what is the effect of increased volatility on investment?[9] The original real options literature predicted a decreasing relationship, but there are opposing effects. Even if the trigger for investment is increased, the probability of reaching it may also be increased, so that the overall effect is ambiguous. Sarkar (2000), Cappuccio and Moretto (2001), Lund (2005), and Wong (2007) study this under various sets of assumptions. Sarkar (2003) extends the analysis to a mean-reverting process. All five papers rely on the CAPM for valuation, at least as one possible alternative. While Sarkar and Wong keep $\rho_{pm}$ constant when $\sigma_p$ is increased, the other authors prefer to keep $\delta$ constant.

The two assumptions can be seen as opposite extremes. It may be more reasonable to invoke an intermediate assumption. Cappuccio and Moretto (2001, p. 11) state that "Which of these viewpoints is more plausible is, in general, an empirical matter ...." To our knowledge the existing real options literature has no references to empirical studies of the possible link between changes in volatility and changes in covariance measures of risk. This paper is a first attempt to link the analysis to such empirics.

Whether the correlation (and covariance) is positive or negative is a separate issue, which plays a somewhat different role under different invariance assumptions, see section 3. Of course, the empirical answer may differ between different underlying assets and between different stock markets.[10] For crude oil the results in section 5 confirm what has been reported previously in the literature, broadly speaking that $\rho_{pm} \leq 0$. Deaves and Krinsky (1992, table 3) find negative, but insignificant correlations with the U.S. stock market in data for crude oil futures 1983–1990. Schwartz (1997) finds negative or insignificant values (for the risk premium, $\lambda$ in his notation) for oil in his one-factor model. Cifarelli and Paladino (2009, p. 364) summarize that "A number of studies, based on different data and estimation procedures, find a negative financial linkage between oil and stock prices i.e. a large negative covariance risk between oil and a widely diversified portfolio of assets." Another broad overview is given by Degiannakis et al. (2014), confirming the same pattern.

Our purpose is not to investigate whether a causal effect exists between changes in volatility and changes in correlation. Research on causal effects are found in Miller and Ratti (2009) and Kilian and Park (2009), who distinguish between different periods and regimes, and find that the correlation varies between negative values and values that are close to zero. Filis et al. (2011) investigate contemporaneous and lagged time varying correlations for oil importing and oil exporting countries. The results confirm that negative correlations are the typical finding for the U.S., although the contemporaneous correlations are positive in the year 2000. For another asset in a resource exporting country, Slade and Thille (1997, p. 634) find that "the rate of copper price appreciation is virtually uncorrelated with the return on the Toronto Stock Exchange".

A recent empirical study by Mohn and Misund (2009) shows that investment in a worldwide sample of oil companies reacted both to $\sigma_p$ and to the volatility of the stock market return, $\sigma_m$. In the short run the reaction to higher volatilities was negative, i.e., reduced investment. In the long run, the estimated reaction was positive for $\sigma_p$ and

[9]With a theoretical approach, this refers to the effect on the optimal investment strategy. It has different interpretations, cf. Lund (2005); the effect on the price at which investment is (optimally) triggered, or, during some defined future period, the expected (optimal) investment or the probability that some real call option is (optimally) exercised.

[10]For a natural resource, the correlation may be different between an importing country's stock market and that of an exporting country, where resource exporting firms will be more prominent.

negative for $\sigma_m$. The first of these was larger in absolute value, so that a combined increase in both volatilities would lead to increased investment in the long run. The authors suggest the existence of compound options to explain this effect, but give no exact theoretical explanation. They give no detailed evidence to distinguish this from other possible explanations, such as that in Sarkar (2000). The separate effect of $\sigma_m$ is not easily incorporated in the theory of real options.

Most of the non-empirical studies cited above neglect the issue of a negative correlation, even though many mention oil as an important application. Some make implicit assumptions about a positive correlation. Dixit and Pindyck (1994, p. 155) state that "each unit increase in $\sigma$ requires an increase in $\delta$ of $\phi\rho_{xm}$ units," without mentioning that the increase could be negative. Sarkar (2000, p. 222) states that "a higher level of uncertainty will increase the critical trigger level" under the same implicit assumption. Pindyck (2001, p. 18) states that "For most industrial commodities such as crude oil and oil products, we would expect the spot price to co-vary positively with the overall economy, because strong economic growth creates greater demand, and hence higher prices, for these commodities." On the other hand, Gutiérrez (2007) and Kanniainen (2009) include the possibility of $\rho < 0$ in their analyses, but they do not suggest that it may be a prominent case for an important underlying asset like oil. Davis (2002, p. 220) explicitly states the condition that $\lambda\rho$ (which corresponds to $\phi\rho_{pm}$ in the notation of this paper) is positive, and pays no attention to the opposite case. Wong (2007, p. 2159) considers $\rho < 0$, but concentrates on "the more plausible case that $\rho > 0$." With reference to the CAPM, Hart and Spiro (2011, p. 7837) assume a substantial positive risk premium for crude oil, implicitly assuming a positive correlation.

For oil in recent decades there is thus a clear dissonance between results of empirical studies and assumptions in the theoretical literature, stronger for some authors, not so strong for others. None of the theorists have adopted $\rho_{pm} < 0$ as their main case. In combination with equation (2), the assumptions of an invariant and negative $\rho_{pm}$ would imply a decreased $\mu$ when $\sigma_p$ is increased. While consistent with the CAPM, such an assumption may be counter-intuitive for many, and is not found in the literature, to our knowledge.

# 3 The model and the unresolved issue(s)

This section shows that there are two versions of the real options model, one in which the underlying asset is a cash flow stream. In that version, the opposing effect of an increased $\delta$ is stronger, provided that $\rho_{pm} > 0$. This strengthens the need to reconsider the traditional view on the value effect of volatility. There is no new theory in this section, just a clarification of the issues.

The model is the real options model of McDonald and Siegel (1986), which is considered by most of the non-empirical studies cited above. A firm has the option to make an investment with cost $I$ which creates an asset with value $V$. For simplicity $I$ is assumed to be fixed,[11] while $V$ is a GBM with drift,

$$dV_t = \alpha V_t dt + \sigma V_t dZ_t, \tag{5}$$

where $t$ is time, $\sigma$ is a constant volatility, and $dZ_t$ is the increment of a standard Wiener process. What is denoted $\sigma$ in this general formulation corresponds to $\sigma_p$ in the previous

[11] McDonald and Siegel (1986) also consider the case where both $V$ and $I$ are GBMs.

section.

The optimal time for the firm to invest is the first time $V_t$ reaches a trigger level $V^*$ from below. Defining the constant $\gamma$ as

$$\gamma = \frac{1}{2} - \frac{r-\delta}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{r-\delta}{\sigma^2}\right)^2 + \frac{2r}{\sigma^2}}, \tag{6}$$

the optimal $V^*$ is

$$V^* = \frac{\gamma}{\gamma - 1} I, \tag{7}$$

cf. equations (14) and (24) in Dixit and Pindyck (1994, pp. 142, 152). They also show that when $V_t$ is below the trigger, the value of the option is

$$F(V_t) = A V_t^{\gamma}, \tag{8}$$

with the constant $A$ defined by

$$A = \frac{V^* - I}{(V^*)^{\gamma}} = I^{1-\gamma}(\gamma - 1)^{\gamma-1}\gamma^{-\gamma}. \tag{9}$$

The model describes a perpetual American call option on an asset with a rate-of-return shortfall, $\delta$. For a financial option, the underlying asset could be a stock with a continuous dividend yield rate $\delta$, in case the option is not payout protected.

For a real option, the underlying asset may be a completed plant, mine or oil field. In many cases market values for such assets are not observable. In a subsequent chapter, Dixit and Pindyck (1994, pp. 177–186) interpret the asset as the present value of a perpetual revenue stream, the sales value of production, first considered with no operating cost. The observable price variable is now the output price, $P_t$, which is assumed to be a GBM. For simplicity output is assumed to be a constant unit flow, so that the value of the completed asset is[12]

$$V_t = \frac{P_t}{\delta}. \tag{10}$$

With this in mind, the trigger level of $P_t$ for investment is the same,

$$P^* = \frac{\gamma}{\gamma - 1}\delta I. \tag{11}$$

The option values before investment are also the same. The difference that occurs when $P_t$ is assumed to be observable instead of $V_t$, is in the comparative-statics results on effects of changes in $\sigma$ (or $\delta$). In one case, $V_t$ is held constant, in the other, $P_t$. The partial derivatives of the two functions

$$F(V_t, I, r, \sigma, \delta) = A V_t^{\gamma} = \left[I^{1-\gamma}(\gamma - 1)^{\gamma-1}\gamma^{-\gamma}\right] V_t^{\gamma} \tag{12}$$

and

$$F_0(P_t, I, r, \sigma, \delta) = A \left(\frac{P_t}{\delta}\right)^{\gamma} = \left[I^{1-\gamma}(\gamma - 1)^{\gamma-1}\gamma^{-\gamma}\right] P_t^{\gamma}\delta^{-\gamma} \tag{13}$$

are not the same, even though the functions yield the same value when (10) holds.

[12]As shown in Dixit and Pindyck (1994, p. 182).

Davis (2002, p. 220) points out that "the derivative $\partial F/\partial\sigma$ is complex and difficult to sign".[13] Both he and Dixit and Pindyck (1994) thus resort to numerical examples in order to illustrate the partial effects. There is a caveat: One cannot be sure to have covered all parameter combinations that may be of interest.

The examples show that $\partial F/\partial\sigma > 0$ and that $\partial F/\partial\delta < 0$ (Dixit and Pindyck, 1994, pp. 154, 156). In order to go further into the effects, it is helpful to write out the total derivatives. In what follows, the definitions of the ten partial derivatives of $F$ and $F_0$ are standard and the same throughout. The total derivatives will depend on what functions and arguments are used in each case.

The total effect of $\sigma$ on $F$ when $V_t$ is assumed constant is

$$\left.\frac{dF}{d\sigma}\right|_{V_t \text{ const.}} = \frac{\partial F}{\partial\sigma} + \frac{\partial F}{\partial\delta}\frac{d\delta}{d\sigma} = \frac{\partial F}{\partial\sigma} + \frac{\partial F}{\partial\delta}\phi\rho_{pm}, \tag{14}$$

where the final expression is taken from (1) and (2), with $\alpha, \phi$, and $\rho_{pm}$ assumed to be constant. Since $\partial F/\partial\delta$ is found to be negative (see above), this total effect may be ambiguous. However, if the covariance is constant because the added risk is independent of the market portfolio, then $\rho_{pm}$ is not constant, and $d\delta/d\sigma$ is zero. Then the total effect of $\sigma$ on $F$ only consists of the partial effect, $\partial F/\partial\sigma$, which is positive.

When, alternatively, $P_t$ is assumed constant, the total effect of $\sigma$ on $F_0$ is

$$\left.\frac{dF_0}{d\sigma}\right|_{P_t \text{ const.}} = \frac{\partial F_0}{\partial\sigma} + \frac{\partial F_0}{\partial\delta}\frac{d\delta}{d\sigma} = \frac{\partial F_0}{\partial\sigma} + \frac{\partial F_0}{\partial\delta}\phi\rho_{pm}. \tag{15}$$

Again, the final expression depends on the fixed $\alpha, \phi$, and $\rho_{pm}$. The difference from (14) is that $\partial F_0/\partial\delta \neq \partial F/\partial\delta$. This can be seen from introducing $P_t/\delta$ as argument instead of $V_t$ in the $F$ function and finding the total derivative for that case:

$$\begin{aligned}\left.\frac{dF}{d\sigma}\right|_{P_t \text{ const.}} &= \frac{\partial F}{\partial\sigma} + \left[\frac{\partial F}{\partial\delta} + \frac{\partial F}{\partial V_t}\frac{d(P_t/\delta)}{d\delta}\right]\frac{d\delta}{d\sigma} \\ &= \frac{\partial F}{\partial\sigma} + \left[\frac{\partial F}{\partial\delta} + \frac{\partial F}{\partial V_t}\left(\frac{-P_t}{\delta^2}\right)\right]\phi\rho_{pm}. \end{aligned} \tag{16}$$

The expression in square brackets is negative and equal to $\partial F_0/\partial\delta$ (when (10) holds), since both are equal to

$$V_t^\gamma\left[\frac{\partial A}{\partial\delta} + A\left(\frac{\partial\gamma}{\partial\delta}\ln(V_t) - \frac{\gamma}{\delta}\right)\right].$$

Accordingly, the total derivatives in (15) and (16) are identical. The new term, compared to (14), is clearly negative when $d\delta/d\sigma > 0$.

Summing up the conclusions to this theoretical discussion:

- There is a direct, positive effect of volatility, $\sigma$, on the value of the option before it is exercised.
- If the rate-of-return shortfall, $\delta$, is invariant to changes in volatility, no counteracting effect to the positive effect has been identified.
- If the rate-of-return shortfall is decreasing in volatility, typically because $\rho_{pm} < 0$ and invariant, then the effect of increased $\sigma$ would be increases in $V$ or $P$, reinforcing the direct, positive effect (Kanniainen, 2009, Figs. 1 and 2).

[13] "Complex" is used in its everyday meaning, not in the mathematical meaning.

- If the rate-of-return shortfall is increasing in volatility, one or two counteracting effects have been identified:
  - If $V_t$ is invariant to changes in volatility, there is one counteracting effect.
  - If $P_t$ is invariant to changes in volatility, there are two counteracting effects.

The first, direct, positive effect is well known and easily explained. The owner of the option may take advantage of higher outcomes of the underlying asset price, but is protected against lower outcomes. More dispersion is then better. The second of the two counteracting effects is also easily explained. The call option value is increasing in the value of the underlying asset. When this asset value is $P_t/\delta$, and $\delta$ is increasing in volatility, there is clearly a counteracting effect.

The first counteracting effect, which appears even when $V_t$ is invariant, is perhaps less obvious. The following intuition is taken from European options with finite maturity, but seems to carry over: The underlying asset with value $V_t$ has a rate-of-return shortfall. A replicating portfolio for the option will not include the underlying asset itself, but "the $P$ asset" (see above), a prepaid forward contract on $V_\tau$ (at maturity, $\tau$). Apart from this, the replicating portfolio has the same composition. Clearly, if $\delta$ is increased, the value of a prepaid forward on $V_\tau$ is decreased, and so is the value of the portfolio and thus the option.

# 4 Empirical specification and data

While the theoretical literature on real options discusses comparative statics results, there is less discussion of the interpretation and relevance of these results. The effect of a higher or lower volatility on the endogenous variables of a real options model will have practical relevance in various circumstances. One is that volatility could change over time for real options on a specific type of assets, such as oil fields. Another could be a comparison between different assets with different volatilities, but this is hardly the typical application, and will not be discussed further.

If a volatility changes over time, this is at odds with the theory as specified here. Valuation and optimal strategy have been derived under the assumption of a constant volatility. If there is the possibility of a changing volatility, this should have been present in the model to begin with. The stochastic process for the price of the underlying asset should have been specified differently, and the value and strategy would have been influenced by this.[14]

In the most common interpretation, changes in volatility over time, the comparative statics results are thus not useful from a purist point of view. A more pragmatic view is chosen because models with constant volatility are easy to solve. The analytical solutions have attracted substantial interest, both from theorists and practitioners, and this is also likely to be the case in the foreseeable future. Thus it is interesting to improve the practical relevance of the comparative statics results of such models, the purpose of the present study.

For an empirical investigation, it is necessary to decide what kind of deviation from a constant volatility one will estimate. One will also have to choose, more generally,

[14] Ting et al. (2013) characterize real options with stochastic volatility. Deterministic volatility is less realistic, but the models are so much simpler that they are likely to be used in most applications for many years to come. This motivates the present study.

Table 1: Structural breaks in volatility

| Time period | | | Standard deviation |
|---|---|---|---|
| July 1, 1993 | – | August 29, 1994 | 0.0191 |
| August 30, 1994 | – | January 8, 1996 | 0.0142 |
| January 9, 1996 | – | June 13, 2005 | 0.0259 |
| June 14, 2005 | – | June 30, 2008 | 0.0191 |

Reproduced from Ewing and Malik (2010).

whether to stick to the specification of the stochastic process that is used in the theory, even if this process can be rejected empirically.

The application here looks at price data for oil, perhaps the most studied underlying asset in applications of real options theory. There will not be an assumption that the volatility changes continuously over time, nor that there are probabilities for switching between various regimes. Instead the volatility changes will be taken from an existing study of structural breaks in volatility. A structural break is an operational concept that seems close to the theoretical notion of an unanticipated change, with no specified probability for any particular magnitude or direction of that change. Under such a process with breaks, it may be imagined that the market's valuation and the decision maker's strategy may as well rely on models with constant volatility. The deviation from a purist view is as little as possible.

The breaks are found in the study by Ewing and Malik (2010) of crude oil spot prices 1993–2008. Based on their method, the data lead to an identification of three breaks in the volatility, i.e., in the standard deviation of relative changes ("returns") in the oil price. Apart from dummy variables that allow for these breaks, the oil price returns are assumed to follow a GARCH process, which is different from the GBM used in most option pricing models.[15] The pragmatic defense of this procedure is that the breaks are taken to exist irrespective of the detailed assumptions made about the stochastic process. Unfortunately, this is not quite true: A subsequent study by Vivian and Wohar (2012) finds fewer breaks in volatility when they allow other parameters of the GARCH model to have breaks as well. Nevertheless, the break points found by Ewing and Malik (2010) will be used here. After all, if many parameters change, this does not correspond well with the theoretical notion of comparative statics, changing $\sigma_p$ while holding everything else constant.

The four periods ($i = 1, \ldots, 4$) delimited by the three breaks are shown in Table 1, including the volatility estimates. The table is reproduced from Table 3 in Ewing and Malik (2010). Fifteen years of daily data are used, and the standard deviation is based on one day as time unit. These are working days, about 250 per year.

The next section is based on oil spot prices and the Standard and Poor's 500 index for the stock market. The question is: Is $\rho_{pm}$ in period $i$ significantly different from $\rho_{pm}$ in period $i-1$ (for $i = 2, 3, 4$)? If the answer is yes, one will have to reject the hypothesis that $\rho_{pm}$ is invariant to changes in volatility. The information here concerns the method in which the CAPM is used for estimating $\mu$ separately. The answers may be different for different breaks.

Spot prices for crude oil are daily data for West Texas Intermediate (WTI), obtained

[15]There also exist GARCH real option models, starting with Duan (1995).

Table 2: Estimates of parameters of the CAPM real options model

| Regime | Years | $\hat{\sigma}_p$ | $\hat{\rho}_{pm}$ | $\hat{\rho}_{pm}\hat{\sigma}_p$ | $\hat{\rho}_{pm}\hat{\sigma}_m\hat{\sigma}_p$ |
|---|---|---|---|---|---|
| 1 | $1993-94$ | 0.0191 | $-0.17875$ | $-3.4141\times 10^{-3}$ | $-1.8604\times 10^{-5}$ |
| 2 | $1994-96$ | 0.0142 | $-0.030461$ | $-4.3255\times 10^{-4}$ | $-2.3332\times 10^{-6}$ |
| 3 | $1996-05$ | 0.0259 | $-0.021816$ | $-5.6503\times 10^{-4}$ | $-6.6787\times 10^{-6}$ |
| 4 | $2005-08$ | 0.0191 | $-0.017739$ | $-3.3881\times 10^{-4}$ | $-3.0779\times 10^{-6}$ |

Source: Own estimates, except $\hat{\sigma}_p$ taken from Ewing and Malik (2010).

from U.S. Energy Information Administration (2012). This is identical to the data in Ewing and Malik (2010), and the same time period was chosen, July 1993–June 2008. Data for the return on a market portfolio was taken from the total return index of Standard and Poor's 500 from the New York Stock Exchange. These data were obtained from the company S&P Dow Jones Indices.

# 5 Empirical results

Within each of the four periods defined by the breaks in Table 1, three variables of interest for the application of the CAPM have been estimated. The results are given in Table 2.

A plot of the point estimates of the values of $\sigma_p$, $\rho_{pm}$ and the covariance (scaled up by a factor of $10^4$) is shown in Figure 1.

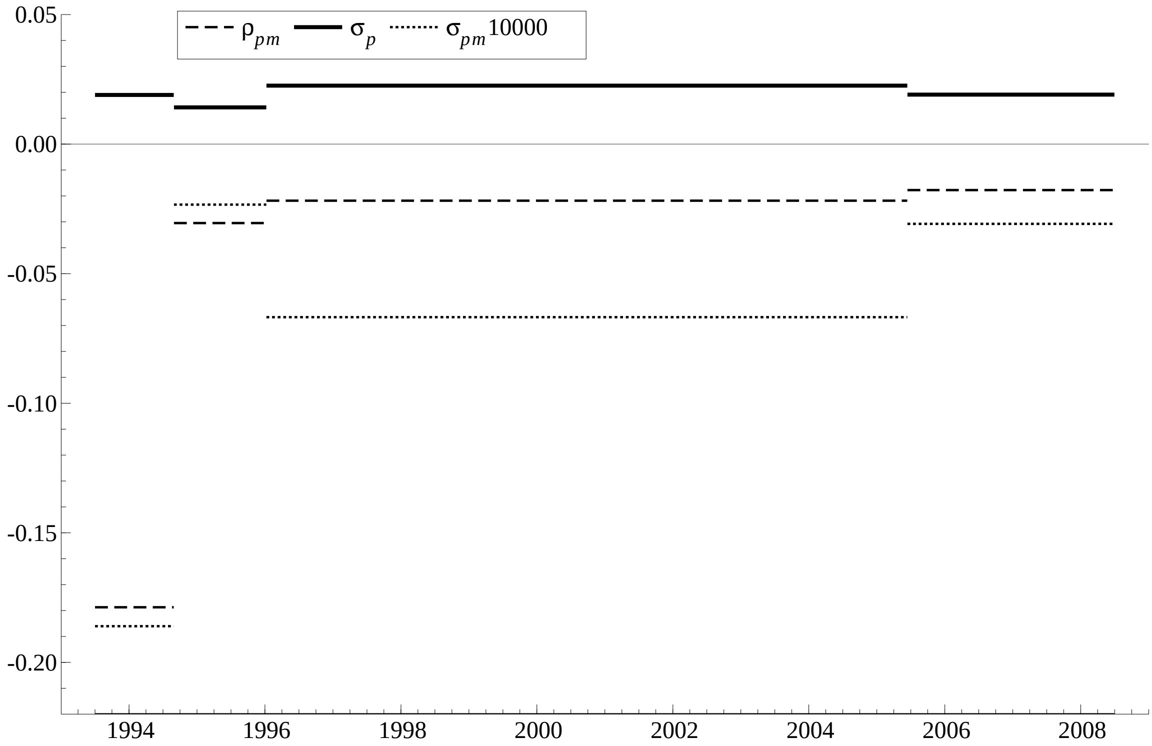
The first notable feature of the estimates is the fact that correlations and covariances are consistently negative throughout the fifteen years. This means that the required expected return according to the CAPM has been less than the risk free interest rate. To get an impression of the magnitudes, consider the product of the point estimates, $\hat{\rho}_{pm}\hat{\sigma}_p$. This should be multiplied by $\sqrt{250}$ to compare with yearly interest rates, under the assumption of a GBM.[16] With the value of $\phi = 0.4$, suggested by Dixit and Pindyck (1994, p. 148), the reduction in annualized $\mu$ below the risk free interest rate, cf. (2), would be 0.0217 in the first subperiod, i.e., 217 basis points. It would practically vanish in the second, third and fourth subperiods, fluctuating between 22 and 36 basis points.

For the solutions of the real options model to be valid, $\delta > 0$ is a necessary requirement. Whether this was the case during 1993–2008 is beyond the scope of this study, since we do not provide any ex ante estimate of $\alpha$. More generally, Roberts (2000) suggests that negative or low correlation is consistent with the flat or downward trend in natural resource prices. $\delta$ can be positive because the ex ante expected price growth, $\alpha$, may have been small.

The estimated correlations and covariances, as well as $\rho_{pm}\sigma_p$, do not appear to be invariant to changes in volatility. In particular, there is a sharp increase in both correlation and covariance at the first break point. At the second and third break point, the covariance changes much more in relative terms than the correlation. For the period after the first break point, i.e., during September 1994 – June 2008, one could draw the preliminary conclusion that the correlation is close to zero and does not change much,

[16] Our data record about 250 working days per year.

Figure 1: Values of $\hat{\sigma}_p$, $\hat{\rho}_{pm}$ and $10^4 \times \hat{\sigma}_{pm}$ in the four subperiods




in particular not at the third break point in June 2005. However, a statistical analysis is necessary to determine whether invariance can be rejected or not. At the first break point, the preliminary conclusion is quite clearly that none of the three parameters are constant. In the first subperiod, the correlation is negative with a substantial absolute value.

Another interesting observation from Table 2 is that the direction of changes in $|\rho_{pm}|$ is not always opposite of the direction of changes in $\sigma_p$. This contradicts the simple intuition from the example in equation (3).

We must assume a probability distribution for the market return and the oil return. We (tentatively) assume identical distributions within volatility regimes, so that the rates of return $\Delta \ln(SP_t)$ and $\Delta \ln(OIL_t)$ are jointly normal with mean $\alpha_i$ and covariance matrix:

$$\Omega_i = \begin{pmatrix} \sigma_{im}^2 & \sigma_{ipm} \\ \sigma_{ipm} & \sigma_{ip}^2 \end{pmatrix},$$

where subscript $i$ "runs over" regimes, and

$$\rho_{ipm} = \frac{\sigma_{ipm}}{\sigma_{im}\sigma_{ip}}.$$

The first hypothesis we want to test is invariance of the correlation coefficient with respect to the structural break in market volatility between regime 1 and 2, against the alternative of a structural break also in the correlation coefficient, formally $H_0$: $\rho_{1pm} = \rho_{2pm}$, against $H_1$: $\rho_{1pm} \neq \rho_{2pm}$.

Since there are three breaks in volatility, we also want to test $H_0$: $\rho_{2pm} = \rho_{3pm}$ against $H_1$: $\rho_{2pm} \neq \rho_{3pm}$, and $H_0$: $\rho_{3pm} = \rho_{4pm}$ against $H_1$: $\rho_{3pm} \neq \rho_{4pm}$.

Table 3: Test of invariance of correlation. Daily data

| Regime | Dates | $\hat{\sigma}_p$ | $\hat{\rho}_{pm}$ | $\psi_T$ [$p$ value] |
|---|---|---|---|---|
| 1 | 1993.07.01 – 1994.08.29 | 0.0191 | −0.17875 | |
| 2 | 1994.08.30 – 1996.01.08 | 0.0142 | −0.030461 | −21.907[0.0000] |
| 3 | 1996.01.09 – 2005.06.13 | 0.0259 | −0.021816 | −2.7410[0.0062] |
| 4 | 2005.06.14 – 2008.06.30 | 0.0191 | −0.017739 | −2.0756[0.0380] |

In our case, although joint normality is a useful reference, there are signs of departures from normality as well, as the Appendix shows. On the other hand the number of observations is large for each sub-sample. Together, this makes it attractive to use an asymptotic test that does not depend on the exact normality of observations. Such a test is suggested in e.g., Omelka and Pauly (2012) . It is a studentized asymptotic test that does not require that $\Delta\ln(SP_t)$ and $\Delta\ln(OIL_t)$ are jointly normal. The statistic is calculated as:

$$\psi_T = \sqrt{\frac{T_1 T_2}{T}} \frac{(\hat{\rho}_{1pm} - \hat{\rho}_{2pm})}{\sqrt{\hat{\upsilon}_Z^2}}, \tag{17}$$

where we use regime 1 and 2 for concreteness, and where $T = T_1 + T_2$. Under the null hypothesis, $\rho_{1pm} = \rho_{2pm}$, $\psi_T$ is $t$-distributed with $T$ degrees of freedom. The variance term $\hat{\upsilon}^2$ is

$$\hat{\upsilon}^2 = \frac{\hat{\sigma}_{Z_1}^2}{T_1} + \frac{\hat{\sigma}_{Z_2}^2}{T_2}. \tag{18}$$

It is an estimate of the asymptotic variance of the difference $\sqrt{\frac{T_1T_2}{T}}(\hat{\rho}_{1pm} - \hat{\rho}_{2pm})$. $\hat{\sigma}_{Z_i}^2$ $(i = 1, 2)$ are the empirical variances,

$$\hat{\sigma}_{Z_i}^2 = \frac{1}{T_1 - 1} \sum_{t=1}^{T_i} (Z_t - \bar{Z}_i)^2,\ i = 1, 2, \tag{19}$$

where $\bar{Z}_i = \frac{1}{T_i} \sum_{t=1}^{T_i} Z_{it}$ $(i = 1, 2)$ and, for $t = 1, \dots, T_1, \dots, T_1 + T_2$,

$$Z_t = S_{mt}S_{pt} - \frac{1}{2}\left(\frac{T_1}{(T_1+T_2)}\hat{\rho}_{1pm} + \frac{T_2}{(T_1+T_2)}\hat{\rho}_{2pm}\right)\left[S_{mt}^2 + S_{pt}^2\right]. \tag{20}$$

$S_{mt}$ and $S_{pt}$ are standardized variables for $\Delta\ln(SP_t)$ and $\Delta\ln(OIL_t)$ that are constructed by subtracting the means and dividing by the empirical standard deviations of the pooled sample. Omelka and Pauly also consider small sample version of the test statistic, but they conclude that the asymptotic test in (17) is the best option for samples larger than 100. With daily data, our shortest sample has 292 observations.[17]

Table 3 gives the $\psi_T$ scores and the $p$ values. Details of the calculations are shown in the appendix. At a 5 percent significant level, the null hypothesis of invariance is rejected for all three break points.

To check the results for robustness, we also did the same empirical investigation and test on weekly data. These were constructed from daily data by two alternative methods,

[17]It might be noted that in the simulations provided by Omelka and Pauly, the size distortions of the asymtotic test are never more that 1–2 percentage points, even for small sample sizes ($< 40$). This suggests that $p$ values of 0.025 and lower are likely to be significant at the 5 % level when the test is applied to our weekly and monthly data sets.

Table 4: Comparing annualized estimates from weekly and daily data

| | | Daily data | | Weekly data | |
|---|---|---|---|---|---|
| Regime | Years | $\sqrt{250}\hat{\rho}_{pm}\hat{\sigma}_p$ | $0.4\sqrt{250}\hat{\rho}_{pm}\hat{\sigma}_p$ | $\sqrt{52}\hat{\rho}_{pm}\hat{\sigma}_p$ | $0.4\sqrt{52}\hat{\rho}_{pm}\hat{\sigma}_p$ |
| 1 | 1993 − 94 | −0.0540 | −0.0215 | 0.0290 | 0.0116 |
| 2 | 1994 − 96 | −0.0068 | −0.0027 | −0.0057 | −0.0023 |
| 3 | 1996 − 05 | −0.0089 | −0.0036 | −0.0099 | −0.0040 |
| 4 | 2005 − 08 | −0.0054 | −0.0021 | −0.0225 | −0.0090 |

averaging the level data for each calendar week, or using only Wednesday data. Both methods produced similar results.

The main results are that correlations are again negative and vary across the subperiods, but with one exception: In weekly data the correlation with the stock market returns is positive for the first period. This is $\hat{\rho}_{pm} = 0.0610$ based on averages, and $\hat{\rho}_{pm} = 0.0986$ based on Wednesday data, as opposed to $\hat{\rho}_{pm} = -0.1788$ from daily data. This raises doubts about the assumption of the returns being correlated GBM's. If they had been, the weekly returns for Wednesday data would also be correlated GBM's with the same correlations.

For subperiods 2–4, however, the results are similar. The point estimates from Wednesday data are summarized in table 4, with annualized estimates to allow comparisons.

Again, the value $\phi = 0.4$ is used, and the columns that include this factor show the CAPM risk premium in the annual required expected returns, annualized $\phi\rho_{pm}\sigma_p$. For the latter three subperiods these are small in absolute values, also when computed from weekly data. For the second and third period, the annualized estimates for the two alternative frequencies are very similar. For the last period, 2005–08, there is again a divergence between estimates based on daily and weekly data. But in this case, both are significantly negative.

A priori one would expect negative correlations in periods when supply variations dominate, but positive correlations when demand variations dominate. Much more detail on this is found in Kilian and Park (2009) and Filis et al. (2011). Even though positive correlations were rejected in daily data for all four subperiods, we have done a further test of robustness of this result by considering correlations for rolling windows of shorter lengths, 80 working days.[18] Among the 3677 windows, 57.8 percent have negative point estimates for $\rho_{pm}$, while the remaining 42.2 percent have positive point estimates. For most windows (89 percent of them, to be precise), 95 percent confidence intervals for $\hat{\rho}_{pm}$ include both positive and negative values. But 9.4 percent of the windows have only negative values in the confidence interval, while 1.6 percent of the windows have only positive values. This suggests that nonpositivity of correlations in daily data is a fairly robust result for these 15 years. An interpretation is that if there were periods with high demand fluctuations, these periods typically also had high supply fluctuations, so that the overall correlation was almost never significantly positive.

Finally, to get an idea of the magnitudes of the estimated changes, consider the numerical effects on call option values and triggers. While a comparative-statics analysis typically looks at changes in one variable at a time, there will now be changes in two

[18] With data for 3757 working days, there are $3757-80 = 3677$ such partly-overlapping windows. Some windows include data from two adjacent subperiods among the four original subperiods.

Table 5: Example of at-the-money call option values and triggers

| Regime | $\sigma_p$ | $\rho_{pm}$ | $\delta$ | $V^*$ | $F(V_t)$ | $P^*$ | $F_0(P_t)$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.3020 | -0.1788 | 0.0084 | 10.7389 | 0.7107 | 0.0903 | 1.8481 |
| 2 | 0.2245 | -0.0305 | 0.0273 | 2.8826 | 0.3722 | 0.0786 | 0.2316 |
| 3 | 0.4095 | -0.0218 | 0.0264 | 5.4067 | 0.5557 | 0.1429 | 0.3948 |
| 4 | 0.3020 | -0.0177 | 0.0279 | 3.6830 | 0.4481 | 0.1026 | 0.2843 |

All data are converted to annual rates. Source: Own calculations.

variables simultaneously, $\sigma_p$ and $\rho_{pm}$. For these two, use the point estimates from daily data during each of the four subperiods defined in Table 1, but keep the other variables fixed. Equations (1) and (2) will be applied to compute the change in $\delta$ that follows from a simultaneous change in $\sigma_p$ and $\rho_{pm}$.

In the first model mentioned in section 3, with $V_t$ as observable, the call option value is defined by (12). The variables $V_t, I, r, \alpha$, and $\phi$ are held fixed. The trigger value for $V$, $V^*$, is defined by (7) and (6). Numbers similar to those in Dixit and Pindyck (1994, p. 153f) will be used as an example. With annual rates, the numbers are $r = 0.04$, $\phi = 0.4$, and $I = 1$ is the required investment. The call option considered here has $V_t = I$, i.e., it is "at the money." Moreover, $\alpha$ is chosen to be 0.01, so that $\delta$ is always positive, although very small when systematic risk is negative. The last term in (2), the product of the three factors $\phi\sigma_p\rho_{pm}$, will have the negative numerical annualized values found in the fourth column of table 4. This leads to a value of $\delta$ from (1). The annualized volatility numbers are $\sqrt{250}$ times the estimates in table 3. The resulting values are given in table 5, all columns except the last two.

The last two columns are computed from the second model, with $P_t$ fixed instead of $V_t$. Since $\delta$ varies between 0.0084 and 0.0279, the fixed $P_t$ value is chosen to correspond to an approximate average of these, $\delta = 0.02$, together with the previously fixed $V_t = 1$. That is, $P_t$ is now fixed at 0.02. The trigger is computed as $P^* = \delta V^*$ (but only one of the two triggers is relevant for a particular decision). This results in the call option values shown in the last column. As mentioned above, the changes in $\delta$ result in strong variations in call option values when the underlying asset is a cash flow stream and $P_t$ is assumed to be fixed.

For all three breaks and both models, the triggers and the call option values do indeed move in the same directions as the changes in volatility. Clearly, this is related to the corresponding changes in $\delta$. For all three breaks, $\delta$ moves in the opposite direction of $\sigma_p$. As noted in the summing-up in section 3, this would have followed from a constant, negative $\rho_{pm}$, but here it coincides with a changing, negative $\rho_{pm}$. But, as noted the same place, the negative correlation means that there is no counteracting effect to the positive effect of volatility on option value. In these data, higher volatility coincides with lower systematic risk, thus a lower rate-of-return shortfall. One could say that this rescues the standard assumption, that higher volatility results in, or at least coincides with, higher triggers and option values. The results should nevertheless warn researchers to be more careful about assumptions underlying comparative statics. In particular, the robust negative correlations contradict what is often assumed in the real options literature.

# 6 Conclusion

This paper has tested two important assumptions underlying a number of studies of real options, that the correlation of the returns on oil and the stock market is positive and invariant to changes in oil price volatility. The theoretical discussion has shown the role of the assumptions when the CAPM is used to estimate the rate-of-return shortfall.

During the 15-year period July 1993 – June 2008, the study by Ewing and Malik (2010) has found three breaks in volatility, i.e., four subperiods with different levels of the volatility of the change in the logarithm of the oil price. We interpret these breaks as the empirical counterparts of the "change in volatility" which is a topic of theoretical comparative statics results. Accordingly, the question about invariance has been, is the correlation invariant to such changes? This is clearly rejected in the tests.

Consistently with previous empirical studies, this study finds that the correlation has been negative in daily data for each of the four subperiods. This indicates that during these periods, there has not been a counteracting (negative) indirect effect of volatility on a real call option on oil. The estimates show that the absolute value of the correlation has fallen both between the first and second subperiod (and also between the third and fourth), when volatility has decreased, and between the second and third, when volatility has increased. As a robustness check, weekly data were also tested. Only for the first subperiod, 1993–94, the results were not confirmed. A positive correlation estimated suggests that the GBM assumption does not hold for that subperiod.

For valuation of real options, a negative correlation removes one source of ambiguity in comparative statics results. But the empirical results indicate that changes in volatility can lead to changes in the same or opposite direction in the correlation, so the ambiguity is still there. An assumption that the covariance is invariant to volatility changes is also not confirmed in the data. Theoretical discussions need to hold all possibilities open.

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# Appendix A Calculation of test statistics from (17)

Results for first regime-break:

$$\hat{v}^2 = \frac{1.34}{292} + \frac{0.893}{341} = 7.2078 \times 10^{-3}$$

$$\psi_{633} = \sqrt{\frac{292 \cdot 341}{633}} \times \frac{(-0.17875 + (0.030461))}{\sqrt{7.2078 \times 10^{-3}}} = -21.907$$

Results for second regime-break:

$$\hat{\upsilon}^2 = \frac{0.835}{341} + \frac{1.21592}{2360} = 2.963\,9 \times 10^{-3}$$
$$\psi_{2701} = \sqrt{\frac{2360 \cdot 341}{2701}} \times \frac{(-0.030461 + (0.021816))}{\sqrt{2.963\,9 \times 10^{-3}}} = -2.7410$$

Results for third regime-break:

$$\hat{\upsilon}^2 = \frac{1.21664}{2360} + \frac{1.30751}{764} = 2.226\,9 \times 10^{-3}$$
$$\psi_{3124} = \sqrt{\frac{2360 \cdot 764}{3124}} \times \frac{(-0.021816 + (0.017739))}{\sqrt{2.226\,9 \times 10^{-3}}} = -2.0756$$

# Appendix B Test of distributional assumptions

In Table B.1 we show two diagnostic tests based on the residual vector. They test for 12th order residual autocorrelation ($\mathsf{F}_{\mathsf{AR}(1-12)}$), and for departure from normality ($\chi^2_{\mathsf{normality}}$). The tests are bivariate versions of the well known single equation diagnostics, see Doornik and Hendry (2009). The respective p-values are in brackets. We also give the number of outliers (larger than 3.5 estimated standard deviations for each variable). In the three first regimes there is no evidence of autocorrelation, but for regime 4 the test is significant. The normality test is significant in all regimes. Although the number of large outliers is relatively small (compared to samples sizes) it is nevertheless clear that a statistical test that allows for departures from normality is relevant for our purpose.

Table B.1: Diagnostics for bivariate distribution of market and oil real returns. Daily data

| Regime | # of obs | $\mathsf{F}_{\mathsf{AR}(1-12)}$ | $\chi^2_{\mathsf{normality}}$ | # of outliers (percent) |
|---|---|---|---|---|
| 1 | 292 | 1.2049[0.1697] | 34.348[0.0000] | 3 (1.0%) |
| 2 | 341 | 1.1967[0.1764] | 133.26[0.0000] | 2 (0.6%) |
| 3 | 2360 | 1.3621[0.0494] | 1061.4[0.0000] | 34 (1.5%) |
| 4 | 764 | 1.8466[0.0004] | 117.06[0.0000] | 6 (0.8%) |

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