A Real Options Approach to Clean Development Mechanism Projects *
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Abstract

Under the Kyoto Protocol, CDM (Clean Development Mechanism) projects are GHG-friendly projects hosted by developing countries that earn carbon credits to overcome financial and economic barriers. In this paper, we model the dynamics of investments in a unilateral, one revenue stream CDM project with the real options method, taking into account the irreversibility and ongoing uncertainty pertaining to the process. The model proposed is a modified version of the Majd and Pindyck (1987) model that allows for a finite horizon of the operating period. We assume that the risks pertaining to the registration period while construction may start expose the project to a catastrophic failure of its carbon revenues. For the solution, a numerical method is implemented with calibrated parameter values. The analyses show that the main threat to the CDM market is the volatility of carbon prices.

Key Words: Clean Development Mechanism, Kyoto Protocol, Real options.

1 Introduction

The strong interest in the Clean Development Mechanism (CDM) projects under the Kyoto protocol has remained persistent throughout the previous decade and is expected to continue. The Clean Development Mechanism is one of the project-based mechanisms set out in the climate change mitigation agreement to reduce the costs of abatement to developed countries while achieving technology transfers to developing countries. It allows sovereign or private entities of developed countries (or Annex I countries, which have committed to caps

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on their emissions) to offset them by investing in projects deemed to be cleaner as regard to
the technology, in that they imply lower CO$_2$ emissions compared to the business-as-usual
scenario.

Certified Emission Reductions units (CERs) achieved by CDM projects clearly dominate
the project-based carbon market supply, representing 65 % of the credit issuance between
2005 and 2013 1.

As the carbon market expands, academic interest in the CDM markets grows. The
overriding concerns are those of additionality and sustainability. Part of the economic lit-
terature on the topic echoes the contentions of many observers that the low cost of mitigation
provided by this mechanism should be deemed to rent-seeking (Muller, 2007) or may not
be suitable for sustainable development objectives (Rive and Rubbelke D. (2009), Muller
2007). As for CDM market project developers and buyers, they voice their concerns about
the regulatory uncertainty surrounding the CDM project cycle and the bureaucracy that
delay project launch or carbon credits issuance.

Some recent papers have taken up the study of the microstructure of the carbon market
and its CDM component. Mansanet-Bataller et al (2010) claims to be the first empiri-
cal study on secondary CER price drivers and the determinants of their spread with the
European Union Allowances (EUAs). Zavodov (2011) uses a cooperative options game me-
thodology to derive the core of the negotiation game that takes place between a project
developer and a carbon firm. This theoretical allocation is then tested against some CDM
market data.

In fact, recent observations suggest that profit sharing is less and less of a concern
for investment in CDM projects. According to the stylized facts uncovered by Lutken and
Michaelowa (2008), in contrast to the primary motivation of the CDM scheme, the CDM
business is largely dominated by unilateral investments. In this context, the regulatory
uncertainties, along with the fluctuations of secondary CER prices, remain central to the
investment decision process in the CDM business.

In this paper, we model the dynamics of investments in a unilateral CDM project with
the real options method, taking into account the irreversibility and ongoing uncertainty
pertaining to the process. The model proposed is close by many aspects to those of Majd
and Pindyck (1987) and Berk, Green and Naik (2004) models for the analysis of the dy-
namics of investment. We assume that the risks pertaining to the registration period while
construction may start expose the project to a catastrophic failure of its carbon revenues.

The remainder of the paper is organized as follows. In the next section, we spell out the
characteristics of a CDM project, giving an overview of the lengthy process leading to the
issuance of CERs and elaborating on the main concepts and the options embedded in such
projects. In the third section, a model is proposed for unilateral, one-revenue-stream CDM
projects. Section 4 presents the results of the numerical solutions obtained with calibrated
parameter values. Section 5 concludes.

1See World Bank (2014, p.39)
2 Characteristics of a CDM project

To ensure the environmental integrity of CDM projects, their emission reductions claims must be real, measurable and verifiable. They must be additional, that is, wouldn’t have taken place without the project. An involved regulatory process ensures that.

An Executive Board (EB-CDM) has been set up by the United Nations to oversee the whole process of the Clean Development Mechanism. At the level of host and investor countries, Designated National Authorities (DNA) are commissioned to guarantee that the projects meet sustainable development objectives. Another group of key actors in CDM projects are the Designated Operational Entities (DOE) which are commercial certification companies accredited by the Executive Board and provide as third parties the mandatory independent expertise on the contribution of the projects to emission reduction.

2.1 A CDM project cycle

Seven (7) stages can be identified in the CDM process: i) elaboration of the Project Design Document (PDD) which can be very costly when it implies methodologies that are not yet approved by the Executive Board, ii) letters of approval requested from the DNAs of host and investor countries, iii) validation whereby the project is subject to an independent expertise by a Designated Operational Entity (DOE) 3, iv) registration (or approval) of the project with the CDM Executive Board as a CDM project activity, v) monitoring, referring to the identification, collection and archiving of information necessary to design and implement a monitoring plan, vi) verification and certification by a DOE that gives a written assurance that, during a specified time period, a project activity achieved the reduction in GHG emissions as verified, vii) issuance of CERs requested from the EB-CDM by the DOE that certified the emissions.

As can be seen, the generation of Certified Emission Reduction units is a complex process. Validation procedures can take up to 12 months while the registration with the Executive Board lasts about 6 months. In the sequel, we will refer to all stages up to the registration (stages 1 to 4) as the registration process.

2.2 Other concepts and definitions

Figure 1 is an example of timeline and financial implications of a CDM investment. Before the operating period, registration fees and expenses for the construction of the plant are incurred. During the operating period, operating revenues and carbon revenue are collected. Their corresponding costs are operating costs and monitoring and verification costs.

Many types of projects do not have operating revenues. For example, industrial gas projects (HFCs, N₂O, SF₆) or other projects that are purported to reduce or eliminate

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2 see for example CDM rulebook 2010 or Farhana (2006)
3 As part of the validation stage, local stakeholders, and accredited NGOs may challenge various aspects of the project that may have been neglected.
4 see Worldbank Carbon Unit (2010, p.24)
existing non-regulated emissions of plants can be considered as stand-alone projects solely relying on their carbon revenues. These are one-revenue-stream projects\(^5\).

The **crediting period** of a CDM project activity is the period for which reductions will count towards the earning of CERs. It is selected by project participants to be either seven years, renewable twice, or ten years. Each renewal is conditional on the reassessment of the baseline and additionality of the project.

**Additionality** is indeed the main concern of the CDM registration process. A project developer must show that the use of the CER credits enables the project activity to overcome at least one of the five categories of barriers that would otherwise prohibit its implementation: financial, political, institutional, technological and economic\(^6\).

The following most important requirement of a CDM project is to contribute to the host country's **sustainable development objectives**. The CDM regulations have given the sustainability check mandate to the host country. But a CDM venture being a voluntary partnership, the investor country usually plays a role in enforcing some sustainability standards. Environmental NGOs also play a role by promoting quality certification to their designed standards.

The implication of developed countries sponsors in a CDM project is not a regulatory requirement. As a consequence, and because of relatively easy financial packaging in certain host countries, **unilateralism** has become a dominant feature of the CDM business. Unilateral CDM projects have minimal or no implication of Annex-I sponsors\(^7\).

\(^5\)See Lutken and Michaelowa (2008, p.85-86) for more details on those types of projects.

\(^6\)(Kenber, 2006, p. 274)

\(^7\)See Lutken and Michaelowa (2008, p.111) for a more elaborate definition of unilateral projects.
2.3 Risk, uncertainty and real options in CDM projects

The real options theory has become an appealing framework to perform economic and financial analysis of projects because it gives better insights in investment and operating activities of the firm than the traditional discounted cash flow methods. In environments where irreversibility and ongoing uncertainty demand a good timing of decisions, it accounts for losses of flexibility in the decision making by adding their value to the cost of the project. The purpose of this subsection is to elicit some of the real options embedded in CDM projects. We start by the sources of risk.

To this end, we first wrap up the features of projects that depict the sources of risk and the flexibility to act:

- project risk, which is not specific to CDM projects;
- "Kyoto risk" or regulatory uncertainty;
- carbon prices volatility;

Regulatory uncertainty is detailed below.

2.3.1 Regulatory uncertainty

Carr and Rosenbuj (2008) coined the term "Kyoto risk" to denote the regulatory risk that can realize in the following cases:

- the project may not be approved;
- there may be delays in the registration of the project or the periodic issuance of CERs;
- CDM standards may change, altering the value of CERs;
- the CDM may be discontinued post-2012;

The latest case is currently perceived as the highest risk and is termed market continuity. Indeed, starting in 2009, some directives of the European Union have taken into account the possibility of the failure of the Copenhagen summit to reach an agreement and have included provisions as to the use of post-2012 carbon projects emission reductions credits. Since 2000, the World Bank has relied on an asset called VERs (Verified Emission Reduction) to deal with such uncertainties even before the unexpected entry into force of the Kyoto protocol in 2005. The Bank is currently using the same assets to deal with the possibility of market discontinuity post-2012.

Another risk pertains to the regulations of quantities and qualities of CERs that are allowed for compliance in domestic carbon markets. The CERs are heterogeneous by nature because they are attached to specific projects. Domestic regulations can target their quality. For example, the European Commission is currently advancing a proposal for the ban of HFC-23 and N2O projects for compliance in the EU-ETS in 2013. This implies that those CERs are no more bankable, which drives down their prices. Concerning quantities, the European Union has set quotas on the CERs that can be used, which should represent at most 13.4% of the EUAs in the current second phase of the domestic market. Predictions of quantities of CERs that will be allowed into the market for compliance during the next
phase rely upon announcements concerning both the quantities of EUAs and the proportion of CERs.

Bottlenecks in the treatment of the request for issuance of CERs or in the registration process are sources of under-delivery risks for carbon funds. Another source of under-delivery risks is project risk. Several carbon funds manage under-delivery risks by allowing for more carbon purchase or projects to be contracted (World Bank Carbon Finance Unit, 2009, p.18). The current practice on the CDM market is to enter into a forward agreement to buy Certified Emissions Reductions. By entering into the so-called Emission Reduction Purchase Agreements (ERPA), the project entity and the buyer lock in a forward price for the CERs.

2.3.2 Real options in CDM projects

Construction of many plants takes time to complete. For most CDM activities to start exactly after the registration date with the CDM Executive Board or a few months later, they have to be under construction before the decision. Thus, big CDM projects, as other types of projects, would imply the following options for the investment process:  

- an option related to the best time to start the construction of the project, or deferral option;
- an option related to defaulting during construction, or time-to-build option;

More precisely, time-to-build option values occur when investment decisions and associated cash outlays occur sequentially over time; there is a maximum rate at which outlays and construction can proceed; the project yields no cash return until it is completed.

- an option of temporary suspension of the project, or mothballing option;
- an option to shut down and restart the operating plant;

Embedded options specific to CDM projects are:

- choice of the best alternative for the crediting period. As already mentioned, the crediting period is either 10 years or 7 years twice renewable.
- the option to decide on the best moment to register the project with the CDM Executive Board.
- the option to delay the beginning of the crediting period for a few months or years: the CDM rules permit to delay the starting of the crediting period for up to 2 years, or even 4 years for projects hosted by least developed countries.

3 A model for unilateral, one revenue stream CDM projects

There are two stages in the model. In the first stage, construction starts under the threat of the failure of the registration. Project sponsors seek registration at the end of the

\footnote{Here, we adopt the terminology of Trigeorgis (1995).}

\footnote{See Majd and Pindyck (1987, p.1)}
construction period at date $\theta$. The second stage is the operating period that corresponds economically to the crediting period, which is fixed. We assume that during construction, the market value of the project, were it completed, can be observed. The project is subject to two types of risks:

1- the stochastic fluctuations of the potential future cash flows of the project if it were completed;

2- The risk of the registration process. We approximate this concept by assuming that because of the hurdles in the registration, there is a fixed probability $\phi$ that the potential cash flows of the project will be extinguished.

Thus, investment decisions are made based on three state variables (1) the level of cash flows the project would be earning were it completed, (2) whether the project potential cash flows have been extinguished by the failure of the registration process, (3) the remaining capital expenditure to complete the project. The model focuses on the deferral and time-to-build options.

The uncertain cash flow stream that the project receives upon completion is made of secondary CER revenues because of the unilateral nature of the project. There is no bargaining or value sharing as would be the case in a true bilateral project. We assume that the amount $q$ of carbon credits produced annually is constant. We also assume that the operating costs, as well as monitoring and verification costs, can be neglected and that during the operating period, there is no option to shut down the project. As in Zavodov (2011), we model the cash flows of the operating CDM project as a constant multiple of CER price in the secondary market, $V = qP_{sCER}$ with $P_{sCER}$ following a standard geometric Brownian motion:

$$dP_{sCER}(t)/P_{sCER} = \alpha dt + \sigma dz, \quad P_{sCER}(0) = P > 0$$

(1)

where
\( \alpha = \mu - \delta \) denotes the expected rate of return net of convenience yield from the sale of carbon credits, \( \sigma \) is the instantaneous volatility of the CER price per unit of time, and \( dz \) is an increment of a Gauss-Wiener process. The risk associated with this process is systematic. Thus, the cash-flow process \( V \) also follows a geometric Brownian motion:\(^{10}\)

\[
dV(t) = \alpha(t) dt + \sigma(t) dz, \quad V(0) > 0.
\] (2)

Construction of the plant takes place in continuous time and does not imply any technical uncertainty. The total expenditure required to complete the project is denoted \( J \). It comprises the necessary capital for investment in the plant and the capital expenditures for registering the project as a CDM activity. At any time, the variable representing the remaining capital expenditure to complete the project is denoted \( K \). To simplify, there is no uncertainty, neither on \( K \) nor on \( J \). The rate of investment is \( I(t) \) with \( 0 \leq I(t) \leq k \). The positive constant \( k \) is the maximum rate of investment. We assume that there is neither decay of capital, nor adjustment cost. The instantaneous cost incurred over an interval of time \( dt \) is \( I(t) dt \). Thus, the dynamics of capital satisfies the equation:

\[
dK = -I dt
\] (3)

Let \( \zeta(t) \) be an indicator function for whether the investment effort is complete.

\[
\zeta(t) = \begin{cases} 
0 & \text{if } \int I(t) dt < J \\
1 & \text{otherwise}
\end{cases}
\] (4)

As long as \( \zeta(t) = 0 \) the promoter has to decide on the investment effort over the next instant.

The risk of a failure of the registration follows a Poisson process such that the probability of failure over the next instant is \( \phi dt \). It is assumed that the process determining the catastrophic failure is independent of all variables in the problem. We track whether the catastrophic failure has occurred by an indicator function denoted \( \xi \), which starts out at one and drops to zero when the failure occurs.

\[
\xi(t) = \begin{cases} 
1 & \text{before the failure} \\
0 & \text{after the failure}
\end{cases}
\] (5)

Finally, by assumption, all registration costs are zero.

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\(^{10}\)See Majd and Pindyck (1987, p.13) for the explanation of why this type of dynamics for \( V \) during the construction period is justified even if the completed project is of finite life.
3.1 Valuation of the CDM project

The riskless interest rate, \( r \), is assumed constant. Suppose the promoter observes the current level of \( V \) and the remaining expenditure to complete the project \( K \).

The possibility of a failure has an add-on impact on the interest rate, that is replaced by \( r + \phi \) over the construction-registration period. It can be shown (see appendix) that the value of the opportunity to invest in the project satisfies the Hamilton Jacobi-Bellman equation:

\[
\frac{1}{2} \sigma^2 V^2 F_{VV} + (r + \phi - \delta_c)VF_V - (r + \phi)F - \max_I \{ I (F_K + 1) \} = 0 \quad (6)
\]

Since the maximand is linear in \( I \) (bang-bang solution), one the following partial differential equations has to be solved depending on the level of the control variable (\( I = 0 \) or \( I = k \)):

\[
\frac{1}{2} \sigma^2 V^2 f_{VV} + (r + \phi - \delta_c)VF_V - (r + \phi)f = 0 \quad (7)
\]

\[
\frac{1}{2} \sigma^2 V^2 F_{VV} + (r + \phi - \delta_c)VF_V - (r + \phi)F - k (F_K + 1) = 0 \quad (8)
\]

The boundary conditions are:

\[
F(V, 0) = V = qP_{CER}(\theta) \left[ (1 - e^{-\delta T})/\delta \right] \quad (39a)
\]

\[
\lim_{V \to \infty} F(V, K) = e^{-\delta K/k} \quad (39b)
\]

\[
f(0, K) = 0 \quad (39c)
\]

\[
f(V^*, K) = F(V^*, K) \quad (39d)
\]

\[
f_V(V^*, K) = F_V(V^*, K) \quad (39e)
\]

Conditions (39d) and (39e) are the well known value matching and smooth pasting conditions. Condition (39c) is straightforward in that for \( V \) tending to zero, the option value to invest in the project also tends to zero. Condition (39b) describes what happens as the value of the completed project becomes very large. As explained in Majd and Pindyck (1987), time to completion is then \( K/k \) as investment will surely proceed on until completion at the rate \( k \). A marginal increase in \( V \) of one dollar increases the value of \( F \) by \( 1 - \int_0^{K/k} \delta e^{(r-\delta)t} dt = e^{-\delta K/k} \). Finally, the first boundary condition (39a) simply translates the fact that when investment is complete, the value of the option is the total value of the expected cash flows previously calculated over the crediting period of length \( T \).

The solution procedure to these equations is numerical. Although equation (7) is in fact a second order ordinary differential equation, equation (8) is a non-homogeneous partial differential equation. Together, they define two regions whose boundary is endogenously
determined by the value matching and smooth pasting conditions in (39d) and (39e). Computational details are given in Dixit and Pindyck (1994, p. 353-356). The code program has been written in the MATLAB software.

4 Numerical solution

The first step for the numerical illustration is to assign some realistic values to the real risk-free interest rate \( r \), the rate \( \phi \) of failure, the convenience yield \( \delta \) and the volatility \( \sigma \) of the secondary CER prices. As well, it is useful to refer to a specific CDM project to get an idea of the magnitude of certain project specific parameters. For that, we take as reference the project number 4166 registered in February 2011 with the CDM Executive Board and titled "SF\(_6\) reduction project in South Korea". This industrial gas project is a one-revenue stream project. Prices and monetary values will be set in euros in order to get a direct reference to the EU-ETS.

The total investment in the project is about \( K = 3 \) million euros at a rate \( k = 1 \) million/year. The total of CERs generated annually are around \( q = 136,000 \).

The nominal interest rate used in the project is the yield of the government bonds which is 5%, one of the lowest in the UNFCCC CDM database. The corresponding real interest rate is somewhere between 1% and 2% since the monthly inflation indicator has been between 2.5% and 4.5% over the period of development of the project. We elect to use a rate of 2% as a benchmark.

The rate \( \phi \) of failure of the registration is not specific to the project. It can be estimated by the number of projects that failed in the total of projects that entered the CDM pipeline. As of the end of 2010, after 7 years of CDM project development, there were 1217 projects that failed at some point before registration or at the very stage of the request for registration while 2708 projects got registered. Another 3118 were in the pipeline with no registration or failure status. Thus, the rate of failure over 7 years is somewhere between 17.3% and 31%. Given those estimations, we think that an annual rate of failure of 3% (that is 21% for 7 years) is appropriate for the calibration. The corresponding continuously compounded rate is roughly the same.

For the convenience yield, it is hard to argue for a constant parameter as in our model in the light of empirical studies. For carbon prices and other commodities, most empirical studies end up supporting a time-varying convenience yield that can take on positive as well as negative values. The reason is that the convenience yield is estimated as a residual parameter as in the equation \( \delta = \mu - \alpha \) (see equation (1)). Dixit and Pindyck (1994, p. 334) explain why it can be considered the other way : \( \delta \) constant and \( \mu \) adjusting. We take the same stance in this paper, as do many other real options papers. One of these, which deals with carbon capture and storage projects (Rammerstorfer and Eisl, 2009), points to values up to 8% for the medium term. This corresponds to the length of the construction period in this paper. Noting that Dixit and Pindyck have usually used 6% as a benchmark in the
illustrations of their chapters, we think that a value of 7% is warranted for the real options analysis under consideration.

The volatility $\sigma$ of the carbon prices (EUAs and CERs) will also be calibrated in reference to some recent papers in the literature. Empirical works suggest that their standard deviations are roughly the same (see for example Mansanet-Bataller et al (2011)). In their paper, Benz and Truck (2009) distinguish in the EUAs of 2005-2006 two regimes for the log returns of the distribution: a base regime where the standard deviation is 1.22% on a daily basis and a spike regime where that standard deviation is 4.76%, that is, roughly four times higher. On an annual basis, these volatilities are respectively 19% and 76%, on the basis of 252 trading days. Studies that do not distinguish the two regimes like Mansanet-Bataller et al (2010) point to annual intermediate values of 39%\footnote{Secondary CER has been estimated in the paper of Mansanet-Bataller et al (2010) to be 2.44\% (p. 4, Table 1, standard deviation of the log returns).}. We elect to use as a benchmark case $\sigma = 0.2$ close to that of the base regime identified by Benz and Truck.

The solution determines the cutoff value $V^*$ for the investment region and is plotted below (left panel figure 3). By the relation $V = qP_{sCER}(\theta) \left[ (1 - e^{-\delta T})/\delta \right]$ one can deduct the cutoff values for the price $P_{sCER}$ of carbon (right panel of figure 3).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{boundary.png}
\caption{Boundary of the optimal investment decision region with $r = 0.05$, $\delta = 0.07$, $\sigma = 0.2$}
\end{figure}

Figure 4 is a lookup table for this numerical solution. The first column displays the value of the completed project. The middle columns are for different levels of the value of the project according to the value of the remaining investment $K$. All values on the same line...
imply the same value of the completed project that is retrieved in the column for \( K = 0 \). Cutoff values for each level of investment are starred. Their corresponding CER prices are in the rightmost column of the table.

<table>
<thead>
<tr>
<th>Value of completed project (million euros)</th>
<th>Remaining Investment K (million euros)</th>
<th>Cutoff carbon prices (euro)</th>
</tr>
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<tbody>
<tr>
<td>10.635</td>
<td>5.9223 6.4834 7.34 8.1289 8.9404</td>
<td>9.7747 10.635</td>
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<tr>
<td>4.8141</td>
<td>2.5445 3.019 1.5099 1.5447* 3.203 3.833 4.4814</td>
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<td>2.0922 2.6682</td>
</tr>
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<td>2.0922 2.6682</td>
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<td></td>
</tr>
</tbody>
</table>

**Fig. 4** – Numerical solution for the optimal investment rule with \( r = 0.05, \delta = 0.07, \sigma = 0.2 \)

As can be seen from Figures 3 and 4, the trigger project value for investment in that project is 8.30 million euros with a corresponding CER price of 8.49 euros. The hyperbolic shape of the optimal investment rule accounts for the dynamics of the option value of the project. As the irreversible investment is under way, the minimum price of carbon required to pursue the project diminishes. This is characteristic of any time-to-build option. Because of irreversibility, promoters are less demanding than at the time the investment starts. Of course, if the price of carbon falls below that cutoff value after some cumulated investment, construction is halted: the project is mothballed. Investment resumes as soon as the price moves back to a value higher or equal to the cutoff price corresponding to that cumulated investment. Since there is no decay of capital in the project and prices follows a geometric
Brownian motion, there is always a possibility that construction will resume as long as there is a market for CERs.

We now perform some sensitivity analyses. The left panel of Figure 5 displays the variations of the cutoff values for $\delta = 0.6$, $\delta = 0.7$ and $\delta = 0.8$. It shows that when the convenience yield $\delta$ is low, investors are more demanding. The reverse is true for the real interest rate $r$ (right panel). This is more intuitive since high real interest rates raise the opportunity cost of capital by improving the return of the alternatives. It is worth noting that $r$ and $\phi$ are not identified in the model. Only $r + \phi$ is identified, so that the sensitivity analysis conclusions for $r$ are also valid for $\phi$. The message of the model relating to the rate $\phi$ of catastrophic failure is intuitive: a decrease in the failure rate fosters investment. This can be accomplished partly by a sound streamlining the regulatory process to be more transparent and less cumbersome in a way that do not hinder other potential investments.

![Figure 5](image_url)  

**Fig. 5** – Sensitivity analysis for $\delta$ (left) and $r$ (right)

The sensitivity analysis for the length of the crediting period $T$ confirms the arguments of the ongoing debate over its length (Figure 6 left panel). Values chosen are 7 (minimum under current regulations), 10 (our benchmark, corresponding to a non renewable crediting period) and 14 (end of the second crediting period of a renewable project). As expected, longer crediting periods meaning longer operating lives for projects lower the minimum price.

In all three cases ($\delta$, $r$ and $T$), CER prices that trigger investment are in the ranges of those observed on the market because prices have been between 8 and 14 euros since the beginning of 2009.

The last sensitivity analysis in the right panel of Figure 6 is with respect to the volatility $\sigma$. It confirms a property of options that a higher volatility raises the option value and thus
the cutoff price. Referring to the above discussion on the empirical values of $\sigma$, it is fair to say that our modest variation of $\sigma$ from 0.2 to 0.25 raises the CER trigger price to 15 euros, a value not experienced in the market for the last two years. Bearing in mind that we have allowed only this modest variation, this is really an alarming and disquieting finding. It implies that the average value of $\sigma = 0.39$ blows up the cutoff trigger value, especially as the impact of $\sigma$ in the model is through $\sigma^2$(notice that $(0.39)^2$ is 2.5 times $(0.25)^2$ and about 4 times $(0.2)^2$). To wrap up, the volatility observed on the CER prices pushes the trigger value of the investment to levels not recently observed on the market.

![Graph showing sensitivity analysis for T (left) and \(\sigma\) (right)](image)

**Fig. 6 – Sensitivity analysis for T (left) and \(\sigma\) (right)**

5 Conclusion

In this paper, we have exposed a model that is purported to account for investment decisions in Clean Development Mechanism (CDM) projects and their dynamics. A unilateral, one revenue-stream project has been featured as a first step before extending the paper to two-revenue-stream projects that were in the mind of the initial designers of that mechanism. A numerical method has been applied in order to get the solutions of the non-homogeneous partial differential equation that characterizes the dynamics of the investment. One of the messages of the model is that a more efficient and less uncertain registration process will foster investment in CDM projects by reducing the "Kyoto risk". Another message is that the crediting period effect on the value of projects is important. But a cautious line should be taken if the CDM has to be reformed so that competitiveness of CDM projects will be preserved. Above all, the main threat on the CDM that is already having its sting is the volatility of the carbon market. It raises investment trigger values of projects to levels that are prohibitive to any further development of the market, especially in the recent period of
a carbon market far from momentum as the Kyoto protocol has expired without a worthy successor.
6 References


Suppose the promoter observes the current level of $V(t)$ and the remaining expenditure to complete the project $K$.

The value of the project is given by:

$$F(t; V(t), K(t), \xi(t)) = \max_{I(s)} E_t \left\{ \frac{\nu_2(t)}{\nu_1(t)} F(\tau, V(\tau), K(\tau), \xi(\tau)) \right\}$$

The terms under the expectation operator are respectively the appropriately discounted salvage value of the project, the total amount invested and the revenue from the operation of the project. $\tau$ is an arbitrary time in the future, and $\nu_1$ and $\nu_2$ are the appropriate stochastic discount factors for the construction and the crediting periods respectively.

We solve the valuation problem conditional on the project being alive, and suppress the dependance on $\xi(t)$ (see also Berk, Green and Naik (2004)):

$$F(t; V(t), K(t)) = \max_{I(s)} E_t \left\{ \frac{\nu_2(t)}{\nu_1(t)} F(\tau, V(\tau), K(\tau)) \right\} - \int_t^\tau \frac{\nu_1(s)}{\nu_1(t)} I(s) \xi(s) ds + \int_t^\tau \frac{\nu_2(s)}{\nu_2(t)} \zeta(s) V(s) ds$$

Under the risk neutral measure, valuation boils down to adjusting the drift $\alpha$ by a fair risk premium $\lambda$, which is equal in equilibrium to $r - \delta$, the risk free rate minus the convenience yield (see Kulatilaka (1995) or Cox, Ingersoll, and Ross (1985)). In addition, the possibility of a catastrophic failure has an add-on impact on the risk-free interest rate, which is replaced by $r + \phi$ over the construction-registration period.

When construction completes at time $\theta$, the value of $F$ is:

$$F(\theta; V(\theta), K(\theta)) = F(\theta) = \int_0^T E_\theta \{ V(s) \} e^{-rs} ds = \int_0^T V(\theta) e^{(r-\delta)s} e^{-rs} ds$$

$$= V(\theta) \left[ (1 - e^{-\delta T}) / \delta \right] = q \Pr(\theta) \left[ (1 - e^{-\delta T}) / \delta \right]$$

This maximization problem can be decomposed over the intervals $[t, \theta]$ and $[\theta, \theta + T]$. The previous relation can be rewritten as:

$$F(t, V(t), K(t)) = \max_{I(s)} E_t \left\{ - \int_t^\theta e^{-(r+\phi)(s-t)} I(s) ds + e^{-(r+\phi)(\theta-t)} F(\theta, V(\theta), K(\theta)) \right\} \quad (9)$$

The multidimensional Ito’s lemma applied to $e^{-(r+\phi)(\theta-t)} F(\theta, V(\theta), K(\theta))$ over the interval $[t, \theta]$ yields:
\[ e^{-\left(r+\phi\right)(\theta-t)} F(\theta, V(\theta), K(\theta)) = F(t, V(t), K(t)) \]
\[ - \int_t^\theta (r + \phi)e^{-\left(r+\phi\right)(s-t)} F(s, V(s), K(s)) ds + \int_t^\theta e^{-\left(r+\phi\right)(s-t)} FV(s, V(s), K(s)) dV + \int_t^\theta e^{-\left(r+\phi\right)(s-t)} FV_V(s, V(s), K(s)) dV^2 \]

or equivalently, after substitution of the dynamics of \( V \) and \( K \):

\[ e^{-\left(r+\phi\right)(\theta-t)} F(\theta, V(\theta), K(\theta)) = F(t, V(t), K(t)) - \int_t^\theta (r + \phi)e^{-r(s-t)} F(s, V(s), K(s)) ds + \int_t^\theta e^{-\left(r+\phi\right)(s-t)} FV(s, V(s), K(s)) \sigma V(s) ds + \int_t^\theta e^{-\left(r+\phi\right)(s-t)} FV_V(s, V(s), K(s)) \sigma V(s) ds + \int_t^\theta e^{-\left(r+\phi\right)(s-t)} FV_V(s, V(s), K(s)) \sigma V(s) ds \]

Substituting back into (9) and rearranging yields the following differential equation:

\[ \frac{1}{2} \sigma^2 V^2 F_{VV} + (r + \phi - \delta) V F_V - (r + \phi) F - \max_{I} \{I(F_K + 1)\} = 0 \]

Since the maximand is linear in \( I \), one finally gets the following differential equations (bang-bang solution):

\[ \frac{1}{2} \sigma^2 V^2 f_{VV} + (r + \phi - \delta) V f_{VV} - (r + \phi) f = 0 \]

\[ \frac{1}{2} \sigma^2 V^2 F_{VV} + (r + \phi - \delta) V F_V - (r + \phi) F - k(F_K + 1) = 0 \]