Valuation of Real Options with Flexible Early Exercise in a Competitive Environment: The Case of Performance Improvement Packages

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Commercial aircraft developments are major endeavors which strain significantly the resources of original equipment manufacturers. These developments represent huge bets for the companies undertaking them due to the assumptions made when business plans are laid out and the abundance of uncertainties both at the technical and market levels. The long development cycles and the long lives of the aircraft once in operations force aircraft manufacturers to speculate regarding future airline needs and future states of the world. One possibility to mitigate these risks is through a continuous optimization of the airframe and its engines after it has entered service. These continuous developments help manufacturers stretch the operating lives of their designs by keeping them up-to-date and therefore relevant in a competitive environment. Performance improvement packages (PIP) represent a means for aircraft and engine manufacturers to offer airlines the ability to infuse new technologies into existing assets at a minimum capital expenditure. Standard capital budgeting methods are not well suited to assess the economic performance of programs subject to significant uncertainty because they fail to account for the flexibility offered to management to steer programs into profitable directions. Similarly, these methods do not usually capture the dynamic nature of markets and the erosion of leadership positions over time. The on-going research tries to overcome some of these challenges by proposing a real-option based method to help substantiate development strategies in the aerospace industry. A new method is proposed to evaluate real-options featuring early exercise possibilities by cross-fertilizing techniques used in the finance industry, in statistics, and in actuarial sciences. It is articulated around the use of an Esscher transform and its non-parametric empirical approximation to perform a risk neutralization, a bootstrapping technique to resample the evolution of the underlying development program value, and finally a regression-based technique to value real-options with early-exercise possibilities. The proposed method is applied to the evaluation of a performance improvement package for a commercial aircraft.

Keywords: Real-options, Simulation, Esscher Transform, Resampling, Bootstrapping, Early Exercise

1 Introduction

Aircraft development cycles are both long and expensive and their lengths force manufacturers to speculate regarding future airline needs and future states of the world. Tremendous risks are associated with these developments and manufacturers are reluctant to develop new airliners from scratch. On the other side of the market, airlines buy aircraft with operating lives exceeding twenty years, and renewing the fleet is a major decision that may affect their global competitiveness for over a decade. Facing unprecedented fuel expenses, airlines pressure aircraft

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manufacturers to produce significantly more efficient designs. The airline fleet selection process is a complex task relying on multi-attribute analyses. According to Paul Clark [1], there are key buying criteria in the decision and these vary slightly from customer to customer. However, these key factors revolve around the economic, the performance, the comfort and the environmental aspects, with the economic and performance aspects accounting for about seventy percent of the decision.

One venue for manufacturers to mitigate these risks is through the continuous optimization and improvement of their product-portfolio even after entry into service. These developments help manufacturers stretch the operating lives of their aircraft designs by keeping them up-to-date and therefore relevant in a competitive environment. Performance Improvement Packages (PIP) present a way for aircraft manufacturers to offer airlines the ability to infuse new technologies into existing assets [2] and to rejuvenate their fleet at a minimum capital expenditure. These packages have been widely used in the aircraft and engine manufacturing industry and have often been proposed to operators as stop-gap measures to improve the economics of aircraft currently on the market. For instance, *McDonnell Douglas* introduced a series of PIP [3] in the 1990's to improve the aerodynamics, reduce the drag, and improve the fuel-burn of its flagship MD-11 aircraft as the aircraft was not meeting promised specifications at entry in service. *CFM International* announced in 2007 the first delivery of a Tech Insertion package and in 2011 announced the availability of a new performance improvement package for the CFM56-5B3 [4] turbofan engine. These aimed at reducing NO_x emissions, improving fuel burn, and extending the time on-wing of the engine. Another example is *Boeing* which introduced in 2009 refinements to the B777 aircraft that airlines could purchase to increase range and payload [5]. These improvements involved reshaping tiny vortex generators on the upper surface of the wing, optimizing the ram air intake system to reduce drag and drooping ailerons by two degrees while in flight. Finally, in 2013 *Airbus* launched a Sharklet retrofit [6] for in-service aircraft of the A320 family. This retrofit brings new advanced wingtip devices to reduce fuel-burn by up to four percent and consequently to reduce carbon emissions.

Still, developing, certifying, testing, and producing a performance improvement package is expensive and manufacturers have to commit scarce engineering resources in return for hypothetical profits. To assess their economic viability, manufacturer estimate development costs and forecast future demand by making assumptions regarding the future state of the airline industry and the future state of the competition. Traditionally, discounted cash flow analysis is used next to assess the economic performance of investments as reported by Graham and Harvey [7]. This is however not well suited for projects involving significant upfront investments such as development programs in the aerospace industry: indeed, with initial investments in the billions and aircraft deliveries starting only several years later, the discounted cash flow analysis over-emphasizes initial cash-outflows and undervalues streams of cash-inflows coming only years later. This leads to an undervaluation of many development programs and possibly the rejection of potentially profitable development programs. Yet, new aircraft developments are undertaken every year.

Part of this problem lays in the fact that discounted cash flow analyses are deterministic and therefore do not handle well projects spanning over multiple years, featuring several decision tollgates and riddled with uncertainties. One method to assess project viability under uncertainty uses real-options [8]. Real-options analysis is an emerging field in corporate finance [9] where it is used to substantiate capital budgeting decisions. It is derived from the financial options analysis pioneered with the seminal work of Black, Scholes [10] and Merton [11]. Real-options analysis may be interpreted as an extension of the discounted cash flow analysis in that it uses the concept of time-value of money but goes beyond and recognizes the fact that managers react to changes in the business environment and actively steer projects into profitable directions. Consequently, a real-options approach accounts for the flexibility offered to management to abandon unprofitable programs. This is particularly well suited for aerospace development programs which usually feature critical tollgate reviews at which programs may be abandoned. In the case of new aircraft developments, the major drivers affecting the profitability of a development program include the growth of air transportation, the retirement of older less-efficient aircraft, as well as the evolution of the energy prices (jet-fuel).

There is little doubt that real-options inspired methodologies present an attractive concept for capital allocation budgeting problems due to their abilities to better mimic the decision processes that take place within companies as uncertainty unfolds. However, as much as option-thinking seems promising for analyzing investments featuring flexibility, the implementation and the adoption of real-options within companies has been slow[12]. There may be several reasons to this and one of them may be the complexity of developing a relevant real-options framework. While simpler models using the closed-form Black-Scholes formula have been attractive initially due to their simplicity, their validity for corporate investment valuation may be questionable. Some of the assumptions underpinning the Black-Scholes model are quite strong and may not be appropriate for corporate investments. More generic methods using Monte Carlo simulations have been proposed over the years and relax some of these assumptions but the explicit formulation of a model for the evolution of the business prospect value remains problematic. Indeed, analysts typically have access to a lot of real data and may be able to model the evolution of one or several source of uncertainty over time. Nevertheless, when several sources of uncertainty impact a development program, fitting a model to simulate the stochastic evolution of the development program value becomes significantly harder.

In this context, the current research proposes a new transparent and integrated methodology aimed at investigating the viability of a strategic investment in a competitive environment. The value-driven methodology will be the foundation for a strategic decision-making framework that facilitates the formulation of robust and competitive solutions. The methodology is applied to the development of a performance improvement package for an aircraft no longer in production. The package includes the addition of advanced wingtip devices to reduce drag and therefore decrease fuel-burn, as well as some refinements to the engine to improve efficiency and further reduce fuel-burn and emissions. The manufacturer has identified a gap in its development stream which makes it possible to develop, certify, and produce the package. As a result, there are three reasons motivating this development: demand by airlines for more efficient aircraft to reduce their exposure to fluctuating energy prices; extension of the operating life of their fleet by making the aircraft more competitive with other offerings from the competition; and identification of a gap in the development stream of the aircraft manufacturer that needs to be filled to keep workforce busy. Decision makers have to identify whether the market conditions are currently optimal for the commercial launch of this product and whether it makes sense to commit resources to this development. If not, the manufacturer can delay the development and wait for trigger events that will ensure a successful development.

2 Development of the Performance Improvement Package

The performance improvement package consists of two subsets of technologies that can be retrofitted to a family of aircraft currently in operations. The first subset consists in an advanced winglet device to be fitted at the wingtip of the aircraft to reduce lift induced-drag. However, some structural strengthening of the wing spar is required to fit the new winglet in order to cope with the increased wing bending moment. The second subset consists in new materials and an advanced shaping of the fan blades for the engine as well as new alloys in the turbine section. This results in increased fuel efficiency for the engine with lower specific fuel consumption as well as fewer maintenance events. The performance improvement packages can be installed by airlines on their aircraft during regular scheduled maintenance in their own facilities. The resulting impacts on several key metrics are quantified in Table 1.

Table 1: Performance Improvement Package Impact on Key Metrics

	SFC	Induced Drag	Vehicle Weight	Maintenance Cost
Advanced Winglet		-5.0%	+0.5%	
Updated Turbofan	-1.0%		-0.5%	-1.0%

2.1 Development timeline

The development process for the Performance Improvement Package may be described as a staggered development program featuring decision tollgates. It is articulated around four main phases, starting with the initial market research and conceptual design, followed by preliminary and detailed developments, followed by certification and testing, and finally ending with production. Each of these phases is separated by a decision tollgate at which point management can exercise some flexibility and decide whether to pursue, delay, or abandon the development altogether if the market conditions are not right. This development timeline is shown in Figure 1.

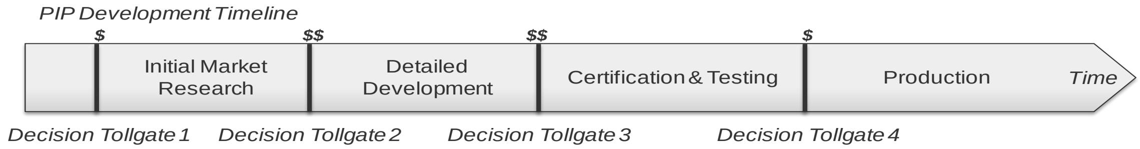

Figure 1: Development timeline and associated milestones

If the development program goes ahead, additional funding is committed and spent during the following phase. All four of these phases do not have the same resource requirements: detailed development as well as certification and testing to a lesser extent are the most expensive phases of the development program. Therefore, it is unlikely that the program will be abandoned at the third or fourth decision tollgate given that the following phases are relatively cheap and that so much has already been spent during the previous phases. Delaying or abandoning the development is nevertheless a possibility at the first and second decision tollgates if conditions are not favorable.

2.2 Windows of possibilities

In this pilot study, the manufacturer has identified a gap in its development stream between two periods of high activity. The first period of high activity is related to a previous development requiring substantial engineering resources to complete the detailed design and to get certification. The second period of high activity concerns a future development for the replacement of a current aircraft design that is getting obsolete. This second program is therefore deemed vital for the profitability of the manufacturer and is projected to tie the engineering resources for several years onwards. In between, there is a development gap during which the manufacturer has currently no projected development and during which engineering resources might be available. This is an unfortunate situation for aircraft and engine manufacturers as they have to retain the workforce to keep skilled and experienced engineers in-house for future programs. In this context, a window of possibility for the development of the PIP program is defined as the ability to undertake a development program. This situation is depicted in Figure 2.

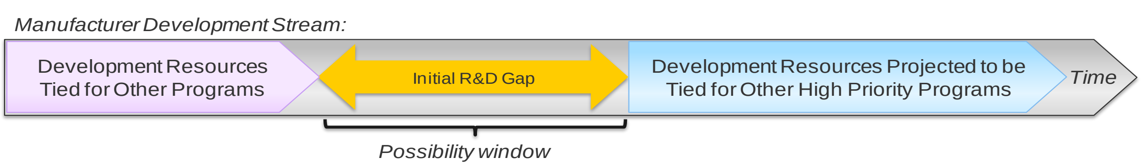

Figure 2: Timeline of manufacturer development stream

2.3 Windows of opportunities

Of interest however are not windows of possibilities but rather windows of opportunities which are defined as the timeframe during which, and the condition for which, launching a new development program is best. If decision-makers invest too early within the window of possibility, they only have limited information and this may be risky as the future realization of uncertainty might undermine the development program. If decision makers invest too late, risk also increases since the target market size is reduced as airlines ground older aircraft and airlines become reluctant to invest in retrofits for an ageing fleet. To be meaningful to decision-makers, a window of opportunity has to be contained within a window of possibility. Therefore, the largest window of opportunity is the window of possibility.

In addition, windows of opportunities are not static: they morph in real time to adjust to the new reality that unfolds. Increasing energy prices drive the demand for more efficient aircraft and a low capital expenditure retrofit to reduce fuel consumption may look like an attractive option for airlines. Alternatively, emerging competitors with new aircraft designs or even competing improvement packages from other manufacturers may impact the demand and therefore the profitability of the program (value leakages). Combined together, these effects may either stretch or constrict the window of opportunity. This dynamic process is depicted in Figure 3 where the impact of progressive aircraft retirement and the impact of competition on the opportunity window are highlighted.

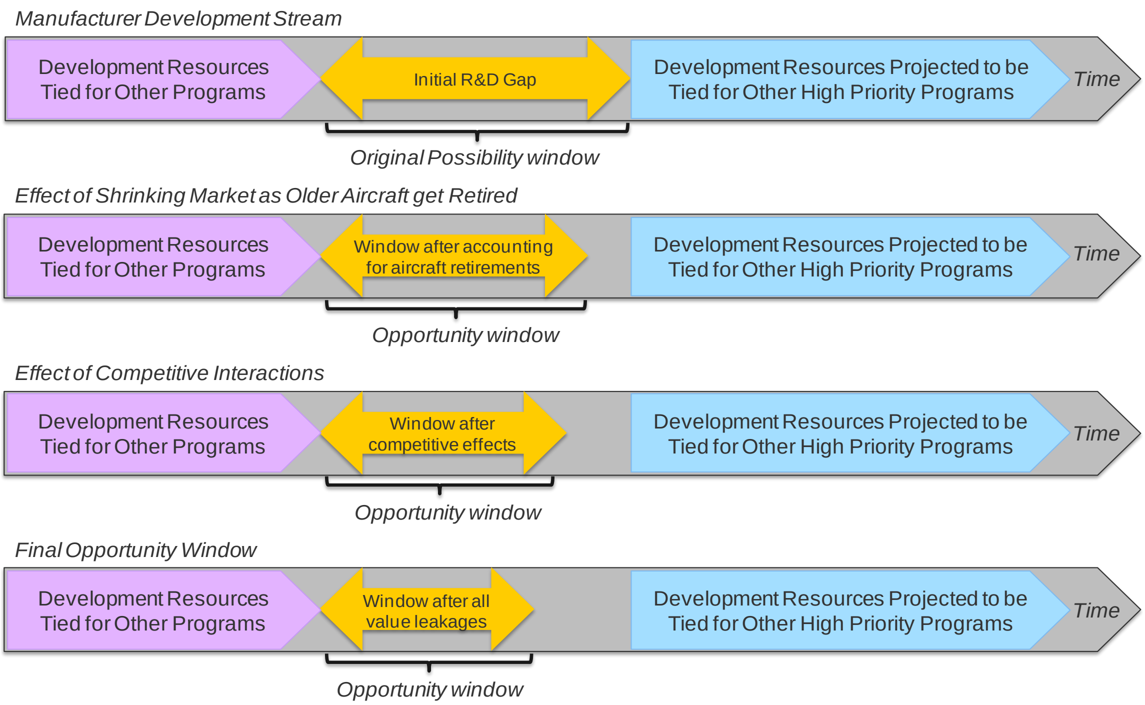

Figure 3: Value leakages and their effects on the opportunity window

2.4 Identification of decision windows

In a staggered investment, decision windows are time-windows during which a decision to fund the following phase of development must be reached. To do so, the overall window of possibility as well as the hard constraints regarding the minimum time required to perform each of the development phases are used to derive sub-windows of possibility. In the PIP development under investigation, four sub-windows of possibility indicate the time-window during which a decision to fund the initial market research, the detailed development, the certification and testing, and finally the production must be made. They are consequently referred to as decision windows. To derive these decision windows, an investigation is carried out to determine the earliest and latest times at which the decisions can be made. The process to identify the decision windows is illustrated in Figure 4.

This is done by first realizing that the detailed development phase is the most critical phase in terms of required engineering resources. Consequently, the detailed development phase (denoted by the black double arrow in the first timeline of Figure 4) cannot end after the start of the following high priority project (in blue shade in the second timeline). Similarly, the detailed development phase cannot start before the previous program is completed (in purple shade in the second timeline). This provides an estimate for the earliest and latest possible time at which the detailed development can take place. Backtracking in time enables to find out the earliest and latest possible times at which the initial market research (denoted by the blue double arrow) can take place. Forward-propagating in time allows the analyst to find out the earliest and latest possible times at which the certification and testing phase (denoted by the double green arrow) and the production phase (denoted by the purple arrow) can occur.

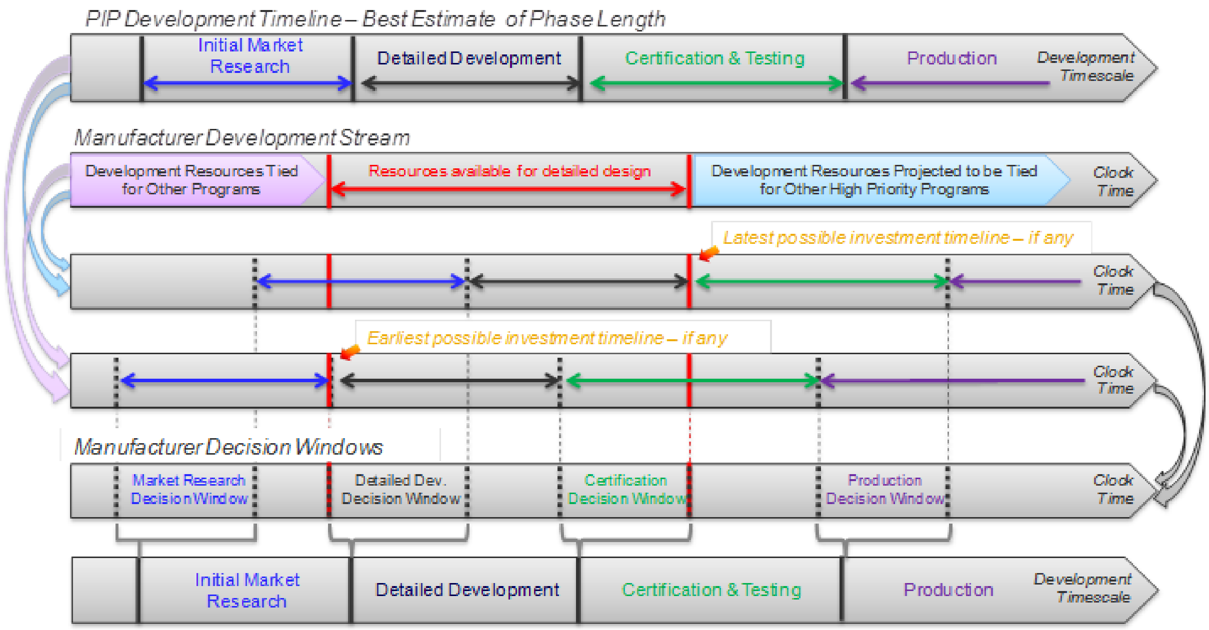

Figure 4: Identifying decision windows

The final objective of this research is to investigate the optimal conditions for the launch of the development program. This includes finding out the optimal timing of decisions and the corresponding state of uncertainties leading to a successful development program. To do so, the *baseline investment timing* is introduced as the latest time at which investment decisions can be made for all four decision windows. Any time a decision is made before this *baseline investment timing*, the decision is called an *early investment decision*. The *investment policy* is defined as the policy of timing investments optimally. In other words, it means that the investment policy maximizes the value of the performance improvement program for the company. In doing so, the investment policy determines an *early investment boundary*. The early investment boundary is the set of external conditions (time and state of uncertainties) that makes investing early optimal. If there is a single uncertainty affecting the value of the development program such as the price of jet-fuel, then the early investment boundary is a curve (price of jet-fuel versus time). If there are two uncertainties affecting the value of the development program, then the early investment boundary is a surface. Notional early investment boundaries are given in Figure 5 for each decision window pertaining to the PIP development program.

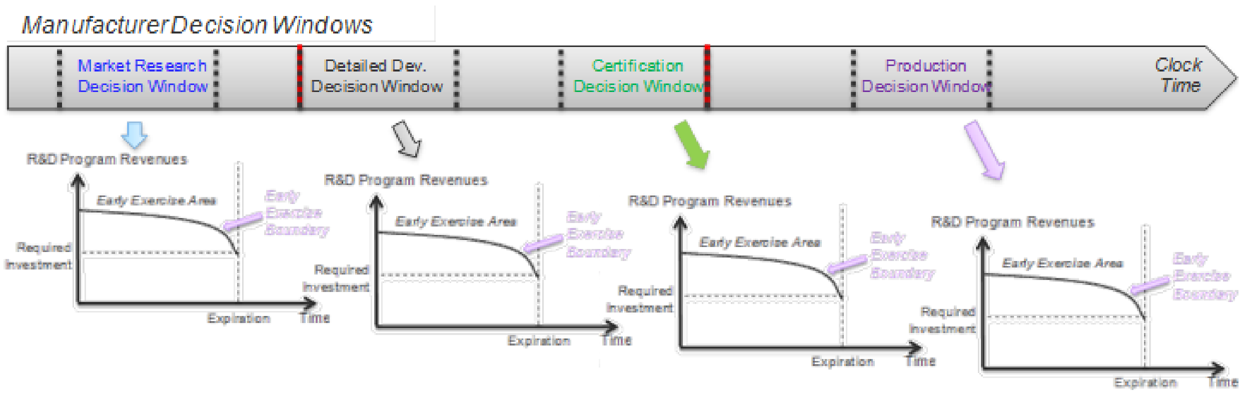

Figure 5: Early investment boundaries at each decision window

The concept of early investment boundary is interesting for decision-makers as it allows to substantiate whether acting now or delaying the decision is optimal: by comparing the current state of the business (current time and current observations of the uncertainties) to the early boundaries, decision-makers are able to identify whether they are in an invest-immediately area or whether they get more value by holding-off and waiting to get more information about the trajectories of the uncertainties. Investigating the shape of the early investment boundary in a parametric environment yields many interesting observations and may help answer the following questions:

- What is the impact of technical uncertainty on the early investment boundary?
- How does exceeding the PIP performance targets impact the early investment boundary?
- How do value leakages impact the shape of the early investment boundary?
- Which combinations of uncertainties substantially impact the shape of the early-investment boundary?
- How can these combinations be classified to yield a list of trigger-events of successful R&D programs?

3 Staggered Investment Analysis and Real-Options Analysis

In the previous section, the timeline for the development of a performance improvement package was highlighted. This development is articulated around four distinct phases, each separated by a decision tollgate. At each of these tollgates, a decision is made to further invest or abandon the development program. There is therefore flexibility offered to management to alter the course of the development program following the realization of uncertainties and the observation of the state of the business. This managerial flexibility is usually not accounted for in traditional capital budgeting methods which assume a deterministic “all or nothing” type of investment [8]. Therefore, traditional capital budgeting methods may undervalue long-term and uncertain investments [13] by not accounting for the value created by active and astute management.

3.1 Borrowing a paradigm from the finance industry

Real-options analysis provides a means of accounting for this managerial flexibility. It is an emerging field in corporate finance where it is used to substantiate capital budgeting decisions when uncertainty abounds. Its emergence at the turn of the 21st century stems mainly from two facts: the realization that a pure discounted cash flow approach does not reflect the flexibility offered to decision makers, and the recent adaptation of option valuation techniques originally developed within the financial industry to capital budgeting problems. Real-options analysis goes beyond discounted cash flow analysis because it recognizes that managers do not stand still while uncertainty unfolds, but rather actively steer projects into profitable directions. Decision makers react to changes in the business environment, abandon projects that are not economically viable, and add resources to those that are promising given the latest realization of uncertainty.

Since the analysis accounts for the abandonment of unprofitable ventures, their value may be understood to be similar to the value of a financial call option which is exercised only if the value of the underlying asset is larger than the exercise price. As such, the value of a research and development (R&D) project may be viewed as the value of the option to fund research and development. In this sense, real-options analysis is an extension of the seminal work pioneered by Black, Scholes and Merton [10][11] regarding financial options: similarly to a financial option which is the right but not the obligation to exercise a predefined action within an allocated timeframe, a real-option is the right but not the obligation to *take action*.

Take action is purposefully a vague term as it encompasses many different notions such as abandoning a research and development investment, continuing the funding of a staggered research and development investment, expanding a promising research and development investment, or finally deferring a R&D investment until the market conditions improve. This ability to better relate to what is actually happening daily within companies has been the driver for most of the research in the real-option fields. Krychowski [12] reports that the literature on real-options has increased exponentially since Myers [9] first coined the term in 1977. Moreover, real-option inspired methodologies have been used in the aerospace industry for many different applications: valuation of aircraft purchase option at *Airbus* [14], valuation of adaptability in aerospace systems [15], investment under uncertainty in air transportation infrastructure [16], and aircraft development investments at *Boeing* [17][18] and *Embraer* [19]. These real-options may have many different shapes and goals but the ones of interest in this research are development programs and more generally investments in the aerospace industry.

3.2 An interesting concept harder to implement in practice

Many of the early applications of real-options theory revolved around the transposition and subsequent use of Black-Scholes inspired formulae to value corporate investments featuring flexibility. In 1998, Luehrman [20] described a step-by-step methodology in the Harvard Business Review to value phased-investment opportunities using the Black-Scholes formula for call options. The application case was the evaluation of a growth option opportunity by a chemical company wishing to expand its production facilities. Later, Shank et al. [21] use the Black-Scholes-Merton model and the resulting call option valuation formula to estimate the value of investing in internet infrastructures to support the potentially growing e-business. However, is there is a risk for model misspecification when using the Black-Scholes formula for real-options? The Black-Scholes model and the Black-Scholes formula rely on several key-assumptions which are summarized in Table 2. Table 3 attempts to translate these assumptions for real-options use.

In a real-options environment, the assumptions related to the dynamics of the underlying asset are directly translated into assumptions related to the dynamics of the value of the underlying project. Consequently, as long as the project value follows a geometric Brownian motion as prescribed in assumption (iv) and as long as the volatility and risk-free rate are constant over time as prescribed in assumption (v), these assumptions should hold. Similarly, if the flexibility offered to management in the underlying project can be modeled as a European-type real-options, then assumption (vii) still holds. Finally, if the project does not lose some of its value over time (no value leakage due for instance to the cost to defer a decision), then assumption (iii) regarding the dividend payments also holds true. If not, a modified Black-Scholes with dividends framework may be used.

Assumptions (i), (ii) and (vi) are more difficult to translate as they relate to the ability to replicate any claim with a self-financing replicating portfolio. Indeed, the Black-Scholes model relies on the assumption that in a complete market, it is possible to replicate every claim with an arbitrary payoff using a self-financing portfolio consisting of a dynamically adjusted linear combination of the basis assets present in the market. Therefore the no-arbitrage price in a complete market can be calculated using this self-financing replicating portfolio. Assumption (i) ensures that, whatever the state of the world, the self-financing portfolio having the same payoff as the claim must have the same price. Assumption (ii) ensures that no loss occurs whenever the replicating portfolio is constructed and continuously adjusted to replicate the claim. Finally, assumption (vi) ensures that the claim is attainable, which means that it is always possible to replicate the claim using a linear combination of assets present in the market. This includes the ability to short some assets and the ability to have fractional quantity of some.

(i)	The market has no arbitrage
(ii)	The market has no fees or trading costs
(iii)	The asset does not pay any dividend
(iv)	The asset follows a Geometric Brownian Motion
(v)	Both volatility of asset and risk-free interest rate are constant
(vi)	Asset and bond may be bought in any quantity, including negative amount and fractions
(vii)	Claim can only be exercised at maturity

Table 2: Main assumptions underpinning Black-Scholes model

(i’)	*Not applicable*
(ii’)	*Not applicable*
(iii’)	The underlying project has no value leakage
(iv’)	The underlying project value follows a Geometric Brownian motion
(v’)	Volatility of underlying project value and risk-free interest rate are constant
(vi’)	*Not applicable*
(vii’)	Taking action to continue or change course can only be made at maturity

Table 3: Translating these assumptions for real-options valuation using Black-Scholes model

Some of these assumptions may be unrealistic for real-options applications because the business prospect value is not traded in any market. Therefore, there is no arbitrage-free price for the underlying project and therefore no guarantee of a single price for the replicating portfolio made up of the underlying project and some other securities. In addition, it is not obvious that the market can be complete. In fact, the market is more likely to be incomplete and the claim is most probably not attainable. This means that its payoff cannot be replicated with a self-financing portfolio made up of a combination of the basis assets in the market. Finally, even if these two assumptions were true, it is not conceptually possible to construct a replicating portfolio with no restrictions on the ability to short sell nor on the ability to take fractional positions: how to borrow and sell half of a project?

3.3 Substantiating real-options thinking: the marketed asset disclaimer

Substantiating the availability of a “twin-security” in the financial markets that can be used to perfectly replicate the value of the business prospect is difficult. There is indeed little reason to believe that the value of a corporate investment which is subject to both private and market risks would exhibit over its entire life a perfect correlation with one particular stock in each and every possible state of the world. This is a weakness facing many real-options methods since the lack of a twin-security to construct a replicating portfolio a priori precludes the use of no-arbitrage arguments for pricing purposes.

Copeland et al. [22] and Copeland and Antikarov [23] argue that in the absence of an explicit market-traded twin-security, the value of the business prospect without flexibility and therefore computed as a net present value is the best known proxy for a traded security having perfect correlation with the corporate investment value. They state that “*We can use the project itself (without flexibility) as the twin-security, and use its NPV (without flexibility) as an estimate of the price it would have if it were a security traded in the open market. After all, what has better correlation with the project than the project itself? […] We shall call this the marketed asset disclaimer.”* The *Marketed Asset Disclaimer* or *MAD* assumption is powerful: by acknowledging that a twin-security probably does not exist in the financial market and by supposing that the best unbiased surrogate for this twin-security is the *subjective* estimation of the business prospect value without flexibility, practitioners can now use this fictitious twin-security to build a replicating portfolio and therefore use the no-arbitrage argument for the economic valuation. The *MAD* assumption also implies that the net present value of the prospect is the best known unbiased estimate of the project's market value if it were a traded asset and that no-one can "arbitrage" this project valuation.

The *MAD* assumption allows practitioners to bridge a gap in the real-options analysis and to transpose a method applied for financial options valuation to corporate investments valuation. It states that when no twin-security can properly be found and used to build a replicating portfolio, then the best *subjective* surrogate is the value of the investment itself. The word *subjective* carries a lot of weight as the net present value of a corporate investment relies on assessments, many of which are subjective. For an aircraft development application, these subjective inputs may be the expected market penetration stemming from the sale of a new more efficient aircraft, the extra revenues generated by these sales, as well as the costs to develop, certify and produce the new aircraft. Borison [24] indicates that the assumption "*ensures that the 'Law of One Price' is maintained internally between the investment and the options*" but that due to the subjective nature of the valuation "*arbitrage opportunities may be available between the corporate investment and traded investments if any traded investments are available.*" In other words, the MAD assumption only ensures that the valuation is internally consistent but arbitrage opportunities may still exist if the investment valuation is biased and if some traded assets that can act as the twin-security are available. Copeland and Antikarov [25] advise analysts to rely primarily on capital markets to substantiate inputs in the prospect valuation since they believe that "*the analysis would be incomplete if it ignored information contained in available market prices."* Borison [24] echoes this statement and argues that "*if investments are evaluated using subjective, non-market assessments of these risks, the possibility of arbitrage is introduced*" and that avoiding arbitrage possibilities requires that practitioners analyze "*relevant spot, future, and option prices to determine the prices that capital markets have already established for an investment's public risks.*"

So, how can this piece of advice be implemented in practice? For the performance improvement package under review, much of the value of the package for an airline is derived from the lower fuel consumption and therefore the lower operating costs which are directly related to the uncertain price of jet-fuel (if the jet-fuel price goes up, so does the value of the performance improvement package; on the other hand, if the jet-fuel price goes down, so does the value of the package). To preclude the possibility of arbitrage, the analyst should closely examine jet-fuel futures that have already established a market price for the jet-fuel at different horizons. By using several jet-fuel prices, each corresponding to a different time horizon and each derived from the jet-fuel futures, the analyst has included as much market information as possible in the construction of the performance improvement package business case.

3.4 What about the dynamics of the underlying real assets value?

A large part of the literature on real-options assumes that the underlying real asset follows a stochastic path best described as a geometric Brownian motion. For financial stocks, the geometric Brownian motion assumption relies on the proof provided by Nobel Memorial Prize in Economic Sciences laureate Paul Samuelson [26] who argues that "*properly anticipated prices fluctuate randomly*". The model is interesting for several reasons. The first is its mathematical simplicity since it is parameterized by only two variables: a drift to account for the long-term evolution and a volatility to characterize the diffusion as shown in Eq. 1.

$$\frac{dS}{S} = \mu dt + \sigma dW \quad \textbf{Eq. 1}$$

For real-options applications, the use of geometric Brownian motion is widespread and applied to many different problems. Kemna [27] uses for instance a geometric Brownian motion to simulate the value of exploiting an off-shore oil field subject to uncertain commodity prices. Weeds [28] also assumes that the value of a technological patent evolves according to a geometric Brownian motion. Despite the widespread use, the case for using geometric Brownian motion in real-options applications is not obvious. Implicit in many applications is the fact that if the uncertainty follows a geometric Brownian motion, so does the business prospect value. This supposition is often made when dealing with prospects deriving their value from the price of an uncertain commodity (coal price, jet-fuel price…)

A closer inspection reveals that this assumption is debatable for two reasons. First, it requires that the uncertainty driving the value of the business prospect indeed follows a geometric random walk or that the geometric Brownian motion be a good enough approximation of the dynamics of these commodities. Next, it also requires that the cash-flows of the project conserve two things: the independence of the increments, as well as the Gaussian nature of the distribution of increments. In many cases, there is no reason to believe that this is true, especially for complex cash-flows that are not simple additions, subtractions or multiplications of uncertain random quantities. Borison [29] argues that "*While there may be good arguments for geometric Brownian motion with respect to equilibrium prices in highly liquid, widely accessible markets, there is no reason to believe that subjective assessments […] of the value of the underlying investment should follow a geometric Brownian motion*". This is because "*the assessed value of the underlying investments may be driven by specific events in specific time periods in a manner that looks nothing like random drift.*" Following this observation, there is a need to extend current real-options methodologies to ensure that the geometric random walk assumption can be relaxed.

3.5 Valuation or real-options using Monte Carlo simulations

In the previous paragraphs, the fundamental assumptions underpinning real-options analysis have been reviewed. The main conclusion is that there is a need for a more versatile analysis framework to handle real-options analysis. Ideally, the framework would be as generic as possible to be able to handle the wide spectrum of applications that real-options practitioners may face while retaining most of the mathematical rigor required by the models and assumptions underpinning these models. There exist many different techniques to establish the value of real-options beyond the closed-form analytical solution previously mentioned. Amongst the most popular ones are partial differential equations [30][31][32][33], lattice methods (binomial trees and trinomial trees) [34][14][35] as well as Monte Carlo simulations [36][37]. The main issue with the partial differential approach and the lattice methods is that their complexity grows significantly as the dimensionality of the problem increases. This presents a major hurdle in many applications where the business prospect value is derived from the realization of several possibly correlated uncertainties. This is in stark contrast with pricing using simulation techniques, which although computationally intensive, scale well with the number of uncertainties and handle well correlation between them.

Monte Carlo simulations originated in the 1940's with Ulam and Metropolis [38]. The approach consists in first randomly generating many numbers following a given probability distribution to perform next some deterministic computations and to finally aggregate the results. The original argument for using Monte Carlo simulations to price options is attributed to Boyle[39]. It is based on the fact that an option value can be expressed as an expectation under a new equivalent martingale probability measure. If the option value can be reduced to an expectation, then it lends itself pretty well for Monte Carlo simulations because it only requires the random generation of many prices for the underlying asset using this new equivalent probability distribution. Indeed, using the strong law of large numbers, it is known that the average of a sample converges almost surely to the expected value. For options pricing purposes, it means that by generating a sufficiently large number of underlying asset price trajectories and therefore a sufficiently large number of option payoffs, it is possible to recover the expected value of the option payoff at maturity.

Pricing options using Monte Carlo methods can be decomposed into four main steps. In the first step, the dynamics of the business prospect value are modeled with a stochastic process using both market and historical information. For options pricing purposes, the underlying asset value process must however be defined under the equivalent martingale measure also known as risk-neutral measure. This is made to ensure that the terminal option payoff can be discounted at the risk-free rate. Therefore, the second step of the analysis is to define this equivalent martingale measure and to express the dynamics of the business prospect under this synthetic probability measure. For some of the most popular stochastic processes, the mathematical expression under the risk-neutral probability measure is known and a closed-form expression can be used. Generally speaking, it requires the removal of the risk premium from the drift of the underlying stochastic process. The numerical implementation is the third step of the analysis. Many simulations are run to generate different trajectories for the value of the business prospect. This step can be implemented in a Monte Carlo simulator as shown in Figure 6 to yield a sampling of the terminal value distribution. In the fourth and final step, the real-options payoff is estimated for each and every trajectory generated during the simulations. This enables the estimation of the average payoff which is then discounted to the present time using the risk-free discount rate.

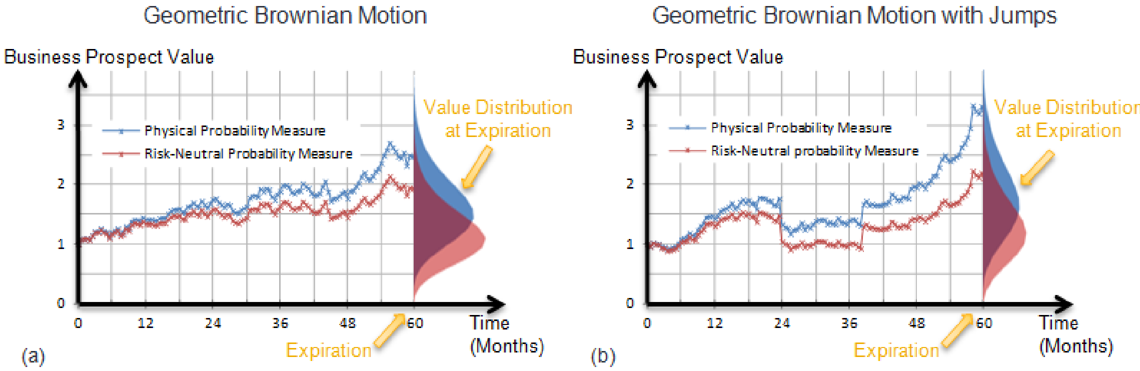

Figure 6: Simulations and resulting business prospect value distributions at expiration under physical and risk-neutral probability measures

Despite the computational flexibility offered by Monte Carlo valuation methods, few academic papers highlight their use and application for real-options valuation. This is both surprising and in stark contrast to the financial industry where Monte Carlo methods have been embraced for valuing financial options [40]. There are still many advantages to the use of Monte Carlo simulations. The first one is that they allow the simulation of complex processes which would prove almost intractable with more conventional partial differential equations and lattice methods. This is especially obvious for multi-dimensional real-options when the value of the underlying real asset is subject to several sources of uncertainties or when the real option depends on the values of several underlying real assets. In these cases, it becomes impractical to code, draw, and visualize lattices whenever the dimension exceeds two or three. The second advantage is that these dimensions may not be independent and some correlations may exist between them. Monte Carlo methods present a simple framework to capture these correlations by generating correlated paths by way of Cholesky decompositions [41]. Justin, Briceno, and Mavris [42] use Monte Carlo simulations to simulate the trajectories representing the evolution of an aircraft development program subject to two correlated uncertainties: jet-fuel price and carbon emission permit prices.

Another advantage of Monte Carlo simulations is the ability to use more complex stochastic models and still implement them with relative ease. More complex models such as those featuring a mean-reverting behavior or those featuring jumps have proven popular in recent years. Mean reverting processes have been proposed to model the price of some commodities because the forced return towards a long-term mean is better suited to account for the demand and supply forces that act when prices get away from an equilibrium level. Besides, while analyzing stock returns, Fama [43] realized that many of them where exhibiting leptokurtic distributions with heavier tails than those predicted by pure diffusive processes. He introduced the idea that jumps may be responsible for those heavy tails representing large and sudden shocks. All in all, there is little doubt that a methodology that can handle these complex processes is superior, for it can be used in more general settings. As a matter of fact, Monte Carlo inspired methodologies can easily simulate trajectories featuring mean reverting behaviors and jumps, and can therefore be useful for real-options valuation. For all these reasons, Monte Carlo simulations are used for the valuation of real-options available during the development of the performance improvement package.

4 Development Programs with Early Investment Possibility

So far, little has been said about the types of options that can be useful for real-options analyses. The most widely studied options are European options which give the option holder the right but not the obligation to undertake an investment at one pre-specified point in time. Let's pause momentarily and remember that one goal of real-options analyses is to leverage the upside potential created by the identification of trigger-events of successful program developments. Managerial flexibility represents the opportunities offered to management to react in real-time to the unfolding of an uncertain future such that decision makers can exercise their managing privileges to substantially alter the course of development programs. In particular, once a trigger-event is observed, managers do not need to wait unnecessarily to launch or abandon a development program. Therefore, European-type options with preset exercise dates may not be the most appropriate type of options to use. In fact, two types of options may be more useful for corporate investment applications: American and Bermudan options.

4.1 American and Bermudan real-options

An American option gives the holder the right but not the obligation to undertake an investment at any time prior to a pre-specified deadline. This is strikingly in line with decision-makers ability to undertake an investment whenever they feel the market is ready and the conditions are optimal for it. A Bermudan option is similar to an American option but exercising the option can only be done at several pre-specified dates up to the expiration of the option. In the context of pricing options via simulations, the time-discretization introduced for the generation of trajectories basically transforms any continuous-time American option into a Bermudan option with exercise possibilities at each time-step. As the number of time-steps in the simulation grows, the Bermudan option tends to be closer and closer to an American option, and its price converges to the price of the American counterpart. The striking similarity between American and Bermudan types of derivative contracts and the flexibility offered to management and decision makers to invest whenever conditions become optimal lead to the following assertion: practitioners could leverage some of the techniques developed for the evaluation of path dependent options to analyze corporate investments featuring timing flexibility.

4.2 Early exercise boundary

American options and their Bermudan approximations are special in that these contracts can be exercised at almost any time prior to the expiration of the option. Quoting Glasserman [41], *"the value of an American option is the value achieved by exercising optimally."* In fact, if this was not the case, arbitrageurs would actually kick-in and enforce a price that is in agreement with an optimally enforced option. Valuing this type of option is therefore equivalent to finding the optimal exercise policy and then computing the expected discounted payoff using this policy to decide whether the option is exercised early or not.

Defining the optimal exercise policy is however not a trivial affair. Indeed, the optimal exercise policy is function of several parameters and can be interpreted as a multi-dimensional surface. Heuristically, it has to be a function of the current asset price and the remaining time before expiration of the option. On the one hand, if the current price of the underlying asset takes extreme values, it might become profitable to exercise early in-the-money options so as to pocket the payoff with certainty. On the other hand, if a significant amount of time remains before expiration, it might not be worth exercising early an option that is barely in-the-money as better opportunities might arise later. Two extra parameters enter into the equation for defining the early exercise boundary. The first one is the risk-free interest rate which indicates how the option's payoff and how the underlying dividend payments will earn interests. The second one is the underlying asset volatility which indicates how likely the underlying asset is to move significantly in the future.

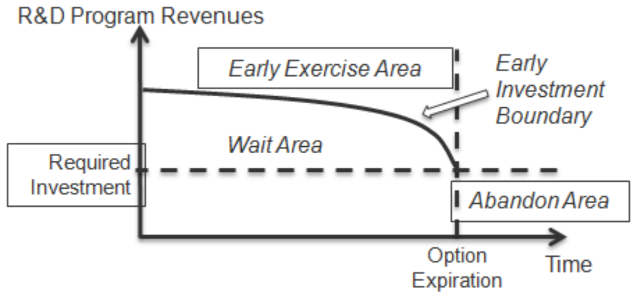

Figure 7: Early exercise boundaries for American call options with dividends

A notional early exercise boundary is given in Figure 7 for an American call option with continuous dividends. For real-options applications, modeling dividend may seem useless at first sight: after all, a real development program usually does not pay any dividend to the company. This is obviously true but dividend-like payments may be useful to model some other aspects that are very relevant in corporate finance. For instance, dividends can be used to model the cost of delays, the entrance of a new competitor in the market, or any value leakage which reduces the expected value of the development program.

5 Proposed Methodology for Real-Options with Early Investment Possibility

In the preceding sections, real-options analysis has been introduced as a means to analyze research and development programs subject to uncertainties and featuring decision tollgates. Subsequently, techniques to perform real-options valuation have been highlighted as suitable for the analysis of complex investments and some specific types of options have been identified as relevant to model managerial flexibility. In this section, the paper proposes a new methodology for the analysis of real-options. The main purpose of this methodology is to remain as generic as possible so that it can be used and adapted to many different types of investments featuring managerial flexibility. Another goal of this methodology is to use techniques widely accepted within companies so that real-options analyses may become more accessible and more accepted by practitioners in the industry.

The methodology is articulated around four main steps which are reviewed individually in the subsequent paragraphs. The first step consists in modeling the uncertainties impacting the value of the development program. This modeling is achieved using potentially correlated stochastic processes which are then simulated with Monte Carlo simulation techniques. At each time-step in the simulation, the value of the business prospect is derived using deterministic parameters as well as the state vector representing the realization of the uncertainties. In a second step, the stochastic process representing the value of the business prospect under the physical probability measure is risk-neutralized using the non-parametric Esscher transform to yield the equivalent martingale measure. In the third step of the analysis, bootstrapping is used to resample the distribution under the new martingale measure and to simulate the risk-neutral evolution of the business prospect. Finally, in the fourth and last step of the analysis, a regression-based technique is used to estimate the value of real-options with early exercise possibilities and to approximate the early-exercise boundary.

5.1 Uncertainty modeling and Monte Carlo simulation

In this step, the market uncertainties that have the most impact on the value of the business prospect are first identified and listed. They are then modeled with stochastic models using data derived from the markets so as to remove as much subjectivity as possible and therefore prevent the possibility of arbitrage in the valuation. If these uncertainties are correlated, the correlations are accounted for so a proper behavior of the uncertain quantities can be used for the valuation. Using these stochastic models, Monte Carlo simulations are performed for each of these uncertainties which leads to a state vector representing the realization of each uncertainty at each time-step in the simulation. Using a "transfer function" representative of the business prospect under review, the corresponding value for the business prospect is assessed at each time-step. This process is illustrated in Figure 8. This entire process is repeated many times to end up with a distribution of business prospect values at each time-step in the simulation.

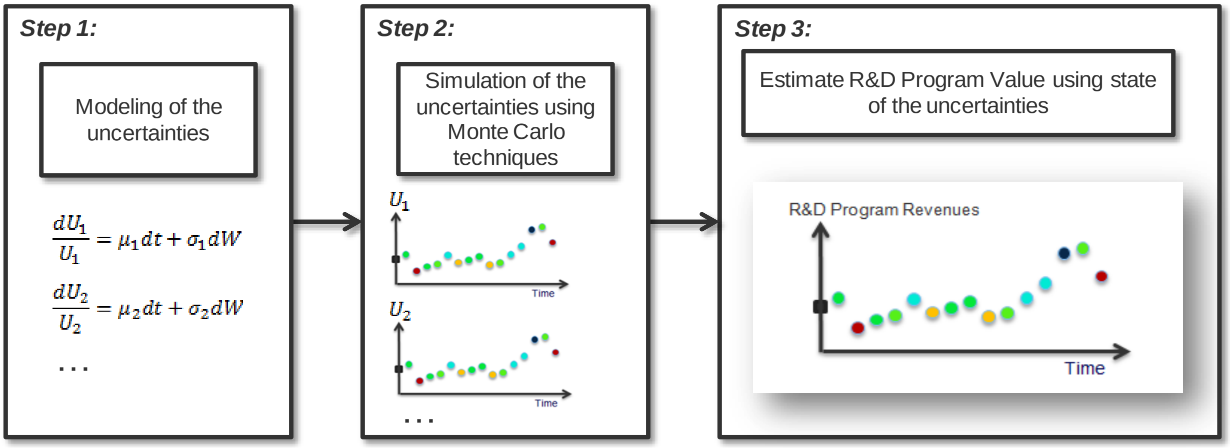

Figure 8: Monte Carlo Simulation

The evolution of the business prospect value is simulated under the physical or historical probability measure since the models used for the evolution of the uncertainties are calibrated using observations from the market. For option valuation purposes, the evolutions must nevertheless be simulated under the equivalent martingale measure or equivalent risk-neutral probability measure. A change of probability measure is therefore required.

5.2 *Risk-neutralization with Esscher transform and its non-parametric approximation*

As previously mentioned, the dynamics of the business prospect value must be specified using the risk-neutral measure. Simply said, the risk-neutral measure is a probability measure for which the returns of all assets are exactly the risk-free rate of return. Mathematically, this is equivalent to subtracting the risk-premium from the expected returns which makes investors indifferent towards risk, hence the name of the measure. A change of probability measure technique was proposed in 1994 by Gerber and Shiu [44] to handle a wide variety of processes featuring stationary and independent increments such as Wiener processes, Poisson processes, Gamma processes and inverse Gaussian processes. A transformation based on the Esscher transform [45], a time-honored tool in actuarial finance pioneered by Swedish mathematician Fredrik Esscher and later publicized by Kahn [46], is used to induce an equivalent probability measure. For a probability density function f and a real number h, the Esscher transform f_{Ess} with parameter h is expressed using the moment generating function of f as shown in Eq. 2:

$$f_{Ess}(x,h) = \frac{e^{hx} f(x)}{M(h)}, \ \ with\ h \in \mathbb{R}\ and\ M(h) = \int_{-\infty}^{\infty} e^{hx} f(x) dx \qquad \textbf{Eq. 2}$$

Looking at this definition, the Esscher transform is the product of an exponential function and a density function, normalized by a moment generating function. As a result, this transformation induces an equivalent probability measure as both distributions agree on sets with probability zero. It also becomes clear why the Esscher transform is sometimes called exponential tilting: the transformation distorts the original probability measure using an exponential function. The goal of this technique is to use the free parameter h introduced by the Esscher transform to ensure that the new probability measure is an equivalent martingale measure. Consequently, the parameter h is determined to ensure that the discounted underlying asset price is a martingale or, better said, that the price of the underlying asset is exactly its expected discounted payout.

When markets are complete, the equivalent martingale measure is unique and therefore the risk-neutral Esscher transform gives the unique arbitrage-free price for the real-options. The marketed asset disclaimer assumption presented earlier ensures that the market is complete and therefore that a unique price for the real-options can be found. On the other hand, when the market is incomplete, the claim is not attainable and there is no possibility for the market and its arbitrageurs to *enforce* a no-arbitrage price. Mathematically, there may be many equivalent martingale measures and the practitioner has to select one of them. Several equivalent measures [47] have been proposed such as the minimal martingale measure [48], the minimal entropy martingale measure[49], the utility martingale measure [49], and of course, the Esscher martingale measure. Each of them corresponds to a different attitude towards risk and consequently some assumptions regarding the preferences and risk attitude of decision makers must be set to pick which utility function and therefore which equivalent martingale measure is most appropriate. In fact, in the discussion pertaining to their paper [50], Gerber and Shiu show that the Esscher martingale measure is consistent with investors or decision-makers exhibiting power utility behaviors[3]. Power utility functions, also known as isoelastic utility functions, have the property of constant relative risk aversion which means that the risk aversion is independent of the level of initial wealth. The power utility assumption also has the advantage of being consistent with some other fundamental results of finance and economics (mutual fund theorem in Cass and Stiglitz [51] and Stiglitz [52] for instance).

A major hurdle is that the Esscher transform as introduced above requires an explicit formulation for the probability density function *f* representing the distribution of the business prospect value at a given point in time. While it may be known to the practitioner in some simple cases, most of the times analysts have little or no information as to the distribution of the business prospect value once all uncertainties are mixed in the business prospect value computation. Surprisingly, the Esscher transformation has never been used for real-options analysis to the author's knowledge. This may be due to the lack of exposure of practitioners to the technique or to the hurdle mentioned above.

Adapting the Esscher transformation technique so that it does not require the explicit formulation of the underlying stochastic process (and its associated distributions at each time-step) would prove particularly useful for real-options analysis. Fortunately, Pereira, Epprecht, and Veiga [53] propose a model-free, non-parametric approximation of the Esscher transform presented previously to transform the behavior of an underlying asset from the physical probability measure to the risk-neutral probability measure. The technique is geared towards the pricing of financial options and therefore needs to be adapted for the economic evaluation of corporate investments featuring flexibility.

The first step of the non-parametric Esscher transform starts with the collection of the business prospect values S_t . This data may have either one of two origins: it can be directly observable and available (such as the market price of the underlying asset) or it can be generated by the practitioner if the underlying asset is synthetic and not publicly traded. These values are used to estimate the continuously compounded rate of return x_t at time *t*. A Monte Carlo simulation is therefore sufficient to generate a distribution of *n* returns at each time-step. Let's now call $\widehat{X_t}$ the vector of size *n* containing these *n* rates of return sampled from the unknown probability distribution at time *t* as shown in Eq. 3:

[3] A power utility function belongs to the class of hyperbolic absolute risk aversion utility functions. It is a special case in that it exhibits a constant relative risk aversion. The power utility function relates the utility U to the level of consumption *c* using the following formula with η a constant measuring risk-aversion:

$$U(c) = \begin{cases} \frac{c^{1-\eta}-1}{1-\eta} & \eta > 0, \eta \neq 1 \\ \ln(c) & \eta = 1 \end{cases}$$

$$\widehat{X_t} = [x_t^1, x_t^2, x_t^3 \dots\ x_t^n] = \left[ln\left(\frac{S_t^1}{S_{t-1}^1}\right), ln\left(\frac{S_t^2}{S_{t-1}^2}\right), ln\left(\frac{S_t^3}{S_{t-1}^3}\right) \dots\ ln\left(\frac{S_t^n}{S_{t-1}^n}\right)\right] \qquad \textbf{Eq. 3}$$

The second step of the analysis consists in the computation of the empirical moment generating function which is estimated using Eq. 4:

$$\widehat{M_t}(h,t) = \frac{1}{n}\sum_{i=1}^{n} e^{hx_t^i} \qquad \textbf{Eq. 4}$$

The third step of the analysis is directly inspired by the work of Gerber and Shiu in that it solves for the specific value of the parameter h such that the asset price is a martingale under the new, to be constructed, probability measure induced by the Esscher transform. The parameter h^* must solves Eq. 5 and in a complete market with no arbitrage, the fundamental theorem of asset pricing [54] ensures that this solution is unique.

$$e^{r_f} = \frac{\sum_{i=1}^{n} e^{(h^*+1)x_t^i}}{\sum_{i=1}^{n} e^{h^* x_t^i}} \qquad \textbf{Eq. 5}$$

With the proper value h^* of the Esscher transform parameter, the final step consists in constructing the new probability measure. This is done by reweighting each observation and ensuring that their probabilities sum to one. The risk-neutral probability vector providing the probability of each observation is given by Eq. 6. This is the set of probabilities that is used for the pricing of options and for the computation of expectations.

$$\mathbb{Q}_t^{h^*} = \left[\frac{e^{h^* x_t^1}}{\sum_{i=1}^{n} e^{h^* x_t^i}},\ \frac{e^{h^* x_t^2}}{\sum_{i=1}^{n} e^{h^* x_t^i}} \quad \dots \quad \frac{e^{h^* x_t^n}}{\sum_{i=1}^{n} e^{h^* x_t^i}}\right] \qquad \textbf{Eq. 6}$$

In summary, the non-parametric Esscher transform enables practitioners to distort or tilt an unknown probability measure into a risk-neutral probability measure. This transformation is done on a sample of simulated observations for real-options pricing purposes and leads to a new risk-neutralized sample. This new sample can then be used to estimate the option payoffs which are finally discounted back to the present time using the risk-free interest rate. All in all, the non-parametric Esscher transform tremendously simplifies the analyses of practitioners who no longer need to estimate, calibrate, and substantiate the choice of one particular stochastic process for the evolution of the business prospect value provided some mild conditions of stationary and independent increments are satisfied. The algorithm to risk-neutralize a sample using the non-parametric Esscher transform is depicted in Figure 9.

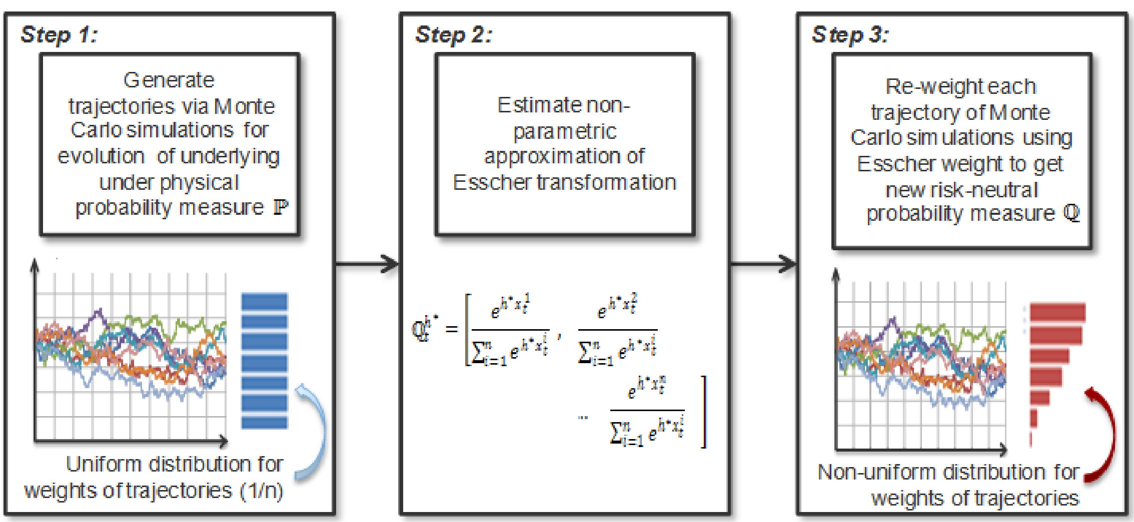

Figure 9: Non-parametric Esscher transform for change of probability measure

5.3 *Resampling using the "Bootstrapping" technique*

The non-parametric Esscher transform enables the change of probability measure and the derivation of a new equivalent risk-neutral measure. By doing so, the technique changes the mean of the terminal distribution of the business prospect value by reweighting the different outcomes. The procedure is however acting only on the terminal distribution of the business prospect value, so what about the distributions at intermediate time-steps? As much as the procedure is sufficient for valuing European-type options whose values depend only on the distribution at expiration, valuing an American or a Bermudan option requires the knowledge of the business prospect value at each and every intermediate step under the risk-neutral measure. With this issue in mind, we propose a way to proceed forward using a resampling technique: resampling consists in drawing with replacement from a sample to generate a new sample. For real-options use, resampling consists in using the previously risk-neutralized terminal distribution of returns to generate new trajectories which are risk-neutral by construction. This technique is quite popular in statistics and finance where it is called bootstrapping.

Bootstrapping is a statistical method whose name was first coined by Efron in his 1979 Rietz Lecture [55] to describe a resampling technique used to estimate the precision of some statistics such as the mean, median, or standard deviation of a distribution. In this case, bootstrap samples are constructed by sampling with replacement a subset of an original distribution. The statistics of interest are then computed for each bootstrap sample and the variability between the results can be analyzed to derive some confidence intervals for the statistics. For the problem under investigation, the essence of the bootstrap method is retained but the application is totally different: similarly to the original application, the bootstrap method is used to sample with replacement from an original distribution but what is new is that the bootstrap sample is used next to generate business prospect value trajectories. In other words, the distribution of asset prices under the historical probability measure is first risk-neutralized using the non-parametric Esscher transform yielding a new re-weighted probability distribution. In turn, this risk-neutral distribution under the new probability measure is sampled with replacement to yield bootstrap subsamples which are used to construct trajectories under the risk-neutral probability measure. Another advantage of the bootstrap technique is that the newly generated trajectories will all carry the exact same weight. The method is illustrated in Figure 10.

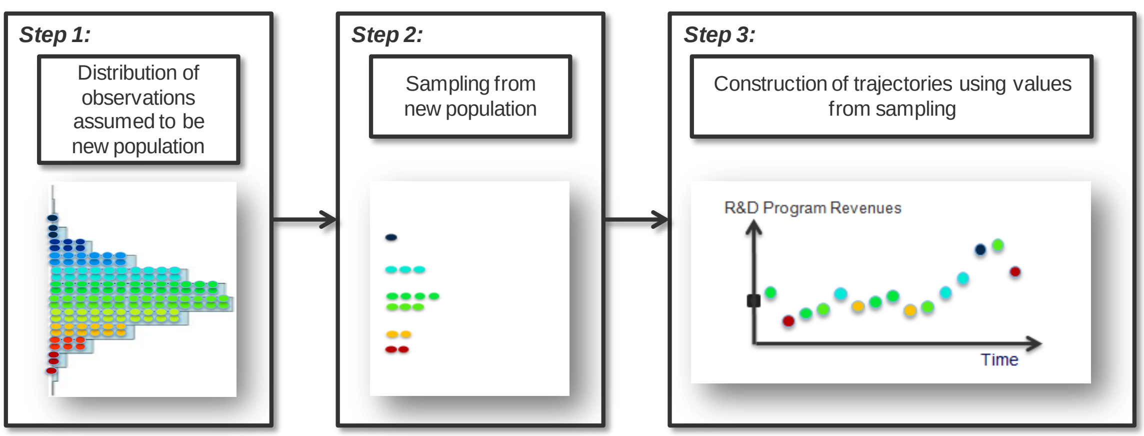

Figure 10: Bootstrap method to generate trajectories

For the pricing of options, special care needs to be paid when bootstrapping: indeed, instead of simply generating distributions, the bootstrapping is applied to simulate the realization of a stochastic process. In fact, bootstrapping will no longer generate distributions but rather trajectories or time-indexed distributions. If the original process to be simulated has some serial correlation properties, these would need to be accounted for in the bootstrapping method since a naïve bootstrapping does not induce any serial correlation. In the following work, the simplifying assumption of lack of serial correlation is imposed.

Provided that no serial correlation exists, another update to the method is required for the purpose of risk-neutral trajectory generation. Indeed, bootstrapping usually starts with a sample of observations for which each individual observation carries the exact same weight. In other words, the original sample is "uniformly distributed" with each outcome carrying a probability of one over the sample size (1/n). However, in the current application the outcomes of the simulation have been reweighted during the risk-neutralization process and each outcome has a specific weight (or probability). As a result, the individual weights (or probabilities) associated with each outcome have to be accounted for when sampling to ensure that the risk-neutral property is preserved and carried over to the trajectories to be generated. An algorithm is proposed in Figure 11 to sample while preserving the risk-neutral property. It consists in figuratively stacking all the weights in a column. The "height of the column" should be exactly one since the weights represent a probability measure. A random number is then drawn to select which "height" in the column is reached and therefore which piece of the stack is selected. By doing so, outcomes with larger weights have a greater chance of being drawn, while outcomes with a smaller weight have less chance of being drawn during the resampling effort.

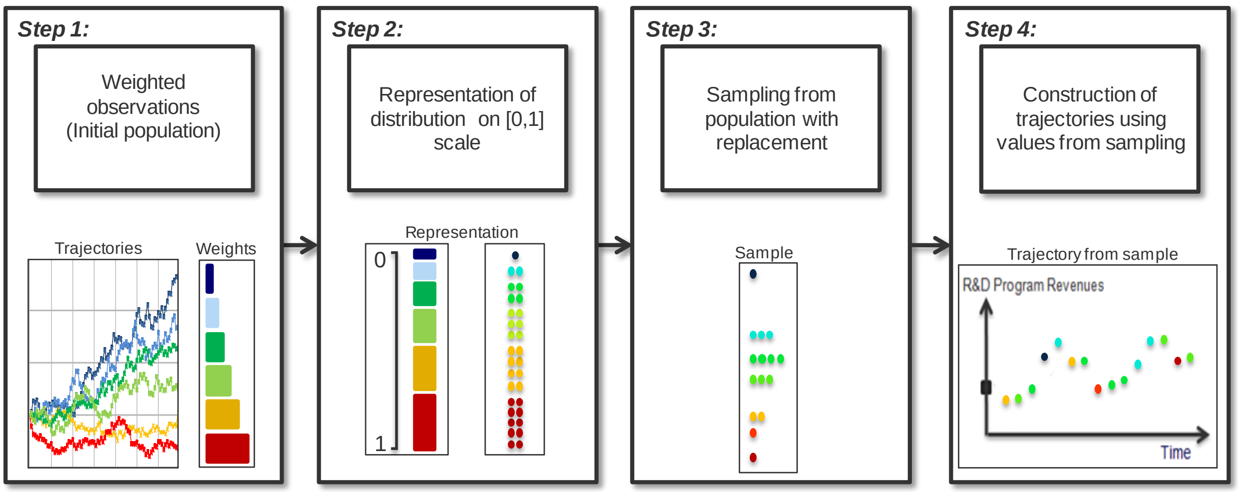

Figure 11: Resampling from weighted observations by first stacking probabilities and then drawing randomly from the stack (mapping between position in the stack and original outcome value is known)

5.4 *American option valuation using Least-Squares Monte-Carlo technique*

For real-options applications, Monte Carlo simulations enable the capture of a multitude of uncertainties and their interdependencies. However, pricing real-options using Monte Carlo simulations has long been hindered by the perceived inability of simulation techniques to correctly handle path-dependant options [56]. The main reason for this difficulty is that simulations will yield an estimate of the option value at a single point defined by the current time and the current business prospect value. The technique does not yield information regarding the option value at future times and for different business prospect values. This is problematic. How then to ensure that the early-exercise policy is not violated? In other words, when moving along a simulated trajectory, one needs to ensure that the optimal early-exercise policy is followed. This means that while marching forward in time, one has to compare the value of holding the option for at least one extra step to the payoff earned by an immediate exercise. Mathematically, the value of the American option at the kth time-step t_k denoted V_{t_k} on an asset S with observed value S_{t_k} and with payoff function P can be expressed as the maximum between exercising immediately and holding the option as shown in Eq. 7. The issue is that there is yet no estimate of the present value of the one-period-ahead option value $V_{t_{k+1}}$.

$$V_{t_k} = max\left[P\left(S_{t_k}\right),\ e^{-r_f(t_{k+1}-t_k)}E_{\mathbb{Q}}\left(V_{t_{k+1}}|S_{t_k}\right)\right] \quad \textbf{Eq. 7}$$

Fortunately, this paradigm has evolved starting in 1993 with the paper of Tilley [57] which aims was to dispel the belief that American-style options could not be valued using simulations. A significant improvement came in 1996 with the work of Carriere [58] regarding the valuation of options with early-exercise properties. Faced with the same problem of estimating the one-period-ahead option value for subsequent comparison with the immediate exercise payoff, Carriere suggests the use of non-parametric regressions to regress the conditional expectation and therefore to estimate the value of holding the options. As noted by Stentoft[59], the reason for this regression is that a conditional expectation is a function and "*any function belonging to a separable Hilbert space may be represented as a countable linear combination of basis-functions for the space*." Consequently, let's introduce $\{\phi_i\}_1^\infty$ as a family of basis-functions for that space. The expectation may be rewritten and approximated using the first M basis-functions $\{\phi_i\}_{i=1}^M$ as shown in Eq. 8:

$$E_{\mathbb{Q}}\left(V_{t_{k+1}}|S_{t_k}\right) = \sum_{i=1}^{\infty} \alpha_i(t_k) \cdot \phi_i\left(S_{t_k}\right) \sim \sum_{i=1}^{M} \alpha_i(t_k) \cdot \phi_i\left(S_{t_k}\right) \quad \textbf{Eq. 8}$$

Any family of basis-functions should work but Carriere suggests using either splines or a polynomial smoother. Next task is the estimation of the coefficients α_i of the linear combination. This is done marching backward, starting at expiration and moving back time-step by time-step until the present time: at expiration, the value of the option is exactly the payoff while for all preceding time-steps denoted t_k, a regression is performed using the observations of the asset price for the N simulated trajectories at that time t_k denoted by S_{t_k} as well as the option value $V_{t_{k+1}}$ at the following time-step t_{k+1}. The regression objective is to select a family of coefficients $\{\alpha_i\}_1^M$ that minimizes the error between the regressed conditional expectations and the option value across all simulated trajectories. This error is defined in Eq. 9:

$$\min_{\{\alpha_i\}_0^M} \sum_{n=1}^{N} \left(\sum_{i=1}^{M} \alpha_i(t_k) \cdot \phi_i\left(S_{t_k}^n\right) - V_{t_{k+1}}^n\right)^2 \quad \textbf{Eq. 9}$$

The immediate exercise value at time t_k denoted $P\left(S_{t_k}\right)$ is then compared to the discounted regressed conditional expectation to find the option value, defined by Eq. 10. The procedure is repeated for each time-step and for each trajectory until the present time to find the value of the American option.

$$V_{t_k} = max\left[P\left(S_{t_k}\right),\ e^{-r_f(t_{k+1}-t_k)} \sum_{i=1}^{M} \alpha_i(t_k) \cdot \phi_i\left(S_{t_k}\right)\right] \quad \textbf{Eq. 10}$$

The algorithm for American option valuation using simulation and regression techniques is depicted in Figure 12. A popular enhancement to this work is the Least-Squares Monte Carlo approach of Longstaff and Schwartz [60]. Dating back to 2001, this approach is very similar to the method of Carriere except for two facts: the algorithm uses a least-squares regression and the regression is made using only in-the-money paths.

In the Longstaff-Schwartz method, the proposed regression uses an ordinary least-squares technique to regress the conditional expectation $E_{\mathbb{Q}}\left(V_{t_{k+1}}|S_{t_k}\right)$ against a set of explanatory variables. The set of explanatory variables is a family of basis-function $\{\phi_i\}_1^M$ valued using the conditioning underlying asset price S_{t_k}. One may use the simple monomial family $\{\phi_i: X \rightarrow X^{i-1}\}_{i=1}^M$ as the family of basis-functions. Furthermore, the regression is performed using only paths that are in-the-money since the decision to exercise or not the option is only relevant whenever the option is in-the-money. According to Longstaff and Schwartz, *"by focusing on the in-the-money paths, [… they…] limit the region over which the conditional expectation must be estimated, and far fewer basis functions are needed to obtain an accurate approximation to the conditional expectation function."*

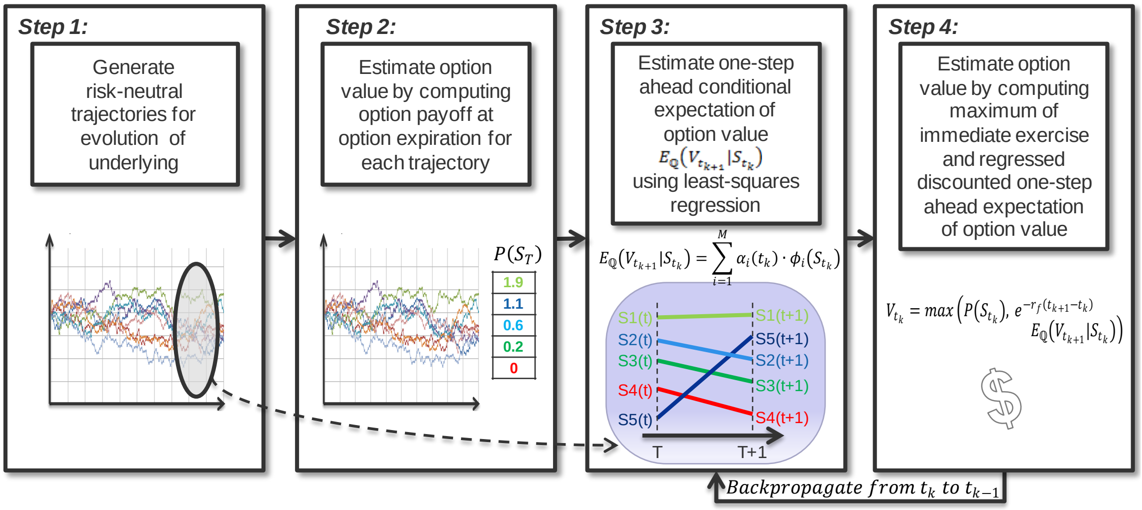

Figure 12: American option valuation with regression

A subtle difference with the works of Carriere is the choice of realized payoffs as dependent variables for the regression instead of using previously computed conditional expectations. These realized payoffs may be resulting from an early exercise at the subsequent time-step t_{k+1} or from an early exercise several steps down-the trajectory, for instance at t_{k+j}. According to the authors, this precludes "*an upward bias in the value of the option*". In other words, the conditional expectation at time t_k denoted by $E_{\mathbb{Q}}\left(V_{t_{k+1}}|S_{t_k}\right)$ is used only once in the entire algorithm: to check whether the value of holding the option is greater than the value of immediate exercise. For all other purposes, such as the estimation of the option value at time t_k denoted as V_{t_k} or the regression of the conditional expectation at time t_{k-1} denoted by $E_{\mathbb{Q}}\left(V_{t_k}|S_{t_{k-1}}\right)$, the conditional expectation at time t_k is not used. This leads to the following exercise rule and option value in Eq. 11. Let's notice the subtle difference with Eq. 10 in what the option value really is (the exercise rule remains the same).

$$V_{t_k} = \begin{cases} P\left(S_{t_k}\right) & , if\ P\left(S_{t_k}\right) \geq\ e^{-r_f(t_{k+1}-t_k)} \sum_{i=1}^{M} \alpha_i(t_k) \cdot \phi_i\left(S_{t_k}\right) \\ e^{-r_f(t_{k+1}-t_k)} \cdot V_{t_{k+1}} & , if\ P\left(S_{t_k}\right) <\ e^{-r_f(t_{k+1}-t_k)} \sum_{i=1}^{M} \alpha_i(t_k) \cdot \phi_i\left(S_{t_k}\right) \end{cases} \qquad \textbf{Eq. 11}$$

5.5 *Summary of the proposed methodology*

Having described extensively the four main steps of the proposed methodology to value real-options with early exercise privileges, the diagram in Figure 13 summarizes the different techniques, highlights the order in which they are used, and finally depicts the flow of information between the various steps. The proposed methodology may seem daunting at first but each of these steps uses techniques widely used and accepted within companies.

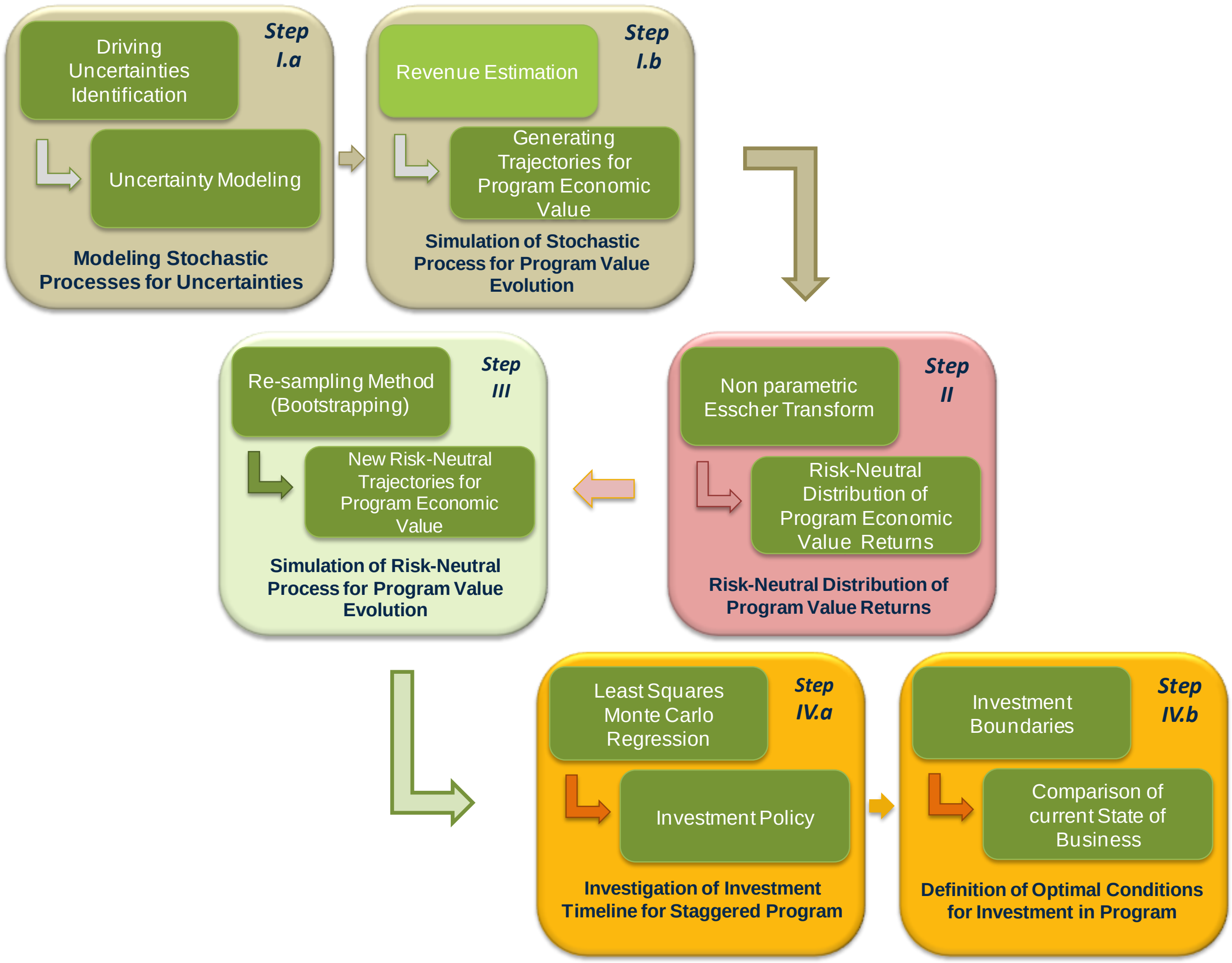

Figure 13: Main steps of proposed methodology

5.6 Meeting the Georgetown Challenge?

In the previous sections, a methodology has been constructed step by step to analyze long-term staggered corporate investments featuring flexibility. Now would be a good time to revisit the key challenges identified earlier in this paper and to verify whether the proposed way-forward would meet the requirements set forth by the *Georgetown Challenge* (Copeland and Antikarov [25]). The *Georgetown Challenge* consists in a series of requirements that real-options analyses have to meet in order to get traction and wide acceptance amongst practitioners in the industry. These set of requirements were agreed upon during a real-options symposium held on the campus of Georgetown University in 2003. Table 4 on the following page maps the requirements of the *Georgetown Challenges* and the specific challenges identified as part of this research to the assumptions, techniques, and solutions shaping the proposed methodology.

Table 4: Addressing the challenges facing the analysis of long-term corporate investment programs featuring flexibility

		Monte Carlo-based and non-parametric Esscher-transformed real-options approach
Georgetown Challenge Requirements (Adapted from Copeland and Antikarov [25])	Intuitively dominates other decision-making methods	• Ability to capture the flexibility in decision making • Recognize the value created by active and astute management
	Capture the reality of the problem	• Ability to handle optimum timing issues related to decision-making using American-type options • Ability to handle staggered investment programs with decision gates using compound options
	Use mathematics that everyone can understand	• Esscher transform ensures that risk-neutralization is performed in a transparent and tractable way • Non-parametric Esscher transform removes the requirement to calibrate complex models
	Rule out the possibility of mispricing by eliminating arbitrage	• Esscher transform provides the price that would be enforced by arbitrageurs in a complete market • Esscher transform provides the price corresponding to the preference of economic agents with iso-elastic utility functions in the case of incomplete markets
	Be empirically testable	• Tough requirements as there are no published transacted price for these investments • Only heuristic argumentation can substantiate whether the method provides acceptable solutions
	Appropriately incorporate risk	• Handling of technical and market risks separately, with technical risk analyzed with decision trees • Possibly difficult to estimate volatilities of some particular risks if no prior history exists
	Use as much market information as possible	• Ability to use market information whenever possible to model the dynamics of the uncertainties driving the development program value
Additional requirements	Ability to capture a complex reality with intertwined uncertainties	• Monte Carlo simulations allow the use of many different stochastic behaviors for uncertainties • Monte Carlo simulations allow the modeling of correlations between some sources of uncertainties
	Ability to visualize uncertainties and the decision process	• Visualization of the evolution of uncertainties affecting the decision process • Visualization of the evolution of the development program value over time
	Ability to handle corporate investments featuring exotic options	• Recent Monte Carlo methods allow analyses of programs with potentially moving decision tollgates and therefore the search for optimum investment timeframes
	Ability to converge to a solution in a timely manner	• Use of bootstrapping methods allow a reduction in computation time to generate trajectories of program values used for Monte Carlo simulations

6 Verification and Validation

In this section, the proposed methodology is validated for different test cases against known results from plain vanilla options. Since the "pricing" part of the proposed methodology is validated, there is no need to set-up a real-options scenario and instead the validation is performed by comparing the results of the proposed methodology to known results for financial options. The test cases involve different sets of put and call options of both European and American types.

6.1 European options

In order to test the methodology using European options, the last step using the Least-Squares Monte Carlo is changed. Indeed, since there is no need to compare the one-step ahead conditional expectation to the value of exercising early, the regression-based Least-Squares Monte Carlo technique is removed and replaced with a simpler payoff estimation at maturity of the option. The results are displayed in Table 5 for an underlying following a geometric random walk with a drift of 8.5% under the physical probability measure. The results from the proposed methodology are based on a simulation of 50,000 paths and are in agreement with the closed-form solution.

Table 5: Comparison of proposed valuation and closed-form formula for European call (a) and put (b) options

(a) European Call Option (Drift=8.50%, Rf=5.00%, Vol=10%)				
Maturity	Strike/Spot	Black-Scholes	Simulation & NP Esscher	Error
0.5	0.8	0.21976	0.21853	0.56%
0.5	0.9	0.12307	0.12299	0.06%
0.5	0.95	0.07832	0.07793	0.50%
0.5	1	0.04192	0.04188	0.10%
0.5	1.05	0.01811	0.01807	0.19%
0.5	1.1	0.00617	0.00594	3.66%
0.5	1.2	0.00035	0.00032	8.06%
1	0.8	0.23910	0.23864	0.19%
1	0.9	0.14629	0.14803	-1.19%
1	0.95	0.10405	0.10338	0.65%
1	1	0.06805	0.06806	-0.02%
1	1.05	0.04046	0.04086	-0.99%
1	1.1	0.02174	0.02164	0.46%
1	1.2	0.00462	0.00424	8.32%
2	0.8	0.27659	0.27839	-0.65%
2	0.9	0.18981	0.19022	-0.22%
2	0.95	0.14994	0.15069	-0.50%
2	1	0.11413	0.11347	0.57%
2	1.05	0.08348	0.08354	-0.07%
2	1.1	0.05861	0.06796	-15.95%
2	1.2	0.02555	0.02570	-0.57%

(b) European Put Option (Drift=8.50%, Rf=5.00%, Vol=10%)				
Maturity	Strike/Spot	Black-Scholes	Simulation & NP Esscher	Error
0.5	0.8	0.00000	0.00000	/
0.5	0.9	0.00085	0.00078	7.85%
0.5	0.95	0.00487	0.00471	3.21%
0.5	1	0.01723	0.01700	1.35%
0.5	1.05	0.04218	0.04222	-0.09%
0.5	1.1	0.07901	0.07957	-0.71%
0.5	1.2	0.17072	0.16913	0.93%
1	0.8	0.00008	0.00008	3.98%
1	0.9	0.00239	0.00247	-3.14%
1	0.95	0.00772	0.00779	-0.90%
1	1	0.01928	0.01937	-0.47%
1	1.05	0.03925	0.03961	-0.91%
1	1.1	0.06809	0.06929	-1.76%
1	1.2	0.14610	0.14655	-0.31%
2	0.8	0.00046	0.00048	-4.48%
2	0.9	0.00416	0.00420	-0.95%
2	0.95	0.00953	0.00953	0.02%
2	1	0.01896	0.01969	-3.83%
2	1.05	0.03356	0.03406	-1.48%
2	1.1	0.05393	0.05340	0.99%
2	1.2	0.11136	0.11060	0.68%

6.2 American options

To test the methodology using American options, the entire methodology is now used. The results are displayed in Table 6. They exhibit on average good results when compared to results obtained from the partial-differential equation approach. There are however some discrepancies, in particular for out-of-the-money options, between the two valuation methods. These discrepancies are probably due to errors introduced by the least-squares regression. Further investigations on the selection of basis-functions and on the number of functions to use might help achieve better results.

Table 6: Comparison of proposed valuation and PDE solution for American call (a) and put (b) options

(a) American Call Option (Drift=8.5%,Div= 4.0%, Rf=5.0%, Vol=10%)				
Maturity	Strike/Spot	PDE Approach	Simulation & NP Esscher	Error
0.5	0.8	0.20490	0.20583	-0.45%
0.5	0.9	0.10870	0.1084	0.28%
0.5	1	0.03260	0.03449	-5.80%
0.5	1.1	0.02800	0.00398	85.77%
0.5	1.2	0.00010	0.00022	-120.00%
1	0.8	0.20960	0.21590	-3.01%
1	0.9	0.11890	0.11944	-0.45%
1	1	0.04840	0.04876	-0.74%
1	1.1	0.01330	0.01325	0.38%
1	1.2	0.00210	0.00225	-7.14%
2	0.8	0.21930	0.22114	-0.84%
2	0.9	0.13700	0.14198	-3.64%
2	1	0.07210	0.08057	-11.74%
2	1.1	0.03180	0.03155	0.79%
2	1.2	0.01180	0.01295	-9.75%

(b) American Put Option (Drift=8.50%, Div=4.0%, Rf=5.00%, Vol=10%)				
Maturity	Strike/Spot	PDE Approach	Simulation & NP Esscher	Error
0.5	0.8	0.00000	0.00001	/
0.5	0.9	0.00140	0.00112	-20.00%
0.5	1	0.02390	0.02405	0.63%
0.5	1.1	0.10000	0.10007	0.07%
0.5	1.2	0.20000	0.19980	-0.10%
1	0.8	0.00020	0.00018	-11.35%
1	0.9	0.00480	0.00460	-4.10%
1	1	0.03150	0.03172	0.70%
1	1.1	0.10090	0.10062	-0.28%
1	1.2	0.20000	0.19973	-0.14%
2	0.8	0.00150	0.00147	-2.00%
2	0.9	0.01050	0.01034	-1.52%
2	1	0.04040	0.03784	-6.34%
2	1.1	0.10360	0.10308	-0.50%
2	1.2	0.20000	0.19923	-0.39%

7 Performance Improvement Package Evaluation

The proposed methodology is applied to the evaluation of the performance improvement package presented previously in this paper. However, the development of the performance improvement package is simplified for this validation so as not to over-complicate the problem with unnecessary burden. Instead of studying the four successive nested real-options, the paper will focus on a single option which is the option to fund the detailed development phase once more information has been obtained after the initial market research. This helps avoids having to perform time-consuming simulations on simulations to evaluate nested options.

7.1 Value of the development of the Performance Improvement Package (PIP)

The computation of the development program value is articulated around two steps. In the first step, an estimate of the sales given the current state of the uncertainties is estimated. This is done using a market model described in Justin, Briceno, and Mavris [42]. This market model estimates the market penetration by first computing the net present value of the aircraft featuring the performance improvement package, and by comparing this value to the value of the same aircraft without the PIP and to the value of other competing aircraft. This evaluation is performed for different missions representing different types of airlines and therefore different market segments. Decision choice analysis is then applied to estimate market preferences on these different segments. In the second step of the analysis, the market preference is used to estimate the revenues and profits stemming from the sales of the Performance Improvement Package to the airlines. This enables the evaluation of the development program value.

Table 7: Market Assumption for Performance Improvement Package

Overall market size	100	Units/Year
Length of PIP program	8	Years
Market value leakages	-15%	per year
Revenues per unit (Normalized)	0.045	2014-US$
Cost per unit (Normalized)	0.040	2014-US$
WACC (Typical Airframer)	13.5%	per year

7.2 *Uncertainty quantification and modeling*

The development of complex aerospace systems-of-systems and the extensive certification processes lead to long development timelines which exacerbate the effects of uncertainty. Uncertainty might be split into two categories. One category is for technical uncertainty over which manufacturers have some control (such as performance estimates and development schedules). Manufacturers may be able to limit the adverse effects of technical uncertainty by implementing mostly mature technologies in their designs or by using conservative estimates, and possibly by using probabilistic techniques during design. The other category is for market uncertainty which manufacturer do not control (such as market size and commodity prices). Owing to this lack of control, aircraft manufacturers must come up with solutions to hedge against these types of uncertainties to ensure that their decision-making process is optimal and robust regardless of the evolution of the underlying uncertain parameters.

Numerous market uncertainties affect manufacturers but only a few have profound effects on the viability of large aerospace development programs. One of them is the price of jet-fuel which drives the need for new more efficient aircraft to replace older ones since fuel-related expenditures account for about forty percent of aircraft direct operating costs. Consequently, increasing fuel prices have dramatic effects on the profitability of airlines and put pressure on airlines to renew their fleets. Another uncertainty which may have some impact in the future is the taxation of carbon dioxide emissions. Little information regarding the effects of such regulations on aircraft manufacturers is available due to the newness of the taxation scheme. Indeed, the European Union has recently set-up the Emissions Trading Scheme (ETS) whereby airlines may have to buy permits for roughly fifteen percent of their carbon dioxide emissions.

7.2.1 Uncertain jet-fuel costs

The jet-fuel price analysis is performed using data from the United States Energy Information Administration representing the historical time series of U.S. Gulf Coast kerosene-type jet-fuel spot price. The time series is plotted in Figure 14 and looks similar to many financial time series with high volatility and no obvious autocorrelation structure.

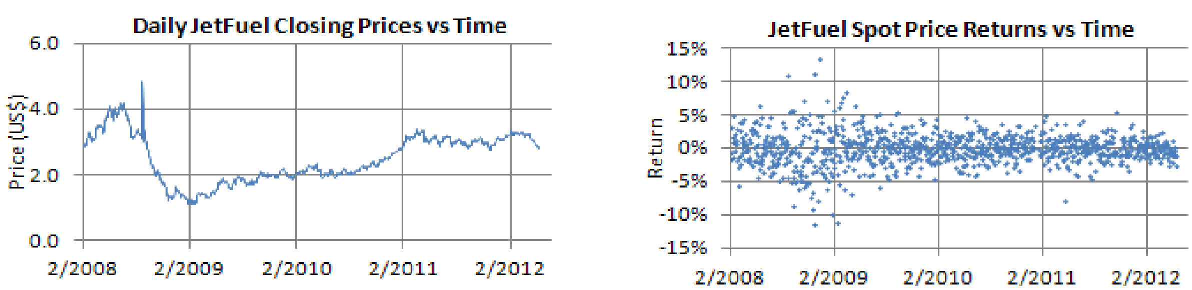

Figure 14: (a) Closing price of jet-fuel; (b) Continuously compounded daily jet-fuel price returns

Inspection of the time series in Figure 14 indicates a bell-shaped distribution of the returns centered on zero with some clustering of high volatility as shown in Figure 15. Despite this heteroscedasticity, a stochastic model similar to a random walk, the Geometric Brownian Motion (GBM) is hypothesized.

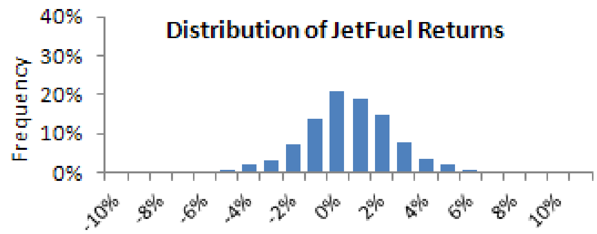

Figure 15: Distribution of daily jet-fuel price returns

Several statistical tests are run to check whether this assumption can be rejected at the usual 5% level of significance. The first test is the variance ratio test as described by Campbell et al.[61]. This test checks the correlation structure of the increments. Under the GBM assumptions, the increments are uncorrelated. The autocorrelation is studied at lags 2, 4, 8 and 16 days, and for each analysis, the test fails to reject the GBM assumption. The second test is the Cowles-Jones Ratio test described again in [61] which checks whether the increments are independent and identically distributed. Under the GBM assumption, the increments are independent and identically distributed. This test also fails to reject the GBM assumption which means that the apparent heteroscedasticity previously observed is not significant enough. Based on these results, a geometric Brownian motion is used to model the stochastic process driving the price of jet-fuel. The stochastic differential equation is given in Eq. 12 with the Wiener process (Wt), the spot price (St), the yearly drift (μ) and the yearly volatility (σ).

$$dS_t = \mu \cdot S_t + \sigma \cdot S_t \cdot dW_t$$
$$S_t = 2.75US\$\, ;\ \mu = 0.005\%\, ;\ \sigma = 43.9\%$$
Eq. 12

7.2.2 Uncertain emission costs

The Emissions Trading Scheme may require airlines to buy permits for about fifteen percent of the airlines' carbon dioxide emissions. These permits are in limited quantity and may be purchased on the carbon market in the form of European Union Allowances (EUA). For instance, *Air France* started using the *BlueNext* exchange platform in 2012 to buy EUAs on the spot market [62]. The carbon emission analysis is therefore performed using the BNS EUA 08-12 time series available on the *Bluenext* exchange website for data from February 2008 to June 2012. Like the previous example, the time series plotted in Figure 16 exhibits high volatility, with no obvious autocorrelation structure but with a downward trend.

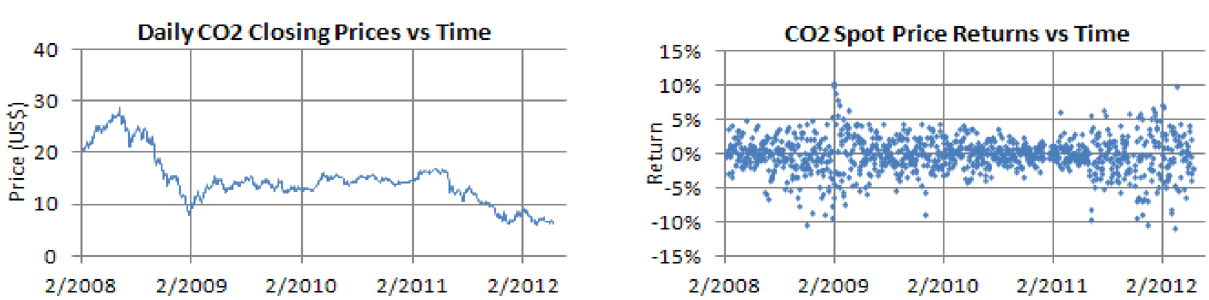

Figure 16: (a) Closing price of EUA; (b) Continuously compounded daily EUA price returns

Inspection of the time series displayed in Figure 16 indicates a bell-shaped distribution of the returns centered on zero with some clustering of high volatility, as shown in Figure 17. Based on these observations, a geometric Brownian motion is hypothesized.

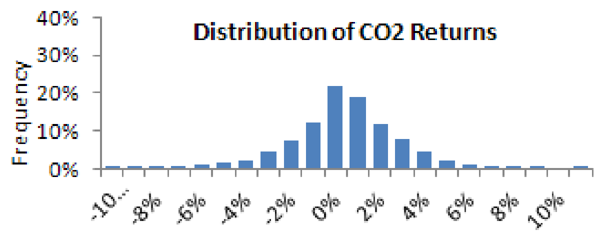

Figure 17: Distribution of daily EUA price returns

The same statistical tests are run to check whether the GBM assumption can be rejected at the 5% level of significance. The variance ratio test is run for lags 2, 4, 8 and 16 days, and each time, the GBM assumption cannot be rejected. The Cowles-Jones Ratio test is run and also fails to reject the GBM assumption. Based on these results, a geometric Brownian motion is used to model the stochastic process driving the price of carbon allowances and its parameters are provided in Eq. 13.

$$dS_t = \mu \cdot S_t + \sigma \cdot S_t \cdot dW_t$$
$$S_t = 6.26US\$ \; ; \; \mu = -28.8\% \; ; \; \sigma = 43.5\%$$

Eq. 13

7.2.3 Treatment of correlations

The two stochastic processes retained for the modeling of jet-fuel price and carbon emission cost uncertainties are independent models. However, a more intricate relationship between the two models is likely. Indeed, a period of strong growth in Europe may result in higher demand for air transportation and therefore higher prices for jet-fuel. Similarly, this higher demand for air transportation may result in more demand for carbon permits and therefore higher emission allowance prices. The relationship between the price of jet-fuel and the price of carbon permits can be captured with the correlation matrix. This matrix is estimated by first cleaning the time series to ensure that quotes are available for both on the same date and then estimating the correlation between the continuous returns of each time series. The correlation matrix is given in Eq. 14 and indicates a correlation of 19% between the two data series.

$$M_{Cor} = \begin{bmatrix} 1 & 0.199 \\ 0.199 & 1 \end{bmatrix}$$

Eq. 14

To include this correlation in the two stochastic models previously defined, correlated numbers need to be sampled from the standard normal distribution used in the geometric Brownian motion. This is performed using a Cholesky decomposition of the correlation matrix as shown in Eq. 15. The positive definite correlation matrix is decomposed to give a lower-triangular matrix which, when applied to a vector of uncorrelated samples, produces a sample vector with the correlation properties of the system being modeled.

$$M_{Corr} = C \cdot C^T \; ; \; C = \begin{bmatrix} 1 & 0 \\ 0.199 & 0.979 \end{bmatrix}$$

Eq. 15

7.3 Results

The results from the real options analysis are summarized in Table 8 and Figure 18. Since simulation is used to estimate the option price and since there is always some randomness left in the results, the computation has been repeated 160 times which helps gauge the spread of results. Using a real-options analysis, the program does have value and would be undertaken. This is in stark contrast with the results from a deterministic discounted cash flow analysis which would lead to an abandonment of the project.

PIP Program Value (Normalized by Initial Investment)	
DCF analysis of PIP program	-0.7002
PIP Program value with options	0.2901
Standard deviation of PIP values	0.0124

Table 8: Value of Performance Improvement Package with and without flexibility

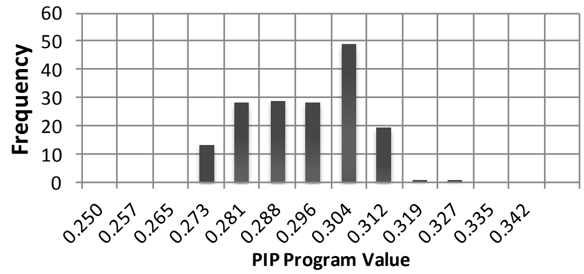

Figure 18: Distribution of PIP program values

8 Conclusion

In this research, a new methodology for the analysis of the economic performance of development programs in the aerospace industry is proposed. By cross-fertilizing elements from financial engineering, actuarial sciences, and statistics, this research has enabled the development of a traceable and transparent framework for the analysis of staggered corporate investments featuring timing flexibility. Most of the techniques shaping this methodology are well accepted and already in use in the aerospace community which may help acceptance by practitioners. The methodology is articulated around four main points: a simulation of the evolution of the value of a business prospect over time under the physical probability measure, a non-parametric risk-neutralization to yield a distribution of the business prospect value under the equivalent martingale measure, a resampling to yield risk-neutral business prospect value evolution over time with equal weight for each trajectory, and finally an option valuation exercise using regression to value a real-option featuring timing flexibility and to estimate the early-exercise boundary. Next, the methodology is tested against different cases of known vanilla options. Finally, the methodology is applied to evaluate the development, certification, and testing of a technology package to improve the economics of commercial aircraft in operations.

Several improvements could be achieved using slight modifications of the proposed methodology. The first one is to use quasi-Monte Carlo simulation to improve the generation of random numbers. The well-known Halton as well as the Sobol sequences could be used to generate random numbers which may be more uniformly distributed than the random number generator implemented in the spreadsheet software used. Another improvement would include the review of other families of basis functions for use in the Least-Squares Monte Carlo regressions. Including more than three terms may also improve the regression fit when computing the conditional expectation.

Furthermore, resampling using the bootstrapping technique precludes the capture of serial correlations, and more generally autocorrelations, when simulating the evolution of the business prospect value. Future work will include further literature review to improve the current bootstrapping technique to be able to capture serial correlations. This may significantly extend the domain of application of the proposed methodology.

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