

# Modeling Flexibilities in Power Purchase Agreements: a Real Option Approach

Rafael Igrejas<sup>a,\*</sup>, Leonardo Lima Gomes<sup>a</sup>, Luiz E. Brandão<sup>a</sup>.

## Abstract

Power purchase and sale contracts in Brazil, have been receiving attention in the last few years due to the high volatility of electricity spot price (PLD) and the regulatory constraints to operate in the Brazilian electricity market. In this scenario, even when contract content is clearly understood by the parties, uncertainties are commonly not well measured and flexibilities are not correctly priced. One reason is the limitation of traditional financial techniques for pricing contracts which often did not adequately take into account some important features, mispricing embedded flexibilities. These flexibilities can be seen as options by the purchasers, in the way to choose the amount of energy to be supplied (amount option) or to reduce the amount or even to interrupt the supply during a predetermined interval (reduction option). In this working progress paper, we present what is intended to investigate in the final study about valuing these flexibilities in bilateral contracts in the Brazilian electricity market, using real options approach. These flexibilities are modeled as compound European call and put options, under uncertainties of monthly energy price and energy demand. In the paper, we also intend to discuss new perspectives for independent stochastic variables modeling to value options embedded in bilateral contracts.

Key words: Contract flexibilities, Electricity, Real options, Power contracts.

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\**Doctoral Student of PUC-Rio University.*

*<sup>a</sup>E-mail addresses: \*rafael.igrejas@iag.puc-rio.br (Rafael Igrejas); brandao@iag.puc-rio.br (L.E. Brandão); leonardolima@iag.puc-rio.br (L.L.Gomes).*

## 1. Introduction

The Brazilian electrical sector, which was basically a government monopoly until 1997, has been receiving attention due to the recent regulation changes and to the high volatility of electricity spot price (PLD). In this scenario, pricing techniques became crucial to value embedded flexibilities in power purchase and sale contracts in Brazil.

These flexibilities can be seen as options by the purchasers in the way to choose the amount of energy to be supplied (amount option) or to reduce the amount or even to interrupt the supply during a predetermined interval (reduction option). Power purchase and sale contracts became the main instrument for negotiation between players.

In this scenario, the Brazilian electricity spot market (ACL) is defined as a free market for bilateral energy contracts transactions, according to specific rules and commercialization procedures. Due to some buyers' demands for electricity, revealed from the inability to accurately predict consumption, bilateral contracts became more sophisticated, embedding flexibilities, such as a range of the amount of energy that can be delivered to the buyer (the option of choice) or; the right of pre-agreed stopping consumption during a given interval (the option to reduce).

The option of choosing the electricity supply arises to the desire of the buyer (Free Consumer in Brazil) to adjust purchased energy to its consumption. This option is valid for all contract months (considering the month as the assessment period). The reduction option appears as a product to adapt the supply to the scheduled or untimely shutdowns of the consumer plant. These flexibilities in commercial transactions can be legally supported by contracts which may function as boundary limits to the flexibility available to the parties involved.

Sykuta (1996) was the first to consider the effects of future negotiations in supply decisions and to explore the variables that impact the future positioning of the companies in the international trade contracts of commodities. In addressing the use of futures contracts in the electricity market, the study also showed that more than hedge strategies, future contracts can contribute as a mechanism for long-term storage. Oum et al. (2006) used forwards contracts to hedge risks in electricity markets, using a combination of forwards, call options and put options to hedge its volumetric risk, drawing attention to regulated firm's difficulties to hedge their position when regulators forbid trade in derivatives that look speculative. In a more recent study, Willems and Morbee (2010) developed an equilibrium model of the electricity market, considering the production process, spot market trading and derivatives.

Although some studies efficiently address contract option modeling, pricing flexibilities in electricity contracts still an issue due to the financial pricing techniques, which often did not adequately take into account some important features of these contracts, mispricing embedded flexibilities. On the other hand, real options approach allows optimal decisions in order to efficiently respond to unforeseen changes in contracting process and uncertainties inherent to the market. In this context, the main point of this paper is to value flexibilities of electricity consumption embedded in energy contracts in Brazil, using real options methods.

We assume the optimal electricity consumption can be decided by the consumer plant on a monthly basis under contracts rules, which allows this

flexibility to be modeled as a bundle of European call and put options under the risk neutral measure. The price of electricity in the Brazilian spot market (PLD) and the electricity demand drive the uncertainties of the model.

The final paper will be organized, considering the discussion about the literature on the use of option pricing methods to value contracts flexibilities in the electricity sector. We will present an overview about contract flexibilities in the Brazilian electricity sector.

## **2. Contract Flexibilities in the Brazilian Electricity Sector**

The presence of flexibilities in the Brazilian electricity sector is reflected in the willingness of agents to reflect their business transactions in contractual terms. An electricity consumer company, for example, needs flexibility to consume, considering the variability of monthly consumption. If this consumer hires a power supply for exactly what it was consumed, the demand will be met. However, the flexibility to pay for what exactly it consumes does not add value to the contract. In a favorable market condition, the consumer can choose the contracted amount.

One of the main flexibilities observed in energy contracts is the permission to choose the amount of energy that will be delivered (option to choose). Typically, contracts with such flexibility specify the range of choice of the contracted amount. For each assessment period (usually monthly), the buyer can choose the quantity to buy between the lower and upper limits at a contracted price.

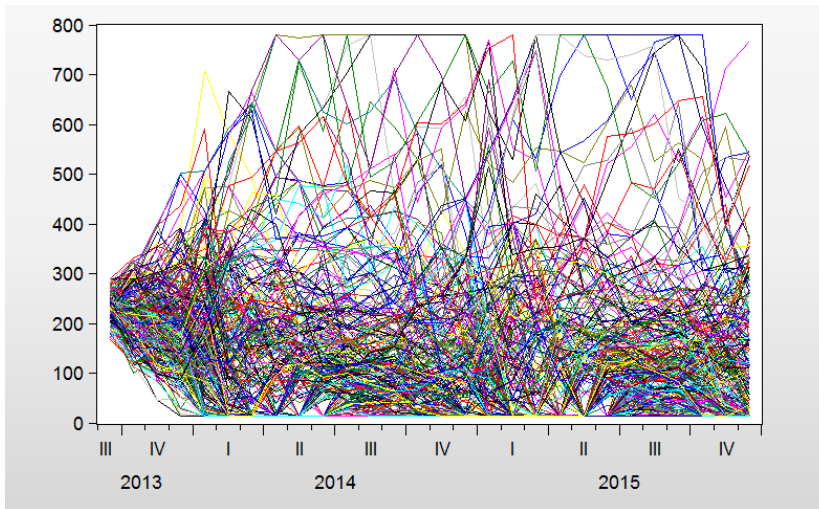
Another important flexibility embedded in energy contracts, is the option to stop or substantially reduce consumption / delivery during certain pre-agreed range (reduction option). This option should include the complete interruption of supply once it is subject to usually scheduled stops of consumer units. However, as the option calculation (exercise) is in monthly basis and in real situations stops occur in less than one month periods, significant reduction could be required without exactly a total interruption of the contract. The option to choose the amount does not exceed 20%. In the case of the reduction option, the reduction is substantial to be conditioned on the interruptions of the consumer units, usually at least 50% until a total outage. If the reduction is partially allowed, the contract should also specify the size, being generally a percentage of the contracted energy.

## **3. Spot Electricity Price Simulation**

The Brazilian Electricity Clearing Chamber (CCEE) analysis determines the Brazilian spot market price (PLD) on weekly basis. Agreements upon electricity price and volumes in bilateral contracts are registered at the CCEE, which also receives power generation and consumption information by the parties. Based on contracts and registered measurement data, the differences between what is produced or consumed and what was initially contracted are determined and the positive or negative differences are settled at the PLD rate.

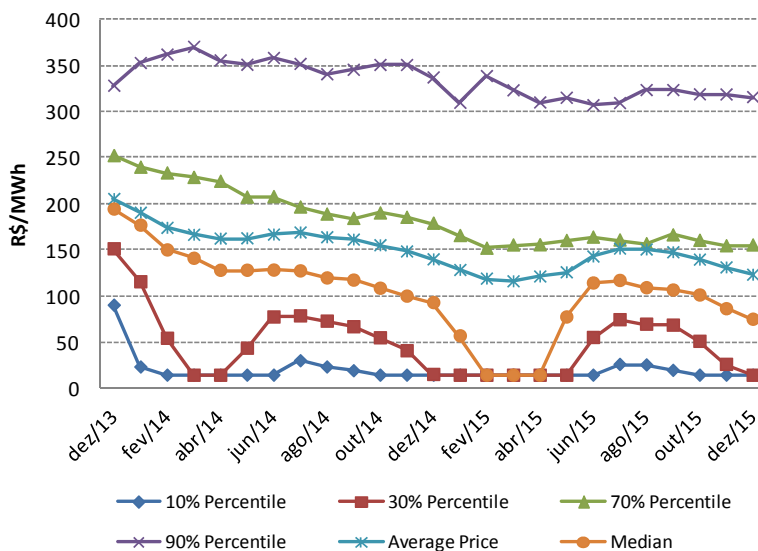
The *Newave* optimization software is used in this process as a centralized decision operational model. The software is available for energy companies and research centers, and allows a link between medium and long term optimization models, with the objective function to minimize the total cost of Brazilian hydrothermal system operation. This software is the most used tool for electricity

trading companies and allows strategies development for short and long position in the market. A Newave 2000 simulation series for PLD, can be seen in Figure 1 for 2013 to 2015.



**Figure 1- Newave Simulation Series of PLD (2013 a 2015).**

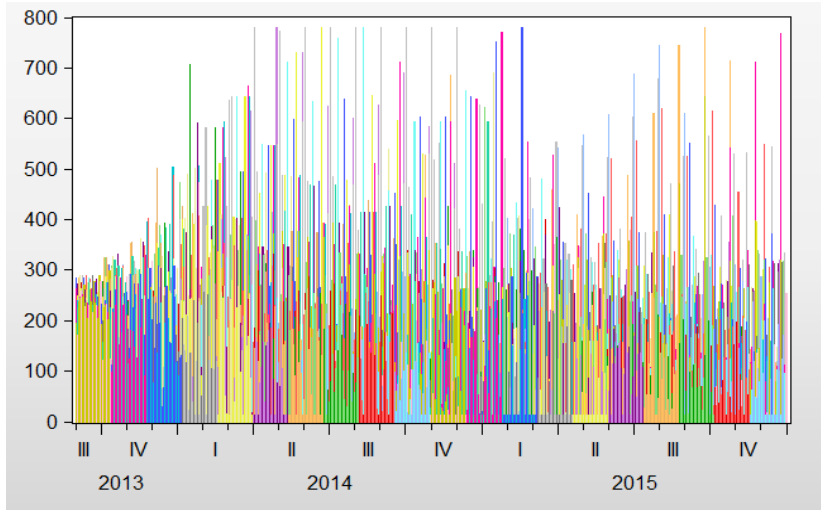
Given the simulated prices, the Figure 2 shows the monthly mean values, as well as some percentiles ranging from 10 to 90%. Comparing series, it is observed that the density distributions of monthly probability are quite asymmetric assigning higher probability to lower price intervals. We also can see the monthly average price is substantially greater than the median for the entire period.



**Figure 2 - Monthly mean values and percentiles of simulated prices.**

Complementing the previous figure, in the Figure we can see four price ranges probability distributions by month. There is great probability of lower

prices. The probability of prices between 100 and 200 R\$/ MWh is about 25% on average for the period, and only 5% of the prices are above 500 R\$/ MWh.



**Figure 3 – Probability Distribution of Monthly Prices.**

To address the issue of contract uncertainty modeling is important to analyze the operational decision under the real option approach.

#### 4. Methodology

In the final paper, we intend to investigate the value of flexibilities in power purchase and sale contracts using real options approach and modeling uncorrelated uncertainties. We propose to model electricity demand and spot price (PLD) as the main uncorrelated uncertainties that impact the dynamic decision of energy purchase by the consumer plant.

For the spot electricity price (PLD) we will run 2000 simulated series for the contract time horizon, using the *Newave* software of the Brazilian Electricity Research Center (CEPEL), which employs an stochastic dual dynamic programming method. The difference between the minimum and the price cap has to be considered due to the conditional operation controls of the hydrothermal system in Brazil.

The electricity demand will be modeled as a mean reverting process, based on Schwartz (1997) model 1, in which the diffusion process is  $dS = \eta(\alpha - \ln S)Sdt + \sigma Sdz$ . It is generally assumed that  $\alpha = \ln(\bar{S})$ , which provides  $dS = \eta(\ln \bar{S} - \ln S)Sdt + \sigma Sdz$ , where:

$S$  is the stochastic variable

$\bar{S}$  is the long term equilibrium level of the stochastic variable

$\eta$  is the reversion speed

$\sigma$  is the volatility of the process

$dz$  is the standard Wiener process with a normal distribution  $dz = \varepsilon\sqrt{dt}$ ,

$\varepsilon \sim N(0,1)$ .

In order to simulate the stochastic variable, a corresponding discrete time equation for this model is required. We adopt the exact discretization equation

proposed by Bastian-Pinto (2009) which allows the use of higher values of  $\Delta t$  as shown in Eq. (1).

$$S_t = \exp \left\{ \ln[S_{t-1}]e^{-\eta\Delta t} + \left[ \ln(\bar{S}) - \frac{\sigma^2}{2\eta} \right] (1 - e^{-\eta\Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}} N(0,1) \right\} \quad (1)$$

Parameter estimation can be made regressing the series  $S_t$ , as show in Eq. (2):

$$\ln(S_t/S_{t-1}) = \underbrace{(1 - e^{-\eta\Delta t}) \left( \ln \bar{S} - \frac{\sigma^2}{2\eta} \right)}_a + \underbrace{(e^{-\eta\Delta t} - 1)}_{b-1} \ln S_{t-1} \quad (2)$$

where the parameters of reversion speed, volatility and long term mean are given by Eq. (3), (4) and (5), respectively:

$$\eta = -\ln(b) / \Delta t \quad (3)$$

$$\sigma = \sigma_\varepsilon \sqrt{\frac{2 \ln b}{(b^2 - 1)\Delta t}} \quad \text{where } \sigma_\varepsilon \text{ is the standard error of the regression} \quad (4)$$

$$\bar{S} = \exp \left[ \frac{a}{(1-b)} + \frac{\sigma^2}{2\eta} \right] \quad (5)$$

Substituting equations (3) and (4) into (5) we arrive at:

$$\bar{S} = \exp \left[ \left( a + \frac{\sigma_\varepsilon^2}{(1+b)} \right) / (1-b) \right] \quad (6)$$

In order to obtain the risk neutral simulation of the process required for option pricing, it is necessary to subtract the normalized risk premium  $[\mu - r/\eta$  or  $\pi/\eta]$  from the long term mean, where  $\mu$  is the risk adjusted discount rate,  $r$  is the risk free interest rate and  $\pi$  is the risk premium, as shown in Eq. (7):

$$S_t = \exp \left\{ \ln[S_{t-1}]e^{-\eta\Delta t} + \left[ \ln(\bar{S}) - \frac{\sigma^2}{2\eta} - \frac{\mu - r}{\eta} \right] (1 - e^{-\eta\Delta t}) + \sigma \sqrt{\frac{1 - e^{-2\eta\Delta t}}{2\eta}} N(0,1) \right\} \quad (7)$$

On the other hand, the option to choose the power supply will be modeled for each month of the contract, through the optimal decision among a set of call and put energy options.

Considering for example, the buyer has in fact to buy 60 MWmed, why he would choose a different amount of supply? Considering the short-term prices (PLDs) in a given month are below the contract price, the buyer could choose to buy 48 MWmed at a contracted price, and the remaining 12 MWmed at a price below the contract price. On the other hand, if the short-term price in a given month is above the contract price, the buyer may choose to buy 72 MWmed at a contracted price, and the excess can be negotiated at the spot market at a higher price, generating an additional gain to the contract.

The value of the option to choose ( $OC_{0,t}$ ) in  $t=0$  for specific time  $t$  of exercise will be equal to the optimal decision between call and put options with the value in  $t=0$ .

$$S_{PLD} > S \therefore \max \left[ \left( E_{C_{\max}} - \tilde{E} \right) * (S_{PLD} - S); \left( E_{P_{\max}} - \tilde{E} \right) * (S - S_{PLD}) \right]$$

$$S_{PLD} < S \therefore \max \left[ \left( E_{C_{\min}} - \tilde{E} \right) * (S_{PLD} - S); \left( E_{P_{\min}} - \tilde{E} \right) * (S - S_{PLD}) \right]$$

where,

$S$  - electricity contracted price in  $t$

$S_{PLD}$  - monthly average electricity spot price

$E_t$  - electricity contracted demand in  $t$

$E_C$  - amount of electricity of the call option

$E_P$  - amount of electricity of the put option

The value of call and put options is obtained as the expected value in  $t$  of the options payment, applying the risk neutral simulation process (Pilipovic, 1998). The flexibility is valid for all ranges of calculation (months) during the contract term. Thus, the total flexibility value of choosing the electricity demand ( $VOC_{0,t}$ ) is equal to the sum of the values of all the options to choose ( $OC_{0,t}$ ) at  $t=0$  to  $t$ , ranging from the instant the contract starts ( $t=i$ ) until the end of the contract.

#### 4. Expected Results

Pricing techniques became crucial to value embedded flexibilities in scenarios of high volatility of electricity spot price and regulatory constraints in the Brazilian electricity market. These flexibilities can be seen as options by the purchasers in the way to choose the amount of energy to be supplied.

In the final paper, we will model these flexibilities as compound European call and put options, under uncertainties of monthly energy price and energy demand. Each option will be assessed simultaneously in relation to some variables. The option value will be calculated in R\$/MWh, which is obtained dividing the original value of the options in Brazilian real currency (R\$), by the amount of nominally contracted energy in MWh. It will also be obtained the conjoint value of options, but not always the whole value of options is equal to the sum of the individual values of each one (Trigeorgis, 1993). Sensitivity analyzes will be processed in relation to the size of flexibility, to the contracted price, and to the volatility parameters.

As contributions of this paper, we believe electricity consumers plants in Brazil, can intensify purchase electricity amount strategies at the Brazilian electricity spot market, using real options approach. Option pricing methods can allow players to get discounts in relation to regulated tariffs applied by distributors and they can get great advantage from flexibilities embedded in trading contracts. Market agents can also negotiate purchase and sale contracts, hedging their position and improving their upside while limiting downside losses relative to other agents with initial expectations under passive management.

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