Entry deterrence and hidden competition

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Abstract

This article studies strategic investment behavior of firms facing an uncertain demand in a duopoly setting. Firms choose both investment timing and the capacity level while facing additional uncertainty about market participants, which is introduced via the concept of hidden competition. We focus on the analysis of possible strategies of the market leader in terms of its capacity choice and on the influence of hidden competition on these strategies.

We show that due to hidden competition the follower is more eager to invest. As a result the entry deterrence strategy of the leader becomes more costly and it can only be implemented for smaller market size, leaving additional room for entry accommodation. The leader has incentives to prevent entry of the hidden competitor stimulating simultaneous investment if the hidden firm has a large capacity, and has more incentives to apply entry deterrence in the complementary case of a small capacity of the hidden player. In the first case overinvestment aimed to deter the follower's entry does not occur for a wide range of parameters values.

1 Introduction

The real options theory is based on the idea that investment timing plays a crucial role in the decisions to undertake irreversible investment in an uncertain world. More precisely, the possibility to delay the investment and, therefore, to access additional information, creates the option value for the market participants. As a result, in the real options framework the optimal investment thresholds turn out to be above the so called Marshallian trigger points, which correspond to a zero net present value (NPV).

Early literature on real options usually focuses on investment decisions of a single firm. However, the analysis can be extended to the case of multiple market participants by incorporating game theoretic elements. For instance, Smets (1993), Dixit and Pindyck (1994), Huisman and Kort (2004), and Pawlina and Kort (2006) study the investment decision in a duopoly setting, while Grenadier (2002) and Bouis *et al.* (2009) are examples of oligopoly real option models.

Another important modification of the basic real options model arises when firms are allowed to choose not only the timing of the investment decision, but also the size of the investment. The main idea here is that investment of a larger size can bring bigger losses or higher gains for the firm depending on the realization of the stochastic profitability shock incorporated in the inverse demand function. Moreover, in a competitive framework capacity choice of a certain firm can influence the decision of the other firm to enter the market. For example, among the early models of capacity choice, Spence (1977) introduces a setting where the firm can deter entry by overinvestment. Wu (2007) studies incentives of the leader in a growing market to preempt the follower by investing in capacity. The main result of his paper is that under the assumption of uncertainty about the date at which the market starts to decline, the leader will choose a smaller capacity in order to take advantage of market decline, i.e. to stay longer in the market than the larger competitor. Huisman and Kort (2013) analyze accommodation and deterrence strategies of the market leader in a duopoly setting. The authors introduce the overinvestment effect that arises due to possibility of the market leader to deter entry of its competitor, as the bigger level of the leader's capacity ensures that the follower invests later. Moreover, the length of the deterrence region becomes larger when market uncertainty is higher. This happens because larger uncertainty generates more incentives for the follower to postpone the investment and, therefore, the leader can enjoy a longer monopoly period when implementing the entry deterrence strategy.

The current research is devoted to the analysis of strategic interactions of firms in a duopoly market in a real options setting. Following Huisman and Kort (2013), we present a model, where in order to enter the market, firms invest in a plant with a certain capacity, i.e. the firms choose the investment scale. In this paper their model is extended by relaxing the assumption that firms are fully informed about all market participants. Similar to Armada *et al.* (2011), we incorporate an additional type of uncertainty in the model by introducing the concept of hidden competition. The key idea behind this approach is that, apart from the two competitors that are well informed about each other, the third, hidden firm, can enter the market at an unknown point in time.

Armada *et al.* (2011) develop a model, where two positioned firms compete in the market with two places available, facing a possibility of a hidden entry. The entrance occasion of the hidden firm is modeled in their paper as an exogenous event driven by a Poisson jump process. They demonstrate that hidden competition can exert significant influence on the firms' investment timing in the limited market. Namely, they show that, as the arrival rate of the hidden competitor rises, we can observe a decrease in the investment trigger for the follower on the one hand, and an increase in the investment trigger for the leader on the other hand. This means that if the probability that the hidden competitor enters the market is higher, the market leader will invest later, while the follower will invest sooner.

In this paper the investment problem on a limited market with hidden competition is approached from a different perspective. We examine how hidden competition affects the optimal strategies of the firms if they are allowed to choose the capacity level. As in Huisman and Kort (2013) we consider deterrence and accommodation strategies for the leader. We show that the deterrence region shrinks with the probability of hidden entry. This happens because the larger the probability that the hidden competitor can enter the market, the more eager the follower is to invest earlier, and therefore it is getting harder for the leader to deter entry. In fact, we show that hidden competition induces the positioned firms to enter the market together.

The rest of this paper is organized as follows. Section 2 is devoted to the analysis of the investment decisions of the positioned firms facing a threat of hidden entry on the market with two places available. We solve the game backwards, first determining the optimal investment trigger and the optimal capacity level for the follower. Then we continue by determining the optimal strategies of the firms when the roles of the leader and the follower are endogenously assigned. Section 3 summarizes the main results and concludes the paper. The proofs of the propositions are presented in the Appendix.

2 Model

In the model two risk-neutral, *ex ante* identical firms make a market entry decision. The entrant becomes active on the market and starts the production process after investing δq in a plant with capacity of size q.

In the presented setting the two firms that have full information about each other are called positioned firms. First we assume that firm roles, i.e. whether they are first or second investor, are exogenously assigned. This creates a benchmark for the situation where both firms are allowed to invest first, which we analyze later on. The positioned firm that invests first is called the leader and the second investor is called the follower.[1]An important feature of the presented approach is that here the standard duopoly model is extended by incorporating the possibility of hidden entry. Like in Armada *et al.* (2011), we assume that at any moment of time the positioned firms face a probability that a third firm can become active on the market. The information about this firm remains hidden from the positioned market players. Therefore, this firm is called the hidden competitor. The entry timing of the hidden firm is modeled as an exogenous event corresponding to a Poisson jump process with arrival rate λ. It is assumed that the hidden firm enters the market with a given capacity level of q_H. We follow Armada *et al.* (2011) by imposing that the market is big enough only for two firms. Therefore, the follower loses the option to invest if the hidden competitor enters the market earlier.[2]

The market price of a unit of output is defined by the multiplicative inverse demand function:

$$p = x(1 - Q), \tag{1}$$

where Q is aggregate market output and x is a stochastic shock process that drives the uncertainty in the firm's profitability. Given this structure of the demand function production optimization results in the fixed optimal quantity irrespectively of the level of x. As a result it is always optimal for the firms to produce up to capacity.

This specific choice of the demand structure is motivated by the desire to reflect the property that the market has limited size, which corresponds to the above assumption hat maximally two firms can enter. A multiplicative demand function implies that, to avoid negative prices, the firms can increase their output only up to a certain level. Without loss of generality, in this model the maximum total market output is normalized to 1. In such a setting it is possible for the firms to acquire a market share large enough to discourage the new entrant from undertaking investment.

[1]If the hidden competitor takes the leader's position there is only one place left on the market, which will be taken by one of the firms. Thus, the outcome of the game is not affected by the hidden competition and therefore we do not consider this case.

[2] Here we can think of the industries where the firms face significant barriers to entry, for example, due to the strict government regulations, exclusive technology, limited resources, patents and licenses.

Denoting the leader's and the follower's capacity levels as q_L and q_F respectively, the total output quantity given that the hidden firm has not entered the market yet can be written as

$$Q = q_L + q_F. \tag{2}$$

It is assumed here that x evolves according to a Geometric Brownian Motion:

$$dx = \alpha x dt + \sigma x dZ, \tag{3}$$

where α is the constant drift, σ – standard deviation, dZ – the increment of a Wiener process.

In the next sections we apply dynamic programming techniques to solve the optimal stopping problem for the positioned firms on the duopoly market described above.

2.1 Exogenous firm roles

In this section it is assumed that the roles of the leader and the follower are assigned beforehand. The problem is solved backwards starting with determining the follower value function, assuming that the leader is the positioned firm and it has already invested. Denote by $x_F^*(q_L)$ the optimal investment threshold for the follower and by $q_F^*(q_L)$ the corresponding capacity level, respectively. This implies that the follower will not enter the market until the stochastic component of the profit flow, x, reaches $x_F^*(q_L)$. On the contrary, for the values of x exceeding $x_F^*(q_L)$ investment becomes optimal and the follower enters the market immediately installing the capacity $q_F^*(q_L)$.

Thus, the range of x such that $x > x_F^*(q_L)$ is called the stopping region, while the one that satisfies $x < x_F^*(q_L)$ is called the continuation or waiting region. The optimal investment trigger is found using the fact that at the threshold value the firm's value of waiting is equal to the value of stopping, i.e. the firm is indifferent between entering the market and waiting for more information.

Under the assumption that the positioned leader is already on the market it is possible to derive the value function for the region where the follower waits with investment. As the optimal behavior of the follower is dependent on the leader's capacity level, different strategies in terms of the leader's capacity choice are considered. This results in the following proposition.

Proposition 1. *The follower's optimal capacity choice for a given level of the stochastic profitability shock, x, and the leader's capacity, q_L, is given by*

$$q_F^*(x, q_L) = \frac{1}{2}\left(1 - q_L - \frac{\delta(r-\alpha)}{x}\right). \tag{4}$$

The value function of the follower takes the following form:

$$F^*(x) = \begin{cases} A(q_L)x^\beta & \textit{if } x < x_F^*(q_L), \\ \\ \dfrac{[x(1-q_L) - \delta(r-\alpha)]^2}{4x(r-\alpha)} & \textit{if } x \geq x_F^*(q_L), \end{cases} \tag{5}$$

with

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}}, \tag{6}$$

$$A(q_L) = \left(\frac{(\beta-1)(1-q_L)}{\delta(r-\alpha)(\beta+1)}\right)^\beta \frac{\delta(1-q_L)}{(\beta-1)(\beta+1)}, \tag{7}$$

and the optimal investment trigger for the follower

$$x_F^*(q_L) = \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)(1-q_L)}. \tag{8}$$

The equations above lead to the following optimal capacity level of the follower given the leader's capacity, q_L,

$$q_F^*(q_L) = \frac{1-q_L}{\beta+1}. \tag{9}$$

It is important to notice that for the given capacity level of the leader, both the optimal capacity level and the investment trigger of the follower are decreasing with λ. The reason is that in the waiting region the follower faces the risk that the hidden competitor might enter the market before $x_F^*(q_L)$. If this is the case, the follower loses his option to invest. The bigger λ is, the more likely it is that such situation can arise. Thus, the follower has an incentive to invest earlier for larger values of λ.

Moreover, both the optimal investment trigger of the follower and its optimal capacity level depend on the capacity that the leader installs. The bigger the capacity level chosen by the leader the later the follower invests, while it will choose a smaller capacity level given the investment timing. Based on this result there are two strategies available for the leader: install such a capacity that the follower enters the market either strictly later or exactly at the same time as the leader.

In order to identify which strategy, and under what conditions, is optimal for the leader, we solve the leader's investment decision problem.

2.1.1 The leader's deterrence strategy

The first strategy for the leader is to choose the capacity level in such a way that the follower will postpone its investment. Following Huisman and Kort (2013) we call this strategy – the entry deterrence strategy.

First, we focus on the region where the follower still waits with investment. The problem is solved backwards starting with determining the leader's value function. It is assumed that the leader is the positioned firm which has already entered the market. We consider the following differential equation:

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 V_L(x)}{\partial x^2} + \alpha x \frac{\partial V_L(x)}{\partial x} - rV_L(x) + xq_L(1-q_L) + \lambda[\Phi_1(x) - V_L(x)] = 0, \tag{10}$$

where V_L denotes the value function and the expected discounted revenue of the leader.

If the hidden competitor enters the market earlier than the follower, the leader's value function will decrease in comparison to the standard case. This loss due to the hidden entry is captured by including the additional term in the differential equation, $\lambda[\Phi_1(x) - V_L(x)]$. As it was mentioned earlier q_H denotes the capacity level chosen by a hidden firm if it enters as a follower. Therefore, in the case of hidden entry the leader's value function will depend on the new total market output $Q_H = q_L + q_H$. Assuming that the leader has already invested and the follower is the hidden firm, the leader's value function can be written as $\Phi_1(x) = \dfrac{xq_L(1-Q_H)}{r-\alpha}$.

Next, using the fact that in the stopping region both positioned firms are present in the market we consider the following boundary conditions:

$$\lim_{x \to 0} V_L(x) = 0, \tag{11}$$

$$\lim_{x \to x_F} V_L(x) = \frac{x_F q_L(1-Q)}{r-\alpha}. \tag{12}$$

Combining these conditions and the expressions for q_F and $x^*(q_L)$, obtained in the previous section, we find the leader's value in the continuation region of the follower:

$$\begin{aligned} L(x) = & x\frac{q_L(1-q_L)}{r-\alpha} - x\frac{q_L \lambda q_H}{(\lambda + r - \alpha)(r-\alpha)} - \delta q_L \\ & - \left(\frac{x(\beta-1)(1-q_L)}{\delta(r-\alpha)(\beta+1)}\right)^\beta \left[\frac{\delta q_L}{(\beta-1)} - \frac{\delta(\beta+1)q_L\lambda q_H}{(\beta-1)(1-q_L)(\lambda+r-\alpha)}\right]. \end{aligned} \tag{13}$$

Recall, that the follower will invest as soon as the stochastic process exceeds the value of the follower's trigger, $x_F^*(q_L)$. Thus, to implement the deterrence strategy the leader choses q_L such that $x < x_F^*(q_L)$ given the current value of x .

Taking into account the expression for x_F^*, the deterrence strategy occurs when the leader chooses the capacity level such that

$$q_L > \hat{q}_L = 1 - \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)x}. \tag{14}$$

Setting the derivative of the value function with respect to q_L to zero results into the following first order condition:

$$\left(\frac{x(\beta-1)(1-q_L)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta}\frac{\delta}{(\beta-1)}\left[-\frac{(1-(\beta+1)q_L)}{(1-q_L)}+\frac{(\beta+1)\lambda q_H}{(\lambda+r-\alpha)}\frac{1-\beta q_L}{(1-q_L)^2}\right]$$
$$+\frac{x(1-2q_L)}{r-\alpha}-\frac{x\lambda q_H}{(r-\alpha)(\lambda+r-\alpha)}-\delta=0. \tag{15}$$

The solution of the equation (15) gives us the optimal capacity level for the leader under the deterrence strategy, q_L^{det}. Further we will show that the leader can use the deterrence strategy if the value of the stochastic process x lies in the interval (x_1^{det}, x_2^{det}), where x_2^{det} is the biggest and x_1^{det} is the smallest possible value of the stochastic process that allows the leader to implement the deterrence strategy. The latter can be found by setting the capacity level to zero in the first order condition for the deterrence problem (equation (15)).

Proposition 2. *The starting point of the deterrence region, x_1^{det}, is implicitly determined by the following equation:*

$$\left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta}\frac{\delta}{(\beta-1)}\left[-1+\frac{(\beta+1)\lambda q_H}{(\lambda+r-\alpha)}\right]$$
$$+\frac{x}{r-\alpha}-\frac{x\lambda q_H}{(r-\alpha)(\lambda+r-\alpha)}-\delta=0. \tag{16}$$

In order to identify the biggest possible value of x for which deterrence is possible, x_2^{det}, recall that the leader uses this strategy only if the followers indeed enters later. This happens for those values of x that satisfy the following inequality: $x < x_F^*(q_L^{det})$. Therefore, x_2^{det} is defined by $x_F^*(q_L^{det}(x_2^{det})) = x_2^{det}$.

Next, x_2^{det} is determined using the expression for $x_F^*(q_L)$ (8) and the first order condition (15):

$$x_2^{det}=\frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-\frac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}+\sqrt{\left(3+\frac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}\right)^2-8}\right]}. \tag{17}$$

Note that x_1^{det} and x_2^{det} determine the feasible region for the deterrence strategy: for the values of the stochastic component of the profit flow, x, that fall into an interval (x_1^{det}, x_2^{det}), the leader will consider implementing the deterrence strategy, i.e. choosing the capacity level that prevents the follower from immediate investment.

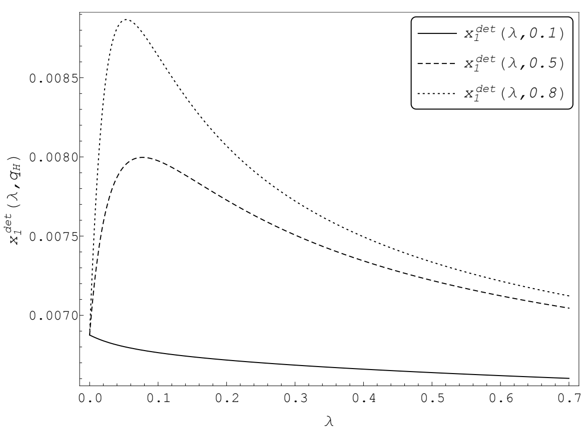
As it can be seen, the endpoints of the deterrence region depend on the parameters λ, the arrival rate of the hidden firm, and q_H, the capacity level of the hidden firm, meaning that introducing an assumption that a hidden firm of an unknown size can enter the market can influence the possibilities to implement the deterrence strategy.

Proposition 3. *An increase in the capacity level of the hidden firm, q_H, leads to an increase in the starting point of the deterrence region, x_1^{det}, and to a decrease in its endpoint, x_2^{det}.*

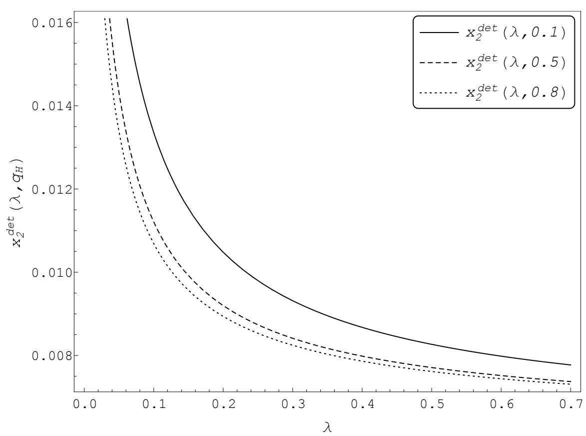
Intuitively, the bigger the hidden firm is, the less is left for the leader after the division of market rents, and thus, the less appealing is the investment opportunity for any given q_L. Therefore on the one hand, the larger x is needed to convince the leader to enter such a market by installing positive capacity. This explains an increasing pattern in x_1^{det}. On the other hand, the less incentives has the leader to deter the follower's entry, causing x_2^{det} to decline.

Proposition 4. *An increase in the arrival rate of the hidden firm, λ, leads to a decrease in its endpoint of the deterrence region, x_2^{det}. If $q_H = 0$ the starting point of the deterrence region, x_1^{det}, also decreases. For $q_H > 0$ the effect of an increase in λ on x_1^{det} is ambiguous.*

The effect of a change in the arrival rate, λ, on the endpoints of the deterrence region is shown in Figure 1.



(a) $x_1^{det}(\lambda, q_H)$



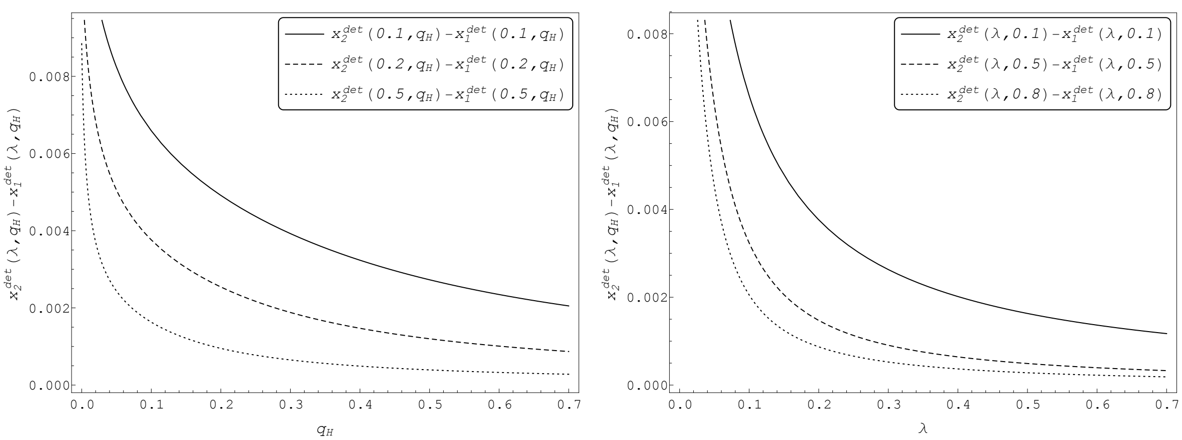
(b) $x_2^{det}(\lambda, q_H)$

Figure 1: $x_1^{det}(\lambda, q_H)$ and $x_2^{det}(\lambda, q_H)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$ and different values of q_H.

In Figure 1a, one can notice two differently directed effects of an increase in λ on x_1^{det}. On the one hand, for small q_H there is only a declining pattern to be observed. The reason is that for larger λ the leader is more willing to invest earlier in order to collect monopoly rents. On the other hand, for larger q_H numerical experiments reveal another effect of an increase in λ. In particular, when the value of q_H is sufficiently large, x_1^{det} is first increasing with λ. This indicates that the effect of declining profitability of the market dominates the advantage of investing earlier and collecting monopoly profits when the probability that the hidden firm enters the market is sufficiently small. However, after a certain point the latter effect becomes more important causing x_1^{det} to decrease with λ.

Considering he influence of a change in the arrival rate of the hidden firm, λ, on the endpoint of the deterrence region, x_2 we can conclude that both the bigger risk of the hidden entry causes x_2^{det} to decline. This means that when the risk that the hidden firm will occupy the last available place on the market is higher the follower is more eager to invest earlier. Hence, in this case, the more complicated it is to ensure that the follower will enter the market strictly later than the leader and the deterrence region becomes smaller. This causes x_2^{det} to decrease with λ. Moreover, for larger values of λ this declining pattern is enhanced by the desire of the leader to invest in smaller capacity due to a decreased profitability of the market. In addition, a smaller capacity does not prevent the follower to enter.

The dependence between the size of the deterrence region and the arrival rate of the hidden competitor and the expected size of the hidden firm is shown in Figure 2.



(a) $x_2^{det}(\lambda, q_H) - x_1^{det}(\lambda, q_H)$

(b) $x_2^{det}(\lambda, q_H) - x_1^{det}(\lambda, q_H)$

Figure 2: $x_2^{det}(\lambda, q_H) - x_1^{det}(\lambda, q_H)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$ for different values of λ and q_H.

Here, the decreasing effect of q_H is a direct implication of Propositions 3 and 4. The relation between the size of the deterrence region and the arrival rate λ cannot be described analytically due to the complexity of the expressions. Thus, numerous numerical experiments were carried out to investigate this dependence, allowing to conclude that a decrease in x_2^{det} dominates a decrease of x_1^{det} for wide range of the parameter values causing the deterrence region to become smaller. The result is presented in Figure 2b. Huisman and Kort (2013) came to the conclusion that the deterrence interval expands with uncertainty. However, in the presented setting another type of uncertainty is involved, namely the uncertainty about the market participants. The region where only deterrence strategy is optimal tends to become smaller if this uncertainty is larger, or in other words if the risk that the hidden firm can enter the market is large. This region also becomes smaller for a larger capacity level of the hidden firm for the reason that the leader has less incentives to overinvest.

2.1.2 The leader's accommodation strategy

An entry deterrence strategy is not the only option for the leader to implement. In fact, the market can be big enough for both positioned firms to invest at once. The leader can choose such an investment scale, that the follower will enter the market immediately after the leader. We call this strategy the accommodation strategy.

Proposition 5. *The starting point of the accommodation region, x_1^{acc}, is determined by*

$$x_1^{acc} = \frac{(\beta+3)}{(\beta-1)}\delta(r-\alpha). \tag{18}$$

Note, that x_1^{acc} does not depend on the capacity level of the hidden firm, because under the assumption of a market with only two places available it is impossible for the third firm of any size to enter the market, given that the leader has entered and applies the accommodation strategy. However, the probability λ still affects the starting point of the accommodation region. Differentiating (18) with respect to λ we get[3]

$$\frac{\partial x_1^{acc}}{\partial \lambda} = -\frac{4\delta(r-\alpha)}{(\beta-1)^2} \cdot \frac{\partial \beta}{\partial \lambda} < 0, \tag{19}$$

The interpretation of the decline in x_1^{acc} with λ is straightforward. The bigger is the chance that the hidden firm can become active on the market, the earlier the positioned firms should undertake their investment, because the follower otherwise faces a high probability to lose its investment option.

2.1.3 The leader's boundary strategy

The main difference between the model presented by Huisman and Kort (2013) and the current modification is that the results of the latter are to a great extent influenced by two additional parameters associated with hidden competition. The key assumption of the presented model is that the positioned firms face a non zero probability of hidden entry, λdt. The investment size of the hidden player is represented by the parameter q_H. Figure 3 depicts the standard scenario with no hidden entries as well as the situation when the positioned firms face a positive probability that a hidden firm can enter the market by investing in a positive capacity. The capacity levels $q_L^{det}(x)$, $q_L^{acc}(x)$ and $\hat{q}(x)$ for both cases are presented as functions of the stochastic profitability shock, x. This specific example point out important differences of the presented setting with a standard duopoly model.

[3] Here we use the observation that $\frac{\partial \beta}{\partial \lambda} = \frac{1}{\sqrt{\left(\alpha - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(\lambda + r)}} > 0.$

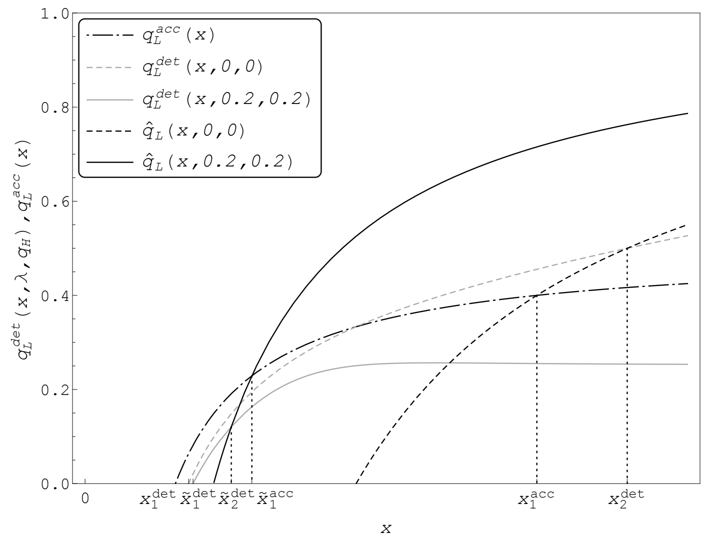


Figure 3: *The capacity levels* $q_L^{det}(x)$, $\hat{q}_L(x)$ *and* $q_L^{acc}(x)$ *for the set of parameter values:* $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$.

Earlier x_1^{det} was defined as the starting point of the deterrence region. Therefore, in this figure x_1^{det} is determined by the intersection of q_L^{det} and horizontal axis. To ensure that the follower invests later than the leader, the condition that the leader's capacity is bigger than $\hat{q}$ has to be satisfied. In contrast, in order to implement the accommodation strategy the leader should choose a capacity level below $\hat{q}$. Thus, the ending point of the deterrence region, x_2^{det}, and the starting point of accommodation region, x_1^{acc}, can be found as intersections of $\hat{q}$ and q_L^{det} or q_L^{acc} respectively.

For the values of λ close to zero the picture resembles the result of Huisman and Kort (2013). Namely, the deterrence and accommodation regions intersect ($x_1^{acc} < x_2^{det}$). In addition, for the values of x below x_1^{acc} only deterrence can occur, in the region above x_2^{det} only accommodation is possible, whereas in the interval (x_1^{acc}, x_2^{det}) the leader chooses the strategy that brings the bigger value. In Figure 3 the functions related to this case are represented by dashed lines, while the endpoints of the deterrence and accommodation regions are marked with tilde.

However, as soon as the parameters associated with hidden competition sufficiently increase, the situation presented above changes. The functions for the case when $\lambda = 0.2$ and $q_H = 0.2$ are represented by the solid lines. As mentioned earlier, the parameters λ and q_H affect the boundaries of the feasible regions both for the deterrence and accommodation strategy. As it can be seen, $\hat{q}_L$ shifts upwards, while q_L^{det} shifts downwards for every value of x, causing x_1^{acc} and x_2^{det} to change in such a way that now $x_1^{acc} > x_2^{det}$. The leader chooses deterrence if x lies in the interval between x_1^{det} and x_2^{det} and the accommodation strategy can only be implemented when x is bigger that x_1^{acc}. Yet, in the interval between x_1^{acc} and x_2^{det} neither a deterrence nor an accommodation optimal capacity level can be installed by the leader and in this region it is optimal for him to invest at the boundary level. This situation is illustrated in the next figure.

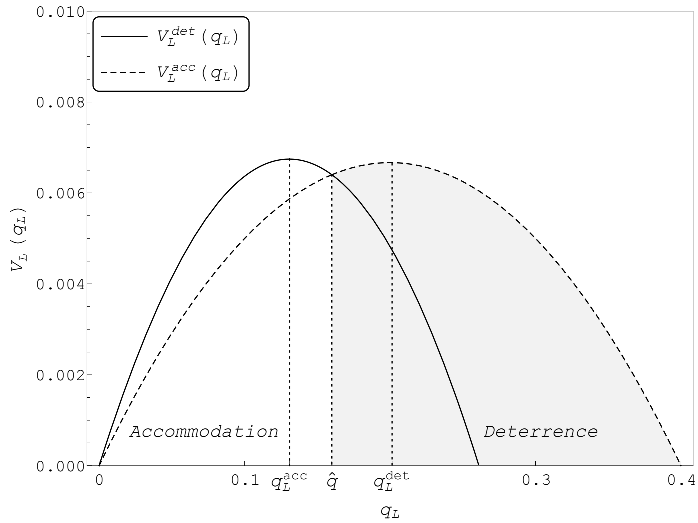


Figure 4: *The value functions $V_L^{det}(q_L)$ and $V_L^{acc}(q_L)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$, $\lambda = 0.22$, $q_H = 0.2$, $x = 0.01$.*

The intuition behind this result is as follows. In the presence of a high risk that the hidden firm will enter the market it is harder to deter the follower from occupying the last available place. Therefore, on the one hand we observe the shrinkage of the deterrence region for large λ, resulting in the fact that the optimal capacity level for which deterrence is optimal falls below $\hat{q}$, which is in the accommodation region (see Figure 4). On the other hand, the optimum in terms of capacity choice cannot be reached for the accommodation strategy either, as the market is not yet big enough. This we see in Figure 4, where maximal accommodation profits are reached for a q being greater than $\hat{q}$, which is in the deterrence region. Therefore, the leader can only invest at the boundary, i.e. choose the capacity level $\hat{q}$ and enter the market simultaneously with the follower. Proposition 6 gives the condition under which this boundary solution occurs.

Proposition 6. *Given that $\lambda(8q_H - 1) > r - \alpha$, it holds that $x_2^{det} < x_1^{acc}$.*

The above condition is sufficient for the boundary region to exist. The obtained result entails that if the parameters reflecting the degree of the hidden competition, λ and q_H, become big enough, while the difference $r - \alpha$ stays relatively low, the boundary region always exists. This can be interpreted in the following way. High λ means larger risk of losing the last place on the market for the follower. Smaller r leads to higher NPV of the future cash flows from an investment, while large α implies better market growth prospects. As a result for smaller r or (and) larger α the follower is more reluctant to lose his investment option. Therefore, for larger λ, larger α and smaller r to secure the place in the market the follower chooses simultaneous investment earlier, before the optimum for the accommodation strategy is reached. This guarantees the existence of the boundary strategy. Larger capacity of the hidden firm, q_H, decreases attractiveness of the investment for the leader. As stated in Proposition 3 the bigger firm is expected to enter, the less incentives has the leader to implement the deterrence strategy, even when the market is not yet big enough to install optimal accommodation capacity. This makes room for the additional entry accommodation, or in other words the boundary strategy.

2.2 Endogenous firm roles

In this section we assume that the roles of the leader and the follower are not preassigned and both positioned firms are allowed to invest first. For low values of x the firm investing second gets the largest value. Thus, in this case the demand level is so small that it is optimal for the positioned firms to postpone their investment.

However, once the market becomes big enough the role of the market leader becomes more beneficial. In the current setting the advantage of the first investor comes from two different sources: investment timing and capacity choice. The timing effect is associated with the fact that increasing market profitability creates incentives for the firms to preempt their rival and thereby to induce the second investor to enter later. In this case the reward for the first entrant is a period of monopoly profits. As a result the firms engage in timing preemption. In particular, as long as the value of the first investor exceeds the value of the second investor, each positioned firm will have an incentive to invest a little earlier in order to become market leader.

On the other hand, if the degree of hidden competition is large the value of the deterrence strategy decreases as it becomes too costly to prevent entry of the second firm. Therefore, in the deterrence region the market is too small to bear such costs and the leader gets lower value. As a result it is optimal for the firms to wait till simultaneous investment is possible. However, even when the market is big for the firms to invest at once, the concept of Stackelberg leadership implies that the leader has an early mover advantage and sets the capacity level first causing a difference in payoffs of the first and second investors. This results into what we have named a capacity preemption game where the firms have incentives to invest in a smaller capacity which allows to undertake earlier investment and take the leader's position.

Both preemption games described above hold, unless the advantage of being the first investor is not present. This is when the market is so small that the first and the second investor get exactly the same value. We denote the value of the stochastic process that corresponds to the preemptive equilibrium by x_p and call it the preemption trigger. This point can be located in three different regions: where the leader applies either deterrence or boundary or accommodation strategy. Given that at the preemption point leader and follower values must match, the next proposition tells us that the preemption point cannot be situated in the accommodation region.

Proposition 7. *If $x \geq x_1^{acc}$, the value of the leader always exceeds the value of the follower.*

Consequently, it is either the deterrence or the boundary capacity level that determines the preemption trigger, which is derived as an intersection of the corresponding follower and leader value functions. These intersections are denoted by x_p^{det} and $\hat{x}_p$ respectively and can be found by solving the equations below:

$$L^{det}(x_p^{det}) = F(x_p^{det}, q_L^{det}), \tag{20}$$

$$\hat{L}(\hat{x}_p) = F(\hat{x}_p, \hat{q}_L). \tag{21}$$

Recall that if λ is small, the boundary strategy is irrelevant. Therefore, as in the benchmark model of Huisman and Kort (2013) where $\lambda = 0$, the preemption equilibrium always occurs in the deterrence region, implying that the first investor enters as soon as the stochastic process x hits the preemption trigger, x_p^{det}, while the second investor postpones its entry till x_F. However, for sufficiently large λ it is also possible that the preemption trigger lies in the boundary region, where it is optimal for the firms to invest simultaneously at $\hat{x}_p$. The latter situation is illustrated in Figure 5.

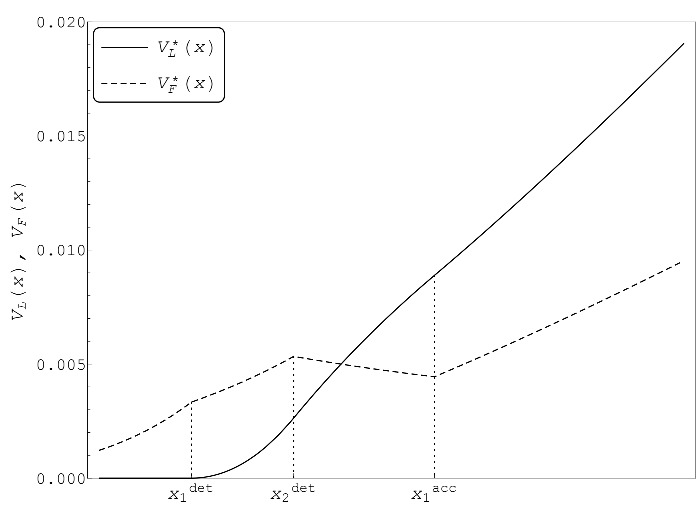


Figure 5: *The value functions $V_L^*(x)$ and $V_F^*(x)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$, $\lambda = 0.22$, $q_H = 0.4$.*

As it can be seen, in contrast to the standard result the follower value declines with x in the boundary region. This result is analytically true, which is stated in the following proposition.

Proposition 8. *The value of the follower declines with x in the boundary region.*

Intuitively this result can be interpreted in the following way. The follower value can be affected by an increase in x in two ways: via investment timing and via capacity choice. In this problem the investment timing of the follower is always given. This is because the boundary capacity level of the leader is determined such that $x = x_F(q_L)$, implying that as the stochastic component of the demand function increases the leader increases its capacity such that the new level of x exactly corresponds to the follower investment threshold. Proposition 8 proves that the follower value is more influenced by the capacity effect, i.e. it declines as the capacity level of the leader increases, than the increase in price for a given capacity due to the growth of x. As a result the follower gets lower value for larger x due to an increase in the leader capacity level. This allows the leader and the follower values to intersect in the boundary region, implying that preemption point is located in an interval where the firms invest simultaneously. In this case the firms engage in capacity preemption.

Proposition 9. *There exists $\lambda_p(q_H)$ such that for $\lambda \geq \lambda_p(q_H)$, preemption always occurs for a boundary capacity level, while for $\lambda < \lambda_p(q_H)$ – for the deterrence capacity level.*

$$x_p(\lambda, q_H) = \begin{cases} x_p^{det}(\lambda, q_H) & \textit{if } \lambda \leq \lambda_p(q_H), \\ \\ \hat{x}_p(\lambda) & \textit{if } \lambda > \lambda_p(q_H). \end{cases} \tag{22}$$

This $\lambda_p(q_H)$ implicitly determined by

$$\frac{\lambda q_H}{\lambda + r - \alpha} - \frac{\beta(\lambda)}{(\beta(\lambda) - 1)(\beta(\lambda) + 2)} = 0, \tag{23}$$

with

$$\beta(\lambda) = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}}. \tag{24}$$

Proposition 9 means that λ_p is the arrival rate of the hidden firm such that for $\lambda \geq \lambda_p(q_H)$ both positioned firms invest simultaneously at the boundary capacity level, $\hat{q}$, while similarly to the original model by Huisman and Kort (2013) for $\lambda < \lambda_p(q_H)$, the first investor implements an entry deterrence strategy acquiring q_L^{det}. Intuitively, the larger is the hidden firm that is expected to enter the market, the more attractive is the boundary strategy for the positioned firms, as it guaranties that the hidden player loses the chance to invest and the moment that both firms enter. Hence, the leader is not exposed to the risk that it has to compete with a large hidden firm since the other positioned firm invests at the same time as the leader.

Proposition 10. *The arrival rate $\lambda_p(q_H)$ declines with the size of the hidden entrant, q_H.*

Thus, a larger capacity of the hidden player implies a larger range of λ and q_H for which simultaneous investment takes place. This is illustrated by the numerical example in Figure 6. As it can be seen, for a larger capacity of the hidden firm the positioned firms are more willing to invest at once, as by doing so they occupy all available places on the market and thus prevent undesirable entry of a large hidden player.

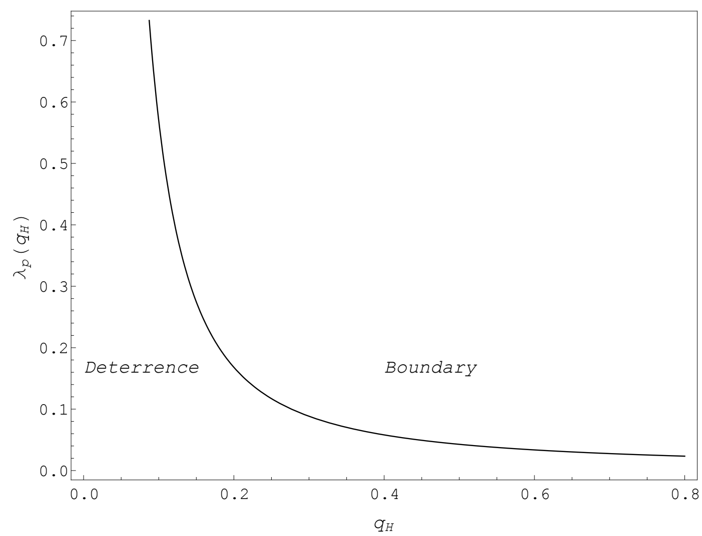


Figure 6: *The possible strategies of the leader depending on λ and q_H for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$.*

The proposition below gives an optimal investment trigger and the corresponding capacity level of the positioned firms when $\lambda \geq \lambda_p(q_H)$, i.e. when they enter the market simultaneously due to the relatively high risk of hidden competition.

Proposition 11. *The preemption trigger determined by the boundary capacity level, $\hat{x}_p$ is given by*

$$\hat{x}_p = \frac{\delta(r-\alpha)(\beta+2)}{(\beta-1)}, \tag{25}$$

with the corresponding capacity level

$$\hat{q}(\hat{x}_p) = \frac{1}{\beta+2}. \tag{26}$$

Note that this investment threshold implies that the positioned firms not only invest at the same time, but also at the same capacity level, as the first mover advantage of the leader disappears due to the rent equalization property in the preemption game.

Differentiating (25) and (26) with respect to λ we get[4]

$$\frac{\partial \hat{x}_p}{\partial \lambda} = -\frac{3\delta(r-\alpha)}{(\beta-1)^2} \cdot \frac{\partial \beta}{\partial \lambda} < 0, \tag{27}$$

$$\frac{\partial \hat{q}(\hat{x}_p)}{\partial \lambda} = -\frac{1}{(\beta+2)^2} \cdot \frac{\partial \beta}{\partial \lambda} < 0. \tag{28}$$

Thus, we observe the negative dependence between arrival rate λ and the preemption trigger $\hat{x}_p$, as well as the capacity level at this preemption point $\hat{q}(\hat{x}_p)$. To interpret this result recall that

$$\hat{q}(x) = 1 - \frac{(\beta+1)\delta(r-\alpha)}{(\beta-1)x}, \tag{29}$$

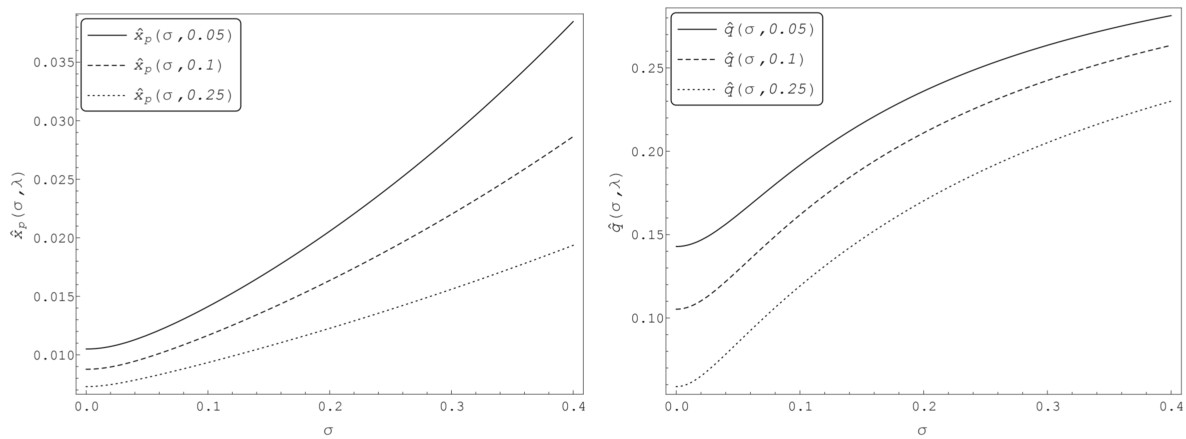
$$\frac{\partial \hat{q}(x)}{\partial \lambda} = \frac{2\delta(r-\alpha)}{(\beta-1)^2 x} \cdot \frac{\partial \beta}{\partial \lambda} > 0, \tag{30}$$

where $\hat{q}(x)$ is the maximal capacity level of the leader such that the follower invests immediately for a given x ($x = x_F$). As it can be seen from (30) the boundary capacity level for a given x is larger if λ increases. In other words the follower facing the threat of loosing the last available place of the market is willing to accommodate for a larger level of the leader's capacity for a given x. Consequently, the bigger is λ the closer is leader's capacity level to the optimum level for the accommodation strategy leading to an increase in the leader value. The follower value on the contrary decreases with λ as a result of an increase in the leader's capacity level. An increase of the leader value together with decrease in the follower value result in the earlier preemption point. That is why $\hat{x}_p$ decreases with λ as well as the capacity level at this point.

Note that the capacity of the hidden firm, q_H, does not exert an influence on the preemption point $\hat{x}_p$ in this case. This happens because applying the boundary strategy implies that both firms invest at once occupying all available places on the market and therefore the third player, the hidden firm, looses the option to invest, i.e. to install capacity.

The analytical expressions for the preemption trigger, (25), and the capacity level, (26), corresponding to the boundary strategy can be used to analyze the effect of uncertainty. The next figure shows how a change in σ affects $\hat{x}_p$ and $\hat{q}(\hat{x}_p)$ for different values of λ.

[4] Given that $\frac{\partial \beta}{\partial \lambda} > 0$.



(a) *The preemption trigger* $\hat{x}_p(\sigma)$ (b) *The capacity level* $\hat{q}(\hat{x}_p(\sigma), \sigma)$

Figure 7: *The preemption trigger* $\hat{x}_p(\sigma)$ *and the corresponding capacity level* $\hat{q}(\hat{x}_p(\sigma), \sigma)$ *for the set of parameter values:* $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$.

As in Huisman and Kort (2013) both the preemption trigger and the corresponding quantity increase with uncertainty. This confirms the standard result in the real options literature that uncertainty creates a value of waiting causing the firms to postpone their investment decisions. In the present model an increase in investment threshold due to uncertainty is less for larger λ, as an increasing probability of the hidden entry induces earlier investment.

Consider now the case when $\lambda < \lambda_p(q_H)$ and the first investor prevents an immediate entry of the second investor by installing the deterrence capacity, q_L^{det}. In this case the leader invests at the moment x hits x_p^{det} , while the follower waits till x_F. These thresholds are described by the following proposition.

Proposition 12. *The preemption trigger for the deterrence capacity level,* x_p^{det} *is determined by*

$$L^{det}(x_p^{det}, q_L^{det}) = F(x_p^{det}, q_L^{det}), \tag{31}$$

with

$$F(x, q_L^{det}) = A(q_L^{det})x^{\beta}, \tag{32}$$

$$\begin{aligned} L^{det}(x, q_L^{det}) =& x\frac{q_L^{det}(1-q_L^{det})}{r-\alpha} - x\frac{q_L^{det}\lambda q_H}{(\lambda+r-\alpha)(r-\alpha)} - \delta q_L^{det} \\ &- \left(\frac{x(\beta-1)(1-q_L^{det})}{\delta(r-\alpha)(\beta+1)}\right)^{\beta}\left[\frac{\delta q_L^{det}}{(\beta-1)} - \frac{\delta(\beta+1)q_L^{det}\lambda q_H}{(\beta-1)(1-q_L^{det})(\lambda+r-\alpha)}\right], \end{aligned} \tag{33}$$

and q_L^{det} implicitly determined by

$$\left(\frac{x(\beta-1)(1-q_L)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta}\frac{\delta}{(\beta-1)}\left[-\frac{(1-(\beta+1)q_L)}{(1-q_L)}+\frac{(\beta+1)\lambda q_H}{(\lambda+r-\alpha)}\frac{1-\beta q_L}{(1-q_L)^2}\right]$$
$$+\frac{x(1-2q_L)}{r-\alpha}-\frac{x\lambda q_H}{(r-\alpha)(\lambda+r-\alpha)}-\delta=0. \tag{34}$$

The second investor enters the market once x reaches

$$x_F=\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)(1-q_L^{det})}, \tag{35}$$

acquiring a capacity of

$$q_F=\frac{1-q_L^{det}}{\beta+1}. \tag{36}$$

Consider now the dependence between the preemption point x_p and the capacity of the hidden firm for a given λ.

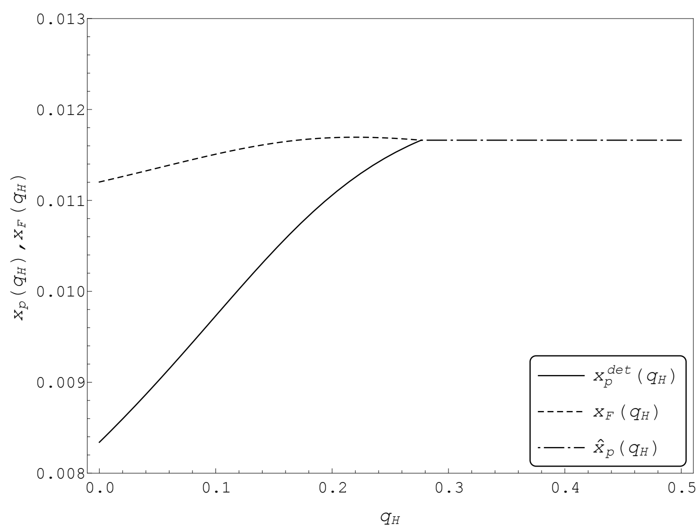


Figure 8: *The preemption trigger $x_p(q_H)$ and the follower trigger $x_F(q_H)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$, $\lambda = 0.1$.*

In the Figure 8 the preemption point for the boundary strategy, $\hat{x}_p$, is not affected by the capacity of the hidden firm, because as mentioned earlier the simultaneous investment of the positioned firms implies that the hidden player loses the chance to install capacity. The effects of an increase in q_H on the deterrence preemption trigger, x_p^{det}, and the follower's investment threshold, x_F, result from changes in the leader and the follower values.

Intuitively, the leader value is lower if the hidden firm is larger. This is because the market becomes less profitable given that when the hidden firm becomes active, it does so by installing a larger capacity. An increase in the capacity level of the hidden firm affects the follower's value only through the leader's capacity choice. The bigger the potential entrant is, the more incentives the leader has to reduce its capacity and as a result to stimulate the follower to enter the market earlier in order to prevent the hidden entry. Hence, the follower value increases for every x. Together with the decrease of the leader value, this shifts the preemption point to the right.

The follower's entry threshold is also influenced only indirectly by the capacity of the hidden firm through the leader's capacity choice at its optimal point in time. Due to the described effects, the bigger is the hidden firm that is expected to enter, the later invests the leader and later investment implies a larger capacity. On the other hand, the leader reduces the capacity for every level of x. The combination of these two effects is the reason for the non-monotonicity in the optimal leader capacity at the preemption point as a function of q_H as demonstrated in the Figure 9.

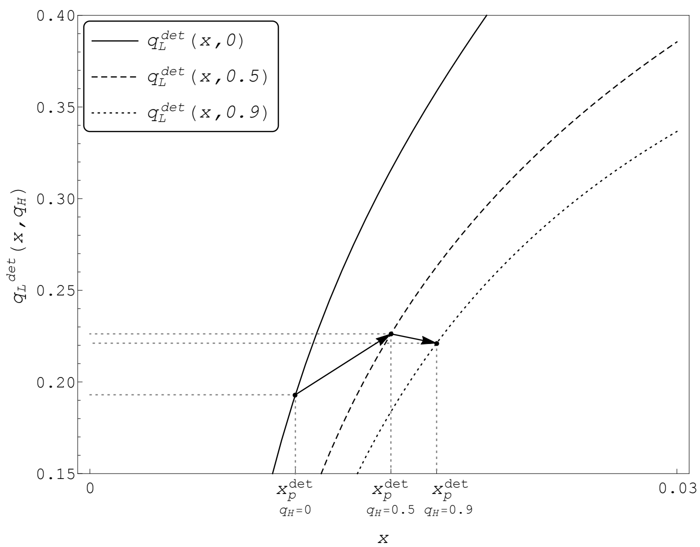


Figure 9: *The capacity level* $q_L^{det}(x, q_H)$ *for different values of* q_H *for the set of parameter values:* $r = 0.05$*,* $\alpha = 0.02$*,* $\sigma = 0.1$*,* $\delta = 0.2$*.*

Due to this non-monotonic relationship between leader and capacity and capacity of the hidden firm, the follower threshold first increases with q_H and then declines until it reaches the threshold for the boundary strategy as depicted in Figure 8.

Now consider the influence of the arrival rate of the hidden firm on the firm's optimal investment thresholds under the deterrence strategy.

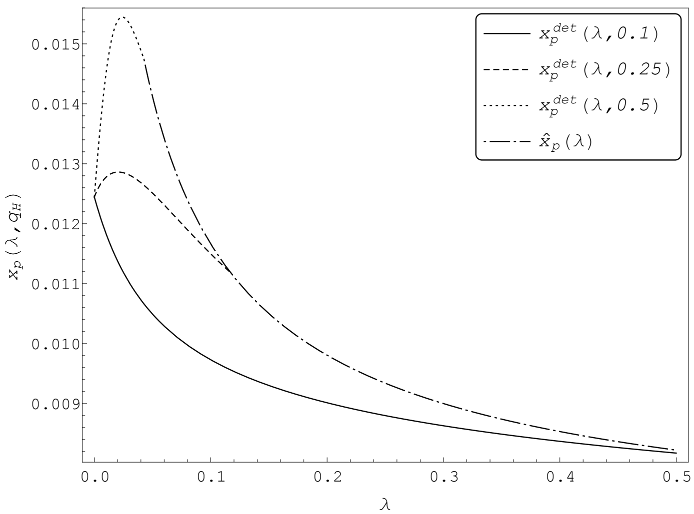


Figure 10: *The preemption trigger* $x_p(\lambda)$ *for the set of parameter values:* $r = 0.05$*,* $\alpha = 0.02$*,* $\sigma = 0.1$*,* $\delta = 0.2$*.*

Figure 10 depicts the dependence between the preemption trigger, x_p, and the arrival rate of the hidden competitor, λ, for different capacity size of the hidden player. The dashed line in the figure depicts the investment trigger related to the boundary region, whereas the solid lines, each corresponding to a different capacity level of the hidden firm, represent the deterrence investment trigger, which is now the focus of our analysis. As it was shown before, for a larger capacity the region where it is optimal for the first investor to use the deterrence strategy shrinks. Moreover, we observe different effects of an increase in the arrival rate of the hidden firm on the investment threshold for different size of the hidden firm. Thus, to interpret this result it is reasonable to consider the scenarios of small and large q_H separately. In what follows we first consider the scenario with $q_H = 0$, followed by an analysis of a situation with large q_H.

2.3 Small hidden firm

If the capacity of the hidden firm is small its entry is beneficial for the leader. In the extreme case of $q_H = 0$ the advantage of hidden entry is particularly big as it implies that the follower loses the option to invest and the leader becomes a monopolist on the market. Thus, an increase in the arrival rate of the hidden firm has a direct effect on the leader value, namely, the value increases for a given x due the attractiveness of the investment opportunity. On the other hand, a higher probability of hidden entry affects the follower's decision, resulting into an indirect effect on the leader value. In fact, facing the threat of losing the last available place on the market the follower is more eager to invest earlier. Therefore, it is more costly for the leader to perform the deterrence strategy, i.e. to induce the follower to invest later. Thus, for each level of x bigger capacity is needed to ensure that the follower indeed invests later.[5] This leads to a decrease in the leader value for a given level of x. The total change in the leader value is determined by the predominance of one of these effects.

Intuitively, for small x, when the investment opportunity is unappealing, the follower is less eager to invest and the direct effect dominates. However once x becomes sufficiently large the investment becomes more attractive, making the credible deterrence of the follower's entry more difficult. Thus, the second effect becomes dominant leading to a decrease in the leader value. Moreover, for larger λ the second effect starts dominating earlier, as the follower becomes more aggressive facing bigger risk of the hidden entry.

As a result of entry deterrence the follower value is always lower for larger λ, as it is forced to postpone its investment, while at the same time the probability to lose its investment option is large. Moreover, numerical experiments show that as a result of an increase in λ the decline in the follower value is always large enough to ensure that the intersection of the leader and follower value curves takes place for lower values of x. This results in the fact that the preemption point always declines with λ for relatively small capacity of the hidden firm, Figure 11 illustrates this result.

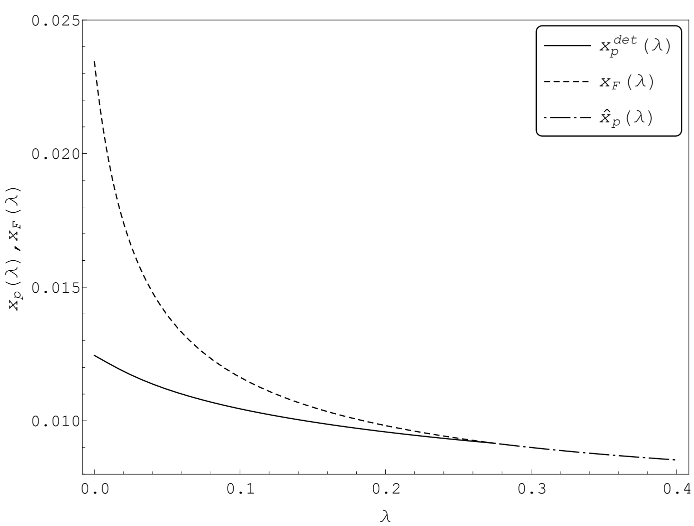


Figure 11: *The preemption trigger $x_p(\lambda)$ and the follower trigger $x_F(\lambda)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$, $q_H = 0.15$.*

The implication for the follower threshold is that it is influenced by a change in λ both by the desire to stay on the market, and by the change in the optimal capacity level of the leader at the

[5] Although analytical expressions for q_L^{det} cannot be obtained, careful numerical simulations confirm that the presented relation holds for the considered range of λ, $\lambda < \lambda_p(q_H)$.

moment of investment. The optimal capacity of the leader is in turn affected by both preemption timing and the capacity choice that insures credible deterrence. As discussed earlier an increase in λ leads to an upward shift in the leader capacity level for each value of x together with a decrease in the leader investment threshold as a result of preemption game. Therefore, if the decrease in investment timing is large enough, the leader capacity at the moment of investment will be lower for larger λ. However, for a relatively small decrease in the investment threshold together with larger upward shift in the capacity curve an increase in λ can result in a larger leader optimal capacity at the moment of investment. This causes the non-monotonicity in the follower investment threshold similar to the one shown on the Figure 9. However, in the considered model the risk associated with later investment is too high, as the opportunity to enter the market might be lost forever. As a result, an increase in λ causes the follower to become more aggressive and invest earlier for higher λ.

2.4 Large hidden firm

Now consider the situation when q_H is sufficiently large. Here we restrict ourselves to scenarios where the firm's revenues are positive. Namely, we exclude the possibility of negative prices by restricting the considered range of the capacity of the hidden firm, namely $q_H \leq \max\{1, 1-q_L\}$. For q_H being large the opposite situation occurs. In the event of the hidden entry the leader is left with a relatively small market share, and thus wants the follower to invest earlier to prevent the hidden entry for bigger values of λ. Therefore, as λ increases the leader reduces its capacity to tempt the follower to enter the market for lower x.

The effect of an increase in λ on the value functions can again be decomposed in two parts. On the one hand, the bigger the arrival rate the more probable it is that the hidden firm enters with large capacity, thus the lower is the leader value due to the direct effect. On the other hand, the bigger is the arrival rate the more eager is the follower to invest, which is good for the leader. Therefore, its value is increasing due to the indirect effect.

For small x investment is not attractive yet, thus, an increase in λ exerts less influence on the follower's investment decision. Therefore, the direct effect associated with the entrance of a large hidden player dominates causing the leader value to decrease. For larger x the investment opportunity is more valuable and as a result the follower is willing to invest sooner to occupy the last available place on the market. This situation is favorable for the leader and, therefore, the second effect of an increasing leader's value becomes more important. As in the previous case the larger is λ, the sooner the indirect effect becomes dominant. This happens because in the presence of bigger hidden entry risk the desire of the follower to invest sooner outweighs the direct negative effect of possible hidden entry for lower x. On the other hand, the follower's value in this case increases with λ as the leader rewards the follower even more for early investment, while the follower is already willing to do so.

Combing these results we conclude that when market is small it is more likely that the leader declines with λ while the follower value increases, resulting in an increase in their intersection point. When the bigger market becomes more profitable, larger λ leads to the increase in both follower and leader values, shifting their intersection point to the left.

As a result, the effect of an increase λ on the preemption point for the deterrence strategy which is determined by the intersection of the value functions is non-monotonic. In particular as we observe in the next Figure $x_p^{det}(\lambda)$ first increases with λ and then starts declining.

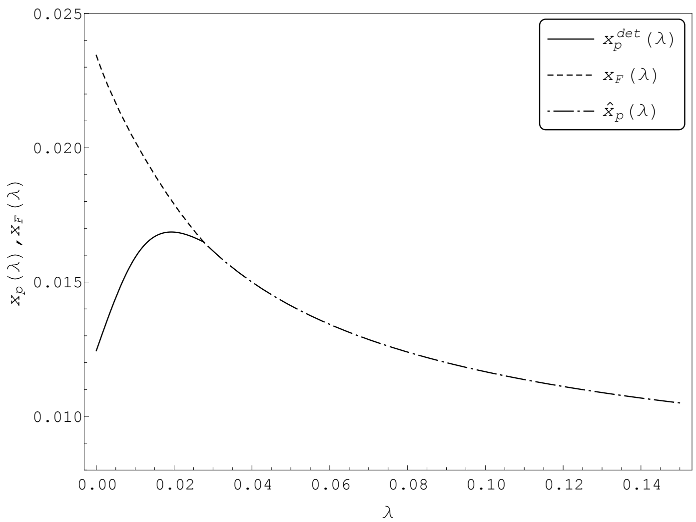


Figure 12: *The preemption trigger $x_p(\lambda)$ for the set of parameter values: $r = 0.05$, $\alpha = 0.02$, $\sigma = 0.1$, $\delta = 0.2$, $q_H = 0.7$.*

For the follower the implication is the same as in the case of zero capacity of the hidden firm. Namely, increasing risk of the hidden entry induces the follower to invest earlier in order not to lose the chance to enter the market and as a result the follower trigger declines with λ.

3 Conclusion

This paper examines firm's strategies when making an investment decision under uncertainty, which includes both timing and the capacity level. After allowing the firms to choose the capacity level in the duopoly model with hidden competition, we found new effects of the possible entry occasion of the hidden firm. As a result, and in contrast with the basic model, the optimum for the accommodation capacity level is not available immediately after the deterrence region ends and for a non-zero probability of the hidden entry a gap between the two strategies is generated. Intuitively, the deterrence region becomes smaller if the uncertainty about the market participants is high, because the follower facing the threat of losing its investment option is more eager to invest earlier and it is getting harder for the leader to deter this entry. On the other hand, the market is not yet big enough to acquire the optimal capacity level for the accommodation strategy. Thus, in the gap region the leader choses the strategy that maximizes his value, namely, invests at the boundary capacity level stimulating immediate investment of the follower. This strategy is equivalent to the entry accommodation in terms of timing. However the usual entry accommodation implies a larger capacity level of the leader. Therefore, in the endogenous roles game when relatively large hidden player is expected to enter with a sufficient probability, the deterrence strategy becomes too costly to implement and the first investor deterring the entry of the second firm always gets a lower value. As a result, the preemption game between positioned firms leads to capacity preemption implying that the leader and follower enter the market simultaneously.

Finally, it is important to point out the possibilities for the further research related to this topic. First, it is worth mentioning that the obtained results are derived for the specific case of the market with only two places available. Thus, it is important to consider the more general setting where the assumption of the limited market places is relaxed in a way that the follower does not lose the chance to invest once the hidden competitor becomes active on this market. Intuitively, as long as the multiplicative demand function is used the firms are limited in the capacity expansion by the condition for non-negative prices. As long as we consider the markets described by this specific demand structure, the positioned firms will always have incentives to install the capacity large enough to prevent the entry of the hidden player. That is why it is interesting to consider how the assumption of hidden competition affects optimal investment behavior of firms on markets described by alternative demand functions with unlimited places available.

Further analysis also needs to be done to examine the influence of the entry decisions of the positioned firms on the arrival rate of the hidden firms. The more profitable is the market, the more attractive is this market for the potential entrants. Therefore, the mean arrival rate of the hidden rivals may decline with every new entry, as the market becomes less profitable.

4 Appendix

Proof of Proposition 1

To determine the optimal quantity, the follower solves the following maximization problem, given the level of Brownian motion, x :

$$V(x) = \max_Q E\left[\int_0^\infty q_F x(1-Q)\exp(-rt)dt - \delta q_F\right]. \tag{37}$$

If the leader has already invested, the value of stopping for the follower, denoted by $F(x)$, is given by [6]

$$F(x) = E\left[\int_0^\infty \pi e^{-rt}dt - \delta q_F\right] = \frac{xq_F(1-Q)}{r-\alpha} - \delta q_F. \tag{38}$$

The first order condition for the follower in this case takes the following form:

$$\frac{\partial}{\partial q_F}\left[\frac{x}{r-\alpha}\left(1-(q_F+q_L)\right)q_F - \delta q_F\right] = 0. \tag{39}$$

Thus, the follower's optimal capacity level is equal to

$$q_F^*(x, q_L) = \frac{1}{2}\left(1 - q_L - \frac{\delta(r-\alpha)}{x}\right). \tag{40}$$

The total quantity, Q, takes now the following form

$$Q(q_L) = Q(q_F^*(q_L), q_L) = q_L + q_F^*(q_L) = \frac{1}{2}\left(1 + q_L - \frac{\delta(r-\alpha)}{x}\right). \tag{41}$$

Substituting the expression for q_F into the follower's value function, we get

$$F(x) = \frac{[x(1-q_L) - \delta(r-\alpha)]^2}{4x(r-\alpha)}. \tag{42}$$

The value function of the follower in the continuation region can be found by solving[7]

$$\frac{1}{2}\sigma^2 x^2 \frac{\partial^2 F(x)}{\partial x^2} + \alpha x \frac{\partial F(x)}{\partial x} - (r+\lambda)F(x) = 0. \tag{43}$$

Denoting x_F is the trigger value for the follower, we consider the following boundary conditions:

$$\lim_{x\to 0} F(x) = 0 \tag{44}$$

[6] The discount factor is assumed to be larger than drift, $r > \alpha$, otherwise waiting would always be an optimal policy.

[7] The problem is solved by applying dynamic programming methods presented in Dixit and Pindyck (1994).

$$\lim_{x \to x_F} F(x) = \frac{x_F q_F (1-Q)}{r-\alpha} - \delta q_F \tag{45}$$

$$\lim_{x \to x_F} \frac{\partial F(x)}{\partial x} = \frac{q_F(1-Q)}{r-\alpha}. \tag{46}$$

Considering the condition (44) we can write the solution of the presented above differential equation as $F(x) = Ax^\beta$ with β equal to[8]

$$\beta = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(-\frac{1}{2} + \frac{\alpha}{\sigma^2}\right)^2 + \frac{2(r+\lambda)}{\sigma^2}} > 1. \tag{47}$$

From the value matching (45) and smooth pasting (46) conditions we get the expression for A

$$A = \frac{1}{x_F^\beta}\left(\frac{x_F q_F(1-Q)}{r-\alpha} - \delta q_F\right), \tag{48}$$

and for the trigger value x_F:

$$x_F = \frac{\beta\delta(r-\alpha)}{(\beta-1)(1-Q)}. \tag{49}$$

From the previous section, at the moment of investment the optimal capacity of the follower q_F^* and the total quantity on the market, Q, are given by

$$q_F^*(x, q_L) = \frac{1}{2}\left(1 - q_L - \frac{\delta(r-\alpha)}{x}\right), \tag{50}$$

$$Q(x, q_L) = \frac{1}{2}\left(1 + q_L - \frac{\delta(r-\alpha)}{x}\right). \tag{51}$$

Hence, the optimal investment trigger, x_F^* and the follower's quantity, q_F^* given the capacity of the leader, q_L, are defined by

$$x_F^*(q_L) = \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)(1-q_L)}, \tag{52}$$

$$q_F^*(q_L) = \frac{1-q_L}{\beta+1}. \tag{53}$$

Substituting the results in the expression for A:

$$A(q_L) = \left(\frac{(\beta-1)(1-q_L)}{\delta(r-\alpha)(\beta+1)}\right)^\beta \frac{\delta(1-q_L)}{(\beta-1)(\beta+1)}. \tag{54}$$

[8]Here β is the larger root of the fundamental quadratic equation $\frac{1}{2}\sigma^2 x\beta^2 + (\alpha - \frac{1}{2}\sigma^2)\beta - (r+\lambda) = 0$.

Therefore, the compact solution for the follower's problem is given by

$$F^*(x) = \begin{cases} A(q_L)x^\beta & \text{if } x < x_F^*(q_L), \\ \dfrac{[x(1-q_L) - \delta(r-\alpha)]^2}{4x(r-\alpha)} & \text{if } x \geq x_F^*(q_L). \end{cases} \tag{55}$$

Proof of Proposition 2

Define $\psi(x)$ as the left hand side of the the equation (34) in the case when $q_L = 0$:

$$\psi(x) = \left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^\beta \frac{\delta}{(\beta-1)}\left[-1 + \frac{(\beta+1)\lambda q_H}{(\lambda+r-\alpha)}\right] + \frac{x}{r-\alpha} - \frac{x\lambda q_H}{(r-\alpha)(\lambda+r-\alpha)} - \delta. \tag{56}$$

Therefore, x_1^{det} is implicitly determined by the equation $\psi(x) = 0$.

Note that

$$\psi(0) = -\delta < 0, \tag{57}$$

$$\psi(x_F^*(0)) = \frac{\delta}{\beta-1} > 0, \tag{58}$$

$$\frac{\partial\psi(x)}{\partial x} = \frac{(\lambda+r-\alpha) - \lambda q_H}{(\lambda+r-\alpha)(r-\alpha)}\left(1 - \frac{(\lambda+r-\alpha)-(\beta+1)\lambda q_H}{(\lambda+r-\alpha)-\lambda q_H}\frac{\beta}{\beta+1}\left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta-1}\right). \tag{59}$$

Differentiating (59) with respect to x we get:

$$\frac{\partial^2\psi(x)}{\partial x^2} = x^{\beta-2}\frac{\beta\delta\left(\frac{(\beta-1)}{(\beta+1)\delta(r-\alpha)}\right)^\beta((\beta+1)\lambda q_H - (\lambda+r-\alpha))}{(\lambda+r-\alpha)}. \tag{60}$$

For $x \geq 0$ the above function is either monotonically increasing or monotonically decreasing depending on the combination of the parameter values.

Consider $x \in (0, x_F^*(0))$. Evaluating the first derivative $\dfrac{\partial\psi(x)}{\partial x}$ at the endpoints of this interval we obtain

$$\left.\frac{\partial\psi(x)}{\partial x}\right|_{x=0} = \frac{\lambda(1-q_H) + r - \alpha}{(\lambda+r-\alpha)(r-\alpha)} \geq 0 \tag{61}$$

$$\left.\frac{\partial\psi(x)}{\partial x}\right|_{x=x_F^*(0)} = \frac{\left(\beta^2-1\right)\lambda q_H + \lambda + r - \alpha}{(\beta+1)(\lambda+r-\alpha)(r-\alpha)} \geq 0. \tag{62}$$

Given the monotonicity of $\dfrac{\partial^2\psi(x)}{\partial x^2}$ we can conclude that for $x \in (0, x_F^*(0))$, $\dfrac{\partial\psi(x)}{\partial x} > 0$. From this fact in combination with the results of (57) and (58) we deduce that x_1^{det} exists.

Proof of Proposition 3

Consider equation (56), which implicitly determines x_1^{det}. Applying implicit function theorem to (56) we get

$$\frac{\mathrm{d}x_1^{det}}{\mathrm{d}q_H} = -\frac{\left.\dfrac{\partial\psi(x,q_H)}{\partial q_H}\right|_{x=x_1^{det}}}{\left.\dfrac{\partial\psi(x,q_H)}{\partial x}\right|_{x=x_1^{det}}}. \tag{63}$$

As $\dfrac{\partial\psi(x)}{\partial x} > 0$ from Proposition 2, in order to show that $\dfrac{\mathrm{d}x_1^{det}}{\mathrm{d}q_H} > 0$ it is sufficient to demonstrate that $\dfrac{\partial\psi(x,q_H)}{\partial q_H} < 0$.

$$\begin{aligned}\frac{\partial\psi(x,q_H)}{\partial q_H} &= \left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta}\frac{\delta(\beta+1)\lambda}{(\beta-1)(\lambda+r-\alpha)} - \frac{x\lambda}{(r-\alpha)(\lambda+r-\alpha)}\\ &= \frac{x\lambda}{(r-\alpha)(\lambda+r-\alpha)}\left(\left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta-1} - 1\right).\end{aligned} \tag{64}$$

For $x < \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)} = x_F^*(0)$, it holds that $\left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta-1} < 1$. Hence, $\left.\dfrac{\partial\psi(x,q_H)}{\partial q_H}\right|_{x=x_1^{det}} < 0$ and $\dfrac{\mathrm{d}x_1^{det}}{\mathrm{d}q_H} > 0$.

Recall that the endpoint of the deterrence region is given by

$$x_2^{det} = \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)} + \sqrt{\left(3+\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}\right)^2 - 8}\right]}. \tag{65}$$

Differentiating with respect to q_H gives

$$\frac{\partial x_2^{det}(q_H)}{\partial q_H} = -\frac{4\delta(r-\alpha)(\beta+1)^2\lambda\left(-1+\dfrac{3+\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}}{\sqrt{\left(3+\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}\right)^2-8}}\right)}{(\lambda+r-\alpha)\left(1-\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}+\sqrt{\left(3+\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}\right)^2-8}\right)^2}. \tag{66}$$

Denote $B = \dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)} > 0$, so that we can rewrite $\dfrac{\partial x_2^{det}(q_H)}{\partial q_H}$ as

$$\frac{\partial x_2^{det}(q_H)}{\partial q_H} = -\frac{4\delta(r-\alpha)(\beta+1)^2\lambda\left(-1+\dfrac{3+B}{\sqrt{(3+B)^2-8}}\right)}{(\lambda+r-\alpha)\left(1-B+\sqrt{(3+B)^2-8}\right)^2}. \tag{67}$$

Given that $\left(-1+\frac{3+B}{\sqrt{(3+B)^2-8}}\right) > 0$ we can conclude that $\dfrac{\partial x_2^{det}(q_H)}{\partial q_H} < 0$.

Proof of Proposition 4

Rewrite x_2^{det} as

$$x_2^{det} = \frac{\dfrac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)}}{1-\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}+\sqrt{\left(3+\dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}\right)^2-8}}. \tag{68}$$

For the simplicity denote the denominator of (68) as $D(B)$ with $B = \dfrac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}$. Then the derivative of with respect to λ takes the following form

$$\begin{aligned}\frac{\partial x_2^{det}(\lambda)}{\partial\lambda} &= \frac{-\dfrac{\partial\beta(\lambda)}{\partial\lambda}\dfrac{8\delta(r-\alpha)}{(\beta-1)^2}D(B)-\left(\dfrac{\partial D(B)}{\partial\beta}\dfrac{\partial\beta}{\partial\lambda}+\dfrac{\partial D(B)}{\partial\lambda}\right)\dfrac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)}}{D(B)^2}\\ &= -\frac{4\delta(r-\alpha)}{(\beta-1)D(B)^2}\left[\frac{\partial\beta(\lambda)}{\partial\lambda}\frac{2D(B)}{(\beta-1)}+\left(\frac{\partial D(B)}{\partial\beta}\frac{\partial\beta}{\partial\lambda}+\frac{\partial D(B)}{\partial\lambda}\right)(\beta+1)\right],\end{aligned} \tag{69}$$

where

$$\frac{\partial D(B)}{\partial\lambda} = \frac{(\beta^2-1)(r-\alpha)q_H}{(\lambda+r-\alpha)^2}\left(-1+\frac{3+B}{\sqrt{(3+B)^2-8}}\right) > 0, \tag{70}$$

$$\frac{\partial D(B)}{\partial\beta} = \frac{2\beta\lambda q_H}{(\lambda+r-\alpha)}\left(-1+\frac{3+B}{\sqrt{(3+B)^2-8}}\right) > 0, \tag{71}$$

and

$$\frac{\partial\beta}{\partial\lambda} = \frac{1}{\sqrt{\left(\alpha-\frac{\sigma^2}{2}\right)^2+2\sigma^2(\lambda+r)}} > 0. \tag{72}$$

Note that $D(B) = 1 - B + \sqrt{1 + 6B + B^2}$ is positive given the range of B, because $D(0) = 2 > 0$ and the function $D(B)$ is monotonically increasing with B:

$$\frac{\partial b(B)}{\partial B} = \frac{B + 3}{\sqrt{1 + 6B + B^2}} - 1 = \sqrt{\frac{9 + 6B + B^2}{1 + 6B + B^2}} - 1 > 0. \tag{73}$$

Thus, all the terms inside the brackets in (69) are positive and we conclude that $\frac{\partial x_2^{det}(\lambda)}{\partial \lambda} < 0$.

Now we want to show that x_1^{det} decreasing with λ if $q_H = 0$. Applying the implicit function theorem to (56) we get

$$\frac{\mathrm{d}x_1^{det}}{\mathrm{d}\lambda} = -\frac{\left.\frac{\partial \psi(x, \beta(\lambda))}{\partial \lambda}\right|_{x=x_1^{det}}}{\left.\frac{\partial \psi(x, \lambda)}{\partial x}\right|_{x=x_1^{det}}}. \tag{74}$$

Using the fact that $\frac{\partial \psi(x)}{\partial x} > 0$ it is enough to prove that $\left.\frac{\partial \psi(x, \beta(\lambda))}{\partial \lambda}\right|_{x=x_1^{det}} > 0$.

$$\frac{\partial \psi(x, \beta(\lambda))}{\partial \lambda} = \frac{\partial \psi(x, \beta(\lambda))}{\partial \beta}\frac{\partial \beta}{\partial \lambda}, \tag{75}$$

where $\frac{\partial \beta}{\partial \lambda} > 0$, so that we only need to consider $\frac{\partial \psi(x, \beta(\lambda))}{\partial \beta}$, which is given by

$$\frac{\partial \psi(x, \beta(\lambda))}{\partial \lambda} = \frac{\partial \psi(x, \beta(\lambda))}{\partial \beta}\frac{\partial \beta}{\partial \lambda}, \tag{76}$$

$$\frac{\partial \psi(x, \beta(\lambda))}{\partial \lambda} = -\frac{\delta \left(\frac{(\beta-1)x}{(\beta+1)\delta(r-\alpha)}\right)^{\beta} \left((\beta + 1) \log\left(\frac{(\beta-1)x}{(\beta+1)\delta(r-\alpha)}\right) + 1\right)}{\beta^2 - 1} \tag{77}$$

Consider

$$g(\beta, x) = (\beta + 1) \log\left(\frac{(\beta - 1)x}{\delta(r - \alpha)(\beta + 1)}\right) + 1 \tag{78}$$

Given that $g(\beta, x)$ function monotonically increases with x, and $g(\beta, \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)} \mathrm{e}^{-\frac{1}{\beta+1}}) = 0$, for $x < \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)} \mathrm{e}^{-\frac{1}{\beta+1}}$ it holds that $g(\beta, x) < 0$ and $\frac{\partial \psi(x, \beta(\lambda))}{\partial \lambda} > 0$. Therefore, demonstrate the negative relation between x_1^{det} and λ for $q_H = 0$ it is sufficient to show that $x_1^{det} < \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)} \mathrm{e}^{-\frac{1}{\beta+1}}$.

If $q_H = 0$ x_1^{det} is implicitely determined by $\psi_1(x_1^{det}, \beta) = 0$ looks with $\psi_1(x, \beta)$ equal to

$$\psi_1(x, \beta) = -\left(\frac{x(\beta - 1)}{\delta(r - \alpha)(\beta + 1)}\right)^{\beta} \frac{\delta}{(\beta - 1)} + \frac{x}{r - \alpha} - \delta \tag{79}$$

$$\psi_1(\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)}\mathrm{e}^{-\frac{1}{\beta+1}},\beta)=\frac{\delta}{(\beta-1)}\left(-\mathrm{e}^{-\frac{\beta}{\beta+1}}+(\beta+1)\mathrm{e}^{-\frac{1}{\beta+1}}-\beta+1\right) \tag{80}$$

$$\lim_{\beta\to\infty}\psi_1(\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)}\mathrm{e}^{-\frac{1}{\beta+1}},\beta)=0 \tag{81}$$

$$\frac{\partial\psi_1(\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)}\mathrm{e}^{-\frac{1}{\beta+1}},\beta)}{\partial\beta}=-\frac{(\beta+3)\left(\left[\mathrm{e}^{\frac{\beta}{\beta+1}}-\mathrm{e}^{\frac{1}{\beta+1}}\right]\beta+\mathrm{e}^{\frac{\beta}{\beta+1}}\right)}{e\left(\beta^2-1\right)^2}<0 \tag{82}$$

Thus, $\frac{\partial\psi_1(\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)}\mathrm{e}^{-\frac{1}{\beta+1}},\beta)}{\partial\beta}>0$, which together with the fact that $\psi_1(x,\beta)$ is increasing with x [9] implies that $x_1^{det}<\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)}\mathrm{e}^{-\frac{1}{\beta+1}}$. As a result $\left.\frac{\partial\psi(x,\beta(\lambda))}{\partial\lambda}\right|_{x=x_1^{det}}>0$ and $\frac{\mathrm{d}x_1^{det}}{\mathrm{d}\lambda}<0$.

Proof of Proposition 5

In the stopping region the value function for the leader looks as follows

$$V_L^{acc}=\frac{xq_L(1-(q_L+q_F^*(q_L))}{r-\alpha}-\delta q_L. \tag{83}$$

Substituting the optimal capacity level for the follower, $q_F^*(x,q_L)=\frac{1}{2}\left(1-q_L-\frac{\delta(r-\alpha)}{x}\right)$ and maximizing with respect to q_L gives the following first order condition:

$$\frac{\partial V_L^{acc}}{\partial q_L}=\frac{x}{2(r-\alpha)}(1-2q_L)-\frac{\delta}{2}=0. \tag{84}$$

Thus, the capacity level of the leader can be written as

$$q_L^{acc}(x)=\frac{1}{2}\left(1-\frac{\delta(r-\alpha)}{x}\right). \tag{85}$$

The next step is to substitute the resulting expression into (83) to obtain the value of the accommodation strategy for the leader

$$V_L^{acc}=\frac{\left[x-\delta(r-\alpha)\right]^2}{8x(r-\alpha)}. \tag{86}$$

The accommodation strategy is implemented by the leading firm when the optimal capacity level of the leader, $q_L^{acc}(x)$, is such that the other positioned firm follows immediately after, that is when

$$x_F^*(q_L^{acc}(x))\leq x. \tag{87}$$

[9] $\frac{\psi_1(x,\beta)}{\partial x}=\frac{1}{(r-\alpha)}\left(1-\frac{\beta}{\beta+1}\left(\frac{x(\beta-1)}{\delta(r-\alpha)(\beta+1)}\right)^{\beta-1}\right)>0$ for $x<\frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)}$.

Let x_1^{acc} denote starting point of the accommodation region, i.e. the level of the stochastic profitability shock such that

$$x_1^{acc} = x_F^*(q_L^{acc}(x_1^{acc})). \tag{88}$$

Given the equation (85), the optimal trigger for the follower can be now written as

$$x_F^*(q_L^{acc}) = \frac{2\delta(r-\alpha)(\beta+1)x}{(\beta-1)[x+\delta(r-\alpha)]}. \tag{89}$$

Solving (88) we get

$$x_1^{acc} = \frac{(\beta+3)}{(\beta-1)}\delta(r-\alpha). \tag{90}$$

Proof of Proposition 6

The endpoint of the deterrence region is given by

$$x_2^{det} = \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-\frac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}+\sqrt{\left(3+\frac{(\beta+1)(\beta-1)\lambda q_H}{(\lambda+r-\alpha)}\right)^2-8}\right]}. \tag{91}$$

Using the notation of Proposition 3 we can rewrite x_2^{det} as

$$x_2^{det} = \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-B+\sqrt{1+6B+B^2}\right]}. \tag{92}$$

We can write the difference between the endpoint of the deterrence region and the starting point of the accommodation region as

$$\begin{aligned}
x_2^{det} - x_1^{acc} &= \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-B+\sqrt{1+6B+B^2}\right]} - \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)} \\
&= \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)}\left[\frac{4\beta+4}{\left[1-B+\sqrt{1+6B+B^2}\right]}-(\beta+3)\right] \\
&= \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-B+\sqrt{1+6B+B^2}\right]}\left[4\beta+4-(\beta+3)\left(1-B+\sqrt{1+6B+B^2}\right)\right] \\
&= \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-B+\sqrt{1+6B+B^2}\right]}\left[(\beta+3)\left(3+B-\sqrt{1+6B+B^2}\right)-8\right].
\end{aligned} \tag{93}$$

From the derivations in Proposition 4 it follows that $\frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1-B+\sqrt{1+6B+B^2}\right]} > 0$. Thus, $x_2^{det} - x_1^{acc} < 0$ when

$$(\beta+3)\left(3+B-\sqrt{1+6B+B^2}\right)-8 < 0. \tag{94}$$

Using the property that $\beta > 1$ we can rewrite the inequality as follows

$$3 + B - \sqrt{1 + 6B + B^2} - \frac{8}{\beta + 3} < 0 \tag{95}$$

$$3 + B - \frac{8}{\beta + 3} < \sqrt{1 + 6B + B^2}. \tag{96}$$

The expressions on both sides of the above inequality are positive as for $\beta > 1$ it holds that $\frac{8}{\beta + 3} < 2$. Therefore, for x_2^{det} to be smaller than x_1^{acc} it is enough to prove that

$$\left(3 + B - \frac{8}{\beta + 3}\right)^2 < 1 + 6B + B^2 \tag{97}$$

$$9 + 6B + B^2 - (3 + B)\frac{16}{\beta + 3} + \frac{64}{(\beta + 3)^2} < 1 + 6B + B^2 \tag{98}$$

$$8 - \frac{48}{\beta + 3} + \frac{64}{(\beta + 3)^2} < B\frac{16}{\beta + 3} \tag{99}$$

$$B > \frac{(\beta - 3)(\beta + 3) + 8}{2(\beta + 3)} \tag{100}$$

$$B > \frac{\beta^2 - 1}{2(\beta + 3)}. \tag{101}$$

Substituting the expesion for B we get

$$\frac{(\beta^2 - 1)\lambda q_H}{(\lambda + r - \alpha)} > \frac{\beta^2 - 1}{2(\beta + 3)} \tag{102}$$

$$\frac{\lambda q_H}{(\lambda + r - \alpha)} > \frac{1}{2(\beta + 3)}. \tag{103}$$

Given the restrictions on β, one can see that $\frac{1}{2(\beta + 3)} < \frac{1}{8}$. Therefore, as long as $\frac{\lambda q_H}{(\lambda + r - \alpha)} > \frac{1}{8}$ (or put differently $\lambda(8q_H - 1) > r - \alpha$), inequality (103) always holds, implying that $x_2^{det} < x_1^{acc}$. Hence, equation (103) is the sufficient condition for the existence of the hysteresis region.

Proof of Proposition 7

Recall that the value of the first investor implementing accommodation and the corresponding capacity level are given by

$$V_L^{acc}(x) = \frac{(x - \delta(r - \alpha))^2}{8x(r - \alpha)}, \tag{104}$$

and

$$q_L^{acc}(x) = \frac{1}{2}\left(1 - \frac{\delta(r-\alpha)}{x}\right). \tag{105}$$

The value of the stochastic process that triggers investment of the follower can be found by substituting $q_L^{acc}(x)$ into the expression for $x_F(q_L)$.

$$x = \frac{2(\beta+1)\delta x(r-\alpha)}{(\beta-1)(\delta(r-\alpha)+x)}. \tag{106}$$

Solving for x we get

$$x_F^{acc} = \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)}. \tag{107}$$

Note that $x_F^{acc} = x_1^{acc}$. This means that it is never optimal for the follower to wait in the accommodation region. Thus, we consider only the value of stopping, which is equal to

$$F(x, q_L^{acc}) = \frac{(x-\delta(r-\alpha))^2}{16x(r-\alpha)}. \tag{108}$$

Thus for $x \geq x_1^{acc}$

$$V_L^{acc}(x) = \frac{(x-\delta(r-\alpha))^2}{8x(r-\alpha)} > \frac{(x-\delta(r-\alpha))^2}{16x(r-\alpha)} = F(x, q_L^{acc}). \tag{109}$$

□.

Proof of Proposition 8

The follower value in the stopping region can be written as

$$F(x, q_L) = q_F^*(x, q_L)\left(\frac{x(1-(q_L(x)+q_F^*(x,q_L)))}{r-\alpha} - \delta\right). \tag{110}$$

Plugging in the boundary capacity level $\hat{q}(x)$ we obtain:

$$F(x, \hat{q}_L) = \frac{\delta^2(r-\alpha)}{x(\beta-1)^2}, \tag{111}$$

so that the value of the follower is clearly decreasing with x.

Proof of Proposition 9

The threshold $\hat{x}_p$ is determined by the interception by the leader's and follower's curves:

$$L(\hat{x}_p, \hat{q}_L) = F(\hat{x}_p, \hat{q}_L), \tag{112}$$

where

$$L(\hat{x}_p, \hat{q}_L) = \frac{\delta}{\beta - 1}\left(1 - \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)x}\right), \tag{113}$$

and

$$\hat{q}_L(x) = 1 - \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)x}. \tag{114}$$

For the capacity level $\hat{q}_L(x)$ the follower's gets exactly the same value in both stopping and continuation regions which is equal to

$$F^*(x, \hat{q}) = \frac{\delta^2(r-\alpha)}{x(\beta-1)^2}. \tag{115}$$

Solving $L(\hat{x}_p, \hat{q}_L) = F(\hat{x}_p, \hat{q}_L)$ for $\hat{x}_p$ we get

$$\hat{x}_p = \frac{\delta(r-\alpha)(\beta+2)}{(\beta-1)}. \tag{116}$$

First, note that the threshold $\hat{x}_p$ is only relevant if it lies in the feasible region of the boundary strategy. In particular, it should hold that $x_2^{det} < \hat{x}_p < x_1^{acc}$. It is easy to see that $\hat{x}_p < x_1^{acc}$, as $\hat{x}_p = \frac{\delta(r-\alpha)(\beta+2)}{(\beta-1)} < \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)} = x_1^{acc}$. Yet the relation between x_2^{det} and $\hat{x}_p$ depends on the hidden competition parameter, λ and q_H.

Using the notation from Proposition 6 we can write

$$\begin{aligned}
x_2^{det} - \hat{x}_p &= \frac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1 - B + \sqrt{1+6B+B^2}\right]} - \frac{\delta(r-\alpha)(\beta+2)}{(\beta-1)} \\
&= \frac{\delta(r-\alpha)}{(\beta-1)}\left[\frac{4\beta+4}{\left[1 - B + \sqrt{1+6B+B^2}\right]} - (\beta+2)\right] \\
&= \frac{\delta(r-\alpha)}{(\beta-1)\left[1 - B + \sqrt{1+6B+B^2}\right]}\left[4\beta + 4 - (\beta+2)\left(1 - B + \sqrt{1+6B+B^2}\right)\right] \\
&= \frac{\delta(r-\alpha)}{(\beta-1)\left[1 - B + \sqrt{1+6B+B^2}\right]}\left[\beta\left(3 + B - \sqrt{1+6B+B^2}\right) + 2 + 2B - 2\sqrt{1+6B+B^2} + 6 - 6\right] \\
&= \frac{\delta(r-\alpha)}{(\beta-1)\left[1 - B + \sqrt{1+6B+B^2}\right]}\left[(\beta+2)\left(3 + B - \sqrt{1+6B+B^2}\right) - 4\right].
\end{aligned} \tag{117}$$

Due to the fact that $\dfrac{4\delta(r-\alpha)(\beta+1)}{(\beta-1)\left[1 - B + \sqrt{1+6B+B^2}\right]}$ is positive, $x_2^{det} - \hat{x}_p < 0$ when

$$(\beta+2)\left(3 + B - \sqrt{1+6B+B^2}\right) - 4 < 0. \tag{118}$$

Using the property that $\beta > 1$ we can rewrite the inequality as follows

$$3 + B - \frac{4}{\beta + 2} < \sqrt{1 + 6B + B^2}. \tag{119}$$

The expressions on both sides of the above inequality are positive as for $\beta > 1$ it holds that $\frac{4}{\beta+2} < \frac{4}{3}$. Therefore, for x_2^{det} to be smaller than $\hat{x}_p$ it is enough to prove that

$$\left(3 + B - \frac{4}{\beta + 2}\right)^2 < 1 + 6B + B^2 \tag{120}$$

$$9 + 6B + B^2 - (3 + B)\frac{8}{\beta + 2} + \frac{16}{(\beta + 2)^2} < 1 + 6B + B^2 \tag{121}$$

$$8 - \frac{24}{\beta + 2} + \frac{16}{(\beta + 2)^2} < B\frac{8}{\beta + 2} \tag{122}$$

$$B > \beta - 1 + \frac{2}{(\beta + 2)} \tag{123}$$

$$B > \frac{\beta(\beta + 1)}{(\beta + 2)}. \tag{124}$$

Substituting the expression for B we get

$$\frac{(\beta^2 - 1)\lambda q_H}{(\lambda + r - \alpha)} > \frac{\beta(\beta + 1)}{(\beta + 2)} \tag{125}$$

$$\frac{\lambda q_H}{\lambda + r - \alpha} > \frac{\beta}{(\beta - 1)(\beta + 2)}. \tag{126}$$

This implies that for $x_2^{det} < \hat{x}_p$.

Note that the left hand hide of (126), $\frac{\lambda q_H}{\lambda + r - \alpha}$, increases with λ. The derivative of the expression on the right hand side of the same inequality with respect to β is negative, $-\frac{\beta^2+2}{(\beta^2+\beta-2)^2} < 0$, meaning that the right hand side is a decreasing function of λ. This implies that there exist unique λ for which these expressions are equal, we denote it by λ_p:

$$\frac{\lambda_p q_H}{\lambda + r - \alpha} = \frac{\beta(\lambda_p)}{(\beta(\lambda_p) - 1)(\beta(\lambda_p) + 2)}. \tag{127}$$

Thus, for $\lambda > \lambda_p$, it holds that $x_2^{det} < \hat{x}_p$ and the preemption trigger lies in the boundary region.

Proof of Proposition 10

The arrival rate λ_p is determined by

$$\frac{\lambda q_H}{\lambda + r - \alpha} = \frac{\beta}{(\beta-1)(\beta+2)}. \tag{128}$$

As reported in Proposition 9 the left hand side of the above equation increases with λ, while the right hand side – decreases. Note that an increase in q_H causes an increase in a function on the left hand side for every value of λ, shifting the intersection point that determines λ_p to the right. Thus, we conclude that λ_p decreases with q_H. $\square$

Proof of Proposition 11

If the preemption triggers lying in the boundary region, $\hat{x}_p$, is determined by

$$\hat{L}(\hat{x}_p) = F(\hat{x}_p, \hat{q}_L), \tag{129}$$

where

$$\hat{q}_L(x) = 1 - \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)x}. \tag{130}$$

By definition installing the capacity $\hat{q}$ implies the equal values of the accommodation and deterrence strategy for the leader. Thus, we can use either of those to find the value function under the boundary strategy.

$$\hat{L}(x) = L^{acc}(x, \hat{q}_L) = \frac{\delta}{\beta-1}\left(1 - \frac{\delta(r-\alpha)(\beta+1)}{(\beta-1)x}\right). \tag{131}$$

For the capacity level $\hat{q}_L(x)$ the follower's gets exactly the same value in both stopping and continuation regions which is equal to

$$F^*(x, \hat{q}) = \frac{\delta^2(r-\alpha)}{x(\beta-1)^2}. \tag{132}$$

Solving $L(\hat{x}_p, \hat{q}_L) = F(\hat{x}_p, \hat{q}_L)$ for $\hat{x}_p$ we get

$$\hat{x}_p = \frac{\delta(r-\alpha)(\beta+2)}{(\beta-1)}. \tag{133}$$

Substituting the above expression into (133) we get

$$\hat{q}_L(\hat{x}_p) = \frac{1}{(\beta+1)}. \tag{134}$$

This threshold is only relevant if it lies in the feasible region of the boundary strategy, $x_2^{det} < \hat{x}_p < x_1^{acc}$. It is easy to see that $\hat{x}_p < x_1^{acc}$, as $\hat{x}_p = \frac{\delta(r-\alpha)(\beta+2)}{(\beta-1)} < \frac{\delta(r-\alpha)(\beta+3)}{(\beta-1)} = x_1^{acc}$. Yet the relation between x_2^{det} and $\hat{x}_p$ depends on the hidden competition parameters through $\lambda_p(q_H)$.

Proof of Proposition 12

The leader implements the deterrence strategy and its capacity choice is such that the follower delays its investment. Therefore, the preemption trigger is defined as the first intersection of the leader's value and the follower's value of waiting. At this point one of the firms enters the market as a leader, whereas its rival waits till the follower's optimal investment moment. The leader's value and its capacity under deterrence strategy are derived in Section 1 while the follower's value of waiting as well as its optimal timing and optimal capacity choice are given in Proposition 1. Due to the complexity of these functions the explicit solution for the preemption trigger cannot be obtained.

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