Valuing Path-Dependent Project-Financed Public-Private Partnerships: MCS vs. GBM

Abstract

Public-private partnerships as a form of financing usually provide solutions to public budget constraints and help improve cost and operating efficiency of large infrastructure project, which contribute to faster economic growth. However, they are not without controversies. Evaluations of financial results of such huge projects involving real options (usually in terms of government guarantees or put options) are performed in a misleading manner, which may severely influence decision-making. In this article we present an illustrative example of valuation of a typical infrastructure project, compare and critically evaluate results obtained by Monte Carlo simulations (MCS) and Geometric Brownian motion (GBM). We argue that any project guarantee (option) with path-dependent payoffs, should always be evaluated using MCS and also show that proper capture of volatility is much less important than proper capture of a growth rate itself. In a typical infrastructure project, GBM underestimates cash flows and overestimates value of guarantees of a public partner. In economic terms, our results imply more implemented projects, which are urgently needed, and thus higher impact on well-being.

JEL Code: G31, H54

Key words: *public-private partnership, real options, capital budgeting decisions, Monte Carlo simulation, Geometric Brownian motion*

1. Introduction

Prosperity and well-being crucially depend on infrastructure projects like ports, highways, water systems, pipelines, hospitals, etc. They facilitate transport, promote communication, provide energy and water, boost the health and education of the workforce and enable the whole economy to flourish. The costs of building infrastructure are vast, but the costs of failing to make such investments are incalculable (Global Agenda Council…, 2013).

The need for infrastructure investment around the globe is climbing and investment is urgently required. In the developed world significant reinvestment in aging infrastructures is becoming urgent. In emerging markets, population growth, increasing urbanization, and rising per capita incomes are driving the demand for new infrastructural spending. This trend, coupled with new financial reality, which severely constrained public budgets in many countries, led to a staggering gap of approximately USD 2 trillion annually between demand and investment in infrastructure, for the period of next 20 years. Dobbs et al. (2013) estimate that USD 57 trillion infrastructure investments are needed until 2030 (a rise of 60% compared to equivalent period up until now). According to World Economic Forum, in order to reach the 7% annual growth required to meet the Millenium Development Goals for Africa, infrastructure project of about 15% of GDP would be needed. Country that has successfully invested into infrastructure projects is China. Government has been pouring money into construction of high-speed rail and urban mass transit systems throughout the county, which resulted in dramatic economic expansion (Miller, 2013). Unfortunately, these investments are adding to a pile of debt and additionally call for new funding of maintenance and operating subsidies.

Solution to this ever-increasing gap can be addressed by public-private partnerships (hereafter PPPs), whereby private partners build, control and operate infrastructure project subject to strict government rules and oversight.[1] Apart from solving the public budget constraint, PPPs can also bring significant discipline to project selection, construction, and operation, when managed effectively (Airoldi et al. 2013-BCG). Further, PPPs improve and speed-up service delivery and importantly transfer project risks to private partners who are better equipped to manage them (Colverson & Pereira, 2012). Dobbs et al. (2013) estimate that substantial savings in range of USD 1 trillion annually are available through improvements in infrastructure productivity. Further, they claim that private partners might deliver 30% boost in the capacity of many ports through more efficient terminal operations. In some cases (e.g. European Union) PPPs are clearly seen as an important vehicle to address the region's needs to boost its growth in the medium to long term, an impossible goal with public investments alone (Dheret et al., 2012).

Colverson and Perera (2012) report that private infrastructure investments are already substantial. Within EU, there were more than 1,300 PPP contracts over EUR 5 million signed between 1990 and 2009, totaling the capital value in excess of EUR 250 billion, with the UK dominating the sum with a 67% share. The PPPs market in developing nations was slow to recover from the shock of the Asian Financial Crisis, taking a full decade to regain pre-crisis levels. Bottoming out in 2003, the decline was then reversed quickly, as total investment commitments to infrastructure projects rose from around USD 70 billion in 2003 to over USD 160 billion in 2007.

In general, huge infrastructure projects like toll-road, power plants, railways and alike are very risky since they involve high investment cost in the beginning and high maintenance and management costs during the long period of concession, which is usually 25-30 years. In such long period quite somewhat might turn wrong, as future changes quantities (demand for the product/service), prices of inputs and outputs, political and economic situation, etc. Benefits of such projects though, are public. Because of their riskiness it might happen that public tenders for infrastructure concession rights do not result in successful bids. In such situations the respective government (i.e. public partner) either postpones granting such a project to later perhaps more convenient moment or grants a project a subsidy or guarantee and thus make project more attractive to potential private partners. Incentives can take various forms like input subsidies, redemption subsidies, exchange rate subsidies, put options, lump-sum subsidies to minimum revenue guarantees etc. (Cheah & Liu (2006), Brandao & Saraiva (2008), Rose (1998), Shan et al. (2010), Ye & Tiong (2000)).

Such features of an infrastructure project are real options, firstly introduced to the literature

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[1] PPPs appear in many variations known as BOT (build-operate-transfer), BOOT (build-own-operate-transfer), BTO (build-transfer-operate), DBFO (design-build-fund-operate) and others, depending upon agreement between public and private partner (Akintoye et al., 2003).

by Myers (1977) as an extension of financial options pricing[2] to non-financial (i.e. real) investment opportunities with managerial flexibility under dynamic market uncertainty (Dixit & Pindyck, 1994). Today, real options literature is rich; from its early days (e.g. see McDonald & Siegel (1986), Majd & Pindyck (1987), Trigeorgis & Mason (1987) Trigeorgis (1995), Amram & Kulatilaka (1999)), to recent contributions (e.g. Galera & Solino (2010), Ashuri et al. (2012), Kim et al. (2013), Park et al. (2013)). Real options valuation methodology is applied onto wide variety of fields and industries (e.g. foreign direct investment – O'Brien et al. (2003), construction – Chiara et al. (2007), Kim et al. (2013), infrastructure maintenance strategy – de Neufville et al. (2006), manufacturing – Miller & Folta (2002), human resource management - Berk & Kase (2010), information technology – Kumar (2002), building clean energy systems – Byungli et.al (2012), installation of clean energy systems – Kim et al. (2012)). The most significant characteristic of a real option is that a limited commitment creates future decision rights (McGrath et al. (2004). The increased managerial flexibility and potential future benefits arising thereof in many cases turn the analyzed project on, even if static NPV yields negative net present value of the project (hereafter NPV).

Valuation of real options embedded into a PPP project is not only of great importance to a private partner, which evaluates the received option in terms of providing it sufficient incentive to make the investment decision. Greater the value of the option, the greater is also the taxpayers' burden. What if a real option value is calculated in a wrong way and in 10 years time government's obligations toward a private partner become much higher? Taxpayers would in such a case perhaps require some unpleasant answers. Hemming et al. (2006) argue that government guarantees can have potential large disruptive fiscal effects. On the other hand, when the embedded option value is overpriced (e.g. due to mistaken valuation technique) public partner will not decide to grant the concession as the estimated impact on fiscal balance is assumingly too large.

The value of such real option thus has substantial impact on decisions on both sides and is as such very important. However, the nature of a problem defines the appropriateness of a certain valuation technique. If the distance to financial markets and the complexity of the strategic investment decision are small and low enough, respectively, then a link between the investment project's cash flows and a tradable financial asset (e.g. stock of some company with similar activities) with similar risk characteristics – *a twin asset* – can be established that enables linking its risk characteristics to the real options valuation model (e.g. Black & Scholes, 1973). In such a setting real option value is established as a function of the value of the underlying twin asset (Trigeorgis 1995). Alternatively, in situations entailing a relatively long distance from financial markets (nonexistence of any comparable stock) and high investment decision complexity, we approach the real-option valuation frontier (Amram and Kulatilaka 1999, p. 99), where no analytical solutions (such as the one gained by using Black & Scholes option pricing model) exist and only sophisticated case-specific numerical models are applicable for valuation purposes. We argue in this paper that additional restrictions should be made regarding the applicability of Black and Scholes formulae in the real options' cases.

Due to highlighted need for new infrastructure project, emergence of phenomenon of PPPs and the need to embed real options (i.e. government guarantees) into such projects, recent

[2] Financial option pricing breaktrough was done by Black & Scholes (1973) and Merton (1973), still perhaps the most famous contributions to modern financial literature. Authors based their models on stochastical movements of an underlying asset in continous time.

literature offers quite some work on valuation of real options in project-financed PPPs. Such papers are Chiara et al. (2007), Ho & Liu (2002), Cheah & Liu (2006, Huang & Chou (2006), Kim et al. (2012), Park et al. (2013), Zhao et al. (2004), Almassi et al. (2013), Brandao et al. (2012), Wibowo et al. (2012), Shan et al. (2010), Caselli et al. (2009), Brandao and Saraiva (2008), Ashuri et al. (2012) and Chiara & Kokkaew (2013). What we notice is that quite some authors in this field do not pay enough attention to characteristics and details of the problem they are modeling. We argue that in a typical PPP project setting, methodology based on stochastic modeling with embedded constant parameter is improper, as it does not correctly capture the cash flow dynamics. Such papers are for example Brandao et al. (2012), Caselli et al (2009), Huang & Chou (2006). We show in this paper that the improper methodology causes relatively large valuation errors, and that due to a typical characteristic of PPP projects, authors overestimate the value of guarantees (i.e. real options). As a typical characteristic of an infrastructure PPP project we consider cash flow dynamics that stems from demand patterns for services offered by the concession private partner.

When analyzing various infrastructure projects all over the world and number of users that are using new-found infrastructure through years, we can notice an underlying common principle regarding yearly growth rate of the users. When new infrastructure (like a toll-road for example) is constructed and put to operation, there is a sudden initial increase of the users of the new infrastructure in the beginning phase. In years that follow, the growth rate of number of users is usually still high, but is then slowly decreasing until it reaches a level that is more or less constant. Reasons for this occurrence are various, but in general it holds that after a crash-start enthusiasm some period is needed until redistribution of the users between existing and new infrastructure is accomplished. For instance, when Taiwan High Speed Rail Project was finished in 2007 (connecting national capital Taipei and southern city Kaohsiung with 345 km of high-speed railroad), the impact of the new infrastructure was so huge, that domestic air traffic was almost halved until 2008. Redistribution of passengers was happening, passengers who were using air traffic or car decided to use train instead (according to Holder & Stover (1972) such redistribution is called converted demand)[3] and that effect contributed to higher growth rates of number of users in the starting years of project which was then slowly decreasing throughout the following years. Similar effect can be seen in everyday life in a case of a new road, where new infrastructure usually takes over part of the users from other roads (i.e. diverted demand). This redistribution takes place for some time causing higher growth rates in the beginning, which are later on slowly decreasing. When we consider underdeveloped countries, growth rates in the initial years of the toll-roads can be very high since new infrastructure is usually the only infrastructure in some part of the state (Brandao & Saraiva 2008) and also takes over all potential users from that part of the country.

Redistribution effect causes that infrastructure project is divided into stages from the aspect of the growth rate. In consequence, the growth rate of number of users is never constant through all its years, but is decreasing or increasing, depending on the project. The traditional ramp-up curves are typically of concave shape, rising sharply initially and tapering off toward the end of the assumed ramp-up period (Review of Traffic Forecasting…, 2011), and

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[3] Holder & Stover (1972) explain six types of demand for a new infrastructure, i.e. diverted demand (e.g. road transposrt divertes to a better road), converted demand (other modes of transportation convert to a new road and drop the existing mode), growth demand (due to a higher growth rate of a population), developed demand (new infrastructure helps develop and change the use of land and thus activities), cultural demand (new infrastructure changes propensity to travel due to socio-economic characteristics), and induced demand (due to added convenience/increased accessibility).

forecasting of such curves quite inaccurate (Bain, 2009; Bain & Polakovic, 2005; Bain & Wilkins, 2002)[4].

Such concave pattern of the demand through a ramp-up period makes demand *path-dependent*. This means that cash flows of a private partner would be importantly different from the ones under the assumption of no-dependency. This has important implications for derivatives pricing. Among financial options such a characteristic have *Asian options* and *look-back options*, which are priced with numerical methods, not analytical formulas like Black & Scholes (1973) (Campbell et al., 1997; Ekstrand, 2011). This fact is very important and is not properly considered in the literature. When analyzing financial effect of the project, one should in many cases not base the model on Geometric Brownian Motion as this approach has a major drawback that does not properly account for the redistribution effect (Podhraski, 2012).

Papers in the field of valuation of PPPs and/or within embedded guarantees focus on providing valuation of a project and/or a guarantee itself, according to their proposed methodology. Our paper is unique as we focus on differences in methodology between MCS and GBM for PPP cases where *firstly*, growth rates are not constant and *secondly*, when public partner's guarantee relevant cash flows are occurring throughout the life of a project. We argue that both characteristics occurring simultaneously are very common in infrastructure projects.

Contribution we make in this paper to the existing literature is in showing superiority of MCS for all cases where cash flows of a project are distributed throughout the project life-time and growth rates are not constant (i.e. growth rates vary by subsequent phases of the project). We show that the magnitude of the valuation error is much higher in case valuation methodology does not properly capture path-dependent cash flows (i.e. due to a specific growth rate pattern), compared to valuation error in case valuation methodology does not properly capture volatility. This makes novel contribution to the literature and builds further on Chiara & Garvin (2008) who argue that valuation methodology should account for changes in project cash flow volatility.

The paper is structured as follows. In the next section a hypothetical toll-road example is introduced, whereby government provides a guarantee in order to make project appealing for a potential private partner. In the section that follows we briefly introduce characteristics of modeling techniques that are the most commonly applied in valuation of contingencies in the PPP projects. Chapter Results contains main part of this paper. It shows the difference between the Monte Carlo simulation and the GBM and discusses financial and economic implications for public-private partnerships. In the next section we further illuminate impacts of main value/risk drivers of an introduced real option problem – volatility and growth rates. Finally, in the last section we conclude.

2. Toll-Road Example

We introduce a realistic hypothetical example of build-operate-transfer (BOT) PPP in an underdeveloped country on a case of a toll-road. Main characteristics are presented in Table 1 below.

[4] They are as well overoptimistic.

Table 1: Characteristics of a project of construction, management and maintenance of a toll-road in a PPP

Concession years	Concession is granted for 30 years. In period 2013-2015 (3 years) toll-road has to be constructed and put into service. In period 2016-2042 (27 years) toll-road is in use and is managed by a private partner. After the concession expiration in 2042, public partner becomes an owner of the infrastructure and is also responsible for further management and maintenance of the toll-road.
Investment value	EUR 600 mil.
Maintenance costs	5 % of the investment cost per year. Maintenance costs are each year increased for 2 % inflation.
Capital structure	Debt-to-Equity ratio is 70:30, meaning that equity stake in the project is worth EUR 180 mil; 35 % (EUR 63 mil) in the 1st year, 35 % (EUR 63 mil) in 2nd and 30 % (EUR 54 mil) in the third.
Liabilities (Debt)	Principal amounts to EUR 420 mil, interest rate is 5 %, maturity is 15 years, yearly annuities are EUR 40,463,761 and are paid once a year.
Number of vehicles	Initial number of vehicles in year 2016 is 12 mil and is distributed normally with standard deviation 3 mil vehicles. Full capacity of the toll-road is 35 mil vehicles a year.
*Traffic growth rate**	Anticipated yearly traffic growth rates in different periods are: – 2016–2020: 9 % a year with standard deviation 6 %, – 2021–2030: 5 % a year with standard deviation 3 %, – 2031–2042: 3 % a year with standard deviation 2 %, until full capacity is reached. Traffic growth rates follow normal distribution.
Average toll per vehicle	EUR 4.80 and each year is increased for 2 % inflation.
Required rate of return on equity	9 %.
Effective profit tax rate	32 %.

Legend (*): Number of vehicles and growth rates are independent and uncorrelated.

Private partners in PPPs have incentive to bid projects that have appropriate relationship between risk and return. At first sight the above toll-road project is relatively attractive as the static discounted cash flow analysis (using only mean values of growth rates without standard deviations to determine future cash flows) yields NPV of EUR 42.1 mil and internal rate of return (hereafter IRR) 10.16%.

However, this project is risky. Firstly, it requires huge resources to be mobilized and a great deal of skills to run the project efficiently. Secondly, its success critically depends on a level and volatility of customers' demand. In order to assess risk of a changing demand, we run a Monte Carlo simulation on 5.000 samples based on anticipated number of vehicles in the first year after the toll-road completion, traffic growth rates and their standard deviations (see inputs in Table 1 above). Our results presented in Table 2 clearly show the toll-road project is very risky. Mean value of the distribution of the NPV is EUR 18 mil, whereby standard deviation is EUR 147.7 mil. Interval of one standard deviation apart from the mean NPV value of the project (with roughly 68% of all simulated NPV values) lies between EUR -129.7 mil and EUR 165.7 mil. Value at risk measure at 10% reaches 30% of investment value of the project and is still negative. Without a sweetener in some kind of form of government support, such project is prone to turn into a failure.

Table 2: Monte Carlo simulation results of the NPV of the project

Number of samples	5,000
Mean value	17,974,875
Standard deviation	147,670,912
Median	31,260,399
Minimum	-639,594,521
Maximum	344,205,251
Percentile 10 %	-183,274,386
Percentile 20 %	-105,776,188
Percentile 30 %	-51,994,151
Percentile 40 %	-5,775,466
Percentile 50 %	31,260,399

In order to make similar projects attractive and feasible, government (or any other public counterparty in a PPP) grants a subsidy or some sort of incentive to private partner. In our hypothetical case a public partner helps the project with minimum revenue guarantee (hereafter MRG). Nevertheless, public partner does not let private partner to lay back, but pushes it towards achieving higher toll-incomes. Namely, MRG document stipulates that a benchmark from which guarantee amount will be derived is represented by the income from the static calculation. And further, that a public partner is obliged to reimburse a private partner any time when actual income falls short of the predetermined income in the static calculation, but in the amounts of only 40% of the missing shortfall between the predetermined income and the actual income. That means that when actual income reaches 90% of predetermined income, private partner is still lacking 6% of the predetermined income.

Such typical MRG concept translates path-dependent cash flows arising from decreasing growth rates into the subsequent lacking amounts that are covered with MRG. In other words, MRG are path-dependent as well.

3. Methodology for Valuing NPV of the Project and Valuing MRG

In this section we provide brief methodological framework used in this paper. We calculate the value of the MRG provided by the public partner, which is defined as a difference between predetermined level of traffic income as calculated in static calculation and actual traffic income. The crucial point for public partner is to accurately anticipate future traffic income and for that reason two different approaches were used in to date theory and practice, Monte Carlo simulation and Geometric Brownian motion.

3.1. Monte Carlo Simulation

Monte Carlo Simulation is a numerical technique, based on the repeated random sampling. It is employed to solve numerous problems, where analytical solutions do not exist. The most common use of MCS in finance is when there is a need to calculate an expected value of a function given a specified distribution density (Jaeckel, 2002).

In our case calculation of the future traffic income is based on mean values and standard deviations of the initial number of vehicles and future growth rates. In our hypothetical case we assumed that initial number of vehicles would be 12 million a year with standard deviation of 3 mil vehicles and with vehicles being distributed normally (see Table 1). Full capacity of the toll road is 35 mil vehicles a year. Regarding future traffic growth rates we anticipated that yearly traffic growth rates in period 2016–2020 is 9 % a year with standard deviation 6 %, in period 2021–2030 is 5 % a year with standard deviation 3 % and in period 2031–2042 is 3 % a year with standard deviation 2 %, until full capacity is reached. Traffic growth rates also follow normal distribution, whereby both variables, number of vehicles and growth rate are independent and uncorrelated.

Based on the above-mentioned mean values and standard deviations, we simulated future traffic cash flow for each of the 27 concession years. In each year of the concession period, when calculated traffic income was smaller than predetermined level (i.e. 40 % of the actual income shortfall) also public partner's MRG was calculated. In the next step present values of public partner's MRGs were calculated, whereby sum of all present values of MRGs through all concession years presents the amount of public partner's MRG obligation for the whole concession period. This procedure was repeated 5,000 times.

3.2. Geometric Brownian motion

Calculation of the future traffic income in second approach is based on the assumption that future traffic income will vary stochastically in time following Geometric Brownian motion. Stochastic process R is said to follow a GBM if it can be described as (Wilmott, 2006):

$$dR = \alpha R_t dt + \sigma_R R_t \varepsilon\sqrt{\delta t}$$

where dR is the incremental change in revenue in time interval dt, α is the revenue growth rate, σ_R is the volatility of the revenue, while $\varepsilon\sqrt{\delta t}$ is standard Wiener process, with $\varepsilon \sim N$ (0,1). First part of the equation (i.e. $\alpha R_t dt$) is deterministic and the second part of the equation (i.e. $\sigma_R R_t \varepsilon\sqrt{\delta t}$) is stohastic.

In case of Monte Carlo Simulation we used data inputs expressed with mean values and standard deviations. In order to use the same data in the GBM approach, data transformations are needed. Namely, GBM operates with *first*, average growth rate and *second*, average volatility. Thus, we had to transform mean values and standard deviations to the average yearly growth rate and average yearly volatility of number of vehicles.

Average geometrical growth rate of number of vehicles was calculated with Monte Carlo simulation from 27 operating years of the project (2016-2042) using 5,000 simulations. The same growth rates that were produced in Monte Carlo simulation when calculating the value of the MRG where then used to calculate average geometrical growth rate that is needed in the GBM approach.

In the same manner (and based on the same data) we computed *average volatility*. We used the same growth rates that were produced in Monte Carlo simulation when calculating the value of the state's MRG and calculated average volatility with procedure known as the Logarithmic Cash Flow Returns Method (Kodukula, Papudesu, 2006, p. 88). Since yearly cash flow in our case consists of number of vehicles a year multiplied with the toll, we adjusted mentioned procedure to the number of vehicles instead of cash flow. The steps used in this procedure were the following:

- Forecast number of vehicles during operative phase of the project at regular time intervals.
- Calculate the growth rate ratio for each time interval, starting with the second time interval, by dividing the current number of vehicles by the preceding one (in case of cash flows this growth rate ratio would be named relative return).
- Take the natural logarithm of each growth rate ratio.
- Calculate the standard deviation of the natural logarithms of the growth rate ratios from the previous step.
- Calculate square of deviations for each year.
- Calculate total of square of deviations for all years.
- Volatility factor of number of vehicles is the square root of (total of squares of deviation, divided by (n-1)), where n is the number of values included.

When we calculated average yearly growth rate and average yearly volatility of revenues, we had all inputs for the calculation of the future traffic income with the GBM. From this point on, the procedure of the MRG calculation was the same as in the case of the Monte Carlo simulation, except that the GBM was used for calculation of future traffic income. Also in this case we repeated the procedure 5,000 times in the course of the calculation, due to different initial numbers of vehicles.

4. Results

In this section we present the calculation of the value of the MRG using two distinct methodologies – MCS and GBM and explain the impact of different growth rates on the methodology choice.

4.1. Value of the MRG under MCS

Under the above mentioned conditions of the PPP agreement we calculated value of the MRG using MCS approach. Results are presented in Table 3.

Table 3: Results of MRG using MCS

Number of samples	5,000
Mean value	41,369,984
Standard deviation	56,818,044
Median	10,646,238
Minimum	0
Maximum	343,685,383

Mean value of the MRG is EUR 41.4 mil. Values of the MRG are not normally distributed, though. Median value amounts to EUR 10.6 mil and is more informative about the economic value of the public partner MRG as 50 % of values are smaller than EUR 10.6 mil and 50% values are larger than this amount.

From table 4 we see that the state's MRG significantly improves cash flows of the project, which makes it more attractive to private partners.

Table 4: Results of improved NPV using Monte Carlo Simulation

Number of samples	5,000
Mean value	58,626,657
Standard deviation	98,217,955
Median	43,112,983
Minimum	-333,233,797
Maximum	338,369,182
Percentile 10%	-55,374,532
Percentile 20%	-20,812,039
Percentile 30%	2,358,399
Percentile 40 %	21,463,860

Comparing Table 2 and Table 4 we see that NPV is now substantially increased and also standard deviation much smaller. We also see that when project was not backed up with the MRG, NPV was still negative at the 40th percentile, while now it is negative at the 20th percentile. Minimum value of the project supported with the MRG, is almost halved.

4.2. Value of the MRG under GBM

Table 5 shows the average growth rate was 4.8578 %, and median was very close to it (4.8481 %). For the purpose of the GBM approach to calculate the value of the MRG we used average growth rate of 4.8578 %.

Table 5: *Results of average geometrical growth rate using MCS*

Number of samples	5,000
Mean value	4.8578
Standard deviation	0.6578
Median	4.8481
Minimum	1.8556
Maximum	7.3299

Pursuant to the described procedure average volatility was calculated as can be seen from Table 6. For the purpose of GBM approach we used average volatility of number of vehicles 3.8021 %.

Table 6: *Results of average volatility using MCS*

Number of samples	5,000
Mean value	3.8021
Standard deviation	0.6793
Median	3.7669
Minimum	1.8673
Maximum	6.2977

All in all, in calculating the average growth rate and average volatility, we transformed information that was until now expressed as mean value and standard deviation to the form of average growth rate and average volatility rate. We are emphasizing this because the same information expressed in a different form produces quite different result. To calculate the value of the MRG using GBM, the above average growth rate and volatility were used to generate project's cash flow. The result of the value of the MRG is presented in Table 7.

Table 7: Results of MRG using GBM

Number of samples	5,000
Mean value	67,344,538
Standard deviation	63,572,713
Median	52,296,074
Minimum	0
Maximum	332,949,929

If we compare results of MRG based on GBM with the ones based on MCS in Table 3, we see that they differ considerably. GBM estimates the median value of guarantee being EUR 52.3 mil, while the median value of the MRG under MCS amounts to EUR 10.6 mil. The value of the MRG calculated with GBM *is almost 5 times higher*, despite the fact that we used the same information (expressed in two different forms) for each procedure. The question is, which number is more realistic and what is the reason for the huge difference.

In order to answer that question we will analyze the impact of growth rates and volatility on the valuation methodology choice.

4.3. Impact of growth rates on the methodology choice

With the purpose of explaining the impact of the growth rates, we provide a simple example, presented in two tables below (Table 8 and Table 9). In the example simplified version of the above project is presented. In Table 8 we present cash flows of a hypothetical 8-year project, where growth rate of number of vehicles in 1st year is 10% and is then descending by 1% point a year (average growth rate is thus 7%). From the top of the Table 8 we see results of actual toll income in accordance to decreasing growth rate as in the case of MCS. From the bottom of the Table 8 we see actual toll rate income pursuant to constant 7% growth rate as in the case of the GBM.

Although 7% growth rate presents mean value (average) of descending growth rates, we can see from Table 8 that both growth rates do not have the same impact on cash flow calculation. In the 1st year, we have EUR 57,600,000 of actual income (initial income) in both cases, but the two actual incomes already differ in the 2nd year. In case of MCS the income is increased by EUR 1,745,280 more than in case of GBM, due to higher growth rate in the 1st year (i.e. 10% vs. 7%). In the 2nd year MCS already has higher basis for EUR 1,745,280, and in addition also has higher growth rate than Geometric Brownian motion, (9% vs. of 7%). Both facts again increase the difference in the actual income in the 3rd year. It's true that in the later stages of the project MCS produces lower incremental income than the GBM, for the reason of lower growth rates, but the latter procedure can not substitute all the losses that were gained in previous years.

Table 8 shows that only in the 8th year of the project both procedures produce similar actual income and when looking from time value of money, earlier cash flows have higher effect on the value of the MRG. In the final consequence and as can be seen from last column, MCS calculated NPV of the MRG being worth EUR 9.7 mil, while GBM calculated NPV of guarantee to be EUR 23.3 mil. The difference between both procedures is EUR 13.6 mil and is a result of the GBM not being able to detect actual (path-dependent) income, due to a constant average growth rate.

We come to an inverse conclusion in case when growth rates are increasing (see Table 9 for a numerical illustration). The effect is here opposite in distinction to decreasing growth rates, since the GBM produces higher actual income as the consequence of average growth rate being higher in the early years comparing to actual growth rates used by the MCS. In the end the MCS calculates NPV of the MRG being worth EUR 37.8 mil, while pursuant to the GBM NPV of the MRG is worth EUR 27.6 mil, which is EUR 10.2 mil less. In the first years of the project the GBM calculated higher income than actual, due to constant growth rate. It detected some income, that really was not there and all this resulted at the end in lower value of the guarantee, as can be also seen from Figure 1.

Table 8: Comparison between MCS and GBM with decreasing growth rates

MONTE CARLO SIMULATION									SUM
Operating years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Calendar years	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	
Predetermined benchmark income in EUR	57,600,000	63,411,840	69,810,095	76,853,933	84,608,495	93,145,492	98,780,794	104,757,033	
Actual toll income in EUR	57,600,000	63,993,600	70,450,554	76,847,465	83,049,055	88,912,318	94,291,513	99,043,806	
Number of vehicles a year	12,000,000	13,200,000	14,388,000	15,539,040	16,626,773	17,624,379	18,505,598	19,245,822	
Yearly growth rate	10.00%	9.00%	8.00%	7.00%	6.00%	5.00%	4.00%		
Toll in EUR	4.80	4.85	4.90	4.95	4.99	5.04	5.10	5.15	
VALUE OF GUARANTEE	**0**	**0**	**0**	**6,469**	**1,559,440**	**4,233,174**	**4,489,281**	**5,713,227**	**16,001,591**
NPV OF GUARANTEE									**9,663,157**

GEOMETRIC BROWNIAN MOTION									SUM
Operating years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Calendar years	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	
Predetermined benchmark income in EUR	57,600,000	63,411,840	69,810,095	76,853,933	84,608,495	93,145,492	98,780,794	104,757,033	
Actual toll income in EUR	57,600,000	62,248,320	67,271,759	72,700,590	78,567,528	84,907,928	91,759,997	99,165,029	
Number of vehicles a year	12,000,000	12,840,000	13,738,800	14,700,516	15,729,552	16,830,621	18,008,764	19,269,378	
Yearly growth rate	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%		
Toll in EUR	4.80	4.85	4.90	4.95	4.99	5.04	5.10	5.15	
VALUE OF GUARANTEE	**0**	**1,163,520**	**2,538,335**	**4,153,343**	**6,040,967**	**8,237,565**	**7,020,797**	**5,592,003**	**34,746,530**
NPV OF GUARANTEE									**23,289,773**

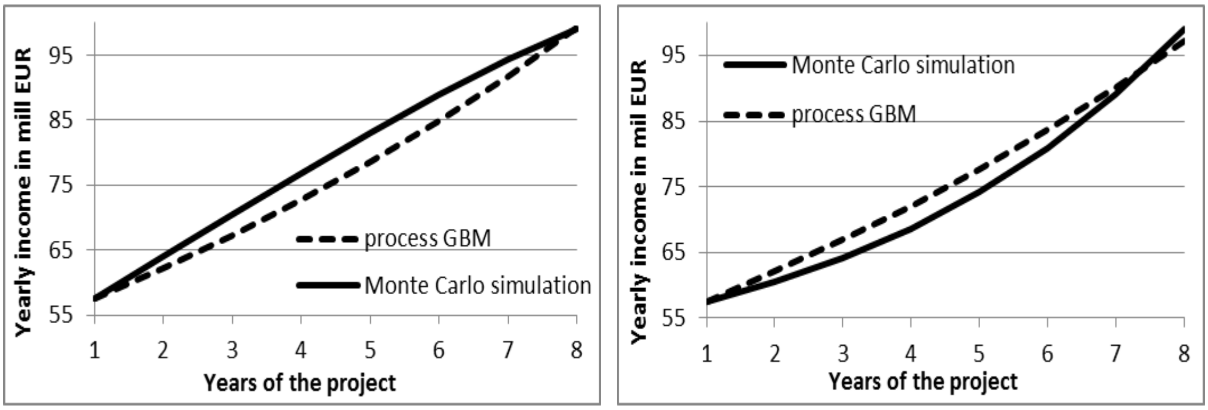
Table 9: Comparison between MCS and GBM with increasing growth rates

MONTE CARLO SIMULATION									**SUM**
Operating years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Calendar years	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	
Predetermined benchmark income in EUR	57,600,000	63,411,840	69,810,095	76,853,933	84,608,495	93,145,492	98,780,794	104,757,033	
Actual toll income in EUR	57,600,000	60,503,040	64,163,474	68,693,415	74,236,974	80,977,691	89,148,340	99,043,806	
Number of vehicles a year	12,000,000	12,480,000	13,104,000	13,890,240	14,862,557	16,051,561	17,496,202	19,245,822	
Yearly growth rate	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%		
Toll in EUR	4.80	4.85	4.90	4.95	4.99	5.04	5.10	5.15	
VALUE OF GUARANTEE	**0**	**2,908,800**	**5,646,621**	**8,160,518**	**10,371,521**	**12,167,801**	**9,632,454**	**5,713,227**	**54,600,943**
NPV OF GUARANTEE									**37,847,220**

GEOMETRIC BROWNIAN MOTION									**SUM**
Operating years	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	
Calendar years	**2016**	**2017**	**2018**	**2019**	**2020**	**2021**	**2022**	**2023**	
Predetermined benchmark income in EUR	57,600,000	63,411,840	69,810,095	76,853,933	84,608,495	93,145,492	98,780,794	104,757,033	
Actual toll income in EUR	57,600,000	62,072,623	66,892,544	72,086,730	77,684,244	83,716,403	90,216,957	97,222,277	
Number of vehicles a year	12,000,000	12,803,759	13,661,353	14,576,390	15,552,715	16,594,434	17,705,928	18,891,869	
Yearly growth rate	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%	7.00%		
Toll in EUR	4.80	4.85	4.90	4.95	4.99	5.04	5.10	5.15	
VALUE OF GUARANTEE	**0**	**1,339,217**	**2,917,551**	**4,767,203**	**6,924,251**	**9,429,090**	**8,563,838**	**7,534,756**	**41,475,905**
NPV OF GUARANTEE									**27,627,120**

The GBM thus produces lower income in cases when growth rates are descending and higher income in cases when growth rates are increasing, compared to the MCS (see Figure 1). From the above finding, we can conclude, that in cases when growth rates are initially higher than the average growth rate, the GBM will underestimate actual income, which will result in higher value of the MRG comparing to the MCS. On the other hand, when growth rates are initially lower than average growth rate, the GBM will overestimate actual income, resulting in lower value of the MRG comparing to the MCS.

Figure 1: Income levels in case of descending (left) and increasing growth rates (right) of a project



5. Sensitivity analysis: Measuring impacts on the value of the MRG

In this section we present magnitude of effects on the value of the MRG provided by different valuation drivers. We are particularly interested in three of them. *Firstly*, we present impact of an existent constant volatility of cash flows and thus make additional step from the simplistic no-volatility case from the previous section. *Secondly*, we present impact growth rate dynamics (decreasing growth rate case) has on the value of the MRG. *Thirdly*, we present impact volatility dynamics has on value of the MRG (decreasing volatility case).

5.1 Impact of volatility

In this section we show that the difference between MCS and GBM result is even greater when the project cash flows (demand for services offered by a PPP project) are more volatile (compared to no-volatility example), and thus GBM even more misleading. When we subsequently increase standard deviation in case of MCS and we transform the data to average growth rate and volatility, we will see that average growth rate is decreasing and volatility is increasing. In order to prove that we prepared a sensitivity analysis that was made with 5,000 simulations for each of the variable. Adjustment of our hypothetical project from Table 1 was done in order to highlight the effect of increasing standard deviation:

- Initial number of vehicles in year 2016 is 12,000,000 and is distributed normally with standard deviation 1,000,000 vehicles. Full capacity of the toll-road is 100,000,000 vehicles a year (this basically means there is no vehicle limitation).

- Anticipated yearly traffic growth rate during projects is 7%. Initial standard deviation is 1% and is in each scenario increased by 1% point. Standard deviation remains constant throughout the life span of the project.
- Average toll per vehicle is 4.40 EUR.

Table 10: Impact of increasing standard deviation on growth rate and volatility within GBM

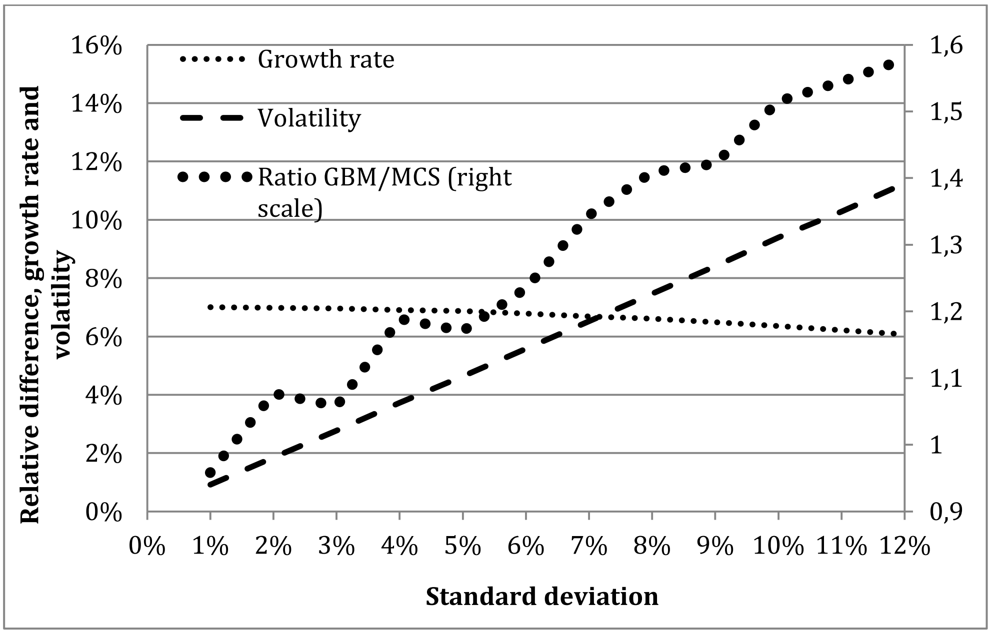
Standard deviation	Growth rate GBM	Volatility GBM	Guarantee MCS	Guarantee GBM	Ratio GBM/MCS
1.00%	7.00%	0.92%	3,249,014	3,112,737	.9581
2.00%	6.98%	1.85%	6,200,500	6,677,718	1.0770
3.00%	6.96%	2.77%	9,877,539	10,450,902	1.0580
4.00%	6.91%	3.73%	12,432,467	14,780,256	1.1888
5.00%	6.87%	4.64%	15,664,698	18,336,824	1.1706
6.00%	6.78%	5.58%	19,928,220	24,604,188	1.2346
7.00%	6.69%	6.52%	22,449,197	30,176,199	1.3442
8.00%	6.61%	7.46%	26,108,224	36,783,531	1.4089
9.00%	6.49%	8.41%	29,673,861	42,189,921	1.4218
10.00%	6.36%	9.39%	33,575,668	50,874,240	1.5152
11.00%	6.22%	10.28%	37,689,802	58,220,608	1.5447
12.00%	6.07%	11.25%	41,444,388	65,426,399	1.5787

From Table 10 we can clearly see that increasing standard deviation reduces growth rate and increases volatility. Average growth rate is descending, as we deal with a geometric mean value. For instance, if we increase 100 units in 1st year for 10 % and decrease in 2nd year for 10 %, the geometrical mean value of 2 years growth rate amounts to -0.50%. However if we increase 100 units in 1st year for 50 % and decrease in 2nd year for 50 %, we have geometric mean value of 2 years growth rates -13.40%. Higher standard deviation causes higher absolute differences between simulated growth rates under the MCS, which transposes to reduction of average growth rate used under the GBM[5].

Figure 2 evidently shows to what an extent an increase of standard deviation reduces growth rate, increases volatility and increases relative difference between the two compared methods. The latter is calculated as ratio between GBM and MCS from Table 10 (also reported in the last column of Table 10).

[5] Our hypothetical case has also standard deviation of 6% in first 5 years of the project, 3% in next 10 years and 2% in last 12 years, and thus one would expect that GBM standard deviation would be less than within the no-volatility example. This is however not the case. GBM standard deviation amounts to 4.86% (see Table 5) and is higher than 4.83% (i.e. within the no-volatility example). Thus, the introduction of standard deviation did not cause a reduction of the growth rate similar to one presented in Table 10. That is because the growth rate and respective standard deviation are actually being constant within the three subperiods, but they both simultaneously decrease twice throughout the project.

Figure 2: Impact of increasing standard deviation on growth rate, volatility and GBM/MSC ratio



From table 10 we see that 12.00% standard deviation transposes assumed growth rate 7.00% to 6.07% and 11.25% standard deviation, which results in guarantee value of EUR 65.4 mil pursuant to Geometric Brownian motion. On the other hand Monte Carlo simulation values the guarantee in case of 7% growth rate and 12% standard deviation to EUR 41.4 mil, which is EUR 24 mil less (58% less). Logical question is, what is responsible for the difference. Is it 6.07% growth rate (instead of 7.00%) or perhaps 11.25% standard deviation (instead of 12.00%). With the purpose of answering this question we made additional GBM valuations, which we present below.

5.2 Impact of a decreasing growth rate

In this section we go back to our initial case (see section 4) and completely exclude effect of decreasing volatility. Doing that, we highlight the true effect of the changing growth rate. We adjust our hypothetical project from Table 1 in the following manner:

- Initial number of vehicles in year 2016 is 12,000,000 with standard deviation 0. Full capacity of the toll-road is 100,000,000 vehicles a year (this basically means there is no vehicle limitation).
- Anticipated yearly traffic growth rates in different periods are:
  – 2016–2020: 9 % a year with standard deviation 0 %,
  – 2021–2030: 5 % a year with standard deviation 0 %,
  – 2031–2042: 3 % a year with standard deviation 0 %,

We see that this first case is actually our static case presented in section 4, of which income is a benchmark for our predetermined level of income. Due to the initial number of vehicles being fixed at 12,000,000 vehicles a year and due to the standard deviation of growth rates being zero, the private partner’s income is equal to the predetermined level of income and hence value of the guarantee according to the MCS is zero.

The corresponding geometric average growth rate within the GBM valuation model in our case is 4.83 %, while standard deviation is zero. According to the GBM, value of the MRG is EUR 50,026,776. This effectively means that due to only the decreasing growth rate GBM is overestimating the value of the MRG by EUR 50,026,776.

5.3 Impact of decreasing standard deviation

In the second case we completely exclude the effect of the decreasing growth rate so that we can transparently present the effect of the changing (decreasing) volatility. In order to achieve this we adjust our hypothetical project from Table 1 in following manner:

- Initial number of vehicles in year 2016 is 12,000,000 with standard deviation 0. Full capacity of the toll-road is 100,000,000 vehicles a year (this basically means there is no vehicle limitation).
- Anticipated yearly traffic growth rates in different periods are:
  – 2016–2020: 4.83 % a year with standard deviation 6 %,
  – 2021–2030: 4.83 % a year with standard deviation 3 %,
  – 2031–2042: 4.83 % a year with standard deviation 2 %,

In this case we have constant growth rate of 4.83 % for MCS and GBM through whole project period, that way removing the effect of reducing growth rate to the value of the MRG. Growth rate averaging will negatively impact the NPV of the project and further positively impact the value of the MRG. Let us also add that in case of GBM we used geometric average growth rate of 4.83 % for the sake of the comparison with the first example. Geometric average growth rate derived from the MCS is actually 4.78 %, which is in line with our finding presented in section 5.1 that the existence of growth rate's standard deviation in MCS causes reduction of the average growth rate that is used in GBM.

In contrast to the first case, we have decreasing standard deviation that results in the geometric average volatility rate of 3.26 % that was used in GBM procedure. Value of the MRG calculated under MCS and under GBM with 5,000 iterations is shown in the Table 12 below.

Table 12: Value of the MRG pursuant to MCS and GBM at 4.83% growth rate and decreasing standard deviation

	MCS	GBM
Number of samples	5,000	5,000
Mean value	54,360,925	52,077,043
Standard deviation	32,090,709	22,488,778
Median	52,793,578	51,115,772
Minimum	0	0
Maximum	160,486,345	130,153,667

Value of the MRG computed with MCS is somewhat higher than the one calculated with GBM, but the difference is only EUR 1.677.806, comparing medians. Those results clearly show that using average volatility rate in GBM procedure in contrast to decreasing standard deviation used in MCS has only minor impact to the value of the MRG.

Based on the results presented above we can thus conclude that using average volatility in the GBM procedure (when standard deviation is in fact decreasing) has small effect on the value of the MRG. Value of the MRG under the MCS is only somewhat higher in contrast to the value of GBM. On the other hand, using average growth rate under the GBM (when cash flows of the project follow decreasing growth rates pattern) has substantial impact on the value of the MRG. Namely, GBM substantially underestimates project's cash flow and overestimates value of the MRG compared to MCS.

Those results build further on the issue addressed by Chiara & Garvin (2008). Authors argue that in order to correctly address financial risk of the project valuation methodology should account for changes in project cash flow volatility. We argue that while accounting for changing volatility has impact on the value of the MRG, magnitude of the valuation error is much higher in case valuation methodology does not properly capture path-dependent cash flows, due to a specific growth rate pattern.

6. Conclusion

In this paper we show the difference between two commonly applied methodologies used to value guarantees, which have path-dependent cash flows, i.e. Monte Carlo simulation (MCS) and Geometric Brownian motion (GBM). We use a hypothetical example of a minimum revenue guarantee (MRG) within a public-private partnership (PPP) project. In such projects, government steps in to provide a sweetener of the project to the private partner, i.e. a guarantee that lowers riskiness of the project. On one hand, such a guarantee improves risk-return profile of the project and in the first place encourages private partners to enter into a PPP, but on the other hand, it imposes a potential burden on taxpayers. As a typical infrastructure PPP project is typically huge, it is of great importance that such a governmental support is calculated consistently.

We show the difference between MCS and GBM used to value a MRG (as well as the projects itself), and show that GBM is not suitable methodology for projects that have path-dependent cash flows. Namely, in all projects with uneven growth rates of demand and thus cash flows (and such growth rate pattern is common for majority of such projects) only MCS can consistently trace present value effects of a project. The reason is path-dependency of cash flows itself. This fact has important implications for guarantee pricing, which is a derivative. Among financial options such a characteristic have *Asian*- and *look-back options*, which are priced with numerical methods, and not with analytical formulas like Black & Scholes. This fact is very important and is only properly considered in financial literature of derivative pricing, but not in the literature of real options and pricing of contingent claims embedded into public-private partnership agreements.

In a typical PPP project with relatively higher growth rates during the subsequent years of a ramp-up period (and lower growth rates later on), decision regarding the approval of such a project, based on the GBM deters a project as public partner systematically overestimates the value of the embedded MRG needed to kick-start the project. The overestimation error increases with volatility of cash flows of the project as it further distorts averaging the growth rates.

Finally and importantly, we also show that the impact of changing volatility on value of the guarantee (issue addressed by Chiara & Garvin, 2008) is important, but much less than impact of changing growth rate of cash flows itself. Namely, usage of the average growth rate in the GBM procedure has considerable effect to the value of the MRG as in that case GBM substantially underestimates project's cash flow and overestimates value of the guarantee.

Our results therefore have very important economic implications for the field of valuation of infrastructure PPP projects, which are lacking in number and size to a large extent throughout the globe. Hopefully, lower valuations of MRG according to a consistent methodology (i.e. MCS, which is able to trace any pattern of cash flows), will help launch more such projects, which will significantly increase economic development and improve well-being.

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