# DO ROLLING OR FIXED EXERCISE DATES INCREASE A COMPOUNDED AMERICAN OPTION'S VALUE?

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## ABSTRACT

Due to the complexities of solving PDEs, a general assumption regarding real options is to discard time-dependency. Nevertheless, this assumption is not necessarily realistic, considering that people tend to simplify problems by self imposing maturity dates.

To study this phenomenon, the value of a compounded American option is compared considering two different exercise rules, using either (1) fixed dates under which a decision must be made, or (2) establishing a fixed lead time after an option exercise where an investor cannot exercise its claim over the asset. These values are then computed using numerical methods, in order to calculate under which circumstances it is worthwhile to establish limited timeframes to maximize a project’s value.

## INTRODUCTION

Behavioral economists have long recognized the fact that people do not always act as predicted by economic theory. In particular, studies by Tversky and Kahneman (1986) have shown that people's decisions are prone to simplification and to biases on how the information is presented (a phenomenon known as framing effect).

These bounds on human rationality also apply to how costly decisions are made. Instead of recognizing the inherent flexibility of making decisions at any moment, people tend to impose expiration dates in order to simplify the decision making process. This goes from major choices such as getting married, to trivial choices, such as starting a diet.

The objective of this paper is to analyze how different exercise rules can affect the value of a project under uncertainty. To this end, a project which is perpetual claim on an underlying asset $S$ is considered, subject to two different exercise rules: (1) either considering fixed dates under which a decision must be made, or (2) establishing a fixed lead time after an option exercise where an investor cannot exercise its claim over the asset. Landing this to an example, this is the choice a university makes between considering a rolling admissions system for receiving graduate applicants compared to establishing a fixed deadline for application. Or the choice one makes between establishing a fixed number of visits to see your parents in a year compared to establishing a minimum time span between visits.

## MODEL

The project's value depends on an underlying asset $S$ that follows a GBM motion process $dS = \mu \cdot S \cdot dt + \sigma \cdot S \cdot dw$, where $\mu$, $\sigma$ and $dw$ represent the instantaneous drift, instantaneous volatility and Weiner process, respectively.

To model the case of fixed dates under which a decisions must be made, the project is modeled as a compound American option with a fixed maturity date of $T$. In case of early exercise at $t < T$, the project owner receives the value of the asset $S$ minus the exercise price $I$, together with a new claim on the underlying asset after the maturity date $T$ (discounted to $t$). On the other hand, modeling the exercise rule as a minimum lead time between options, a perpetual compound American option is considered with the same exercise price $I$. However, in this case, a fixed lead time of $T$ once the option is exercised, in order to regain the claim over $S$.

It is important to emphasize at this point that this stochastic process ensures that both projects share a common lower bound of zero (when $S$ reaches the sinking point $S = 0$) and a common upper bound (when $S_0 \rightarrow \infty$), given by the discounted cash flow of exercising the option at the beginning of each year $\frac{S_0}{r-\mu} - \frac{I}{r}$.

Using Ito's Lemma to determine the value of the project *F* for the first case, the following partial differential equation in terms of *t* and *S* is reached, assuming the project's owner is risk neutral:

$$\frac{1}{2}\cdot\sigma^2\cdot S^2\cdot F_{SS}(S,t)+\mu\cdot S\cdot F_S(S,t)+F_t(S,t)-r\cdot F(S,t)=0 \tag{1}$$

Considering that both projects (with fixed maturity dates and fixed lead times) possess the same dynamics, it must be the border conditions that differ. In the case of considering a fixed maturity date of *T*, the project's border conditions are given by:

$$F(S,T)=\max\{S-I,0\}+F(S,0) \tag{2}$$

$$F(\bar{S},t)=\bar{\mathrm{S}}-\mathrm{I}+\mathrm{F}(\bar{S}\cdot e^{\mu\cdot(T-t)},0)\cdot e^{-r\cdot(T-t)} \tag{3}$$

$$F_S(\bar{S},t)=1+\mathrm{F_S}(\bar{S}\cdot e^{\mu\cdot(T-t)},0)\cdot e^{-(r-\mu)\cdot(T-t)} \tag{4}$$

$$F(0,t)=0 \tag{5}$$

The first condition shows that, if the final expiration date has been reached, the option for that year must be exercised. However, besides this payoff, a renewed claim on the underlying asset is establish for the following year, and must be included in the payoff at *T*. The second and third conditions are the value matching and smooth pasting conditions in case the option is exercised prior to its expiration date. Regarding this condition, it is important to emphasize that the option can be exercised only once per period. Hence, the renewed claim over the underlying asset is activated after *T*, which explains the second expression in equation (3). Finally, the final equation indicates that *S* = 0 is a sinking point due to the nature of the GBM process.

On the other hand, if a perpetual American option with a fixed lead time of *T* is considered, the project value in this case solely depends on the value of *S*. Hence, the project's value dynamics in (1) change to an ordinary differential equation given by:

$$\frac{1}{2}\cdot\sigma^2\cdot S^2\cdot F_{SS}(S)+\mu\cdot S\cdot F_S(S)-r\cdot F(S)=0 \tag{6}$$

And the boundary conditions in this case correspond to the value matching and smooth pasting conditions when exercising the option and the sinking point at *S* = 0:

$$F(\bar{S})=\bar{\mathrm{S}}-\mathrm{I}+\mathrm{F}(\bar{S}\cdot e^{\mu\cdot T})\cdot e^{-r\cdot T} \tag{7}$$

$$F_S(\bar{S})=1+\mathrm{F_S}(\bar{S}\cdot e^{\mu\cdot T})\cdot e^{-(r-\mu)\cdot T} \tag{8}$$

$$F(0)=0 \tag{9}$$

## MODEL RESOLUTION

In order to solve both alternatives, different resolution techniques are considered:

For the perpetual fixed maturity date option, a numerical method technique must be used, given that it is a partial differential equation with boundary conditions that depend on the value function itself. In this case, the Longstaff-Schwartz Least Squares Method (LSM) technique is used iteratively within the following algorithm:

**Algorithm: Modified LSM to estimate value function of perpetual fixed maturity date option**

1. Initialize value function $F^0(S,0)\ \forall\ S = \{S_{min}, S_{min} + dS, \dots, S_{max} - dS, S_{max}\}$

Do while $F^i(S,0) - F^{i-1}(S,0) > \varepsilon$ for some $S \in \{S_{min}, S_{min} + dS, \dots, S_{max}\}$

2. Perform LSM for each $S \in \{S_{min}, S_{min} + dS, \dots, S_{max}\}$ to determine $F^{i+1}(S)$.
   Use $F^i(S)$ in estimating early exercise payoffs.

Exit Do

3. Output: $F(S,0)\ \forall\ S = \{S_{min}, S_{min} + dS, \dots, S_{max} - dS, S_{max}\}$

In order to avoid covering a large support for $S$, we restrict ourselves to cases where the drift of the stochastic process ($\mu$) is arbitrarily small.

For the compound American option with lead time, this project value can be solved using traditional techniques for ordinary differential equations. The chosen resolution (for the moment) relates to Adkins and Paxson (2012) approach in evaluating analytically American perpetual compound options.

## CURRENT STATE

The LSM algorithm for this model has already been implemented and tuned by comparing results with the American put options calculated in Longstaff and Schwartz (2001). The iterative LSM algorithm is currently being tested under different parameters of $\mu$ and $\sigma$ in order to map the regions where it is more valuable to establish fixed maturity dates compared to establishing fixed lead times. This analysis is to be completed by March.

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