M&A Target Portfolio Selection: A Real Options Approach

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Abstract. In this paper we present a two-component portfolio selection problem under two types of uncertainties, i.e., probabilistic risk and possibilistic risk. We study the portfolio selection problem in mergers and acquisitions, M&As, and show the usability of the presented mixed model in portfolio selection of corporate acquisition targets. We view the total M&A value consisting of a stand-alone of a target value plus a synergistic strategic (real options) value. We illustrate, through a numerical example, how the portfolio model can be applied to M&As from an acquirer's perspective, in the case, where some targets are valued probabilistically using Datar-Mathews real options approach (Datar and Mathews, 2004) to value the strategic part and other targets possibilistically using the fuzzy real options approach to value the strategic part as presented by Kinnunen (2010) and Collan and Kinnunen (2011). The portfolio problem corresponds to a situation in which some return rates on M&A investments are described by random variables, while others by fuzzy numbers. We discuss the setup of an acquirer facing a situation in which some acquisition targets are reasonable to be valued probabilistically and others possibilistically. Markowitz probabilistic model and a possibilistic portfolio selection model are unified resulting in the optimal solution of the mixed portfolio problem with the minimum of the unified portfolio risk.

Keywords: portfolio optimization; mergers and acquisitions; real options; risk theory; fuzzy numbers; possibility theory; probability theory

1 Introduction

Mergers and acquisitions are typically hard to value, particularly, because they are often unique (Bower, 2001; Bruner, 2004, 2005) and potentially arising synergistic benefits are rarely realized as expected (KPMG, 1999, Bower, 2001, Bruner, 2004). This calls for appropriate valuation methods to correspond the type of uncertainty faced by an acquirer.

From acquirer's perspective, the return rates on M&A investments need to be estimated ex-ante and determined by the ex-ante valuation of targets. The way the return rates are described depends on the applied valuation method, i.e., when acquisition targets are analyzed in probabilistic terms, the return rates will be described by random variables; when they are analyzed in possibilistic terms, the return rates will be described possibilistically, e.g., by fuzzy numbers.

For the former case, probability theory represents the standard mathematical instrument to study the uncertainty phenomena. However, as for the latter case, there are types of uncertainty, which cannot be approached probabilistically, e.g., due to lack of data, lack of comparable assets/companies, and, as is often the case with synergies in M&As, due to non-stochasticity of future cash flows in the sense that an acquirer can have significant effect on synergy realization through its post-merger actions (Kinnunen, 2010; Collan and Kinnunen, 2011). Zadeh's (1978) possibility theory offers an alternative to the treatment of such uncertainty situations.

For an acquirer, an M&A can be valued using real option models as argued by Kinnunen (2010) and Collan and Kinnunen (2011). They apply the fuzzy pay-off method for real option valuation of Collan et al (2009) to value the strategic part of M&As. Traditionally such valuation is done by a probabilistic approach. The probabilistic counterpart of the fuzzy pay-off method is Datar-Mathews method for real option valuation (Datar and Mathews, 2004). In this paper, we will use the two approaches for synergy valuation, while the stand-alone parts are valued using standard methods. Hence, the total value of an M&A (NPV_{TOTAL}) is the stand-alone net present value ($NPV_{STAND\text{-}ALONE}$) of a target company plus the real option value of synergies ($ROV_{SYNERGY}$):

$$NPV_{TOTAL} = NPV_{STAND\text{-}ALONE} + ROV_{SYNERGY}. \tag{1}$$

When we consider the transition from probabilistic to possibilistic models, two components are concerned:

1. random variables (probability distributions) are replaced with possibility distributions (particularly, fuzzy numbers);

2. typical probabilistic indicators (e.g., expected value, variance, and covariance) are replaced with appropriate possibilistic indicators.

Portfolio selection is one of the crucial problems, which appear in financial decision-making. In M&A context an acquirer may be acquiring more than one company at the same time. This situation can be viewed as a portfolio selection problem: an acquirer needs to decide how much to invest to each potential target company under analysis. Acquirer's actions are specifically limited by its budget constraint, i.e., the solution of the portfolio problem can be presented as the optimal shares of its budget to be invested to each target company. The optimal shares should lead to a best available profit with a minimum risk level in line with the traditional Markowitz portfolio selection model (Markowitz, 1952, 1959), which considers asset returns as (probabilistic) random variables. Markowitz model uses two probabilistic indicators: mean value for return and variance for risk (see Markowitz, 1952, 1959; Altar, 2002).

Following Zadeh (1978), various possibilistic portfolio selection models have been studied (e.g., Carlsson et al, 2002; Georgescu, 2012; Huang, 2007, 2009, 2010, 2011; Inuiguchi and Ramik, 2000; Tanaka et al, 2000; Wang and Zhu, 2002). We have presented the idea of the portfolio selection problem combining probabilistic and possibilistic methods (Georgescu and Kinnunen, 2011, 2012a, 2012b) arising from the complexity of financial decision-making situation.

We have presented a portfolio model with discrete returns, where the possibilistic part is analyzed in credibilistic terms (Georgescu and Kinnunen, 2011) based on Liu and Liu (2002) and Liu (2007). We have shown how this case reduces to a probabilistic portfolio problem and how it can be applied to corporate acquisitions from a venture capitalist's perspective.

In Georgescu and Kinnunen (2012a; 2012b) we have approached the possibilistic part by fuzzy numbers and we have not restricted the analysis to discrete returns. In this paper, we will study an M&A portfolio selection problem characterized by the two components: some returns are mathematically described by random variables and others by fuzzy numbers. For the first component the *probabilistic indicators* associated with random variables are used, and for the second component corresponding *possibilistic indicators* associated with fuzzy numbers are used.

The paper is organized as follows.

In Section 2 the definition of a fuzzy number and operations with fuzzy number are recalled together with intuitive comparison to their probabilistic counterparts. The focus is on three indicators associated with fuzzy numbers: expected value, variance, and covariance (cf. Appadoo and Thavaneswaran, 2010; Carlsson and Fullér, 2001; Carlsson et al., 2002, 2005; Paseka et al., 2011; Wang and Tian, 2010; Zhang and Wang, 2007). They will be used in the subsequent sections to build a mixed portfolio selection model (Section 4) and with the M&A application (Section 5).

In Section 3 two portfolio selection models are compared: Markowitz's and a possibilistic one, the latter being derived from the former according to 1. and 2. The difference between the two approaches consists in indicators' interpretation:

- for the first model the return is evaluated by probabilistic mean value, while for the second model the return is evaluated by the possibilistic mean value;
- for the first model the risk is evaluated by probabilistic variance, while for the second model the risk is evaluated by the possibilistic variance.

The mixed portfolio is introduced in Section 4. Rentability of some assets is mathematically represented by random variables, while rentability of other assets is represented by fuzzy numbers. Two types of indicators are associated with a portfolio:

- a possibilistic mean value and a probabilistic mean value, and a total mean value;
- a possibilistic variance and a probabilistic variance, and a total variance.

The two-component mixed portfolio selection problem is formulated using these indicators in Section 4. The main result of the section is the optimal solution of portfolio selection problem and the calculation of the minimum risk.

M&A portfolio selection problem is presented in Section 5, which firstly discusses the problem setup, where the acquisition portfolio consists of the two types of companies, which will be valued either by a probabilistic or by a possibilistic approach. Secondly, it shortly presents the chosen methods, the Datar-Mathews method for the former, and the fuzzy pay-off method for the latter case and illustrates their use in valuing acquisition targets with a numerical example of the faced portfolio selection problem.

Section 6 concludes the paper with discussion of limitations and suggestions for future research.

2 Possibilistic Indicators

In this section we recall the possibilistic indicators (Carlsson and Fullér, 2001; Carlsson et al., 2002, 2005) associated with fuzzy numbers (Carlsson and Fullér, 2002; Dubois and Prade, 1980, 1988). They are needed in our subsequent analysis of Sections 3-4, where the probabilistic indicators are replaced with their possibilistic counterparts, and in Section 5, which presents the M&A application. For the purpose of the subsequent analysis, we also reflect the presented notions with the normal triangular fuzzy number (and very shortly with the normal probability distribution).

Let X be a set of states. A *fuzzy subset* of X is a function $A: X \to [0,1]$. For any state $x \in X$, the real number $A(x)$ is the degree of membership of x to A. The support of a fuzzy set A is $supp(A) = \{x \in X | A(x) > 0\}$. A fuzzy set A is *normal*, if there exists $x \in X$ such that $A(x) = 1$. On the right side of Figure 1, $A(x) = 1$ is described at the peak/center a (i.e., $A(a)$=1) of the triangular fuzzy number. The probabilistic "counterpart" of degree of membership $A(x)$ is the frequentist probability P(x) on the left side of Figure 1. (Note that P(x) is never 1 unless the variance is 0, in Figure 1 the probabilistic variance is 0.5).

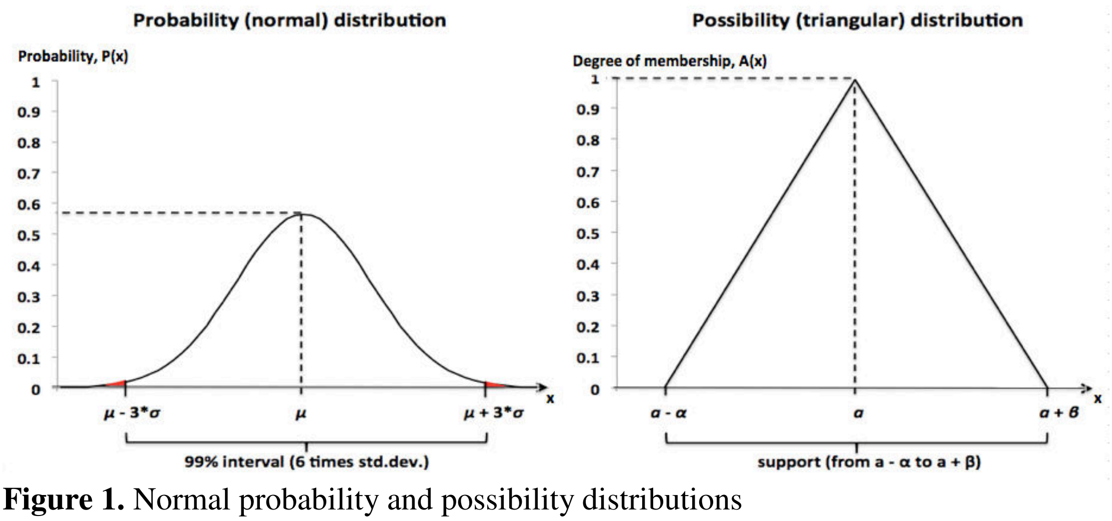

Figure 1. Normal probability and possibility distributions

Probability is interpreted in frequency terms, while possibility is interpreted as a degree of belief. For M&A valuation (presented in Section 5), we note that efficient market hypothesis assumes that future values of target companies are process-generated, e.g., by geometric Brownian motion (which suggest that markets determine the values), while relaxation of the assumption leads to non-generate processes. In practice, the former valuation problem is typically handled probabilistically using, e.g., Black-Scholes or binomial real options models, while in case of a non-generate process, (where acquiring company may be able to affect the values), expert estimates can be necessary in which case a probabilistic Datar-Mathews method or a possibilistic pay-off method for real option valuation can be found useful as discussed in Section 5.

In the following we consider $X = \mathbb{R}$.

Let A be a fuzzy subset of $\mathbb{R}$ and $\gamma \in [0,1]$. The γ–level set of A is defined by

$$[A]^{\gamma} = \begin{cases} \{x \in \mathbb{R} | A(x) \geq \gamma\} & if \quad \gamma > 0 \\ cl(supp(A)) & if \quad \gamma = 0 \end{cases} \tag{2}$$

$cl(supp(A))$ is the topological closure of the set $supp(A) \subseteq \mathbb{R}$.

A is called *fuzzyconvex* if $[A]^{\gamma}$ is a convex subset of $\mathbb{R}$ for any $\gamma \in [0,1]$.

A *fuzzy number* is a fuzzy set of $\mathbb{R}$, normal, fuzzyconvex, continuous, and with bounded support.

Let A be a fuzzy number and $\gamma \in [0,1]$. Then $[A]^{\gamma}$ is a closed and convex subset of $\mathbb{R}$. We denote $a_1(\gamma) = \min[A]^{\gamma}$ and $a_2(\gamma) = \max[A]^{\gamma}$. Hence $[A]^{\gamma} = [a_1(\gamma), a_2(\gamma)]$ for all $\gamma \in [0,1]$. On the right side of Figure 1 the total (closure of) support of triangular fuzzy number (from a-alpha to a+beta) corresponds to the support from $a_1(\gamma) = \min[A]^{\gamma}$ to $a_2(\gamma) = \max[A]^{\gamma}$, where $\gamma = 0$. On the left side of Figure 1, the probabilistic counterpart, the range of the distribution is limited to 99% interval, which is typical for practical purposes, when one prefers to neglect the asymptotic tales of the distribution.

Let A and B be two fuzzy numbers and $\lambda \in \mathbb{R}$. We define the functions $A + B: \mathbb{R} \rightarrow [0,1]$ and $\lambda A: \mathbb{R} \rightarrow [0,1]$ by

$$(A + B)(z) = sup\{A(x) \wedge B(y) | x + y = z\}; \tag{3}$$
$$(\lambda A)(z) = sup\{A(x) | \lambda x = z\}. \tag{4}$$

Then $A + B$ and λA are fuzzy numbers.

If $A_1, \ldots, A_n$ are fuzzy numbers and $\lambda_1, \ldots, \lambda_n \in \mathbb{R}$, then one can consider the fuzzy number $\sum_{i=1}^{n} \lambda_i A_i$.

A non–negative and monotone increasing function $f: [0,1] \rightarrow \mathbb{R}$ is a *weighting function* if it satisfies the normality condition $\int_0^1 f(\gamma) d\gamma = 1$.

We fix a fuzzy number A and a weighting function *f*. Assume that $[A]^{\gamma} = [a_1(\gamma), a_2(\gamma)]$ for all $\gamma \in [0,1]$.

The *f–weighted possibilistic expected value* of A is defined by

$$E(f, A) = \frac{1}{2} \int_0^1 (a_1(\gamma) + a_2(\gamma)) f(\gamma) d\gamma. \tag{5}$$

The *f–weighted possibilistic variance* of A is defined by

$$Var(f,A) = \frac{1}{2}\int_0^1 [\left(a_1(\gamma) - E(f,A)\right)^2 + (a_2(\gamma) - E(f,A))^2] f(\gamma) d\gamma. \tag{6}$$

Assume now that A and B are two fuzzy numbers such that $[A]^\gamma = [a_1(\gamma), a_2(\gamma)]$ and $[B]^\gamma = [b_1(\gamma), b_2(\gamma)]$ for any $\gamma \in [0,1]$.

On the right side of Figure 1, the possibilistic expected value of the normal (symmetric, i.e., when alpha=beta) triangular fuzzy number is at the peak value a. On the left side of Figure 1, μ represents the probabilistic expected value of the normal probabilistic distribution. In Section 5, we will see how both the possibilistic expected value, as well as, the possibilistic variance are calculated in our simplified M&A case.

The *f–weighted possibilistic covariance* of A and B is defined by

$$\begin{aligned} Cov(f,A,B) = \frac{1}{2}\int_0^1 &[\left(a_1(\gamma) - E(f,A)\right)\left(b_1(\gamma) - E(f,B)\right) \\ &+\left(a_2(\gamma) - E(f,A)\right)\left(b_2(\gamma) - E(f,B)\right)] f(\gamma) d\gamma. \end{aligned} \tag{7}$$

In the following, when we write $E(f,A)$, $Var(f,A)$, or $Cov(f,A,B)$, the weighting function f will be apriori fixed.

Proposition 2.1 *(Appadoo and Thavaneswaran, 2010; Fullér and Majlender, 2003) Let $A_1, \dots, A_n$ be fuzzy numbers and $\lambda_1, \dots, \lambda_n \in \mathbb{R}$.*

(i) $E(f, \sum_{i=1}^n \lambda_i A_i) = \sum_{i=1}^n \lambda_i E(f, A_i)$;
(ii) *If* $\lambda_1, \dots, \lambda_n \geq 0$, *then* $Var(f, \sum_{i=1}^n \lambda_i A_i) = \sum_{k,l=1}^n \lambda_k \lambda_l Cov(f, A_k, A_l)$.

3 Approaches to Portfolio Selection

In this section we present two approaches to portfolio selection, i.e., Markowitz's (1952, 1958) probabilistic portfolio selection model and a possibilistic model (Huang, 2007, 2010; Inuiguchi and Ramik, 2000).

Probabilistic Portfolio Selection Model

One considers m assets $j = 1, \dots, m$. Assume that the returns of the m assets are random variables $X_1, \dots X_m$. The following elements are known:

- probabilistic mean returns $\mu_j = M(X_j)$, $j = 1, \dots, m$;
- probabilistic covariances $\sigma_{s,t} = Cov(X_s, X_t)$, $s, t = 1, \dots, m$.

A portfolio is a vector $(y_1, \dots, y_m) \in \mathbb{R}^m$ with $\sum_{j=1}^m y_j = 1$ and $y_j \geq 0$ for any $j = 1, \dots, m$. The return of the portfolio $(y_1, \dots, y_m)$ is the random variable $\sum_{j=1}^m y_j X_j$. One associates with each portfolio $(y_1, \dots, y_m)$:

- mean return $M\left(\sum_{j=1}^{m} y_j X_j\right) = \sum_{j=1}^{m} y_j \mu_j$;
- probabilistic variance $Var\left(\sum_{j=1}^{m} y_j X_j\right) = \sum_{s,t=1}^{m} y_s y_t \sigma_{s,t}$.

The probabilistic portfolio selection problem is:

$$\begin{cases} min \frac{1}{2} \sum_{s,t=1}^{m} y_s y_t \sigma_{s,t} \\ \sum_{j=1}^{m} y_j \mu_j = \rho \\ \sum_{j=1}^{m} y_j = 1 \\ y_j \geq 0, j = 1, \ldots, m. \end{cases} \tag{8}$$

Possibilistic Portfolio Selection Model

One considers n assets $i = 1, \ldots, n$. Assume that the returns of the n assets are represented by the fuzzy numbers $A_1, \ldots A_n$. The following elements are known:

- possibilistic mean returns $\gamma_i = E(f, A_i), i = 1, \ldots, n$ (, where, according to the convention from Section 2, we assume that the weighting function f is apriori fixed);
- possibilistic covariances $\delta_{k,l} = Cov(f, A_k, A_l), k, l = 1, \ldots, n$.

In this case a portfolio is a vector $(x_1, \ldots, x_n) \in \mathbb{R}^n$. The return of the portfolio $(x_1, \ldots, x_n)$ is the fuzzy number $\sum_{i=1}^{n} x_i A_i$. One associates with each portfolio:

- possibilistic mean return $E(f, \sum_{i=1}^{n} x_i A_i) = \sum_{i=1}^{n} x_i \gamma_i$;
- possibilistic variance $Var(f, \sum_{i=1}^{n} x_i A_i) = \sum_{k,l=1}^{n} x_k x_l \delta_{k,l}$.

The possibilistic portfolio selection problem is:

$$\begin{cases} min \frac{1}{2} \sum_{k,l=1}^{n} x_k x_l \delta_{k,l} \\ \sum_{i=1}^{n} x_i \gamma_i = \delta \\ \sum_{i=1}^{n} x_i = 1 \\ x_i \geq 0, i = 1, \ldots, n. \end{cases} \tag{9}$$

4 Mixed Portfolio Model to Unify Probabilistic and Possibilistic Approaches

In this section we introduce the *mixed portfolio* and its indicators, and establish the mixed portfolio selection problem form with its optimal solution and the value of its minimum risk. These notions correspond to a financial situation in which some assets are modeled by fuzzy numbers and others by random variables.

We consider $n + m$ assets. We make the following assumptions:

- The returns of the first n assets are fuzzy numbers $A_1, \dots A_n$.
- The returns of the other m assets are random variables $X_1, \dots X_m$.

We know the following elements:

- possibilistic mean returns $\gamma_i = E(f, A_i), i = 1, \dots, n$;
- possibilistic covariances $\delta_{k,l} = Cov(f, A_k, A_l), k, l = 1, \dots, n$;
- probabilistic mean returns $\mu_j = M(X_j), j = 1, \dots, m$;
- probabilistic covariances $\sigma_{st} = Cov(X_s, X_t),\ s, t = 1, \dots, m$;

A *mixed portfolio* has the form $(x_1, \dots, x_n, y_1, \dots, y_m) \in \mathbb{R}^{n+m}$, where $\sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j = 1$ and $x_i, y_j \geq 0$ for any $i = 1, \dots, n, j = 1, \dots, m$. The real numbers $x_1, \dots, x_n$ represent the investment proportions of the first n assets and $y_1, \dots, y_m$ represent the investment proportions of the other m assets.

We consider the mixed portfolio $(x_1, \dots, x_n, y_1, \dots, y_m)$ and define the following indicators of portfolio $(x_1, \dots, x_n, y_1, \dots, y_m)$:

- possibilistic mean returns $A_P = \gamma_P = \sum_{i=1}^{n} x_i E(f, A_i)$;
- probabilistic mean returns $X_P = \mu_P = \sum_{j=1}^{m} y_j M(X_j)$;
- portfolio's (total) mean return $R_P = \gamma_P + \mu_P$:
  $R_P = \sum_{i=1}^{n} x_i E(f, A_i) + \sum_{j=1}^{m} y_j M(X_j)$

We further define:

- portfolio's possibilistic variance $\delta_P = \sum_{k,l=1}^{n} x_k\, x_l \delta_{kl}$;
- portfolio's probabilistic variance $\sigma_P = \sum_{s,t=1}^{m} y_s\, y_t \sigma_{st}$;
- portfolio's (total) variance $Var_P = \delta_P + \sigma_P$:

  $Var_P = \sum_{k,l=1}^{n} x_k\, x_l \delta_{kl} + \sum_{s,t=1}^{m} y_s\, y_t \sigma_{st}$.

Var_P is a risk indicator associated with the portfolio $(x_1, \dots, x_n, y_1, \dots, y_m)$. It comprises the possibilistic risk component δ_P and the probabilistic risk component σ_P of the portfolio.

Mixed Portfolio Selection Problem

Next, we establish the mixed portfolio selection problem form. We compute its optimal solution and the value of the associated minimum risk.

Our approach to this problem subscribes to the ideas of, e.g., Altar (2002) and Markowitz (1952, 1959). We keep the notations from the previous section.

The mixed portfolio problem has the form:

$$\begin{cases} min\frac{1}{2}[\sum_{k,l=1}^{n} x_k\, x_l \delta_{kl} + \sum_{s,t=1}^{m} y_s\, y_t \sigma_{st}] \\ \sum_{i=1}^{n} x_i \gamma_i = \rho_1 \\ \sum_{j=1}^{m} y_j \mu_j = \rho_2 \\ \\ \sum_{i=1}^{n} x_i + \sum_{j=1}^{m} y_j = 1 \\ x_i, y_j \geq 0, \quad i = 1, \ldots, n, j = 1, \ldots, m. \end{cases} \tag{10}$$

To solve the mixed portfolio selection problem means to find a portfolio $(x_1, \ldots, x_n, y_1, \ldots, y_m)$ of minimum risk, which ensures a possibilistic mean return ρ_1 and a probabilistic mean return ρ_2.

We follow Georgescu and Kinnunen (2012a; 2012b) and denote:

$$x = (x_1, \ldots, x_n)^T; y = (y_1, \ldots, y_m)^T;$$
$$\gamma = (\gamma_1, \ldots, \gamma_n)^T;\ \mu = (\mu_1, \ldots, \mu_m)^T;$$
$$e_n = (\underbrace{1, \ldots, 1}_{n-times})^T;\ e_m = (\underbrace{1, \ldots, 1}_{m-times})^T;$$
$$\Omega_1 = (\delta_{kl})_{k,l=1,\ldots,n};\ \Omega_2 = (\sigma_{st})_{s,t=1,\ldots,m}.$$

With these notations problem (10) is written in matrix form:

$$\begin{cases} min\frac{1}{2}[x^T\Omega_1 x + y^T\Omega_2 y] \\ x^T\gamma = \rho_1 \\ y^T\mu = \rho_2 \\ \\ x^T e_n + y^T e_m = 1 \\ x, y \geq 0. \end{cases} \tag{11}$$

If $n = 0$, then Markowitz model is obtained from (11); if $m = 0$, then (11) is exactly the possibilistic portfolio selection model presented in Section 3. It follows that that the mixed model (11) extends both Markowitz probabilistic model and the possibilistic model of Section 3.

We denote:

$$A = \gamma^T\Omega_1^{-1}\gamma;\ B = \mu^T\Omega_2^{-1}\mu;$$
$$C = e_n^T\Omega_1^{-1}\gamma; D = e_m^T\Omega_2^{-1}\mu;$$
$$E = e_n^T\Omega_1^{-1}e_n + e_m^T\Omega_2^{-1}e_m; \text{ and}$$

$$F = ABE - BC^2 - AD^2.$$

If $F \neq 0$, then the system (11) has a solution (Georgescu and Kinnunen, 2012a).

Remark 4.1 We note now that the system (11) has no solution in case m=n=e=1, i.e., when there is only one asset of both (possibilistic and probabilistic) classes.

It is straightforward to show that $F = 0$, if m=n=e=1 as then we denote:

$$A = \gamma^2 \Omega_1^{-1};\ B = \mu^2 \Omega_2^{-1};$$
$$C = \Omega_1^{-1}\gamma; D = \Omega_2^{-1}\mu;$$
$$E = \Omega_1^{-1} + \Omega_2^{-1}.$$

Replacing the above in $F = ABE - BC^2 - AD^2$, we get $\frac{F}{\gamma^2\mu^2} = 0$, which is true only when $F = 0$.

We have the optimal solution of (10) (Georgescu and Kinnunen, 2012a):

$$x = \frac{(BE - D^2)\rho_1 + DC\rho_2 - BC}{F}\Omega_1^{-1}\gamma + \frac{-BC\rho_1 - AD\rho_2 + AB}{F}\Omega_1^{-1}e_n; \qquad (12)$$
$$y = \frac{(AE - C^2)\rho_2 + DC\rho_1 - AD}{F}\Omega_2^{-1}\mu + \frac{-BC\rho_1 - AD\rho_2 + AB}{F}\Omega_2^{-1}e_m. \qquad (13)$$

We have Var_P , i.e., the value of the minimum risk of the mixed portfolio $(x_1, \dots, x_n, y_1, \dots, y_m)$, which assures a possibilistic mean return ρ_1 and a probabilistic mean return ρ_2 (proof in Georgescu and Kinnunen, 2012a):

$$Var_P = \frac{1}{F}((BE - D^2)\rho_1^2 + (AE - C^2)\rho_2^2 + 2CD\rho_1\rho_2 - 2BC\rho_1 - 2AD\rho_2 + AB) \qquad (14)$$

5 M&A Portfolio Selection

We suppose an active acquirer analyzing several, at least three (cf. Remark 4.1: there exists no solution of the portfolio model in the case of only two elements), mergers and acquisitions, M&As simultaneously, i.e., it needs to make a selection from a set of acquisition target companies, which have gone through an initial target company search and screening process, they may be already in a due diligence process, but they are still under an ex-ante analysis, i.e., an agreement for the closure of the deal has not been signed (cf. Kinnunen, 2010; Collan and Kinnunen, 2011). M&As can be motivated, e.g., by efficiency increases, economies of scale and scope, increase the market share, or enhance presence in new markets (Bradley, et al., 1983; Bruner, 2004; DePamphilis, 2009; Krishnamurti and Vishwanath, 2008; Pablo and Javidan, 2004; Seth, 1990; Walker, 2000) to mention a few. Target companies under analysis can have different motives behind their acquisitions, some may be to support service and maintenance functions in established markets, some may be seen interesting for establishing geographical presence in a new market.

Regardless of the motivation for acquisitions, companies want them to be wealth creating and an obvious problem is to invest to companies, which together as a portfolio maximize the return from realized M&As. For the purpose of this paper we don't focus on qualitative strategic rationales behind M&As, instead, we consider pure economic return expected from the transactions. We assume that an acquirer wants to gain the maximum profit with a minimum risk. We further assume that some potential targets are *hard-to-value companies*, e.g., they are privately owned small companies with limited publicly disclosed financial statements information, without publicly priced valuations, there may not be comparable firms in the market against which their value could be determined, their value may depend largely on intangibles or they own novel technologies or novel business plans built on strategic investment options arising in the future; the other potential targets are *easier-to-value companies* in the sense that they may be publicly traded in a stock exchange, where their prices are determined by, more or less, efficient markets or there are comparable market-priced companies, which can be used in valuation, or their business may be simple, e.g., their revenues and costs are determined by publicly available market prices of their inputs and outputs.

Traditional probabilistic portfolio optimization techniques are easily implemented in the case the portfolio of potential M&A targets consists only of easy-to-value firms, but as hard-to-value firms are assumed as a part of the portfolio, the faced situation may require new techniques, firstly to value such companies, and secondly to optimize such portfolio. We assume that a fuzzy (possibilistic) valuation method is used for latter type of firms. In this paper, the M&A case example to follow uses the fuzzy pay-off method for real option valuation of Collan et al (2009) to value the strategic part (synergies) of such M&As, i.e., such targets as a part of an acquirer, which is in line what has been suggested by Kinnunen (2010) and Collan and Kinnunen (2011). In line with their view the total value of an M&A is presented by equation (1), i.e., $NPV_{TOTAL}=NPV_{STAND\text{-}ALONE} + ROV_{SYNERGY}$; for the former types of firms, we use the probabilistic counterpart of the fuzzy pay-off method, the Datar-Mathews method for real option valuation to value the strategic part of their total value. It is important to note that *any probabilistic and any possibilistic techniques could be used* (either discrete or continuous) and they would fit to the two-component portfolio selection approach. The selected methods are used for their practicality and simplicity for both illustrative purposes and to demonstrate the practical potential of our two-component portfolio approach.

Next cash flow based real option valuation is discussed. This is done, firstly, to construct the inputs required by the presented real option valuation models and, secondly, to together give us required inputs for the numerical example of the application of the portfolio selection model.

Cash Flows and Real Option Valuation

[Cash flow valuation of Datar-Mathews and fuzzy pay-off methods presented here]

Numerical Example

Suppose we have four target companies under analysis and two of them are analyzed using a possibilistic technique (fuzzy pay-off method, FPOM) and the other two using a probabilistic technique (D-M). Particularly, the synergy part, $\mathrm{ROV_{SYNERGY}}$, which comes above the stand-alone value, $\mathrm{NPV_{STAND\text{-}ALONE}}$, of a target (see Equation 1), is analyzed through the process as discussed in Kinnunen (2010) in the possibilistic case and analogously in the probabilistic case. The strategic synergy real options value is added to the stand-alone value to receive the expected total value of a target. After assuming a price to be paid for each, the expected return rates on investments of the size of the prices can be determined as:

$$Return = \frac{\mathrm{NPV_{STAND-ALONE}} + \mathrm{ROV_{SYNERGY}}}{Price} - 1$$

The data presented in Table 1 includes the expected returns and elements in variance-covariance matrix of the expected total cash-flows from four potential M&A targets. Table 1 further presents required rates of return for both possibilistic and probabilistic portfolios set and varied by an acquirer. This completes the required inputs for the portfolio analysis.

	Possibilistic portfolio		**Probabilistic portfolio**	
Target (*n*=*m*=2)	x_1	x_2	y_1	y_2
Expected return $\boldsymbol{\gamma, \mu}$	30%	10%	30%	30%
Covariance $\boldsymbol{\delta, \sigma}$ $(\boldsymbol{\Omega_{1,2}})$	4%		3%	
Variance $\boldsymbol{\delta, \sigma}$ $(\boldsymbol{\Omega_{1,2}})$	3%	6%	5%	6%
Required return $\boldsymbol{\rho_{1,2}}$	10%		20%	

Table 1. Expected and required returns and (co)variances of M&A investments

According to the data, we have a 2-component mixed portfolio problem, which can reduce to a pure (Markowitz) probabilistic problem containing only targets in probabilistic portfolio if the analysis leads to zero shares for targets in possibilistic portfolio, or vice versa, to a pure possibilistic problem, if targets of probabilistic portfolio get zero shares.

The needed matrices (cf. notions on p. 9) can be written as:

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix};$$

$$\gamma = [0.3 \quad 0.1]^T \text{ ; } \mu = [0.3 \quad 0.3]^T;$$

$$\gamma^T = [0.3 \quad 0.1]; \mu^T = [0.3 \quad 0.3];$$

$$e_n = [1 \quad 1]^T; e_m = [1 \quad 1]^T;$$

$$\Omega_1 = \begin{bmatrix} 0.03 & 0.04 \\ 0.04 & 0.06 \end{bmatrix}; \Omega_2 = \begin{bmatrix} 0.05 & 0.03 \\ 0.03 & 0.06 \end{bmatrix};$$

$$\Omega_1{}^{-1} = (1/(0.03 * 0.06 - 0.04 * 0.04)) \begin{bmatrix} 0.06 & -0.04 \\ -0.04 & 0.03 \end{bmatrix} = \begin{bmatrix} 300 & -200 \\ -200 & 150 \end{bmatrix};$$

$$\Omega_2{}^{-1} = (1/(0.05 * 0.06 - 0.03 * 0.03)) \begin{bmatrix} 0.06 & -0.03 \\ -0.03 & 0.05 \end{bmatrix} = \begin{bmatrix} 28.57 & -14.29 \\ -14.29 & 23.81 \end{bmatrix},$$

and now we can compute (cf. p. 9):

$$A = \gamma^{\mathrm{T}}\Omega_1^{-1}\gamma = [0.3 \quad 0.1]\begin{bmatrix}300 & -200\\ -200 & 150\end{bmatrix}\begin{bmatrix}0.3\\ 0.1\end{bmatrix} = 16.50;$$

$$B = \mu^{\mathrm{T}}\Omega_2^{-1}\mu = [0.3 \quad 0.3]\begin{bmatrix}28.57 & -14.29\\ -14.29 & 23.81\end{bmatrix}\begin{bmatrix}0.3\\ 0.3\end{bmatrix} = 2.14;$$

$$C = \mathrm{e}_{\mathrm{n}}^{\mathrm{T}}\Omega_1^{-1}\gamma = [1 \quad 1]\begin{bmatrix}300 & -200\\ -200 & 150\end{bmatrix}\begin{bmatrix}0.3\\ 0.1\end{bmatrix} = 25.00;$$

$$D = \mathrm{e}_{\mathrm{m}}^{\mathrm{T}}\Omega_2^{-1}\mu = [1 \quad 1]\begin{bmatrix}28.57 & -14.29\\ -14.29 & 23.81\end{bmatrix}\begin{bmatrix}0.3\\ 0.3\end{bmatrix} = 7.14;$$

$$E = \mathrm{e}_{\mathrm{n}}^{\mathrm{T}}\Omega_1^{-1}e_n + \mathrm{e}_{\mathrm{m}}^{\mathrm{T}}\Omega_2^{-1}e_m$$
$$= [1 \quad 1]\begin{bmatrix}300 & -200\\ -200 & 150\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} + [1 \quad 1]\begin{bmatrix}28.57 & -14.29\\ -14.29 & 23.81\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = 73.80.$$

The mixed portfolio problem (10), (11) has a solution only if F is nonzero:

$$F = ABE - BC^2 - AD^2 = 16.5*2.14*73.8 - 2.14*25^2 - 16.5*7.14^2 = 427.21 \neq 0,$$

which ensures that the portfolio problem has a solution and allows us to compute the optimal shares and the related minimum variance.

The required rates of return, $\rho_1 = 0.1$ and $\rho_1 = 0.2$, set to possibilistic and probabilistic portfolios (Table 1), respectively, are assured with the minimum variance of the total portfolio in equation (14), Var_P

$$Var_P = (1/F)((BE - D^2)\rho_1^2 + (AE - C^2)\rho_2^2 + 2CD\rho_1\rho_2 - 2BC\rho_1 - 2AD\rho_2 + AB)$$
$$= (1/427.21)*((2.14*73.8 - 7.14^2)*0.1^2 + (16.5*73.8-25^2)*0.2^2$$
$$+2*25*7.14*0.1*0.2 -2*2.14*25*0.1 - 2*16.5*7.14*0.2 + 16.5*2.14)/427.21$$
$$= 0.022.$$

The optimal solution to our mixed portfolio problem (10) using equation (12) for targets belonging to the possibilistic portfolio, n=2:

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix} = \frac{(BE - D^2)\rho_1 + DC\rho_2 - BC}{F}\Omega_1^{-1}\gamma + \frac{-BC\rho_1 - AD\rho_2 + AB}{F}\Omega_1^{-1}\mathrm{e}_{\mathrm{n}}$$
$$= (((2.14*73.8-7.14^2)*0.1+7.14*25*0.2-2.14*25)/427.21)\begin{bmatrix}300 & -200\\ -200 & 150\end{bmatrix}\begin{bmatrix}0.3\\ 0.1\end{bmatrix}$$
$$+ ((-2.14*25*0.1 - 16.5*7.14*0.2 + 16.5*2.14)/427.21)\begin{bmatrix}300 & -200\\ -200 & 150\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}$$
$$= \begin{bmatrix}0.33\\ 0.00\end{bmatrix},$$

where only the row of the inverse covariance matrix, Ω_1^{-1}, makes the difference in the computation according to which target's share is calculated (the first row determines x_1 and the second one x_2). Similarly, plugging into equation (13) and computing the shares of targets in the probabilistic portfolio, m=2, give:

$$\begin{bmatrix}y_1\\ y_2\end{bmatrix} = \begin{bmatrix}0.40\\ 0.27\end{bmatrix}.$$

The optimal mixed portfolio is (x_1 = 33%; x_2 = 0%; y_1 = 40%; y_2 = 27%). It is, x_2, a target company in the possibilistic portfolio, gets a 0% share suggesting that it will not be acquired. The other possibilistically analyzed target has a share of 33% of the given budget. The companies in the probabilistic portfolio have shares of 40% and 27% (y_1 and y_2, respectively). In line with intuition, the possibilistic company is dropped out due to significantly lower (10%) expected return compared to the other three companies (30%), while it has relatively high variance; The companies in the probabilistic portfolio have equal expected returns and one percentage point difference in their variance and the larger share (40%) is attached to the company with lower variance within the taken approach.

Discussion

We added the synergy real option values to the related stand-alone values of the targets in the above example (as suggested in Kinnunen, 2010, allowing to take into account possible modification to stand-alone values due to inter-relation of synergy development and stand-alone operations, eg., some operating assets can be liquidated during the process), but for simplification purposes, as well as, to make an interesting point, we could have considered only the strategic value arising from synergies. The interesting note in this case is that we can assume that in efficient markets the stock value of a target reflects its fundamental value, i.e., when stock pricing is used as the basis of the stand-alone valuation and a probabilistic approach is used, the assumption would be in line with the standard efficient markets hypothesis; and for simplification, we could have assumed the same for the possibilistically handled targets, although, they can be hard-to-value private targets without market prices.

According to the efficient market assumption, it can be argued that targets are bought by the price, which leads to zero returns from the stand-alone operations and then only the synergy real options part, $ROV_{SYNERGY}$, is the interesting for the analysis and decision metrics. This would lead to a simple interpretation of the solution of the above analysis: after the returns on each target had been determined purely by synergies over the expected costs of integrating the targets and the corporate development to create the synergies, the shares would indicate the shares of the corporate development budget optimal to be spent to each target's synergy development processes (excluding the stand-alone part, $NPV_{STAND\text{-}ALONE}$, and the paid price from the analysis). Interesting future research can arise from such approach.

The data in Table 1 means that we have a 2-component mixed portfolio problem, which could have reduced to a pure (Markowitz) probabilistic problem containing only targets in probabilistic portfolio if the analysis leads to zero shares for targets in possibilistic portfolio (equal to analysis under n=0), or vice versa, to a pure possibilistic problem, if targets of probabilistic portfolio get zero shares (equal to analysis under m=0). Particularly, if we would have taken the efficient market hypothesis as the starting point, the $ROV_{SYNERGY}$ values would determine the possible reduction, e.g., if they are zero for all targets in a (probabilistic or possibilistic) class, the reduction takes place to another class (possibilistic or probabilistic, respectively).

6 Conclusions

The two-component portfolio selection problem treated in this paper allows an analysis of financial management situation, where return rates on assets fall into different uncertainty types, i.e., into both probabilistic and possibilistic types. The expected returns of the former type are analyzed as random variables and the latter as fuzzy numbers. Markowitz probabilistic model and a possibilistic portfolio selection model were unified resulting to a formula combining probabilistic and possibilistic indicators for the calculation of the optimal mixed portfolio and for the minimum risk associated with it.

It was shown in M&A context that an active acquirer can face such complex situation, when it has under analysis several potential M&A target companies of which some are handled traditionally in probabilistic terms and others possibilistically, when uncertainty is very high regarding due to, e.g., there is no statistical information available for the use of frequentist probabilistic approach, targets are privately owned small companies with limited publicly information without market values and when there are no comparable firms to allow comparables-based valuations, or when their value largely depends on intangibles or strategic future actions.

Real options valuation approach to M&A of Kinnunen (2010) and Collan and Kinnunen (2011) was extended from fuzzy target analysis to a portfolio setting, where some targets are analyzed possibilistically using the fuzzy pay-off method (FPOM) of Collan et al. (2009), and some targets probabilistically using Datar-Mathews (D-M) method to value the strategic part of the total value of an M&A. The strategic part was defined widely as synergy, which comes above the stand-alone value of a target. The synergy arising from M&A, whose stand-alone part is valued probabilistically, is also valued probabilistically (D-M method), and for M&A with possibilistically valued stand-alone part, the synergy is correspondingly valued possibilistically (FPOM).

The usefulness of the two-component portfolio selection approach to M&A was discussed and demonstrated by a numerical example, which was based on a setup in which an acquirer had three potential target companies in ex-ante analysis, where one of them was analyzed probabilistically and the other two possibilistically. The procedure for a real options evaluation of targets was discussed and shown to result to inputs for the presented mixed portfolio model. It was noted that other probabilistic and possibilistic valuation methods can be used. The applied methods were chosen for their practicality and intuitivity. Finally, the optimal portfolio shares were computed together with the minimum variance of the total portfolio.

For future research topics, we notice that the two components, probabilistic and possibilistic, of the mixed portfolio model are considered independent, i.e., an asset belonging to a class is not correlated with an asset from another class, i.e., covariances between possibilistic versus probabilistic assets are unknown, although the covariances of assets of the same type are accounted. We are developing a more general model with interdependences between asset types. Also, analysis of adequacy of our model in real situations compared to other portfolio selection models will be researched. The real options approach in this paper is limited to two components, possibilistic and probabilistic. This model can be extended to include also a credibilistic part as presented by Georgescu and Kinnunen (2012b) to allow more options for M&A analysts.

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